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TORI OF COLORED CUBES

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ABSTRACT

A lxlxn prism of touching colored cubes with the two ends identified is called a torus. Using the 30 colored cubes of MacMahon's set, we construct tori of six cubes which display all the six colors on each of their four sides and which have the same color on any pair of touching internal faces. There are 34 color invariant classes of the six-tori. The classes are 'isomers' of 21 color invariant patterns on the Conway matrix. MacMahon's 30 cubes can be used to build various complete sets of six-tori.

АННОТАЦИЯ

Призму 1x1xn, состоящуюся из цветных кубов с совмещающимися сторонами и имеющую два идентифицированных конца, мы назвали тором. Из 30 цветных кубов комплекта Макмахона мы конструируем шестикубические торы так, чтобы на каждой из четырех сторон проявлялись все цвета и цвет любых внутренних совмещающихся сторон был одинаковый. Шеститоры имеют 34 цветовоинвариантных класса. Эти классы являются "изомерами" 21 цветовоинвариантной структуры в матрице Конвейа. Комплект Макмахона может быть применен для построения различных полных систем шеститоров.

KIVONAT

A lapjukkal illeszkedő kockákból felépitett lxlxn-es hasábot tórusznak nevezzük, ha a hasáb két végét azonositjuk. A MacMahon-készlet 30 szinezett kockájából hatkocka-tóruszokat készitünk ugy, hogy négy oldaluk mindegyikén valamennyi szin megjelenik, és bármely illeszkedő belső lappár szine megegyezik. A hat-tóruszoknak 34 szininvariáns osztálya van. Ezek az osztályok a Conway mátrixon 21 szininvariáns ábra "izomérjei". A MacMahon készletből a hat-tóruszoknak különféle teljes rendszereit késziti jük el.

1. PROBLEM STATEMENT

There are 30 different ways to paint a cube by six colors such that each face receives a distinct color. This set of 30 colored cubes is known also as MacMahon's cubes after the British mathematician and artillerist who described them in detail [1]. MacMahon's cubes feature in recreational puzzle games; but they are also a resource of intriguing combinatorial problems [2]. In these applications it is common to adopt what is called the generalized domino condition: two cubes are joined such that the pair of touching faces have the same color.

One might seek to represent colored cubes in convenient ways. However, such cubes represent quite suitably, in a sense, what they are: they provide a generalized blob notation for certain tensorial quantities. The blob notation has been introduced by Penrose [3] for displaying invariant structures involving tensors. (Fig. 1). One can think of a colored cube as a kernel symbol of a tensorial object with its six sides representing each a tensor index. Contraction of a pair of indices is carried out by joining the corresponding sides. Sides of different color are associated with distinct indices which are not contractible. Thus the generalized domino condition holds.

n colored cubes selected from a MacMahon set and assembled according to the generalized domino condition to form a 1x1xn prism with the two ends considered touching will be called an n-torus. The purpose of the present note is to investigate and classify 6-tori having the properties that

(1) all the six colors appear on each of their four sides [4];

(2) the six colors also occur on the six pairs of touching faces.

In the blob notation, these 6-tori represent cyclic contractions of six tensors, each of rank 6, with like indices completely skew-symmetrized.

It is possible to construct a large number of different 6-tori with properties (1) and (2). The enumeration of 6-tori might seem a frightful task. We shall, however, accomplish this by considering first the properties of 6-tori under re-coloring.

In section 2 we introduce Conway's matrix for labeling the colored cubes. We then investigate the structure of charts of 6-tori in Sec. 3. We find a characterization of this structure by a 24-ary number system. This enables us to list all the classes of tori invariant under re-coloring and under parity inversion (section 4). Finally, in Sec. 5 we consider complete sets of 6-tori (A complete set makes up all the 30 MacMahon cubes). We give examples of complete sets and we encounter some open problems.

2. CHARTING THE TORI

The 30 colored MacMahon cubes can be conveniently labeled by arranging them in Conway's matrix, a 6x6 array of cubes with the diagonal left empty (Fig. 2). Each row and each column of the matrix is a one-by-one-by-five prism which satisfies

(i) Touching faces have the same color (domino condition)

(ii) The two ends of the prism have the same color

(iii) One side of the prism can be arranged to display just one of the 6 colors

(iv) The other three sides then each have all the five remaining colors.

A permutation of colors carried out systematically on each of the 30 cubes interchanges some rows of the matrix and simultaneously interchanges the same way some columns. The color permutation group is generated by swops of two rows (such as a with b) simultaneous with the corresponding pair of columns (A with B). This gives a pairwise interchange of colors [5] $\underline{l} \leftrightarrow \underline{g}$, $\underline{o} \leftrightarrow \underline{b}$ and $\underline{r} \leftrightarrow \underline{y}$. Conversely, the systematic interchange of just two colors is achieved by exchanging 3 pairs of rows and the corresponding columns.

A torus of cubes can be pictured by cutting its surface along one of the four long edges. Thus we obtain a chart of the torus. For a 6-torus, the chart is a 4x6 array of squares. The external surface of a cube is represented here as a column of 4 squares. The hidden color $\underline{c}$ of a pair of touching faces is easily read off this chart noting that the color $\underline{c}$ is missing from both columns of the chart representing the adjacent external faces (Fig. 3). The color $\underline{c}$ must appear in each of the remaining four columns since $\underline{c}$ cannot occur once again on touching faces by rule (2).

The four columns in which color $\underline{c}$ does appear constitute a 4x4 matrix which is possibly cut in two parts by the two ends of the prism. This matrix will be called the matrix of color c. It contains the color $\underline{c}$ precisely once in each row and in each column. The four positions at which color $\underline{c}$ appears form a pattern corresponding to a term in the determinant of the 4x4 matrix.

3. RE-COLORING AND PARITY

By re-coloring the cubes systematically, we can easily produce new tori from an old one. Each torus represents really a class of tori: those available by re-coloring. The order in which the colors are assigned to the six determinant-term patterns is then rendered unimportant. What we are concerned with instead is the division of the chart into the six patterns. Nevertheless we shall use a coloring of the chart in order to indicate which squares belong to a given determinant-term pattern.

We may number the determinant-term patterns in the order which they follow in the usual determinant expansion rule. The determinant expansion of a 4x4 matrix consists of 24 terms. This leads us to consider numbers in a 24-ary number system. To avoid a cumbersome plethora of new symbols, we introduce the following notation. We cyclically permute the four rows until color c appears in the upper left corner. Then we delete the first row and column and we number the terms in the remaining 3x3 determinant expansion consecutively, according to Fig. 4. We put the original number of the deleted row in a subscript. The subscript 1 will be conveniently dropped. For example, the 24-ary digit describing the configuration when all four c's occur in the main diagonal is written as 1_1 or just 1. Counting in our notation goes as follows: $1,2,3,\ldots,6,1_2,2_2,\ldots,6_4$, 11, 12,...

We can read the sequence of six 24-ary digits on a chart in several equivalent ways. This is because the two mutually orthogonal lines along which the surface of the torus is cut are chosen arbitrarily. In addition, the resulting 4x6 chart can be read in two equivalent positions (turning it upside down). A reading proceeds by representing each of the six overlapping color matrices from left to right as a digit of a 24-ary number. Altogether we obtain 24 six-digit, 24-ary numbers for a torus. We then choose the smallest of these 24 numbers to characterize the torus.

We define our classes of tori to include the parity images as well as the re-colored copies of a torus in the same class. The mirror image of a cube in Conway's matrix is at the transpose position. (The mirror image can be taken by exchanging a pair of opposite faces). From a given torus we obtain another by parity reflection. Select the six touching pairs of faces along the torus for exchange under parity reversal. The mirror image cubes make up a torus with the sequence of cubes reversed. The resulting color matrices on the chart are left-right reflected. They provide 24 six-digit numbers which characterize the mirror image torus.

A class of tori will be characterized by the smallest of the possible 48 readings of a representative chart. It may appear to be a tiresome task to carry out a large number of these readings. However, the task is easier if we remember the upside-down or T-reverse reading and the mirror image or P-reverse of a determinant term. This information is given in Table 1.

4. THE CLASSES OF SIX-TORI

The 4x6 chart of a six-torus introduced in section 2 characterizes the torus somewhat redundantly. We can omit, for example, a column without loss of information. The colors of the four omitted squares are just the ones missing from each row. Somewhat surprizingly, in many cases we can still complete the chart uniquely when omitting two adjacent columns. All this requires is a kind of simple crossword-logic. In a small number of exceptional configurations the chart can be completed in two different ways. We shall use this phenomenon to enumerate all the classes of six-tori by completing a single color matrix in all possible ways.

Let the order of (overlapping) color matrices from left to right be chosen

$$\underline{l},\ \underline{r},\ \underline{o},\ \underline{b},\ \underline{y},\ \underline{g}.$$

The matrix of color $\underline{r}$ is in the center of the chart. First we want to color this matrix, then to complete the chart. For some colors, however, the completion becomes obvious before the matrix of $\underline{r}$ is entirely filled out. The following sequence of rules proves practical for mass production of our charts:

1. Color the four $\underline{r}$ squares consecutively, from left to right, striving to fill in the highest possible position in each column
2. Color the four $\underline{o}$ squares as in 1
3. Color the four $\underline{l}$ squares as in 1
4. Put the color $\underline{b}$ in the highest possible position of column 4 of the $\underline{r}$ matrix
5. Put color $\underline{g}$ in highest position of column 1 of $\underline{r}$ matrix
6. Complete the chart.

We seek for smallest characteristic numbers. By cyclic permutation of the six cubes we see that the value of the first digit from left must be minimum among the digits of the characteristic number. By moving the horizontal cut of the torus, the first digit can be made to have subscript 1. The values 4, 5 and 6 are excluded for the first digit since for these values a T or P reverse would start with a smaller digit (Cf. Table 1). The 165 charts with first characteristic digit 1, the 57 charts with 2 and the 2 charts with 3 are given in Table 2. Several charts listed in the Table are killed by some of the T or P reverse readings or by a cyclic permutation (C) of the six figures of the characteristic number. Tori in which a given cube appears repeatedly (those Conway coordinates encircled) are impossible.

The surviving tori are each displayed also on Conway's matrix. The number of cubes lying in the same row (or column) in Conway's matrix is a color invariant concept. It is used to characterize the classes of tori. Another color invariant is the number of mirror image pairs of cubes.

The distribution of the cubes over the 6 rows defines a partition of the number 6. For example, the structural formula $1,(\overline{22}11)(\overline{1}11111)$ of the second entry in Table 2 signifies that this torus has 2 cubes in two rows, 1 cube in two other rows and it has 1 cube in each column. The figure (1) in front of the symbol is the number of parity pairs. The clamp $\sqcap$ connects the two rows (or columns) where a parity pair occurs. Clamps are color invariant.

The tori representing classes in Table 2 exhibit 21 different color invariant structures on Conway's matrix. These structures each are given a Greek symbol. They are enlisted in Table 3. Some color invariant structures have '<u>isomers</u>'. That is to say that their six cubes can be assembled to form a torus in more than one ways. Isomers are distinguished by starring the Greek class symbol in Table 3. Three of the invariant structures (α,λ and τ) have 3 isomers and six structures (ε,ν,σ,ψ, and ω) have 2 isomers. In conclusion, the number of torus classes is 34.

5. COMPLETE SETS OF SIX-TORI

A <u>complete set</u> of 6-tori is a division of the 30 MacMahon cubes into five 6-tori. Thus far, not even the existence of a complete set has been established. But no longer is a difficult task to construct complete sets of tori by using our tables.

We first give an example in which we partition Conway's matrix into four blocks, each block being a 3x3 array (Fig. 5). There is just one class of tori (β) containing 3 parity pairs. By systematic re-coloring (i.e., interchanging pairs of rows and pairs of columns symmetrically across the main diagonal of Conway's matrix), we can jam the structure β in either of the two blocks on the main diagonal. The remaining two off-diagonal blocks can be filled by tori in several ways. For example, two re-colored versions of α and one copy of γ suffice to give the pattern of Fig. 5. An assembled complete set is dispayed on Fig. 6. A further source of diversity is that the structure α has 3 isomers.

We are now in position to ask a more difficult question: Is it possible to prepare a complete set by using just <u>one</u> class of tori? The answer is again affirmative. Figure 7 shows a division of Conway's matrix into ψ structures.

The insight our classification allows into the amazing world of 6-tori enables us to formulate more new problems than to answer old ones. How many complete sets of tori there are? Then, which are the classes of tori that can be used alone to form a complete set? And, importantly, do the classes represent any structure of the physical world - perhaps via their rôle in the blob notation?

REFERENCES

[1] P.A. MacMahon: New Mathematical Pastimes (Cambridge, University Press, 1921)

[2] M. Gardner, Scientific American, September 1978

[3] R. Penrose: Applications of Negative Dimensional Tensors, in <u>Combinatorial Mathematics and its Applications</u>, Ed. D.J.A. Welsh (Academic, 1971)

[4] Thus we exclude twisted tori which have a smaller number of sides.

[5] One may associate the following colors with the color symbols we use: <u>l</u>ilac, <u>r</u>ed, <u>o</u>range, <u>b</u>lue, <u>y</u>ellow, <u>g</u>reen.

Det. term	1	2	3	4	5	6
T-reverse	1	4_2	3	2_2	5_3	6_3
P-reverse	6_4	2_3	5_4	4_3	3_2	1_2
Det. term	1_2	2_2	3_2	4_2	5_2	6_2
T-reverse	1_4	4	3_4	2	5_2	6_2
P-reverse	6	2_4	5	4_4	3_3	1_3
Det. term	1_3	2_3	3_3	4_3	5_3	6_3
T-reverse	1_3	4_4	3_3	2_4	5	6
P-reverse	6_2	2	5_2	4	3_4	1_4
Det. term	1_4	2_4	3_4	4_4	5_4	6_4
T-reverse	1_2	4_3	3_2	2_3	5_4	6_4
P-reverse	6_3	2_2	5_3	4_2	3	1

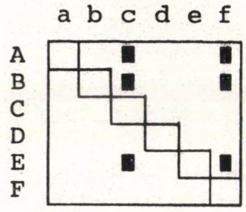
Table 1. The T-reverse (upside down) and P-reverse (right to left) readings of determinant-term patterns

Chart	Killed by	Characteristic number	Alternative completion

	Bf	Ec	Af	Bc	Ef	Ac
α	ℓ	r	o	b	y	g
	g	ℓ	r	o	b	y
	y	g	ℓ	r	o	b
	b	y	g	ℓ	r	o

r matrix

1 1 1 1 1 1



Fd Fd

y	r	o	ℓ	b	g
ℓ	g	r	o	y	b
b	ℓ	g	r	o	y
g	y	ℓ	b	r	o

g	r	o	ℓ	y	b
ℓ	y	r	o	b	g
b	ℓ	g	r	o	y
y	g	ℓ	b	r	o

T

$1 \ 1 \ 3_4 \ 5 \ 6_2 \ 1_2$

$T{:}1 \ 1_{\hookleftarrow} \ 3_2 \ 5_3 \ 6_2 \ 1_4$

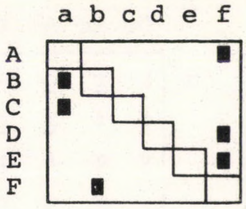
Ba Ba

g	r	o	ℓ	b	y
ℓ	g	r	o	y	b
b	y	ℓ	r	o	g
y	ℓ	g	b	r	o

	Fb	Ba	Af	Df	Ef	Ca
ι	g	r	o	ℓ	y	b
	ℓ	g	r	o	b	y
	b	y	ℓ	r	o	g
	y	ℓ	g	b	r	o

$1 \ 1 \ 3_4 \ 2 \ 5_3 \ 3_2$

$T{:}1 \ 1 \ 3_2 \ 4_2 \ 5 \ 3_4$



b	r	o	ℓ	y	g
ℓ	g	r	o	b	y
g	y	ℓ	r	o	b
y	ℓ	g	b	r	o

T

$1 \ 1 \ 5_4 \ 2 \ 3 \ 3_2$

$T{:}1 \ 1_{\hookleftarrow} \ 5 \ 4_2 \ 3 \ 3_4$

Table 2. Complete enumeration of charts of 6-tori. The listing proceeds by coloring first the r matrix according to the rules 1-6 (Cf. text). The 24-ary characteristic numbers begin with the r digit. Their T and P reverses are to be read backwards. The symbols encircled kill a chart. Conway coordinates of cubes are given above the corresponding column. The Conway matrix is displayed only for surviving charts representing classes

g	r	o	ℓ	y	b	T
ℓ	y	r	o	b	g	
b	g	ℓ	r	o	y	
y	ℓ	g	b	r	o	

1 1 3_4 5 4_2 3_2

T:1 $1_{\hookleftarrow}$ 3_2 5_3 2 3_4

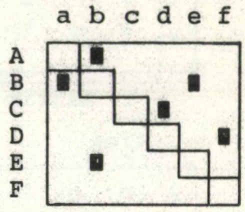
	Cd	Ab	Be	Df	Ba	Eb
σ	g	r	o	ℓ	b	y
	b	ℓ	r	o	y	g
	ℓ	y	g	r	o	b
	y	g	ℓ	b	r	o

1 1 2_4 6_2 6_2 4_3

T:1 1 4_3 6_2 6_2 2_4

P:6_4 6_4 2_2 1_3 1_3 4

PT:6_4 6_4 4 1_3 1_3 2_2

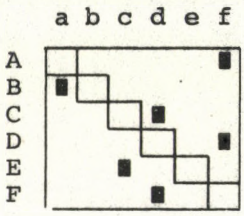


b	r	o	ℓ	y	g	T
g	ℓ	r	o	b	y	
ℓ	y	g	r	o	b	
y	g	ℓ	b	r	o	

1 1 5_4 2 2 4_3

T:1 $1_{\hookleftarrow}$ 5_4 4_2 4_2 2_4

	Cd	Ec	Af	Df	Ba	Fd
ω	y	r	o	ℓ	b	g
	g	ℓ	r	o	y	b
	b	g	ℓ	r	o	y
	ℓ	y	g	b	r	o



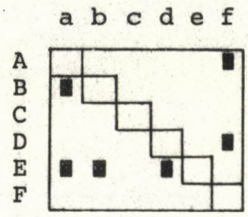
1 1 1_4 2_2 1 5_4

T:1 1 1_2 4 1 5_4

P:6 . 6 . 6 . 2 . 6 . 3 .

PT:6 . 6 . 6 . 4 . 6 . 3 .

	Ea	Ed	Af	Df	Ba	Eb
λ	g	r	o	ℓ	b	y
	b	ℓ	r	o	y	g
	y	g	ℓ	r	o	b
	ℓ	y	g	b	r	o



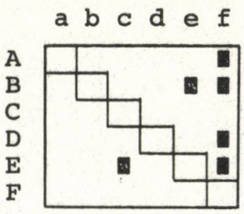
1 1 2_4 4_2 4_2 5_4

T:1 1 4_3 2 2 5_4

P:6 . 6 . 2 . 4 . 4 . 3 .

PT:6 . 6 . 4 . 2 . 2 . 3 .

	Bf	Ec	Af	Df	Ef	Be
ε	g	r	o	ℓ	y	b
	y	ℓ	r	o	b	g
	b	g	ℓ	r	o	y
	ℓ	y	g	b	r	o



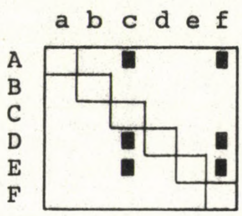
1 1 3_4 3 4_2 5_4

T:1 1 3_2 3 2 5_4

P:6 . 6 . 5 . 5 . 2 . 3 .

PT:6 . 6 . 5 . 5 . 4 . 3 .

	Dc	Ec	Af	Df	Ef	Ac
α*	b	r	o	ℓ	y	g
	g	ℓ	r	o	b	y
	y	g	ℓ	r	o	b
	ℓ	y	g	b	r	o



1 1 5_4 1 1 5_4

T:1 1 5_4 1 1 5_4

P:6 . 6 . 3 . 6 . 6 . 3 .

PT:6 . 6 . 3 . 6 . 6 . 3 .

Table 2 ctd. Charts of tori

		Be			Be
ℓ	r	o	b	y	g
b	g	r	ℓ	o	y
y	ℓ	g	r	b	o
g	y	ℓ	o	r	b

η

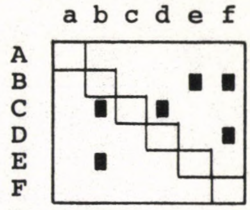
Bf	Df	Be	Cb	Cd	Eb
ℓ	r	o	b	y	g
g	y	r	ℓ	o	b
y	ℓ	g	r	b	o
b	g	ℓ	o	r	y

1 4 3 6 2 5

T:1 2_2 3 6_3 4_2 5_3

P:6_4 4_3 5_4 1_2 2_3 3_2

PT:6 . 2 . 5 . 1 . 4 . 3 .



ξ

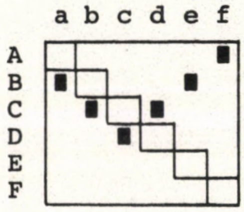
Dc	Ba	Af	Cb	Cd	Be
ℓ	r	o	b	y	g
b	g	r	ℓ	o	y
g	y	ℓ	r	b	o
y	ℓ	g	o	r	b

1 4 5 2 3 6

T:1 2_2 5_3 4_2 3 6_3

P:6 . 4 . 3 . 2 . 5 . 1 .

PT:6 . 2 . 3 . 4 . 5 . 1 .



Cd				Cd	
ℓ	r	o	b	y	g
y	g	r	ℓ	o	b
g	y	ℓ	r	b	o
b	ℓ	g	o	r	y

λ*

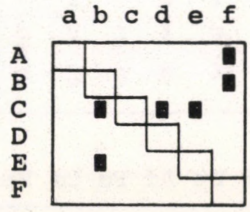
Bf	Ce	Af	Cb	Cd	Eb
ℓ	r	o	b	y	g
g	y	r	ℓ	o	b
y	g	ℓ	r	b	o
b	ℓ	g	o	r	y

1 4 3 6 1 6

T:1 2 . 3 . 6 . 1 . 6 .

P:6 . 4 . 5 . 1 . 6 . 1 .

PT:6 . 2 . 5 . 1 . 6 . 1 .

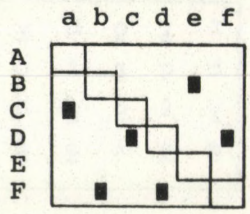


Dc	Fd	Be	Fd	Dc	Be
y	r	o	ℓ	b	g
ℓ	g	r	b	o	y
b	ℓ	g	r	y	o
g	y	ℓ	o	r	b

Table 2 ctd. Charts of tori

	Fb	Df	Be	Fd	Dc	Ca
ω*	g	r	o	ℓ	b	y
	ℓ	y	r	b	o	g
	b	ℓ	g	r	y	o
	y	g	ℓ	o	r	b

1 4 6_2 3_3 6_2 1_2
T:1 2_2 6_2 3_3 6_2 1_4
P:6_4 4_3 1_3 5_2 1_3 ·6
PT:6 . 2 . 1 . 5 . 1_3 6_3



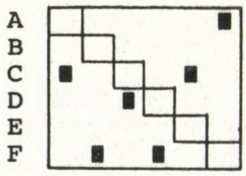
	Fd	Fd			
g	r	o	ℓ	y	b
ℓ	g	r	b	o	y
y	ℓ	g	r	b	o
b	y	ℓ	o	r	g

	Fd		Fd		
b	r	o	ℓ	y	g
ℓ	g	r	b	o	y
y	ℓ	g	r	b	o
g	y	ℓ	o	r	b

	Df				Df
g	r	o	ℓ	y	b
ℓ	y	r	b	o	g
y	ℓ	g	r	b	o
b	g	ℓ	o	r	y

	Fb	Ce	Af	Fd	Dc	Ca
π	g	r	o	ℓ	b	y
	ℓ	y	r	b	o	g
	b	g	ℓ	r	y	o
	y	ℓ	g	o	r	b

1 4 6_2 3_3 4_2 3_2
T:1 2_2 6_2 3_3 2 3_4
P:6_4 4_3 1_3 5_2 4_4 5
PT:6 . 2 . 1 . 5 . 2 . 5 .



	Bf	Ba	Af	Fd	Cd	Be
λ**	b	r	o	ℓ	y	g
	ℓ	g	r	b	o	y
	g	y	ℓ	r	b	o
	y	ℓ	g	o	r	b

1 4 1_2 2 3 3_2
T:1 2_2 1_4 4 . 3 . 3_4
P:6 . 4 . 6 . 2 . 5 . 5 .
PT:6 . 2 . 6 . 4 . 5 . 5 .

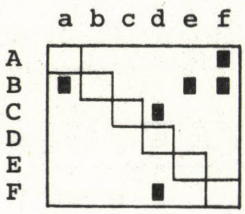


Table 2 ctd. Charts of tori

κ

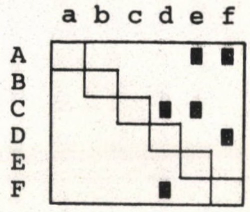
Ae	Ce	Af	Fd	Cd	Df
g	$\underline{r}$	$\underline{o}$	ℓ	$\underline{y}$	b
ℓ	$\underline{y}$	$\underline{r}$	b	$\underline{o}$	g
$\underline{y}$	g	ℓ	$\underline{r}$	b	$\underline{o}$
b	ℓ	g	$\underline{o}$	$\underline{r}$	$\underline{y}$

$1 \quad 4 \quad 2_2 \quad 6 \quad 4_2 \quad 3_2$

T: $1 \quad 2_2 \quad 4 \,.\, 6 \,.\, 2 \quad 3_4$

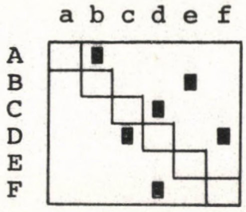
P: $6 \quad 4 \quad 2_4 \quad 1_2 \quad 4_4 \quad 5$

PT: $6 \quad 2 \quad 4_3 \quad 1_4 \quad 2_3 \quad 5$



σ^*

Dc	Ab	Be	Fd	Cd	Df
g	$\underline{r}$	$\underline{o}$	ℓ	$\underline{y}$	b
$\underline{y}$	ℓ	$\underline{r}$	b	$\underline{o}$	g
ℓ	$\underline{y}$	g	$\underline{r}$	b	$\underline{o}$
b	g	ℓ	$\underline{o}$	$\underline{r}$	$\underline{y}$



Be Be

b	$\underline{r}$	$\underline{o}$	ℓ	$\underline{y}$	g
g	ℓ	$\underline{r}$	b	$\underline{o}$	$\underline{y}$
ℓ	$\underline{y}$	g	$\underline{r}$	b	$\underline{o}$
$\underline{y}$	g	ℓ	$\underline{o}$	$\underline{r}$	b

$1 \quad 4 \quad 2_2 \quad 4 \quad 6_2 \quad 4_3$

T: $1 \quad 2_2 \quad 4 \quad 2_2 \quad 6_2 \quad 2_4$

P: $6_4 \quad 4_3 \quad 2_4 \quad 4_3 \quad 1_3 \quad 4$

PT: $6 \,.\, 2 \,.\, 4 \,.\, 2 \,.\, 1_3 \quad 2_2$

π^*

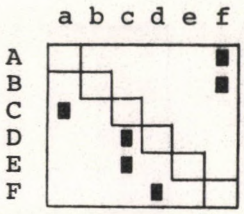
Bf	Ec	Af	Fd	Dc	Ca
g	$\underline{r}$	$\underline{o}$	ℓ	b	$\underline{y}$
$\underline{y}$	ℓ	$\underline{r}$	b	$\underline{o}$	g
b	g	ℓ	$\underline{r}$	$\underline{y}$	$\underline{o}$
ℓ	$\underline{y}$	g	$\underline{o}$	$\underline{r}$	b

$1 \quad 4 \quad 6_2 \quad 5_3 \quad 4_2 \quad 5_4$

T: $1 \quad 2_2 \quad 6_2 \quad 5 \quad 2 \quad 5_4$

P: $6_4 \quad 4_3 \quad 1_3 \quad 3_4 \quad 4_4 \quad 3$

PT: $6 \quad 2. \quad 1_3 \quad 3_2 \quad 2 \quad 3$



P

$\underline{y}$	$\underline{r}$	$\underline{o}$	ℓ	b	g
g	ℓ	$\underline{r}$	b	$\underline{o}$	$\underline{y}$
b	g	ℓ	$\underline{r}$	$\underline{y}$	$\underline{o}$
ℓ	$\underline{y}$	g	$\underline{o}$	$\underline{r}$	b

$1 \quad 4 \quad 6_2 \quad 6_3 \quad 1 \quad 5_4$

T: $1 \quad 2_2 \quad 6 \quad 6 \quad 1 \quad 5$

P: $6_4 \quad 4_3 \quad 1_3 \quad 1_4 \cdot 6_4 \quad 3$

C

b	$\underline{r}$	$\underline{o}$	ℓ	$\underline{y}$	g
g	ℓ	$\underline{r}$	b	$\underline{o}$	$\underline{y}$
$\underline{y}$	g	ℓ	$\underline{r}$	b	$\underline{o}$
ℓ	$\underline{y}$	g	$\underline{o}$	$\underline{r}$	b

$1 \quad 4 \quad 1_2 \quad 1 \quad 1 \quad 5_4$

PT

ℓ	$\underline{r}$	$\underline{o}$	b	$\underline{y}$	g
$\underline{y}$	g	$\underline{r}$	ℓ	b	$\underline{o}$
b	ℓ	g	$\underline{r}$	$\underline{o}$	$\underline{y}$
g	$\underline{y}$	ℓ	$\underline{o}$	$\underline{r}$	b

$1 \quad 6 \quad 2 \quad 3 \quad 4 \quad 5$

T: $1 \quad 6_3 \quad 4_2 \quad 3 \quad 2_2 \quad 5_3$

P: $6_4 \quad 1_2 \quad 2_3 \quad 5_4 \quad 4_3 \quad 3_2$

PT: $6_4 \quad 1_4 \quad 4_4$

Table 2 ctd. Charts of tori

ℓ	$\underline{r}$	$\underline{o}$	b	$\underline{y}$	g	PT
g	$\underline{y}$	$\underline{r}$	ℓ	b	$\underline{o}$	
$\underline{y}$	ℓ	g	$\underline{r}$	$\underline{o}$	b	
b	g	ℓ	$\underline{o}$	$\underline{r}$	$\underline{y}$	

$1\ \ 6\ \ 1\ \ 6\ \ 2\ \ 5$

T: $1\ \ 6\,.\,1\,.\,6\,.\,4\,.\,5\,.$

P: $6_4\ \ 1_2\ \ 6_4\ \ 1_2\ \ 2_3\ \ 3_2$

PT: $6_4\ \ 1_4\ \ 6_4\ \ 1_4\ \ 4_4\ \ 3_4$

ℓ	$\underline{r}$	$\underline{o}$	b	$\underline{y}$	g	PT
g	$\underline{y}$	$\underline{r}$	ℓ	b	$\underline{o}$	
b	ℓ	g	$\underline{r}$	$\underline{o}$	$\underline{y}$	
$\underline{y}$	g	ℓ	$\underline{o}$	$\underline{r}$	b	

$1\ \ 6\ \ 2\ \ 5\ \ 2\ \ 5$

T: $1\ \ 6\,.\,4\,.\,5\,.\,4\,.\,5_3$

P: $6_4\ \ 1_2\ \ 2_3\ \ 3_2\ \ 2_3\ \ 3_2$

PT: $6_4\ \ 1_4\ \ 4_4\ \ 3_4\ \ 4_4\ \ 3_4$

ℓ	$\underline{r}$	$\underline{o}$	b	$\underline{y}$	g	C
$\underline{y}$	g	$\underline{r}$	ℓ	b	$\underline{o}$	
g	$\underline{y}$	ℓ	$\underline{r}$	$\underline{o}$	b	
b	ℓ	g	$\underline{o}$	$\underline{r}$	$\underline{y}$	

$1\ \ 6\ \ 1\ \ 4\ \ 3\ \ 6$

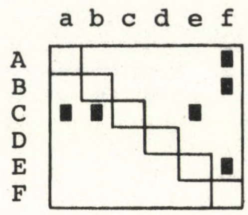
	Bf	Ce	Af	Cb	Ef	Ca
γ	ℓ	$\underline{r}$	$\underline{o}$	b	$\underline{y}$	g
	g	$\underline{y}$	$\underline{r}$	ℓ	b	$\underline{o}$
	$\underline{y}$	g	ℓ	$\underline{r}$	$\underline{o}$	b
	b	ℓ	g	$\underline{o}$	$\underline{r}$	$\underline{y}$

$1\ \ 6\ \ 1\ \ 6\ \ 1\ \ 6$

T: $1\ \ 6_3\ \ 1\ \ 6_3\ \ 1\ \ 6_3$

P: $6_4\ \ 1_2\ \ 6_4\ \ 1_2\ \ 6_4\ \ 1_2$

PT: $6_4\ \ 1_4\ \ 6_4\ \ 1_4\ \ 6_4\ \ 1_4$



ℓ	$\underline{r}$	$\underline{o}$	b	$\underline{y}$	g	C
g	$\underline{y}$	$\underline{r}$	ℓ	b	$\underline{o}$	
b	g	ℓ	$\underline{r}$	$\underline{o}$	$\underline{y}$	
$\underline{y}$	ℓ	g	$\underline{o}$	$\underline{r}$	b	

$1\ \ 6\ \ 2\ \ 5\ \ 1\ \ 6$

	Fd		Fd		
g	$\underline{r}$	$\underline{o}$	ℓ	b	$\underline{y}$
ℓ	g	$\underline{r}$	b	$\underline{y}$	$\underline{o}$
$\underline{y}$	ℓ	g	$\underline{r}$	$\underline{o}$	b
b	$\underline{y}$	ℓ	$\underline{o}$	$\underline{r}$	g

y	r	o	ℓ	b	g	PT
ℓ	g	r	b	y	o	
b	ℓ	g	r	o	y	
g	y	ℓ	o	r	b	

$1\ \ 6\ \ 6_2\ \ 1_2\ \ 4\ \ 1_2$

T: $1\ \ 6_3\ \ 6_2\ \ 1_4\ \ 2_2\ \ 1_4$

P: $6_4\ \ 1_2\ \ 1_3\ \ 6\ \ 4_3\ \ 6$

PT: $6_4\ \ 1_4\ \ 1_3\ \ 6_3\ \ 2_4\ \ 6\ .$

Table 2 ctd. Charts of tori

	Ba			Ba	
y	r	o	ℓ	b	g
ℓ	g	r	b	y	o
g	y	ℓ	r	o	b
b	ℓ	g	o	r	y

g	r	o	ℓ	b	y	P
ℓ	g	r	b	y	o	
b	y	ℓ	r	o	g	
y	ℓ	g	o	r	b	

$1 \quad 6 \quad 6_2 \quad 6_2 \quad 5_3 \quad 3_2$

T:$1 \quad 6 . 6 . 6 . 5 \quad 3_4$

P:$6 \quad 1 \quad 1_3 \quad 1_3$

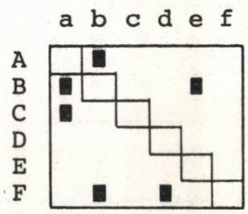
	Fb	Ab	Be	Fd	Ba	Ca
τ	y	r	o	ℓ	b	g
	g	ℓ	r	b	y	o
	ℓ	y	g	r	o	b
	b	g	ℓ	o	r	y

$1 \quad 6 \quad 4_2 \quad 5_2 \quad 2 \quad 4_3$

T:$1 \quad 6_3 \quad 2 \quad 5_2 \quad 4_2 \quad 2_4$

P:$6_4 \quad 1_2 \quad 4_4 \quad 3_3 \quad 2_3 \quad 4$

PT:$6_4 \quad 1_4 \quad 2_3 \quad 3_3 \quad 4_4 \quad 2_2$



y	r	o	ℓ	b	g	T
g	ℓ	r	b	y	o	
b	g	ℓ	r	o	y	
ℓ	y	g	o	r	b	

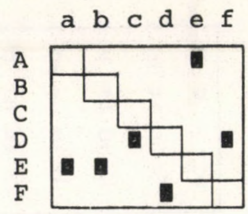
$1 \quad 6 \quad 6_2 \quad 2_2 \quad 1 \quad 5_4$

T:$1 \quad 6 . 6 . 4 \quad 1 \quad 5_4$

	Ae	Fd	Eb	Ea	Dc	Df
ζ	ℓ	r	g	o	b	y
	y	g	r	ℓ	o	b
	b	ℓ	o	r	y	g
	g	y	ℓ	b	r	o

$1 \quad 5_3 \quad 1_4 \quad 5_3 \quad 2_3 \quad 5$

T:$1 \quad 5 \quad 1_2 \quad 5 \quad 4_4 \quad 5_3$



Ea			Ea		
ℓ	r	g	o	b	y
g	y	r	ℓ	o	b
b	ℓ	o	r	y	g
y	g	ℓ	b	r	o

Dc				Dc	
ℓ	r	g	o	b	y
b	y	r	ℓ	o	g
g	ℓ	o	r	y	b
y	g	ℓ	b	r	o

Table 2 ctd. Charts of tori

ℓ	$\underline{r}$	g	$\underline{o}$	$\underline{y}$	b	C
b	g	$\underline{r}$	ℓ	$\underline{o}$	$\underline{y}$	
$\underline{y}$	ℓ	$\underline{o}$	$\underline{r}$	b	g	
g	$\underline{y}$	ℓ	b	$\underline{r}$	$\underline{o}$	
1	5_3	4_4	$\underline{1}$	2_3	5	

ℓ	$\underline{r}$	g	$\underline{o}$	$\underline{y}$	b	T
b	$\underline{y}$	$\underline{r}$	ℓ	$\underline{o}$	g	
g	ℓ	$\underline{o}$	$\underline{r}$	b	$\underline{y}$	
$\underline{y}$	g	ℓ	b	$\underline{r}$	$\underline{o}$	

1 5_3 4_4 5 1_2 5
T: $\underline{1}$ 5 2_3 5_3 1_4 5_3

g	$\underline{r}$	ℓ	$\underline{o}$	b	$\underline{y}$	T
$\underline{y}$	ℓ	$\underline{r}$	b	$\underline{o}$	g	
ℓ	g	$\underline{o}$	$\underline{r}$	$\underline{y}$	b	
b	$\underline{y}$	g	ℓ	$\underline{r}$	$\underline{o}$	
1	5_3	4_2	5_3	4_2	6_3	
T: $\underline{1}$					6	

$\underline{y}$	$\underline{r}$	ℓ	$\underline{o}$	b	g	T
g	ℓ	$\underline{r}$	b	$\underline{o}$	$\underline{y}$	
ℓ	g	$\underline{o}$	$\underline{r}$	$\underline{y}$	b	
b	$\underline{y}$	g	ℓ	$\underline{r}$	$\underline{o}$	
1	5_3	4_2	6_3	1	6_3	
T: $\underline{1}$					6	

g	$\underline{r}$	ℓ	$\underline{o}$	$\underline{y}$	b	T
$\underline{y}$	ℓ	$\underline{r}$	b	$\underline{o}$	g	
ℓ	g	$\underline{o}$	$\underline{r}$	b	$\underline{y}$	
b	$\underline{y}$	g	ℓ	$\underline{r}$	$\underline{o}$	

1 5_3 2_2 3 4_2 6_3
T: $\underline{1}$ 6

g	$\underline{r}$	ℓ	$\underline{o}$	b	$\underline{y}$	T
b	$\underline{y}$	$\underline{r}$	ℓ	$\underline{o}$	g	
ℓ	g	$\underline{o}$	$\underline{r}$	$\underline{y}$	b	
$\underline{y}$	ℓ	g	b	$\underline{r}$	$\underline{o}$	

1 5_3 2_4 3_3 4_2 1_3
T: $\underline{1}$ 2 1_3

g	$\underline{r}$	ℓ	$\underline{o}$	$\underline{y}$	b	T
b	g	$\underline{r}$	ℓ	$\underline{o}$	$\underline{y}$	
ℓ	$\underline{y}$	$\underline{o}$	$\underline{r}$	b	g	
$\underline{y}$	ℓ	g	b	$\underline{r}$	$\underline{o}$	

1 5_3 1_3
T: $\underline{1}$ 1_3

Table 2 ctd. Charts of tori

g	r	ℓ	o	y	b	T
b	y	r	ℓ	o	g	
ℓ	g	o	r	b	y	
y	ℓ	g	b	r	o	

1 5_3 1_3

T:1 1_3

b	r	ℓ	o	y	g	T
g	y	r	ℓ	o	b	
ℓ	g	o	r	b	y	
y	ℓ	g	b	r	o	

1 5_3 1_3

T:1 1_3

g	r	ℓ	o	b	y	T
y	g	r	ℓ	o	b	
b	ℓ	o	r	y	g	
ℓ	y	g	b	r	o	

1 5_3 4_4

T:1 2_3

y	r	ℓ	o	b	g	T
b	g	r	ℓ	o	y	
g	ℓ	o	r	y	b	
ℓ	y	g	b	r	o	

1 5_3 4_4

T:1 2_3

g	r	ℓ	o	y	b	T
b	g	r	ℓ	o	y	
y	ℓ	o	r	b	g	
ℓ	y	g	b	r	o	

1 5_3 4_4

T:1 2_3

b	r	ℓ	o	y	g	T
y	g	r	ℓ	o	b	
g	ℓ	o	r	b	y	
ℓ	y	g	b	r	o	

1 5_3 4_4

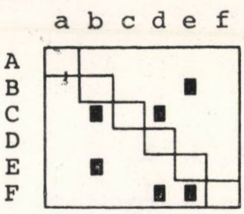
T:1 2_3

The 12 charts whose characteristic number begins with 16_3... are not listed since the characteristic number of their P-reverse begins with 16...

	Cd	Fd	Eb	Cb	Fe	Be
κ*	ℓ	r	g	b	o	y
	y	g	r	ℓ	b	o
	g	ℓ	o	r	y	b
	b	y	ℓ	o	r	g

1 1_3 1 5_3 6_4 5

T:1 1_3 1 5 6_4 5_3



	Fd				Fd
ℓ	r	g	b	o	y
y	g	r	ℓ	b	o
b	ℓ	o	r	y	g
g	y	ℓ	o	r	b

	Ea	Df	Eb	Cb	Fe	Fd
χ	ℓ	r	g	b	o	y
	g	y	r	ℓ	b	o
	b	ℓ	o	r	y	g
	y	g	ℓ	o	r	b

1 1_3 2 3_3 4_3 5

T:1 1_3 4 . 3 . 2 . 5 .

P:6 6

PT:6 6

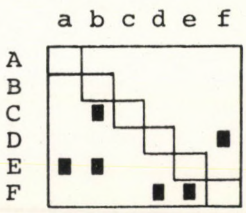


Table 2 ctd. Charts of tori

	Fd				Fd
ℓ	r	g	b	o	y
b	g	r	ℓ	y	o
y	ℓ	o	r	b	g
g	y	ℓ	o	r	b

y	r	g	ℓ	o	b
ℓ	g	r	b	y	o
g	ℓ	o	r	b	y
b	y	ℓ	o	r	g

T

$1 \quad 1_3 \quad 2_2 \quad 2_2 \quad 6_4 \quad 1_2$

T: $1 \quad 1_3 \quad 4 \quad 4 \quad 6_4 \quad 1_4$

b	r	g	ℓ	o	y
ℓ	g	r	b	y	o
y	ℓ	o	r	b	g
g	y	ℓ	o	r	b

T

$1 \quad 1_3 \quad 1_2 \quad 4_2 \quad 2_3 \quad 1_2$

T: $1 \quad 1_3 \quad 1_4 \quad 2 \quad 4_4 \quad 1_4$

		Eb			Eb
y	r	g	ℓ	o	b
g	ℓ	r	b	y	o
ℓ	g	o	r	b	y
b	y	ℓ	o	r	g

y	r	g	ℓ	o	b
g	ℓ	r	b	y	o
ℓ	y	o	r	b	g
b	g	ℓ	o	r	y

T

$1 \quad 1_3 \quad 2_2 \quad 5_2 \quad 4_3 \quad 4_3$

T: $1 \quad 1_3 \quad 4 \quad 5_2 \quad 2_4 \quad 2_4$

b	r	g	ℓ	o	y
g	ℓ	r	b	y	o
ℓ	y	o	r	b	g
y	g	ℓ	o	r	b

C

$1 \quad 1_3 \quad 1_2 \quad 6_2 \quad 4_3 \quad 4_3$

α**

Fb	Ce	Fa	Cb	Fe	Ca
y	r	ℓ	b	o	g
g	y	r	ℓ	b	o
ℓ	g	o	r	y	b
b	ℓ	g	o	r	y

$1 \quad 1_3 \quad 1 \quad 1_3 \quad 1 \quad 1_3$

T: $1 \quad 1_3 \quad 1 \quad 1_3 \quad 1 \quad 1_3$

P:6

PT:6

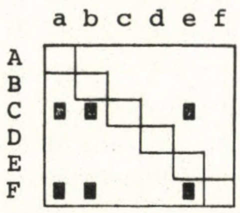


Table 2 ctd. Charts of tori

g	r	ℓ	b	o	y
b	g	r	ℓ	y	o
ℓ	y	o	r	b	g
y	ℓ	g	o	r	b

C

1 1_3 5 6_2 5_3 1_3

Fd Fd

g	r	ℓ	b	o	y
y	g	r	ℓ	b	o
b	ℓ	o	r	y	g
ℓ	y	g	o	r	b

g	r	ℓ	b	o	y
b	g	r	ℓ	y	o
y	ℓ	o	r	b	g
ℓ	y	g	o	r	b

T

1 1_3 5 4_2 5_3 4_4
T:1 1_3 5 2 5 2_3

Cd Cd

ℓ	r	g	b	y	o
y	g	r	ℓ	o	b
g	ℓ	o	r	b	y
b	y	ℓ	o	r	g

y	r	ℓ	b	o	g
b	g	r	ℓ	y	o
g	ℓ	o	r	b	y
ℓ	y	g	o	r	b

T

1 1_3 5 2_2 3 4_4
T:1 1_3 5 4 3 2_3

Eb Eb

ℓ	r	g	b	y	o
b	g	r	ℓ	o	y
y	ℓ	o	r	b	g
g	y	ℓ	o	r	b

τ**

Bf	Df	Eb	Cb	Cd	Be
ℓ	r	g	b	y	o
g	y	r	ℓ	o	b
y	ℓ	o	r	b	g
b	g	ℓ	o	r	y

1 2_3 3 6 4_3 5
T:1 4 . 3 . 6 . 2 . 5_3
P:6 2 5_4 1_2 4 3_2
PT:6 4_2 5_4 1_4 2_2 3_4

	a	b	c	d	e	f
A						
B					■	■
C		■		■		
D						■
E		■				
F						

τ*

Dc	Df	Eb	Cb	Cd	Ac
ℓ	r	g	b	y	o
b	y	r	ℓ	o	g
g	ℓ	o	r	b	y
y	g	ℓ	o	r	b

1 2_3 5 5 1_2 5
T:1 4 5 5_3 1_4 5_3
P:6

	a	b	c	d	e	f
A			■			
B						
C		■		■		
D			■			■
E		■				
F						

Table 2 ctd. Charts of tori

y	r	g	ℓ	b	o
ℓ	g	r	b	o	y
g	ℓ	o	r	y	b
b	y	ℓ	o	r	g

T

1 2_3 4_2 6_3 6_4 1_2
T:1 4 2 6 6_4 1_4

y	r	g	ℓ	b	o
ℓ	g	r	b	o	y
b	ℓ	o	r	y	g
g	y	ℓ	o	r	b

T

1 2_3 6_2 6_3 2_3 1_2
T:1 4_4 1_4

y	r	g	ℓ	b	o
ℓ	y	r	b	o	g
g	ℓ	o	r	y	b
b	g	ℓ	o	r	y

C 1 2_3 1_2 1_2

b	r	g	ℓ	y	o
ℓ	g	r	b	o	y
y	ℓ	o	r	b	g
g	y	ℓ	o	r	b

T

1 2_3 1_2 1 2_3 1_2
T:1 4_4 1_4 1 4_4 1_4

b	r	g	ℓ	y	o
ℓ	y	r	b	o	g
g	ℓ	o	r	b	y
y	g	ℓ	o	r	b

C 1 2_3 1_2 1_2

y	r	g	ℓ	b	o
g	ℓ	r	b	o	y
ℓ	g	o	r	y	b
b	y	ℓ	o	r	g

P

1 2_3 4_2 6_3 5_4 4_3
T:1 4 2 6 4 2_4
P:6 2 4_4 1_4 3 2

Table 2 ctd. Charts of tori

b	r	g	ℓ	y	o
y	ℓ	r	b	o	g
ℓ	g	o	r	b	y
g	y	ℓ	o	r	b

T $\quad 1 \; 2_3 \; 1_2 \; 3 \; 3_2 \; 4_3$

T:1 $4_4 \; 1_4 \; 3 \; 3 \; 2$

b	r	g	ℓ	y	o
g	ℓ	r	b	o	y
ℓ	y	o	r	b	g
y	g	ℓ	o	r	b

T $\quad 1 \; 2_3 \; 1_2 \; 2 \; 4_3 \; 4_3$

T:1 $4_4 \; 1_4 \; 4_2 \; 2_4 \; 2_4$

g	r	ℓ	b	y	o
y	g	r	ℓ	o	b
ℓ	y	o	r	b	g
b	ℓ	g	o	r	y

C

$1 \; 2_3 \; 3 \; 4 \; 5_3 \; 1_3$

g	r	ℓ	b	y	o
b	g	r	ℓ	o	y
ℓ	y	o	r	b	g
y	ℓ	g	o	r	b

C

$1 \; 2_3 \qquad 1_3$

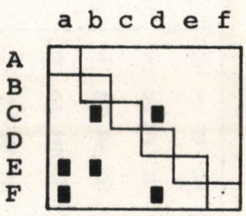
g	r	ℓ	b	y	o
b	y	r	ℓ	o	g
ℓ	g	o	r	b	y
y	ℓ	g	o	r	b

C $\quad 1 \; 2_3 \qquad 1_3$

	Ea	Fd	Fa	Cb	Cd	Eb
ν	g	r	ℓ	b	y	o
	b	g	r	ℓ	o	y
	y	ℓ	o	r	b	g
	ℓ	y	g	o	r	b

$1 \; 2_3 \; 5 \; 1 \; 5_3 \; 4_4$

T:1 $4_4 \; 5_3 \; 1 \; 5 \; 2_3$



The 34 charts whose characteristic number begins with 11_4... or 12_4... are not listed since the characteristic number of their T-reverse begins with less than 11_4...

Table 2 ctd. Charts of tori

ℓ	r	g	b	o	y
y	ℓ	r	o	b	g
b	g	ℓ	r	y	o
g	y	o	ℓ	r	b

C 1 3_4 1

ℓ	r	g	b	o	y
g	ℓ	r	o	y	b
y	g	ℓ	r	b	o
b	y	o	ℓ	r	g

C

1 3_4 1

ℓ	r	g	b	o	y
b	ℓ	r	o	y	g
y	g	ℓ	r	b	o
g	y	o	ℓ	r	b

C

1 3_4 1

ℓ	r	g	b	o	y
b	ℓ	r	o	y	g
g	y	ℓ	r	b	o
y	g	o	ℓ	r	b

C 1 3_4 1

y	r	ℓ	b	o	g
ℓ	g	r	o	b	y
b	ℓ	g	r	y	o
g	y	o	ℓ	r	b

T 1 3_4 2_2

T:1 4

g	r	ℓ	b	o	y
ℓ	y	r	o	b	g
b	ℓ	g	r	y	o
y	g	o	ℓ	r	b

T 1 3_4 2_2

T:1 4

Table 2 ctd. Charts of tori

g	r	ℓ	b	o	y
ℓ	g	r	o	y	b
y	ℓ	g	r	b	o
b	y	o	ℓ	r	g

T ·

$1 \quad 3_4 \qquad\qquad\qquad 2_2$

T:$1_{\hookleftarrow}$ $\qquad\qquad\qquad$ 4

Dc $\quad$ Dc

y	r	g	ℓ	o	b
ℓ	y	r	o	b	g
b	g	ℓ	r	y	o
g	ℓ	o	b	r	y

b	r	g	ℓ	o	y
ℓ	g	r	o	y	b
g	y	ℓ	r	b	o
y	ℓ	o	b	r	g

P

$1 \quad 3_4 \quad 6_4 \quad 6_2 \quad 6_4 \quad 3_2$

T:$1_4 \quad 3 \quad 6 \quad 6 \quad 6 \quad 3_4$

P:$6_4 \quad 5_3 \quad 1 \quad 1_3 \quad 1_{\hookleftarrow} \quad 5$

y	r	ℓ	b	o	g
g	ℓ	r	o	y	b
ℓ	y	g	r	b	o
b	g	o	ℓ	r	y

T

$1 \quad 3_4 \qquad\qquad\qquad 6_3$

T:$1_{\hookleftarrow}$ $\qquad\qquad\qquad$ 6

g	r	ℓ	b	o	y
b	ℓ	r	o	y	g
ℓ	y	g	r	b	o
y	g	o	ℓ	r	b

T

$1 \quad 3_4 \qquad\qquad\qquad 6_3$

T:$1_{\hookleftarrow}$ $\qquad\qquad\qquad$ 6

	Cd	Ec	Dc	Df	Fe	Fd
ρ	y	r	g	ℓ	o	b
	g	ℓ	r	o	b	y
	b	g	ℓ	r	y	o
	ℓ	y	o	b	r	g

$1 \quad 3_4 \quad 3_4 \quad 6_3 \quad 5_4 \quad 5_4$

T:$1 \quad 3_2 \quad 3_2 \quad 6 \quad 5_4 \quad 5_4$

PT:$6_4 \quad 5 \quad 5 \quad 1_2 \quad 3 \quad 3$

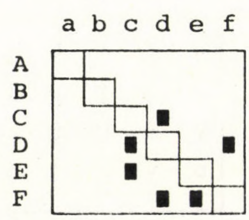


Table 2 ctd. Charts of tori

	b	$\underline{r}$	g	ℓ	$\underline{o}$	$\underline{y}$
	g	ℓ	$\underline{r}$	$\underline{o}$	$\underline{y}$	b
	$\underline{y}$	g	ℓ	$\underline{r}$	b	$\underline{o}$
	ℓ	$\underline{y}$	$\underline{o}$	b	$\underline{r}$	g
	1	3_4	6_4	4_2	5_4	5_4
T:	1	3	6	2	5_4	5_4
P:	6	5_3	1	4	3	3

P

	$\underline{y}$	$\underline{r}$	g	ℓ	$\underline{o}$	b
	b	ℓ	$\underline{r}$	$\underline{o}$	$\underline{y}$	g
	g	$\underline{y}$	ℓ	$\underline{r}$	b	$\underline{o}$
	ℓ	g	$\underline{o}$	b	$\underline{r}$	$\underline{y}$
	1	3_4	4_4	5_2	1_2	5_4
T:	1			5_2	1_4	5_4

T

ℓ	$\underline{r}$	g	b	$\underline{y}$	$\underline{o}$
g	ℓ	$\underline{r}$	$\underline{o}$	b	$\underline{y}$
$\underline{y}$	g	ℓ	$\underline{r}$	$\underline{o}$	b
b	$\underline{y}$	$\underline{o}$	ℓ	$\underline{r}$	g
1	5_4				1

C

ℓ	$\underline{r}$	g	b	$\underline{y}$	$\underline{o}$
$\underline{y}$	ℓ	$\underline{r}$	$\underline{o}$	b	g
b	g	ℓ	$\underline{r}$	$\underline{o}$	$\underline{y}$
g	$\underline{y}$	$\underline{o}$	ℓ	$\underline{r}$	b
1	5_4				1

C

ℓ	$\underline{r}$	g	b	$\underline{y}$	$\underline{o}$
$\underline{y}$	ℓ	$\underline{r}$	$\underline{o}$	b	g
g	$\underline{y}$	ℓ	$\underline{r}$	$\underline{o}$	b
b	g	$\underline{o}$	ℓ	$\underline{r}$	$\underline{y}$
1	5_4				1

C

ℓ	$\underline{r}$	g	b	$\underline{y}$	$\underline{o}$
g	ℓ	$\underline{r}$	$\underline{o}$	b	$\underline{y}$
b	$\underline{y}$	ℓ	$\underline{r}$	$\underline{o}$	g
$\underline{y}$	g	$\underline{o}$	ℓ	$\underline{r}$	b
1	5_4				1

C

g	$\underline{r}$	ℓ	b	$\underline{y}$	$\underline{o}$
ℓ	g	$\underline{r}$	$\underline{o}$	b	$\underline{y}$
$\underline{y}$	ℓ	g	$\underline{r}$	$\underline{o}$	b
b	$\underline{y}$	$\underline{o}$	ℓ	$\underline{r}$	g
1	5_4			1	1

C

Table 2 ctd. Charts of tori

g	$\underline{r}$	ℓ	b	$\underline{y}$	$\underline{o}$
ℓ	$\underline{y}$	$\underline{r}$	$\underline{o}$	b	g
$\underline{y}$	ℓ	g	$\underline{r}$	$\underline{o}$	b
b	g	$\underline{o}$	ℓ	$\underline{r}$	$\underline{y}$

T

1 5_4 2_2

T:1

4

g	$\underline{r}$	ℓ	b	$\underline{y}$	$\underline{o}$
ℓ	$\underline{y}$	$\underline{r}$	$\underline{o}$	b	g
b	ℓ	g	$\underline{r}$	$\underline{o}$	$\underline{y}$
$\underline{y}$	g	$\underline{o}$	ℓ	$\underline{r}$	b

T

1 5_4 1 2_2

T:1

4

$\underline{y}$	$\underline{r}$	g	ℓ	b	$\underline{o}$
ℓ	g	$\underline{r}$	$\underline{o}$	$\underline{y}$	b
b	$\underline{y}$	ℓ	$\underline{r}$	$\underline{o}$	g
g	ℓ	$\underline{o}$	b	$\underline{r}$	$\underline{y}$

T 1 5_4 3_2

T:1

3_4

b	$\underline{r}$	g	ℓ	$\underline{y}$	$\underline{o}$
ℓ	g	$\underline{r}$	$\underline{o}$	b	$\underline{y}$
g	$\underline{y}$	ℓ	$\underline{r}$	$\underline{o}$	b
$\underline{y}$	ℓ	$\underline{o}$	b	$\underline{r}$	g

T 1 5_4 3_2

1

3_4

b	$\underline{r}$	g	ℓ	$\underline{y}$	$\underline{o}$
ℓ	$\underline{y}$	$\underline{r}$	$\underline{o}$	b	g
$\underline{y}$	g	ℓ	$\underline{r}$	$\underline{o}$	b
g	ℓ	$\underline{o}$	b	$\underline{r}$	$\underline{y}$

T 1 5_4 3_2

T:1

3_4

g	$\underline{r}$	ℓ	b	$\underline{y}$	$\underline{o}$
$\underline{y}$	ℓ	$\underline{r}$	$\underline{o}$	b	g
ℓ	$\underline{y}$	g	$\underline{r}$	$\underline{o}$	b
b	g	$\underline{o}$	ℓ	$\underline{r}$	$\underline{y}$

T 1 5_4 1 6_3

T:1

6

Table 2 ctd. Charts of tori

y	r	g	ℓ	b	o
g	ℓ	r	o	y	b
b	g	ℓ	r	o	y
ℓ	y	o	b	r	g

C

$1 \quad 5_4 \hookrightarrow 1_4 \quad 2_2 \quad 5_4 \quad 5_4$

y	r	g	ℓ	b	o
g	ℓ	r	o	y	b
b	y	ℓ	r	o	g
ℓ	g	o	b	r	y

C

$1 \quad 5_4 \hookrightarrow 1_4 \quad 5_2$

y	r	g	ℓ	b	o
b	ℓ	r	o	y	g
g	y	ℓ	r	o	b
ℓ	g	o	b	r	y

C

$1 \quad 5_4 \qquad 5_2 \hookrightarrow 1_2 \quad 5_4$

Dc Dc

b	r	g	ℓ	y	o
g	ℓ	r	o	b	y
y	g	ℓ	r	o	b
ℓ	y	o	b	r	g

b	r	g	ℓ	y	o
y	ℓ	r	o	b	g
g	y	ℓ	r	o	b
ℓ	g	o	b	r	y

C

$1 \quad 5_4 \quad 5_4 \qquad \hookrightarrow 1_2 \quad 5_4$

Table 2 ctd. Charts of tori

Charts whose characteristic number begins with 2

/Kernel symbols 1 and 6 are not allowable/

Ea Ea

ℓ	r	o	b	y	g
g	ℓ	r	o	b	y
b	y	g	ℓ	r	o
y	g	ℓ	r	o	b

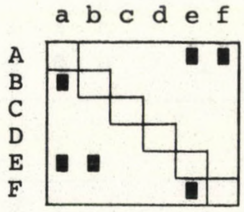
ψ

Ea	Ba	Af	Ae	Fe	Eb
y	r	o	ℓ	b	g
ℓ	g	r	o	y	b
g	y	ℓ	b	r	o
b	ℓ	g	r	o	y

$2 \quad 2 \quad 5_3 \; 5_2 \; 3 \quad 3_2$

$T{:}4 \,.\, 4 \,.\, 5 \quad 5 \,.\, 3 \quad 3 \,.$

$P{:}2_3 \; 2_3 \; 3_4 \; 3_3 \; 5_4 \; 5$



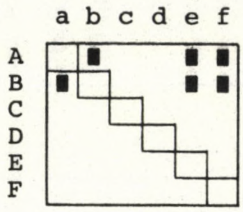
δ

Bf	Ba	Af	Ae	Ab	Be
b	r	o	ℓ	y	g
ℓ	g	r	o	b	y
g	y	ℓ	b	r	o
y	ℓ	g	r	o	b

$2 \quad 2 \quad 4_3 \; 2 \quad 3 \quad 3_2$

$T{:}4 \,.\, 4 \,.\, 2 \,.\, 4 \,.\, 4 \,.\, 3 \,.$

$P{:}2_3 \; 2_3 \; 4 \quad 2 \,.\, 5 \,.\, 5$



y	r	o	ℓ	b	g
g	ℓ	r	o	y	b
ℓ	y	g	b	r	o
b	g	ℓ	r	o	y

P

$2 \quad 2 \quad 5_3 \; 5_2 \; 2 \quad 4_3$

$P{:}2_3 \; 2_3 \qquad\qquad 2_3 \; 4$

Ae Ae

b	r	o	ℓ	y	g
g	ℓ	r	o	b	y
ℓ	y	g	b	r	o
y	g	ℓ	r	o	b

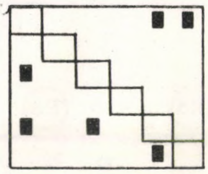
Table 2. ctd. Charts of tori

	Ea	Ec	Af	Ae	Fe	Ca
ψ^*	g	r	o	ℓ	b	y
	b	ℓ	r	o	y	g
	y	g	ℓ	b	r	o
	ℓ	y	g	r	o	b

$2\ \ 2\ \ 3_3\ \ 4_2\ \ 4_2\ \ 5_4$

T: $4_2\ \ 4_2\ \ 3_4\ \ 2\ \ 2\ \ 5_4$

P: $2_3\ \ 2_3\ \ 5_2\ \ 4_4\ \ 4_4\ \ 3$



Fd					Fd
ℓ	r	o	b	y	g
y	g	r	ℓ	o	b
b	ℓ	g	o	r	y
g	y	ℓ	r	b	o

ℓ	r	o	b	y	g
g	y	r	ℓ	o	b
b	ℓ	g	o	r	y
y	g	ℓ	r	b	o

PT

$2\ \ 3\ \ 4\ \ 5\ \ 2\ \ 5$

T: $4_2\ \ 3\ \ 2_2\ \ 5_3\ \ 4_2\ \ 5_3$

P: $2_3\ \ 5_4\ \ 4_3\ \ 3_2\ \ 2_3\ \ 3_2$

PT: $4_4\ \ 5_4\ \ 2_4\ \ 3_4\ \ 4_4\ \ 3_4$

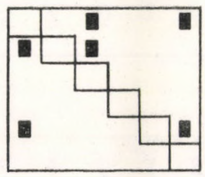
	Ea	Ba	Af	Bc	Ef	Ac
ν^*	y	r	o	ℓ	b	g
	ℓ	g	r	b	o	y
	g	y	ℓ	o	r	b
	b	ℓ	g	r	y	o

$2\ \ 3\ \ 4_2\ \ 3_4\ \ 3\ \ 3_2$

T: $4_2\ \ 3\ \ 2\ \ 3_2\ \ 3\ \ 3_4$

P: $2_3\ \ 5_4\ \ 4_4\ \ 5_3\ \ 5\ \ 5$.

PT: $4_4\ \ 5_4\ \ 2_3\ \ 5$



	Ae	Ce	Af	Bc	Ef	Eb
o	g	r	o	ℓ	b	y
	ℓ	y	r	b	o	g
	y	g	ℓ	o	r	b
	b	ℓ	g	r	y	o

$2\ \ 3\ \ 4_2\ \ 2_4\ \ 4_2\ \ 3_2$

T: $4_2\ \ 3\ \ 2\ \ 4_3\ \ 2\ \ 3_4$

P: $2_3\ \ 5_4\ \ 4_4\ \ 2_2\ \ 4_4\ \ 5$

PT: $4\ .\ 5\ \ 2_3\ \ 4\ \ 2_3\ \ 3_3$

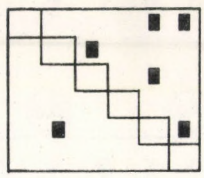


Table 2 ctd. Charts of tori

	Ba			Ba	
g	r	o	ℓ	y	b
ℓ	g	r	b	o	y
b	y	ℓ	o	r	g
y	ℓ	g	r	b	o

	Ba			Ba	
b	r	o	ℓ	y	g
ℓ	g	r	b	o	y
g	y	ℓ	o	r	b
y	ℓ	g	r	b	o

μ

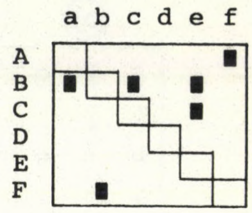
Fb	Ce	Af	Bc	Ba	Be
g	r	o	ℓ	y	b
ℓ	y	r	b	o	g
b	g	ℓ	o	r	y
y	ℓ	g	r	b	o

$2 \quad 3 \quad 5_2 \quad 5 \quad 4_2 \quad 3_2$

T: $4 \quad 3 \quad 5 \quad 5 \quad 2 \quad 3$.

P: $2 \quad 5 \qquad\qquad\qquad 5$

PT: $4 \quad 5 \quad 3 \quad 3 \quad 2 \quad 5$



T

y	r	o	ℓ	b	g
g	ℓ	r	b	o	y
ℓ	y	g	o	r	b
b	g	ℓ	r	y	o

$2 \quad 3 \quad 4_2 \quad 3_4 \quad 2 \quad 4_3$

T: $4_2 \quad 3 \quad 2 \quad 3. \quad 4. \quad 2_4$

C

b	r	o	ℓ	y	g
g	ℓ	r	b	o	y
ℓ	y	g	o	r	b
y	g	ℓ	r	b	o

$2 \quad 3 \quad 3_2 \quad 2 \quad 2 \quad 4_3$

ε*

Bf	Ec	Af	Bc	Ba	Be
g	r	o	ℓ	y	b
y	ℓ	r	b	o	g
b	g	ℓ	o	r	y
ℓ	y	g	r	b	o

$2 \quad 3 \quad 5_2 \quad 3 \quad 4_2 \quad 5_4$

T: $4_2 \quad 3 \quad 5_2 \quad 3 \quad 2 \quad 5_4$

P: $2_3 \quad 5_4 \quad 3_3 \quad 5_4 \quad 4_4 \quad 3$

PT: $4_4 \quad 5 \quad 3 \quad 5 \quad 2_3 \quad 3$

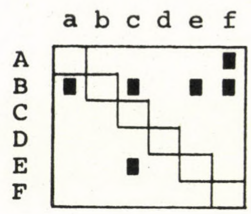


Table 2 ctd. Charts of tori

ℓ	r	o	b	y	g
y	g	r	ℓ	b	o
b	ℓ	g	o	r	y
g	y	ℓ	r	o	b

C

$2 \quad 5 \quad 2 \quad 3$

Df Df

ℓ	r	o	b	y	g
g	y	r	ℓ	b	o
b	ℓ	g	o	r	y
y	g	ℓ	r	o	b

y	r	o	ℓ	b	g
ℓ	g	r	b	y	o
g	y	ℓ	o	r	b
b	ℓ	g	r	o	y

PT

$2 \quad 5 \quad 4_2 \quad 5_2 \quad 3 \quad 3_2$

T: $4 \quad 5_3 \quad 2 \quad 5 \,.\, 3 \quad 3\,.$

P: $2_3 \quad 3 \,.\, 4_4 \quad 3_3 \quad 5_4 \quad 5$

PT: $4 \quad 3 \quad 2_3 \quad 3_3 \quad 5 \quad 5$

y	r	o	ℓ	b	g
g	ℓ	r	b	y	o
ℓ	y	g	o	r	b
b	g	ℓ	r	o	y

P

$2 \quad 5 \quad 4_2 \quad 5_2 \quad 2 \quad 4_3$

T: $4_2 \quad 5_3 \quad 2 \quad 5_2 \quad 4_2 \quad 2_4$

P: $2_3 \quad 3_2 \quad 4_4 \quad 3_3 \quad 2_3 \quad 4$

ℓ	r	g	o	y	b
y	ℓ	r	b	o	g
b	g	o	ℓ	r	y
g	y	ℓ	r	b	o

C

$2 \quad 5_3 \qquad\qquad 2$

Table 2 ctd. Charts of tori

ℓ	r	g	o	y	b
g	ℓ	r	b	o	y
b	y	o	ℓ	r	g
y	g	ℓ	r	b	o

C

2 5_3 2

ℓ	r	g	o	b	y
g	y	r	ℓ	o	b
y	ℓ	o	b	r	g
b	g	ℓ	r	y	o

P

$2\ \ 5_3\ \ 5_3\ \ 2_4\ \ 4_3\ \ 5$

T: $4_2\ \ 5\ \ 5\ \ 4_3\ \ 2_4\ \ 5$.

P: $2_3\ \ 3\ .\ 3_4\ \ 2_2\ \ 4\ \ 3_2$

y	r	ℓ	o	b	g
ℓ	g	r	b	o	y
g	y	o	ℓ	r	b
b	ℓ	g	r	y	o

T

$2\ \ 5_3\ \ 4_2$

T: 5 2

g	r	ℓ	o	b	y
ℓ	y	r	b	o	g
y	g	o	ℓ	r	b
b	ℓ	g	r	y	o

T

$2\ \ 5_3\ \ 4_2$

T: 5 2

g	r	ℓ	o	y	b
ℓ	g	r	b	o	y
b	y	o	ℓ	r	g
y	ℓ	g	r	b	o

P

$2\ \ 5_3\ \ 5_2\ \ 2\ \ 5_3\ \ 5_2$

T:

P: $2_3\ \ 3_4\ \ 3_3\ \ 2_3\ \ 3_4\ \ 3_3$

b	r	ℓ	o	y	g
ℓ	g	r	b	o	y
g	y	o	ℓ	r	b
y	ℓ	g	r	b	o

C

$2\ \ 5_3\ \ 2\ \ 3$

Table 2 ctd. Charts of tori

g	$\underline{r}$	ℓ	$\underline{o}$	$\underline{y}$	b
ℓ	$\underline{y}$	$\underline{r}$	b	$\underline{o}$	g
b	g	$\underline{o}$	ℓ	$\underline{r}$	$\underline{y}$
$\underline{y}$	ℓ	g	$\underline{r}$	b	$\underline{o}$

P

$2\ \ 5_3\ \ 5_2\ \ 5\ \ 4_2\ \ 5_2$

T: $4_2\ \ 5\ \ 5_2\ \ 5_3\ \ 2\ \ 5_2$

P: $2_3\ \ 3\ \ 3\ \ 3\ \ 4\ \ 3_3$

g	$\underline{r}$	ℓ	$\underline{o}$	$\underline{y}$	b
$\underline{y}$	ℓ	$\underline{r}$	b	$\underline{o}$	g
b	g	$\underline{o}$	ℓ	$\underline{r}$	$\underline{y}$
ℓ	$\underline{y}$	g	$\underline{r}$	b	$\underline{o}$

P

$2\ \ 5_3\ \ 5_2\ \ 3\ \ 4_2\ \ 3_4$

P: $2_3\ \ 3\ \ 3\ \ 5\ \ 4\ \ 5_3$

b	$\underline{r}$	ℓ	$\underline{o}$	$\underline{y}$	g
$\underline{y}$	g	$\underline{r}$	ℓ	$\underline{o}$	b
g	ℓ	$\underline{o}$	b	$\underline{r}$	$\underline{y}$
ℓ	$\underline{y}$	g	$\underline{r}$	b	$\underline{o}$

C

$2\ \ 5_3\ \ 2_3\ \ 3\ \ 3\ \ 4_4$

ℓ	$\underline{r}$	g	$\underline{o}$	b	$\underline{y}$
g	ℓ	$\underline{r}$	b	$\underline{y}$	$\underline{o}$
$\underline{y}$	g	$\underline{o}$	ℓ	$\underline{r}$	b
b	$\underline{y}$	ℓ	$\underline{r}$	$\underline{o}$	g

C

$2\ \ 3_3\ \ 4_2\qquad 2$

	Fd		Fd		
ℓ	$\underline{r}$	g	$\underline{o}$	b	$\underline{y}$
b	g	$\underline{r}$	ℓ	$\underline{y}$	$\underline{o}$
$\underline{y}$	ℓ	$\underline{o}$	b	$\underline{r}$	g
g	$\underline{y}$	ℓ	$\underline{r}$	$\underline{o}$	b

Table 2 ctd. Charts of tori

y	r	ℓ	o	b	g
ℓ	g	r	b	y	o
g	y	o	ℓ	r	b
b	ℓ	g	r	o	y

P

$2 \quad 3_3 \quad 4_2 \quad 5_2 \quad 3 \quad 5_2$

T: $4_2 \quad 3_3 \quad 2 \quad 5_2 \quad 3 \quad 5_2$

P: $2_3 \; 5 \, . \, 4 \, . \, 3 \, . \, 5 \, . \, 3_3$

g	r	ℓ	o	b	y
b	g	r	ℓ	y	o
y	ℓ	o	b	r	g
ℓ	y	g	r	o	b

T

$2 \quad 3_3 \quad 3_3 \quad 4_2 \quad 5_3 \quad 4_4$

T: $4_2 \quad 3_3 \quad 3_3 \quad 2 \quad 5 \quad 2_3$

Fd Fd

y	r	ℓ	o	b	g
b	g	r	ℓ	y	o
g	ℓ	o	b	r	y
ℓ	y	g	r	o	b

b	r	ℓ	o	y	g
y	g	r	ℓ	b	o
g	ℓ	o	b	r	y
ℓ	y	g	r	o	b

T

$2 \quad 3_3 \quad 4_3 \quad 3 \quad 3 \quad 4_4$

T: $4_2 \quad 3_3 \quad 2_4 \quad 3 \quad 3 \quad 2_3$

ℓ	r	g	b	y	o
y	ℓ	r	o	b	g
b	g	o	ℓ	r	y
g	y	ℓ	r	o	b

C

$2 \quad 4_3 \qquad 2$

ℓ	r	g	b	y	o
g	ℓ	r	o	b	y
b	y	o	ℓ	r	g
y	g	ℓ	r	o	b

C

$2 \quad 4_3 \qquad 2$

Table 2 ctd. Charts of tori

g	r	ℓ	b	y	o
ℓ	g	r	o	b	y
b	y	o	ℓ	r	g
y	ℓ	g	r	o	b

C

$2 \quad 4_3 \quad 2 \quad 2$

g	r	ℓ	b	y	o
ℓ	y	r	o	b	g
b	g	o	ℓ	r	y
y	ℓ	g	r	o	b

C

$2 \quad 4_3 \quad 2 \quad 5$

y	r	g	ℓ	b	o
g	ℓ	r	o	y	b
ℓ	g	o	b	r	y
b	y	ℓ	r	o	g

T

$2 \quad 4_3 \quad 5_3 \quad 2_2 \quad 5_4 \quad 4_3$

T: $4 \quad 2 \quad 5 \quad 4 \quad 5_4 \quad 2_4$

y	r	g	ℓ	b	o
b	ℓ	r	o	y	g
ℓ	g	o	b	r	y
g	y	ℓ	r	o	b

C

$2 \quad 4_3 \quad 3_3 \quad 2_2 \quad 3_2$

y	r	g	ℓ	b	o
g	ℓ	r	o	y	b
ℓ	y	o	b	r	g
b	g	ℓ	r	o	y

T

$2 \quad 4_3 \quad 5_3 \quad 2_2 \quad 4_3 \quad 4_3$

T: $2_4 \quad 2_4$

b	r	g	ℓ	y	o
y	ℓ	r	o	b	g
ℓ	g	o	b	r	y
g	y	ℓ	r	o	b

T

$2 \quad 4_3 \quad 4_3 \quad 3 \quad 3_2 \quad 4_3$

T: $4_2 \quad 2_4 \quad 2_4 \quad 3 \quad 3_4 \quad 2_4$

Table 2 ctd. Charts of tori

b	r	g	ℓ	y	o
g	ℓ	r	o	b	y
ℓ	y	o	b	r	g
y	g	ℓ	r	o	b

T

$2\quad 4_3\quad 4_3\quad 2\quad 4_3\quad 4_3$

T: $4_2\quad 2_4\quad 2_4\quad 4_2\quad 2_4\quad 2_4$

g	r	ℓ	b	y	o
y	ℓ	r	o	b	g
b	g	o	ℓ	r	y
ℓ	y	g	r	o	b

C

$2\quad 4_3\quad 2\quad 3$

y	r	ℓ	b	o	g
ℓ	g	r	o	b	y
b	y	g	ℓ	r	o
g	ℓ	o	r	y	b

T

$2\quad 3_4\quad 2\quad 3_4\quad 4\quad 5_2$

T: $4_2\quad 3_2\quad 4_2\quad 3_2\quad 2_2\quad 5$

y	r	ℓ	b	o	g
ℓ	g	r	o	y	b
b	y	g	ℓ	r	o
g	ℓ	o	r	b	y

T

$2\quad 3_4\quad 4\quad 5_2\quad 4$

T: $4\quad 3\quad 5_2\quad 2_2$

b	r	g	ℓ	o	y
ℓ	y	r	o	b	g
y	g	ℓ	b	r	o
g	ℓ	o	r	y	b

C

$2\quad 3_4\quad 4_3\quad 2_4\quad 3_2\quad 3_2$

Table 2 ctd. Charts of tori

y	r	ℓ	b	o	g
g	ℓ	r	o	b	y
b	y	g	ℓ	r	o
ℓ	g	o	r	y	b

P

$2 \quad 3_4 \quad 2 \quad 3_4 \quad 2 \quad 3_4$

T: $4 \quad 3 \quad 4 \quad 3 \quad 4 \quad 3$

P: $2_3 \quad 5_3 \quad 2_3 \quad 5_3 \quad 2_3 \quad 5_3$

y	r	ℓ	b	o	g
g	ℓ	r	o	y	b
b	y	g	ℓ	r	o
ℓ	g	o	r	b	y

C

$2 \quad 3_4 \quad 4 \quad 5_4 \quad 2 \quad 3_4$

b	r	g	ℓ	o	y
g	ℓ	r	o	y	b
y	g	ℓ	b	r	o
ℓ	y	o	r	b	g

P

$2 \quad 3_4 \quad 2_3 \quad 4_2 \quad 5_4 \quad 5_4$

T: $4_2 \quad 3_2 \quad 4_4 \quad 2 \quad 5_4 \quad 5_4$

P: $2. \quad 5_3 \quad 2 \quad 4\ . \quad 3 \quad 3$

y	r	g	ℓ	o	b
ℓ	g	r	b	y	o
b	y	ℓ	o	r	g
g	ℓ	o	r	b	y

C

$2 \quad 4_4 \quad 5_2 \quad 5_2 \quad 2_3 \quad 3_2$

y	r	g	ℓ	o	b
g	ℓ	r	b	y	o
b	g	ℓ	o	r	y
ℓ	y	o	r	b	g

C

$2 \quad 4_4 \quad 5_2 \quad 2_2 \quad 5_4$

b	r	g	ℓ	o	y
g	ℓ	r	b	y	o
y	g	ℓ	o	r	b
ℓ	y	o	r	b	g

T

$2 \quad 4_4 \quad 3_2 \quad 4_2 \quad 5_4 \quad 5_4$

T: $4_2 \quad 2_3 \quad 3_4 \quad 2 \quad 5_4 \quad 5_4$

Table 2 ctd. Charts of tori

y	r	g	ℓ	o	b
g	ℓ	r	b	y	o
b	y	ℓ	o	r	g
ℓ	g	o	r	b	y

T

$$\begin{array}{l} \;\;\;2 \;\; 4_4 \;\; 5_2 \;\; 5_2 \;\; 4_3 \;\; 5_4 \\ T{:}4_2 \;\; 2_3 \;\; 5_2 \;\; 5_2 \;\; 2_4 \;\; 5_4 \end{array}$$

Cd		Cd			
y	r	ℓ	b	o	g
g	y	r	ℓ	b	o
b	ℓ	g	o	r	y
ℓ	g	o	r	y	b

Charts whose characteristic number begins with 3

/Only kernel symbols 3 and 5 are allowable/

Cd		Cd			
ℓ	r	o	b	y	g
y	g	ℓ	r	o	b
g	ℓ	r	o	b	y
b	y	g	ℓ	r	o

	Ea	Ba	Be	Ae	Ab	Eb
β	y	r	ℓ	b	o	g
	ℓ	g	o	r	y	b
	g	y	r	ℓ	b	o
	b	ℓ	g	o	r	y

$$\begin{array}{l} \;\;\;3 \;\; 5_2 \;\; 3 \;\; 5_2 \;\; 3 \;\; 5_2 \\ T{:}3 \;\; 5_2 \;\; 3 \;\; 5_2 \;\; 3 \;\; 5_2 \\ P{:}5_4 \;\; 3_3 \;\; 5_4 \;\; 3_3 \;\; 5_4 \;\; 3_3 \end{array}$$

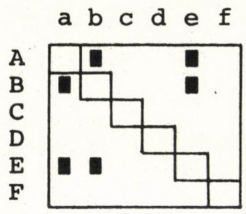


Table 2 ctd. Charts of tori

Tori with no parity pairs

			Page
ε	0	(411)(2211)	8
ε*	0	(411)(2211)	27
α	0	(33)(222)	7
α*	0	(33)(222)	8
α**	0	(33)(222)	16
ι	0	(321)(111111)	7
γ	0	(3111)(3111)	12
λ	0	(3111)(2211)	8
λ*	0	(3111)(2211)	9
λ**	0	(3111)(2211)	10
μ	0	(3111)(21111)	27
ν	0	(222)(222)	19
ν*	0	(222)(222)	26
ο	0	(2211)(2211)	26
π	0	(2211)(111111)	10
π*	0	(2211)(111111)	11

Tori with 1 parity pair

			Page
δ	1	(33)(2211)	25
κ	1	(222)(2211)	11
κ*	1	(222)(2211)	15
ψ	1	(2211)(2211)	25
ψ*	1	(2211)(2211)	26
η	1	(2211)(2211)	9
τ	1	(2211)(2211)	13
τ*	1	(2211)(2211)	17
τ**	1	(2211)(2211)	17
χ	1	(2211)(21111)	15
ω	1	(2211)(111111)	8
ω*	1	(2211)(111111)	10
ξ	1	(2211)(111111)	9

Tori with 2 parity pairs

			Page
ρ	2	(2211)(2211)	21
ζ	2	(2211)(111111)	13
σ	2	(21111)(21111)	8
σ*	2	(21111)(21111)	11

The tori with 3 parity pairs: β 3 (222)(222) 35

Table 3. Classification of tori

FIGURE CAPTIONS

Fig. 1. Blob notation. The tensor is drawn as some kernel symbol (a blob, say) and its indices as 'legs' of the symbol. The Kronecker delta becomes a line segment. Contraction of a pair of indices is achieved by connecting the corresponding legs

Fig. 2. Conway's matrix

Fig. 3. The 4x6 chart of torus sides. A color c̲ occurs within a 4x4 matrix of squares, which is possibly cut in two parts by the vertical cut surface of the torus

Fig. 4. Numbering the determinant-term patterns of some color c̲

Fig. 5. A division of Conway's matrix into five tori

Fig. 6. A complete set of tori. Primes denote re-colored representative of a class

Fig. 7. A division of Conway's matrix into five re-colored version of the ψ structure

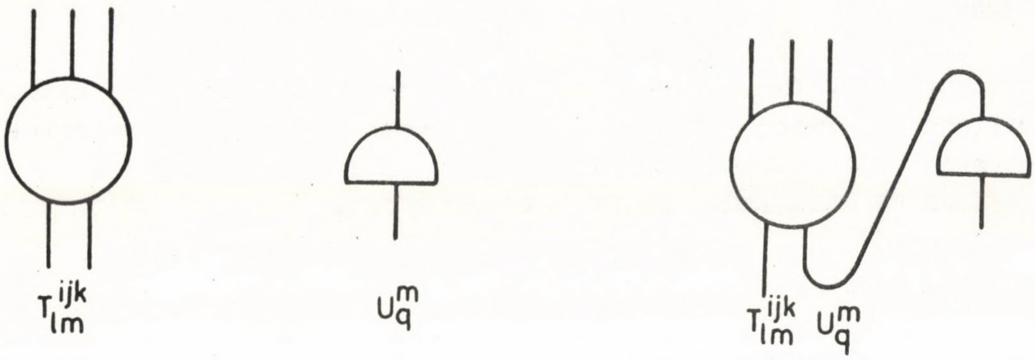


Fig. 1.

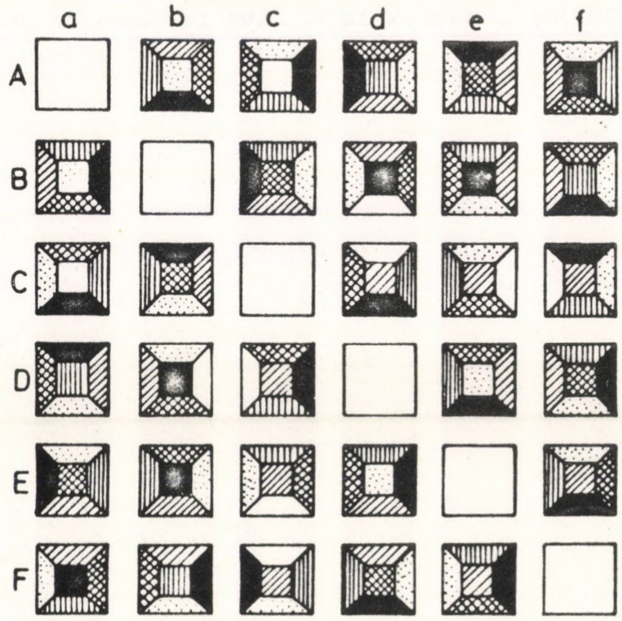


Fig. 2.

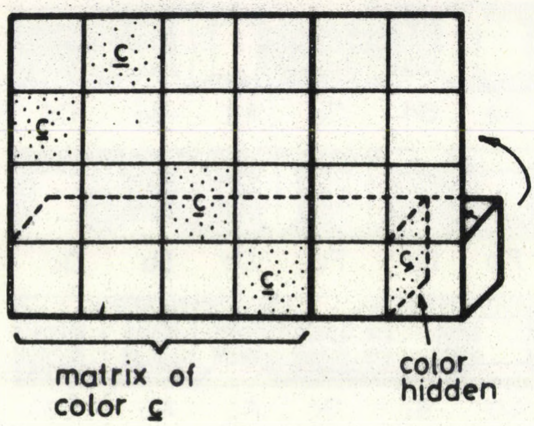


Fig. 3.

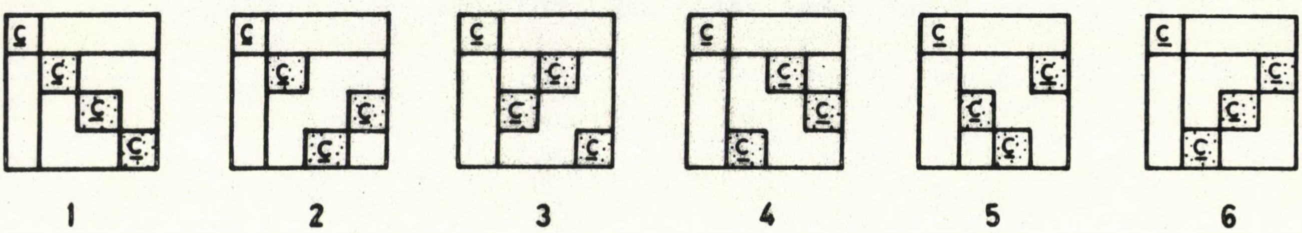


Fig. 4.

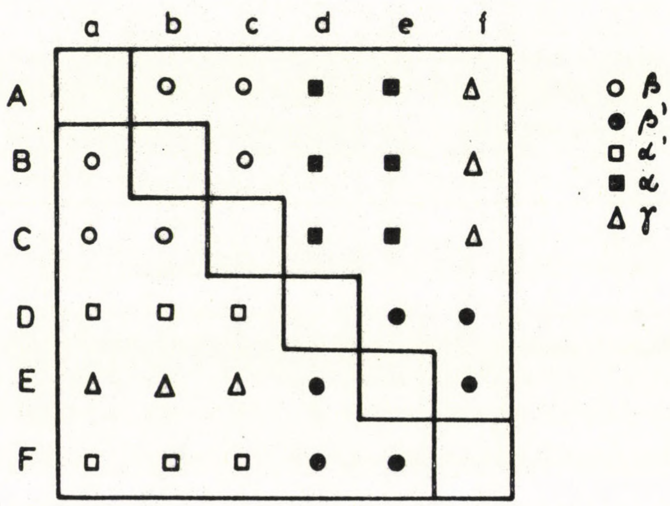


Fig. 5.

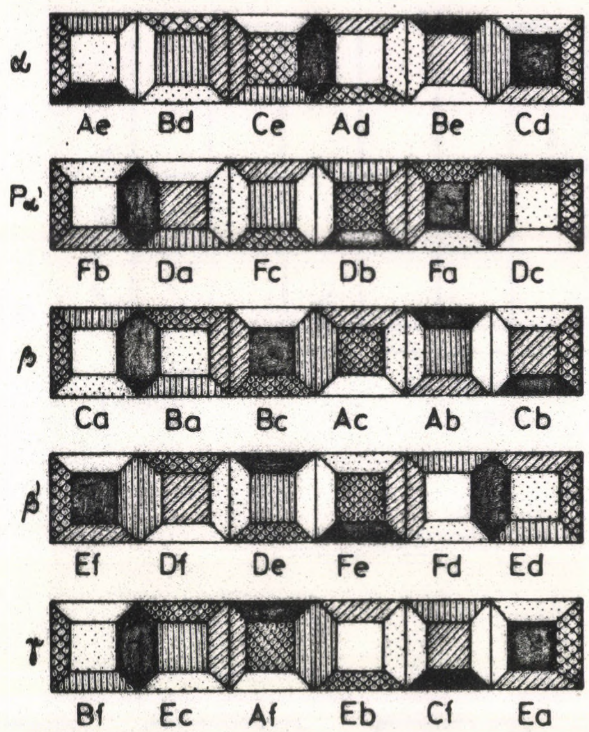


Fig. 6.

	1	2	3	4	5
1		5	2	1	5
2	1		4	2	1
3	5	2		2	3
4	3	4	3		1
5	3	5	4	4	

Fig. 7.

Kiadja a Központi Fizikai Kutató Intézet
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Szakmai lektor: Lukács Béla
Nyelvi lektor: Forgács Péter
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