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ABSTRACT

In the paper some question of the acoustic boiling detection in investigated for PWR reactors. The physical origin of subcooled boiling acoustic noise, its statistical features and a possible way of data evaluation for boiling detection are described. It is shown, that the boiling noise can be treated as a series of independent acoustic events caused by volume change of bubbles, and the relative flatness of the boiling signal amplitudo-probability-density function can be used as a criterion of boiling detection.

АННОТАЦИЯ

В работе рассматриваются некоторые вопросы акустического детектирования недогретого кипения в реакторах с водой под давлением. Описан механизм возникновения акустического шума при недогретом кипении, его статистический характер, а также возможный метод обработки сигнала с целью детектирования кипения. Показано, что акустический шум кипения можно рассмотреть как последовательность независимых акустических событий, вызванных изменением объема паровых пузырей, а коэффициент эксцесса распределения вероятности амплитуды сигнала можно использовать в качестве критерия детектирования.

KIVONAT

A dolgozatban megvizsgáltuk az aláhütött forrás akusztikus detektálásának néhány kérdését nyomottvizes reaktorok esetében. Leirjuk a forrás zajának fizikai eredetét, a zaj statisztikai jellemzőit, bemutatunk egy forrásdetektálás céljára alkalmas jelfeldoldozási módszert. Megmutatjuk, hogy a forrás zaja egymástól független akusztikus események sorozataként tekinthető, amelyeket a gőzbuborékok térfogatváltozása okoz, a forrászaj amplitudójának valószinüségi sürüségfüggvényére kiszámitott relativ lapultsági tényező pedig a forrásdetektálás kritériumaként használható fel.

INTRODUCTION

In the core of PWR reactors subcooled boiling may exist during normal operation in the so-called hot channel. In case of a failure subcooled boiling may appear in other parts of the core too, or in the whole core.

From the view-point of safe operation, the recognition of subcooled boiling can be an important auxiliary information.

The existence of subcooled boiling can be recognised by the analysis of boiling acoustic noise. In the paper some question of the acoustic boiling detection is investigated for PWR reactors.

For the recognition of reactor coolant boiling by in-core detected acoustic noise, the background noise, the noise propagation and the acoustic effect of boiling must be known. In the paper the mechanism of acoustic emission in case of subcooled forced boiling is investigated, its statistical features, and a possible way of data evaluation is described.

A significant advancement was reached during the last decade in the field of sodium boiling acoustic emission analysis, and sodium boiling detection. It is appropriate to utilise the results of sodium boiling acoustic detection for the acoustic detection of subcooled boiling of water, as the basic physical phenomenon is similar.

Carey et. al published a representative summary about the acoustic boiling detection for LMFBR [1]. The autors experimentally investigated the acoustic noise of sodium boiling for steady state and transient (loss of flow) cases and analysed the physical origin of boiling noise. They found that pulse like acoustic events are related with the boiling, which are triggered by bubbles (see *Fig. 1*).

An elementary acoustic event is related to a pressure perturbation, caused by the volume change of a bubble. The autors surveyed the basic results of bubble dynamics and acoustics. It is stated, that the collapse means the major acoustic source during the different parts of life for a bubble (nucleation, growth, detachment, condensation, collapse). The important statistical components of boiling acoustic signal were determinated, so the frequency of elementary events, the waiting time between two events, and the probability distribution of peak pressure of events. Concerning the badkground noise they stated that it consist from pump induced vibroacoustic signal, structural induced vibration, and locally induced pressure variation (flow noise). The last one is partially composed of local turbulent induced pressure variations, pseudo-sound.

The subcooled boiling acoustic emission of water has been investigated for examply Bessho and Nishihara [2]. They examined the pressure fluctuation in the liquid caused by a bubble at distande "ℓ" from the bubble center by the Gilmore momentum equation. The initial data of numerical calculations has been the bubble radius, which has been measured as a function of time by Nishikawa for atmospheric subcooled boiling. On the *Fig. 2* a typical result of the measurement and calculation is shown. It seems, that in this case a single bubble causes pressure perturbation similar to that of *Fig. 1*. As it is seen on *Fig. 2*, the most significant pressure perturbation has been caused by the vapour bubble growth in the given case. Meanwhile it is clear from Carey s paper [1] that depending on the given conditions, the main source of acoustic energy might be different from the collapse.

According to our own experimental results, we concluded that the acoustic noise of forced subcooled boiling is not different basically from the boiling noise of [1] and [2].

THE ACOUSTIC EMISSION OF SUBCOOLED BOILING IN WATER FORCED FLOW

We accept about the subcooled boiling acoustic noise of water like in [1] the following presumptions:

1. The boiling noise is composed by acoustic events triggered by bubbles.

2. Similar to that of [2] we presume that a bubble causes pressure perturbations $\tilde{p}_E(\ell,t)$ at "ℓ" distance from its centre, described by the Gilmore momentum equation; that is

$$\tilde{p}_E(\ell,t) = p_E(\ell,t) - p_\infty \qquad (1)$$

$$\frac{d\tilde{p}_E(\ell,t)}{dt} + \frac{\ell}{c}\,\tilde{p}_E(\ell,t) = \rho_\ell \left[\frac{R^2}{\ell}\,\ddot{R} + \frac{2R}{\ell}\,(\dot{R})^2 - \frac{R^4}{2\ell^4}\,(\dot{R})^2\right] \qquad (2)$$

where $p_E(\ell,t)$ is the actual pressure at "ℓ" distance from the centre of bubble;

p_∞ the pressure far away from the bubble;

$R,\dot{R},\ddot{R}$ the bubble radius and its derivatives by time;

ρ_ℓ the liquid density;

c the sound velocity.

The bubble radius must be given as a function of time to the solution of the equation.

3. The acoustic emission of bubbles generated by a cavity might be given in the following form:

$$\tilde{p}(t) = \sum_i \tilde{p}_{E_i}(t-t_i) \qquad (3)$$

where $\tilde{p}_{Ei}(t)$ is the elementary acoustic event generated by i-th bubble; t_i is the nucleation time of the given bubble.

4. The acoustic noise of boiling is the superposition of independent phenomenons for all cavities, described by Eq. 3.

THE ACOUSTIC EMISSION OF BUBBLES GENERATED BY AN ACTIVE CAVITY

In order to describe the phenomenon, the bubble generation period and its statistical features must be known, and the mechanism of bubble growth and condensation (that is the R(t) function) to determine the $\tilde{p}_{E_i}(t)$ pressure perturbation. In order to reply for the abovementioned questions, well known sources [3-9] are applied for these physical problems considering the specific PWR parameters.

The generation of bubbles is started at the surface cavities of the heated wall in the so called generating centres, when the superheat in the thermal boundary layer reaches a critical value. The minimum superheat according to Griffith and Wallis for boiling initiation is

$$T_w - T_s(p_\infty) = \frac{2\sigma T_s v_{fg}}{h_{fg} R_c} \qquad (4)$$

where T_w is the wall temperature; T_s is the saturation temperature belonging to p_∞ pressure; σ is the surface tension; $v_{fg} = v_g - v_f$ the difference of specific volumes on the saturation line; h_{fg} is the evaporation heat; R_c is the radius of bubble in the cavity.

This criteria gives the minimal size of bubbles for a given superheat too.

A cavity of R_c size becomes active when the wall temperature attains the T_c activation value. This t_i time is regarded the initiation of an elementary acoustic event. The bubble grows on the wall, while the forces acting upon it tear it off. The growth time t_g is the elapsed time between the initiation and detachment. When the bubble departed, its place is filled by liquid, and there is a t_w delay time while the liquid temperature attains again the T_c temperature value. The frequency of bubble generation is

$$\nu = \frac{1}{t_g + t_w} \qquad (5)$$

That is the bubble generation is a strictly periodical phenomenon if the process is ideally fluctuation free, and the departure size of bubbles is constant.

In fact both t_g and t_w are random variables as a consequence of turbulent temperature and velocity fluctuations. Afgan [7] investigating the temperature fluctuations of the two phase boundary layer states that the wall voidage and the related temperature noise component are Poisson processes. That is the probability density function of the elapsed time between two nucleation

$$\tau = t_g + t_w \tag{6}$$

has the following form:

$$\varphi(\tau) = \nu\, e^{-\nu\tau} \tag{7}$$

where $\nu = \frac{1}{\bar{\tau}}$ is the average frequency of bubble generation.

Therefore the probability density function of growth time is:

$$\varphi(t_g) = \frac{1}{\bar{t}_g}\, e^{-\frac{1}{\bar{t}_g} t_g} \tag{8}$$

and the delay time probability density function is:

$$\varphi(t_w) = \frac{1}{\bar{t}_w}\, e^{-\frac{1}{\bar{t}_w} t_w} \tag{9}$$

where $\bar{t}_g$, $\bar{t}_w$ is the average growth and delay time.

According to the convolution theorem

$$\nu = \frac{1}{\bar{t}_g} + \frac{1}{\bar{t}_w}\ . \tag{10}$$

If the R(t) radius-time relation is known, so the acoustic noise caused by a cavity may be interpreted as a respons of a system having $h(\alpha)$ transfer function to the Poisson impulses:

$$z(t) = \sum_i \delta(t-t_i). \tag{11}$$

So we shall examine the dynamic behaviour of bubbles further on. The growth of a spherical or hemispherical bubble on the heated surface in incompressible and infinit liquid is described by the

$$R\cdot\ddot{R} + \frac{3}{2}\,(\dot{R})^2 = \frac{1}{\rho_\ell}\,(p_v - p_\infty - \frac{2\sigma}{R}) \tag{12}$$

momentum equation, where the effect of viscosity is neglected, and the

$$\frac{\partial T}{\partial t} = a\left(\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r}\right) - \frac{R^2\cdot\dot{R}}{r}\frac{\partial T}{\partial r} \tag{13}$$

energy equation. On the vapour- liquid interface the

$$\frac{\partial T}{\partial t} = \frac{h_{fg}}{4\pi\ R^2\ k} \left(\frac{4}{3}\ \pi\ R^3\ \rho_v\right) \qquad (14)$$

relation is applied (see [3-9]).

In a PWR channel in case of subcooled coolant boiling the behaviour of vapour bubbles is determined by the energy transport between the heated surface and the thermal boundary layer around the bubble. In the core of a PWR the superheat of the thermal boundary layer on the fuel elements may reach the liquid inertia controlled growth only in special circumstances. This is for example a sudden power excursion, or a blow down [10].

The growth velocity of bubbles are examined in details by Afgan [7]. We accept the result of Plesset and Zwick under thermal control,

$$\dot{R}_{therm} = \left(\frac{3}{\pi}\right)^{1/2} \cdot \frac{k}{h_{fg}\ \rho_v\ (T_w)} \frac{\Delta T_{SH}}{(D\ t)^{1/2}} \qquad (15)$$

where ΔT_{SH} is the superheat of liquid;
k is the heat concuction coefficient of the liquid;
h_{fg} is the evaporation heat;
D is the thermal diffusivity of the liquid.

The bubble grows on the surface until t_g time, then reaching the R_d departure radius, leaves the surface.

The bubble getting into the subcooled liquid may grow further, while the excess enthalpy of its own thermal boundary layer used up partly for evaporation, partly for the heating of the bulk liquid, than the collapse is initiated. We accept that the collapse is a thermally controlled phenomena for our qualitative analysis (see [6] for the collapse of vapour bubbles in subcooled liquid).

Whith this presumption bubble radius as a function of time is determined by the temperature difference between the bubble surface and the ambient liquid from the nucleation to the full collapse. If the ambient liquid temperature is known always, that is deterministic, than R(t) is deterministic too.

In this case the process Eq. 3 is a shot noise.

Although in the reality the bubble growth and collapse rate depends upon the turbulent fluctuation of liquid velocity and temperature. It was shown earlier, that the turbulent ambient has influence upon the frequency of bubble generation and the growth time too. The collapse of bubbles is effectuated by the tubulence by two means; on one hand the bubble moves in a turbulent velocity field, so it is enclosed in a changing temperature field; on the other hand the local temperature itself fluctuates.

The motion of bubbles in turbulent velocity field is studied in [11-14] papers. In this paper we give the simplified description of the bubble motion in turbulent velocity field in order to show the statistic characteristics of subcooling, determining the thermally controlled collapse rate.

The flow is regarded as unidirect, and the turbulent velocity field in the channel is supposed to be homogeneous and isotrop. The axial velocity of bubble is taken equal to that of liquid axial velocity $u_\ell = u_b$.

The crosswise bubble motion is determined by the crosswise velocity fluctuations.

The abovementioned presumptions are acceptable if the bubble size is extremely small, that is in case of high pressures [15].

During the qualitative description of motion the change of bubble radius is neglected.

Presuming the abovementioned simplifications, the crosswise motion of bubble might be treated as a one-dimensional Brown motion from a coordinate system, moving with $u_\ell = u_b$ velocity, therefore it is described by the Langevin equation [16].

Let x(t) the crosswise position of the bubble, and

$$V_b(t) = \frac{dx(t)}{dt} \tag{16}$$

the velocity of this motion.

The equation of motion is

$$\frac{d\,V_b(t)}{dt} + \beta\,V_b(t) = n(t) \tag{17}$$

in this case, where $\beta = \frac{f}{m}$ is the ratio of friction factor and bubble mass, and the n(t) term notes the effect of turbulence having

$$E\{n(t)\} = 0 \tag{18}$$

mean value and

$$S_n(\omega) = \text{const} = \varepsilon \tag{19}$$

spectrum.

One may neglect the acceleration term in the Eq. 17 if $t \gg 1/\beta$ so the integration of Eq. 17 is simple:

$$x(t) = \int_0^t n(t')\,dt'. \tag{20}$$

As homogeneous turbulent velocity field was presumed, the n(t) is a normal white noise, its integral is a Wiener-Levy process with zero mean, and

$$E\{x^2(t)\} = \frac{1}{\beta^2}\,\varepsilon t = D_b^2\,t \tag{21}$$

variancy, where $D_b = \varepsilon/\beta^2$ the diffusivity factor.

Let s suppose furthermore, that the bubble moves only towards the subcooled bulk liquid, never returning to the superheated boundary layer. In this case the probability for the bubble, being at $x \le x_o$ distance from the wall at a t_o time is:

$$\Phi\{x(t_o) \leq x_o\} = \frac{2}{\sqrt{2\pi\ \sigma^2}} \int_o^{x_o} e^{-\frac{x^2}{2\sigma^2}} dx \qquad (22)$$

where $\sigma^2 = D_b^2\ t_o$.

The D_b diffusivity factor according to Téchy and Szabados [13] is:

$$D_b \sim \sqrt{\pi}\ \frac{\langle \tilde{u}_\ell^2 \rangle}{\omega_L} \qquad (23)$$

where ω_L is the characteristic frequency of turbulence.

Summing up the abovmentioned presumptions; the bubble moves axially with $u_\ell = u_b$ velocity, and crosswise according to Eq. 20.

The bubble is surrounded by subcooled liquid, specified for a given point. This subcooling determines the thermally controlled bubble collapse rate.

The spatial distribution of time averaged subcooling is given in the following form, presuming steady-state subcooled boiling

$$\Delta T_{sc}(z,x) = \overline{\Delta T}_{sc}(z)\cdot g(x) \qquad (24)$$

where $\overline{\Delta T}_{sc}(z)$ is the average subcooling in the channel cross section at z axial co-ordinate. (See [17] for the shape of $\overline{\Delta T}_{sc}(z)$ function.) The g(x) represents the crosswise distribution of the subcooling in the liquid.

This presumption is acceptable for boiling initiation, for very small qualities. The axial and crosswise distribution of liquid subcooling has been experimentally investigated for example in [18].

The condensation rate of bubbles is determined by the subcooling described by the following equation:

$$T_{sc}(t) = \overline{\Delta T}_{sc}(u_b\cdot t)\cdot g(x(t)) \qquad (25)$$

based upon Eq. 24 and the presumptions concerning the bubble motion.

In Eq. 25 g(x(t)) is the function of the process, given by Eq. 20. In this manner alone the stochastic bubble motion is turns stochastic phenomenon the bubble collapse.

It is conceivable according to the abovementioned, that the change of bubble radius versus time might be regarded as an independent, non interacting phenomenon, having identical distribution (independently from the departure size).

Therefore the process starting at t_i, defined by Eq. 3 might be interpreted as a series of independent acoustic events having identical distribution, where t_i Poisson distributed. The process given by Eq. 3 might be interpreted as a respons of a stochastic system for impulse series determined by Eq. 11, where the impulses correspond to a single bubble.

It is presumed, that the quantity of emitted acoustic energy during the bubble life might be related to a characteristic bubble size. Therefore Eq. 11 is transformed into the following form

$$z(t) = \sum_i R_{d_i} \; \delta(t-t_i) \tag{26}$$

where R_{d_i} is the departure radius of i-th bubble, characterising the emitted acoustic energy.

THE ACOUSTIC NOISE OF BUBBLES GENERATED BY ACTIVE CAVITIES WORKING SIMULTANEOUSLY ON THE HEATED SURFACE

It is presumed, that the active cavities are working independently at the boiling initiation. Similar assumptions are made by the authors of [19,20] papers too. Therefore the acoustic noise, emitted by active cavities on the heated surface is assumed as a superposition of independent processes, described by Eq. 3. (The bubbles have no interactions, which is acceptable at small steam qualities.)

Let us examine at first the statistical characteristic of the bubble generation on the heated surface.

The bubbles, generated by a single isolated cavity were considered as a Poisson impulse series, according to Eq. 26. Consequently the bubble generation of N active cavity on the heated surface will be the sum of N independent Poisson processes, so Poisson process in itself, and the characteristic frequency will be the sum of individual process frequencies.

The average frequency of bubble generation in an active cavity depends on the local wall superheat, and the cavity size [21]. For reasons of simplicity, let us assume that the variation of wall superheat on the surface is negligible, and spatially constant. In this case the size of active cavity determines the average frequency of bubble generation. The relation between frequency and cavity size is the clearest in case of pool boiling [21,22] (see *Fig. 3*). In the case of convective subcooled boiling this relation is less known and presumably no so simple, but here the average frequency of bubble generation depends on the cavity size similarly. Let us note this relationship with $\nu(R_c)$. Knowing the size distribution of active cavities, the bubble generation frequency can be defined. The size distribution of active cavities on the boiling surface was investigated in [23,24] papers. Here we rely on information obtained for pool boiling too. (About flow boiling see [25].) Let N active cavity on a unit surface area, and let denote with $n(R_c)$ the number of cavities having radius within $(R_c; R_c + dR_c)$. Starting from the size distribution of cavities, $n(R_c)$ is given by

$$n(R_c) = N\varphi(R_c)dR_c \tag{27}$$

where $\varphi(R_c)$ is the probability density function of size distribution.

Therefore the average frequency of bubble generation on the surface A according to Eq. 27 and the $\nu(R_c)$ function is follows:

$$\bar{\nu} = A \cdot N \int_0^\infty \nu(R_c) \cdot \varphi(R_c) dR_c. \qquad (28)$$

Summing up the abovementioned: the acoustic signal of subcooled boiling may be written into a form analogous with Eq. 3, that is

$$\tilde{p}_B(t) = \sum_i \tilde{p}_{E_i}(t-t_i) \qquad (29)$$

where $\tilde{p}_{E_i}(t)$ is independent elementary acoustic process, of a given distribution, and the t_i Poisson time points, having $\bar{\nu}$ average frequency according to Eq. 28.

ANALYSIS OF THE APD FUNCTION OF THE ACOUSTIC EMISSION SIGNAL

In the previous chapter we described the acoustic emission signal of the subcooled boiling as a stochastic process, corresponding to the Eq. 3, according to physical considerations.

Now we analyse the amplitude probability density (APD) function of the process.

We assume, that the process given by Eq. 3 is ergodic with zero mean, its second central moment exists and it is finite, and the same holds true for the elementary acoustic events $\tilde{p}_{E_i}(t)$.

Utilizing the ergodicity the r-th central moment of $\tilde{p}_B(t)$ is calculated as follows:

$$\mu_r = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} [\tilde{p}_B(t)]^r \, dt. \qquad (30)$$

(It was considered here, that the $\tilde{p}_B(t)$ has zero mean.)

The abovementioned relation will be used in a modified form:

$$\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} [\tilde{p}_B(t)]dt = \left\langle \frac{1}{T} \int_0^T [\tilde{p}_B(t)]^r \, dt. \right\rangle \qquad (31)$$

The central moments of signal will be given below:

1. According to our presumption the first central moment is zero.

$$\mu_{1B} = 0. \qquad (32)$$

2. According to Eq. 29 and Eq. 31 the second central moment is

$$\mu_{2B} = \left\langle \frac{1}{T} \int_0^T [\sum_i \tilde{p}_{E_i}(t-t_i)]^2 \, dt \right\rangle \qquad (33)$$

that is $\mu_{2B} = \bar{\nu}\ \overline{\Delta\tau}\ \sigma_E^2 = \bar{\nu}\ \overline{\Delta\tau}\ \mu_{2E} = \sigma_B^2$ (34)

where $\bar{\nu}$ is the frequency of elementary acoustic events,
$\overline{\Delta\tau}$ is the average duration of elementary events,
μ_{2E} is the averaged second central moment of elementary events.

Physically this has the following meaning:

the RMS of boiling acoustic signal is proportional to the RMS of pressure perturbations caused by single bubbles, and with the $\bar{\nu}\ \overline{\Delta\tau}$, the coverage of time axis with elementary acoustic events.

3. The 3rd central moment is assumed to be zero.

4. The 4-th central moment of $\tilde{p}(t)$ is:

$$\mu_{4B} = \left\langle \frac{1}{T} \int_0^T \sum_i \tilde{p}_{E_i}(t-t_i)^4\, dt \right\rangle \tag{35}$$

therefore

$$\mu_{4B} = \bar{\nu}\ \overline{\Delta\tau}\ \mu_{4E} + 3\ \bar{\nu}^2\ \overline{\Delta\tau}^2\ \mu_{2E}^2\ . \tag{36}$$

According to the central limit theorem the $\tilde{p}(t)$ process (Eq. 29) in case of $\bar{\nu}\to\infty$ – that is in case of proper overlapping of elementary acoustic events – turns into a normal or Gauss process. This marginal case must be appear on the APD of $\tilde{p}(t)$ signal too.

In case of normal distribution the relative flatness of APD function is:

$$\gamma_{2B} = \frac{\mu_{4B}}{\mu_{2B}^2} - 3 = 0. \tag{37}$$

Therefore in our case the following presumption must be fulfilled:

$$\lim_{\bar{\nu}\to\infty} \frac{\bar{\nu}\ \overline{\Delta\tau}\ \mu_{4E} + 3\ \bar{N}^2\ \overline{\Delta\tau}^2\ \mu_{2E}^2}{(\bar{\nu}\ \overline{\Delta\tau}\ \mu_{2E})^2} - 3 = 0. \tag{38}$$

This is easily conceivable.

Eq. 37 is transformed into the following form (see Eq. 34 and Eq. 36 too):

$$\gamma_{2B} = \frac{\bar{\nu}\ \overline{\Delta\tau}\ \mu_{4E} + 3\ \bar{\nu}^2\ \overline{\Delta\tau}^2\ \mu_{2E}^2}{(\bar{\nu}\ \overline{\Delta\tau}\ \mu_{2E})^2} - 3 = \frac{1}{\bar{\nu}\ \overline{\Delta\tau}}(\gamma_{2E} + 3) \tag{39}$$

where

$$\gamma_{2E} = \frac{\mu_{4E}}{\mu_{2e}^2} - 3 \tag{40}$$

is the relative flatness of the APD of an elementary acoustic event.

According to Eq. 39 it seems, that the relative flatness of $\tilde{p}(t)$ signal APD in the case of low $\bar{\nu}$ (that is extremely low quality) won't be zero even if the elementary acoustic event $\tilde{p}_{E_i}(t)$ is normal process, that is $\gamma_{2E} = 0$ (see Eq. 40). The relative flatness γ_{2B} is zero only if the elementary acoustic events are overlapped repeatedly, coverning the time axis repeatedly, or otherwise:

$$\frac{1}{\bar{\nu}\ \overline{\Delta\tau}} \sim \frac{T}{S\ \overline{\Delta\tau}} = \frac{T}{T_B} \rightarrow 0 \qquad (41)$$

where T is the duration of sampling time;

S is the number of elementary events during T;

T_B is the time interval coveriable with this elementary events without overlapping.

The relative flatness can't be zero in case of rare events.

The relative flatness of APD function of $\tilde{p}(t)$ signal as a consequence of the abovementioned characteristics may give a proper criterion for the identification of boiling start up.

THE RELATIVE FLATNESS, AS A CRITERION OF THE BOILING DETECTION

In the practice in case of convective subcooled boiling in the detected signal one can find the flow noise component, that is:

$$\tilde{p}_D(t) = \tilde{p}_N(t) + \tilde{p}_B(t) \qquad (42)$$

where $\tilde{p}_D(t)$ is the detected acoustic signal,

$\tilde{p}_N(t)$ is the background noise,

$\tilde{p}_B(t)$ is the boiling acoustic noise (see Eq. 29).

In this case, presuming $\tilde{p}_B(t)$ and $\tilde{p}_N(t)$ independent from each another and $\tilde{p}_N(t)$ is ergodic, steady-state with zero mean, the central moments will be follows:

$$\mu_1 = 0. \qquad (43)$$

$$\mu_2 = \mu_{2N} + \mu_{2B}. \qquad (44)$$

$$\mu_4 = \mu_{4N} + 6\mu_{2N}\,\mu_{2B} + \mu_{4B}. \qquad (45)$$

(see Appendix 1)

The relative flatness of APD function of detected acoustic signal is:

$$\gamma_2 = \frac{\mu_4}{\mu_2^2} - 3 = \frac{\mu_{4N} + 6\mu_{2N}\,\mu_{2B} + \mu_{4B}}{(\mu_{2B} + \mu_{2N})^2} - 3. \qquad (46)$$

Introducing the

$$\Theta = \frac{\mu_{2B}}{\mu_{2N}} = \frac{\sigma_B^2}{\sigma_N^2} = \frac{\overline{\nu}\ \overline{\Delta\tau}\ \sigma_E^2}{\sigma_N^2} \tag{47}$$

signal to noise relation, γ_2 is expressed as follows (see Eq. 46):

$$\gamma_2 = \frac{\gamma_{2N} + \gamma_{2B}\cdot\Theta^2}{(1 + \Theta)^2} \tag{48}$$

where γ_{2N} is the relative flatness of background noise APD function, and γ_{2B} is defined by Eq. 39.

Therefore

$$\gamma_2 = \frac{\gamma_{2N} + \frac{1}{\overline{\nu}\ \overline{\Delta\tau}} (\gamma_{2E} + 3)\ \Theta^2}{(1 + \Theta)^2} \ . \tag{49}$$

Let's consider the case when the $\tilde{p}_N(t)$ noise has normal distribution, which is practically acceptable regarding its physical origin.

Than

$$\gamma_2 = \frac{1}{\overline{\nu}\ \overline{\Delta\tau}} (\gamma_{2E} + 3) \frac{\Theta^2}{(1 + \Theta)^2} = \gamma_{2B} \frac{\Theta^2}{(1 + \Theta)^2} \ . \tag{50}$$

That is, the signal to noise relation has a decisive effect upon the value of relative flatness of detected signal APD function.

In case of boiling initiation, when the frequency of elementary acoustic events are small, the signal to noise relation is small too (see Eq. 34). Therefore the factor

$$\frac{\Theta^2}{(1 + \Theta)^2}$$

renders more difficult the boiling detection by the value of γ_2.

But in the same case – at the low frequency of elementary events – γ_{2B} has its peak value, the difference from zero is maximal (see Eq. 39-41).

Regarding to this counteracting effects, one has to identify experimentally the smallest bubble generation intensity, when the relative flatness of the detected acoustic signal APD function might be unambiguous criteria of boiling detection.

According to the abovementioned peculiarities, the γ_2 criteria is not applicable for boiling detection even if the frequency of elementary acoustic events is very high, as according to Eq. 38 γ_2 will be zero. The upper limit of applicability of γ_2 for boiling detection must be determined experimentally too. The dependence of γ_2 on the Θ and $(\overline{\nu\Delta\tau})$ is shown on the *fig. 4*.

To illustrate these, some experimental results will be shown [26].

The measurements were carried out on a directly heated 10 mm I.D. 2.5 m long tube test section.

The subcooled boiling acoustic noise has been investigated for PWR parameter range, that is for ~ 100 bar pressure, ~ 2500 kg/m^2 sec mass flux, ~ 250 $^{\circ}$C inlet temperature. The piezo-electric detector has been fixed near to the outlet, the membran of it was in direct contact with the coolant. The signal, after recording to a tape recorder was filtered by a high-pass filter, and analysed by a HP analysator in order to obtain the signal APD function.

Here some experimentally gained APD functions will be shown. On the *Figure 6, 6,* only the positive side is given, and it is plotted log APD - versus $\tilde{p}_D^2$ for the sake of descriptiveness.

As it seems on the figures, the single phase flow acoustic noise has normal or Gauss amplitude distribution. But the APD function of subcooled boiling acoustic noise differs from the Gauss distribution.

At the first glance the figure, the subcooled boiling acoustic noise appears to be a sum of two processes, but one of them doesn't cover the time axis only periodically exists.

The acoustic noise that is, the series of elementary events is added to a continuous ever present background noise, but it doesn't cover the time axis at boiling initiation.

It is obvious, that the relative flatness of boiling acoustic noise APD function will differ from zero in the given cases.

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APPENDIX

Let us assume, that the detected signal is a superposition of two stochastic processes. One of them, we denoted with $\tilde{p}_N(t)$, represents the background noise, the other, denoted $\tilde{p}_B(t)$, represents the noise component being connected with the given physical phenomenon, that is:

$$\tilde{p}_D(t) = \tilde{p}_N(t) + \tilde{p}_B(t). \tag{A.1}$$

Let $\tilde{p}_D(t)$, $\tilde{p}_N(t)$ and $\tilde{p}_B(t)$ be stationary and ergodic. Let us accept $\tilde{p}_N(t)$ and $\tilde{p}_B(t)$ being statistically independent of each other, that is in case of an ansamble in any point of time, t_i, the following equation is valid for the probability density function:

$$\varphi\{\tilde{p}_N(t_i);\ \tilde{p}_B(t_i)\} = \varphi\{\tilde{p}_N(t_i)\}\varphi\{\tilde{p}_B(t_i)\}. \tag{A.2}$$

The moments are:

$$\left.\begin{array}{ll} \mu_{1N} = 0 & \\ \mu_{2N} = \sigma_N^2 & \text{exists and finite} \\ \mu_{3N} = 0 & \\ & \text{exists and finite} \\ \mu_{4N} & \end{array}\right\} \tag{A.3}$$

We do not have to make a specal restriction for the APD function of $\tilde{p}_N(t)$.

The impulse-like component, $\tilde{p}_B(t)$ is to be imagined as a series of certain impulse-like events in the time:

$$\tilde{p}_B(t) = \sum_k \tilde{p}_{Ek}(t-t_k) \tag{A.4}$$

where t_k are the points of time following the Poisson distribution with parameter $\bar{\nu}$; these are the points of time when the certain impulse-like events ensue;

$\tilde{p}_{Ek}(t)$ is the process in time of the k-th event.

We don't put restrictions on the functions $\tilde{p}_{Ek}(t)$, these are always determined by the given physical process.

Let $\tilde{p}_{Ek}(t)$ and $\tilde{p}_{E\ell}(t)$ be independent of each other by pairs at the same time, i.e. it is valid for the segment of the ansamble taken in the point of time, t_i, that:

$$\varphi\{\tilde{p}_{Ek}(t_i);\ \tilde{p}_{E\ell}(t_i)\} = \varphi\{\tilde{p}_{Ek}(t_i)\}\varphi\{\tilde{p}_{E\ell}(t_i)\}. \tag{A.5}$$

Let it be true besides, that

$$\left.\begin{aligned} &\langle\tilde{p}_{Ek}(t)\rangle = 0 \\ &\langle\tilde{p}^2_{Ek}(t)\rangle = \sigma^2_{E,k} = \mu_{2E,k} \\ &\langle\tilde{p}^3_{Ek}(t)\rangle = 0 \\ &\langle\tilde{p}^4_{Ek}(t)\rangle = \mu_{4Ek} \end{aligned}\right\} \qquad (A.5a)$$

where $\tilde{p}^r_{Ek}(t)$ is the average of the ansamble

$$\frac{1}{\Delta\tau_k}\int_{t_k}^{t_k+\Delta\tau_k} \tilde{p}_{Ek}(t)\,dt \qquad (A.5b)$$

and t_k is the beginning of the k-th elementary event;
$\Delta\tau_k$ is the time of life ot the k-th elementary event.

Let us interpret the average of the moments of the elementary events as below:

$$\left.\begin{aligned} &\langle\mu_{2Ek}\rangle = \mu_{2E} = \sigma^2_E \\ &\langle\mu_{4Ek}\rangle = \mu_{4E} \end{aligned}\right\} \qquad (A.6)$$

THE CENTRAL MOMENTS OF $\tilde{p}_D(t)$

By using the ergodicity we can write the r-th central moment on the basis of the equation:

$$\mu_r = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} [\tilde{p}_D(t)]^r\,dt. \qquad (A.7)$$

Here we have considered besides, that the mean of $\tilde{p}_D(t)$ is zero.
We shell use the above formula in a modified form, namely as it follows:

$$\lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} [\tilde{p}_D(t)]^r\,dt = \left\langle\frac{1}{T}\int_{0}^{T} [\tilde{p}_D(t)]^r\,dt\right\rangle. \qquad (A.8)$$

On the basis of the accepted assumptions and the formula we get the following for the central moments of $\tilde{p}_D(t)$:

A) $\mu_1 = 0$ as it derives from the assumptions.

B) According to the definition and using A.1.:

$$\left\langle \frac{1}{T} \int_0^T [\tilde{p}_N(t) + \tilde{p}_B(t)]^2 \, dt \right\rangle =$$

$$= \underbrace{\left\langle \frac{1}{T} \int_0^T \tilde{p}_N^2(t)dt \right\rangle}_{I.} + 2\underbrace{\left\langle \frac{1}{T} \int_0^T \tilde{p}_N(t) \cdot \tilde{p}_B(t)dt \right\rangle}_{II.} + \underbrace{\left\langle \frac{1}{T} \int_0^T \tilde{p}_B^2(t)dt \right\rangle}_{III.}. \qquad (A.10)$$

Let us define the value of the parts I-III.

ad I.

$$\left\langle \frac{1}{T} \int_0^T \tilde{p}_N^2(t)dt \right\rangle = \mu_{2N} = \sigma_N^2. \qquad (A.11)$$

ad II.

$$\left\langle \frac{1}{T} \int_0^T \tilde{p}_N'(t)\tilde{p}_B(t)dt \right\rangle = 0. \qquad (A.12)$$

These can be easily seen considering the condition A.2, and the assumptions concerning the expectation values of $\tilde{p}_N(t)$ and $\tilde{p}_B(t)$.

ad III.

$$\left\langle \frac{1}{T} \int_0^T \left(\sum_k \tilde{p}_{Ek}(t-t_k) \right)^2 dt \right\rangle =$$

$$= \left\langle \frac{1}{T} \int_0^T \left(\sum_k \sum_\ell^k \tilde{p}_{Ek}(t-t_k)\tilde{p}_{E(k-\ell)}(t-t_{k-\ell}) \right) dt \right\rangle. \qquad (A.13)$$

Let us use condition A.5, so in every case when $k \neq \ell$ in the formula

$$\left\langle \frac{1}{T} \int_0^T \left(\sum_k \sum_\ell^k \tilde{p}_{Ek}(t-t_k)\tilde{p}_{E(k-\ell)}(t-t_{k-\ell}) \right) dt \right\rangle$$

we get zero, that is instead of A.13 it remains

$$\left\langle \frac{1}{T} \int_0^T \left(\sum_m \tilde{p}_{Em}^2(t-t_m) \right) dt \right\rangle. \qquad (A.14)$$

We change the succesion of integration and summation in A.14.

$$\left\langle \frac{1}{T} \sum_m \int_0^T \tilde{p}_{Em}^2(t-t_m)dt \right\rangle. \qquad (A.15)$$

Because the span of life of the elementary event $\tilde{p}_{Em}(t)$ is $\Delta\tau_m$, that is why

$$\int_0^T \tilde{p}_{Em}^2(t-t_m)dt = \int_{t_m}^{t_m+\Delta\tau_m} \tilde{p}_{E_m}^2(t-t_m)dt = E_{E,m} \qquad (A.16)$$

the right hand side of the formula is equal to the "energy" of the elementary event $\tilde{p}_{Em}$.

On the basis of A.5a and A.5b

$$E_{E,m} = \Delta\tau_m \cdot \sigma_{Em}^2.$$

Considering the condition A.6 in addition to this it is valid for

$$\left\langle \frac{1}{T} \sum_m \int_0^T p_{E,m}^2(t-t_m)dt \right\rangle = \bar{\nu}\ \overline{\Delta\tau}\ \sigma_E^2 \qquad (A.17)$$

whete $\overline{\Delta\tau}$ is the average span of life of elementary events.

On the basis of the abovementioned we get the following value for the second central moment:

$$\mu_2 = \mu_{2N} + \mu_{2B} = \sigma_N^2 + \bar{\nu}\ \overline{\Delta\tau} \cdot \sigma_E^2 \qquad (A.18)$$

where $\bar{\nu}\ \overline{\Delta\tau}\cdot\sigma_E^2$ is the second central moment of $\tilde{p}_B(t)$, i.e.

$$\sigma_B^2 = \mu_{2B} = \bar{\nu}\ \overline{\Delta\tau}\ \sigma_E^2 = \bar{\nu}\ \bar{E}_E. \qquad (A.19)$$

C) The third central moment of $\tilde{p}_D(t)$ will be automatically zero on the basis of the introduced conditions.

D) The fourth central moment, or the relative flatness: According to the equations A.1, A.4 and A.8 the fourth central moment will be equal to the formula below:

$$\left\langle \frac{1}{T} \int_0^T [\tilde{p}_N(t) + \tilde{p}_B(t)]^4 dt \right\rangle = \underbrace{\left\langle \frac{1}{T} \int_0^T \tilde{p}_N^4(t)dt \right\rangle}_{a)} +$$

$$+ 4\underbrace{\left\langle \frac{1}{T} \int_0^T \tilde{p}_N^3(t)\tilde{p}_B(t)dt \right\rangle}_{b)} + 4\underbrace{\left\langle \frac{1}{T} \int_0^T \tilde{p}_N(t)\tilde{p}_B^3(t)dt \right\rangle}_{c)} + \qquad (A.20)$$

$$+ 6\underbrace{\left\langle \frac{1}{T} \int_0^T \tilde{p}_N(t)\tilde{p}_B(t)dt \right\rangle}_{d)} + \underbrace{\left\langle \frac{1}{T} \int_0^T \tilde{p}_B^4(t)dt \right\rangle}_{e)}.$$

Let us determine the value of the parts a), b), c), d) and e).

a)

$$\left\langle \frac{1}{T} \int_0^T \tilde{p}_N^4(t)dt \right\rangle = \mu_{4N}. \qquad (A.21)$$

The parts b) and c) give zero according to A.2, A.3, A.5a.

d) This part can be written according to A.2 as follows:

$$6\left\langle\frac{1}{T}\int_{0}^{T}\tilde{p}_N^2(t)\tilde{p}_B(t)dt\right\rangle = 6\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\tilde{p}_N^2\cdot\tilde{p}_B^2\varphi\{\tilde{p}_N;\ \tilde{p}_B\}d\tilde{p}_N d\tilde{p}_B =$$

$$= 6\left\langle\frac{1}{T}\int_{0}^{T}\tilde{p}_N^2(t)dt\right\rangle\left\langle\frac{1}{T}\int_{0}^{T}\tilde{p}_B^2(t)dt\right\rangle. \tag{A.22}$$

This will be equal to the following formula on the basis of A.19 and A.3:

$$6\ \mu_{2N}\cdot\mu_{2B} = 6\ \sigma_N^2\cdot\bar{\nu}\ \overline{\Delta\tau}\cdot\sigma_E^2. \tag{A.23}$$

The part denoted by e) is actually a formula of the fourth central moment of $\tilde{p}_B(t)$, i.e.:

$$\mu_{4B} = \left\langle\frac{1}{T}\int_{0}^{T}\tilde{p}_B^4(t)dt\right\rangle = \left\langle\frac{1}{T}\int_{0}^{T}\left(\sum_m \tilde{p}_{Em}(t-t_m)\right)^4 dt\right\rangle. \tag{A.24}$$

As a result of raising to a power, behind the sign of integration we shall have the sum of parts of the following type:

$$\tilde{p}_{E,k}(\cdot)\cdot\tilde{p}_{E,\ell}(\cdot)\cdot\tilde{p}_{E,m}(\cdot)\ \tilde{p}_{E,n}(\cdot).$$

If $n\neq\ell\neq m\neq k$, it can be seen on the basis of the condition A.5 that:

$$\left\langle\frac{1}{T}\int_{0}^{T}\tilde{p}_{Ek}(t-t_k)\tilde{p}_{E\ell}(t-t_\ell)\tilde{p}_{Em}(t-t_m)\tilde{p}_{En}(t-t_n)dt\right\rangle = 0$$

In the same way give the $\tilde{p}_{E,k}^3(\cdot)\tilde{p}_{E,\ell}(\cdot)$ /$k\neq\ell$/ - type terms zero after completing the assigned operations.

According to our assumptions the following terms give results other than zero:

$$\tilde{p}_{E,k}^4(\cdot) \quad\text{and}\quad \tilde{p}_{Ek}^2(\cdot)\cdot\tilde{p}_{E\ell}^2(\cdot)$$

i.e.

$$\left\langle\frac{1}{T}\int_{0}^{T}\left(\sum_k \tilde{p}_{Ek}^4(t-t_k)\right)dt\right\rangle \tag{A.25}$$

and

$$\left\langle\frac{1}{T}\int_{0}^{T}\left(\sum_{k,\ell}\tilde{p}_{Ek}^2(t-t_k)\tilde{p}_{E\ell}^2(t-t_\ell)\right)dt\right\rangle. \tag{A.26}$$

We get simply for the formula A.25.

$$\left\langle\frac{1}{T}\int_{0}^{T}\sum_k \tilde{p}_{Ek}^4(t)dt\right\rangle = \frac{1}{T}\left\langle\int_{t_1}^{t_1+\Delta\tau_1}\tilde{p}_{E1}^4(t-t_1)dt + \int_{t_2}^{t_2+\Delta\tau_2}\tilde{p}_{E2}^4(t-t_2)dt+\ldots+\int_{t_k}^{t_k+\Delta\tau_k}\tilde{p}_{Ek}^4(t-t_k)dt+\ldots\right\rangle \tag{A.27}$$

Since we have defined $\mu_{4E,k}$ according to A.5b so A.27 will have the following form

$$\frac{\bar{N}}{T}\left\langle \Delta\tau_k \cdot \mu_{4E,k}\right\rangle \tag{A.28}$$

where $\bar{N}$ is the average number of the events ensuring during the time interval [o, T].

We can rewrite the formula A.28 considering A.6 and according to the interpretation of the parameter of the Poisson distribution in the following form:

$$\bar{\nu}\ \overline{\Delta T}\ \mu_{4E}. \tag{A.29}$$

While analysing the formula A.26 let us consider the condition A.5, so the sum consisting of the following type terms

$$\left\langle \frac{1}{T}\int_{o}^{T} \tilde{p}^2_{Ek}(t-t_k)\cdot\tilde{p}^2_{E\ell}(t-t_\ell)dt\right\rangle \qquad k \neq \ell$$

can be written in the form below:

$$\iint_{-\infty}^{\infty} \tilde{p}^2_{Ek}\cdot\tilde{p}^2_{E\ell}\varphi\{\tilde{p}_{Ek};\ \tilde{p}_{E\ell}\}d\tilde{p}_{Ek}d\tilde{p}_{E\ell} \tag{A.30}$$

that is further equal to the product formula of

$$\left\langle \frac{1}{T}\int_{o}^{T} \tilde{p}^2_{Ek}(t-t_k)dt\right\rangle \left\langle \frac{1}{T}\int_{o}^{T} \tilde{p}^2_{E\ell}(t-t_\ell)dt\right\rangle \tag{A.31}$$

according to A.5 and A.8, or moreover to the formula

$$\frac{1}{T^2}\left\langle \int_{t_k}^{t_k+\Delta\tau_k} \tilde{p}^2_{Ek}(t-t_k)dt\right\rangle \left\langle \int_{t_\ell}^{t_\ell+\Delta\tau_\ell} \tilde{p}^2_{E\ell}(t-t_\ell)dt\right\rangle \tag{A.32}$$

which is equal to

$$\frac{1}{T^2}\left\langle \Delta\tau_k\ \sigma^2_{Ek}\right\rangle \left\langle \Delta\tau_\ell\ \sigma^2_{E\ell}\right\rangle \tag{A.33}$$

considering A.5b and A.6.

Since in A.26 the number of the $\tilde{p}^2_{Ek}(\cdot)\tilde{p}^2_{E\ell}(\cdot)/k\neq\ell/$ - type terms is 3N(N-1), in case of $N \gg 1$ according to the already applied considerations (see A.28, A.29) A.33 equals to the following formula:

$$3\ \bar{\nu}^2\ \overline{\Delta\tau}^2\ \sigma^4_E. \tag{A.34}$$

Accordingly, the fourth central moment of $\tilde{p}_B(t)$ can be written as follows:

$$\mu_{4B} = \bar{\nu}\ \overline{\Delta\tau}\ \mu_{4E} + 3\bar{\nu}^2\overline{\Delta\tau}^2\ \sigma_E^4 \tag{A.35}$$

or

$$\mu_{4B} = \bar{\nu}\ \overline{\Delta\tau}\ \mu_{4E} + 3\bar{\nu}^2\overline{\Delta\tau}^2\ \mu_{2E}^2. \tag{A.35a}$$

According to the central limit theorem the process $\tilde{p}_B(t)$ converges to a normal or Gauss process while $\bar{\nu}\to\infty$ - or otherwise in case of a sufficiently dense overlapping of the elemantary events. This marginal case has to be shown by the parameters of distribution determined for $\tilde{p}_B(t)$. As it is known, in case of a normal distribution the following relation is valid between the fourth and the second central moment

$$\frac{\mu_4}{\mu_2^2} = 3 \tag{A.36}$$

that is, in our case

$$\lim_{\bar{\nu}\to\infty} \frac{\bar{\nu}\ \overline{\Delta\tau}\cdot\mu_{4E} + 3\bar{\nu}^2\overline{\Delta\tau}^2\mu_{2E}^2}{(\bar{\nu}\ \overline{\Delta\tau}\mu_{2E})^2} = 3 \tag{A.37}$$

as it was expected.

Thus the wanted value of the fourth central moment of the process

$$\begin{aligned}\mu_4 &= \mu_{4N} + 6\mu_{2N}\mu_{2B} + \mu_{4B} = \\ &= \mu_{4N} + 6\mu_{2N}(\bar{\nu}\ \overline{\Delta\tau}\mu_{2E}) + \bar{\nu}\ \overline{\Delta\tau}\mu_{4E} + 3\bar{\nu}^2\overline{\Delta\tau}^2\mu_{2E}^2\ .\end{aligned} \tag{A.38}$$

We can formulate the relative flatness of the amplitude probability density function of the resultant process

$$\tilde{p}_D(t) = \tilde{p}_N(t) + \tilde{p}_B(t)$$

with full knowledge of the moments.

According to the definition the relative flatness is

$$\gamma_2 = \frac{\mu_4}{\mu_2^2} - 3. \tag{A.39}$$

On the basis of this in case of the studied process

$$\gamma_2 = \frac{\mu_{4N} + 6\mu_{2N}\mu_{2B} + \mu_{4B}}{(\mu_{2N} + \mu_{2B})^2} - 3. \tag{A.40}$$

The quantity γ_2 can be expressed by means of the signal to noise ratio

$$\Theta = \frac{\mu_{2B}}{\mu_{2N}} = \frac{\bar{\nu}\ \overline{\Delta\tau}\ \mu_{2E}}{\mu_{2N}} = \frac{\sigma_B^2}{\sigma_N^2} \tag{A.41}$$

as follows:

$$\gamma_2 = \frac{\gamma_{2N} + \gamma_{2B}\,\theta^2}{(1 + \theta)^2} = \frac{\gamma_{2N} + \frac{1}{\bar{\nu}\,\overline{\Delta\tau}}(\gamma_{2E} + 3)\theta^2}{(1 + \theta)^2} \tag{A.42}$$

where γ_{2N} and γ_{2B} according to A.39 is the relative flatness concerning $\tilde{p}_N(t)$ and $\tilde{p}_B(t)$ respectively.

In the case of normal background noise

$$\gamma_2 = \frac{1}{\bar{\nu}\,\overline{\Delta\tau}}(\gamma_{2E} + 3)\frac{\theta^2}{(1 + \theta)^2} = \gamma_{2B}\frac{\theta^2}{(1 + \theta)^2} . \tag{A.43}$$

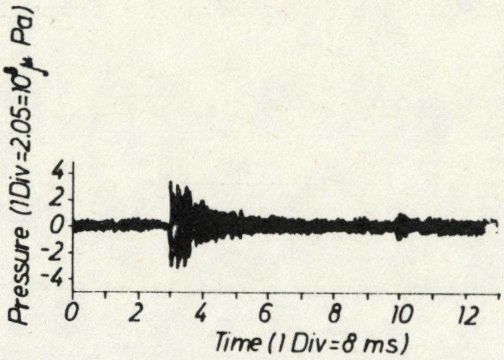

Fig. 1. (Ref [1])

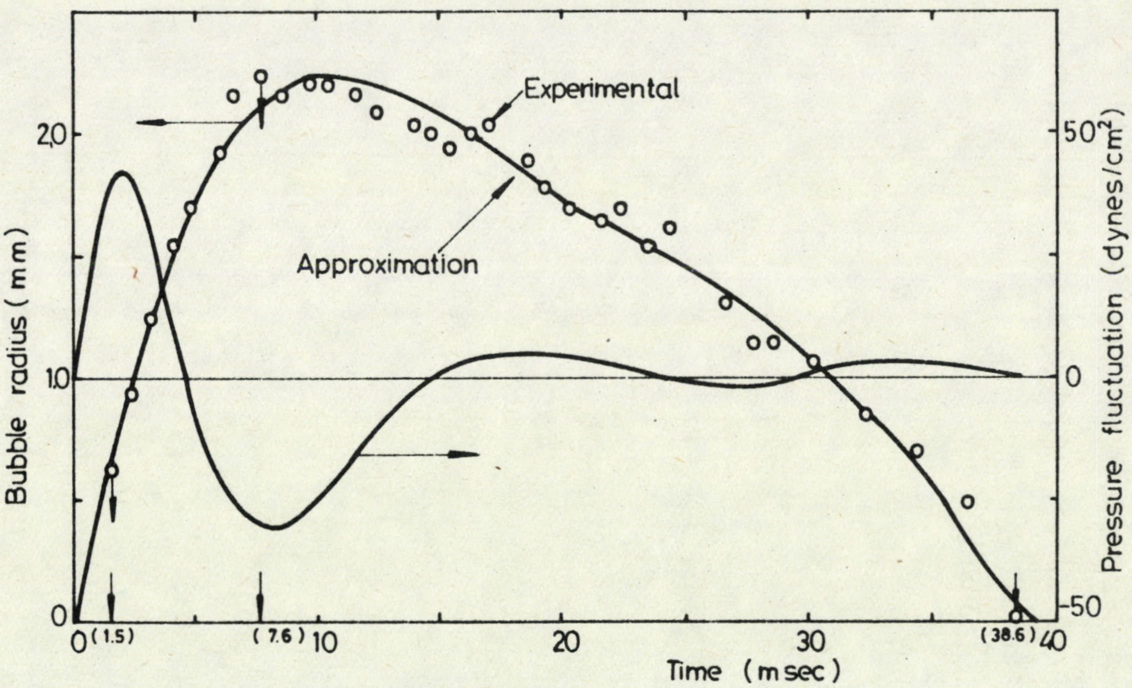

Fig. 2. Bubble history and associated pressure fluctuation (**Ref [2]**)

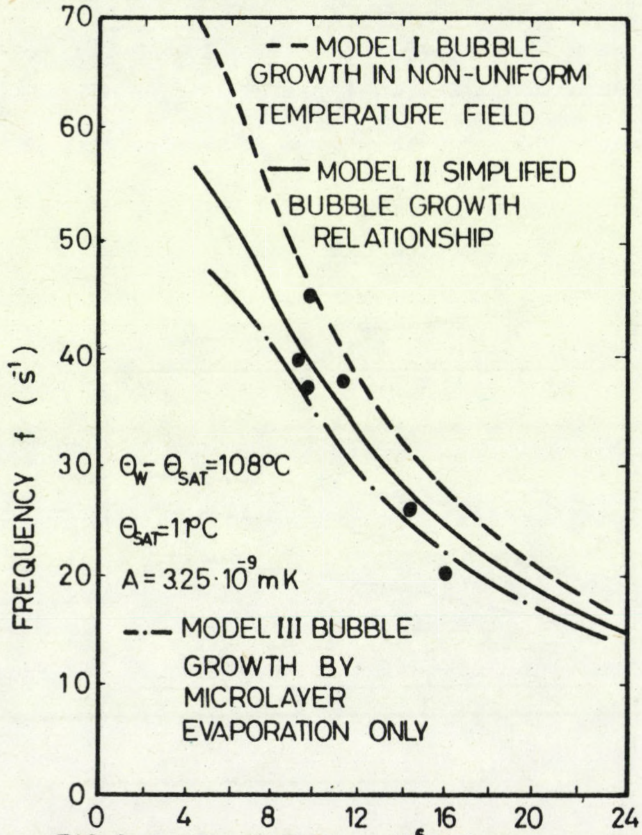

FIG. 3. CAVITY RADIUS $r_c \cdot 10^{6}$ (m)
COMPARISON BETWEEN EXPERIMENT AND THEORY FOR WATER BOILING ON A FINE MACHINED SURFACE (REF [22])

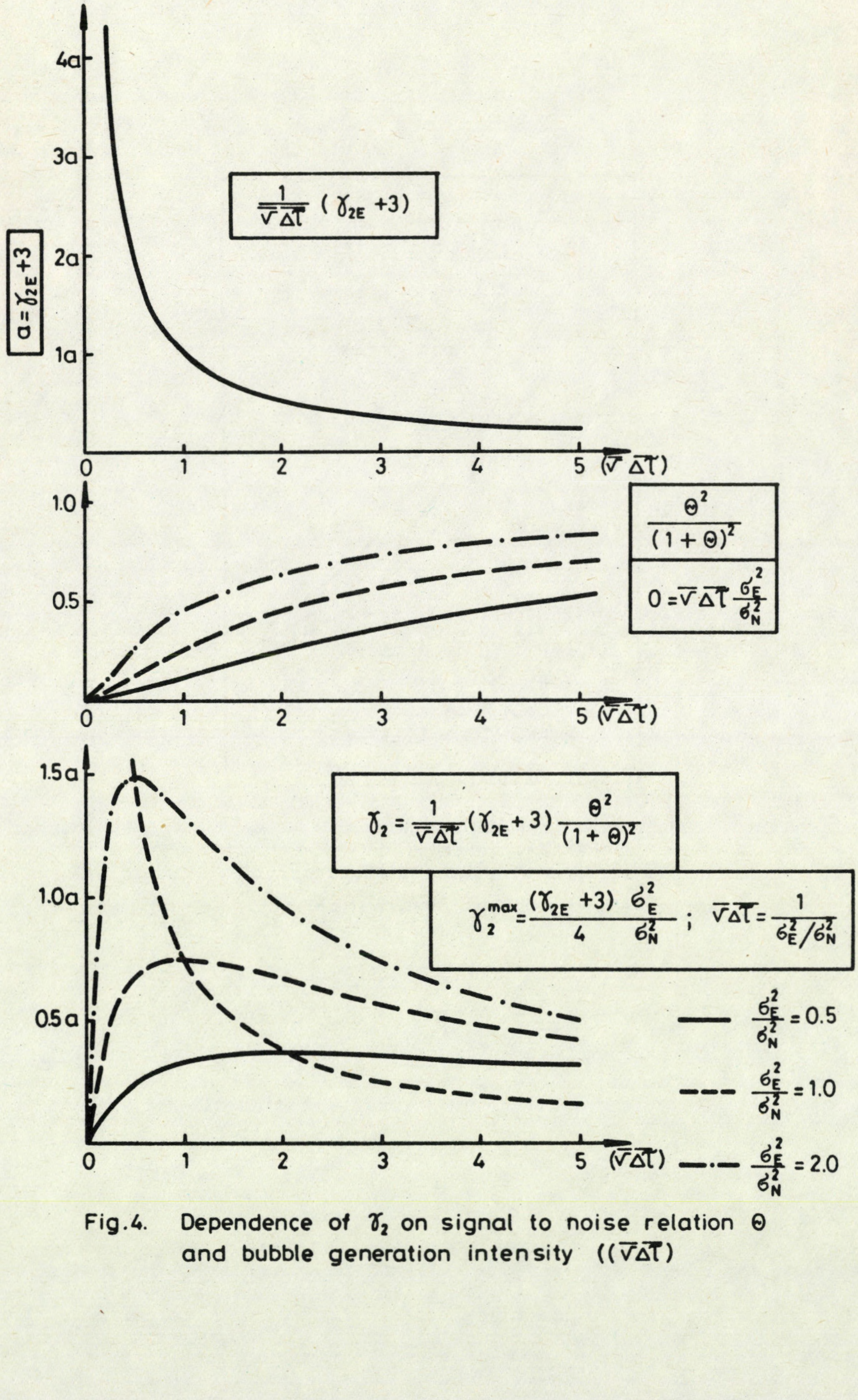

Fig.4. Dependence of γ_2 on signal to noise relation Θ and bubble generation intensity $((\overline{v}\,\overline{\Delta\tau})$

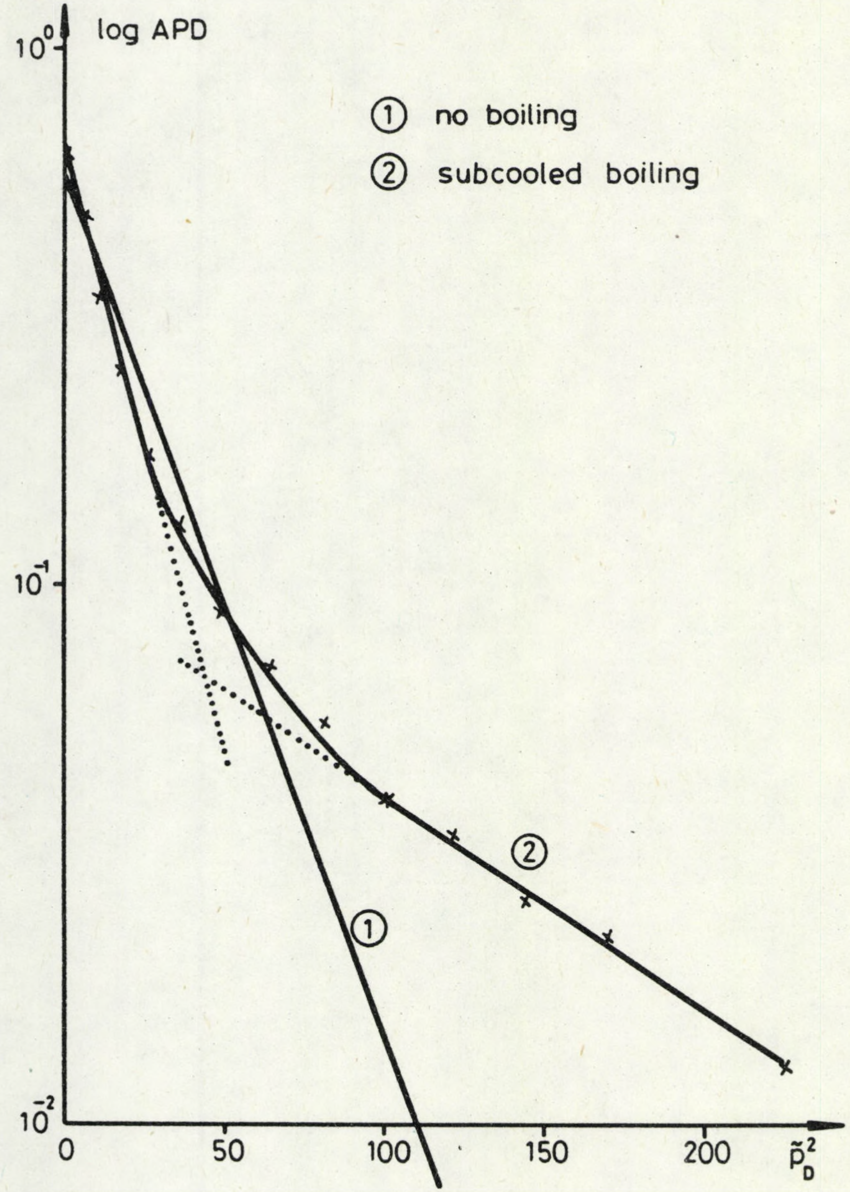
log APD
10^0
10^{-1}
10^{-2}
① no boiling
② subcooled boiling
①
②
0
50
100
150
200
$\tilde{p}_D^2$

Fig. 5.

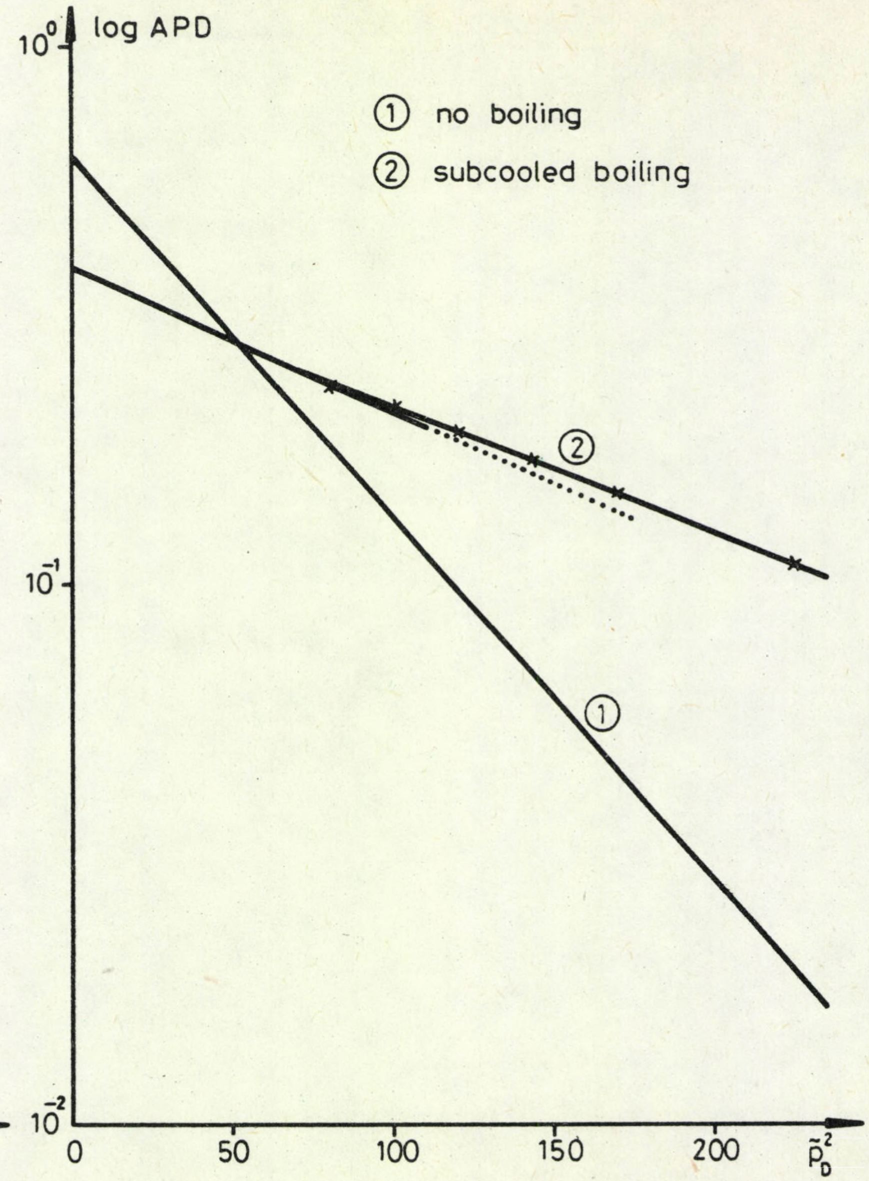
log APD
10^0
10^{-1}
10^{-2}
① no boiling
② subcooled boiling
①
②
0
50
100
150
200
$\tilde{p}_D^2$

Fig. 6.

Kiadja a Központi Fizikai Kutató Intézet
Felelős kiadó: Gyimesi Zoltán
Szakmai lektor: Vigassy József
Nyelvi lektor: Egeli György
Gépelte: Végvári Istvánné
Példányszám: 285 Törzsszám: 81-113
Készült a KFKI sokszorositó üzemében
Felelős vezető: Nagy Károly
Budapest, 1981. julius hó