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ABSTRACT

Force-field theory is studied in 3-dimension using the full diffusion tensor incorporating drift effects. An analytical approximate solution is deduced under some assumptions which include a flat neutral sheet and a non-uniform density distribution at the outer boundary. By contrast with the usual force-field theory, our solution gives a large and charge dependent latitudinal gradient and near perfect isotropy, even corotation disappears. The results are in general agreement with the numerical calculations of Jokipii and Kopriva.

АННОТАЦИЯ

Изучается теория "форс-филд" модуляции галактических космических лучей в трехмерном пространстве, используя полный тензор диффузии, включающий и дрейфовые эффекты. Выведено аналитическое решение при предположении плоского межпланетного нейтрального слоя и неоднородного распределения космических лучей на границе объема модуляции. Вопреки обычному решению дается крупный гелиоширотный градиент, зависящий от заряда частиц, и полная изотропия с отсутствием коротации. Наши результаты согласуются с расчетами, проведенными Иокипии и Копривой.

KIVONAT

Az erőtér közelitést vizsgáljuk 3 dimenzióban. A teljes diffuziós tenzort használjuk, vagyis a drift-hatásokat is figyelembe vesszük. Analitikus közelitő megoldást vezetünk le bizonyos feltevések mellett: a bolygóközi semleges réteget siknak vesszük, továbbá feltesszük, hogy a kozmikus sugárzás sürüségeloszlása nem egyenletes a modulációs tartomány külső határán. A szokásos erőtér közelitéstől eltérően az általunk kapott megoldás nagy és töltéstől függő zenit irányu sürüséggradienst és teljes izotrópiát ad, az együttforgási anizotrópia is eltünik. Eredményeink - fő vonásait tekintve - összhangban vannak Jokipii és Kopriva numerikus számitásaival.

INTRODUCTION

It has been known for some time that galactic cosmic ray transport in the heliosphere cannot be treated as spherically symmetric. The importance of curvature and gradient drifts has been pointed out and discussed in detail in a series of works by the University of Arizona group [1]-[6]. The concept of drift is neither new nor is it 'ad hoc' introduced into the modulation theory: it is incorporated in the antisymmetric term of the diffusion tensor -- in the term that has incorrectly been disregarded earlier. Since, at least at the GeV energies, drift is capable to provide considerable particle transport across the magnetic field lines an ambitious 3-dimensional calculation clearly has to operate with the full diffusion tensor. Such numerical calculations have been carried out by Jokipii and Kopriva [6] and Gleeson et al. [7]. The predictions of the two works are at variance due to the different boundary conditions used at the solar equator.

In the light of these recent developments it may be worth asking how force-field theory will change if the full diffusion tensor is used i.e. drift is included. The force-field solution derived by Gleeson and Axford [8] has been the most successful analytical approximate solution to the modulation equation. It is, however, essentially one-dimensional in the sense that it applies under the condition of either spherical symmetry or strictly field-aligned diffusion -- in both cases only one spatial co-ordinate enters the calculations. Thus, a modification due to drift would not be surprising.

In this work, we deduce a 3-dimensional force-field solution under several simplifying assumptions. Among these the most important is the azimuthal symmetry i.e. a flat interplanetary neutral sheet which coincides with the solar equator. Of course, the real neutral sheet is wavy, and this waviness may have profound effects in producing the 11-year variation (e.g. Kóta [9], Jokipii and Thomas [10], Tverskoi [11]). The effect of a wavy neutral sheet is, however, beyond the scope of this paper. A particular feature of the calculations to be presented is that, at the outer boundary of the modulation region, we set a non-uniform density distribution imposed by the exterior electric field as suggested by Jokipii and Levy [12].

INTERPLANETARY MAGNETIC AND ELECTRIC FIELD

We use an azimuthally symmetric Parker-spiral Interplanetary Magnetic Field (IMF) within a sphere of radius R:

$$\tilde{B} = B_o(|\Theta|)(2H(\Theta)-1)\left(\frac{a}{r}\right)^2\left[\hat{e}_r - \frac{\Omega r \cos\Theta}{V}\hat{e}_\varphi\right] , \qquad (1)$$

where B_o is the radial field strength at a = 1 AU, $\Omega=3\cdot10^{-6}$ $\sec^{-1}$ is the angular velocity of the sun, V is the solar wind speed. r, Θ and φ represent heliocentric radius, solar latitude and longitude, respectively; $\hat{e}_r$, $\hat{e}_\Theta$, $\hat{e}_\varphi$ stand for the unit vectors pointing along respective directions. H is Heaviside step function. B_o is assumed to be symmetric with respect to the neutral sheet at $\Theta = 0$. The sign of B_o reverses at the polarity reversal of the sun: B_o is positive for the 1969-80 solar cycle and negative for the previous cycle.

The appropriate electric field, $\tilde{E} = (\tilde{B}\times\tilde{V})/c$, can be derived from a scalar potential, Φ, since the magnetic field is steady state.

$$\tilde{E} = B_o(|\Theta|)(2H(\Theta)-1)\cdot\frac{a}{r}\,\frac{\Omega a \cos\Theta}{c}\hat{e}_\Theta = -\,\mathrm{grad}\Phi \qquad (2)$$

with

$$\Phi(\Theta) = -\frac{a^2\Omega}{c}\int_0^{|\Theta|} d\Theta' B_o(\Theta')\cos\Theta' + \Phi_o , \qquad (3)$$

where c is the velocity of light and Φ_o is an additive constant. As seen from equation (3) Φ is even function of Θ.

The potential Φ applies for $r < R$, the field exterior to $r = R$ has been calculated by Jokipii and Levy [12] for various models. Here, we take the simplest case i.e. that of a completely neutralized plasma beyond $r = R$. We need not use the actual form of the exterior field, it is enough to know that Φ should be continuous at $r = R$. Then, the boundary condition at $r = R$ is directly obtained from Liouville's theorem:

$$F(T, r=R, \Theta) = F_\infty(T + Ze\Phi(\Theta)), \tag{4}$$

where F is the particle distribution in phase space, T is kinetic energy. F_∞ refers to the undisturbed galactic spectrum. Here we set Φ_o so that the potential be zero at infinity.

The value of Φ_o is

$$\Phi_o = \frac{a^2\Omega}{c}\int_0^{\pi/2} d\Theta B_o(\Theta)\cos\Theta(1-\sin\Theta) - \frac{Q}{R} \quad , \tag{5}$$

where Q is the net charge of the solar system.

FORCE-FIELD SOLUTION

We follow the line of the force-field theory [8] according to which the net particle streaming

$$S_i = -p^2\left(K_{ij}\frac{\partial F}{\partial x_j} + \frac{p}{3}\frac{\partial F}{\partial p}V_i\right) \tag{6}$$

can be taken as divergence-free

$$\mathrm{div}\tilde{S} = -\frac{1}{3}\frac{\partial}{\partial p}(p^3\tilde{V}\mathrm{grad}F) \approx 0 \quad , \tag{7}$$

where p is the particle momentum and K_{ij} is the diffusion tensor.

The following further simplifying assumptions will be made:

(i) The BGK relaxation time approximation of scattering process [13] is used which gives the inverse diffusion tensor as

$$K^{-1}_{ij} = \frac{3}{v}\left[\frac{1}{\lambda}\delta_{ij} - \epsilon_{ijk}\frac{Ze}{pc}B_k\right] \tag{8}$$

where λ is the mean free path while v and Ze are the particle velocity and electric charge, respectively.

(ii) Separable diffusion tensor is assumed. This demands that λ be proportional to the momentum, p.

(iii) The mean free path and the radial solar wind are spherically symmetric:

$$\lambda = \lambda_1(r)(p/p_o) \quad \text{and} \quad \tilde{V} = V(r)\hat{e}_r$$

(iv) Reflecting inner boundary is taken. This -- though may be unrealistic -- is expected to give minor effect on the cosmic ray distribution everywhere but near the sun.

Under assumptions (i)-(iv) and boundary condition (4) the solution to the force-field equations (6) and (7) is

$$S_i = 0 \qquad \text{instead of merely} \qquad S_r = 0 \tag{9}$$

and

$$\frac{\partial F}{\partial x_i} = -\frac{p}{3}\frac{\partial F}{\partial p}K^{-1}_{ij}V_j = -\frac{\partial F}{\partial T}\left[\frac{\tilde{V}}{\lambda_1}p_o + Ze\tilde{E}\right] \tag{10}$$

whence

$$F(T,r,\Theta) = F_\infty\left(T + \int_r^R (Vp_o/\lambda_1)\,dr' + Ze\Phi(\Theta)\right) \tag{11}$$

In deriving (10) and (11) we made use of $\tilde{E} = (\tilde{B}\times\tilde{V})/c$ and relation (8). It should be pointed out that S = 0 i.e. isotropy is not a trivial solution to div$\tilde{S}$ = 0, it holds only if the vector $K^{-1}_{ij}V_j$ is a gradient-vector. This is not met in general.

DISCUSSION AND CONCLUSION

The 3-dimensional force-field solution obtained represents a very specific solution which relies upon a number of assumptions including a non-uniform boundary condition at r = R. If the exterior field is disregarded (11) will not hold in its form. Still, we may have a fair approximation replacing Φ in (11) by the potential difference between the observer and the place where particles entered the heliosphere; this latter is fairly well defined being around the solar polar or equatorial region depending on the sign of B_o [6], [7], [9].

The present work predicts a large and charge dependent latitudinal gradient in accordance with the results of Jokipii and Kopriva [6]. At the same time, also there are some deviations in the diffusion tensors and inner boundary conditions used in the two works.

The large-scale electric field is not directly connected with the rate of energy loss. Yet, surprisingly it appears explicitely in (11). This expresses that drift is governed by the large-scale IMF which, in turn, is associated with the electric field.

A seemingly controversial result is the complete isotropy. Of course, we do not intend to deny corotational anisotropy. Corotation would probably appear if, violating azimuthal symmetry, a wavy interplanetary neutral sheet were considered. At least, wavy neutral sheet was shown to produce corotation at energies as high as ~50 GeV [14].

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