KFKI-75-35

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BUDAPEST

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LOW ENERGY BEHAVIOUR OF A ONE DIMENSIONAL FERMI MODEL WITH SHORT AND LONG RANGE INTERACTION

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Submitted to Solid State Communications

ISBN 963 371 037 5

ABSTRACT

A one dimensional Fermi model is investigated for a general four parameter interaction, containing both short and long range components. Using the second order renormalization approach it is shown that the type of interaction with all the four participating electrons around the same Fermi point has a drastic effect. It tends to suppress the superconducting type instability and favours a normal metal, antiferromagnetic or charge density wave state.

РЕЗЮМЕ

Исследована одномерная модель фермионов, в которой взаимодействие описывается 4-мя параметрами и, таким образом, учтены как короткодействующие, так и дальнодействующие силы. Проводя ренормализацию во втором порядке показано, что взаимодействие в случае, когда все четыре электрона находятся вблизи одной точки Ферми, приводит к существенному эффекту. При этом наблюдается тенденция исчезнования неустойчивости сверхпроводящего типа и основное состояние, повидимому, соответствует нормальному металлу, антиферромагнетику или пайерлсовскому состоянию.

KIVONAT

Egy dimenziós Fermi rendszert vizsgálunk, melyben a kölcsönhatást négy paraméter jellemzi és igy mind a rövid, mind a hosszu hatótávolságu részt figyelembe vesszük. Másodrendü renormálást használva megmutatjuk, hogy az a kölcsönhatási tipus, melynél mind a négy, a kölcsönhatásban résztvevő dektron ugyanazon Fermi pont közelében van, igen lényeges befolyással van a rendszer viselkedésére. Elnyomni igyekszik a szupravezető tipusu instabilitást és ugyanakkor kedvez a normál fém, antiferromágneses vagy Peierls állapot kialakulásának

The recent interest in the behaviour of quasi one dimensional highly conducting systems (like TCNQ salts) has led to an extensive study of the properties of the one dimensional interacting Fermi systems[1-11]. Various methods such as parquet diagram summation[1,2], renormalization group approach[3-7], exact Ward identities[8], Tomonaga boson transformation[9,10] and equation of motion method[11] have been used to investigate the Green functions and response functions for such systems.

Supposing that the interaction is important only for electrons with momenta lying near the Fermi surface, which here consists of two points, $\pm k_F$, and that the matrix elements are nearly independent of the momenta in a range characterized by a cut-off k_c around the Fermi momentum, the general form of the interaction Hamiltonian is

$$
\begin{aligned}
H_{int} = {} & \frac{g_1}{L} \sum_{k_i,\alpha,\beta} a^+_{k_1\alpha} b^+_{k_2\beta} a_{k_3\beta} b_{k_1+k_2-k_3\alpha} \\
& + \frac{g_2}{L} \sum_{k_i,\alpha,\beta} a^+_{k_1\alpha} b^+_{k_2\beta} b_{k_3\beta} a_{k_1+k_2-k_3\alpha} \\
& + \frac{g_3}{2L} \sum_{k_i,\alpha,\beta} \left(a^+_{k_1\alpha} a^+_{k_2\beta} b_{k_3\beta} b_{k_1+k_2-k_3-4k_F\alpha} + b^+_{k_1\alpha} b^+_{k_2\beta} a_{k_3\alpha} a_{k_1+k_2-k_3+4k_F\alpha} \right) \\
& + \frac{g_4}{2L} \sum_{k_i,\alpha,\beta} \left(a^+_{k_1\alpha} a^+_{k_2\beta} a_{k_3\beta} a_{k_1+k_2-k_3\alpha} + b^+_{k_1\alpha} b^+_{k_2\beta} b_{k_3\beta} b_{k_1+k_2-k_3\alpha} \right),
\end{aligned}
\tag{1}
$$

where $a^+_{k_i\alpha}$ and $b^+_{k_i\alpha}$ create electrons with momenta $k_F-k_c \leq k_i \leq k_F+k_c$ and $-k_F-k_c \leq k_i \leq -k_F+k_c$, respectively. The term with g_1 is a large momentum transfer interaction (back-scattering), the terms with g_2 and g_4 describe small momentum transfer, while the term with g_3 is effective only if the band is half-filled and it corresponds to Umklapp processes.

In the case $g_1=g_3=0$, the Hamiltonian reduces to that of the Tomonaga model, for which exact results are available[8,9]. The Hubbard model is equivalent to $g_1=g_2=g_3=g_4$. In the presence of the large momentum transfer interaction g_1 only a special case can be solved exactly[10] and in general we have to have recourse to some approximate treatment as in [1-7].

Until now there has been no attempt to investigate the one dimensional Fermi model with this general Hamiltonian. In the renormalization group treatments the long range interaction g_4 has been completely neglected, although its importance can be easily seen. Even if the bare coupling g_4 is zero, in higher orders g_1 and g_2 can generate g_4 type couplin. Therefore in a consequent renormalization procedure the invariant coupling of g_4 type has to be introduced.

From the exact treatment of the g_4 coupling it is known[8] that at very low energies a branch cut appears in the Green function, instead of the usual pole behaviour. The spin of the fermions is very important in this respect, as has been pointed out by Nozières[12]. In this work we will neglect this branch cut and will sum up logarithmic contributions only. This may modify our results at energies very near the Fermi energy, where our approximation is questionable anyway.

As we have shown earlier[3,4], first order renormalization in the renormalization group approach corresponds to summing up leading logarithmic corrections, while second order renormalization corresponds to considering next to leading logarithmic corrections as well and thereby to taking into account fluctuation effects.

We have performed second order renormalization for the general 1D Fermi model, and after a straightforward but tedious calculation we have got the Lie equations for the invariant couplings in the following form

$$\frac{\partial g_1'(x)}{\partial x} = \frac{1}{x}\left\{\frac{1}{\pi v} g_1'^2 + \frac{1}{2\pi^2 v^2}\left(g_1'^3 + g_1'^2 g_4'\right) + \dots\right\}, \tag{2}$$

$$\frac{\partial g_2'(x)}{\partial x} = \frac{1}{x}\left\{\frac{1}{2\pi v}\left(g_1'^2 - g_3'^2\right) + \frac{1}{4\pi^2 v^2}\left[g_1'^3 + g_1'^2 g_4' - \left(g_1' - 2g_2' - g_4'\right) g_3'^2\right] + \dots\right\} \tag{3}$$

$$\frac{\partial g_3'(x)}{\partial x} = \frac{1}{x}\left\{\frac{1}{\pi v}\left(g_1' - 2g_2'\right) g_3' + \frac{1}{4\pi^2 v^2}\left[\left(g_1' - 2g_2'\right)^2 - 2\left(g_1' - 2g_2'\right) g_4' + g_3'^2\right] g_3' + \dots\right\} \tag{4}$$

$$\frac{\partial g_4'(x)}{\partial x} = \frac{1}{x}\left\{\frac{1}{4\pi^2 v^2}\left[-3 g_1'^3 + 3\left(g_1' - 2g_2'\right) g_3'^2\right] + \dots\right\} \tag{5}$$

where $x = k_c'/k_c$ is the ratio of the cut-offs in the renormalized and original systems. For the definitions and notations used we refer to Refs. 3-4.

This system of equations can be best analyzed by taking the combination $g_1 - 2g_2$.

$$\frac{\partial(g_1' - 2g_2')}{\partial x} = \frac{1}{x}\left\{\frac{1}{\pi v} g_3'^2 + \frac{1}{2\pi^2 v^2}(g_1' - 2g_2' - g_4')g_3'^2 + \dots\right\} \qquad (6)$$

The solutions for x 0, i.e. the fixed points of the invariant couplings can be obtained from the zeros of the right hand sides of eqs. (2-6). This consideration gives a plane of fixed points and two isolated fixed points, namely

1.) $g_1'(0)=0$, $g_3'(0)=0$, $g_2'(0)$ and $g_4'(0)$ have non-universal value, (7)
2.) $g_1'(0)=-2\pi v$, $g_2'(0)=0$, $g_3'(0)=2\pi v$, $g_4'(0)=0$, (8)
3.) $g_1'(0)=-2\pi v$, $g_2'(0)=0$, $g_3'(0)=-2\pi v$, $g_4'(0)=0$. (9)

The domain of attraction of the plane of fixed points consists at least of the whole subspace $g_3=0$, i.e. the case when the band is not half-filled. In this case, starting from any value for the bare coupling g_1, it will always be renormalized to zero, in contrast to the case when g_4' has been neglected. This shows, that the inclusion of g_4', even if $g_4=0$, changes drastically the behaviour of the invariant couplings. This comes from the fact that, as can be seen from the the Lie equations (2-5), the large momentum transfer interaction g_1 can generate an effective g_4, which, in turn, will modify the behaviour of g_1' itself.

The importance of g_4' can be seen from comparing our results with that of Kimura[7]. He performed second order renormalization with g_1', g_2' and g_3' only (in his notation the corresponding invariant couplings are Ψ_1, Ψ_2, Ψ_3) and he got six fixed points (or better to say two lines of fixed points and four isolated fixed points). As we see, the inclusion of g_4' modifies the invariant couplings in such a way that only one plane of fixed points and two isolated fixed points will persist.

By determining the generalized susceptibilities corresponding to the charge density, spin density and Cooper pair fluctuations, the respective Lie equations are as follows:

$$\frac{\partial \ln \bar{N}(x)}{\partial x} = \frac{1}{x}\left\{ \frac{1}{\pi v}\left[2g_1' - g_2' + g_3'\right] + F(x) + \dots \right\}, \quad (10)$$

$$\frac{\partial \ln \bar{\chi}(x)}{\partial x} = \frac{1}{x}\left\{ -\frac{1}{\pi v}\left[g_2' + g_3'\right] + F(x) + \dots \right\}, \quad (11)$$

$$\frac{\partial \ln \bar{\Delta}(x)}{\partial x} = \frac{1}{x}\left\{ \frac{1}{\pi v}\left[g_1' + g_2'\right] + F(x) + \dots \right\}, \quad (12)$$

with
$$F(x) = \frac{1}{2\pi^2 v^2}\left[g_1'^2 - g_1' g_2' + g_2'^2 + \frac{1}{2} g_3'^2 \right],$$

where $\bar{N}$, $\bar{\chi}$ and $\bar{\Delta}$ are the auxiliary susceptibilities of the charge density, spin density and Cooper pair fluctuations as defined in Ref. 4. $x=\omega/\omega_D$ for these susceptibilities are calculated as a function of the energy variable ω and ω_D is the cut-off in energy space. The behaviour at $\omega \sim 0$ or at $T \sim 0$ is governed by the fixed point value of the invariant couplings. Inserting the results of eqs. (7)-(9) into eqs. (10)-(12), we get that

1.) when $g_1'(0)=0$ and $g_3'(0)=0$, the fixed point values of g_2' and g_4' are not universal, therefore the critical exponents of the susceptibilities are not universal either and no general statement can be made as to which of the susceptibilities will diverge,

2.) when $g_1'(0)=-2\pi v$, $g_3'(0)=2\pi v$ and $g_2'(0)=g_4'(0)=0$, none of the susceptibilities diverges and the system will tend to a normal metal ground state,

3.) when $g_1'(0)=-2\pi v$, $g_3'(0)=-2\pi v$ and $g_2'(0)=g_4'(0)=0$, the charge density fluctuation N diverges and the ground state of the system will be a period doubled charge density wave state.

The phase diagram, as can be expected from the above considerations is drastically changed by the inclusion of g_4'. In the last two cases the superconducting type instability is suppressed completely. In the first case it might appear depending on $g_2'(0)$, but even in this case the tendency for this type of instability is weakened because $g_1'(0)$ whose large value was mainly responsible for its appearance is now renormalized to zero.

The results obtained here are as yet tentative, because the fixed point values in the last two cases are of the order of unity, and so second order renormalization is not sufficient any more, the lower order logarithmic corrections being equally important. Nevertheless this calculation shows the importance of the coupling of g_4 type and the tendency that this coupling might suppress the superconducting instability and favour a normal metal, antiferromagnetic or charge density wave state.

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62220

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Készült a KFKI sokszorosító üzemében
Budapest, 1975. május hó