KFKI
6 / 1969

THEORY OF ELECTRON-ELECTRON INTERACTION AND SUPERCONDUCTIVITY IN DILUTE MAGNETIC ALLOYS ABOVE THE KONDO TEMPERATURE

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BUDAPEST

THEORY OF ELECTRON-ELECTRON INTERACTION AND SUPERCONDUCTIVITY IN DILUTE MAGNETIC ALLOYS ABOVE THE KONDO TEMPERATURE

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Summary

The effective electron-electron interaction resulting from the virtual polarization of the impurity spin is investigated. Abrikosov's pseudofermion representation of spin operators has been applied. It is pointed out that the effective electron-electron interaction consists of two parts: 1, elastic scattering 2, inelastic scattering. The second part shows a singularity of new type as $(\omega-\omega')^{-1}$, where $\omega - \omega'$ is the energy transfer between the two electrons. Both parts have been calculated in logarithmic approximation. In spite of the fact that the second one is found to be lower by one order of the typical logarithmic Kondo terms than the first one, both terms can be of the same order of magnitude. Studying the superconducting transition temperature Abrikosov and Gor'kov's calculation is extended to include these interactions in any order of perturbation theory keeping the leading logarithmic terms. Our results are restricted to the temperature region well above the Kondo temperature. Considering the decrease of the superconducting transition temperature due to the magnetic impurities in the antiferromagnetic case the effect of the elastic scattering can be essentially reduced by the inelastic one.

[+] The main part of A.Z.'s contribution to the paper was carried out during a visit.

[++] Permanent adress.

I. Introduction

In the recent years there have been numerous attempts to investigate the influence of magnetic impurities on the critical temperature of superconducting alloys. In the majority of cases, except for some dubious cases, the experimental situation is that the transition metal impurities depress the superconducting transition temperature T_c. The first theoretical work is due to Abrikosov and Gor'kov [1]. Using a ladder approximation and the Born approximation for the effective electron--electron interaction they have determined the decrease of T_c. After the discovery of the Kondo anomaly [2] and the Abrikosov-Suhl resonance [3,4] in the conduction electron - magnetic impurity scattering, Griffin [5] discussed the effect of this resonance on T_c.

Investigating the electron-electron interaction induced by the virtual polarization of the magnetic impurity spin, we have shown [6,7] that this interaction consist of two parts. The first one plays role in the elastic scattering processes, while the second one in the inelastic processes. The result in second and third order [6] is

$$V^{(2)}_{\alpha\beta\gamma\delta}(\omega,\omega') = \frac{1}{T}\frac{1}{3}S(S+1)\left(\frac{J}{N}\right)^2(\bar{\sigma}_{\alpha\gamma}\,\bar{\sigma}_{\beta\delta})\,\delta_{\omega,\omega'} \qquad /1.1/$$

and

$$V^{(3)}_{\alpha\beta\gamma\delta}(\omega,\omega') = \left[V^{(3)}_{\text{elastic}}(\omega,\omega') + V^{(3)}_{\text{inelastic}}(\omega,\omega')\right]\bar{\sigma}_{\alpha\gamma}\,\bar{\sigma}_{\beta\delta} \qquad /1.2/$$

where

$$V^{(3)}_{\text{elastic}}(\omega,\omega') = -\frac{1}{T}\frac{2}{3}S(S+1)\left(\frac{J}{N}\right)^2 2\frac{J}{N}\rho_o \ln\frac{D}{|\omega|}\,\delta_{\omega,\omega'} \qquad /1.3/$$

$$V^{(3)}_{\text{inelastic}}(\omega,\omega') = \frac{2}{3}S(S+1)\left(\frac{J}{N}\right)^2 2\frac{J}{N}\rho_o\frac{\pi}{\omega-\omega'}(\text{sg}\omega - \text{sg}\omega') \qquad /1.4/$$

The diagram representation of the effective electron-electron interaction is given in Fig. 1. The results (1, 1-4) correspond to the diagrams of second and third order in Fig.2.a-c., where the dotted lines stand for the spin pseudofermions proposed by Abrikosov [3]. As we have pointed out the

inelastic processes show a new type of singularity $(\omega - \omega')^{-1}$ which can be seen from eq. (1.4). In the case of antiferromagnetic s-d coupling the inelastic interaction (1.4) is an attractive one. The elastic part of the interaction, given by eq.s (1.1) and 1.3) is always repulsive. In Griffin's calculation [5] of T_c only the elastic scattering has been taken into account. We have called attention to the possible importance of this scattering in the change of the transition temperature T_c and calculated this change up to third order [6]. We have concluded that the inelastic part of this interaction can essentially reduce the effect of the inelastic scattering.

Recently Zuckermann [8] has given a general expression for the superconducting transition temperature in the limit of low impurity concentration. In Green function formulation he has used Nagaoka's decoupling scheme [9,10] for treating the Kondo effect and Hamann's solution [11] is applied so his result is thought to be valid for whole region of temperature. He compared his results with our previous one and concluded that the inelastic part of the effective electron-electron interaction is involved, too. In the second and third order, however there are numerical differencies by a factor 2 and 4 between his and our earlier calculation considering the effect of the elastic and inelastic electron-electron interaction, respectively. These differencies might be due to some algebraic mistakes or the decoupling procedure applied to the Green functions, where it is not clear which processes have been taken into account.

Making use of scattering theory on the base of generalized Suhl's equations [4] Maki and Fowler [12] and generalizing Maleyev's method [13] Ginsburg [14] have treated the problem. With the exception of Maki's very recent result [15] for the gappless superconductivity these results do not contain the effect of the inelastic electron-electron scattering processes.

There are two aims of the present paper: 1, to reinvestigate our previous results [7] for the effectiv electron-electron interaction, where some analytical properties of the electron -pseudofermion vertex function have been overlooked which results in a numerical mistake by a factor two, 2, to calculate the change in T_c using an extension of Abrikosov and Gor'kov's method [1]. The expressions of self-energy and vertex function derived by Abrikosov [3] will be used, therefore our results are valid only above the Kondo temperature T_K ($T_c >> T_K$). In this limit the obtained formula agrees with the one given by Zuckermann [8] except for the factors mentioned above.

In Sec. II. the formalism used in the paper is presented and it is shown that the effective electron-electron interaction must be calculated. Sec. III. contains this calculation. Both the elastic and inelastic scatterings are investigated. In each case only the highest order logarithmic terms are collected. In Sec. IV. these expressions are substituted into the formulas given in Sec. III. to determine the change in T_c. In Sec. V. this result is discussed and compared with Zuckermann's results [8] and a few speculations are given for the case $T_c \ll T_k$. The Appendices contain the investigation of the vertex function.

II. The formulation of the problem

The considered system is a superconductor containing magnetic impurities with spin $\bar{S}$ at the positions $\bar{R}_n$. The Hamiltonian of this system is given by

$$H = H_o + H_{sd} + H_{BCS} \qquad /2.1/$$

where

$$H_o = \sum_\alpha \int \frac{d^3k}{(2\pi)^3} \xi_k \, a^+_{k\alpha} \, a_{k\alpha} \qquad /2.2/$$

$$H_{sd} = -\frac{J}{N} \sum_{\substack{n \\ \alpha,\alpha'}} \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3k'}{(2\pi)^3} e^{-i(\bar{k}-\bar{k}')\bar{R}_n} \, a^+_{\bar{k}\alpha} \, \bar{\sigma}_{\alpha\alpha'} \, a_{\bar{k}'\alpha'} \, \bar{S}_n \qquad /2.3/$$

and

$$H_{BCS} = -|g| \int \frac{d^3k'}{(2\pi)^3} \int \frac{d^3k'}{(2\pi)^3} a^+_{\bar{k}\uparrow} \, a^+_{-\bar{k}\downarrow} \, a_{-\bar{k}'\downarrow} \, a_{\bar{k}\uparrow} \qquad /2.4/$$

H_o is the Hamiltonian of a free electron gas, where $a^+_{\bar{k}\alpha}$ and $a_{\bar{k}\alpha}$ are creation and annihilation operators for a conduction electron of momentum $\bar{k}$, energy ξ_k and spin α. H_{sd} describes the s-d exchange interaction between the conduction electrons and impurity spins with the coupling constant J. $\bar{\sigma}_{\alpha\alpha'}$ is the Pauli matrix and $\bar{S}_n$ is the impurity spin operator for which a pseudofermion representation [3] will be used. contains the effective attractive electron-electron interaction via phonons.

As it was mentioned in the Introduction we will closely follow the formalism given by Abrikosov and Gorkov [1] extending the calculation to the higher order processes. After having linearized the Gorkov equations

for Green functions $\mathcal{G}$ and $\mathcal{F}$ in the vicinity of the transition temperature, they obtained (see eq. 11 of Ref. [1])

$$\overline{\Delta^*_{\alpha\beta}} = |g|\, T \sum_{\omega} \int \overline{\mathcal{G}'_{-\omega,\gamma\,\alpha}(\bar{s},\bar{r})\, \mathcal{G}'_{\omega,\lambda\beta}(\bar{s},\bar{r})}\; \overline{\Delta^*_{\gamma\lambda}}\; d^3s \qquad /2.5/$$

where $\mathcal{G}'$ is the Green function of a normal metal in the presence of magnetic impurities, $\overline{\Delta^*_{\alpha\beta}(r)}$ is the space dependent gap, T is the temperature and $\omega = (2n+1)\,\pi T$ in the thermodynamic Green function technique [16] used throughout this paper. The bar denotes an averaging over the random distribution of the magnetic impurities.

The transition temperature T_c is determined as the highest temperature at which eq. (2.5) has a nonvanishing solution. Abrikosov and Gorkov [1] have elaborated a special diagram technique to average over the positions of the impurities. A quantity $K_{\omega,\alpha\beta}(\bar{p}_1,\bar{p}_2)$ has been introduced by the definition

$$\overline{\mathcal{G}'_{-\omega,\gamma\alpha}(\bar{s},\bar{r})\, g_{\gamma\lambda}\, \mathcal{G}'_{\omega,\lambda\beta}(\bar{s},\bar{r}')} =$$

$$= \frac{1}{(2\pi)^6} \int K_{\omega,\alpha\beta}(\bar{k}_1,\bar{k}_2)\, \exp\{ik_1(\bar{s}-\bar{r}) + ik_2(\bar{s}-\bar{r}')\}\, d^3k_1\, d^3k_2 \qquad /2.6/$$

where

$$g_{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}_{\alpha\beta} \qquad \text{and} \qquad \Delta^*_{\alpha\beta} = \Delta^*\, g_{\alpha\beta}\ .$$

$K_{\omega,\alpha\beta}(p_1,p_2)$ can be represented by the sum of the diagrams in Fig. 3., where the lines correspond to the Green function $\tilde{\mathcal{G}}(k)$ of the normal metal in the presence of impurities averaged over their positions and the shaded square corresponds to the irreducible part of the effective electron-electron interaction mediated by the magnetic impurities. $\tilde{\mathcal{G}}_\omega(k)$ has the form

$$\tilde{\mathcal{G}}_\omega(k) = \frac{1}{i\omega\eta_1 - \xi} \qquad /2.7/$$

where $\xi = \frac{k^2}{2m} - \varepsilon_F$ and $\eta_1 = 1 + \frac{1}{2\tau_1|\omega|}$ and τ_1 is the electronic life time due to the s-d scattering. According to Abrikosov's [3] calculation

$$\frac{1}{2\tau_1} = N_i \left(\frac{J}{N}\right)^2 S(S+1)\, \pi\rho_o \left[1 + \frac{2J\rho_o}{N} \ln\frac{D}{|\omega|}\right]^{-2} \qquad /2.8/$$

where N_i is the number of impurities in unit volume, $\rho_o = \frac{p_o m}{2\pi^2}$ is the density of states per atom for one spin orientation and $\bar{S}$ is the value of the localized spine and D is the cut-off energy. This form of the life time is appropriate only well above the Kondo temperature T_K. We will restrict our considerations to this temperature interval and to logarithmic approximation, therefore eq. (2.8) can be applied. We will see that if the superconducting transition temperature is of the same order of magnitude as the Kondo temperature, a more precise expression of the life time is necessary.

The diagrammatic equation for $K_{\omega\alpha\beta}$ given by Fig. 3. can be written analytically as

$$K_{\omega,\alpha\beta}(\bar{p}_1,\bar{p}_2) = \tilde{\mathcal{G}}_{\omega}(\bar{p}_1)\,\tilde{\mathcal{G}}_{-\omega}(\bar{p}_2)\left[g_{\alpha\beta} + \right.$$

$$\left. + \frac{N_i}{(2\pi)^3} T \sum_{\omega'} \int V_{\alpha\beta\lambda\rho}(\omega,\omega')\, K_{\omega';\lambda\rho}(\bar{p}_1',\bar{p}_1 + \bar{p}_2 - \bar{p}_1')\, d^3p_1'\right] \qquad /2.9/$$

where $V_{\alpha\beta\lambda\rho}(\omega,\omega')$ stands for the shaded square. The momentum variables are omitted because the exchange coupling constant J in eq. (2.3) is independent of the momenta.

Our main task in the following is the calculation of the electron--electron interaction due to the magnetic impurities.

The pseudofermion representation of spin operators proposed by Abrikosov [3] will be used. In this language the irreducible effective interaction $V(\omega',\omega')$ is represented by two solid lines connected by a closed loop of dotted lines. The crossing of two dotted line loops can be neglected as it has been pointed out by Abrikosov and Gorkov [1].

The lowest order diagram in V is of second order (see Fig.2.). The simplest way to take into account diagrams of higher order is to put the whole vertex instead of the bare one. This procedure leads to the diagram in Fig. 4. The simplest diagrams obtained in this way are shown in Fig. 2. Another class of diagrams is illustrated in Fig. 5.a-c. In these diagrams the two solid lines are connected more than twice by the pseudofermion lines. It will be shown studying a few examples in Appendix I that these diagrams yield logarithmic contribution of lower power than the diagrams represented in Fig. 2., therefore their contribution will be neglected.

III. The effective electron-electron interaction

As we have shown in Sec. II. the effective electron-electron interaction due to the virtual polarization of the impurity spin must be calculated in the special case, when the energies of the incoming electrons are ω and $-\omega$ respectively. Here we will deal with this interaction. The effect of H_{BCS} has been taken into account in the deviation of eq. (2.5) so in the present calculation only the influence of H_{sd} given by eq. (2.8) will be treated. The general irreducible vertex part for the electron-electron scattering which is to be calculated in the logarithmic approximation, is given in Fig. 4. Its analytical form is

$$V_{\alpha\beta\gamma\delta}(\omega,\omega') = -\frac{e^{\beta\lambda}}{2S+1} T \sum_{\omega_1} \Gamma_{\alpha\varepsilon\gamma\eta}(i\omega, i\omega_1; i\omega', i(\omega + \omega_1 - \omega'))$$

$$\mathcal{G}_\varepsilon(i\omega_1)\, \mathcal{G}_\eta(i(\omega + \omega_1 - \omega'))\, \Gamma_{\beta\eta\delta\varepsilon}(-i\omega, i(\omega + \omega_1 - \omega'); -i\omega', i\omega_1)$$

where Γ is the renormalized vertex of the s-d interaction and $\mathcal{G}(i\omega)$ is the pseudofermion propagator

$$\mathcal{G}(i\omega) = \frac{1}{i\omega - \lambda} \qquad /3.2/$$

The limit $\lambda \to \infty$ must be taken to eliminate the contribution of the nonphysical states. After the analytic continuation $i\omega_1 \to z$ the summation over the imaginary frequencies $i\omega_1 = \pi T(2n+1)$ can be transformed to a contour integral around the poles in those regions where the continuation is analytic. If one pole lies on a cut then it has to be treated separately.

Investigating the analytic properties of the continuation of the vertex function $\Gamma(i\omega, i\omega_1; i\omega', i\omega_1')$ and its spectral representation Eliashberg [17] has proved that there are several cuts parallel to the real axis, where the imaginary part of one of the following quantities is zero:

a/ The frequencies of the incoming and outgoing lines
$i\omega$, $i\omega_1$, $i\omega'$, $i(\omega + \omega_1 - \omega') = i\omega_1'$

b/ the sum of the frequencies of the two incoming lines
$i(\omega + \omega_1) = i(\omega' + \omega_1')$

c/ the difference of the frequencies of an incoming and an outgoing line $i(\omega - \omega_1') = i(\omega' - \omega_1)$ and $i(\omega - \omega') = i(\omega_1' - \omega_1)$

We are interested in the cuts corresponding to the variable $i\omega_1 = z$, which are situated on the horizontal lines

$$\mathrm{Im}z = 0 \quad , \quad \mathrm{Im}z = (\omega' - \omega) \quad , \quad \mathrm{Im}z = -\omega \quad , \quad \mathrm{Im}z = \omega' \qquad /3.3/$$

The cuts of the other vertex $\Gamma\,(-i\omega,\ i(\omega+\omega_1-\omega');-i\omega;i\omega_1)$ are the same as determined by eq. (3.3). The cuts of the pseudofermion propagators occuring in eq. (3.1) are at the lines $\mathrm{Im}z = 0$ and $\mathrm{Im}z = (\omega' - \omega)$.

The sum in (3.1) can be transformed to integrals along these four cuts and still the contribution of the points $z = -i\omega$, and $z = i\omega'$ must be considered as it is shown in Fig. 6. The cuts $\mathrm{Im}z = 0$ and $\mathrm{Im}z = (\omega'-\omega)$ will be called cuts of Bose type because they lie on lines $\mathrm{Im}z = 2n\,\pi T$, while the cuts $\mathrm{Im}z = -\omega$ and $\mathrm{Im}z = \omega'$ will be called cuts of Fermi type. Denoting the contours around the cuts in the order given in (3.3) by C_1, C_2, C_3 and C_4 respectively, we get

$$V_{\alpha\beta\gamma\delta}\,(\omega,\omega') =$$

$$= \frac{e^{\beta\lambda}}{2S+1} \sum_{j=1}^{4} \int_{C_j} \frac{dz}{2\pi i}\, n_F(z)\, \Gamma_{\alpha\varepsilon\gamma\eta}\,(i\omega,z;\ i\omega',\ i(\omega-\omega') + z)\ \mathcal{G}_\varepsilon(z)\ .$$

$$.\ \mathcal{G}_\eta(i(\omega-\omega') + z)\ \Gamma_{\beta\eta\delta\varepsilon}\,(-i\omega,\ i(\omega-\omega')+z;\ -i\omega',\ z)$$

$$- \frac{e^{\beta\lambda}}{2S+1}\, T\, \Gamma_{\alpha\varepsilon\gamma\eta}\,(i\omega,-i\omega;\ i\omega';-i\omega')\ \mathcal{G}_\varepsilon(-i\omega)\ \mathcal{G}_\eta(-i\omega')\ \Gamma_{\beta\eta\delta\varepsilon}\,(-i\omega,-i\omega';-i\omega';-i\omega)$$

$$- \frac{e^{\beta\lambda}}{2S+1}\, T\, \Gamma_{\alpha\varepsilon\gamma\eta}\,(i\omega,i\omega';i\omega',i\omega)\ \mathcal{G}_\varepsilon\,(i\omega')\ \mathcal{G}_\eta\,(i\omega)\ \Gamma_{\beta\eta\delta\varepsilon}\,(-i\omega,\ i\omega;\ -i\omega',\ i\omega')\ .$$

/3.4/

The integrals on the contour C_j can be performed using

$$\int_{C_j} dz\ f(z) = \int_{-\infty}^{\infty} dx\ [f(x + i\mathrm{Im}z + i\varepsilon) - f(x + i\mathrm{Im}z - i\varepsilon)]\ , \qquad /3.5/$$

where $x=\mathrm{Re}z$ and $\mathrm{Im}z$ is one of the four values given in (3.3).

In eq. (3.5) the Fermi distribution function, the vertex function or the propagators may have cuts. If two cuts coincide, the following identity can be used

$$\Delta\,\{F(x)\ G\ (x)\} = \Delta F(x)\ \overline{G(x)} + \Delta G(x)\ \overline{F(x)} \qquad /3.6/$$

where

$$\Delta F(x) = F(x + i\varepsilon) - F(x - i\varepsilon)$$

$$\overline{F(x)} = \frac{1}{2}\left[F(x + i\varepsilon) + F(x - i\varepsilon)\right] \qquad /3.7/$$

This formula can be generalized to arbitrary number of function with the result that in a product the cut of each factor must be taken multiplied by the average of the remaining part.

With the help of this transformation it is easy to show that the last two terms in eq. (3.4) will be cancelled by the contribution of these cuts of the Fermi distribution function. The contribution of this cut from the integral on C_3 is

$$\frac{e^{\beta\lambda}}{2S+1}\int_{-\infty}^{\infty}\frac{dx}{2\pi i}\left[n_F(x - i\omega + i\varepsilon) - n_F(x - i\omega - i\varepsilon)\right]$$

$$\Gamma_{\alpha\varepsilon\gamma\eta}(i\omega, x-i\omega; i\omega', -i\omega'+x)\,\mathcal{G}_\varepsilon(x-i\omega)\,\mathcal{G}_\eta(x-i\omega')\,\Gamma_{\beta\eta\delta\varepsilon}(-i\omega, x-i\omega'; -i\omega', x-i\omega) \qquad /3.8/$$

Substituting the identity

$$n_F(x + i\omega + i\varepsilon) - n_F(x + i\omega - i\varepsilon) = i\,\frac{2\pi}{\beta}\,\delta(x) \qquad /3.9/$$

where $\beta=1/T$ and ω is an odd frequency, into (3.8) the expression obtained in this way cancels the second term in (3.4). Similarly the cut of the distribution function on C_4 cancels the third term in eq. (3.4). This means that in eq. (3.4) only the first term must be kept and only the cuts of the vertex functions and the propagators must be taken into account.

We have shown [6] that the interaction in the second order given by (1.1) is purely elastic, in third and higher order the inelastic processes contributes, too (see eq.s (1.2-4)). Their contributions contains lower power of the typical logarithmic term $\ln\frac{D}{|\omega|}$, than that of the elastic processes. Although we will use a logarithmic approximation, also the inelastic scattering will be investigated. It will be shown that their contribution can be of the same order of magnitude as the corresponding term of the elastic scattering.

A. Elastic part of the scattering

For the elastic scattering $\omega = \omega'$, according to eq. (3.4) we have

$$V_{\alpha\beta\gamma\delta}(\omega) = \frac{e^{\beta\lambda}}{2S+1} \sum_{j=1}^{4} \int_{C_j} \frac{dz}{2\pi i} n_F(z) \Gamma_{\alpha\varepsilon\gamma\eta}(i\omega,z;i\omega,z) \mathcal{G}_\varepsilon(z) \mathcal{G}_\eta(z) . \Gamma_{\beta\eta\delta\varepsilon}(-i\omega,z;-i\omega,z)$$

where only the cuts of the vertex functions and the propagators must be considered. The second and third order contributions are given in (1.1-4). In the actual calculation it is easy to see that in logarithmic approximation, i.e. when in each order of the perturbational calculation only the highest power of the typical logarithmic term $\ln \frac{D}{|\omega|}$ is kept, only the poles of the propagators must be considered. The contribution of the cut of the vertex function is of lower order than the vertex function itself. In other words this means that the imaginary part of the vertex function contains lower power of $\ln \frac{D}{|\omega|}$ than the real part.

The propagators have double pole at $z=\lambda$. Its contribution is given by

$$V_{\alpha\beta\gamma\delta}(\omega) = - \frac{e^{\beta\lambda}}{2S+1} \frac{d}{dz} \left[n_F(z) \Gamma_{\alpha\varepsilon\gamma\eta}(i\omega,z;i\omega,z) \Gamma_{\beta\eta\delta\varepsilon}(-i\omega,z;-i\omega,z) \right]_{z=\lambda} \quad /3.11/$$

The derivative of Γ with respect to z can be neglected in logarithmic approximation and after taking the limit $\lambda \to \infty$ we get

$$V_{\alpha\beta\gamma\delta}(\omega) = \frac{1}{T} \frac{1}{2S+1} \Gamma_{\alpha\varepsilon\gamma\eta}(i\omega,\lambda;i\omega,\lambda) \Gamma_{\beta\eta\delta\varepsilon}(-i\omega,\lambda;-i\omega,\lambda) \quad /3.12/$$

According to Abrikosov's calculation [3]

$$\Gamma_{\alpha\varepsilon\gamma\eta}(i\omega,\lambda;i\omega,\lambda) = \Gamma(\omega)(\bar{\sigma}_{\alpha\gamma} \bar{S}_{\varepsilon\eta}) =$$

$$= \frac{\frac{J}{N}}{1+2\frac{J}{N}\rho_o \ln \frac{D}{|\omega|}} (\bar{\sigma}_{\alpha\gamma} \bar{S}_{\varepsilon\eta}) \sim \frac{\frac{J}{N}}{1+2\frac{J}{N}\rho_o \ln \frac{D}{T}} (\bar{\sigma}_{\alpha\gamma} \bar{S}_{\varepsilon\eta}) = \Gamma . (\bar{\sigma}_{\alpha\gamma} \bar{S}_{\varepsilon\eta}) \quad /3.13/$$

where in the last approximation it is supposed that $|\omega| \sim T$, the thermal smearing is taken into account and a new notation Γ is introduced. This expression is valid well above the Kondo temperature T_K. Inserting eq. (3.13) into (3.12) we obtain

$$V_{\alpha\beta\gamma\delta}(\omega) = \frac{1}{T} \frac{1}{3} S(S+1) \frac{\left(\frac{J}{N}\right)^2}{\left(1 + 2\frac{J}{N}\rho_o \ln \frac{D}{|\omega|}\right)^2} (\bar{\sigma}_{\alpha\gamma} \bar{S}_{\beta\delta}) \quad /3.14/$$

where the identity

$$(\bar{\sigma}_{\alpha\gamma}\,\bar{S}_{\varepsilon\eta})(\bar{\sigma}_{\beta\delta}\,\bar{S}_{\eta\varepsilon}) = \frac{1}{3}\,S\,(S+1)\,(2S+1)\,(\bar{\sigma}_{\alpha\gamma}\,\bar{\sigma}_{\beta\delta}) \qquad /3.15/$$

has been used.

It must be emphasized that eq. (3.14) is valid only in logarithmic approximation. In better approximation the vertex function must be replaced by a more precise form and its cuts must be taken into account, too.

B. Inelastic part of the scattering

From the third order calculations given by eq. (1.4) it is known that the inelastic contribution contains lower power of $\ln\frac{D}{|\omega|}$ than the elastic one. Nevertheless, the inelastic contributions can be of importance in the determination of the change in T_c. It is due to the fact that this contribution /see eq. (1.4)/ has quite different structure exhibiting a new type of singularity $(\omega' - \omega)^{-1}$

In the present case $\omega \neq \omega'$ and the integrals in eq. (3.4) must be taken along the four horizontal lines. The vertices and the propagators may have cuts along the same line, but according to eq. (3.6) their contributions can be taken separately.

a/ First we investigate the contribution from the cuts of the vertex functions.

In the logarithmic approximation only the parquet diagrams are considered. The vertex can be separeted into three parts

$$\Gamma = \Gamma_o + \Lambda_1 + \Lambda_2 \qquad /3.16/$$

$\Gamma_o = J/N(\bar{\sigma},\bar{S})$ is the bare vertex, Λ_1 and Λ_2 are vertex parts which by cutting two parallel or antiparallel electron and pseudofermion lines fall into two parts. They are shown in Fig. 7.a. and b.

The cuts of Λ_1 and Λ_2 are investigated in Appendix II. It is shown that the important cut of Λ_1 which gives the highest logarithmic contribution is along $\mathrm{Im}z=-\omega$, where z is the analytical continuation of $i\omega_1$. Similarly in Λ_2 the important cut can be found along $\mathrm{Im}z = \omega'$. These contributions are calculated in Appendix III. and according to eq.s (A.III.3) and (A.III.4) they are for $|x| < D$

$$\Delta\Lambda(x)_{1\alpha\varepsilon\gamma\eta} = 2i\ \mathrm{Im}\ \Lambda(x+i\varepsilon)_{1\alpha\varepsilon\gamma\eta} = 2\pi i\rho_o\ \mathrm{Re}\ \Gamma(0,\ x+\ ;\ x,\lambda)_{\alpha\varepsilon\alpha'\varepsilon'} \ .$$

$$\left(1 - n_F(x)\right)\ \mathrm{Re}\ \Gamma_{\alpha'\varepsilon'\gamma\eta}(x,\ \lambda;\ 0,\ x+\lambda) \qquad /3.17/$$

and

$$\Delta\Lambda(x)_{2\alpha\varepsilon\gamma\eta} = 2i\ \mathrm{Im}\ \Lambda(x+i\varepsilon)_{2\alpha\varepsilon\gamma\eta} = -2\pi\ i\rho_o\ \mathrm{Re}\ \Gamma_{\alpha\varepsilon'\alpha'\eta}(0,\lambda;-x,x+\lambda)$$

$$\left(1 - n_F(x)\right)\ \mathrm{Re}\ \Gamma_{\alpha'\varepsilon\gamma\varepsilon'}(-x,x+\lambda;0,\lambda) \qquad /3.18/$$

where $x = z + i\omega - \lambda$ and $x = z - i\omega' - \lambda$, respectively.

First let us calculate the contribution of the vertex part Λ_1 to the effective interaction given by eq. (3.4). Considering eq.s (3.2), (3.4), (3.5) and (3.7) and substituting the expression of Λ_1 given by eq. (3.17) instead of the two vertex function one after the other, in eq. (3.4) one obtains

$$V^{(\Lambda_1)}_{\alpha\beta\gamma\delta}(\omega,\omega') = \frac{e^{\beta\lambda}}{2S+1}\ \frac{1}{2\pi i}\int dx \ .$$

$$\Big\{ n_F(x+\lambda-i\omega)\ \Delta\Lambda_{1\alpha\varepsilon\gamma\eta}(x)\ \frac{1}{x-i\omega'}\ \frac{1}{x-i\omega'}\ \Gamma_{\beta\eta\delta\varepsilon}(2) +$$

$$+ n_F(x+\lambda-i\omega')\ \Gamma_{\alpha\varepsilon\gamma\eta}(2^x)\ \frac{1}{x+i\omega'}\ \frac{1}{x+i\omega}\ \Delta\Lambda_{1\beta\eta\delta\varepsilon}(x)\Big\} =$$

$$= \frac{1}{2S+1}\ e^{\beta\lambda}\ \rho_o \int dx\ \frac{1}{-e^{(x+\lambda)\beta}+1}\ \left(1 - n_F(x)\right) \ .$$

$$\Big\{ \mathrm{Re}\ \Gamma_{\alpha\varepsilon\alpha'\varepsilon'}(1)\ \mathrm{Re}\ \Gamma_{\alpha'\varepsilon'\gamma\eta}(\bar{1})\ \frac{1}{x-i\omega}\ \frac{1}{x-i\omega'}\ \Gamma_{\beta\eta\delta\varepsilon}(2) +$$

$$+ \Gamma_{\alpha\varepsilon\gamma\eta}(2)\ \frac{1}{x+i\omega'}\ \frac{1}{x+i\omega}\ \mathrm{Re}\ \Gamma_{\beta\eta\alpha'\varepsilon'}(1)\ \mathrm{Re}\ \Gamma_{\alpha'\varepsilon'\delta\varepsilon}(\bar{1})\Big\} \qquad /3.19/$$

where for the sake of brevity the following notations are introduced for the arguments

$$(1) = (0,\ x+\lambda;\ x,\ \lambda) \qquad (\bar{1}) = (x,\lambda\ ;\ 0,\ x+\lambda)$$

$$(2) = (-i\omega,\ -i\omega' + x + \lambda; -i\omega',\ -i\omega + x + \lambda) \qquad /3.20/$$

$$2^x = (i\omega,\ i\omega' + x + \lambda;\ i\omega',\ i\omega + x + \lambda)$$

The spin factors can be evaluated by using the first of the following two identities

$$(\bar{\sigma}_{\alpha\alpha'}\ \bar{S}_{\varepsilon\varepsilon'})(\bar{\sigma}_{\alpha'\gamma}\ \bar{S}_{\varepsilon'\eta}) = S(S+1)\ \delta_{\alpha\gamma}\ \delta_{\varepsilon\eta} - (\bar{\sigma}_{\alpha\gamma}\ \bar{S}_{\varepsilon\eta}) \qquad /3.21/$$

$$(\bar{\sigma}_{\alpha\alpha'}\ \bar{S}_{\varepsilon'\varepsilon})(\bar{\sigma}_{\alpha'\gamma}\ \bar{S}_{\eta\varepsilon'}) = S(S+1)\ \delta_{\alpha\gamma}\ \delta_{\varepsilon\eta} + (\bar{\sigma}_{\alpha\gamma}\ \bar{S}_{\varepsilon\eta}) \qquad /3.22/$$

and eq. (3.15). The spin dependence of the vertex function can be given by the following representation

$$\Gamma_{\alpha\beta\gamma\delta} = \Gamma^{(0)}\ \delta_{\alpha\gamma}\ \delta_{\beta\delta} + \Gamma^{(\sigma)}(\bar{\sigma}_{\alpha\gamma}\ \bar{S}_{\beta\delta}) \qquad /3.23/$$

Similar representetion holds for Λ_1 and Λ_2 , too.
It is easy to show that each term in eq. (3.19) can be written into the form

$$A\ \delta_{\alpha\gamma}\ \delta_{\beta\delta} + B\bar{\sigma}_{\alpha\gamma}\ \bar{\sigma}_{\beta\delta} \qquad /3.24/$$

The spin factors of the two terms in (3.19) are very similar, because one can be obtained from the other one by replacing $\alpha \leftrightarrow \beta$, $\varepsilon \leftrightarrow \eta$, $\gamma \leftrightarrow \delta$. As the final form (3.24) shows this symmetry, the spin indices can be dropped for a time.

Furthermore, the symmetry relation

$$\Gamma(i\omega_1,\ i\omega_2;\ i\omega_3,\ i\omega_4) = \left|\Gamma(-i\omega_1,\ -i\omega_2;\ -i\omega_3,\ -i\omega_4)\right|^x \qquad /3.25/$$

will be of importance, which is a consequence of the fact that in the thermodynamic Green function technique the energy variable $i\omega$ is the only complex quantity. In our case $\Gamma(2) = \left(\Gamma(2^x)\right)^x$.

In eq. (3.19) the expression between the curly brackets can be transformed as

$$\mathrm{Re}\Gamma(1)\ \mathrm{Re}\Gamma(\bar{1})\left[\frac{1}{x-i\omega}\ \frac{1}{x-i\omega'}\ \Gamma(2)+\frac{1}{x+i\omega'}\ \frac{1}{x+i\omega}\ \Gamma(2^{x})\right]=$$

$$=2\mathrm{Re}\Gamma(1)\ \mathrm{Re}\Gamma(\bar{1})\ \mathrm{Re}\left[\frac{1}{x-i\omega}\ \frac{1}{x-i\omega'}\ \Gamma(2)\right]=$$

furthermore

$$=2\mathrm{Re}\Gamma(1)\ \mathrm{Re}\Gamma(\bar{1})\ .$$

$$.\left[\mathrm{Re}\Gamma(2)\left(\frac{\omega}{x^2+\omega^2}-\frac{\omega'}{x^2+\omega'^2}\right)\frac{1}{\omega-\omega'}-\mathrm{Im}\Gamma(2)\ \frac{1}{x^2+\omega^2}\ \frac{1}{x^2+\omega'^2}\ .\ x\ .(\omega+\omega')\right] \qquad /3.26/$$

In logarithmic approximation the imaginary part of the vertex function can be neglected comparing to its real part, therefore only the first part of the expression (3.26) must be considered. As the vertex function is a smooth logarithmic function on the energy scale $i\omega$ (or $i\omega'$), we can apply the following approximation

$$\frac{\omega}{x^2+\omega^2}\sim \mathrm{sg}\omega\ \lim_{\varepsilon\to+0}\ \frac{\varepsilon}{x^2+\varepsilon^2}=\mathrm{sg}\omega\ \pi\delta(x) \qquad /3.27/$$

In this way, the first term in (3.26) can be written as

$$2\pi\ \frac{1}{\omega-\omega'}\ (\mathrm{sg}\omega-\mathrm{sg}\omega')\ \mathrm{Re}\Gamma(1)\ \mathrm{Re}\Gamma(\bar{1})\ \mathrm{Re}\Gamma(2)\ \delta(x) \qquad /3.28/$$

Inserting this result into eq. (3.19) and in the case of small ω and ω' neglecting these variables according to logarithmic approximation we get an expression between the curly bracket which is an even function of the variable x. Therefore, $n_F(x)$ can be replaced by 1/2, which arises from the expression $\exp(-\beta x)\,(1-n_F(x))=n_F(x)$. Considering (3.28) and (3.19) we get

$$V_{1\,\alpha\beta\gamma\delta}^{(\Lambda_1)}(\omega,\omega')=-\frac{1}{2S+1}\ \pi\ \frac{1}{\omega-\omega'}\ (\mathrm{sg}\omega-\mathrm{sg}\omega')\ . \qquad /3.29/$$

$$.\ \mathrm{Re}\Gamma_{\alpha\varepsilon\alpha'\varepsilon'}(0,\lambda;0,\lambda)\ \mathrm{Re}\Gamma_{\alpha'\varepsilon'\gamma\eta}(0,\lambda;0,\lambda)\ \mathrm{Re}\Gamma_{\beta\eta\delta\varepsilon}(0,\lambda;0,\lambda)$$

According to (3.13) and evaluating the spin factors by using the identities (3.21) and (3.15) we get

$$V_{1\,\alpha\beta\gamma\delta}^{(\Lambda_1)}(\omega,\omega')=\frac{S(S+1)}{3}\ \pi\ \frac{1}{\omega-\omega'}\ (\mathrm{sg}\omega-\mathrm{sg}\omega')\ \Gamma^3\ (\bar{\sigma}_{\alpha\gamma}\ \bar{\sigma}_{\beta\delta}) \qquad /3.30/$$

This calculation can be repeated to obtain the contribution $V_{\alpha\beta\gamma\delta}^{(\Lambda_2)}$ due to the other vertex part Λ_2 and the same result is yielded as (3.30)

$$V^{(\Lambda_1)} = V^{(\Lambda_2)} \qquad /3.31/$$

The only differencies are that eq.s (3.18) and (3.22) must be used instead of eq.s (3.17) and (3.22) and the two changes in sign cancel each other.

The final expression of the effective interaction arising from the cuts in the analytical continuation of the vertex functions is

$$V^{(\Lambda)}_{\alpha\beta\gamma\delta}(\omega,\omega') = V^{(\Lambda_1)} + V^{(\Lambda_2)} = \frac{2S(S+1)}{3}\,\pi\,\frac{1}{\omega-\omega'}\,(sg\omega - sg\omega')\,\Gamma^3(\bar{\sigma}_{\alpha\gamma}\,\bar{\sigma}_{\beta\delta}) \qquad /3.32/$$

b/ Let us turn to the contribution of the cuts corresponding to the Green function.

$V^{(\mathcal{G})}$ We have two cuts C_1 and C_2 in Fig. 6. and the contribution can be written into a similar form to eq. (3.19) making use of eq.s (3.4), (3.5), (3.7)

$$V^{(\mathcal{G})}_{\alpha\beta\gamma\delta}(\omega,\omega') = \frac{e^{\beta\lambda}}{2S+1}\,\frac{1}{2\pi i}\int dx\; n_F(x+\lambda)\;\cdot$$

$$\Big\{\Gamma_{\alpha\varepsilon\gamma\eta}(3)\,\Delta\mathcal{G}(x)\,\frac{1}{i(\omega-\omega')+x}\,\Gamma_{\beta\eta\delta\varepsilon}(4) + \qquad /3.33/$$

$$+\,\Gamma_{\alpha\varepsilon\gamma\eta}(4^x)\,\frac{1}{i(\omega'-\omega)+x}\,\Delta\mathcal{G}(x)\,\Gamma_{\beta\eta\delta\varepsilon}(3^x)\Big\}$$

where the arguments are

$$(3) = \left(i\omega,\; x+\lambda;\; i\omega',\; i(\omega-\omega') + x+\lambda\right)$$

$$(3^x) = \left(-i\omega,\; x+\lambda;\; -i\omega',\; -i(\omega-\omega') + x+\lambda\right) \qquad /3.34/$$

$$(4) = \left(-i\omega,\; i(\omega-\omega') + x+\lambda;\; -i\omega',\; x+\lambda\right)$$

$$(4^x) = \left(+i\omega,\; -i(\omega-\omega') + x+\lambda;\; +i\omega',\; x+\lambda\right)$$

From eq.s (3.2) and (3.7)

$$\Delta\mathcal{G}(x) = -2\pi i\;\delta(x)$$

and inserting this into eq. (3.34) we get

$$V^{(g)}_{\alpha\beta\gamma\delta}(\omega,\omega') = -\frac{1}{2S+1}\frac{1}{i(\omega-\omega')}\left(\Gamma_{\alpha\varepsilon\gamma\eta}(3)\,\Gamma_{\beta\eta\delta\varepsilon}(4) - \Gamma_{\alpha\varepsilon\gamma\eta}(3^x)\,\Gamma_{\beta\eta\delta\varepsilon}(4^x)\right) \qquad /3.36/$$

The spin factors of both terms can be calculated in a similar way as before. We get to the representation (3.24) again, which does not change due to the application of the operation $\alpha\leftrightarrow\beta$, $\gamma\leftrightarrow\eta$. Applying this operation to the second term of eq. (3.37), using the identity (3.25) we obtain

$$V^{(g)}_{\alpha\beta\gamma\delta}(\omega,\omega') = -\frac{1}{2S+1}\frac{1}{\omega-\omega'}\,\mathrm{Im}\left[\Gamma_{\alpha\varepsilon\gamma\eta}(3)\,\Gamma_{\beta\eta\delta\varepsilon}(4)\right]_{x=0} =$$

$$= -\frac{1}{2S+1}\frac{1}{\omega-\omega'}\left(\mathrm{Im}\,\Gamma_{\alpha\varepsilon\gamma\eta}(3)\,\mathrm{Re}\Gamma_{\beta\eta\delta\varepsilon}(4) + \mathrm{Re}\Gamma_{\alpha\varepsilon\gamma\eta}(3)\,\mathrm{Im}\Gamma_{\beta\eta\delta\varepsilon}(4)\right)_{x=0} \qquad /3.37/$$

Supposing that ω and ω' are small, we can take the limit $\omega \to 0$ and $\omega' \to 0$ in the vertex functions. The limit of the real part of the vertex is given by eq. (3.31). Making use of the representation (3.23) for Im Γ and calculating the spin factors according to (3.15) we obtain

$$V^{(g)}_{\alpha\beta\gamma\delta}(\omega,\omega') = -\frac{1}{\omega-\omega'}\frac{S(S+1)}{3}\,\Gamma\left(\mathrm{Im}\,\Gamma^{(\sigma)}(3) + \mathrm{Im}\,\Gamma^{(\sigma)}(4)\right)_{x=0}\cdot(\bar\sigma_{\alpha\gamma}\,\bar\sigma_{\beta\delta}) \qquad /3.38/$$

where the limit of Im Γ must be determined. There are cuts in the analytical continuations of the vertex parts Λ_1 and Λ_2. These cuts are on the real axis in the variables $i(\omega+z)$ and $i(z-\omega')$, where the notations can be seen in Fig. 7.a.-b. and $i\omega_1 \to z$. According to (3.17), (3.18) and (3.34)

$$\mathrm{Im}\Gamma^{(\sigma)}(3)\Big|_{x=0} \simeq \mathrm{Im}\,\Lambda_1^{(\sigma)}(0+i\varepsilon\,\mathrm{sg}\omega)\Big|_{x=0} + \mathrm{Im}\Lambda_2^{(\sigma)}(0-i\varepsilon\,\mathrm{sg}\omega')\Big|_{x=0} = -\pi\rho_0(\mathrm{sg}\omega - \mathrm{sg}\omega')\,\Gamma^2$$

and

$$\mathrm{Im}\Gamma^{(\sigma)}(4)\Big|_{x=0} \simeq \mathrm{Im}\Lambda_1^{(\sigma)}(0-i\varepsilon\,\mathrm{sg}\omega')\Big|_{x=0} + \mathrm{Im}\Lambda_2^{(\sigma)}(0+i\varepsilon\,\mathrm{sg}\omega')\Big|_{x=0} = -\pi\rho_0(\mathrm{sg}\omega - \mathrm{sg}\omega')\,\Gamma^2 \qquad /3.39/$$

where in the last steps (3.21), (3.22) and (3.13) are used. Inserting this results into (3.38) and comparing with the eq. (3.32) we can find that we obtain just the same contribution as before

$$V^{(g)}_{\alpha\beta\gamma\delta}(\omega,\omega') = V^{(\Lambda)}_{\alpha\beta\gamma\delta}(\omega,\omega') \qquad /3.40/$$

c/ The inelastic effective electron-electron interaction is according to eq.s (3.40) and (3.32)

$$V_{\alpha\beta\gamma\delta}(\omega,\omega') = \frac{4S(S+1)}{3}\,\pi\,\frac{1}{\omega-\omega'}\,(sg\omega - sg\omega')\,\Gamma^3\,(\bar{\sigma}_{\alpha\gamma}\,\bar{\sigma}_{\beta\delta}) \qquad /3.41/$$

We have seen that the contribution of the cuts of the vertex functions and the propagators give the same result. In our previous paper [7] we did not realize the importance of the cuts of the vertex functions and only the contribution from the propagators was taken. Therefore the result obtained there for the inelastic part must be completed by a factor 2.

Inserting instead of Γ the bare vertex J/N into eq. (3.41) we get back our previous result [6] (1.4) obtained in the third order of the perturbation theory.

Finally, it can be mentioned that all of the analytical properties can be taken into account in a formal way, if the contribution of the diagram in Fig. 8. is calculated with three vertices and only these poles are considered which arise from the denominators of the Green functions corresponding to the lines in Fig. 8.

IV. Change in the superconducting transition temperature

After having determined the effective electron-electron interaction due to the magnetic impurities, the next step is to calculate the change in T_c. Starting from the formulas given by (2.5) and (2.6) the superconducting transition temperature can be obtained from the solution of

$$g_{\alpha\beta} = |g|\,T_c \sum_{\omega} \frac{1}{(2\pi)^6} \int K_{\omega,\alpha\beta}(\bar{p}_1, \bar{p}_2)\, e^{-i(\bar{p}_1+\bar{p}_2)(\bar{r}-\bar{s})}\, d^3p_1\, d^3p_2\, d^3s =$$

$$= |g|\,T_c \sum_{\omega} \frac{1}{(2\pi)^6} \int K_{\omega,\alpha\beta}(\bar{p}_1, -\bar{p}_1)\, d^3p_1 \qquad /4.1/$$

With the supposition $K_{\omega',\alpha\beta} = K_{\omega} \cdot g_{\alpha\beta}$ we get

$$1 = \frac{|g|T_c}{2\pi^3} \sum_{\omega} \int K(\bar{p}_1 - \bar{p})\, d^3p \qquad /4.2/$$

After some algebraic manipulation the change in T_c can be given by the following expression

$$\ln\left(\frac{T_{co}}{T_c}\right) = \sum_{\omega_n} \frac{\pi T_c}{|\omega_n|}\left(1 - 2|\omega_n|\, K_{\omega_n}\right) \qquad /4.3/$$

where

$$K_\omega = \frac{1}{2\pi}\int K_\omega\left(\bar{p}_1 - \bar{p}\right) d\xi \qquad /4.4/$$

Making use of eq.s (2.7) and (2.9) and the identity

$$\left(\bar{\sigma}_{\alpha\lambda}\,\bar{\sigma}_{\beta\rho}\right) g_{\lambda\rho} = -3g_{\alpha\beta}$$

we obtain

$$K_\omega = \frac{1}{2|\omega|\eta_1}\left[1 - N_i\,\sigma\pi\rho_o \sum_{\omega'} V(\omega,\omega')\, K_{\omega'}\right] \qquad /4.5/$$

$V(\omega,\omega')$ denotes the interaction without the spin factors and N_i is the number of impurities. In the small concentration limit this equation can be solved by iteration. In the present case the first step of the iteration coincides with the logarithmic approximation. Inserting the elastic and inelastic part of the interaction from eq.s (3.14) and (3.41) into (4.5) and keeping only the terms linear in the impurity concentration at the expansion of η_1 finally we get

$$K_\omega = \frac{1}{2|\omega|}\left\{1 - 4\pi\, N_i\, S(S+1)\rho_o\, \Gamma^2 \frac{1}{2|\omega|} - \right.$$

$$\left. - 8\pi\, N_i\, S(S+1)\,\rho_o^2\,\Gamma^3\, T \sum_{\omega'} \frac{\pi}{\omega-\omega'}\left(\mathrm{sg}\omega - \mathrm{sg}\omega'\right)\frac{1}{2|\omega'|}\right\} \qquad /4.6/$$

where on the right hand side the second and third term corresponds to the elastic and inelastic scattering, respectively. The notation Γ is given by eq. (3.13). Considering the identity [8]

$$T \sum_{\omega_n'} \frac{\pi}{\omega_n - \omega_n'}\left(\mathrm{sg}\omega_n - \mathrm{sg}\omega_n'\right)\frac{1}{2|\omega_n'|} = \frac{1}{2|\omega_n|}\,\bar{\Psi}\,(2n+1) \qquad /4.7/$$

where

$$\bar{\Psi}\,(2n+1) = \Psi\,(n+1) - \Psi\,(1/2) \qquad /4.8/$$

and $\Psi(x)$ is the digamma function, we get for K_ω

$$K_{\omega_n} = \frac{1}{2|\omega_n|}\left\{1 - 4\pi N_i S(S+1)\rho_o \Gamma^2 \frac{1}{2|\omega_n|}\left(1 + 2\rho_o \Gamma \bar{\Psi}(2n+1)\right)\right\} \qquad /4.9/$$

Inserting eq. (4.9) into (4.3) we get the final formula for T_c

$$\ln \frac{T_{co}}{T_c} = \frac{N_i S(S+1)\rho_o J^2}{T_c N^2} \sum_{n=o}^{\infty} \frac{1}{(2n+1)^2} \left|1 + \frac{2J\rho_o}{N} \ln \frac{D}{T_c}\right|^{-2} \cdot$$

$$\cdot \left\{4 + 8 \frac{J\rho_o}{N} \Psi(2n+1) \left|1 + \frac{2J\rho_o}{N} \ln \frac{D}{T_c}\right|^{-1}\right\} \qquad /4.10/$$

The first term in the curly brackets arises from the elastic scattering, while the other one from the inelastic scattering. The expression for the change in T_c is essentially that obtained by Zuckermann [8] if the approximation $\log \frac{D}{(2n+1)\pi T_c} \sim \log \frac{D}{T_c}$ is applied. This approximation is correct in the framework of logarithmic approximation if n is a small integer. Due to the factor $(2n+1)^{-2}$ in eq. (4.10) only the small values of n play important role in the sum. There are some differences in the numerical factors. In Zuckermann's notation the elementary vertex is $\frac{J}{2N}$ instead of $\frac{J}{N}$ and comparing the results we must take this difference into account. So we obtain that in our calculation the elastic contribution is 2 times, the inelastic contribution 4 times larger than those in Zuckermann's paper. The reason of this difference is not clear, it might be the result of the decoupling procedure used in Ref. 8. or some algebraic mistake. In the second and third order eq. (4.10) reduces to the result of the perturbational calculation [6] and the contribution of the elastic scattering is in agreement with Griffin's result [5].

V. Conclusion

Investigating the effective electron-electron interaction due to the virtual excitation of quasiparticles e.g. phonons, in the second order of the perturbation theory the interaction is shown to be attractive, if the energy of the quasi-particle $h\omega_o >> |\varepsilon_k - \varepsilon_{k+\kappa}|$, where ε_k and $\varepsilon_{k+\kappa}$ are the energies of the initial and final electron states. In the case of magnetic impurities without external magnetic field the excitation energy is zero and the interaction is repulsive in the second order. In higher orders, however, a new term appears which depending on the sign of the s-d interaction gives attraction or repulsion between the electrons.

Our result in eq. (4.10) is valid only above the Kondo temperature, as we have used the vertex function proposed by Abrikosov [3]. For other temperatures the vertex function given by eq. (3.13) must be replaced by an expression which is valid not only in the logarithmic approximation. Nevertheless, we can make a remark comparing our result with that of Zuckermann [8].

Using the usual notation for the Kondo temperature

$$T_K = D\, e^{\frac{N}{2J\rho}} \qquad /5.1/$$

eq. (4.10) can be written in the form

$$\ln \frac{T_{co}}{T_c} = \frac{N_i\, S(S+1)}{\rho_o T_c} \sum_{n=o}^{\infty} \frac{1}{(2n+1)^2} \frac{1}{\ln^2(T_k/T_c)} \left[1 + \frac{\bar{\Psi}(2n+1)}{\ln(T_k/T_c)}\right] \qquad /5.2/$$

Zuckermann [8] obtained a similar expression (see.eq. (4.18) in Ref. [8]) even for $T_c \ll T_K$. This shows that there is a symmetry in the change of T_c much above or much below T_K.

Eq. (5.2) is thought to be valid, when $\ln^2 \frac{T_k}{T_c} \gg S(S+1)\pi^2$, which can be seen from e.g. Hamann's solution [11]. Estimating the behaviour of $\ln \frac{T_{co}}{T_c}$ we obtain that T_c is always smaller than T_{co}, but the inelastic processes can essentially reduce (near to T_K almost the half of) the effect of the elastic scattering. This is a consequence of the fact that the inelastic contribution has quite different structure from the elastic one. This shows also that the strict logarithmic approximation, where the inelastic processes are neglected compared to the elastic ones, is appropriate only to estimate the order of magnitude of the effect. When $T_c \sim T_K$, the problem is not only to find an appropriate expression for the vertex function valid not only in logarithmic approximation. Another problem arises in this temperature region, namely to investigate the contribution of the diagrams (see Fig. 4.), where the two electron lines are connected more than twice by the pseudofermion lines. It is not clear, whether these processes are taken into account or not in a Nagaoka type decoupling procedure of the Green functions and therefore it cannot be taken for sure that the decoupling gives the proper change in T_c. The case $T_c \sim T_K$ needs further investigations.

Acknowledgment

We are grateful to Prof. L. Pál for his continuous interest during this work. One of the authors (A.Z.) expresses his gratitude to Prof. P. Fulde and Dr. H. Schmidt for the hospitality in the Laue-Langevin Institute, where the essential part of this work was achived and for stimulating discussions. He would like to thank Prof. K. Maki for an interesting discussion.

Appendix I.

The contribution of a few diagrams of fourth and fifth order

The aim of this calculation is to show that the contribution of the diagrams in Fig. 5. can be neglected compared to the leading logarithmic contributions of the diagrams in Fig. 4., which are proportional to $J^n \ln^{n-2} \frac{D}{|\omega|}$ and $J^n \ln^{n-3} \frac{D}{|\omega|}$ for the elastic and inelastic scattering, respectively.

First we calculate the contribution of the fourth order diagram in Fig. 9. Except for the spin factors the contribution is as follows

$$I^4 = -e^{+\lambda/T} T^3 \sum_{\substack{\omega_1 \omega_2 \\ \omega_3}} \rho_o \int d\xi_1 \, \rho_o \int d\xi_2 \, \frac{1}{i\omega_1 - \lambda} \, \frac{1}{i\omega_2 - \lambda} \, \frac{1}{i\omega_3 - \lambda}$$

$$\frac{1}{i(-\omega+\omega'-\omega_1+\omega_2+\omega_3)-\lambda} \, \frac{1}{i(\omega+\omega_1-\omega_2)-\xi_1} \, \frac{1}{i(-\omega-\omega_1+\omega_3)-\xi_2} \qquad /A.I.1/$$

Performing the summations over the frequencies, taking the limit $\lambda \to \infty$ and integrating over ξ_1

$$I^{(4)} = \rho_o^2 \int d\xi_2 \, \frac{1}{i\omega+\xi_2} \, \frac{1}{i\omega'+\xi_2} \left\{ n_F(\xi_2) \ln \left| \frac{D-\xi_2-i(\omega+\omega')}{D-\xi_2} \right| + n_F(-\xi_2) \ln \left| \frac{D+\xi_2+i(\omega+\omega')}{D+\xi_2} \right| - \right.$$

$$\left. - \ln \left| \frac{\xi_2+i(\omega+\omega')}{\xi_2} \right| + n_F(\xi_2) \ln \left| \frac{D+i\omega}{D-i\omega} \right| + n_F(-\xi_2) \ln \left| \frac{D-i\omega'}{D+i\omega'} \right| \right\} \qquad /A.I.2/$$

Neither in the elastic $(\omega = \omega')$ nor in the inelastic $(\omega \neq \omega')$ part a term of type $\ln \frac{D}{|\omega|}$ appears in the limit $T_c >> T_k$. It means that the contribution of this diagram of fourth order can be neglected compared to the diagrams shown in Fig. 4. which in the fourth order yields a contribution $J^4 \ln^2 \frac{D}{|\omega|}$ and $J^4 \ln \frac{D}{|\omega|}$ for the elastic and inelastic parts, respectively.

Next we investigate the diagram of fifth order in Fig. 10. Its contribution is proportional to

$$I^{(5)} = -T^4 \sum_{\substack{\omega_1 \omega_2 \\ \omega_3 \omega_4}} \rho_o^3 \int d\xi_1 \int d\xi_2 \int d\xi_3 \, \frac{1}{i\omega_1 - \lambda} \, \frac{1}{i\omega_2 - \lambda} \, \frac{1}{i\omega_3 - \lambda} \, \frac{1}{i\omega_4 - \lambda}$$

$$\frac{1}{i(-\omega+\omega'+\omega_1-\omega_2+\omega_3)-\lambda} \, \frac{1}{i(\omega-\omega_1+\omega_4)-\xi_1} \, \frac{1}{i(-\omega+\omega_1-\omega_2)-\xi_2} \cdot \frac{1}{i(\omega-\omega_1+\omega_2-\omega_3+\omega_4)-\xi_3}$$

After a straightforward calculation we obtain

$$I^{(5)} = \rho_o^3 \int d\xi_1 \int d\xi_2 \int d\xi_3 \left(1 - n_F(\xi_3)\right) \left\{ \frac{n_F(\xi_1)}{i\omega - \xi_1} \frac{1 - n_F(\xi_2)}{i\omega + \xi_2} \cdot \right.$$

$$\cdot \frac{1}{i(\omega' - \omega) + \xi_1 - \xi_3} \frac{1}{i\omega - \xi_1 + \xi_2 + \xi_3} + \frac{1}{2} \frac{1 - n_F(\xi_1)}{i\omega - \xi_1} \frac{1 - n_F(\xi_2)}{i\omega' - \xi_3} \frac{1}{\xi_1 + \xi_2} \frac{1}{\xi_2 + \xi_3} +$$

$$+ \frac{n_F(\xi_1)}{\xi_1 + \xi_2} \frac{n_F(\xi_2)}{i\omega + \xi_2} \frac{1}{i\omega' + \xi_1 + \xi_2 - \xi_3} \frac{1}{\xi_1 - \xi_3} +$$

$$+ \{ \text{similar terms with the change } \omega \leftrightarrow \omega' . \} \qquad /A.I.4/$$

Performing the integration in the logarithmic approximation we keep only the highest power of the typical $\ln \frac{D}{|\omega|}$, $\ln \frac{D}{|\omega'|}$ terms and the others like $\ln \left|\frac{\omega}{\omega'}\right|$ $\ln \left|\frac{D + \omega}{D - \omega}\right|$, $\ln \frac{D + |x|}{D}$ will be neglected. In the elastic case the first term gives $\frac{1}{2D} \ln^2 \frac{D}{|\omega|}$. The second term gives the same contribution with a negative sign, while the third term gives no $\ln^2 \frac{D}{|\omega|}$ term. So the contribution of this diagram is proportional to $J^5 \ln \frac{D}{|\omega|}$, which can be again neglected.

As an example we will show the calculation of the second term. In the T=0 limit, after analytical continuation $i\omega \to \omega$

$$I = \frac{1}{2} \rho_o^3 \int_0^D d\xi_1 \int_0^D d\xi_2 \int_0^D d\xi_3 \frac{1 - n_F(\xi_1)}{\omega - \xi_1} \frac{1 - n_F(\xi_2)}{\xi_1 - \xi_2} \frac{1 - n(\xi_3)}{\omega - \xi_3} \frac{1}{\xi_2 + \xi_3} =$$

$$= \frac{1}{2} \rho_o^3 \int_0^D d\xi_1 \int_0^D d\xi_2 \int_0^D d\xi_3 \frac{1}{(\omega + \xi_2)^2} \left(\frac{1}{\omega - \xi_1} + \frac{1}{\xi_1 + \xi_2}\right)\left(\frac{1}{\omega - \xi_3} + \frac{1}{\xi_2 + \xi_3}\right) \simeq$$

$$\simeq \frac{1}{2} \rho_o^3 \int_0^D \alpha\xi_2 \frac{1}{(\omega + \xi_2)^2} \ln^2 \left|\frac{\omega}{\xi_2}\right| \qquad /A.I.5/$$

Here the terms of type $\ln \frac{D + |x|}{D}$ have been neglected. In the logarithmic approximation this integral can be calculated as follows

$$I \simeq \frac{1}{2} \rho_o^3 \int_0^{|\omega|} d\xi_2 \frac{1}{\omega^2} \ln^2 \left|\frac{\omega}{\xi_2}\right| + \frac{1}{2} \rho_o^3 \int_{|\omega|}^D \frac{d\xi_2}{\xi_2^2} \ln^2 \left|\frac{\omega}{\xi_2}\right| \simeq - \frac{1}{2D} \ln^2 \frac{D}{|\omega|}$$

It is easy to show that the inelastic part is also proportional at last to $J^5 \ln \frac{D}{|\omega|}$ and can be neglected. This will be demonstrated on the second term

$$I = \frac{1}{2}\rho_o^3 \int_0^D d\xi_1 \int_0^D d\xi_2 \int_0^D d\xi_3 \frac{1}{\omega-\omega'}\left(\frac{1}{\omega'-\xi_2} - \frac{1}{\omega-\xi_2}\right)\left(\frac{1}{\omega-\xi_1} + \frac{1}{\xi_1+\xi_2}\right)\left(\frac{1}{\omega'-\xi_3} + \frac{1}{\xi_2+\xi_3}\right) \simeq$$

$$\simeq \frac{1}{2}\rho_o^3 \int_0^D d\xi_2 \frac{1}{\omega-\omega'}\left(\frac{1}{\omega'-\xi_2} - \frac{1}{\omega-\xi_2}\right)\left(\ln\frac{|\omega|}{D} + \ln\frac{D}{\xi_2}\right)\left(\ln\frac{|\omega'|}{D} + \ln\frac{D}{\xi_2}\right) \simeq$$

$$\simeq \frac{1}{2}\rho_o^3 \frac{1}{\omega-\omega'}\left[\left(\ln\frac{|\omega'|}{D} - \ln\frac{|\omega|}{D}\right)\ln\frac{|\omega|}{D}\ln\frac{|\omega'|}{D} - \right.$$

$$- \left(\ln\frac{|\omega|}{D} + \ln\frac{|\omega'|}{D}\right)\frac{1}{2}\left(\ln^2\frac{|\omega'|}{D} - \ln^2\frac{|\omega|}{D}\right)$$

$$\left. + \frac{1}{3}\left(\ln^3\frac{|\omega'|}{D} - \ln^3\frac{|\omega|}{D}\right)\right] \simeq$$

$$\simeq \frac{1}{2}\rho_o^3 \frac{1}{\omega-\omega'}\left(\ln\frac{|\omega'|}{D} - \ln\frac{|\omega|}{D}\right)\left[-\frac{1}{6}\ln^2\frac{|\omega'|}{D} + \frac{1}{3}\ln\frac{|\omega'|}{D}\ln\frac{|\omega|}{D} - \frac{1}{6}\ln^2\frac{|\omega|}{D}\right] \simeq$$

$$\simeq -\frac{1}{12}\rho_o^3 \frac{1}{\omega-\omega'}\left(\ln\frac{|\omega'|}{D} - \ln\frac{|\omega|}{D}\right)^3 = -\frac{1}{12}\rho_o^3 \frac{1}{\omega-\omega'}\ln^3\left|\frac{\omega'}{\omega}\right|$$

The $\ln \frac{D}{|\omega|}$ terms disappear.

The fifth order diagram shown in Fig.5.c. can be obtained from the fourth order diagram in Fig. 5.a. by inserting a vertex of second order instead of a bare vertex and therefore its contribution is at last $J^5 \ln \frac{D}{|\omega|}$.

So all the diagrams with more than two connecting pseudofermion lines will be neglected.

Appendix II.

Investigation of the cuts of the vertex function

The simplest diagrams in Λ_1 are the chain diagrams (see Fig.11.) Its contribution is given by

$$\Lambda_1^{(n)} = \left(\frac{J}{N}\right)^n \rho_o^{n-1} T^{n-1} \sum_{\omega_2 \dots \omega_n} \int d\xi_1 \dots d\xi_{n-1}$$

$$\mathcal{G}(i\omega_2)\, \mathcal{G}\left(\xi_1, i(\omega+\omega_1-\omega_2)\right) \dots \mathcal{G}(i\omega_n)\, \mathcal{G}\left(\xi_{n-1}, i(\omega+\omega_1-\omega_n)\right) =$$

$$= \frac{J}{N}\left(\frac{J}{N}\rho_o\right)^{n-1} \int d\xi_1 \dots d\xi_{n-1} \frac{n_F(\xi_1)-1}{i(\omega+\omega_1)-\lambda-\xi_1} \dots \frac{n_F(\xi_{n-1})-1}{i(\omega+\omega_1)-\lambda-\xi_{n-1}} \qquad /A.II.1./$$

This vertex has a cut only along $Imz = -\omega$.

This is not true for arbitrary vertices, but it is shown that only this cut gives important contribution in logarithmic approximation. Let us see the third order vertex shown in Fig. 12. The contribution of this diagram is as follows

$$\Lambda_1^{(3)} = \left(\frac{J}{N}\right)^3 \rho_o^2 T^2 \sum_{\omega_2,\omega_3} \int d\xi_2\, d\xi_3$$

$$\mathcal{G}(i\omega_2)\, \mathcal{G}(i\omega_3)\, \mathcal{G}\left(\xi_1, i(\omega+\omega_3-\omega_2)\right) \mathcal{G}\left(\xi_2, i(\omega+\omega_1-\omega_2)\right) \qquad /A.II.2/$$

Performing the summation over the frequencies ω_2 and ω_3, in the limit $\lambda \to \infty$ we get

$$\Lambda_1^{(3)} = -\left(\frac{J}{N}\right)^3 \rho_o^2 \int d\xi_1\, d\xi_2 \frac{n_F(\xi_1)}{i\omega_1+\xi_1-\xi_2-\lambda} \frac{1-n_F(\xi_2)}{i(\omega+\omega_1)-\xi_2-\lambda} \qquad /A.II.3/$$

The cuts are at $Imz=0$ and $Imz = -\omega$. At the first cut

$$\Delta\Lambda_1^{\,3} = -\left(\frac{J}{N}\right)^3 \rho_o^2 \int d\xi_1\, d\xi_2\, n_F(\xi_1) \left| \frac{1}{x+i\varepsilon+\xi_1-\xi_2} - \frac{1}{x-i\varepsilon+\xi_1-\xi_2} \right| \frac{1-n_F(\xi_2)}{i\omega+x-\xi_2} =$$

$$= 2i\pi \left(\frac{J}{N}\right)^3 \rho_o^2 \int d\xi_1\, d\xi_2\, n_F(\xi_1) \frac{1-n_F(\xi_2)}{i\omega-\xi_1} \delta\left(x+\xi_1-\xi_2\right) \qquad /AII.4/$$

where $x = Re z - \lambda$ and the operation Δ is given by eq. (3.7). Substituting this expression into the effective interaction, the contribution of this cut in the corresponding fourth order diagram (diagram in Fig. 4., where the upper vertex is given in Fig. 12. and lower one is a bare vertex) is proportional to

$$V \sim \frac{e^{\beta\lambda}}{2S+1} \int_{-\infty}^{\infty} \frac{dx}{2\pi i}\, 2\pi i \left(\frac{J}{N}\right)^3 \rho_o^2 \int d\xi_1\, d\xi_2\, n_F(\xi_1) \frac{1-n_F(\xi_2)}{i\omega-\xi_1} \cdot$$

$$\left(x+\xi_1-\xi_2\right) n_F\left(x+\lambda\right) \frac{1}{x} \frac{1}{i(\omega-\omega')+x} =$$

$$= \frac{1}{2S+1} \left(\frac{J}{N}\right)^3 \rho_o^2 \int d\xi_1\, d\xi_2\, n_F(\xi_2)\left(1-n_F(\xi_1)\right) \frac{1}{i\omega-\xi_1} \frac{1}{i(\omega-\omega')} \left(\frac{1}{\xi_2-\xi_1} - \frac{1}{i(\omega-\omega')+\xi_2-\xi_1} \right)$$

$$\simeq \frac{1}{2S+1} \left(\frac{J}{N}\right)^3 \rho_o^2 \int d\xi_1 \left(1-n_F(\xi_1)\right) \frac{1}{i\omega-\xi_1} \frac{1}{i(\omega-\omega')} \left| \ln\left|\frac{\xi_1}{\xi_1+i(\omega'-\omega)}\right| - \ln\left|\frac{D-\xi_1}{D-\xi_1+i(\omega'-\omega)}\right| \right|$$

/A.II.5/

This integral gives no term of $\ln\frac{D}{|\omega|}$ type. On the other hand the cut at $Im z = -\omega$ will give logartihmic contribution. Along this cut

$$\Delta\Lambda_1^{\,3} = -\left(\frac{J}{N}\right)^3 \rho_o^2 \int d\xi_1\, d\xi_2\, \frac{n_F(\xi_1)}{x-i\omega+\xi_1-\xi_2} \left(1-n_F(\xi_2)\right)\left(\frac{1}{x+i\varepsilon-\xi_2} - \frac{1}{x-i\varepsilon-\xi_2}\right) =$$

$$= 2i\pi \left(\frac{J}{N}\right)^3 \rho_o^2 \int d\xi_1 \quad d\xi_2\, \frac{n_F(\xi_1)}{\xi_1-i\omega} \left(1 - n_F(\xi_2)\right) \delta\left(x - \xi_2\right) \qquad /A.II.6./$$

The integration with respect of ξ_1 yields the typical logarithmic term. Supposing that $\omega = (2n+1)\ \pi T \sim T$ i.e. n is a small integer we get in logarithmic approximation

$$\Delta\Lambda_1^{\,3}(x) = \begin{cases} -2i\pi\left(\frac{J}{N}\right)^3 \rho_o^2 \log\frac{D}{T} & \text{if} \quad D > |x| > 0 \\ 0 & \text{otherwise} \end{cases} \qquad /A.II.7/$$

Inserting this expression into the effective interaction (3.4) this logarithmic term will not be affected.

From this calculation we can learn that in Λ_1 the important cut, which gives the logarithmic term of highest order, is along $\mathrm{Im}z = -\omega$ and the contribution of the other cuts can be neglected. We can see further that in logarithmic approximation only the imaginary part of Λ_1 contributes to $\Delta\Lambda_1$, as

$$\Delta\Lambda_1 = 2i \ \mathrm{Im}\ \Lambda_1 (x + i\varepsilon) \qquad /A.II.8/$$

what can be proved very easily.

Similarly in Λ_2 the important cut belongs to the variable $i(\omega-\omega')$ and in the z plane it lies along the line $\mathrm{Im}z = \omega'$.

Appendix III

Calculation of the vertices at the important cut

As it is shown in Appendix II the important cut of Λ_1 is in the variable $i(\omega + \omega_1)$ and arises from those pairs of an electron and pseudofermion lines at which Λ_1 can be cut into two parts. One can verify in logarithmic approximation that $\Delta\Lambda_1$ at the cut is pure imaginary. In the following the imaginary part of Λ_1 will be calculated. To get the highest order of the logarithmic term we must calculate the imaginary part of the contributions of each pairs of two lines and the real part of the other parts of the diagram. It can be carried out in a formal way calculating the diagram in Fig. 7.a. In this calculation only the real parts of the vertices are to be considered.

The contribution of the diagram in Fig. 7.a. is

$$T\rho_o \sum_{\omega_2} \int d\xi_1 \, \Gamma_{\alpha\varepsilon\alpha'\varepsilon'}\left(i\omega, i\omega_1; i(\omega + \omega_1 - \omega_2), i\omega_2\right) \cdot$$

$$\mathcal{G}_\varepsilon(i\omega_2)\,\mathcal{G}_{\alpha'}\left(\xi_1, i(\omega + \omega_1 - \omega_2)\right)\Gamma_{\alpha'\varepsilon'\beta\eta}\left(i(\omega + \omega_1 - \omega_2), i\omega_2; i\omega', i\omega_1'\right)$$

and performing the summation /A.III.1/

$$-\rho_o \int d\xi_1 \, \Gamma_{\alpha\varepsilon\alpha'\varepsilon'}\left(i\omega, i\omega_1; \xi_1, i(\omega + \omega_1) - \xi_1\right) \cdot$$

$$\cdot \frac{1 - n_F(\xi_1)}{i(\omega+\omega_1) - \lambda - \xi_1} \Gamma_{\alpha'\varepsilon'\beta\eta}\left(\xi_1, i(\omega + \omega_1) - \xi_1; i\omega', i\omega_1'\right) \qquad /A.III.2/$$

Supposing that $i\omega$ and $i\omega'$ are of order T they can be replaced by zero if the thermal smearing is considered. We are interested in the imaginary part of the analytical continuation of Λ_1 $(i\,\omega_1 \to z)$ along the line $z = -i\omega + x\;\lambda + i\varepsilon$. According to the consideration given here we obtain

$$\operatorname{Im} \Lambda_{1\,\alpha\varepsilon\gamma\eta}(x \pm i\varepsilon) = \begin{cases} \pm \rho_o \pi \operatorname{Re} \Gamma_{\alpha\varepsilon\alpha'\varepsilon'}\left(0, x+\lambda; x, \lambda\right) & \text{for } |x| < D \\ \quad \cdot \left(1 - n_F(x)\right) \cdot & \\ \quad \cdot \operatorname{Re} \Gamma_{\alpha'\varepsilon'\gamma\eta}\left(x, \lambda; 0, x+\lambda\right) & \\ 0 & \text{for } |x| > D \end{cases} \qquad /A.III.3/$$

Similar calculation for Λ_2 on the line $z=i\omega' + x + \lambda \pm i\varepsilon \quad (i(\omega_1-\omega') \to z)$ yields

$$\mathrm{Im}\,\Lambda_{2\,\alpha\varepsilon\gamma\eta}(x\pm i\varepsilon) = \mp \rho_o \pi\, \mathrm{Re}\;\Gamma_{\alpha\varepsilon'\alpha'\eta}(0,\lambda;-x,x+\lambda)\,.$$

$$\cdot\left(1 - n_F(x)\right)\mathrm{Re}\;\Gamma_{\alpha'\varepsilon\gamma\varepsilon'}(-x,x+\lambda;0,\lambda) \qquad \text{/A.III.4/}$$

$$\text{if} \quad |x| < D$$

where the identity $n_F(-x)=1-n_F(x)$ is used.

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Figure Captions

Fig. 1. Diagram of the effective electron-electron interaction

Fig. 2. Electron-electron interaction diagrams of second (a) and of third (b-c) order

Fig. 3. Ladder diagram

Fig. 4. Diagram of the electron-electron interaction to be calculated in logarithmic approximation

Fig. 5. Diagrams of the electron-electreon interaction to be neglected in logarithmic approximation

Fig. 6. The cuts on the complex energy plane

Fig. 7. Vertex corrections of parquet type

Fig. 8. Diagram which can be ordered to the correct result in a formal way

Fig. 9. Correction of fourth order

Fig.10. Correction of fifth order

Fig.11. Vertex correction of ladder type

Fig.12. Third order vertex correction of parquet type.

FIG. 1.

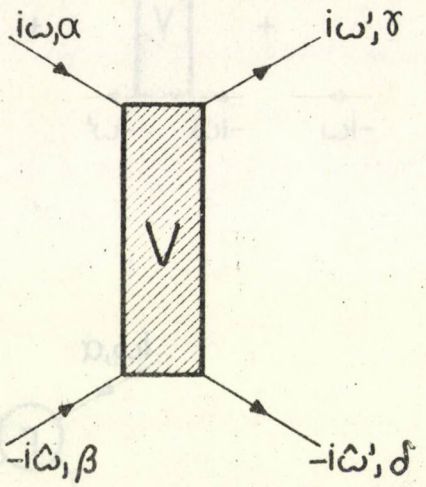


FIG. 2.a-c.

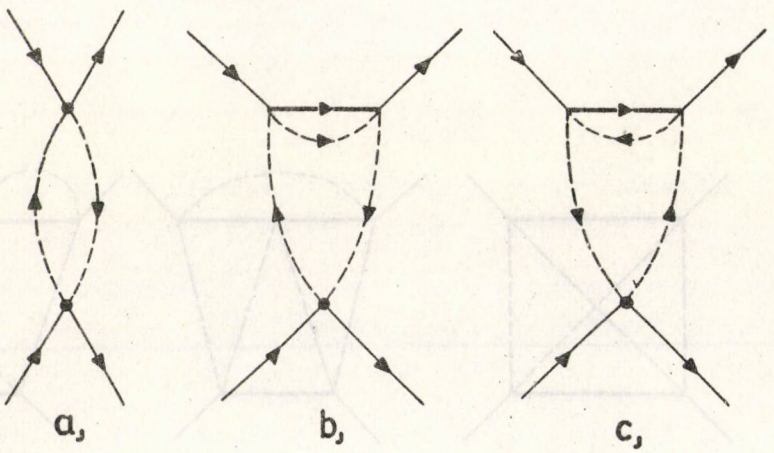


FIG. 3.

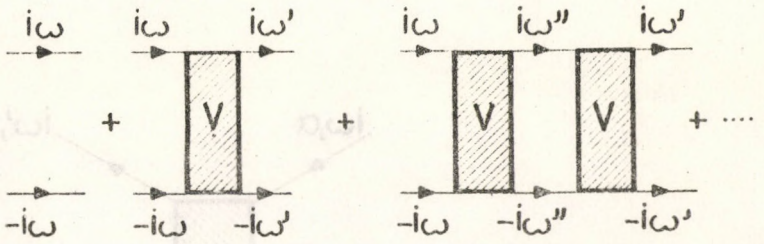


FIG. 4.

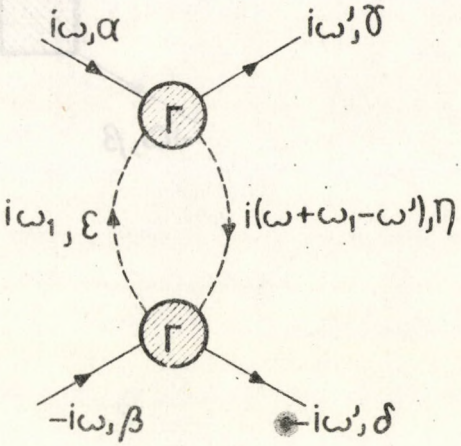
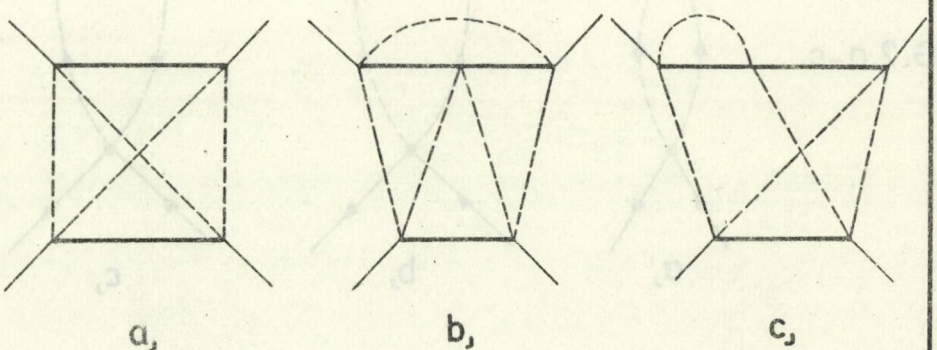


FIG. 5



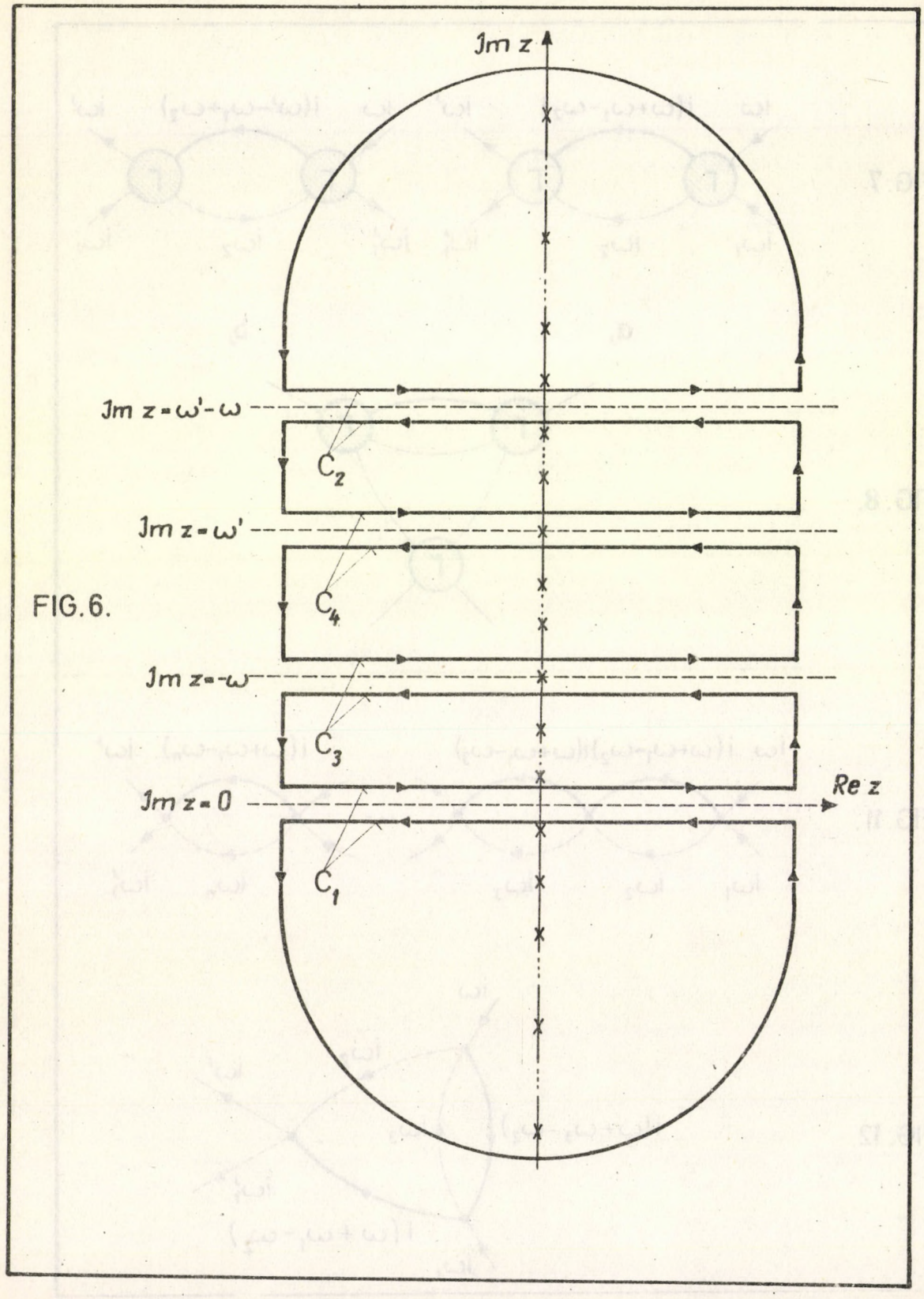


FIG. 6.

FIG. 7.

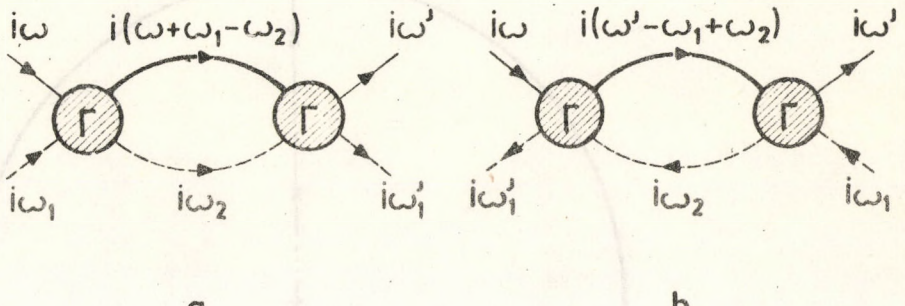


FIG. 8.

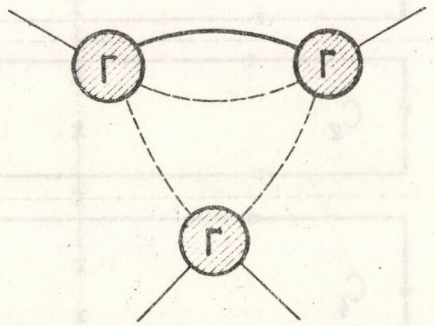


FIG. 11.

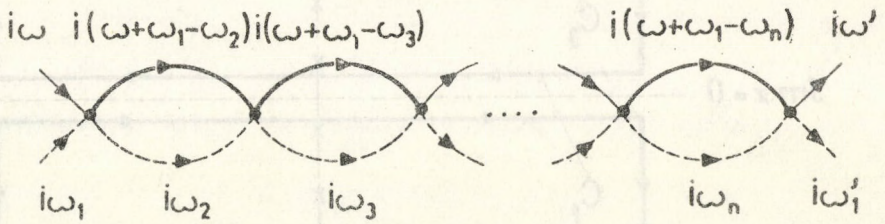
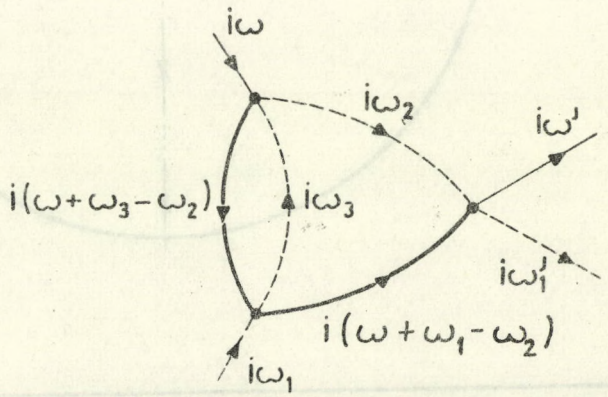


FIG. 12.



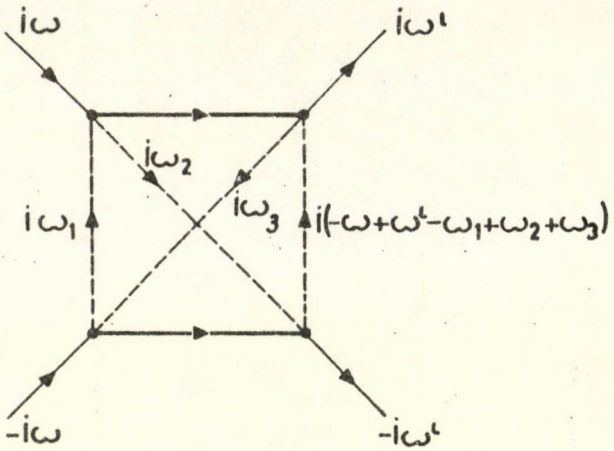


FIG. 9.

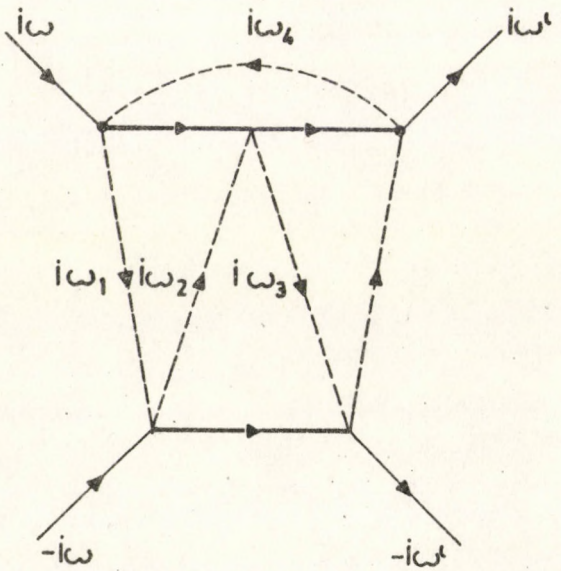


FIG. 10.

Printed in the Central Research Institute for Physics, Budapest

Kiadja a KFKI Könyvtár- és Kiadói Osztály. Gv.: Dr.Farkas Istvánné
Szakmai lektor: Hargitai Csaba. Nyelvi lektor: Menyhárd Nóra
Példányszám: 225 Munkaszám: KFKI 4189 Budapest, 1969. január 20.
Készült a KFKI házi sokszorositójában. Fv.: Gyenes Imre

01.807