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BUDAPEST

π-N RESONANCE WIDTHS IN THE BROKEN SL/2,C/ MODEL

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Abstract

The elastic decay width of some πN resonances is evaluated in the SL/2,C/ model of Regge-poles. Two families of resonances are examined in the first order of symmetry breaking, one of them has isotopic spin $I = \frac{1}{2}$, the other $I = \frac{3}{2}$. The width of other resonances along the trajectories is calculated in symmetry limit and the differential cross section is examined for πN backward scattering. The results are in a good agreement with experiments.

I

The analiticity problems of the Regge-theory at $u = 0$ for unequal mass scattering led the physicists to the discovery of a higher symmetry of the Regge-poles, the SL/2,C/ one [1]. This symmetry manifests itself in grouping the poles into families at $u = 0$. Near $u = 0$ the SL/2,C/ symmetry is broken, the breaking mechanism was elaborated by Domokos and Surányi [2]. They have applied it successfully to classify the πN resonances [3]. The aim of this paper is to calculate the elastic widths of those resonances.

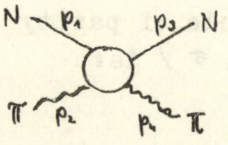
II

The elastic decay width of π-N resonances is given by the formula

$$\Gamma_{el} = \frac{1}{2\pi} \int dp\, dq\, \delta^4(P-p-q) \left| \langle N^*(P) | T | \pi(q) N(p) \rangle \right|^2 . \qquad (1)$$

In what follows we shall calculate the transition matrix elements making use of the SL/2,C/ symmetry of the Regge-poles. $\langle N^* | T | \pi N \rangle = g_J(P^2)$ can be continued analytically not only in P^2 but in J, the spin of N^* as well/the kinematical singularities are separable/, and it is evident that this quantity is nothing else but the vertex function of a N^* type Regge-pole at $J = \alpha(P^2)$. The fact that the Regge-poles are grouped into families makes possible to connect the residua of the daughters.

Let us consider now the πN backward-scattering amplitude at $u=0$, $s \gg 0$.



$$s = (p_1 + p_2)^2 \qquad u = (p_1 + p_4)^2 = P^2$$

$$2q = p_1 - p_4 \qquad 2q' = p_2 - p_3$$

The Lorentz-pole terms, giving the main contribution to the amplitude can be written as [4]

$$F_{\lambda\mu}(s, u=0) \sim \sum_{i, j_0} \frac{1}{8\pi} \frac{(\sigma_i^2 - j_0^2)}{\cos\pi\sigma_i} \xi_i \mathcal{D}^{j_0 \sigma_i}_{\frac{1}{2}\lambda \frac{1}{2}\mu} \left(L_q L_{q'}^{-1} \right) T^{\sigma_i j_0} \qquad (2)$$

$\xi_i = \frac{1}{2}\left(1 + \tau \exp(i\pi(\sigma - \frac{1}{2}))\right)$, τ is the signature factor, $\Gamma^{j_0\sigma}$ and $\Gamma^{*j_0\sigma}$ are the factorized residua of the Lorentz-poles, $T^{j_0\sigma} = \Gamma^{j_0\sigma} \Gamma^{j_0\sigma *}$.

Further we can write

$$T^{j_0\sigma}\mathcal{D}^{j_0\sigma}_{s\lambda s'\lambda'}(L_q L_{q'}^{-1})=\sum_{jm}\langle j_0\sigma\, jm|T|j_0\sigma\, jm\rangle \mathcal{D}^{j_0\sigma}_{s\lambda jm}(L_q)\,\mathcal{D}^{j_0\sigma}_{jm s'\lambda'}(L_{q'}^{-1})$$

$$=\sum_{jm}\langle j_0\sigma\, jm|T|j_0\sigma\, jm\rangle d^{j_0\sigma}_{s\lambda j}\left(\frac{q_0}{q}\right) d^{j}_{\lambda\lambda'}\left(\frac{q\cdot q'}{|q||q'|}\right) d^{j_0\sigma}_{j\lambda' s}\left(\frac{q_0'}{q'}\right). \qquad (3)$$

Here $(j_0\,\sigma\; jm)$ are the Lorentz quantum numbers of the Regge-poles what "intermediate" between the initial and final states. The parity is a good quantum number of the poles, so we diagonalize in it. The parity operator P acts on a state $|j_0\sigma\, jm\rangle$ as $P|j_0\sigma\, jm,s\rangle=\eta(-1)^{j-s}|-j_0\sigma\, jm,s\rangle$. So we introduce the parity eigenstates as

$$|j_0\sigma\, jm,s,\pm\rangle=\frac{1}{\sqrt{2}}\left(|j_0\sigma\, jm,s\rangle\pm\eta(-1)^{j-s}|-j_0\sigma\, jm,s\rangle\right).$$

We need not label the reduced matrix element of T with the parity quantum number because

$$\langle j_0\sigma\, jm,+|T|j_0\sigma\, jm,+\rangle=\langle j_0\sigma\, jm,-|T|j_0\sigma\, jm,-\rangle=\eta(-1)^{j-s}\langle -j_0\sigma\, jm,+|T|j_0\sigma\, jm,+\rangle=-\eta(-1)^{j-s}\langle j_0\sigma\, jm|T|-j_0\sigma\, jm,-\rangle.$$

This is not miraculous, the III. class conspiracy means the same: the residua of the parity doublets are equal at $u=0$. Introducing the parity quantum number into eq. /3/ we can write:

$$T^{j_0\sigma}\mathcal{D}^{j_0\sigma}_{s\lambda s'\lambda'}+T^{-j_0\sigma}\mathcal{D}^{-j_0\sigma}_{s\lambda s'\lambda'}=$$

$$\sum_{jm}\frac{1}{\sqrt{2}}\Big\{\left(d^{j_0\sigma}_{s\lambda j}+\eta(-1)^{j-s}d^{-j_0\sigma}_{s\lambda j}\right){}_s\langle j_0\sigma\, jm,+|T|j_0\sigma\, jm,+\rangle_{s'}\left(d^{j_0\sigma}_{j\lambda' s'}+\eta'(-1)^{j-s'}d^{-j_0\sigma}_{j\lambda' s'}\right)+$$
$$+\left(d^{j_0\sigma}_{s\lambda j}-\eta(-1)^{j-s}d^{-j_0\sigma}_{s\lambda j}\right){}_s\langle j_0\sigma\, jm,-|T|j_0\sigma\, jm,-\rangle_{s'}\left(d^{j_0\sigma}_{j\lambda' s'}-\eta'(-1)^{j-s'}d^{-j_0\sigma}_{j\lambda' s'}\right)\Big\}. \qquad (4)$$

We suppose that the Lorentz-residuum $T^{j_0\sigma}_{ss'}$ is factorisable: $T^{j_0\sigma}=\Gamma^{j_0\sigma}_s\Gamma^{j_0\sigma *}_{s'}$, hence if we compare eqs. /2/ and /4/ to the ordinary Regge-decomposition we obtain that the residuum of a pole of parity P, being the $\varkappa$-th member of a family, labelled by /j_0, σ/ is:

$$\beta^{\pm}_{\sigma-1-\varkappa}=\Gamma^{j_0\sigma}_s\left(d^{j_0\sigma}_{s\lambda\,\sigma-1-\varkappa}\pm\eta(-1)^{\sigma-1-\varkappa-s}d^{-j_0\sigma}_{s\lambda\,\sigma-1-\varkappa}\right) \qquad (5)$$

where s is the total spin and λ the total helicity of the in /out/ going state what the pole is coupled to.

Up to now we are stuck to the point $u=0$; we apply the SL/2,C/ symmetry breaking method [2] to go to the region of resonances. We shall work in the first order of the symmetry breaking. So we write the scattering amplitude as it is done in eq. 14 of [2a] and separate the residue in the same way as we did in the symmetry limit. The result is:

$$\beta_{\mu\lambda}(N^* \to N\pi) = \frac{1}{\sqrt{2}} C_I(N^*, N\pi)\Big\{ A\Big(d^{j_0\,\sigma}_{J\lambda\frac{1}{2}}(x) \pm (-1)^{J+\frac{1}{2}} d^{-j_0\,\sigma}_{J\lambda\frac{1}{2}}(x)\Big) +$$

$$+W\Big[B\, C^{j_0\,\sigma+1}_{SL(2,C)}\Big(d^{j_0\,\sigma+1}_{J\lambda\frac{1}{2}}(x) \pm (-1)^{J+\frac{1}{2}} d^{-j_0\,\sigma+1}_{J\lambda\frac{1}{2}}(x)\Big) + C\, C^{j_0\,\sigma-1}_{SL(2,C)}\Big(d^{j_0\,\sigma-1}_{J\lambda\frac{1}{2}}(x) \pm (-1)^{J+\frac{1}{2}} d^{-j_0\,\sigma-1}_{J\lambda\frac{1}{2}}(x)\Big) +$$

$$+D\, C^{j_0-1\,\sigma}_{SL(2,C)}\Big(d^{j_0-1\,\sigma}_{J\lambda\frac{1}{2}}(x) \pm (-1)^{J+\frac{1}{2}} d^{-j_0+1\,\sigma}_{J\lambda\frac{1}{2}}(x)\Big)\Big]\Big\} d^{J}_{\mu\lambda}(\vartheta) \qquad (6)$$

and

$$\Gamma(N^* \to N\pi) = \frac{1}{2\pi}\frac{1}{2J+1}\frac{m_N}{M}\, p \sum_{\mu\lambda} |\beta_{\mu\lambda}|^2 .$$

In the case of πN system we have only three breaking terms because of the constraint for the symmetry limit: $|j_0 = \frac{1}{2}$. In eq. /6/ J is the spin-parity of the resonance, μ is its spin-projection quantized along the z-axis of a coordinate system in which the three-momentum of the N^* is zero, λ is the helicity of the nucleon. The index σ is a half integer denoting the actual family to which the resonance belongs. W stands for the mass of the resonance, $W=M$ for the resonances of natural parity, and $W=-M$ for those of unnatural parity. As it can be easily seen [5]:

$$x = \frac{1}{4sq^2}\Big[m_N^2 - m_\pi^2 - \sqrt{-4sq^2 + (m_N^2 - m_\pi^2)^2}\Big]^2 = \frac{(m_N^2 - m_\pi^2)^2}{4\,sq^2}\Big[1 - \frac{2Wp}{m_N^2 - m_\pi^2}\Big]^2$$

where $s = W^2 = M^2$, $4q^2 = 2/m_N^2 + m_\pi^2/ - s$ and p is the magnitude of the three-momentum of the pion and nucleon in the final state.

Now we have to speak a few words about the "reduced matrix elements" A, B, C, D. As an example we take A. It consists of a $\sqrt{\sigma^2 - j_0^2}$ factor, and a function $A'/s/$. For compensating the singularity of the $d^{j_0\sigma}$ functions at the point $s = 2/m_N^2 + m_\pi^2/$ we write $A'/s/$ as

$$A'(s) = \Big(\frac{4q^2}{s_0}\Big)^{\frac{1}{2}(\sigma - 1)} g(s) \qquad (7)$$

and suppose $g/s/$ to be a smooth function of s.

Finally we notice the factor $1 \pm (-1)^{J+\frac{1}{2}}\sqrt{x}$ in $\beta_{\mu\lambda}$ coming from the combination $d^{\frac{1}{2}\sigma} \pm (-1)^{J+\frac{1}{2}} d^{-\frac{1}{2}\sigma}$. To have the well known threshold behaviour we define the physical sheet by the prescription:

$$\sqrt{x} = \frac{m_N^2 - m_\pi^2}{2W\sqrt{q^2}}\Big(1 - \sqrt{-4sq^2 + (m_N^2 - m_\pi^2)^2}\Big)$$

for the resonances of natural parity / W=M/, and

$$\sqrt{x} = \frac{m_N^2 - m_\pi^2}{-2W\sqrt{q^2}}\left(1-\sqrt{-4sq^2+(m_N^2-m_\pi^2)^2}\right)$$

for the resonances of unnatural parity /-W=M/.

C_I and $C^{j_0\sigma}_{SL(2,C)}$ in eq. /6/ are isospin and SL/2,C/ Clebsch-Gordan coefficients, $C^{j_0\sigma'}_{SL(2,C)} = \langle \sigma' j_0' j m ; 1\,0\,0\,0 | \sigma j_0 j m \rangle$.

The following interpretation is differing from that of eq. /6/ in [2]; however it was pointed out for us by the authors of [2]. In the original form of eq. /6/ every quantity is to be taken at u=0. But this is not necessary, as can be seen considering the following.

An $F_{\lambda\mu\lambda'\mu'}$ scattering amplitude is the function of the six invariants P^2, Pq, Pq', qq', q^2, q'^2. When introducing $F_{\lambda\mu\lambda'\mu'}$ over a group, we sought a group G so that if $g \in G$, $gP=P$, but $gq \neq q$. If $P=0$, this group is the SL/2,C/. If $P \neq 0$, only the Pq, Pq' type quantities break the invariance but P^2 does not. This way, we expand $F_{\lambda\mu\lambda'\mu'}$ into Taylor-series in Pq, Pq', but in P^2 not; that is to say in eq. /6/ every quantity has a P^2-dependence. The further steps are the same as in the previous case, so the final form remains the same.

The five unknown functions what would be in the general case, in eq. /6/ can be chosen to be real: at u=0 where only the symmetric term is not zero, the trajectory is real so the residue is real as well. As we neglect the imaginary part of the trajectory throughout our calculation, it is consistent to take the residua to be real. /There is another argumentation, leading to the same result. The first derivative of the residue-function, that gives the first order symmetry breaking term, transforms as a vector. But only two types of vectors can be composed out of the operators we have: q_μ and Γ_μ ; each yields a complex parameter, so the total number of the parameters is four./

III

After summarizing the main points we apply the method for getting the elastic decay width of πN resonances. For numerical calculations we have chosen the $I=\frac{3}{2}$, $\sigma=\frac{9}{2}$ and $I=\frac{1}{2}$, $\sigma=\frac{7}{2}$ families classified in [3]. To reduce the work we have taken degenerate masses in the families, except for calculating the phase spaces. The central masses were got from the symmetry limit of the trajectory formula fitted in [3].

a. $I = \frac{3}{2}$.

The central mass is $M_0 = 1,94$ GeV. From a least squares fit we got the following values for the parameters being defined as $a = \frac{1}{\sqrt{2\pi}} x^{\frac{i}{2}(\sigma-1)} A$, $b = \frac{1}{\sqrt{2\pi}} x^{\frac{i}{2}\sigma} B$ etc.: $a = 2,15$, $b = -0,40$, $c = 0,00$, $d = -0,45$.
In the symmetry limit $a = 2,29$. The results for the widths, summarized in Table I, are in a good agreement with the experiment. The prediction for the width of the missing G_{37} resonance is done with the same mass value as that of F_{37}.

b. $I = \frac{1}{2}$.

The central mass is $M_0 = 1,66$ GeV. Parameters: $a = 2,60$, $b = 1,25$, $c = -0,48$, $d = 0,53$. In the symmetry limit $a = 2,56$. As it can be seen from Table II, there is problem about the S_{11} resonance. Either the $\Gamma_{el} = 186$ MeV is right for the $N^*/1550/$, or the resonance $N^*/1700/$ belongs to the $\sigma = \frac{7}{2}$ family.

To get informations on the s-dependence of the function g/s/ in /7/, we evaluated some other elastic widths in symmetry limit supposing s to be constant. We got:

c. $I = \frac{3}{2}$

	$\Gamma_{theo.}$ (MeV)	$\Gamma_{exp.}$ (MeV) [7]
$P_{33}/1236/$	$6.29\, g^2\, s_o^2 = 120$	120
$H_{311}/2420/$	$2.30 \cdot 10^5 g^2\, s_o^{-2} = 30$	34
$J_{315}/2850/$	$1.28 \cdot 10^8 g^2\, s_o^{-4} = 80$	13
$L_{319}/3230/$	$9.79 \cdot 10^{10} g^2\, s_o^{-6} = 300$	2

For the neighbours P_{33} and H_{311} of the fitted family taking the same value of g/s/ as it is at $s = 1,94^2$ GeV^2, $\sqrt{20}\, g(1,94^2) = 0,22$ we got nearly the right widths if $s_o = 20$ GeV^2. If we hope a qualitatively nice picture in the symmetry limit, g/s/ must decrease when s is increasing. For getting the cross section of $N\pi$ backward scattering the g/s/ function has to decrease again at small s values

d. $I = \frac{1}{2}$.

	theo. (Mev)	exp. (MeV) [7]
G_{17} /2190	$16.3 \cdot 10^3\ \tilde{g}^2 s_o^{-2} = 68$	75
I_{11}/2650/	$61.2 \cdot 10^5\ \tilde{g}^2 s_o^{-4} = 170$	27
K_{115}/3030/	$12.4 \cdot 10^8\ \tilde{g}^2 s_o^{-6} = 240$	2,5

For the πN coupling constant we have taken: $\frac{g^2}{4\pi} = 15$.

Again taking $\tilde{g}/s/$ at $s = 1,66^2\ GeV^2$, $s_o = 12\ GeV^2$. We evaluated the width of S_{11}/1550/ supposing it to be the Mac Dowell-pair of the nucleon, with $s_o = 12$, $\tilde{g}/1,55^2/=\tilde{g}/1,66^2/$. The result is wrong /3 GeV/. However, the results are wonderful for $\pi^+ p \to \pi^+ p$ backward scattering if $\tilde{g}/0/ \approx \tilde{g}/1,66^2/$. /We left out the small contribution of the Δ-trajectory./ [6].

p_{lab}/GeV/c/		5.9	9.9.	13.7	17.1
$\frac{d\sigma}{du}(\pi^+ p \to \pi^+ p)\big\rvert_{u=0}$ $\frac{\mu barn}{GeV^2}$	theo.	16	4	1.75	1
	exp.	21±1	6±0.5	3±0.5	2±1

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Table 1.

I = 3/2 resonance widths, /Elastic width in MeV/

	exp.[7]	in symmetry limit	with first order symmetry breaking
F_{37}/1920/	85	53.5	84.5
D_{35}/1954/	47	46	35
P_{33}/1688/	28	64	53
S_{31}/1670/	50	66	41
G_{37}/1920/	-	53	20
F_{35}/1913/	57	45	46
D_{33}/1690/	37	64	60
P_{31}/1934/	101	74	91

Table 2.

I = 1/2 resonance widths. /Elastic width in MeV/

	exp.[7]	in symmetry limit	with first order symmetry breaking
D_{15}/1680/	68	81	77
P_{13}/1530/	-	105	130
S_{11}/1550/	39 /Rosenfeld/ 186 /Lovelace/	156	182
S_{11}/1710/	240	180	205
F_{15}/1690/	85	81	92
D_{13}/1530/	76	105	76
P_{11}/1466/	138	144	133

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