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# APPLICATION OF THE RENORMALIZATION GROUP TECHNIQUE TO THE PROBLEM OF PHASE TRANSITION IN ONE-DIMENSIONAL METALLIC SYSTEMS II.

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# APPLICATION OF THE RENOMRALIZATION GROUP TECHNIQUE TO THE PROBLEM OF PHASE TRANSITION IN ONE-DIMENSIONAL METALLIC SYSTEM

## II. Response functions and the ground state problem


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## 1. Introduction

In the first of this series of papers[1] /hereafter referred to as I/ on the problem of phase transitions in one-dimensional metallic systems, the renormalization group technique was used to determine the invariant coupling. A particular Hamiltonian with two independent bare coupling constants was considered. In this model, electrons with momentum near to $+k_o$ /where $k_o$ is the Fermi momentum/ interact only with electrons on the opposite side of the Fermi "surface", i.e. with electrons of momentum near to $-k_o$. It was shown that the divergence obtained by Bychkov et al.[2] in the vertex fuction is an artifact of the approximation used. Going beyond the parquet approximation the invariant coupling remains finite and the singularity of the vertex is pushed down to $\omega=0$; there is no singularity whatsoever at finite energies or finite temperatures.

Although the smooth behaviour of the invariant couplings indicates that there is no phase transition in the system at finite temperatures, in agreement with general arguments[3], the nature of the conjectural order that sets in at T=0 has still to be clarified. For that reason, response functions characteristic for the appearance of long-range order will be calculated. Three generalized susceptibilities, corresponding to three different types of long-range order, will be investigated: namely, superconductivity, magnetic /ferro- or antiferromagnetic/ ordering, and static density waves. A singularity in any of these susceptibilities implies the onset of the corresponding ordered phase.

These quantities can be considered at finite temperature T, as a function of T, so that a singularity at $T_c$ would yield directly the transition temperature. As the typical logarithmic terms in the response functions are expected to depend symmetrically on T and on the frequency variable $\omega$, for the sake of convenience the calculations will be made at T=0, keeping $\omega$ as variable.

The generalized susceptibilities describing the fluctuation of Cooper pairs and the propagation of magnetic and density waves are defined in Sec. 2. If a few terms of the perturbation series of these susceptibilities are known, this result can be improved by means of solving a Lie differential equation in which the bare couplings in the series expansion are replaced by the invariant ones. It will be shown that the susceptibilities, as defined, however, are not appropriate directly for such a treatment, and therefore auxiliary functions will be introduced which satisfy the requirement of multiplicative renormalization and which are closely related to the generalized susceptibilities.

That the solution of the Lie equation generates a better approximation than those terms of the series expansion which we start from, is due partly to the integration of the differential equation and partly to the use of the invariant coupling. In diagrammatic language it corresponds to the summation of a series of diagrams, starting from some elementary diagrams for the susceptibilities. In Sec. 3 the first-order corrections coming from the interaction of electrons are calculated. Using the result obtained in I for the invariant couplings, this first-order renormalization of the susceptibilities gives a fairly good approximation for repulsive /$g_1 > 0$/ interaction. The case of attractive /$g_1 < 0$/ interaction is more interesting as it was here that Bychkov et al.[2] found a phase transition. It was shown in I that for $g_1 < 0$ first-order scaling is a poor approximation; via second-order scaling, though, it was possible to go beyond the parquet approximation for the invariant coupling. Thus in order to be consistent, the susceptibilities must likewise be investigated in an approximation in which the next leading logarithmic terms are also collected. This is done in Sec. 4, where it is demonstrated that the susceptibilities diverge at $\omega=0$ only. Depending on the sign and the relative value of the bare coupling constants, one or two of the susceptibilities diverges, indicating that at T=0 the system tends to a superconducting or antiferromagnetic state in which, in certain cases, the period is doubled due to the formation of a density wave. These results as well as further problems are discussed in Sec. 5.

## 2. Response functions and Lie equations

To investigate whether there is any singularity in the response of the system to generalized external forces and to see what the ground state of the system may be, Dzyaloshinsky and Larkin[4] considered three generalized forces: 1/ an external field creating Cooper pairs, 2/ a magnetic field with arbitrary wave vector, 3/ an external field inducing density waves with wave vector k.

Corresponding to these are three generalized susceptibilities. Superconductivity is related to the formation of Cooper pairs, which in turn can be described by the following pair fluctuation Green's function

$$\Delta(\omega) = -i\int_{-\infty}^{\infty} dt\, e^{i\omega t} \langle T\{ \int \frac{dp}{2\pi} c_{p\uparrow}(t)\, c_{-p\downarrow}(t) \int \frac{dp'}{2\pi} c^{+}_{-p'\uparrow}(0)\, c^{+}_{p'\downarrow}(0) \} \rangle . \qquad /2.1/$$

The usual dynamical magnetic susceptibility the singularity of which indicates the appearance of a ferro- or antiferromagnetic state is given as

$$\chi(k,\omega) = -i\int_{-\infty}^{\infty} dt\, e^{i\omega t} \langle T\{ \int \frac{dp}{2\pi} c^{+}_{p\uparrow}(t)\, c_{p+k\downarrow}(t) \int \frac{dp'}{2\pi} c^{+}_{p'\downarrow}(0)\, c_{p'-k\uparrow}(0) \} \rangle . \qquad /2.2/$$

Here the transversal susceptibility has been chosen though it is easy to see that the longitudinal one would give the same result. Finally, the response function describing the propagation of density waves in the system has the form

$$N(k,\omega) = -i\int_{-\infty}^{\infty} dt\, e^{i\omega t} \langle T\{ \sum_{\alpha,\beta} \int \frac{dp}{2\pi} c^{+}_{p\alpha}(t)\, c_{p+k\alpha}(t) \int \frac{dp'}{2\pi} c^{+}_{p'\beta}(0)\, c_{p'-k\beta}(0) \} \rangle . \qquad /2.3/$$

We want to use the renormalization group method to get information on these response functions. As was remarked in I, once the invariant couplings are known any physical quantity can be determined by solving a Lie equation, provided this quantity obeys the multiplicative renormalization condition. Let us suppose that for a quantity $A/\omega/$, which may be a response function or a related quantity, the change of the energy scale given by the cut-off $\omega_D$ to $\omega_D'$ and the simultaneous variation of the coupling constants in the way determined in I are equivalent to multiplication by a constant z, independent of $\omega$.

$$A\left(\frac{\omega}{\omega_D'}, g_1'\left(\frac{\omega_D'}{\omega_D}\right), g_2'\left(\frac{\omega_D'}{\omega_D}\right)\right) = z\,A\left(\frac{\omega}{\omega_D}, g_1, g_2\right), \qquad /2.4/$$

where z depends on $g_1$, $g_2$, $\omega_D$ and $\omega_D'$ only. $g_1'$ and $g_2'$ are given by the relations /I.3.11/. Introducing the notations $\omega/\omega_D = x$, $\omega_D'/\omega_D = t$ and differentiating the logarithm of eq. /2.4/ with respect to x, taking t=x we get

$$\frac{\partial}{\partial x} \ln A(x, g_1, g_2) = \frac{1}{x}\frac{\partial}{\partial \xi}\left[\ln A(\xi, g_1'(x), g_2'(x))\right]_{\xi=1}. \qquad /2.5/$$

In the same way as for the invariant coupling itself, this Lie differential equation generates a reasonable solution for $A/\omega/$ when a few terms of the perturbational expansion at the cut-off are known, provided, of course, the invariant coupling is small. This is not the case in the present problem for $g_1 < 0$, as the dimensionless invariant couplings become of the order of unity when $x \to 0$. Nevertheless some inferences can be drawn concerning the existence or non--existence of phase transitions and the symmetry of the ground state.

## 3. First-order scaling for the susceptibilities

We are interested in the possible singularity of the system's response to external forces and so must aim to pick up the most singular contributions in the susceptibilities. The wave vector k in $\chi/k,\omega/$ and $N/k,\omega/$ is fixed correspondingly. Since both functions are most singular for $k=2k_o$, only this particular value will be investigated. Neglecting the electron-electron interaction, all three susceptibilities as defined in eqs. /2.1/-/2.3/ show logarithmic singularity for a one-dimensional electron gas. Diagrammatically, they are represented by the simple bubbles shown in Fig. 1. These bubbles are related to the Cooper- and zero-sound-type vertex corrections and, as was shown in I, both are logarithmic in one dimension.

In first order in the coupling constants the susceptibility diagrams are given by two successive bubbles, as displayed in Fig. 2. Here, as in I, the solid line stands for the propagator of electrons with momentum near to $+k_o$, and the dotted line represents electrons with momentum near to $-k_o$. The respective contributions of these processes are

$$\Delta(\omega) = -\frac{1}{\pi v}\ln\frac{\omega}{\omega_D}\left[1+\frac{1}{2\pi v}(g_1+g_2)\ln\frac{\omega}{\omega_D}+\ldots\right], \qquad /3.1/$$

$$\chi(\omega) \equiv \chi(2k_0,\omega) = \frac{1}{2\pi v}\ln\frac{\omega}{\omega_D}\left[1-\frac{1}{2\pi v}g_2\ln\frac{\omega}{\omega_D}+\ldots\right], \qquad /3.2/$$

$$N(\omega) \equiv N(2k_0,\omega) = \frac{1}{\pi v}\ln\frac{\omega}{\omega_D}\left[1+\frac{1}{2\pi v}(2g_1-g_2)\ln\frac{\omega}{\omega_D}+\ldots\right]. \qquad /3.3/$$

Due to the fact that the zeroth-order term depends logarithmically on $\omega/\omega_D$, neither of these susceptibilities satisfies the criterion of multiplicative renormalization in eq. /2.4/. As Zawadowski[5] has pointed out in a similar problem, for the susceptibility of the X-ray absorption[6], instead of the susceptibilities, auxiliary quantities can be introduced by differentiating with respect to $\ln\omega$. These are the proper quantities for the application of the renormalization group method, since they satisfy eq. /2.4/. In order to obtain series expansions starting with unity and normalized to unity at the cut-off, the following quantities will be defined

$$\bar{\Delta}(\omega) = -\pi v\,\omega\,\frac{\partial\Delta(\omega)}{\partial\omega}, \qquad /3.4/$$

$$\bar{\chi}(\omega) = 2\pi v\,\omega\,\frac{\partial\chi(\omega)}{\partial\omega}, \qquad /3.5/$$

$$\bar{N}(\omega) = \pi v\,\omega\,\frac{\partial N(\omega)}{\partial\omega}. \qquad /3.6/$$

Using the expansions of eqs. /3.1/-/3.3/, we get

$$\bar{\Delta}(\omega) = 1+\frac{1}{\pi v}(g_1+g_2)\ln\frac{\omega}{\omega_D}+\ldots \qquad /3.7/$$

$$\bar{\chi}(\omega) = 1-\frac{1}{\pi v}g_2\ln\frac{\omega}{\omega_D}+\ldots \qquad /3.8/$$

$$\overline{N}(\omega) = 1 + \frac{1}{\pi v}(2g_1 - g_2)\ln\frac{\omega}{\omega_D} + \dots \qquad /3.9/$$

The imaginary parts are not considered here as this low--order scaling is not adequate to account for them. /The same situation occured in I in calculating the Green's function and vertices./

Applying eq. /2.5/ to these auxiliary functions, we have

$$\frac{\partial}{\partial x}\ln\overline{\Delta}(x) = \frac{1}{x}\frac{1}{\pi v}\left(g_1'(x) + g_2'(x)\right), \qquad /3.10/$$

$$\frac{\partial}{\partial x}\ln\overline{\chi}(x) = -\frac{1}{x}\frac{1}{\pi v}g_2'(x), \qquad /3.11/$$

$$\frac{\partial}{\partial x}\ln\overline{N}(x) = \frac{1}{x}\frac{1}{\pi v}\left(2g_1'(x) - g_2'(x)\right). \qquad /3.12/$$

The right-hand sides of these equations contain the invariant couplings which were calculated in I.

Using the results of first-order renormalization as given in eqs. /I.4.11/ and /I.4.12/

$$g_1'(x) = \frac{g_1}{1 - \frac{g_1}{\pi v}\ln x}, \qquad /3.13/$$

$$g_2'(x) = g_2 - \frac{1}{2}g_1 + \frac{1}{2}\frac{g_1}{1 - \frac{g_1}{\pi v}\ln x}, \qquad /3.14/$$

and inserting them into eqs. /3.10/-/3.12/, simple integration gives

$$\overline{\Delta}(\omega) = \left(1 - \frac{g_1}{\pi v}\ln\frac{\omega}{\omega_D}\right)^{-3/2}\cdot\left(\frac{\omega}{\omega_D}\right)^{-\alpha}, \qquad /3.15/$$

$$\overline{\chi}(\omega) = \left(1 - \frac{g_1}{\pi v}\ln\frac{\omega}{\omega_D}\right)^{1/2}\cdot\left(\frac{\omega}{\omega_D}\right)^{\alpha}, \qquad /3.16/$$

$$\bar{N}(\omega) = \left(1 - \frac{g_1}{\pi v} \ln \frac{\omega}{\omega_D}\right)^{-3/2} \left(\frac{\omega}{\omega_D}\right)^{\alpha}, \qquad /3.17/$$

where $\alpha = (g_1 - 2g_2)/2\pi v$ . Although the integration to get the susceptibilities themselves cannot be performed analytically, these forms are none the less sufficient for us to be able to discern the singularities. Our result does not agree completely with that obtained by Dzyaloshinsky and Larkin[4] when Umklapp processes are neglected in their paper. The reason of this discrepancy will be discussed in the last section.

First-order scaling yields a fairly good approximation for $g_1 > 0$ only, as in this case the invariant couplings decrease from their bare value when the scaling energy approaches the Fermi energy. For $g_1 > 0$ a singularity can come from the factors $(\omega/\omega_D)^{\pm\alpha}$ only and thus, so far as the dominant singularity is concerned, the susceptibilities and these auxiliary functions behave similarly. In the case $g_1 > 2g_2$ $\Delta(\omega)$ exhibits a power law singularity at $\omega = 0$, while for $g_1 < 2g_2$ $\chi(\omega)$ and N/$\omega$ / are singular. That means, in the first case, that the system tends towards a superconducting phase, while in second the ground state is antiferromagnetic with a period-doubled stationary density wave. The wave vector of both the antiferromagnetic and density wave states is $k = 2k_o$.

For $g_1 < 0$ the results given in eqs. /3.15/-/3.17/ are not satisfactory, since they have a singularity at finite $\omega$ .

This is a consequence of the singularity of the invariant couplings as given in eqs. /3.13/-/3.14/. It was shown in I, however, that this singularity is spurious, being due to the logarithmic approximation alone. Going beyond the parquet diagrams, in second-order scaling the invariant couplings are smooth functions of the scaling energy and tend to a constant value which, in the weak coupling limit $g_i/\pi v \ll 1$ , is independent of the bare couplings

$$\lim_{\omega\to 0} \frac{g_1'(\omega)}{\pi v} = -2 , \qquad /3.18/$$

$$\lim_{\omega\to 0} \frac{g_2(\omega)}{\pi v} \approx -1 . \qquad /3.19/$$

The analytic expression of the invariant couplings is not known explicitly in this approximation, nor can the susceptibilities be given in the whole range of energies. Nevertheless, for $\omega \to 0$ an asymptotic form can be obtained by inserting eqs. /3.18/ and /3.19/ into eqs. /3.10/-/3.12/.

$$\Delta(\omega) \propto \bar{\Delta}(\omega) \propto \left(\frac{\omega}{\omega_D}\right)^{-3} \quad \text{for} \quad \omega \to 0, \qquad /3.20/$$

$$\chi(\omega) \propto \bar{\chi}(\omega) \propto \frac{\omega}{\omega_D} \quad \text{for} \quad \omega \to 0, \qquad /3.21/$$

$$N(\omega) \propto \bar{N}(\omega) \propto \left(\frac{\omega}{\omega_D}\right)^{-3} \quad \text{for} \quad \omega \to 0. \qquad /3.22/$$

This result shows that for $g_1 < 0$, too, the singularity can appear at $\omega=0$ only. As the dimensionless invariant couplings are of the order of unity, higher-order corrections as well should be considered: this will be done in the next section.

## 4. Second-order scaling for the susceptibilities

In the preceding section the generalized susceptibilities have been determined by means of a Lie equation in which the right-hand side was replaced by a first-order expansion in the invariant couplings. For an attractive interaction this restriction to the first-order term is highly insufficient. Since in the Lie equation for the invariant coupling we had to go to at least the second order, to be consistent, the susceptibilities, too, have to be calculated in the same order.

The second-order diagrams of the generalized susceptibilities are presented in Fig. 3. The vertices corresponding to the interactions $g_1$ and $g_2$ are commonly represented by a point; moreover, as the spin orientations are not denoted, the same diagrams /Fig. 3b/ can represent both $\chi(k,\omega)$ and $N(k,\omega)$. The first two diagrams in Fig. 3a and 3b have three typical logarithmic integrations and are proportional to $\ln^3 \omega/\omega_D$. They are already accounted for by the first-order scaling, because it sums up the leading logarithmic terms; these graphs can in fact, be generated from the first--order diagrams of Fig. 2 by replacing the elementary vertex with first-order vertices. This corresponds to the replacement of the coupling constants by the invariant couplings in the Lie equation.

The new contribution comes from the self-energy-type corrections. These are proportional to $\ln^2 \omega/\omega_D$ and therefore are not negligible compared to the contribution of bubbles, when the invariant couplings are of the order of unity. Neglecting the imaginary parts, a straightforward calculation gives up to second order

$$\Delta(x) = -\frac{1}{\pi v} \ln x \Big[ 1 + \frac{1}{2\pi v}(g_1+g_2)\ln x + \frac{1}{4\pi^2 v^2}(g_1+g_2)^2 \ln^2 x + \frac{1}{12\pi^2 v^2}(2g_1^2 - 2g_1 g_2 - g_2^2)\ln^2 x + \frac{1}{4\pi^2 v^2}(g_1^2 - g_1 g_2 + g_2^2)\ln x + \dots \Big], \quad /4.1/$$

$$\chi(x) = \frac{1}{2\pi v} \ln x \Big[ 1 - \frac{1}{2\pi v} g_2 \ln x + \frac{1}{4\pi^2 v^2} g_2^2 \ln^2 x - \frac{1}{12\pi^2 v^2}(g_1^2 + g_2^2)\ln^2 x + \frac{1}{4\pi^2 v^2}(g_1^2 - g_1 g_2 + g_2^2)\ln x + \dots \Big], \quad /4.2/$$

$$N(x) = \frac{1}{\pi v} \ln x \Big[ 1 + \frac{1}{2\pi v}(2g_1 - g_2)\ln x + \frac{1}{4\pi^2 v^2}(2g_1 - g_2)^2 \ln^2 x - \frac{1}{12\pi^2 v^2}(g_1^2 - 4g_1 g_2 + g_2^2)\ln^2 x + \frac{1}{4\pi^2 v^2}(g_1^2 - g_1 g_2 + g_2^2)\ln x + \dots \Big]. \quad /4.3/$$

The series expansion of the auxiliary functions follows directly from these equations;

$$\bar{\Delta}(x) = 1 + \frac{1}{\pi v}(g_1+g_2)\ln x + \frac{3}{4\pi^2 v^2}(g_1+g_2)^2 \ln^2 x + \frac{1}{4\pi^2 v^2}(2g_1^2 - 2g_1 g_2 - g_2^2)\ln^2 x + \frac{1}{2\pi^2 v^2}(g_1^2 - g_1 g_2 + g_2^2)\ln x + \dots, \quad /4.4/$$

$$\bar{\chi}(x) = 1 - \frac{1}{\pi v} g_2 \ln x + \frac{3}{4\pi^2 v^2} g_2^2 \ln^2 x - \frac{1}{4\pi^2 v^2}(g_1^2 + g_2^2)\ln^2 x + \frac{1}{2\pi^2 v^2}(g_1^2 - g_1 g_2 + g_2^2)\ln x + \dots, \quad /4.5/$$

$$\bar{N}(x) = 1 + \frac{1}{\pi v}(2g_1 - g_2)\ln x + \frac{3}{4\pi^2 v^2}(2g_1 - g_2)^2 \ln^2 x - \frac{1}{4\pi^2 v^2}(g_1^2 - 4g_1 g_2 + g_2^2)\ln^2 x + \frac{1}{2\pi^2 v^2}(g_1^2 - g_1 g_2 + g_2^2)\ln x + \dots \quad /4.6/$$

The third and fourth terms give no contribution in the Lie equations, because, as was mentioned above, they are already accounted for by the first-order scaling.

The Lie equations in this approximation have the form

$$\frac{\partial}{\partial x} \ln \bar{\Delta}(x) = \frac{1}{x}\left[\frac{1}{\pi v}\left(g_1'(x)+g_2'(x)\right)+\frac{1}{2\pi^2 v^2}\left(g_1'^2(x)-g_1'(x)g_2'(x)+g_2'^2(x)\right)\right] \qquad /4.7/$$

$$\frac{\partial}{\partial x} \ln \bar{\chi}(x) = \frac{1}{x}\left[-\frac{1}{\pi v} g_2'(x)+\frac{1}{2\pi^2 v^2}\left(g_1'^2(x)-g_1'(x)g_2'(x)+g_2'^2(x)\right)\right], \qquad /4.8/$$

$$\frac{\partial}{\partial x} \ln \bar{N}(x) = \frac{1}{x}\left[\frac{1}{\pi v}\left(2g_1'(x)-g_2'(x)\right)+\frac{1}{2\pi^2 v^2}\left(g_1'^2(x)-g_1'(x)g_2'(x)+g_2'^2(x)\right)\right] \qquad /4.9/$$

Here the invariant couplings obtained in the second-order scaling have to be used. The latter being smooth functions, a singularity can come only from the factor $1/x$ at $x=0$, i.e. at $\omega=0$. For $\omega \to 0$ we have the asymptotic expressions

$$\Delta(\omega) \propto \bar{\Delta}(\omega) \propto \left(\frac{\omega}{\omega_D}\right)^{-3/2} \qquad /4.10/$$

$$\chi(\omega) \propto \bar{\chi}(\omega) \propto \left(\frac{\omega}{\omega_D}\right)^{5/2} \qquad /4.11/$$

$$N(\omega) \propto \bar{N}(\omega) \propto \left(\frac{\omega}{\omega_D}\right)^{-3/2} \qquad /4.12/$$

In this approximation $\Delta(\omega)$ and $N/\omega/$ are singular at $\omega=0$, indicating that the system tends to a ground state which is superconducting with a period-doubled density wave present. Two features are worth mentioning. First, a similar result would be obtained if the calculation were carried out at finite temperatures, replacing $\omega$ by T; this would lead directly to the finding that there is no phase transition in the system at finite temperatures. Second, the exponents in the susceptibilities are universal numbers, independent of the bare coupling constant values for weak bare coupling. This scaling behaviour is analogous to the Kondo problem[7,8], where, similarly, the invariant coupling tends to a value of the order of unity which depends only on the spin. For $g_1 > 0$ in the present model and in the X-ray absorption problem, on the contrary, where the coupling remains weak, the exponents depend explicitly on the bare coupling constants.

## 5. Discussion

In this paper the response of a one-dimensional system to external forces has been investigated by using the renormalization group method. Three generalized susceptibilities have been considered: namely, the propagation of Cooper pairs and of magnetic and density waves with wave number $k=2k_o$. Singularity in these propagators would indicate the formation of superconductivity, antiferromagnetic or density wave states, respectively.

Making use of the results of I for the invariant couplings, it turns out that these susceptibilities can have singularity at $\omega=0$ only; in other words, in our model system no phase transition can occur at finite temperatures.

The singularity at T=0 and $\omega=0$ is of power law type. Analogously to the X-ray absorption problem[6], the logarithmic terms in the perturbation expansion sum up to give a power law behaviour. Depending on the sign and relative value of the bare coupling constants, the system tends to a superconducting or antiferromagnetic state as $T\to 0$. In some region of the coupling constants a static density wave is also present, leading to a doubling of the period of the system. The phase diagram, displaying the response functions which are singular in a given range of the couplings, is shown in Fig. 4.

First-order scaling works well for $g_1>0$, and expressions /3.15/-/3.17/ yield a reasonable approximation. For $g_1<0$, however, the invariant couplings do not remain small and arbitrarily high order terms in the Lie equations can give important contributions. We went up to second order in the Lie equations for both the invariant couplings and the susceptibilities, and though our result is of only qualitative nature, due to the neglection of higher-order terms, we believe that the calculation indicates correctly that there is no phase transition at finite temperatures, and that at T=0 the singularity at $\omega=0$ is of power law type with exponents independent of the bare couplings. The exponents given in eqs. /4.10/-/4.12/, however, are not precise. We can only claim that in second-order scaling the system seems to become superconducting at T=0 with a doubled period.

No comparison can be made with exactly solvable one-dimensional models, because there is no exact result for the ground state problem. In turn our method is not suitable for determining whether or not there is a gap in the excitation spectrum, for which exact statements exist. Dzyaloshinsky and Larkin[4] investigated the ground state problem in the parquet approximation, taking into account Umklapp processes as well. The discrepancy that they get a normal metallic phase for $g_1 > 0$ and $g_1 > 2g_2$ probably stems for their neglection of the factors $(\omega/\omega_D)^{\pm\alpha}$ in eq. /3.15/. The parquet approximation is clearly insufficient for $g_1 < 0$.

In the present calculation the electron-electron interaction matrix elements have been chosen in a particular form /see eq. /I.2.4/ /, that neglects the scattering processes in which both incoming electrons are on the same side of the Fermi surface, /i.e. their momenta are around either $+k_o$ or $-k_o$/, so that Umklapp processes have also been ignored. The effect of these processes will be investigated in a later paper, going beyond the parquet approximation by use of a second-order scaling.

Another problem open to question is the relation of the renormalization group technique to direct diagram summation. First-order scaling is undoubtedly equivalent to the logarithmic approximation, but the comparison of second-order scaling with diagrams, and the attempt to determine the vertex as a function of several variables, needs further investigations.

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## References


1. N. Menyhárd and J. Sólyom, J. Low Temp. Phys. /preceding paper/.
2. Yu. A. Bychkov, L.P. Gorkov and I. E. Dzyaloshinsky, Zh. Eksperim. i Teor. Fiz. 50, 738 /1966/ [English transl. Soviet Phys.-JETP 23, 489 /1966/].
3. L. D. Landau and E. M. Lifshitz, Statistical Physics p. 482 /Pergamon Press, New York, 1958/.
4. I. E. Dzyaloshinsky and A. I. Larkin, Zh. Eksperim. i Teor. Fiz. 61, 791 /1971/ [English transl. Soviet Phys.--JETP 34, 422 /1972/].
5. A. Zawadowski, private communication.
6. B. Roulet, J. Gavoret and P. Noziéres, Phys. Rev. 178, 1072 /1969/.
7. A. A. Abrikosov and A. A. Migdal, J. Low Temp. Phys. 3, 519 /1970/.
8. M. Fowler and A. Zawadowski, Solid State Comm. 9, 471 /1971/.

Figure captions

Fig. 1. Zeroth-order diagram of response functions: a/ Cooper-pair fluctuation, b/ transverse magnetic susceptibility, c/ density fluctuation. The arrows show the spin direction. Solid /dotted/ line stands for the propagator of electrons with momentum near to $+k_o$ /$-k_o$/.

Fig. 2. First-order diagrams of response functions: a/ Copper--pair fluctuation, b/ magnetic susceptibility, c/ density fluctuation.

Fig. 3. Second-order diagram of response functions: a/ Cooper-pair fluctuation, b/ magnetic susceptibility or density fluctuation. The interaction vertices $g_1$ and $g_2$ are commonly represented by a point.

Fig. 4. Phase diagram of the system at T=0. S=superconductor, PD S= period-doubled superconductor, PD AF= period--doubled antiferromagnet.

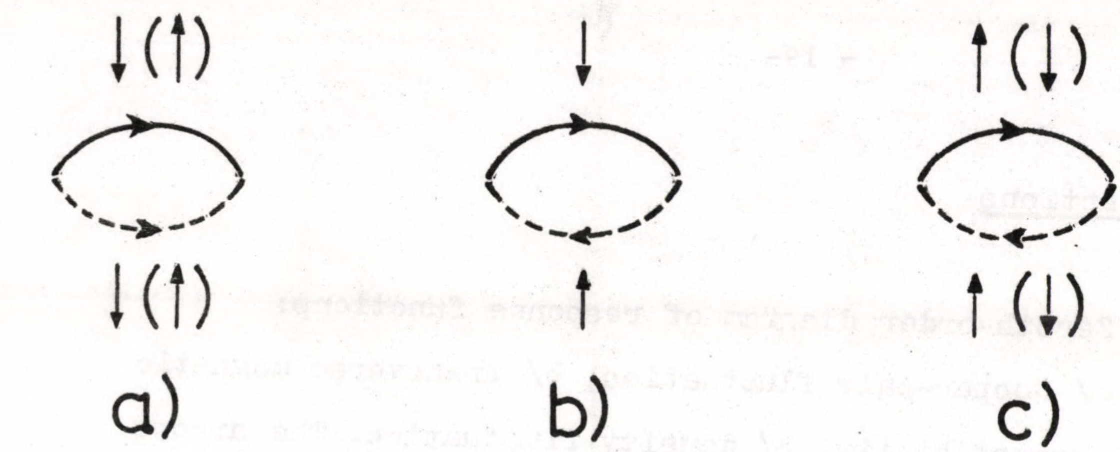



Fig.1.

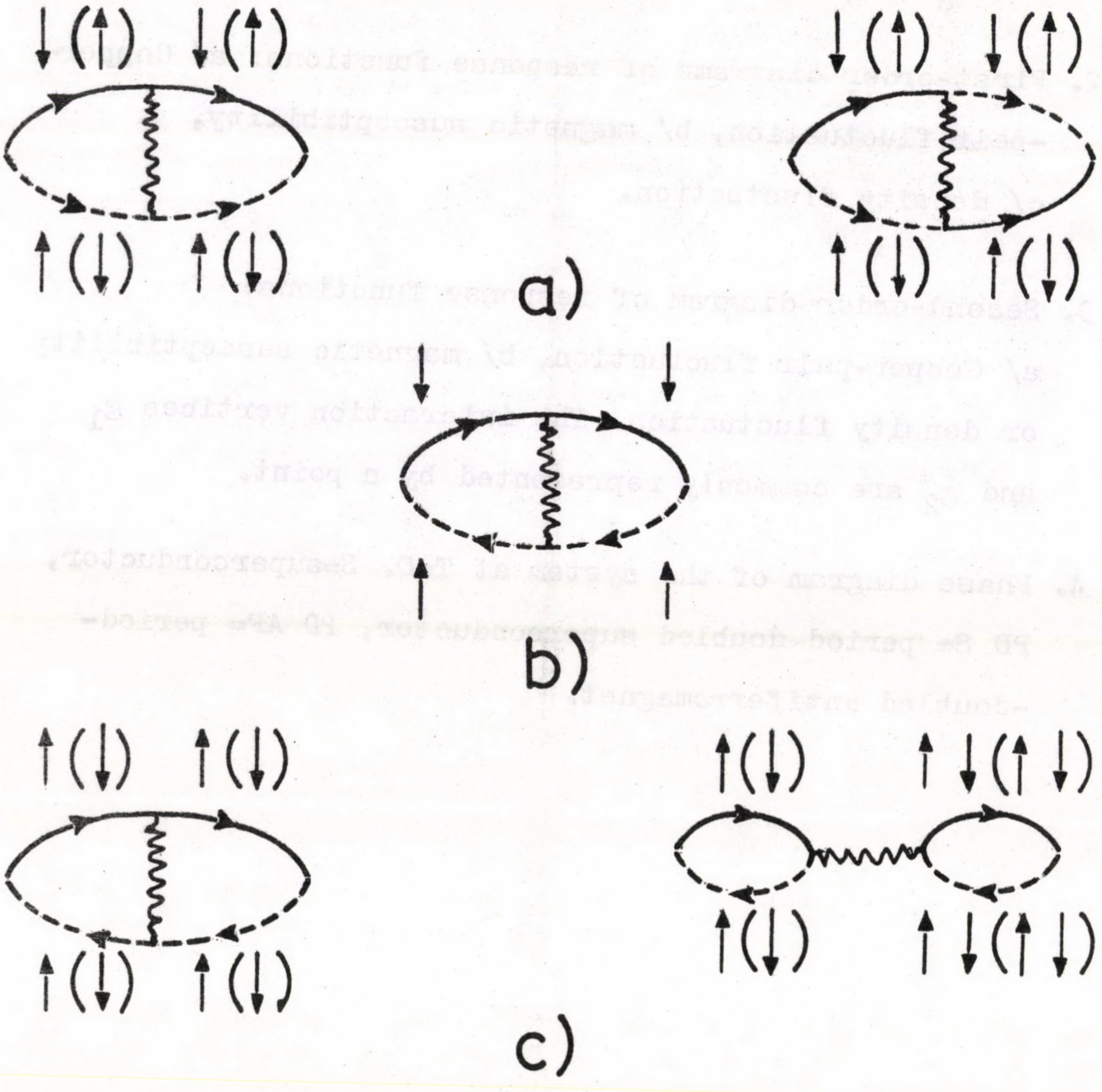
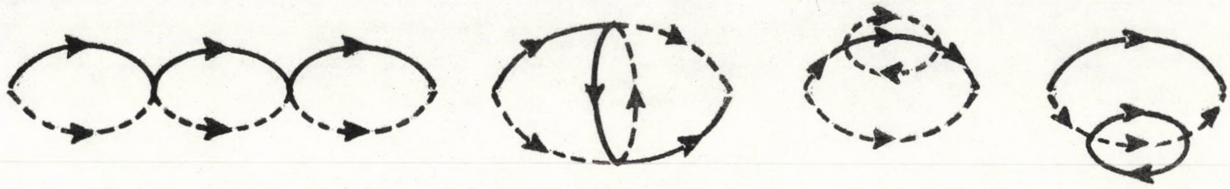
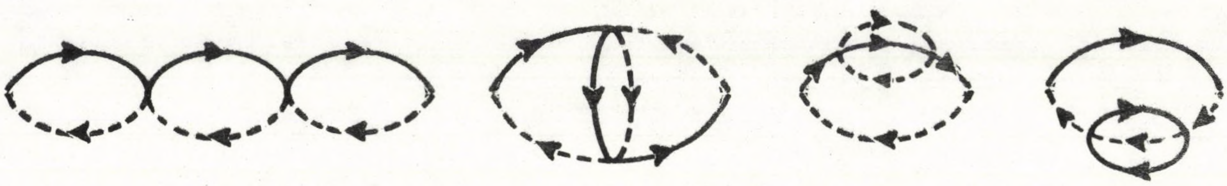



Fig. 2.

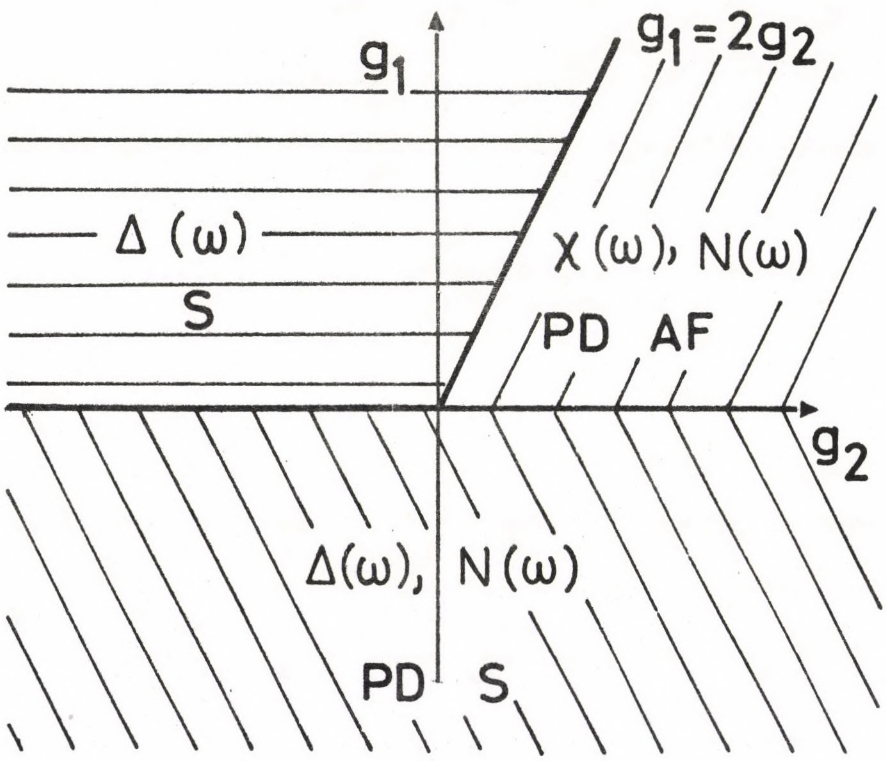


a)



b)

Fig.3.



$g_1 = 2g_2$
$g_1$
$\Delta(\omega)$
S
$\chi(\omega)$, $N(\omega)$
PD AF
$g_2$
$\Delta(\omega)$, $N(\omega)$
PD S


Fig.4.

ABSTRACT

The results of the preceding paper for the invariant coupling are used to calculate some response functions in a one-dimensional metallic system. Three generalized susceptibilities, characterizing the possible superconducting, or antiferromagnetic, behaviour of the system and the appearance of density waves, are calculated by means of the Lie equations of the renormalization group. Due to the non-singular behaviour of the invariant couplings, the response functions can diverge at $\omega = 0$ only, and this singularity is of power law type. Depending on the sign and relative value of the bare coupling constants, the model system tends to superconducting or antiferromagnetic order at $T = 0$. In certain cases the period of the system is doubled.

РЕЗЮМЕ

В предыдущей работе были получены выражения для инвариантных констант связи, которые используются в настоящей работе для определения функций отклика одномерной металлической системы. Исследованы три обобщенных восприимчивости на основе уравнения Ли группы ренормировки, которые могут охарактеризовать возможное сверхпроводящее или антиферромагнитное поведение системы или же они могут указать на появление волн плотности. Так как инвариантные константы связи не имеют сингулярностей, функции отклика могут иметь расходимость только при $\omega = 0$, и появляющаяся сингулярность в этом случае имеет степенный характер. В зависимости от знака и от относительного значения констант связи в модели появляется или сверхпроводящий или антиферромагнитный порядок при $T = 0$ $^{\circ}$K. В некоторых случаях период системы удваивается.

KIVONAT

Az első részben az invariáns csatolásra kapott eredményeket felhasználva válaszfüggvényeket határozunk meg egydimenziós fémes rendszerekre. A renormálási csoport Lie egyenlete segitségével három általánositott szuszceptibilitást vizsgálunk, melyek a rendszer esetleges szupravezető vagy antiferromágneses viselkedésére jellemzők, vagy sürüséghullámok megjelenésére utalnak. Az invariáns csatolások nem szinguláris volta miatt a válaszfüggvények csak $\omega = 0$-nál divergálhatnak és a szingularitás itt hatvány jellegü. A csatolási állandók előjelétől és relativ értékétől függően a modell vagy szupravezető vagy antiferromásneses rendet mutat $T = 0$ -nál. Bizonyos esetekben a rendszer periódusa megkétszereződik.

61.997

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