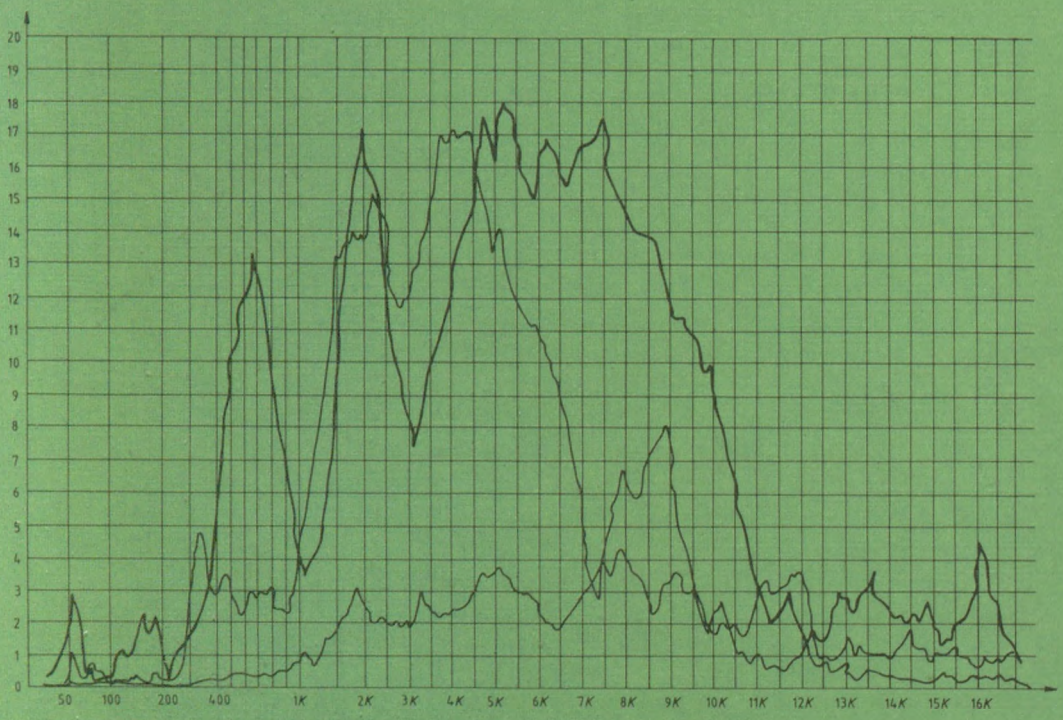
Vibration Testing of Machines and their Maintenance

by

György Lipovszky
Károly Sólyomvári
and
Gábor Varga



Akadémiai Kiadó, Budapest

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AND THEIR MAINTENANCE

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by

GYÖRGY LIPOVSZKY
Technical University of Budapest, Hungary

KÁROLY SÓLYOMVÁRI
Technical University of Budapest, Hungary

and

GÁBOR VARGA
Telecommunication Machine Factory, Budapest, Hungary

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PREFACE

The growth of manufacturing industry and technical and scientific progress demand the constant improvement of the level of plant operations and of maintenance. Technical diagnostics is an important tool in achieving this aim. Numerous processes and methods of technical diagnostics are known, some of which are already widespread in practice. One of them is vibration test, which enables faults to be identified early in the maintenance process without dismantling of the machine being necessary. However, vibration analysis is not yet practised widely.

The aim of this book is to promulgate this new test method to a wider audience. It deals briefly with the general maintenance problems of machines, the guiding principles of modernization and of technical condition tests, including technical diagnostics, with the instruments of vibrometry and in more detail with the testing of a few characteristic machine elements and main components, as well as the organization of machine control. Of practical use is the part of the book which discusses the causes of imbalance, and the principle, the methods and the machines of balancing.

The application of vibrometry and balancing to maintenance is, as far as we are concerned, in its early stages. We hope that it will prove to be a useful aid to specialists dealing with the modernization of maintenance. Comments and criticisms conceived during the use of this book are invited and would greatly assist us in our further work. We recommend this book in the first place to upper and middle level managers who are in charge of plant maintenance.

Finally we wish to express our thanks to the reviewer of this book, Dr. Győző Ludvig, whose conscientious expert work contributed greatly to the uniform approach and phraseology of this book.

György Lipovszky
Károly Sólyomvári
Gábor Varga

1 THE MAINTENANCE OF MACHINES

The process of maintenance includes all the technical and organizational activities which ensure that machines and vehicles operate with the prescribed reliability. Maintenance and repair works performed within this framework prevent unexpected failures and so reduce downtime. The costs of maintenance are influenced by the characteristic features of the particular types of machine and the conditions of their commissioning. Details of maintenance activities are shown in Fig. 1.1.

The procurement and selection of the most satisfactory types of machine is of outstanding importance from the point of view of their economic operation and maintenance.

The mode of commissioning may determine the operational reliability of the machine and the amount of wear it suffers, which ultimately influences the maintenance operations.

The first commissioning, the so-called O-revision, is performed by the customer service, whose function is to check the soundness of the machine, or vehicle, and to see if it was damaged during transport.

The protective coverings, packagings, etc. are removed, and after being fuelled and lubricated the machine is started; both its operation and whether it meets the prescribed safety requirements are checked.

Putting the machine into operation is the responsibility of the operating plant, which must provide the necessary power supply and connections, obtain permission for its operation, and create the conditions for the correct operation of the machine. The amount of professional skill employed in commissioning new machines and recommissioning used ones doubtless affects their useful life.

The maintenance process comprises general care, inspection and testing. The care of machines extends to their handling, tending, cleaning lubrication, oil changes, etc. The objectives of inspection are the early detection of irregularities in the operation, structure and accuracy control of the machine, the making of necessary adjustments and the performance of occasional small repairs. During the guarantee period all this is the responsibility of the customer service. The prescriptions for the regular inspection and checks are contained in the maintenance instructions. In the case of machines and vehicles whose operation may be dangerous, official technical inspec-

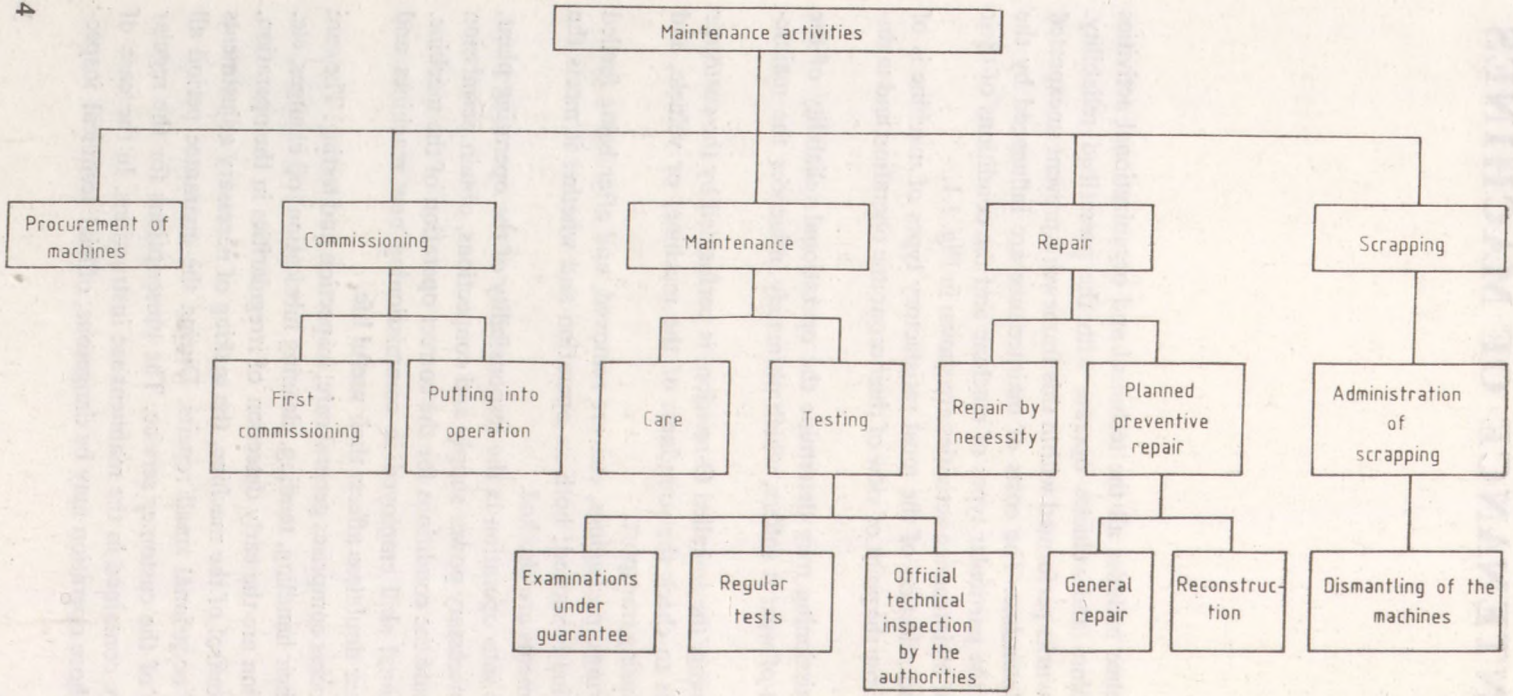

Fig. 1.1 Maintenance activities

tion by the supervising authority is also prescribed. Examples of this would be the so-called official technical inspections of motor vehicles, depending on their age and type (cars, lorries), or the contact protection control of cranes.

Maintenance works are characterized by the fact that, apart from small repairs, they are generally performed without dismantling the machines. So if simple components are damaged (e.g. carburetors, ignition coils, *V*-belts, etc.), they are replaced during maintenance. If the machine must be dismantled more completely because of a breakdown, we speak of a repair.

Repairs may be performed to eliminate unexpected failures, or as prescribed by the planned cycle of preventive maintenance. In the latter case the machines or vehicles are partly or fully dismantled, their structural units and parts are thoroughly cleaned then checked and their defects recorded. The damaged parts are repaired, reconstructed, or replaced.

After assembly, the adjustments, regulations and checks — the so-called functional tests — are performed frequently on test benches during trial runs. The repair is accompanied by corrosion protection.

The various types of repair are designated by different names in the individual branches and sub-branches of the national economy, depending on the quantity of work required (e.g. small, medium and large repairs, reconstruction, general inspection and general repair). In other industrial branches the repairs are called inspections, with various adjectives being applied.

In general three types of repair may be distinguished:

— the repair of unexpectedly occurring failures; the so-called repair by necessity is carried out immediately, but it is restricted to the damaged part only;
— general repair, i.e. repairs foreseen by the plan, with the objective of restoring the working capacity and reliability of the machine, or vehicle, to a similar (approximately identical) level to the original one;
— full reconstruction to attain the technical level of a new machine, or vehicle, also involving modernization in frequent cases.

The maintenance of the machine ends when it is scrapped. Modern repair methods are characterized namely by the replacement of parts or main components (non-specific repair). As a result of this practice some important parts or main components of the machine to be scrapped which were built-in earlier as new parts will not be worn completely. After scrapping, these parts and components may be used again without repair, if they are removed in a workmanlike way, or after repairing or reconstructing them in a specialized repair workshop according to the information in their damage control records.

Technical progress and the requirements for productivity and economy demand ever increasing reliability of machines and vehicles. Maintenance is a basic tool of economic production and transport. These requirements cannot be met by convention-

al methods any more; the attitude towards maintenance must change and the new approach must gain wide acceptance.

One of the development trends aims at creating harmony between the investment and the maintenance and management of fixed assets.

A new production process starts with the investment, which is then followed by operation of the purchased fixed assets. These must be maintained and repaired in the interest of their safe operation. Outdated machines and vehicles which are incapable of meeting the technical requirements involved must be scrapped and replaced by new ones. The objective of the acquisition or replacement may be to keep up with the previous levels of production and productivity or to increase them.

The responsibility of the management of fixed assets is to keep the assets in evidence, study the exploitation of the capacities, evaluate the costs of operation and repair, observe the characteristics (physical, technical condition, up-to-dateness) of machines, harmonize the maintenance plans and finally make proposals for scrapping and replacement.

The experts dealing with maintenance are in possession of most of the information required for the above activities (e.g. the condition of the equipment, the maintenance costs, etc.). Therefore close connections must be established between the departments dealing with the development, management, the management of fixed assets and maintenance, whereby those items whose operation and maintenance is uneconomical can be scrapped in due time, and due attention can be devoted to the maintainability and repairability of the new machines to be acquired.

Recently maintenance has been interpreted in a more general sense. Accordingly, all factors influencing the safe operation of machines must be taken into account, from the design stage to the scrapping stage, with a view to promoting the success of the enterprise. The phrase "maintenance and repair" cannot express these efforts sufficiently, and as a result several new designations have appeared in the professional literature — for example, complex maintenance, integrated maintenance, total preventive maintenance, terotechnology, etc. [67, 69, 70].

Though the listed technical expressions are not yet used uniformly, the term "terotechnology" has become widespread in foreign literature. The meaning of "terotechnology" according to the addenda to the Shorter Oxford English Dictionary, whose new edition was published in 1973, is as follows: "The branch of technology and engineering concerned with the installation, maintenance, and replacement of industrial plant and equipment and with related subjects and practices." On the other hand, according to the Commission of Maintenance Terotechnology of the British Standards Institute (BSI), "Terotechnology is an activity extending to several professional areas related to the production plant, the equipment, the installation and organization with the objective that the costs arising during the life of the establishments achieve an economical value."

According to the above description, the essence of terotechnology is that it contains all the technical and organizational measures aimed at achieving the minimum operational costs of the system over its design, production, commissioning, operation, maintenance and scrapping stages.

1.1 MAINTENANCE SYSTEMS

The objective of maintenance is generally the prevention of failures. Therefore the starting point for a maintenance system is the extent of the damage. Various maintenance systems have been developed, taking into account the different types of failure of the machines and their individual parts, the possibilities of their repair and reconditioning, the different technological possibilities, and the consequences of breakdown.

Figure 1.2 presents three basic maintenance systems, with reference to the advisability and economy of their application [57, 90].

1.1.1 REPAIR BY NECESSITY

The care and lubrication of machines must be carried out within the maintenance framework, but a repair is done only in the case of a breakdown. The damaged parts or main components are repaired, or replaced by new ones. The advantage of this method is that the individual parts and components may be used until they wear out. A disadvantage is that, as corrective intervention cannot be preplanned, the downtime is generally longer, depending on the availability of the repair shop, the work load of which is unplanned and variable. Replacing the damaged machine immediately is generally impossible and this fact may cause production problems.

If an unexpected failure cannot be detected in time, it may lead to further damage. For rapid fault elimination a relatively large stock of spare parts must be stored.

In the case of repairs by necessity the maintenance costs are high.

1.1.2 PLANNED PREVENTIVE MAINTENANCE

The industrially developed countries have recognized in good time the importance of maintenance and have introduced, instead of the older, simple corrective maintenance, i.e. the repair of faults only as they appear (a "fire fighting" mode of maintenance), planned preventive maintenance (PPM), with the main objective of preventing faults and avoiding unexpected breakdowns.

A developing trend is that, while the number of people employed in production continues to decrease, the maintenance of the complex and automated facilities requires ever more labour and ever increasing professional skill. Items of equipment demanding engineering or technical qualifications for their maintenance are already in operation.

Planned preventive maintenance was instituted in Hungary by the decree No. 2022/1950 MT; this ruled that in the interest of better exploitation of machines, and to decrease the maintenance costs, the regular maintenance of machines must be organized in every plant, determining the time interval between two maintenance operations and the standard of the repair costs for all types of machines and vehicles.

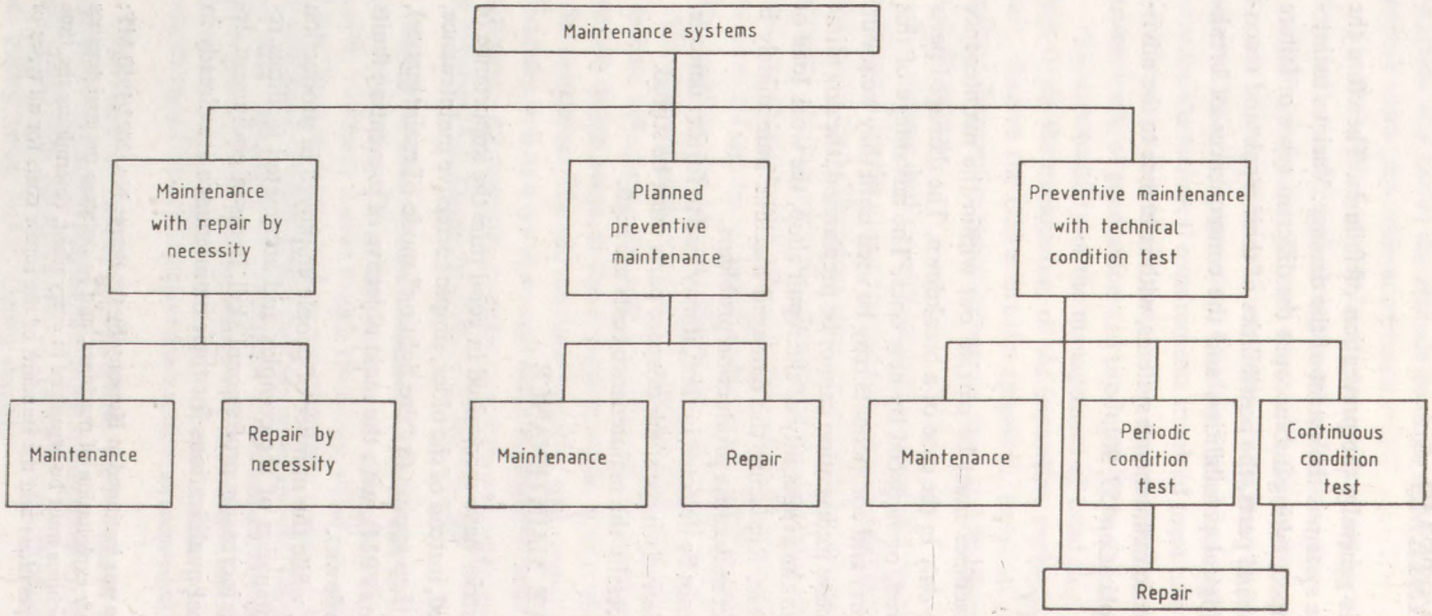

Fig. 1.2 Maintenance systems

As a consequence of the central management of industry, planned preventive maintenance was organized in every industrial enterprise.

In the framework of the PPM-system developed on the basis of the above decree the permanent operable condition of machines is ensured by regularly repeated, planned controls and repairs, carried out according to a cyclic order within the framework of the maintenance system, prescribing the mode and extensions of the tasks to be carried out, their sequence and other parameters to be observed between the controls and the repairs (time, capacity, etc.).

The cyclic order is elaborated on the basis of the useful life of the individual parts. In this system the machines are maintained and repaired at rigidly determined intervals.

The objective of PPM is to ensure a constant reliability, the purposeful measure of which is determined on the basis of safety and economic considerations.

The advantage of the PPM system is, assuming a well-founded, correct cyclic order, the possibility of planning the repairs for a longer time interval, thereby reducing the downtime and increasing the useful lives of the machines or vehicles.

The order of the maintenance cycle may be time dependent, or capacity dependent. In the former case the prescriptions apply to calendar periods (month, year) and have the advantage of simple planning. However, the machines are not all used to the same extent; so they fail at different stages during a given time period. The capacity-dependent cycle serves to ensure that repairs are carried out on the basis of a uniform technical condition, assessed by a measurable parameter which best characterizes the operation of the machine or vehicle.

Example. For vehicles, this parameter is generally the distance covered in units of km; for electrical equipment, the electrical energy consumption in kWh; for motor vehicles, the fuel consumed. In the case of machine tools the most widely used maintenance cycle is based on the working hours logged; though no true picture is obtained in this way, as in most cases it is the time that the machine is available that is logged. Recently, working-hour counters are increasingly being mounted on machines, switching on when the machine is started, or when the starting key is turned, and counting until the machine is stopped. On hydraulic machines a pressure switch may be used, starting the counter only after a certain working pressure is attained.

Another solution is to fit independent hour counters to the components of a machine which experience different amounts of use.

1.1.3 PREVENTIVE MAINTENANCE BASED ON THE EXAMINATION OF TECHNICAL CONDITIONS

Preventive maintenance realized according to a rigid cycle relies on their being regular wear of the machine's parts. Unfortunately the load bearing capacity and useful life of the parts cannot be assessed accurately in every case, even by a detailed multiple analysis of the operational data, in spite of the rapid development of technology.

The useful life of each part is influenced by numerous factors (e.g. the quality of production and maintenance, the greatly varying plant conditions, early dismantling, etc.). Prediction of the useful life of parts produced from less well-known materials is also uncertain.

The quality of both production and repair is influenced by the accuracy of the processing, the fineness of the surface, the heat treatment and the accuracy of the mounting.

The plant conditions may depend on the weather, the environment (dust content, relative humidity, etc.), a variable and often unexpected load, as well as on the unskilled handling of the machine.

With regard to these uncertain effects it is advisable to institute a maintenance system that ensures the operable condition of the machines and vehicles on the basis of periodic or continuous instrumental examination of their technical condition during their operation. Instrumental technical diagnostics permits the assessment of the parameters of the individual parts and components which characterize their wear (e.g. compression pressure, vibration, state of insulation of the electrical components, bearings, conductors, other electrical faults, etc.) without having to dismantle the machine.

The systematic observation, and documentation, of the technical condition of a machine enables one to determine, in the light of the factors governing its wear, the time and expected extent of necessary future repairs.

Example. The final compression pressure of internal combustion engines decreases in proportion to the wear, depending on the useful life of the cylinder and the piston and on the capacity of the motor. The date of the repair can be predicted from a knowledge of the wear rate and the degree of wear permitted from the aspect of economic and reliable operation. In other cases a conclusion may be drawn concerning the degree of wear from the quantity of wear products found in the lubricating oil, or its increase since the preceding measurement. The condition of the oil filter is characterized by the difference between the early and late measurements of the pressure values. The condition of built-in rolling bearings and plain bearings may be examined by measuring their vibration.

Technical progress has resulted in higher productivity, but also in more highly complex machines. The breakdown of these machines causes great losses in production. Their downtime may be reduced by a maintenance system based on examination of their technical condition.

The systematic examination of the technical condition of a machine may lead to a reduction in the number of unexpected failures, an increase in the interval between general repairs, or the abandoning of general repair, as understood in its traditional meaning, depending on the type of machine (or vehicle) and its role in production.

The system offers sufficient information for planning the repairs. From a knowledge of the expected failures, the choice and quantity of spare parts and the repair capacity required can be determined, enabling the optimum stock to be kept.

The organization of repairs may also be improved, and modern correction methods (e.g. replacing the main components on site) may be introduced.

On the basis of the above analysis the maintenance system based on examination of the technical condition of the machines seems to be the most economical, but its efficiency depends on the level of the diagnostic methods and the correctness of the predictions [34].

There are many methods and procedures for the examination of the technical condition (diagnostics). Some of the examination methods may already be regarded as conventional and are in general use; others are of limited application. Conventional methods include the measurement of length, pressure, temperature, speed, mass (mass flow), force, capacity, torque, hardness, the measurement of electrical quantities, etc.

The following are new methods that are not yet in general use: measurement of vibrations, vibration analysis, pulse measurement, ultrasonic techniques, noise–source analysis, noise measurement, stethoscopy, radiography, the use of radioactive isotopes, the use of potential probes, the magnetic powder process, endoscopy, spectral analysis, pulse counting, photography, etc.

A third group contains methods whose theoretical foundations are understood, but require further laboratory development to make them technically and economically applicable in practice (e.g. the four main branches of holography, i.e. holography with photo-optical presentation, holography with interferometry, holography with data processing, acoustic and microwave holography).

The widespread application of technical diagnostics is impeded by the fact that entirely suitable diagnostic means are few and that examination is also made difficult by other circumstances. Hence:

— in most cases the diagnostic examination can be performed only after dismantling the machine,
— the connection points of the diagnostically relevant parts are often nearly inaccessible,
— often no connection points for the examining instruments were provided for in the design,
— the dimensions and the form of the connection points are not standardized,
— the quality and safety characteristics which are important for the operation of the machine can be checked only partly by the methods of technical diagnostics.

Such an examination may be performed periodically, or continuously; the periodic examination is also known as "search".

In the case of advanced systems the measurements are performed automatically according to a given programme the automatic control device first senses the main characteristics and the positions of any faults and then proceeds to carry out detailed testing.

Example. Such systems are used, amongst others, by VW for examining cars and in the USSR for examining railway diesel engines and underground trains.

In the case of the so-called pointer method, used for continuous examination, the characteristic parameters of the technical condition are measured and recorded continuously (e.g. on magnetic tape) by instruments and data recording devices mounted on the machine. These characteristics are evaluated periodically (e.g. daily) by a computer, which displays the expected date and extent of the breakdown.

1.1.4 FAULT LOCATION DURING OPERATION

Wear during operation may result in partial loss of serviceability or even breakdown of the machine or one of its components.

The partial loss of serviceability means an alteration of the fundamental operational parameters within the specified values, or a maladjustment of the set values. In such cases immediate intervention, i.e. elimination of the fault, is not necessary.

If failure is due to breakage of the main structural parts, electrical short circuit, serious faults in the seals of the hydraulic system, or considerable deviation of the operational parameters from the specified values, further operation is possible only after elimination of the fault.

The fundamental purpose of inspections during operation is to decide on the serviceability by determination of the operational parameters.

Checking the running of the machines without dismantling and setting the operational parameters — depending on recognition of the faults — are performed using tests that employ the sensory organs, universal gauges, various instruments and diagnostic apparatuses and suitably developed test benches.

1.1.4.1 TESTS EMPLOYING THE SENSORY ORGANS (SUBJECTIVE TESTS)

These include visual inspection, listening, smelling, feeling.

Serious faults, breakage, deformation, external damage, seizures due to corrosion, damage to the protective coatings can all be detected with the naked eye (by inspection).

For the detection of faults difficult to see with the naked eye (e.g. cracks) a hand magnifier may be used. The endoscope facilitates inspection of interiors.

Faulty functioning, the failure of machine elements (e.g. rolling bearings, gears), the expansion of joint gaps, etc. can be detected by *listening* for abnormal noise and knocking during operation (with the ear or a stethoscope). The stethoscope is particularly suitable in noisy surroundings for listening to the sound of the machine and to sources of abnormal noise. A probe is placed in contact with the part of the machine to be checked and the vibrations picked up by the probe are amplified electronically and transmitted to headphones.

Detection of the inaudible leakage of gas or fluid and corona discharge is possible by use of the so-called leakage-finder. The signals received by the ultrasonic micro-

phone or sensing probe are converted to a visual signal by means of the luminous diode indicator of the instrument. At the same time sound impulses corresponding to the intensity of the leakage are audible in the headphones.

Smoke and smells due to chemical reactions and overheating (the smell of oil, grease, electrical insulations) are sensed by *olfaction.*

Feeling with the hand is a suitable means for checking the heating and vibration of structural elements (e.g. bearings).

Mocolours (heat sensitive paints) are used for the subjective checking of temperatures.

1.1.4.2 MEASURING THE OPERATIONAL PARAMETERS (OBJECTIVE TESTS)

Universal instruments or diagnostic devices are used for determining the fundamental operational indices, i.e. parameters referring to the technical condition of the machine. These are suitable for testing the machine and its structural units without the need for dismantling.

The operational parameters to the measured are temperature, pressure, speed, quantity of fluid delivered or consumed, moment, power, acceleration or deceleration, noise, vibration, level of oil impurity.

Various groups of instruments (e.g. universal instruments) are available for measuring the different parameters. With the aid of suitable instruments test benches have been developed for complex tests, enabling several parameters to be measured at the same time,

The *temperature* of oil, cooling water and bearings and the heating of electrical equipment can be checked with thermometers or by using thermovision (infra-red television).

The principle of *thermovision* measuring is that the wavelength of the infra-red radiation emitted from the surface to be tested varies according to the temperature of the surface. The intensity of the radiation from the surface of the tested object appears on the monitor as a black-and-white picture. The parts of the picture which have the same tone of grey indicate those areas on the surface which have the same temperature, provided the surface is homogeneous. The thermovision equipment may be provided with an auxiliary colour monitor, where ten different colours correspond to the different radiation intensities. The coloured picture appearing on the monitor can be photographed. The actual values of the temperatures corresponding to the different colours on the photograph can be determined by the use of reference measurements. The light colours on the photograph represent higher surface temperatures.

Manometers are used for measuring the pressure of the lubricating oil, hydraulic oil, or air as well as the compression of the engine.

The speeds of internal combustion engines and electric motors, or other rotors, are measured with tachometers and stroboscopes.

Flow calorimeters are used for measuring the quantity of fluid delivered.

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Faulty functioning, the failure of machine elements (e.g. rolling bearings, gears), the expansion of joint gaps, etc. can be detected by *listening* for abnormal noise and knocking during operation (with the ear or a stethoscope). The stethoscope is particularly suitable in noisy surroundings for listening to the sound of the machine and to sources of abnormal noise. A probe is placed in contact with the part of the machine to be checked and the vibrations picked up by the probe are amplified electronically and transmitted to headphones.

Detection of the inaudible leakage of gas or fluid and corona discharge is possible by use of the so-called leakage-finder. The signals received by the ultrasonic micro-

phone or sensing probe are converted to a visual signal by means of the luminous diode indicator of the instrument. At the same time sound impulses corresponding to the intensity of the leakage are audible in the headphones.

Smoke and smells due to chemical reactions and overheating (the smell of oil, grease, electrical insulations) are sensed by *olfaction.*

Feeling with the hand is a suitable means for checking the heating and vibration of structural elements (e.g. bearings).

Mocolours (heat sensitive paints) are used for the subjective checking of temperatures.

1.1.4.2 MEASURING THE OPERATIONAL PARAMETERS (OBJECTIVE TESTS)

Universal instruments or diagnostic devices are used for determining the fundamental operational indices, i.e. parameters referring to the technical condition of the machine. These are suitable for testing the machine and its structural units without the need for dismantling.

The operational parameters to the measured are temperature, pressure, speed, quantity of fluid delivered or consumed, moment, power, acceleration or deceleration, noise, vibration, level of oil impurity.

Various groups of instruments (e.g. universal instruments) are available for measuring the different parameters. With the aid of suitable instruments test benches have been developed for complex tests, enabling several parameters to be measured at the same time,

The *temperature* of oil, cooling water and bearings and the heating of electrical equipment can be checked with thermometers or by using thermovision (infra-red television).

The principle of *thermovision* measuring is that the wavelength of the infra-red radiation emitted from the surface to be tested varies according to the temperature of the surface. The intensity of the radiation from the surface of the tested object appears on the monitor as a black-and-white picture. The parts of the picture which have the same tone of grey indicate those areas on the surface which have the same temperature, provided the surface is homogeneous. The thermovision equipment may be provided with an auxiliary colour monitor, where ten different colours correspond to the different radiation intensities. The coloured picture appearing on the monitor can be photographed. The actual values of the temperatures corresponding to the different colours on the photograph can be determined by the use of reference measurements. The light colours on the photograph represent higher surface temperatures.

Manometers are used for measuring the pressure of the lubricating oil, hydraulic oil, or air as well as the compression of the engine.

The speeds of internal combustion engines and electric motors, or other rotors, are measured with tachometers and stroboscopes.

Flow calorimeters are used for measuring the quantity of fluid delivered.

The torque of an engine or drive is measured with a measuring shaft or belt balance.

Accelerometers and decelerometers are used for measuring accelerations and decelerations — for example to test brake efficiency.

For testing the condition of *bearings, gears* or *drives,* noise-level meters, vibrometers, frequency analysers, or bearing-test sets are used.

The *impurity* of engine, drive, or hydraulic oils is determined by spectral analysis, particle analysis, activation analysis, or ferrography.

The *electrical parameters* (voltage, current intensity, insulation resistance etc.) are measured with universal electrical instruments.

1.2 THE SELECTION OF THE MAINTENANCE SYSTEMS

The choice of one of the above maintenance systems is determined by the structure, the complexity and the role of the machines involved in the production [57, 72]. For simple machines repair by necessity is sufficient, if the faults can be corrected in a relatively short time without any considerable downtime. A preliminary condition is that no further damage ensues because of delay in fault detection.

Planned preventive maintenance (PPM) is used for mechanical machines of average complexity, the breakdown of which leads to a halt in production for a longer time, with high material losses, further destruction of the machine and possibly the risk of accidents and threats to life.

For improving the efficiency of the PPM-system, it is advisable to employ a cyclic maintenance of the main parts, or of the components. The suitability of the maintenance cycle under the given plant conditions must be ensured by a systematic fault analysis. The conditions for the capacity-rated maintenance cycle must be created, and the performance of the machine or its main units must be recorded.

Preventive maintenance based on the examination of the technical condition is advisable mainly for complex machines and in cases where the diagnostic examination and the unequivocal evaluation of the measured data can be realized.

In certain cases PPM and maintenance based on the examination of the technical condition may be applied together. The essence of this system is that the repairs that become due in shorter intervals than those arising from the systematic maintenance are carried out depending on the result of the diagnostic examination. At longer intervals — once or twice only, depending on the useful life of the machine — a general repair or reconditioning involving the complete dismantling of the machine may also be performed.

Also other combinations of the systems described above are possible, serving always the requirements of economical operation.

2 THE PROCESS OF MACHINE WEAR

The wear to which a machine is subjected depends on the process of deterioration of its parts. Under the effect of varying loads during operation the parts gradually deteriorate in their abilities to meet the functional requirements of the machine; their utility decreases continuously until they become completely useless.

Therefore there are two domains of faults appearing during the operation of the machine which must be investigated:

— the faults of the parts and
— the faults of the machine.

The faults of the parts may be due to material flaws or macrogeometrical and microgeometrical faults. Material flaws occur on the one hand during production (casting, forging, heat treatment, welding) because of an unsuitable chemical composition, mechanical properties which deviate from the description, or gas and slag inclusions; on the other hand they may be caused by changes in the chemical composition and the mechanical properties during operation (e.g. sulphating of battery plates, variations in the composition of the lubricant; changes in the material properties due to the working temperature, decreasing elasticity, etc.).

Faults occur in a machine during its operation and change its operational characteristics (including, for example, reduction of the capacity, loosening of the fixing bolts, etc., shifting of the adjusted values, the appearance of dynamic loads, vibrations, etc.).

Macrogeometrical faults include the dimensions, the form, and the positions of individual surfaces of a part relative to each other (e.g. coaxiality faults), and deformations, cracks and breaks. Microgeometrical faults are changes in the surface roughness and defects of the surface layer.

Faults may arise because of omissions in the design, carelessness in production (e.g. faults of processing, machining, positioning, measurement, mounting, adjustment, etc.), repairs of an unsatisfactory quality and, finally, the mode of operation.

Some faults may even occur during the prescribed operational mode. These are slowly developing faults due to natural wear in use, material fatigue and corrosion. Their appearance cannot be prevented, but their progress may be slowed down.

Another group of operational faults is caused by the incorrect use and negligent handling of the machine.

The causes of faults may be grouped according to primary, secondary and indirect sources.

Example. If a shaft is deformed by an overload, the primary source of the fault is the overload — an incorrect operation. But the deformation leads to imbalance and increasing vibration, which may become a secondary source of material fatigue and cracking.

2.1 THE WEAR OF MACHINE PARTS

The parts of a machine are worn by friction as they move over each other, thereby reducing the dimensions and weight of each part. Wear is a gradual separation of particles of material from the surface under the effect of mechanical forces, frictional forces and other influences (e.g. heating, oxidation, etc.).

The process of wear may cause much consequential damage (e.g. seizures, cracks, factures, etc.), leading to deterioration of the parts and the inoperability of the machine.

Various kinds of wear occur, depending on the material characteristics of the pair of elements in frictional contact, the load, the conditions under which the friction occurs (dry or wet friction, limit friction) and the effects of the lubricant. Hence we have:

— cold-welding (adhesive) wear,
— oxidation wear,
— hot-welding wear,
— abrasive wear,
— fatigue wear,
— fretting wear [95].

Cold-welding (adhesive) wear occurs when the pressure between the contacting surfaces is high and they move over each other relatively slowly, without a film of liquid or oxide on the surfaces. Because of the high surface-pressure and the friction the surface layers are subjected to plastic deformation and hardening as a function of the distance measured from the surface (Fig. 2.1).

At the points of contact a molecular–metallic (adhesive) bond is formed. During further displacement the harder metal rips out particles from the surface of the softer metal as a result of their being welding together. The softer surface is scratched and worn by the harder surface and the ripped-out particles, which act as abrasive material.

Some of these particles are smeared onto the surface, causing further unevenness. The coefficient of friction for cold-welding wear is the highest among those for the various kinds of wear. Thus the wear rate is also the highest, causing considerable

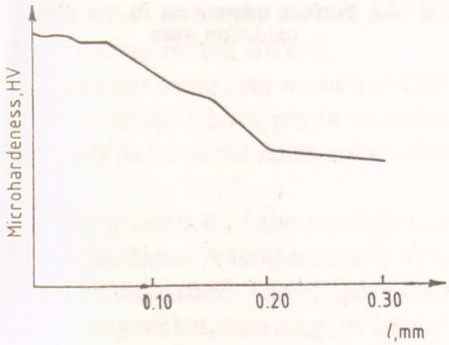


Fig. 2.1 Surface hardening

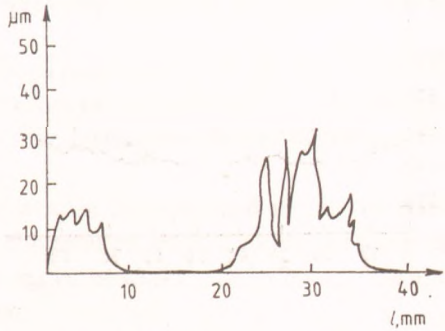


Fig. 2.2 Surface unevenness due to cold-welding wear
The surface may get easily creased and smeared during softening

dimensional changes, seizures and finally rapid destruction of the part. Figure 2.2 illustrates the measured surface-unevenness produced by cold-welding wear.

Oxidation wear occurs during faster relative movements of the contacting surfaces, where the preliminary conditions for forming and reforming oxide layers prevail. The heat produced by the rapid friction promotes the oxidation. The rate of oxidation is higher than the rate of formation of a possible metallic bond in cold-welding wear. The surface pressure is lower than in the case of cold-welding wear, it does not break the lubricant film nor destroy the surface oxide layer, so no welding occurs. Oxidation wear is nearly independent of the state of lubrication.

The oxidation results in the formation of metal-oxide compounds on the surface with 0.2...0.5% oxygen content, i.e. about 10–50 times the initial oxygen content of the surfaces. The metal oxide (e.g. FeO) forms a solid solution with the metal of about 650 HV hardness, compared with hardnesses of the oxides in the range 1100... ...1400 HV. These rigid solid-solutions peel off slowly, as the contacting surfaces move over each other, in the form of oxide powder and oxide film. The separation process is very slow; the oxides protect the surfaces against intense welding wear and increase the wear resistance of the surface considerably. So oxidation wear is a slow and, usually, a minor process.

Figure 2.3 shows the type and degree of surface unevenness formed during oxidation wear.

The wear extends to the whole surface of the part at a nearly uniform rate, depending on the load. As the process progresses, microcracks appear on the surface under the stress concentrating effect of the rigid compounds leading to fatigue pitting.

Hot-welding wear appears during rapid sliding friction when the surfaces are loaded by a high specific pressure. The surfaces in frictional contanct are heated up very rapidly to 800...900 °C, or even higher; moreover, the temperature of the surface layer may even attain 1500 °C. Under the effect of the frictional heat, the tensile

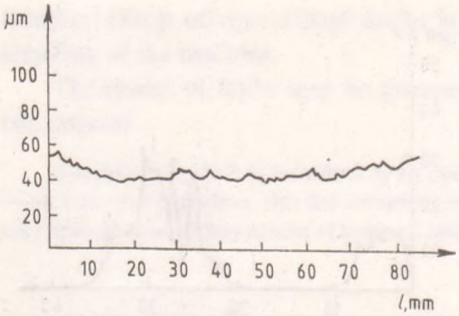


Fig. 2.3 Surface unevenness in the case of oxidation wear

strength, the yield limit and the hardness of the surface layer decrease, i.e. this layer softens. In addition transcrystallization, recrystallization, tempering, secondary hardening and even partial melting may occur, with the formation of local metallic bonds between the frictioning surfaces, as in the case of cold-welding wear. In the course of further friction the metallic parts weld together and the ripped-out particles are destroyed.

The development of the wear is determined by the strength of the surface layers and the conditions of the formation of metallic bonds. This explains the high resistance to this type of wear of heat-resistant steels of high hot strength.

As a consequence of the high surface temperature (600...1500 °C) the rate of oxidation is also high. The oxide layer formed may reduce the rate of wear. The hot-welding wear of a loaded surface is less uniform than oxidation wear. In some parts of the surface deeper seizures may be present (Fig. 2.4).

Figure 2.5 shows the degree of wear per unit length and the oxygen content of the surface versus the displacement rate for the three types of wear described above.

Abrasive wear is caused by hard, small grains, which have a grinding effect — the so-called abrasives found between the surfaces moving over each other. The hard,

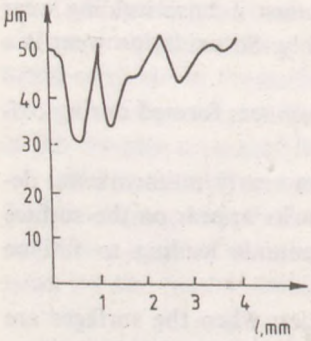


Fig. 2.4 Surface unevenness in the case of hot-welding wear

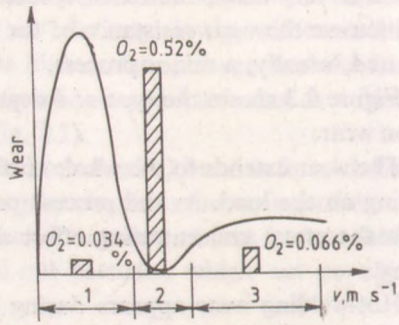


Fig 2.5 Specific wear versus the displacement rate

1 primary adhesive wear ; *2* oxidation wear; *3* hot-welding wear

sharp grains scratch the surface and remove microchips from it; this effect is considerable during sliding friction.

Abrasive wear may occur at widely varying speeds and loads (in this respect it differs from the other types of wear described above) and is influenced by the temperature only as far as the mechanical properties of the surface are affected by the temperature.

The resistance of the metals to abrasive wear is characterized mainly by their scratch-hardness. A harder surface is worn to a greater extent by an abrasive material than a softer surface. The abrasive grains become embedded in the softer surface and move along with it, abrading the harder surface.

The abrasive material may consist of grains from various sources (e.g. dust, sand), or friction-wear products (e.g. metal grains), or oxides peeled off during friction. If an oxide film adheres well, it increases the wear resistance of the metal, even against abrasive wear, but when it peels off it has a highly abrasive action.

Fatigue wear is met with mainly in rolling bearings. The repeated compressive load brings about the plastic deformation of the surface layer. The volume of the latter is increased by plastic deformation and hardening. The increase of the volume leads to a considerable shear stress at the boundary between the deformed and undeformed layers of material. The surface layer is fatigued by the repeated stresses, and microcracks appear in it. Under the effect of the latter and a high sliding stress, the surface layer splits and peels off in floculated form.

Figure 2.6 shows the stresses arising when cylindrical and plane surface are in contact.

The formation of cracks is accelerated by the residual stresses found in the surface layers (caused by processing, heat treatment, etc.). The surface may be fatigued also by the pressure of the lubricant. In elastic lubricants the pressure distribution does not follow the Hertz-distribution, but pressure peaks develop on the outgoing side. The pressure peaks are the starting points for fatigue wear and pitting.

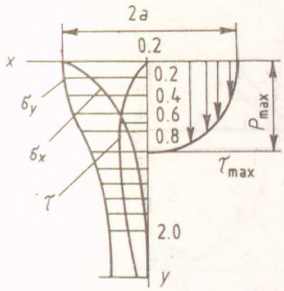


Fig. 2.6 The distribution of stresses over the contacting surfaces

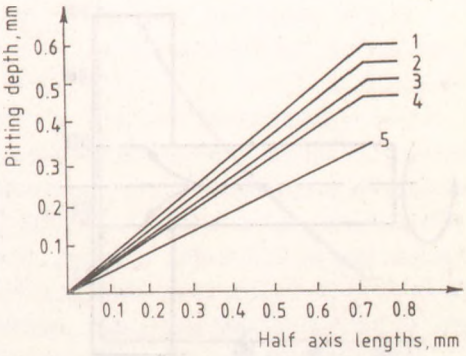


Fig. 2.7 The depth of pitting

According to observations the maximum pitting depth coincides with the point of maximum shear stress. The depth of pitting or flaking — depending on the shape and dimensions of the contacting surfaces — is shown in Fig. 2.7. This theoretical depth depends on the ratio of contacting surface and semi-axes *a*, *b* of the contacting ellipse.

The pits, as places of stress accumulation, may be sources of further damage, initiating secondary cracks and splitting.

Fretting wear may occur even at vibrations of low amplitude (1...5 μm). The friction process is accompanied by heavy oxidation; on the surface of steel parts in contact, black or red-brown deposits, i.e. oxide layers, may be observed. The abraded metal surfaces show traces of metal transfer (e.g. by welding effects), abrasive wear, impressions and pitting. The wear products that remain between the fretting frictional surfaces cause rapid wear and seizure, greatly reduce the fatigue limit of the parts and, finally, may cause cracking and fracture.

Where vibration is present this type of wear occurs in all the mechanical parts (e.g. motors, vehicle bodies, airplanes, machine tools, machine elements, etc.).

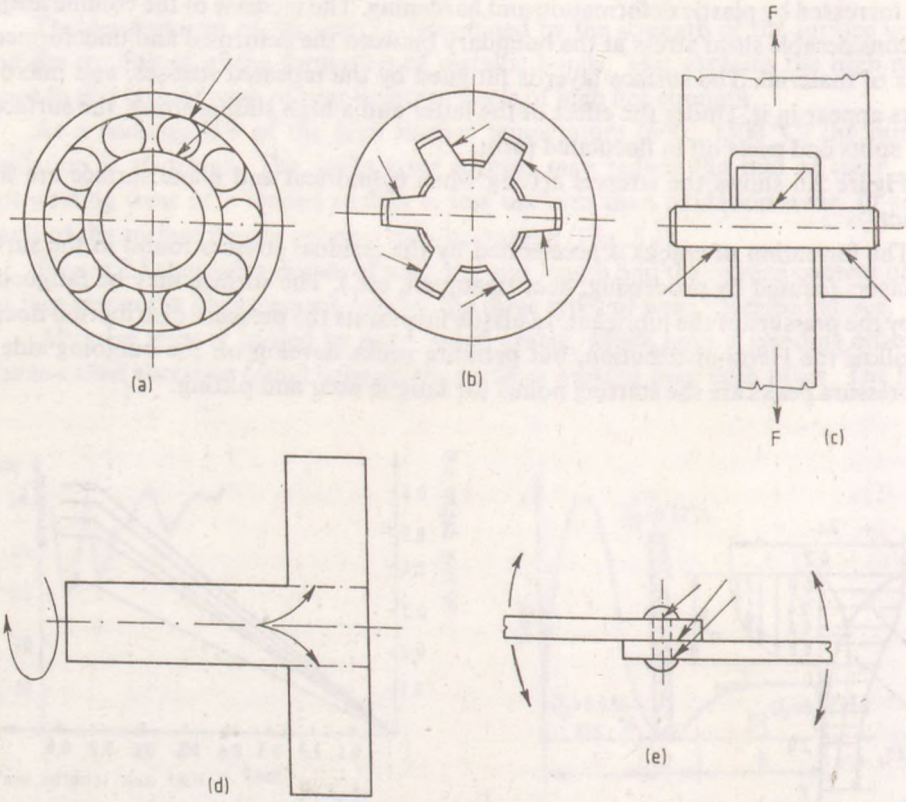


Fig. 2.8 Fretting wear

(a) on a rolling bearing ; (b) in ribbed junctions; (c) on the contact surfaces of a pin and bore-hole; (d) of a hub and hub seat; (e) in the case of riveted connections

Fretting wear may also be found in machine parts which are assembled by solid or transition matching [68]. Figure 2.8 illustrates the appearance of fretting wear (indicated by arrows) on some characteristic machine elements.

The rate of fretting wear is influenced mainly by the material properties of the contacting parts, the load applied on their surfaces, the lubrication characteristics and the amount of vibration.

Generally the harder the surface, the less wear it suffers. But, besides the hardness, the properties of the oxide layers and their adhesion to the surfaces are also of high importance. The rate of fretting wear is increased by the presence of a load; but the load does not affect the destruction of the surface if both surfaces are displaced only by vibration (e.g. in the case of solid fits), while the sliding of matched pairs is impeded by large loads.

The amount of wear is increased by the oxygen in the atmosphere and the humidity of the air.

The wear rate is greatly influenced by the properties of the lubricants and the mode of lubrication. Fretting wear is decreased by stable lubrication oils, which are resistant to high pressure and do not incline to oxidation, their oxide solubility is low and they adhere well to the surfaces. The development of fretting is impeded more by high viscosity oils and thick lubricant layers than by thin lubricant films. A large quantity of lubricant circulating between the surfaces (forced lubrication) reduces the rate of fretting wear, even with low viscosity oils, by removing the wear products from between the surfaces.

Figure 2.9 shows that the greater the amplitude of the vibration the greater the wear [94]. But the maximum wear attained by increasing the amplitude is followed

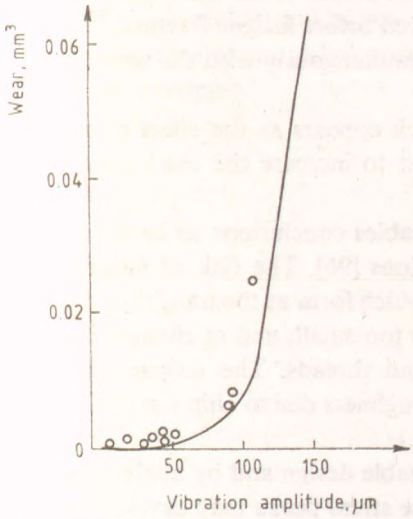


Fig. 2.9 Fretting wear versus the amplitude

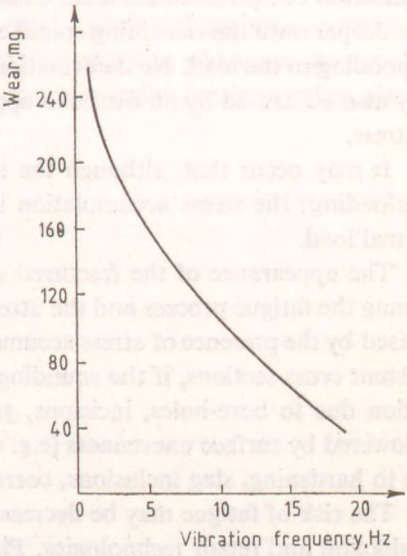


Fig. 2.10 Fretting wear versus the frequency

by a reduction. The relationship between the amplitude and the maximum wear depends on the quality of the materials, on the load, the lubricant, etc.

The frequency of the vibration does not generally affects the wear rate, but the results of some research have shown that the destruction of the surface decreases with increasing frequency — as shown in Fig. 2.10 [96]. Here the surface pressure is 50 MPa (5 kp/mm^2) and the amplitude is 0.5 mm.

2.2 MATERIAL FATIGUE

Under the effect of continually repeated large tensile or compressional bending, or torsional loads, cracking begins in steel components and structures after a certain period of use; this is followed by fracture. Fatigue fracture may even occur at a considerably lower load than that equal the static strength [57].

This property of metals is known as fatigue. The overwhelming majority of the fractures occurring during operation are fatigue fractures. This is experienced especially in rotating shafts. Structural materials may be fatigued not only by varying loads, experienced during operation, but also by the imbalance of the rotating masses, or by vibrations caused by other types of load.

The useful life of parts loaded by variable loads depends on the fatigue limit of the material from which they are fabricated. The fatigue limit is the maximum continuously varying stress tolerated by the material without fracture occurring at any number of times.

Fatigue fracture always starts with the appearance of haircracks at the point of application of the maximum load. Under the effect of repeated loads, the crack grows ever deeper until the remaining sound cross-section breaks under the static stress corresponding to the load. No deformation is observed before fatigue fracture. Fractures may also be caused by an overload appearing simultaneously with the accumulation of stress.

It may occur that, although the initial crack appears as the effect of repeated overloading; the stress accumulation is sufficient to increase the crack even at the normal load.

The appearance of the fractured surface enables conclusions to be drawn concerning the fatigue process and the stress conditions [96]. The risk of fatigue is increased by the presence of stress accumulations which form at the transitions between different cross-sections, if the rounding radius is too small, and at changes in cross-section due to bore-holes, incisions, grooves, and threads. The fatigue limit may be lowered by surface unevenness (e.g. surface roughness due to chip removal, cracks due to hardening, slag inclusions, corrosion, etc.).

The risk of fatigue may be decreased by suitable design and by applying correct production and repair technologies. Places where stress peaks may develop must be avoided in the design. From the technological point of view finer surface processing

and suitable heat treatment (e.g. cementing, high-frequency hardening, nitriding, etc.) give more favourable results. The fatigue limit may be increased by the plastic forming of the surface or by surface hardening (e.g. by rollers, shot cleaning, surface hardening by impacting bodies, etc.).

2.3 CORROSION

Corrosion is the chemical and electrochemical decomposition of metals resulting from exposure to environmental effects.

Depending on the surface attacked, the corrosion may be uniform in appearance or involve pitting, it may be intercrystalline and, in the case of alloys, selective.

Corrosion, which starts on the surface of the metal, decreases the dimensions, the weight and the strength of the part. Chemical corrosion may occur in all sorts of circumstances. The preliminary condition for electrolytic corrosion is two metals of different electric potential in contact in the presence of an electrolyte, which dissolves the more electrically negative metal. Electrolytic corrosion may occur, amongst other processes, when two materials of different quality and different crystalline structure are put in contact.

In a corrosive medium the fatigue limits of the materials change. After a time, a fracture may occur even under a low load, as the materials loose their strength by corrosion even if they are unloaded. The cracks caused by corrosion fatigue are intercrystalline phenomena in nearly every case.

Corrosion, which is independent of the mode of operation, causes very great losses. Machines and vehicles exposed to extreme weather conditions must be especially protected against corrosion. The metals may be protected against corrosion by alloying (chrome, nickel) and by the use of protective coatings, which may be metallic (e.g. zinc, tin, chromium, nickel, copper, etc.) or non-metallic (various paint, plastic and enamel coatings).

3 VIBRATION AND ITS MEASURING INSTRUMENTS

3.1 THE BASIC CONCEPTS OF VIBROMETRY

If a body, of mass m, connected to a spring is removed from its equilibrium position then released, it will perform a vibratory motion (Fig. 3.1). If the body may only be deflected in the y direction, its position can be given by a single coordinate. This sample system is called a vibratory system with one degree of freedom. The central equilibrium position of the mass corresponds to the position in which the spring is in a relaxed state. The deflection of the mass is measured from this position. When the mass is displaced the spring acts on it with a force which is proportional to the displacement (deflection); this force is of opposite sense to the displacing force and tries to return the mass to its original (central) position. In most cases the characteristic of the spring is assumed to be linear.

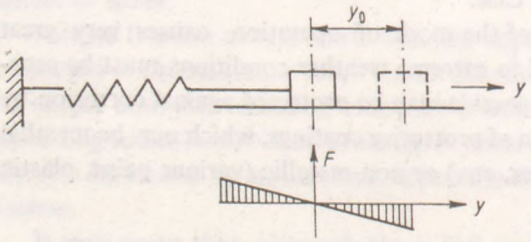


Fig. 3.1 A single-mass vibratory system of one degree of freedom

After releasing the mass, which has been displaced by an external force to a distance y_0, it is returned by the force of the spring to its central position. If there is no loss of energy (damping), the mass moves to the opposite side of the equilibrium position to a distance y_0. The motion is a simple sinusoidal oscillation about the point of equilibrium. The displacement, the velocity and the acceleration of the mass point vary periodically.

According to our assumption, the restoring force exerted by the spring on the mass point is proportional to the magnitude of the deflection, but its sense is opposite; therefore the acceleration of the mass point is also proportional to the deflection. Such a system is called a simple harmonic oscillator. According to a basic law of dynamics the movement of the mass is described by the following differential equa-

tion:

$$m\frac{d^2y}{dt^2} = -\frac{y}{c},$$

where

$\frac{d^2y}{dt^2}$ is the acceleration of the mass,

$-\frac{y}{c}$ is the spring force corresponding to the deflection y, c is a constant of the spring.

The negative sign refers to the fact that the restoring force is in the opposite sense to that of the deflection. The spring constant expresses the deflection brought about by a unit force, so its dimensions are m/N, or mm/N.

Introducing the notation $\alpha^2 = \frac{1}{mc}$, we obtain, after rearranging,

$$\frac{d^2y}{dt^2} + \alpha^2 y = 0.$$

For the linear combination of the independent particular solutions of this homogeneous differential equation the following general solution is obtained:

$$y = A \sin \alpha t + B \cos \alpha t.$$

The constants A and B are obtained on taking into account the starting conditions.

If, for example, at the instant $t=0$ the mass is in its equilibrium position $(y=0)$ and its velocity is v_0, the accurate form of the equation describing the motion may be determined as

$$y = \frac{v_0}{\alpha} \sin \alpha t \quad \text{or} \quad y = A \sin \alpha t,$$

where

A is the maximum deflection, i.e. the amplitude of the harmonic vibration,

α is the angular frequency of the natural vibration of the vibrating system, which depends on the spring constant and the magnitude of the mass; therefore

$$\alpha = \frac{1}{\sqrt{mc}}.$$

If the harmonic vibration is regarded as the projection of a rotating vector, then α is the angular velocity of the rotation; so α is also called the "projectional angular velocity".

The deflection of the harmonic oscillation may also be plotted as a function of time (Fig. 3.2).

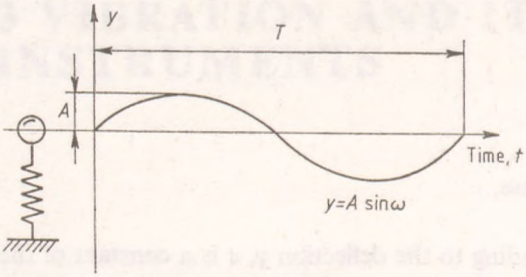


Fig. 3.2 Deflection of a harmonic oscillation versus time

The kinetic characteristics of the moving mass point (deflection, velocity, acceleration) after a time T are the same as they were at $t=0$. T is called the period of vibration or period time. The number of complete oscillations performed in unit time (1 second) is the frequency of the vibration, denoted by f or ν; it is measured in Hz.

In terms of the angular frequency, α, the frequency is

$$f = \frac{\alpha}{2\pi} \text{ Hz}$$

and the period is

$$T = \frac{1}{f} = \frac{2\pi}{\alpha} \text{ s};$$

the angular frequency is

$$\alpha = 2\pi f = \frac{2\pi}{T}.$$

The velocity is the first derivative with respect to time of the deflection $y=A \sin \alpha t$ of the harmonic oscillation, the acceleration is the second:

$$\text{velocity} = v = \frac{\mathrm{d}y}{\mathrm{d}t} = \alpha A \cos \alpha t = v_0 \cos \alpha t = v_0 \sin\left(\alpha t + \frac{\pi}{2}\right),$$

where

$v_0 = \alpha A =$ the amplitude of the velocity;

$$\text{acceleration} = a = \frac{\mathrm{d}^2 y}{\mathrm{d}t^2} = \frac{\mathrm{d}v}{\mathrm{d}t} = -\alpha^2 A \sin \alpha t = -\alpha v_0 \sin \alpha t =$$

$$= A\alpha^2 \sin(\alpha t + \pi) = a_0 \sin(\alpha t + \pi),$$

where

$a_0 = A\alpha^2 =$ amplitude of the acceleration.

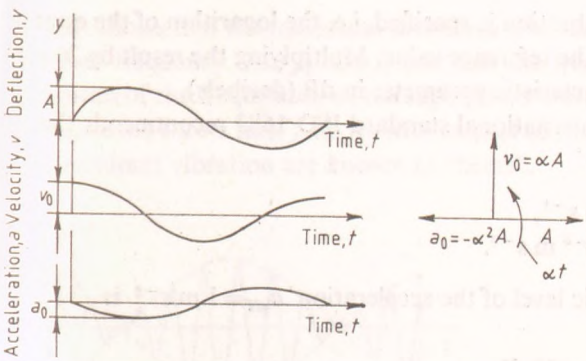


Fig. 3.3 Variation with time of the parameters of harmonic vibratory motion

In Figure 3.3, the deflection, the velocity and the acceleration are plotted with respect to time. The velocity and acceleration are seen to lead the deflection by one quarter of the period and by half of the period, respectively, as demonstrated also by the rotating vectors on the right of the figure. The instantaneous values of the deflection, velocity and acceleration are obtained by forming the vertical projections of the vectors rotating at angular velocity α. The figure shows the positions of the vectors corresponding to the time $t=0$.

Harmonic vibratory motion may be described unequivocally by the maximum deflection, the value of the velocity or acceleration and the angular frequency or period of oscillation; therefore it is advisable to measure peak values (A, v_0, a_0). But if the vibration is not purely sinusoidal, it cannot be characterized by peak values, as they disclose nothing about the nature of the motion between the two peak values. In this situation somewhat more may be learned from the arithmetic mean, the quadratic mean and the effective value. The arithmetic mean is not used in practice.

The effective value of the acceleration is given by

$$a_{\text{eff}} = \sqrt{\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} a^2(t)\,\mathrm{d}t};$$

and for a single wave,

$$a_{\text{eff}} = \sqrt{\frac{1}{T} \int_0^T a^2(t)\,\mathrm{d}t}.$$

The effective value may also be indicated by the suffix RMS (Root Mean Square). The effective value of the acceleration of a point experiencing sinusoidally varying vibrations is

$$a_{\text{eff}} = \frac{\sqrt{2}}{2}\, a_0 = 0.707\, a_0.$$

Here a_0 is the amplitude of acceleration which also represents the peak value of the acceleration of the harmonic vibration. Frequently the logarithmic level of the

characteristic parameter of the vibration is specified, i.e. the logarithm of the quotient of the effective value divided by the reference value. Multiplying the result by 20 gives the logarithmic level of the characteristic parameter in dB (decibels).

The second edition of the international standard ISO 1683 recommends the following reference values:

for the velocity, $v_r = 10^{-9}\ \mathrm{m\,s^{-1}}$,
for the acceleration, $a_r = 10^{-5}\ \mathrm{m\,s^{-2}}$.

For example, the logarithmic level of the acceleration $a_{\mathrm{eff}} = 1\ \mathrm{m\,s^{-2}}$ is

$$L_a = 20 \log \frac{1}{10^{-5}} = 20 \times 5 = 100\ \mathrm{dB}.$$

Note: In numerous countries the reference values recommended by ISO have not yet been standardized, so other values are used instead.

Therefore the logarithmic level given in dB units of a characteristic parameter of vibration can be interpreted only if the value of the reference quantity is known.

For information, Table 3.1 shows the reference values used in some countries.

Table 3.1
Values of reference quantities used in some countries

Country	v_r	a_r	Remark
	$\mathrm{m\,s^{-1}}$	$\mathrm{m\,s^{-2}}$	
Hungary	5×10^{-8}	1×10^{-6}	Hungarian standard, MSz 3391
France	1×10^{-8}	1×10^{-5}	—
GDR	5×10^{-8}	1×10^{-6}	—
USSR	4.81×10^{-8}	3.102×10^{-4}	—

Frequently several vibrations are added up, i.e. superimposed. Vibrations of a uniform deflection A and different frequencies, f_1, f_2, are superimposed in the same direction.

Let us assume, for the sake of simplicity, that at the instant $t=0$ both vibrations have the same phase, there is no phase shift between them $(\varphi=0)$.

$$y_1 = A \sin \alpha_1 t = A \sin 2\pi f_1 t;$$

$$y_2 = A \sin \alpha_2 t = A \sin 2\pi f_2 t.$$

The resultant superposition of both vibrations is:

$$y = y_1 + y_2 = A(\sin 2\pi f_1 t + \sin 2\pi f_2 t) =$$

$$= 2A \cos 2\pi \frac{f_1 - f_2}{2} t \sin 2\pi \frac{f_1 + f_2}{2} t.$$

This shows that the maximum deflection will vary according to a cosine function at a lower frequency than f_1 and f_2 and that the frequency of vibration is the arithmetic mean of the frequencies of the superposed vibrations.

Figure 3.4 shows how y varies with respect to time. These variations in amplitude of the resultant vibration are known as "beats".

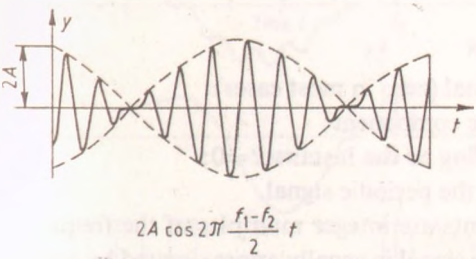


Fig. 3.4 The resultant of two vibrations which have frequencies f_1 and f_2 and identical amplitudes and phases

If the quotient of the frequencies of the two components is rational, then the resultant vibration is periodic. Figure 3.5 illustrates a few combined vibrations of this type.

If two or more vibrations are combined, the resultant motion may be highly variable, as we have seen. The opposite process to superposition can also be carried

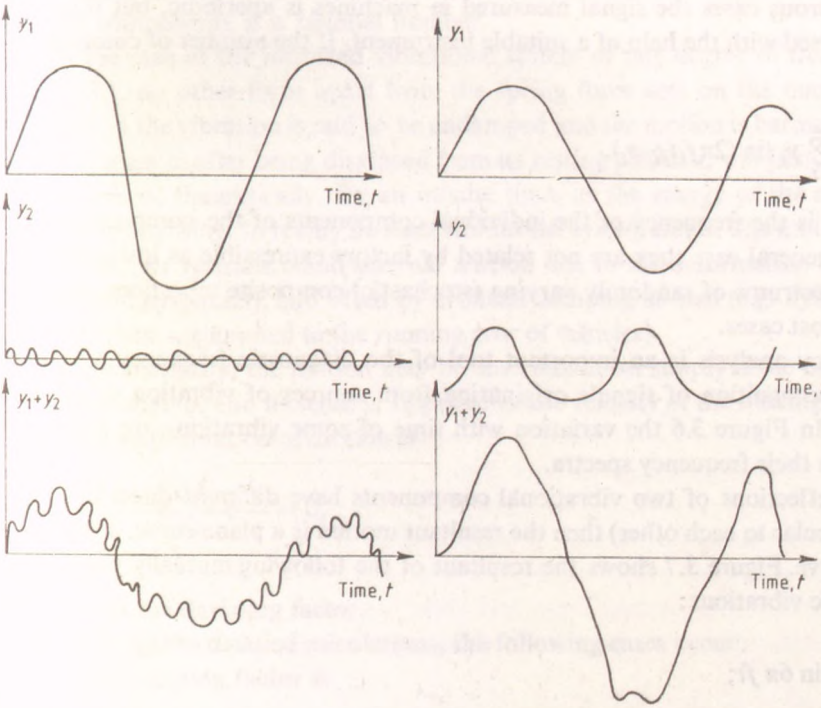


Fig. 3.5 Various component vibrations and their resultants

out: any periodic motion may be decomposed into several harmonic components and may be assumed to have been formed from these components.

According to the mathematically proved thesis of Fourier, any arbitrary periodic vibration $y(t)$ can be decomposed into harmonic components:

$$y(t) = y_0 + \sum_{i=1}^{\infty} y_i \sin(2\pi f_i t + \varphi_i)$$

where

y_0 is the arithmetic mean of the signal (zero in most cases);
y_i is the amplitude of the harmonic component;
φ_i is the phase position corresponding to the instant $t=0$;
f_0 is the fundamental frequency of the periodic signal.

The frequencies of the other components are integer multiples of the frequency of the fundamental harmonic. The periodic signal is usually approximated by a finite number (n) of components. So the combined periodic vibration may also be given by the components.

This can be illustrated by plotting a frequency spectrum. In this case the frequency is measured on the abscissa and the amplitudes of the components, or their effective values, on the ordinate of an orthogonal coordinate system. In the case of a periodic signal a line spectrum is obtained.

In numerous cases the signal measured in machines is aperiodic, but this may also be analysed with the help of a suitable instrument. If the number of components is n, then

$$y(t) = \sum_{i=1}^{n} y_i \sin(2\pi f_i t + \varphi_i).$$

Here f_i is the frequency of the individual components of the composite vibration (in the general case they are not related by factors expressible as integers). The frequency spectrums of randomly varying (stochastic) composite vibrations are continuous in most cases.

Frequency analysis is an important tool of the diagnostic frequency test, permitting the recognition of signals originating from sources of vibration of various frequencies. In Figure 3.6 the variation with time of some vibrations are presented together with their frequency spectra.

If the deflections of two vibrational components have different directions (e.g. are perpendicular to each other) then the resultant motion is a plane curve. This is the Lissajous curve. Figure 3.7 shows the resultant of the following mutually perpendicular harmonic vibrations:

$$x = \frac{A}{2} \sin 6\pi ft;$$

$$y = A \cos 2\pi ft.$$

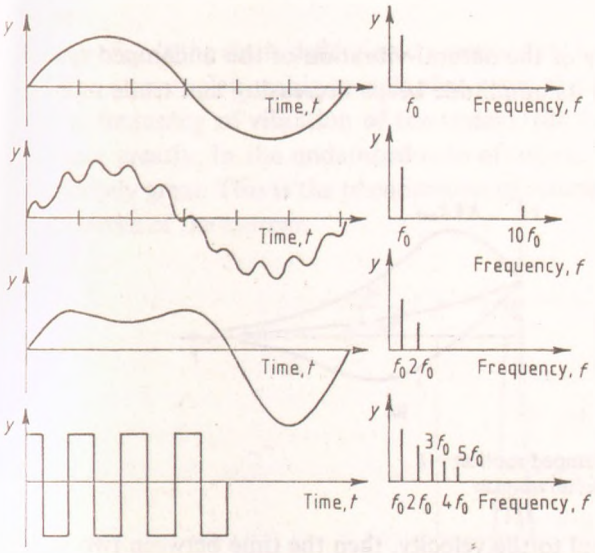


Fig. 3.6 Frequency spectra of some vibrations

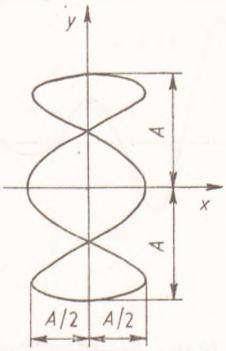


Fig. 3.7 The resultant of two mutually perpendicular vibrational components

A continuously closed curve is obtained, because the ratio of the frequencies of the two components is a rational number.

In the case of the idealized vibrational system of one degree of freedom shown in Fig. 3.1, no other force apart from the spring force acts on the mass. With this assumption the vibration is said to be undamped and the motion is harmonic. Accordingly the mass m after being displaced from its resting position, will perform its vibrational motion, theoretically, for an infinite time, as the energy of the system is not reduced by anything. In reality no such vibrational system exists. The motion is braked by friction, air resistance and internal friction due to the deformation of the spring (mechanical hysteresis), and often by artificial damping as well (e.g. hydraulic vibration absorbers are applied to the running gear of vehicles).

Mathematically, the motion may be described most simply if the damping force is proportional to, and its sense is opposite to the velocity of the moving mass; hence the differential equation of motion is

$$m\frac{d^2y}{dt^2}+k\frac{dy}{dt}+\frac{y}{c}=0,$$

where

k is the damping factor.

Omitting the detailed calculations, the following cases occur:

if the damping factor is

$k < 2m\alpha,$

where m is the mass

α is the angular frequency of the natural vibration of the undamped system, the motion remains periodic, but its amplitude keeps decreasing and tends monotonously to zero (Fig. 3.8(a)).

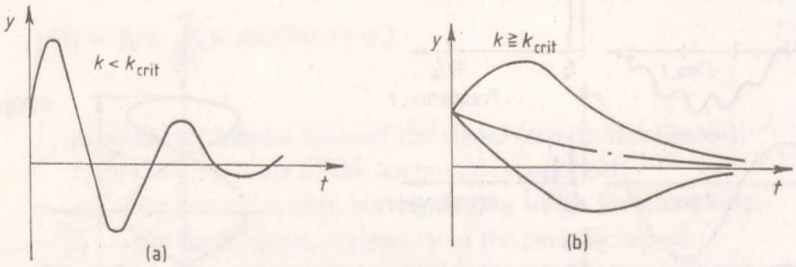


Fig. 3.8 Damped motion
(a) periodic; (b) aperiodic

If the damping is proportional to the velocity, then the time between two maximum deflections is the same and the logarithm of the quotient of two adjacent deflections of the same sense is constant. This is called "logarithmic decrement".

If the damping factor is

$$k \geqq 2m\alpha = k_{\text{crit}},$$

i.e. it exceeds the critical value, then the motion is aperiodic, the deflection tends continuously to zero, and the mass returns to its equilibrium position.

So far, we have assumed that no energy is transmitted to the system, so no excitation exists (free vibration). If a harmonic excitation, depending exclusively on the time, $F = F_0 \sin(\omega t + \varphi)$ is assumed, then the undamped and the damped motions, respectively, are described by the following differential equations:

$$m\frac{d^2y}{dt} + \frac{y}{c} = F_0 \sin(\omega t + \varphi);$$

$$m\frac{d^2y}{dt} + k\frac{dy}{dt} + \frac{y}{c} = F_0 \sin(\omega t + \varphi).$$

Omitting the mathematical solution, the mass of the system will move according to the resultant of the natural vibration and the excited vibration. As damping always exists in practice, the natural vibration will be damped after a time and the maswill perform a harmonic oscillation corresponding to the angular frequency ω of the excitation. The amplitude of the excited vibration depends not only on the excitation but also on the characteristics of the system under study.

In the undamped case the magnitude of the deflection may be expressed as

$$y = y_0 \frac{\alpha^2}{\alpha^2 - \omega^2} \sin(\omega t + \varphi),$$

where

$y_0 = F_0 c$ is the static deflection corresponding to the force F_0.

The above relationship reveals that when the angular frequency is close to the natural frequency of vibration of the system, the amplitude of the excited vibration increases greatly. In the undamped case of $\omega = \alpha$, the deflection, in principle, may be infinitely great. This is the phenomenon of resonance. Figure 3.9 presents the resonance curve of the system.

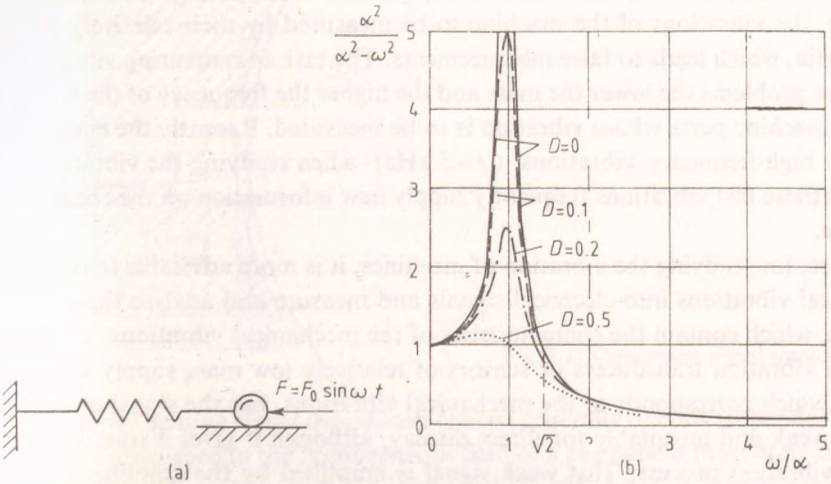


Fig. 3.9 Excited vibrational system

A large amplitude and a high vibration velocity implies a large potential energy, and a large kinetic energy. This energy is supplied by the work done by the exciting force. Performing the work requires time, so in the case of resonance the amplitude of the excited vibration increases in proportion with the time elapsed; this permits our machines to pass the resonance frequency at the start without damage.

The conditions are more complex in the case of a damped system. Neglecting the natural vibration of the damped system, the angular frequency of the stabilized vibration agrees with the angular frequency of the excitation, but the phase of the forced vibration lags behind that of the excitation. The phase lag depends on the damping factor D and the ratio ω/α (Fig. 3.9(b)).

The vibration of machines is very complex, as not only periodic but also stochastic excitation effects may be observed, so the spectrum of the vibration is often not a line spectrum, but a continuous one.

3.2 VIBRATION-MEASURING INSTRUMENTS

Instruments that operate purely on mechanical principles are used for measuring the mechanical vibrations of machines only where the amplitude of the vibration is relatively large and its frequency is low. These instruments are not always capable of following truly the vibrational motion of the machine to be tested, partly because of the inertia of their mass and partly owing to the friction, or damping, which appears between their elements. They also have the important disadvantage of changing, i.e. detuning, the vibrations of the machine to be measured by their relatively great mass, or inertia, which leads to false measurements. The task of measuring vibrations involves more problems the lower the mass and the higher the frequency of the vibrating body or machine parts whose vibration is to be measured. Recently the emphasis has been on high-frequency vibrations ($f > 5$ kHz) when studying the vibration of machines, as these fast vibrations frequently supply new information on the condition of a machine.

Therefore, for studying the vibration of machines, it is more advisable to convert the mechanical vibrations into electrical signals and measure and analyse these electrical signals, which contain the characteristics of the mechanical vibrations.

Modern vibration transducers or sensors of relatively low mass supply an electrical signal which corresponds to the mechanical vibrations, but the signal obtained is relatively weak and unsuitable for direct display, although it gives a true description of the vibration process. This weak signal is amplified by the amplifier of the vibration-measuring instrument to make it suitable for display on an analogue or digital instrument. Figure 3.10 shows the principle of the construction of a simple electrical vibrometer.

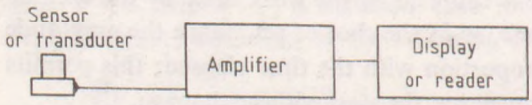


Fig. 3.10 Principle of the construction of a vibration-measuring instrument

3.2.1 THE CONVERSION OF MECHANICAL VIBRATIONS TO ELECTRICAL SIGNALS. TRANSDUCERS, SENSORS

The vibration process of a machine is characterized by the magnitude of the deflection, the path (amplitude), the frequency and the phase shift, which may be sensed during the vibration.

In some cases it is necessary to know the values of the velocity and acceleration for the accurate determination of the vibration processes. Fortunately the conversion of the velocity and acceleration to electrical signals is much simpler than sensing the magnitude of the deflection; therefore the majority of vibration transducers operate with velocity or acceleration sensors. In order to obtain an electrical signal which is proportional to the characteristics of the motion, the sensors are coupled to, or at

least pressed against the body to be tested. Contactless signal transducers which sense the velocity or the amplitude are also available.

Contactless signal transducers generally sense a relative movement (e.g. the distance between the sensor fixed on the bearing block and the vibrating shaft). Their principle of operation may be based on electrodynamics or on variations in capacitance. Figure 3.11 shows an example of a contactless sensor.

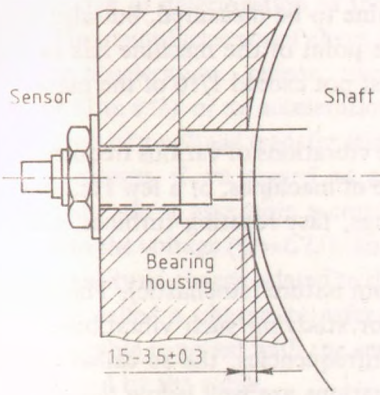


Fig. 3.11 Contactless signal transducer

Velocity-sensing signal transducers consist generally of coils suspended on springs. The coil is displaced in the homogeneous field of a permanent magnet by the mechanical vibration. The voltage induced in the coil, i.e. the signal voltage, is proportional to the number of intersections of the lines of force per unit time, i.e. proportional to the velocity of the coil. The standard specifications (ISO 2954) prescribe generally the measurement of the effective value of the vibration velocity when studying rotating machines.

Acceleration-sensing signal transducers employ, in most cases, a piezo-electric crystal, placed between the cover of the sensing headpiece and the mass in the sensor. During an acceleration the mass exerts — by its inertia — a force on the crystal, and the potential difference appearing between the terminals connected to the crystal is proportional to the measured acceleration (Fig. 3.12).

Piezo-electric acceleration sensors measure the absolute acceleration of the motion to a good approximation. The characteristic of the vibration (path, velocity, or acceleration) to be measured is determined by the measurement task or the type of

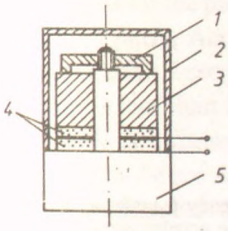


Fig. 3.12 Design principle of the piezo-electric accelerometer
1 clamp spring; *2* case; *3* mass; *4* piezo-electric plates; *5* base block

sensor available. Modern electronic devices, with the help of differentiating and integrating circuits, enable any parameter to be displayed.

Today the most widely used instruments are the acceleration-sensing devices, which also permit values of the velocity deflection to be read off from their displays. The main advantages of acceleration sensors are their relatively low mass and their applicability over a wide frequency range; this is because of their relatively uniform frequency transfer. The low mass is important not only from the point of view of power input and for avoiding detuning of the machine to be measured, but also because there are tasks where the vibration of a single point of the machine has to be determined. The mass of the acceleration sensor must not exceed 1/10 of the mass of the machine being measured.

A wide frequency range is necessary also because vibrations of various frequencies are met with in the course of a diagnosis, from those of machines, of a few Hz, up to the rapid vibrations (f>15 kHz) of rolling bearings, fast rotating turbines, compressors, etc.

The vibrations of some machines have a random nature (stochastic). The wide frequency range is an indispensable requirement for studying such vibrations. An acceleration sensor has naturally one or more eigenfrequencies, the so-called resonance frequencies, as well. When the measured vibrations are well below the eigenfrequency, the sensor follows the vibration of the machine faithfully, so the inert mass induces a voltage, which is proportional to the acceleration in the piezo-electric crystal.

In practice, the distortion-free transfer of the acceleration sensors has an upper limit determined by the eigenfrequency. The lower frequency limit is determined generally by the characteristics of the signal amplifier and not by the acceleration sensor. Figure 3.13 presents the characteristic curve of the frequency of an accelerometer operating on the basis of the piezo-electric principle. The accelerometer was vibrated at a varying frequency, but with the same acceleration, and the intensity of the signal ttained was recorded. Naturally, the greater the mass of the acceleration sensor, obe larger the inertia, so the more intense the signal supplied at a given acceleration. thut the resonance frequency of an accelerator or large mass is low, thereby reducing Bhe useful frequency range. The mass of an acceleration sensor which may be used

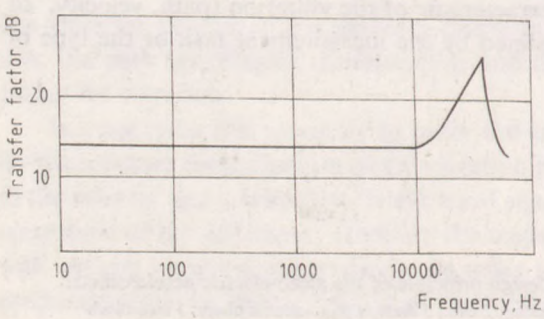


Fig. 3.13 Frequency transfer of the piezo-electric accelerometer

over a wide frequency range is small, and this is why they supply a weak electrical signal.

Acceleration sensors of suitable construction are able to measure accelerations ranging from very low to very high values, even of several thousand g, depending on their construction. The appropriate type is always selected according to the actual measuring task. A limit to the lowest measurable value is placed by an unfavourable signal to noise ratio; the upper limit is set by the distortion of the characteristic curve or the risk of deterioration of the sensor.

The manufacturers of these instruments specify the electrical signal supplied by the acceleration sensor usually relative to the gravitational acceleration ($g=9.81$ m s^{-1}) or an acceleration of 1 m s^{-2}. If the given output signal is a voltage, we speak of a voltage transfer factor (e.g. 45 mV/g).

However, if we regard an acceleration sensor as a high capacitance electrical generator, then, assuming a constant capacitance, the quantity of charge is proportional to the voltage ($Q=CU$), so the acceleration sensor may also be characterized by the quantity of charge related to the given acceleration (e.g. 45 pC/g, or 4.5 pC/m s^{-2}). This is called a charge transfer factor. The voltage transfer factor is only valid for the cable supplied with the sensor but the charge transfer factor is practically not influenced by the cable.

The ideal acceleration sensor would be sensitive to the acceleration only in a defined direction, but in practice most accelerometers are sensitive also to accelerations which are perpendicular to the specified direction (cross-sensitivity) [12].

The cross sensitivity is given by the sense factor, which is the quotient of the transfer factor of the acceleration perpendicular to the main direction by the transfer factor of the acceleration occurring in the main direction. The causes of cross sensitivity are the irregular structure of the piezo-electric crystal and the incorrect connection between the crystal and the metallic parts. The cross sensitivity of modern acceleration sensors is low. Certain manufacturers indicate the direction of the minimum cross-sensitivity by a red point on the sensors.

The acceleration sensors used for measuring the acceleration of machines may be used without damage generally up to 250...260 °C. With increasing temperature the electrical signal obtained becomes weaker. The firms producing the sensors supply a diagram showing the variation of the signal with temperature.

The relative humidity of the air does not considerably affect the operation of carefully produced sensors, as materials with nearly identical thermal expansion coefficients are selected for the parts and the seals, so the moisture cannot penetrate into the delicate structural parts. Acceleration sensors can be used for many years, as their parts are not exposed to wear. According to experience the reduction in their transfer factor due to aging is less than 2% per year.

Table 3.2 shows the parameters of some piezo-electric acceleration sensors in general use. More detailed data on the acceleration sensors of various types and of various manufacturers may be found in the publications of the manufacturing firms.

Table 3.2
Main parameters of some general-purpose accelerometers [12, 82]

Type	Transfer factor		Resonance frequency	Sense factor	Mass
	mV/m s²	pC/m s²	kHz	%	g
B&K 4368	≈4.0	≈4.8	22	<4	30
B&K 4339	≈1.0	≈1.0	45	<3	16
B&K 4344	≈0.25	≈0.25	70	<5	2
RFT KD 35a	≈4.0	≈5	>20 *k*	<3	28

3.2.2 AMPLIFIERS, DISPLAY AND READER DEVICES

The task of the amplifiers (preamplifiers) is to amplify the weak signal of the sensors and to transform the high impedance of the sensor to a more favourable, lower impedance to improve the evaluation and further processing of the signals.

The signal of the sensor is amplified directly by the preamplifier and amplified further by the measuring amplifiers. The measuring amplifiers are generally assembled within the display instrument. The preamplifiers may be constructed in two ways. In the case of voltage amplifiers the voltage of the output signal is proportional to the voltage of the input signal; if the voltage of the output signal is proportional to the charge of the sensor, we speak of a charge amplifier.

Using a charge amplifier is generally more advantageous, inasmuch as a charge amplifier is insensitive to the capacitance of the connecting cable, and its frequency transfer is not notably affected by the input impedance either. This is especially favourable in the case of low frequencies. However, the construction of voltage amplifiers is simpler; they are cheaper and more reliable, and operate unsatisfactorily only in the direct vicinity of the sensor [9, 12, 17].

As was mentioned before, frequently a knowledge of the velocity or the deflection of the vibration is also necessary. These are derived, by means of integrating circuits, from the acceleration signals. The integrating electronic circuits are usually assembled within the measuring amplifier, as in this way a simple switch-over system enables the output to deliver a signal which is proportional either to the velocity or to the deflection. Also the rate of amplification may be varied here, usually in a decade system.

The output signal of the preamplifier is amplified further by the measuring amplifier up to the level required for the display instrument and other instruments that may be required. The display instrument may show the peak value or the effective value of any parameter. On measuring complex vibrations the phase conditions of the component vibrations of various frequencies may vary because of errors in the amplifiers. This means, that the shape of the input and output signals will not ne uniform.

This fault is especially significant in the case of components with frequencies close to the lower or the upper limit of the measuring system. Therefore, it is generally advisable that in the case of complex vibrations the frequency of the lowest frequency components should amount to at least 10 times the value of the lower limit frequency of the instrument and the frequency of the highest frequency component should equal 1/10 of the upper limit frequency of the measuring instrument at the most [12].

On measuring the effective value the errors due to the phase shift may be neglected.

An analogue instrument can only indicate vibrations of nearly constant intensity with respect to time; they are unsuitable for recording shock-type, transient phenomena because of their inertia. In most cases oscilloscopes are used for this purpose, enabling these phenomena to be observed and recorded (by photography). The slower variations of the vibration may be studied with the help of the diagrams produced by level recorders.

In the following are presented some instruments for the use of technicians studying the technical condition of machines on the basis of vibration measurements in the plant.

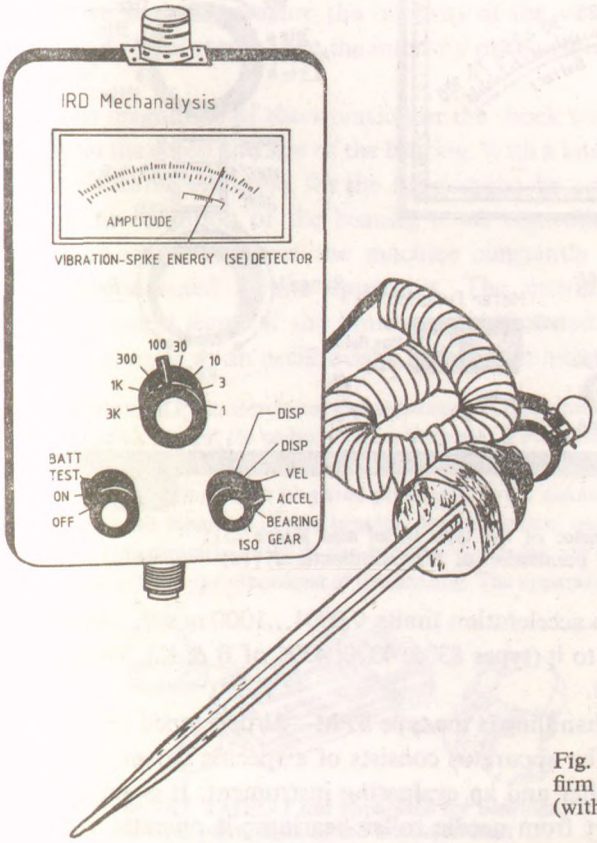


Fig. 3.14 The Model 811 vibrometer of the firm Reutlinger, type IRD-Mechanalysis (with the kind permission of the manufacturer) [81]

Figure 3.14 shows a small instrument, produced by Reutlinger, IRD–Mechanalysis (Model 811) consisting of a sensor, an amplifier and display unit. The apparatus is suitable for the general examination of machines, the early recognition of bearing and drive-gear failures and for periodic balance control. It measures the path, the velocity and the acceleration of the vibrations, as well as the "spike energy" in 8 measurement ranges. Various devices may be connected to its output, analysers, oscilloscopes, vibration recorders — for example, etc. The instrument may be used in plants because the mass employed in the sensor is large; its mass is 0.95 kg.

Figure 3.15 illustrates the apparatus type 2511, operating with a piezo-electric acceleration sensor, produced by the firm Brüel and Kjaer. The instrument also contains a charge amplifier and display. It measures, with the help of its integrating circuits, the velocity and the displacement in addition to the acceleration. Its frequency range is 0.3 Hz...15 kHz.

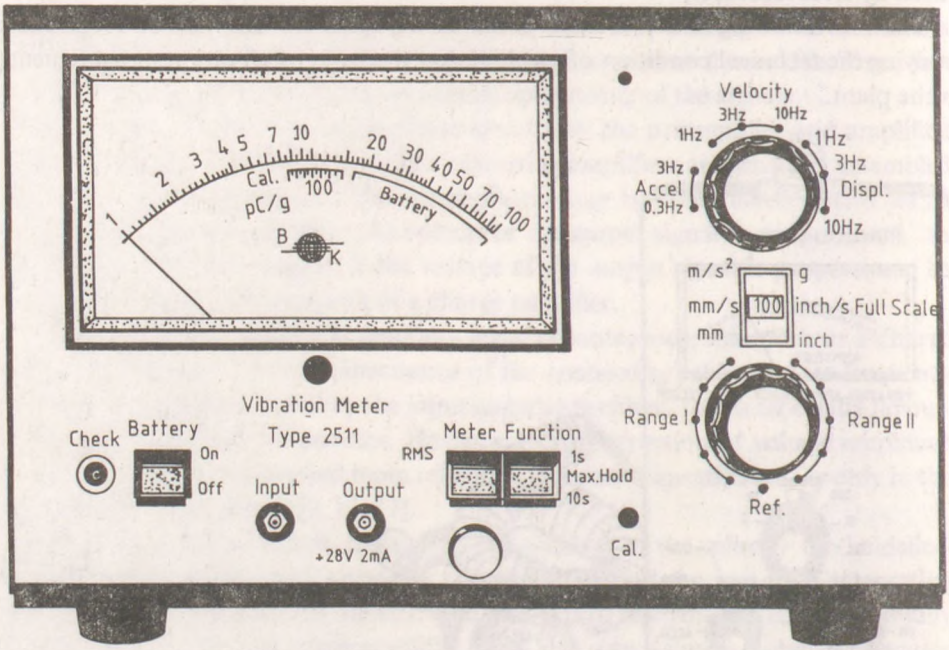


Fig. 3.15 Vibrometer of the type Brüel and Kjaer 2511 (with the kind permission of the manufacturer) [18]

It may be used between the acceleration limits 0.0001...1000 m s^{-2}, depending on the type of sensor connected to it (types 8306, 4370, 4371 of B & K). The display indicates peak or effective values.

A simple apparatus of easy handling is the type SPM—21 developed for examining rolling bearings (Fig. 3.16). The apparatus consists of a specific sensor, a battery operated portable amplifier, a filter and an evaluating instrument. It is suitable for testing any rolling bearing apart from needle roller bearings; it operates with the

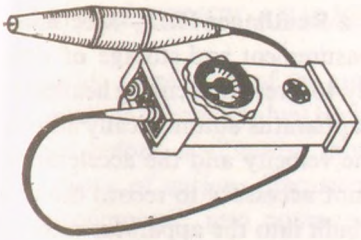


Fig. 3.16 Vibrometer of the type SPM—21 (with the kind permission of the manufacturer)

SPM (Shock Pulse Measurement) process, i.e. it measures the magnitude of the shock pulses produced by faults in the bearings. The sensor is piezo-electric, assembled in conjugation with a mechanical resonance mechanism. Hence the sensor is sensitive only to shock waves, and the measurement is not affected by other vibrations of the machine or its environment. The sensor is tuned to a frequency of 32 kHz.

For measurement positions which are directly accessible, a sensor of the type SPM 43A is used. The evaluating unit compares the signal, which is proportional to the amplitude of the shock wave, with a threshold value set on the scale of the instrument. If the measured value exceeds this threshold value, the apparatus issues short sound pulses (a chirping sound).

If we wish to measure the intensity of the vibration, the scale is rotated until the sound signal stops; then the intensity of the vibration or the shock wave may be read off.

The magnitude of the vibration or the shock waves originating in a bearing depends on the speed and size of the bearing. With a knowledge of these factors, the value of the tolerable vibration for the bearing can be set on the instrument, thereby enabling the condition of the bearing to be controlled easily and relatively rapidly. A sensor which observes the machine constantly from a distance of 100 m may also be connected to the apparatus. The instrument mentioned above gives a warning sound signal if the limit value is exceeded. A recording apparatus, such as a level recorder or an oscilloscope, may be connected to the instrument.

The firm SKF has developed the apparatus SKF Maintenance Product TMED 1, operating with a piezo-electric sensor, for testing rolling bearings. It evaluates the high-frequency vibrations of the bearing by the Shock Pulse Method (SPM). To try to ensure the best transfer of the vibrations, the sensor is pressed into a nest prepared previously in the machine to be examined. After adjusting the speed and bore diameter of the bearing, the instrument indicates the characteristic dB_C (Carpet) and dB_M (Maximum) values of the bearing. A headphone may be attached to the apparatus for listening directly to the vibrations of the bearing. The apparatus is shown in Fig. 3.17.

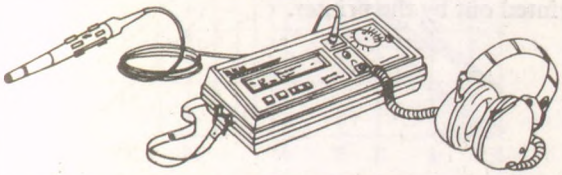


Fig. 3.17 Type SKF TMED 1 test apparatus for bearings (with the kind permission of the manufacturer)

The apparatus of the type "Data Collector 816", a Reutlinger IRD—Mechanalysis product, has been developed for the periodic measurement and storage of data concerning machine vibrations. The operator has only to press the sensing headpeice into the previously prepared measuring position. The apparatus automatically adjusts the measuring limits and measures the amplitude, the velocity and the acceleration of the vibration, as well as the "Spike Energy". It is not necessary to record the displayed analogue or digital values, as the storage unit built into the apparatus records 400 measurement results; assuming 8 measurement positions per machine, this is sufficient for collecting data from 50 machines. The apparatus indicates when the measured value attains or exceeds the previously set reference value. Even additional information concerning the individual measuring positions can be recorded, e.g. the warming up of the bearing, oil leaks, etc. A sensor of type 970 permits 5.8 Hz–10 kHz to be measured and on measuring Spike Energy the frequency range is 5 kHz... ...50 kHz. The apparatus is operated by a rechargeable battery of 2 kg (Fig. 3.18).

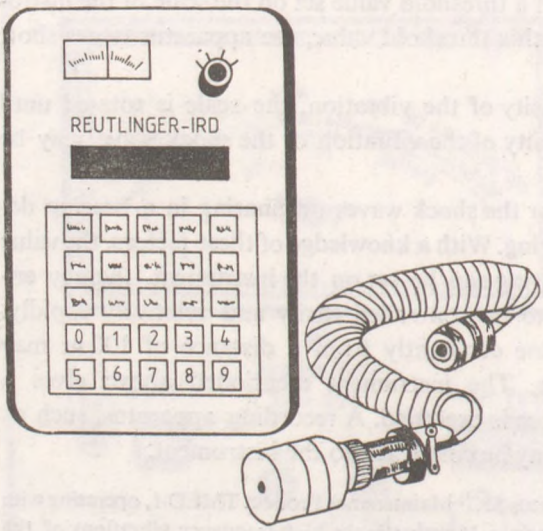


Fig. 3.18 Vibrometer of the Data Collector type (with the kind permission of Reutlinger, IRD Mechanalysis)

Similar devices are the models 817 and 818, which are also supplied with a small screen. These devices, which may be connected to a type 1410 Data Processor, or a type 1420 Data Analyser and a Printer, permit the vibration characteristics of the examined machines, the variations of the vibrations with time, etc. to be studied either displayed on the screen or printed out by the printer.

3.2.3 FREQUENCY ANALYSERS

Knowing some value of the amplitude, velocity, or acceleration of the vibration (e.g. the peak or effective value) is not sufficient in the case of complex vibrations. As mentioned before, a vibration mix may be produced, consisting of the sum of several vibrations of various frequencies, amplitudes and phases, i.e. the vibration may be decomposed into component vibrations of different frequencies, phases and amplitudes.

The devices used for decomposing the vibrations in terms of their frequency are called frequency analysers. They consist of a tunable vibrating circuit, or low-pass and high-pass filters, which pass only vibrations of a desired frequency or a given bandwidth. For example, the pass bandwidth may be narrowed down by bringing the limit frequencies of a high-pass and a low-pass filter closer to each other. In this way signals of a given frequency, or of an adjusted bandwidth, are introduced into the evaluating instrument.

With regard to the bandwidth two types of frequency analysers are used. The first type consists of analysers with a constant relative bandwidth, which permit the bandwidth to be expressed as a percentage of the examined frequency, generally in relation to the mean frequency of the band; i.e. on examining high frequencies the absolute bandwidth increases (e.g. in the case of octave and tierce band-pass filters).

The quotient of the lower limit frequency divided by the upper one is 2 in the case of the octave filter, $\sqrt{2}$ in the case of the half-octave filter and $\sqrt[3]{2}$ in the case of the third-octave filters. The characteristic curves of band-pass filters are naturally not ideal parabolas, as they gradually filter out lower and higher frequencies compared

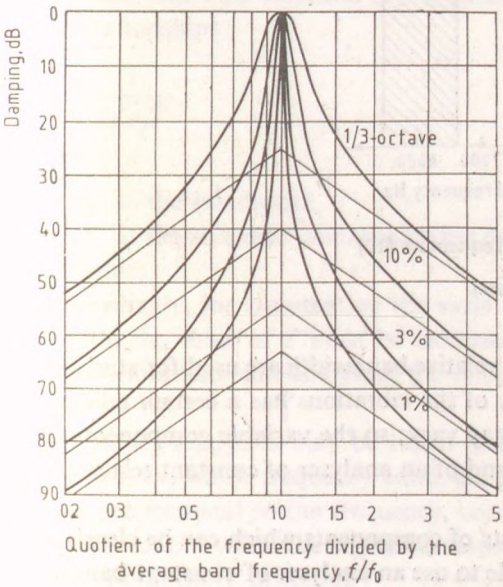


Fig. 3.19 Characteristic curves of a band-pass filter of constant relative bandwidth [12]

with the set frequency. Figure 3.19 illustrates the characteristic curves of a filter of constant relative bandwidth.

The bandwidth of the second type of analyser (analysers with a constant bandwidth), i.e. the difference between the lower and upper limit frequencies of the band, is constant. The bandwidth is specified in frequency units (e.g. a bandwidth of 1 Hz). These devices are generally rather complex and are used mainly for laboratory measurements.

Figure 3.20 illustrates the bandwidths of analysers with constant bandwidth and those with constant relative bandwidth.

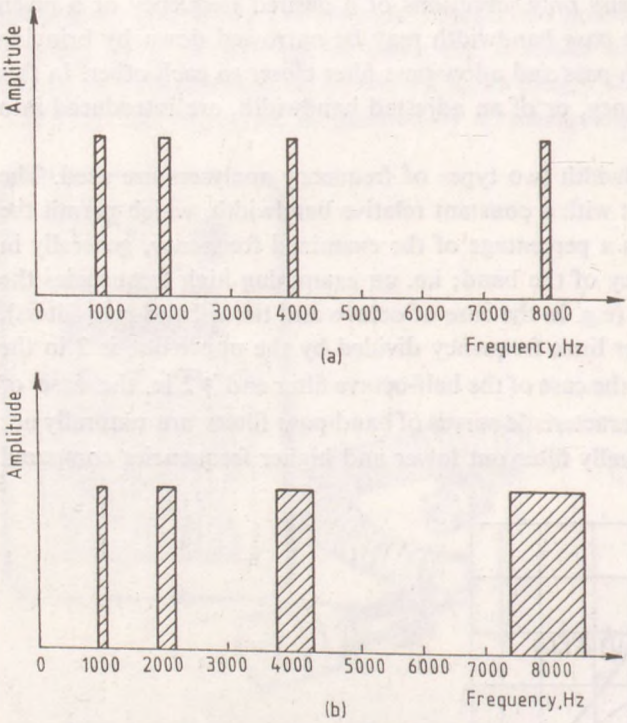


Fig. 3.20 Bandwidths of analysers at various frequencies [12]
(a) analysers of a constant bandwidth;
(b) analysers of a constant relative bandwidth

In most cases analysers of constant relative bandwidth are used for studying the vibrations of machines, as the frequency of the vibrations has a certain relationship with the speed of the machines, which may vary; so the variable components of the frequency are likely to stay within the band of an analyser of constant relative bandwidth.

If the vibration being studied consists of components which can be clearly separated from each other, it is more advisable to use an analyser of constant bandwidth,

as in this way the risk of allowing two inseparable components into the band at higher frequencies is lower.

The narrower the bandwidth, the truer the image obtained from the components of the vibration. Naturally the time taken for the measurement is longer if the bandwidth is narrower.

Figure 3.21(a) presents the spectrum of a vibration source recorded by an analyser of 1 Hz constant bandwidth. The discrete vibrations appearing at 1600 Hz and 2500 Hz are visible only on the diagrams (a) and (d), prepared with a constant-bandwidth analyser.

For analysing the stochastic vibrations of a continuous frequency spectrum it is more advisable to use analysers of constant bandwidth, as in this case the vibrations are expressed as density functions of the quadratic-mean value, and the frequency spectrum given by such an analyser is proportional to the same.

When using analysers of relative bandwidth (e.g. at higher frequencies), the absolute bandwidth s_f is wider, so the quadratic-mean value differs from that which would be obtained if s_f were constant over the whole range. Therefore the frequency spectrum obtained with the help of an analyser of relative bandwidth must be reconstructed appropriately. This may be done automatically by the specific filter circuits of the instruments [12].

When examining stochastic vibrations the determination of the effective value takes a certain time, during which the intensity of the vibrations, and therefore the value indicated by the instrument, varies. The longer the time taken for the measurement, the better the accuracy of the effective value obtained. The variation of the effective value increases if the bandwidth is narrowed and it decreases if the bandwidth is widened. This variation of the effective values may be expressed by the following relationship:

$$\varepsilon = \frac{1}{2\sqrt{s_f T}},$$

where

s_f is the bandwidth,

T is the so-called communication period needed for determining the effective value.

Accordingly, for maintaining the variation at a constant value by reducing the bandwidth, the value of T must be increased. More detailed guidance for studying stochastic vibrations and analysing the related spectra is given in the literature [12, 15, 19].

Frequency analysers may be tuned manually or automatically, making it possible to scan the whole frequency band. On plotting the spectrum, the output signal, which is proportional to the frequency, controls the deflection of an X–Y recorder in the X-direction, while the signal corresponding to the effective intensity controls its deflection in the Y-direction.

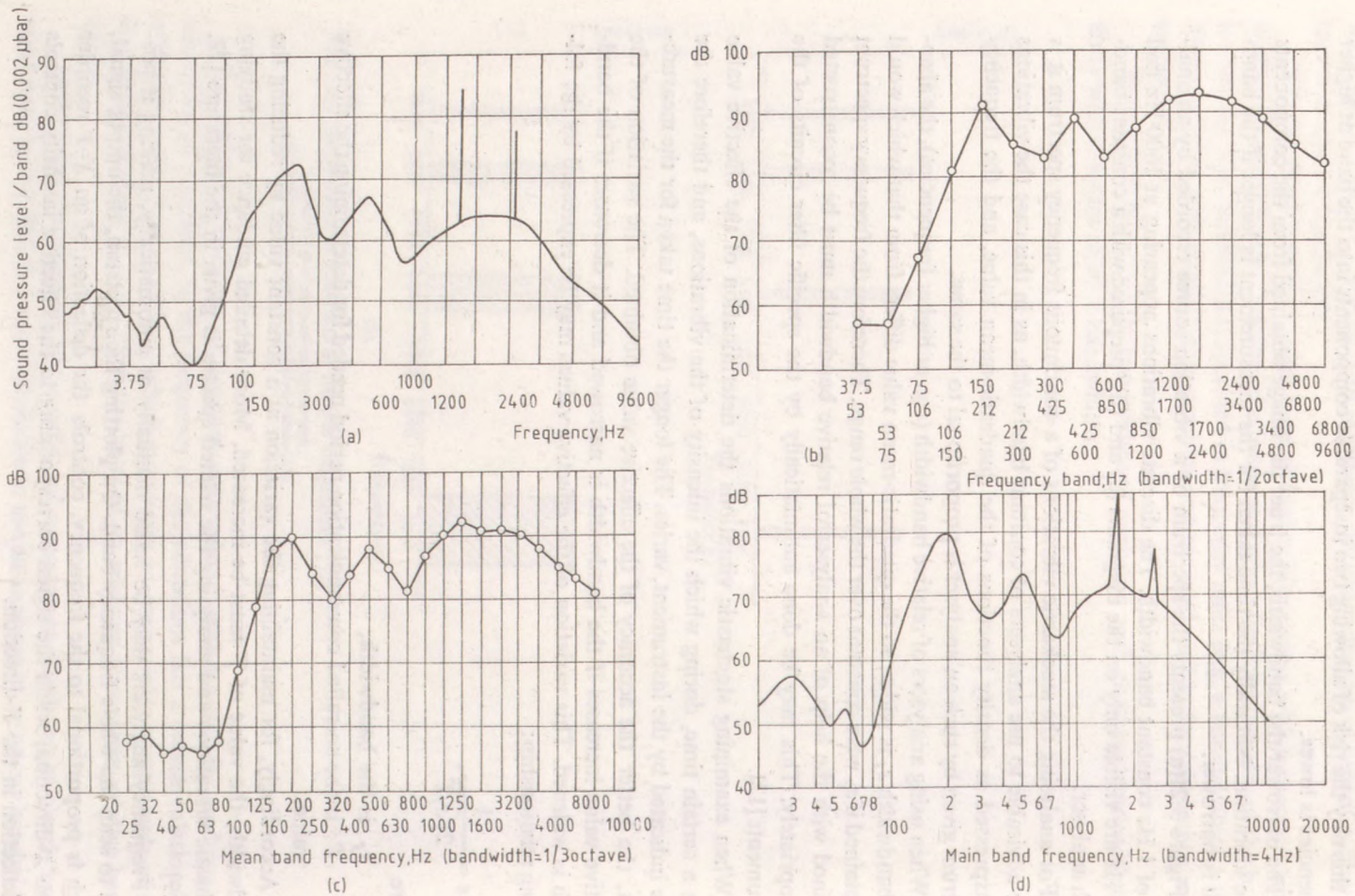


Fig. 3.21 Vibration spectra obtained by frequency analysers of various bandwidths

(a) constant bandwidth = 1 Hz; (b) relative bandwidth = 1/2 octave; (c) relative bandwidth = 1/3 octave; (d) constant bandwidth = 4 Hz [5]

With other analysers, the frequency spectrum is displayed on a screen, or by some other means (e.g. by built-in diagram-plotting recorders, etc.). In the following a few filter devices and frequency analysers, and the instruments supplied with the latter, are presented.

The RFT filter devices consist of two low-pass and two high-pass filters each. The desired value of the bandwidth to be passed may be adjusted by setting the lower and upper band values. Figure 3.22 shows the characteristic curves of high-pass and low-pass filters of the RFT types SM 22 and SM 26.

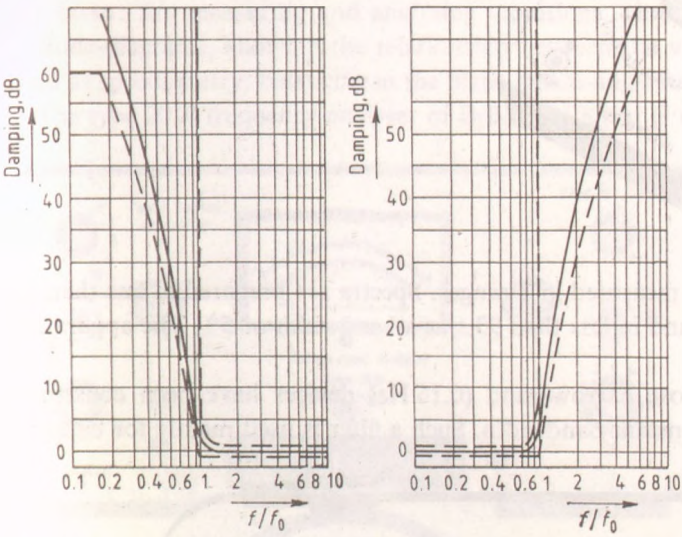


Fig. 3.22 Characteristic curves of the RFT instruments types SM 22 and SM 26 high-pass and low-pass filters [82]

Vibration-measuring and analysing devices of small size are produced for making rapid measurements during servicing and maintenance. The apparatus shown in Fig. 3.23 measures the usual v_{eff}-value (according to ISO—R 2372) and the magnitude of the deflection in four ranges. The bandwidth of the vibration analyser is 3%; the frequency may be adjusted manually. The mass of the apparatus is 0.8 kg only.

The "IRD–Mechanalysis" and the Model 820 "Spike Energy Analysator" manufactured by Reutlinger were produced for plant measurements, permitting the rapid analysis of narrow-band vibrations; they have built-in diagram recorders, which plot the spectrum of the vibration and also the intensity of the vibration, the "Spike Energy", as a function of time. The control system is a microprocessor, and special accessories, such as speedometers, may be used to supply additional data. The measurement range for the frequency is 3...10 kHz, and 5 kHz...50 kHz for measuring the Spike Energy. The amplitude of the vibration, the speed, the acceleration and the

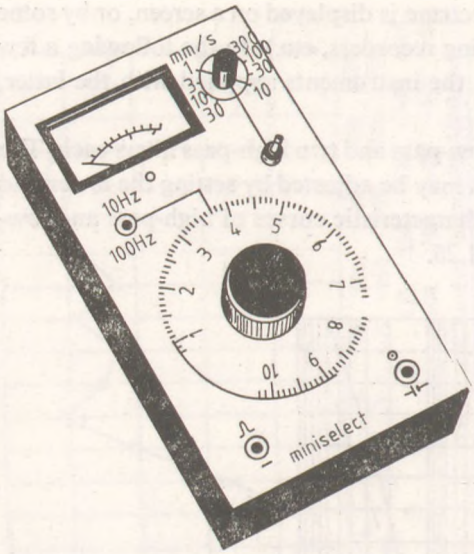


Fig. 3.23 "Reutlinger Miniselect" vibration analyser (with the kind permission of the manufacturer) [81]

Spike Energy values are measured in 8 ranges. Spectra are prepared in less than 36 s at a bandwidth of 10% and in less than 93 s at a bandwidth of 5%. The apparatus is depicted in Fig. 3.24.

Other special purpose narrow-band (0.16 Hz) devices have been constructed with built-in filters of constant bandwidth. Such a filter is used mainly for balancing

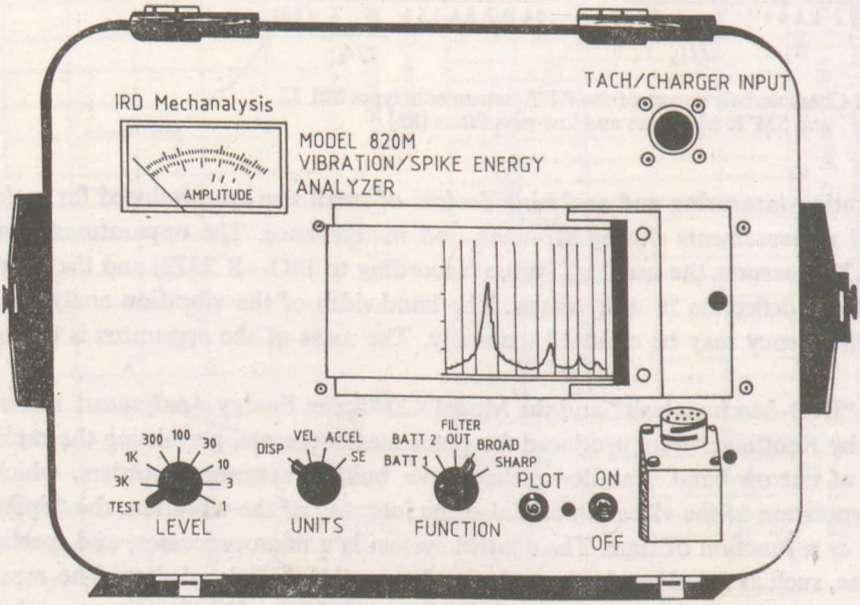


Fig. 3.24 The vibrometer Model 820 IRD—Mechanalysis of Reutlinger (with the kind permission of the manufacturer) [81]

purposes. Its band is too narrow for manual adjustment or use as a follower; therefore a built-in follower circuit keeps the filter synchronized with the signal of the rotating machine. As a result the handling of the apparatus is also simple and its measuring accuracy is high. The follower circuit is capable of following the vibrations of a rotating machine even when there are rapid changes in its speed, thus permitting the indication of critical speeds, resonances and eigenfrequencies as well. Also, a stroboscopic lamp, connected to the apparatus, is very useful for balancing rotors.

For plant measurements, the "Spectroselect" apparatus, produced by Reutlinger, is also useful. It is suitable — with an attached *X–Y* plotter — for numerous measurement tasks: for measuring and analysing vibrations, plotting resonance diagrams and Bode-diagrams, showing the relationship between the vibration and the speed, as well as speedometry, balancing in the plant, phase-angle measurement, etc.

The type 2010 frequency analyser of Brüel and Kjaer (Fig. 3.25) may be used in

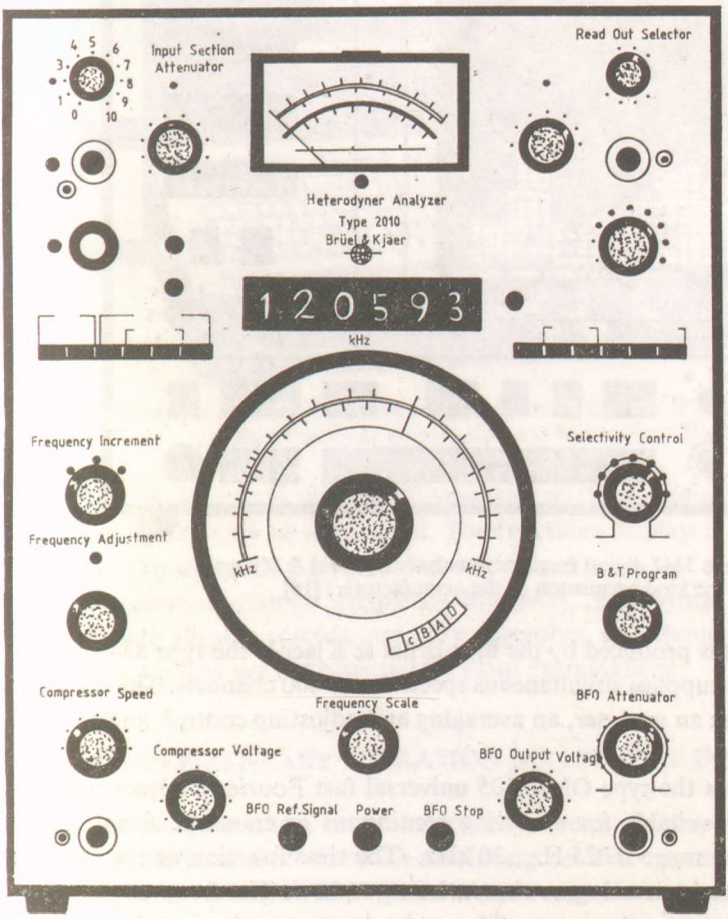


Fig. 3.25 Type 2010 frequency analyser of Brüel & Kjaer (with the kind permission of the manufacturer) [18]

the frequency range 2 Hz...20 kHz. It is produced with the following constant bandwidths: 3.16 Hz, 10 Hz, 31.6 Hz, 100 Hz, 316 Hz, and 1000 Hz. The frequency adjustment may be automated at the four lower bandwidths.

The tuning of the frequency may be achieved manually, or by means of an external flexible shaft drive, or a d. c. voltage. The adjusted frequency may be read off the numerical indicator.

The apparatus shown in Fig. 3.26 contains digital third and octave filters, performing in the I mode of operation 1/3-band analyses simultaneously in 42 channels between 1.6 Hz...20 kHz, and in the II mode of operation, with octave band-pass filters in 14 bands between 2 Hz...16 kHz. It can also be used to compare spectra as it possesses two memories. For such purposes the two spectra may also be displayed alternately.

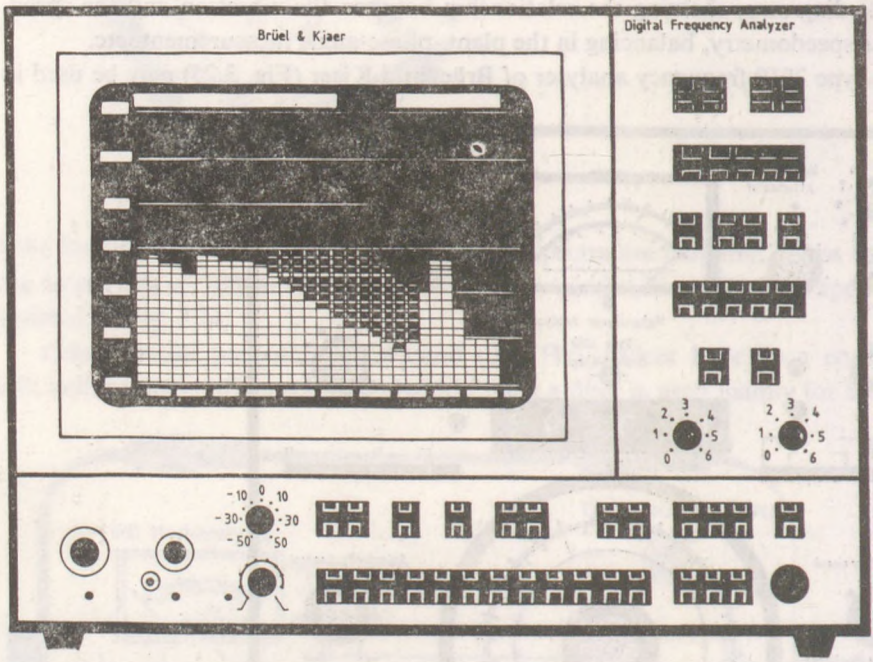


Fig. 3.26 Type 3347 digital frequency analyser of Brüel & Kjaer (with the kind permission of the manufacturer) [18]

Another apparatus produced by the firm Brüel & Kjaer is the type 3348 narrow-band analyser, which supplies simultaneous spectra over 400 channels. The apparatus consists of three units: an analyser, an averaging and adjusting control, and a control and display unit.

Figure 3.27 shows the type OMC 105 universal fast Fourier-analyser produced in Hungary which is suitable for analysing continuous or transient signals in any frequency band in the range 0.025 Hz...20 kHz. The time function or the spectrum may be stored for later processing or comparison purposes. The frequency spectrum appears in 400 columns on the screen. The measurement results may be processed with the help of the alphanumerical keyboard built into the instrument and a com-

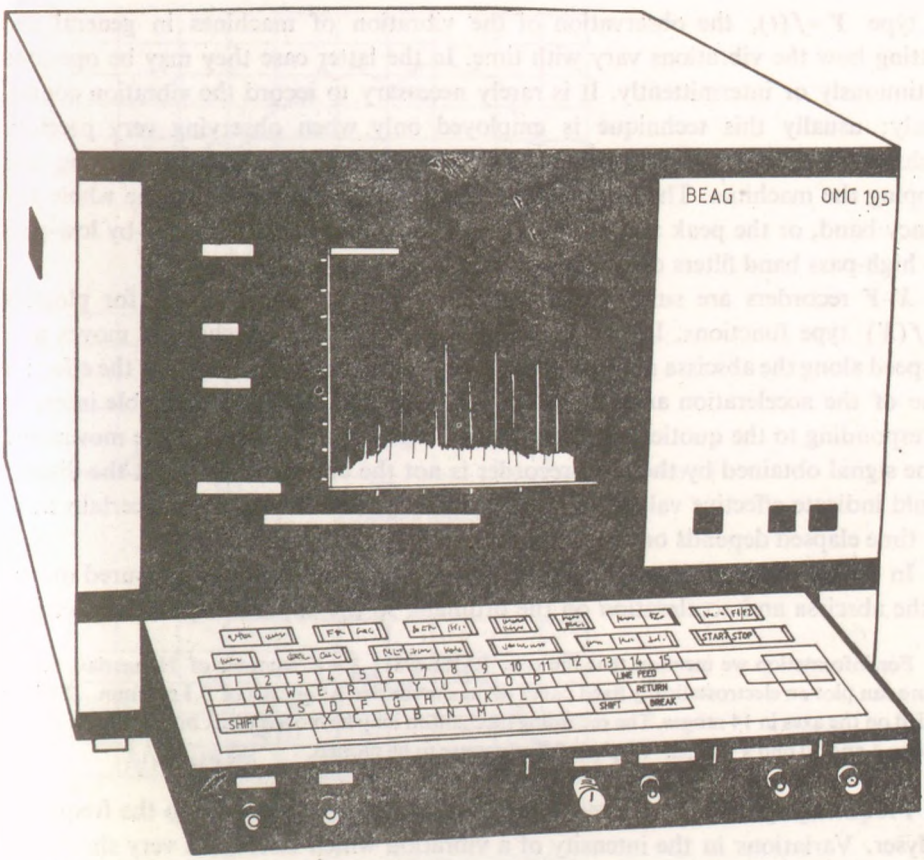


Fig. 3.27 The Hungarian type OMC 105 analyser (with the kind permission of the Cooperative Enterprise Point Automation and Measuring Technics)

puter program written in the BASIC-language, so the measurements and the processing of the results may be automated. The functions displayed on the screen may be printed out by means of a graphical matrix printer.

The above-mentioned frequency analysers, employing digital systems and supplied with display screens, are very expensive, and though they are convenient and fast in use they are not essential for plant examinations.

3.2.4 COMPLEMENTARY VIBRATION-MEASURING INSTRUMENTS

Numerous complementary instruments have been developed for recording and evaluating the results of vibration-measuring devices, the most frequently used of which are level recorders, *X–Y* plotters, and magnetic band recorders.

Level recorders, X–Y plotters. Level recorders are generally suitable for recording the variation of some measured parameter with time, the description of functions of

the type $Y=f(t)$, the observation of the vibration of machines in general and plotting how the vibrations vary with time. In the latter case they may be operated continuously or intermittently. It is rarely necessary to record the vibration continuously; usually this technique is employed only when observing very precious machines. Intermittent operation of these devices occurs mostly on starting and stopping the machines. The level recorder may be used for recording the whole frequency band, or the peak and effective vibration values of the band set by low-pass and high-pass band filters or by the analyser.

X–Y recorders are suitable for recording $Y=f(t)$ functions or for plotting $Y=f(X)$ type functions. In the former case the recording mechanism moves at a set speed along the abscissa and records the time-dependent processes (e.g. the effective value of the acceleration and its variation with respect to time) at suitable intervals corresponding to the quotient of the diagram length and the speed of the movement. If the signal obtained by the level recorder is not the effective value, but the display should indicate effective values, the determination of the latter takes a certain time. The time elapsed depends on the feeding speed of the diagram paper.

In the case of functions of the type $Y=f(X)$ the frequency is measured mostly on the abscissa and acceleration on the ordinate, so the apparatus plots a spectrum.

For information we mention that the type EMG 79811 $X-Y$ recorder of Hungarian manufacture can plot on electrostatically fixed paper of maximum size A3 signals of 0.1 mV/mm...2 V/mm applied on the axes in 14 ranges. The recording mechanism may be moved at six built-in speed ranges between 1 and 50 mm s^{-2} when $Y=f(t)$ functions are to be plotted.

Frequency spectra are plotted by connecting an X–Y recorder to the frequency analyser. Variations in the intensity of a vibration which lasts for a very short time only and pulse-type vibrations cannot be plotted by level recorders and X–Y plotters. The shape of the plotted curve depends on the time during which the instrument supplying the measurement signals communicates these signals, the so-called response time of the level recorder, together with the writing speed of the recording pen and other properties of the apparatus.

Figure 3.28 shows some spectra of the same vibration recorded at various speeds. If the recording speed is faster in the X-direction, the pen, because of its inertia, is unable to follow the variations in the intensity of the vibration, so the curve will be smoother than that plotted at a slower speed and similar to the enveloping curve of the latter.

Level recorders are suitable for recording d.c. and a.c. voltage variations over quite a wide frequency range (e.g. a range of 2 Hz...200 kHz with the type 2305 level recorder of Brüel and Kjaer). Level recorders generally record peak values, but the newer devices also record quadratic-mean values. The peak factor, which is the quotient of the peak value and the quadratic-mean value, is known to depend on the shape of the vibration. In the case of sinusoidally varying vibrations its value is $\sqrt{2}$, while if non-sinusoidal vibrations are examined, this quotient is set on the instrument

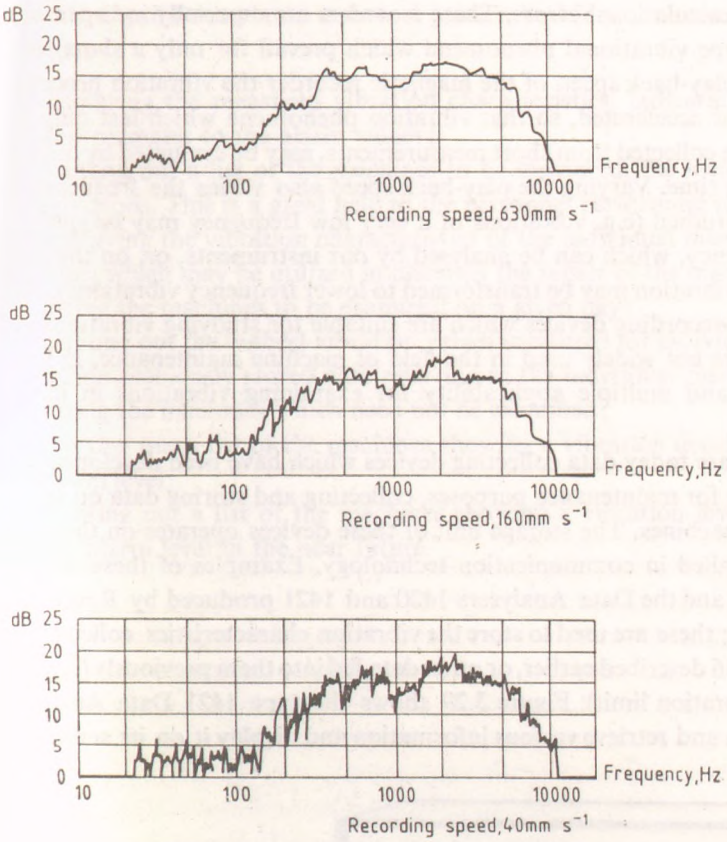


Fig. 3.28 Spectra recorded at various speeds [12]

according to the characteristics of the vibration. If this is neglected, serious deviations and errors are to be expected in plotting the diagram, especially in the case of pulse-type vibrations.

Recorders of the the magnetic-band type are essentially high quality tape recorders with a uniform, high signal to noise ratio for the frequency transfer within a wide band and are of a uniform beatless operation (free from mewing). These recorders are generally used for:

— repeated recording of vibrations, convenient analyses and other tests, and
— transforming the vibrations in the interest of the applicability of our instruments over wider bands.

Recording the vibrations of machines on magnetic tape is a fast means of collecting such data, permitting the execution of plant tests in a relatively short time, without disturbing production. On playing back the tape in the laboratory, the vibrations may be analysed in detail, compared, etc. This may be repeated several times, thereby

reducing the risk of calculational errors. These recorders are especially indispensable for studying pulse-type vibrational phenomena which prevail for only a short time.

By varying the play-back speed of the magnetic recorder the vibration processes can be decelerated, or accelerated, so that vibration phenomena which last only for a short time, and data collected from short measurements, may be evaluated by observing them for a longer time. Varying the play-back speed also varies the frequency of the vibration being studied (e.g. vibrations of a very low frequency may be speeded up to attain a frequency, which can be analysed by our instruments, or, on the contrary, the examined vibration may be transformed to lower frequency vibrations).

Magnetic signal-recording devices which are suitable for studying vibrations are expensive, so they are not widely used in the field of machine maintenance, in spite of their advantages and multiple applicability for examining vibrations in industrial plant.

However, there are today data collecting devices which have been developed and produced specifically for maintenance purposes, collecting and storing data on vibrations measured on machines. The storage unit of these devices operates on the basis of the principles applied in communication technology. Examples of these are the Data Processor 1410 and the Data Analysers 1420 and 1421 produced by Reutlinger IRD—Mechanalysis; these are used to store the vibration characteristics collected by the Data Collector 816 described earlier, or other data fed into them previously (e.g. the acceptable upper vibration limit). Figure 3.29 shows the type 1421 Data Analyser, which is able to store and retrieve various information and display it on its screen, or

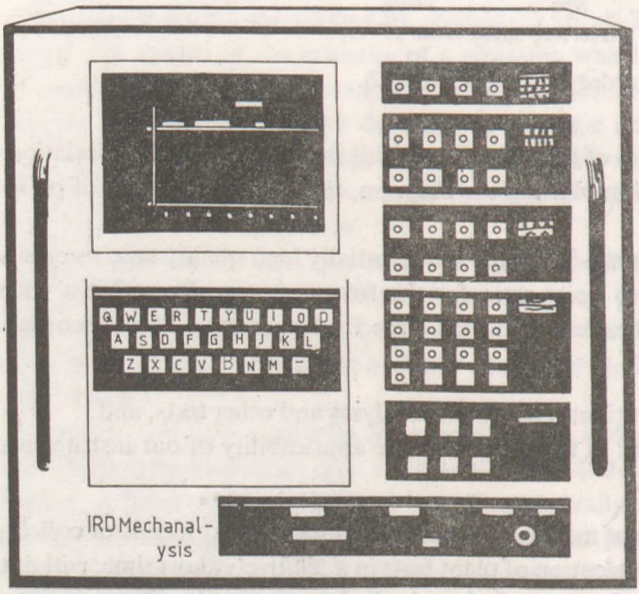


Fig 3.29 The type 1421 Data Analyser produced by Reutlinger, IHR Mechanalysis (with the kind permission of the manufacturer)

provide a print-out via an attached printer, and fulfil a number of functions. These include:

— displaying the measured vibration characteristics, indicating those whose value approximates to the alarm values;
— printing out a list of the machines to be measured, the measuring positions and directions. This is a great help to the personnel performing the measurements;
— specifying the vibration characteristics of the individual machines with respect to time, which may be utilized in planning the repair of the machines;
— listing the machines to be examined on a given day;
— printing out the highest vibration values measured for individual machines;
— listing previously stored data referring to the individual machines;
— listing the machines which need not be examined;
— printing out a list of the machines showing a vibration density which exceeds the alarm level;
— printing out a list of the machines showing a vibration level which may exceed the alarm level in the near future.

4 EXAMINING THE CONDITION OF MACHINES

4.1 THE VIBRATION OF MACHINES

Machines that work with moving components vibrate during their operation, or they may emit sounds by moving the adjacent air. These phenomena can be sensed in most cases by our sensory organs: the vibration by placing a hand on the machine, while a sound is heard if its intensity and frequency are within the range of the human ear. Machines which operate with alternating movements (e.g. internal combustion engines) excite the whole assembly to move at a rate which corresponds generally to the motion of the moving components.

Pulse-type vibration waves are formed if one part impacts against another (e.g. the valve seat is hammered by the valve in internal combustion engines). Vibrations produced in this way often have frequencies in the ultrasonic range — depending on the mass forces, the velocity and the frequency of the impacts — as in this case the upper harmonics of the excitation frequency also appear.

Generally, the vibration with frequency corresponding to the excitation frequency has the largest amplitude; the amplitude then decreases with increasing frequency. But it may happen that some upper harmonic causes a vibration of a larger amplitude than that of the basic vibration because the frequency of the upper harmonic may coincide with one of the eigenfrequencies of the equipment, leading to resonance. Frequently, the seventh upper harmonic, or even higher ones, can be oserved.

Vibrations are produced even by machines which operate without alternating motions, because of play between their parts, leading to impact-induced vibrations, or because they slide, rub, or roll against each other. Rotating, sliding, or rolling bodies cause vibrations because they are unbalanced, irregularly shaped and their surfaces are rough. But vibrations occur even when bodies of a regular form are sliding or rolling (e.g. balls), because their surfaces are never perfectly smooth. The protrusions impact against each other, or the bodies moving over each other, because of their uneven surfaces, follow a forced track which deviates from the ideal track, causing the formation of vibrational waves.

Vibrations may be produced even by new, faultless machines, because of the inevitable geometrical faults which occur during the manufacture of their individual parts. Also the surfaces of the component parts are never perfectly smooth. The

initial intensity of the vibration depends generally on the construction of the machine and mainly on the tolerance limits of its individual parts.

In the course of operating a machine the intensity of the vibration varies, owing to wear, in a similar way to the curve of general wear. After the initial running-in (t_1) the vibration level increases nearly uniformly, then at the end of the useful life of the machine (t_2) it increases rapidly. Figure 4.1 shows the variation of the vibration intensity. During the time of the initial running-in (t_1) it does not always increase, moreover it may decrease according to curve *2* as the traces of machining disappear and are smoothed out, and the individual parts are matched by wear. Also a contrary effect may appear as the wear of the protruding rough surfaces leads to the growth of gaps and increasing play, causing more intense vibrations. Usually, the resultant of both effects is a slight decrease in the initial vibrations of the running-in phase, as shown in Fig. 4.2.

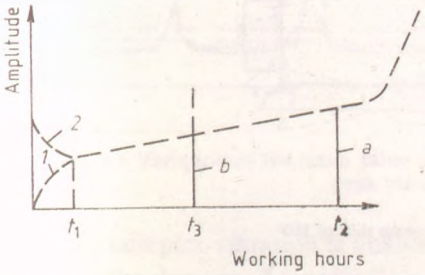


Fig. 4.1 Variation of the vibration intensity during the period of operation of a machine

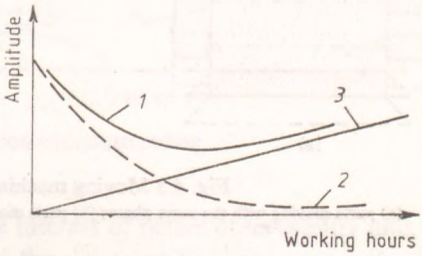


Fig. 4.2 Changes in the vibration in the course of running-in

1 initial vibration; *2* vibration caused by surface unevennesses; *3* vibration caused by increasing play

During their operation, machines are worn by usage, the play between components increases, and the individual parts impact against each other ever harder, citing ever stronger waves. Rotating shafts are displaced from their original axis because of wear in the bearings and become unbalanced. Worn rolling and sliding surfaces show fatigue, cracks, and seizures, and the geometrical irregularities brought about thereby can change considerably the way the parts of the machine move. Vibrations are essentially increased by even smaller changes occurring in the technical condition of the machine.

The vibrations generally increase uniformly over a long period (t_2-t_1), from a calm, uniform running-in to a state of coarse unbalanced running and deterioration.

If the vibration intensity of an operating machine deviates hardly at all from that of a new, faultless machine for a long time, or it increases only moderately, it is in a good, operable condition; but even a tenfold change in the vibration intensity may belong to the uniformly increasing stage, depending on the characteristics of the machine, while the intensity of the vibration increases rapidly when the machine or its individual parts approach the end of their useful life.

Machines operated in a bad condition may produce — according to some sources in the literature — vibrations 2...3 times greater than those produced by a faultless machine; according to other sources this increase may be 10...15-fold, and, at certain frequencies, even higher, amounting to several hundred times the original vibration amplitude [4, 38, 40].

The individual points of a vibrating machine perform motions which deviate from each other. This is possible even if we regard the machine as a perfectly rigid body.

In Figure 4.3(a) all points of the machine (e.g. points *1* and *2*) perform deflections of the same phase in the same direction; in Fig. 4.3(b) the right and left sides deflect in opposite directions, with a phase shift of 180°, as measured at the two points.

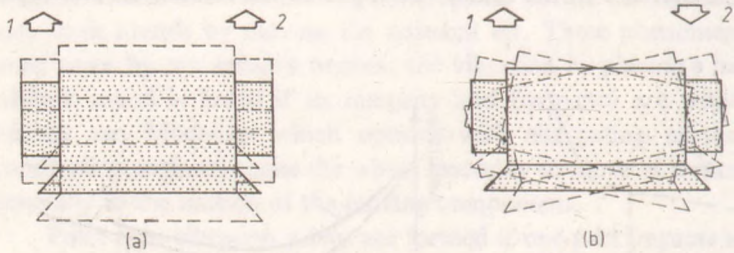


Fig. 4.3 Moving machine parts
(a) parts moving with the same phase; (b) parts moving with a phase shift of 180°

Naturally the machines are not rigid bodies, so their parts may each vibrate differently. Standing waves may also form in the frames of the machine, so there will be points where the vibrations are very large and points where no deflection or displacement can be observed at all. These are the nodal points.

The vibration of a machine may have several causes (e.g. a piston-mechanism raises vibrations of a relatively low frequency, while the roller bearing of the corresponding crank mechanism may excite vibrations of a very high frequency), so the vibration is always complex; it is the resultant of numerous vibrations. As the individual periodic vibrations can be decomposed into sinusoidal components (frequency analysis), so in the final resort the vibration of a machine is the resultant of numerous harmonic vibrations.

If two vibrations of nearly the same frequency arise in a machine, then the phenomenon of "floating" (beats) appears. The two vibrations cannot even be separated instrumentally, because the bandwidth of the frequency analyser is, for example, larger than the separation of the two frequencies. Floating occurs frequently when two machines that stand on a common foundation are operated at nearly the same speed. Naturally, such a vibration is not an indicator of damage, but often it may be misleading.

The observable and measurable vibration of a machine is, as mentioned before, the resultant of various vibrations of various intensities, frequencies and phase angles

caused by various local faults due to damage. If only the intensity of the resultant vibration is measured when examining the condition of a machine, any vibrations of large amplitude which appear at various frequencies are left unobserved, as our instrument indicates the mean vibration intensity. The peak value of the vibrations appearing at a given frequency does not change this mean value considerably (as shown in Fig. 4.4), so the vibration is hardly observable, and possibly it is not indicated by our instrument at all.

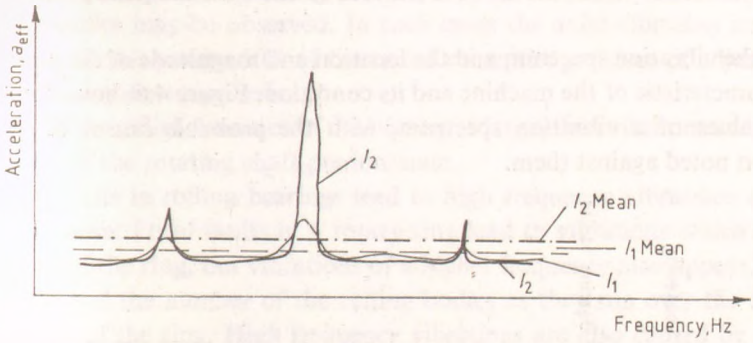


Fig. 4.4 Variation of the mean value of a vibration with outstanding peak values

The complex vibration is analysed in the interest of better observability and for locating the damage. Accordingly a curve of the vibration intensity versus the frequency is plotted, which permits observation of the variation of the vibration intensity in the desired bandwidth. See the example in Fig. 4.5, where a significant deviation between the effective values of the vibration of a sound and a faulty machnie can be observed in the indicated bandwidth. This variation of the vibration, indicating changes in the condition of a machine, can be more easily detected by this method.

The measurement of vibrations using band-pass filters and by plotting the curve of the vibration intensity versus the frequency (spectrum) is also necessary for delimiting the location of the fault. A damaged part produces vibrations of a characteristic

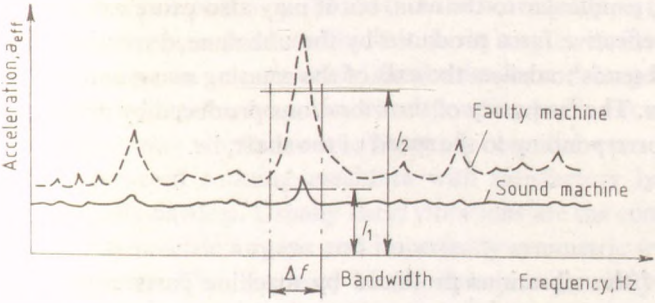


Fig. 4.5 Variation of the effective value of the vibration depending on the bandwidth

frequency. Therefore the spectrum of a faultless machine is first plotted, and this is then compared with the spectrum of the worn or, possibly, damaged machine. Conclusions concerning the possible frequency of the vibrations which could be produced by the individual parts may also be drawn. If we know the frequency of the vibrations caused and increased by the wear of individual parts of the machine, we may mark out in advance the frequency ranges in which it is worthwile following up the variation of the vibrations. Accordingly it is not necessary to plot the whole spectrum of the vibration, only the certain bands which are to be observed, so the measurement becomes simpler and faster.

The shape of the vibration spectrum and the location and magnitude of the peaks found in it are characteristic of the machine and its condition. Figure 4.6 shows some significant peak values of a vibration spectrum, with the probable causes of the increased vibration noted against them.

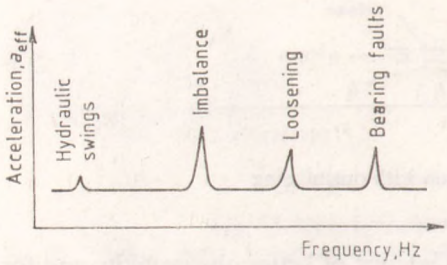


Fig. 4.6 The most frequent causes of the peak values appearing in the vibration spectrum

Generally, three parameters of the vibration are examined: the amplitude, the frequency and the phase shift. The magnitude of the amplitude characterizes the general state of the machine, the frequency permits determination of the cause of failure, while the phase shift assists in the accurate determination of the imbalance, or other faults. In the following, the vibrational phenomena which are characteristic of the individual types of machine or damage are discussed, assuming a machine speed of n_0 [38, 86].

Imbalance is a consequence of the unsatisfactory dimensioning, production, mounting, or maintenance of rotating machine parts and piston mechanisms; generally it causes vibrations perpendicular to the axis, but it may also cause axial vibrations. The direction of the effective force produced by the unbalanced rotating mass varies with the rotation and tends to deflect the axis of the rotating mass, and, in this way, also loads the bearings. The frequency of the vibrations produced by imbalance agrees with the frequency corresponding to the speed of the shaft, i.e.

$$f_0 = \frac{n_0}{60} \text{ Hz.}$$

The basic frequency of the vibrations produced by machine parts exposed to pulse-type forces (e.g. valves) agrees with the rhythm of the impacts, i.e. it corresponds to the basic speed of the equipment. In the case of such vibration sources, upper

harmonic, the so-called "conducted sounds" of very high frequency, may also appear. If the frequency of an upper harmonic approximates to an eigenfrequency of a part of the machine, it appears with especial intensity.

Crooked, deformed shafts and coaxiality faults cause imbalance; the basic frequency of the vibrations produced in this way corresponds to the number of revolutions.

Incorrectly adjusted bearings and couplings lead to the formation of vibrations of a frequency corresponding to the speed, but frequently twofold and even threefold frequencies may be observed. In such cases the axial vibration may also be considerable. If this attains 50% of the radial value, the presence of adjustment or mouting faults may be suspected.

If the radial clearance of the bearing is large, then vibrations corresponding to the speed of the rotating shaft predominate.

Faults in rolling bearings lead to high frequency vibrations of 5...15 kHz and even more. Local faults in a rotary ring lead to vibrations which correspond to the speed of the ring, but vibrations of a higher frequency also appear, depending on the speed and the number of the rolling bodies as they run over the cracks and surface defects of the ring. High frequency vibrations are also caused by faults in the balls and other rolling bodies. Faults in the bearing cages are characterized usually by vibrations of a lower frequency than the basic frequency corresponding to the speed.

Impurities (contaminated, dusty grease, separated peelings, scale) on the bearings produce vibrations of very high frequency depending on the quantity and fineness of the deposited impurity.

The vibrations raised by gears can be easily recognized, generally by their frequency, which is proportional to the product of the number of revolutions, n_0, and the number, z, of teeth on the gear. The vibrations corresponding to the rate of meshing of the teeth are usually strong. On analysing the frequency spectrum, the peak corresponding to the product $n_0 z$ is wide, because vibrations which correspond to the products $n_0(z-1)$ and $n_0(z+1)$ may also appear.

Loosening of the bearing bushes, bearing covers and rings produces vibrations which correspond to twice the speed (e.g. in this case the phase angle of the vibrations measured at two points on the shaft at the bearing support also fluctuates).

The vibrations produced by belt drives may be caused by variations in the cross-sectional area, an uneven mass distribution or a damaged belt. The fault is characterized by a vibration frequency corresponding to the circulation of the belt.

Very strong vibrations are frequently observed even in very well-balanced electrically powered rotating machines with satisfactory bearing supports which are mechanically flawless. Usually these vibrations are the consequence of magnetic-force effects. Asymmetric airgaps and imperfectly symmetric windings frequently occur in such machines. The frequency of the vibrations generally agrees with the frequency, or twice the frequency, of the a. c.-current supply to the machine, but it may also equal the frequency of the rotating magnetic field. These frequencies are often decep-

Table 4.1

Vibration characteristics of machine faults

Cause of the vibration	Deflection	Frequency, Hz	Phase shift	Remarks
Imbalance	Proportional to the magnitude of the imbalance	$f_0 = \frac{n_0}{60}$, where n_0 = number of revolutions/min.		A very frequent cause of vibration.
Swinging, alternating mass forces, impacts	Variable	f_0, $2f_0$		Generally, in the case of machines operating with a crank mechanism it may be reduced by insulating the vibration by balancing.
Non uniaxial coupling	A function of the eccentricity	f_0		Observable generally in measuring positions, which are close to the location of the fault. The latter can be determined by decoupling the individual units of the aggregates.
Loosening of bearings	Radial	$\frac{1}{2}f_0$, $\frac{1}{3}f_0$		
Incorrect shaft adjustment, coupling faults, incorrect bearing adjustment	Radial and axial. The axial deflection exceeds 50% of the radial deflection	f_0, $2f_0$ and $3f_0$		The faults are characterized by a large axial vibration. The faulty part may be located by examining the individual parts.
Faulty rolling bearing	Not constant (the velocity should be measured if possible)	$z \times f_0$ (a multiple of f_0) and very fast vibrations		The strongest high-frequency signal may be characteristic for the type of bearing (number of balls, rollers, etc.). (z = number of rolling bodies)

Faulty, damaged gear	Relatively slight (the velocity should be measured if possible)	$z \times f_0$ (product of the number of teeth and the fundamental frequency) and very high upper harmonic		
Mechanical looseness	Depends on the mode of operation of the machine and the amount of looseness	$2f_0$		It is observed generally together with imbalane and incorrect shaft adjustment.
Faulty drive belt	Variable, pulsating	The rotational frequency of the belt is f_{01} and 1, 2, 3, $4 \times f_{01}$	Not constant	The fault may be detected with the help of a stroboscope.
Magnetic coiling faults of electrically powered rotating machines	Observed only in the switched-on state	$f_1 \times 2f_0$ and the rotational frequency of the magnetic field		On switching off it disappears.
Vibrations caused by oil films	Radial	$0.42...0.48 f_0$		Especially in the case of turbogenerators.
Hydraulic, aerodynamic faults	Variable	f_0 and $n_1 \times f_0$, where n_1 = number of blades		Generally it is the consequence of resonance.
Resonance of pipelines	Relatively large	Generally < 5 Hz		Elimination by detuning, stopping the excitation.
Vibrations due to cavitation	Relatively slight	Very low, < 1 Hz and very high, > 15 kHz		

tive, referring seemingly to imbalance or loosening; but these vibrations disappear when the current is switched off.

Loosening in transformers causes similar phenomena to those described for electric machines, as a consequence of pulsating magnetic fields. This fault can be detected easily as there are no moving or rotating parts here.

The vibrations of hydraulic and aerodynamic equipment (pumps, fans) due to imbalance, bearing faults, or incorrect installation may be recognized in the same way as in the case of other machines. A frequent fault is the imbalance caused by the deposition of transported material onto the blades. In the case of centrifugal pumps and compressors, frequencies which are proportional to the product of the speed and the number of blades, as well as frequencies originating from other oscillations, may also be observed.

Pipelines of hydraulic and pneumatic systems may vibrate at the eigenfrequencies of the individual pipe sections. These are relatively low-frequency vibrations (sometimes less than 1 Hz) and are especially dangerous if the rate of the pulsating flow of the liquid or the air is close to the eigenfrequency (resonant frequency) of the construction. These vibrations are excited generally by the feedpump or the air compressor, and can be eliminated or reduced by detuning the system (by reinforcements, dimensional variations), or by varying the speed of the exciting machine, respectively.

Very slow or very fast vibrations can be caused by the phenomena of cavitation in hydraulic installations. The vibrations of the suction pipes are of a relatively low frequency but of a very large bandwidth. The frequency of cavitation vibrations arising in machines (e.g. in pumps) may amount to 50...60 kHz.

The probable causes of the formation of vibrations with various frequencies are summed up in Table 4.1.

4.2 PERMISSIBLE VIBRATIONS OF MACHINES

Every machine vibrates during operation. The intensity of the vibration depends on the design, the situation, the foundation, the conditions of operation, the state of the machine and other factors. In the case of a given machine a vibration of a certain intensity may be said to be normal. But if the mechanical condition of the machine deteriorates from usage, the intensity of its vibration increases. The long lasting, so-called working vibrational level of a machine belongs generally to its acceptable technical condition, and its tolerated value is determined by several factors.

1. The excited vibrations must not exceed the level which permits the flawless operation of the machine (e.g. the vibration of a grinding machine must not interfere with the prescribed surface quality of the workpiece).

2. The machine must be able to tolerate the excited vibrations until the end of its intended useful life without any too early damage to its parts.

3. The vibration of the machine must not affect the satisfactory operation of other machines situated nearby to a greater extent than permitted (e.g. a forging machine influencing a grinding machine).

4. The machine must not harm, by its vibrations, other installations found in its vicinity, even if the latter are not being operated (e.g. the rolling bearings of stopped machines, etc.).

5. The machine must not, as a result of its vibration, have any harmful effect on humans (work and health protection).

The initial working vibration of a machine is that vibration whose intensity was measured during the first flawless operation of the machine, when the technical condition of the machine is satisfactory and needs no maintenace or repair work whatever.

Naturally the initial working vibration of the machine can be specified only within a certain tolerance limit. The lower and upper limit values of the initial working vibration significantly affect the production and maintenance costs of the machine.

It is not advisable to select initial working vibration values which are too low, as in this case the production costs of the machine will be too high. Yet the manufacturing firms try to prove the satisfactory quality of their products by reference to their relatively low initial working vibrations. Often the firms operating the machines also regard the favourable initial working vibration level as a guarantee of acceptable quality.

The working vibration is the vibration of a correctly operating machine, in case of which the signs of use and wear are already discernible, i.e. the process of damage is in development. These machines require extra attention, and if their vibration level keeps increasing and surpasses the tolerated or permitted vibration limit, the resulting reduction in operational safety means that maintenance intervention is essential. A repair performed in due time may prevent further damage and decrease the maintenance costs considerably [40].

If the intensity of the vibration varies in accordance with the conventional wear curve during the period of use of the machine, then the working vibration corresponds to the uniformly rising section and the tolerated vibration limit is found at the end of the uniformly rising section and at the beginning of the rapidly rising section (see line *a* in Fig. 4.1).

In some industrial plants the vibration is monitored in accordance with the planned preventive maintenance cycles and the tolerated vibration intensity is determined with a view to ensuring the flawless operation of the machine or component until the following repair cycle. According to Fig. 4.1, the tolerated vibration intensity is found in the uniformly rising section (see line *b* in Fig. 4.1).

If the vibration level is higher than this, the expected useful life will not be long enough to last till the next repair cycle and the machine part must be replaced or repaired. Work carried out about 40 years ago on levels of vibrations to be tolerated resulted in the elaboration of vibration diagrams and tables, together with acceptable guide values for machines [7, 30]. Since that time the tolerable vibration levels have

been specified in standards and recommendations, taking not only technical and economic requirements into account, but also aspects of health and safety. So, for example, the Hungarian standard MSZ 19 426–77 prescribes the requirements of the instruments to be used for monitoring the vibration of machines operating at a working speed of 10–200 s^{-1} (600...1200 rev/min.), the conditions, the mode of testing and the evaluation of the test results.

The instrument measures the effective value of the vibration velocity in the frequency band 10–1000 Hz. If the angular frequencies, ω_i, of the sinusoidal components of a complex vibration, the amplitudes, $\hat{s}_i$, of the corresponding displacements, the velocity amplitudes, $\hat{v}_i$, and the acceleration amplitudes, $\hat{a}_i$, are known, then the effective value of the vibration velocity is

$$v_{\text{eff}} = \sqrt{\frac{1}{2}\sum_{i=1}^{n}(\hat{s}_i\omega_i)^2} = \sqrt{\frac{1}{2}\sum_{i=1}^{n}\hat{v}_i^2} = \sqrt{\frac{1}{2}\sum_{i=1}^{n}\left(\frac{\hat{a}_i}{\omega_i}\right)^2}.$$

The relative sensitivity, or the relative transfer factor, of the applied measuring device should fall between both dashed lines of the diagram shown in Fig. 4.7.

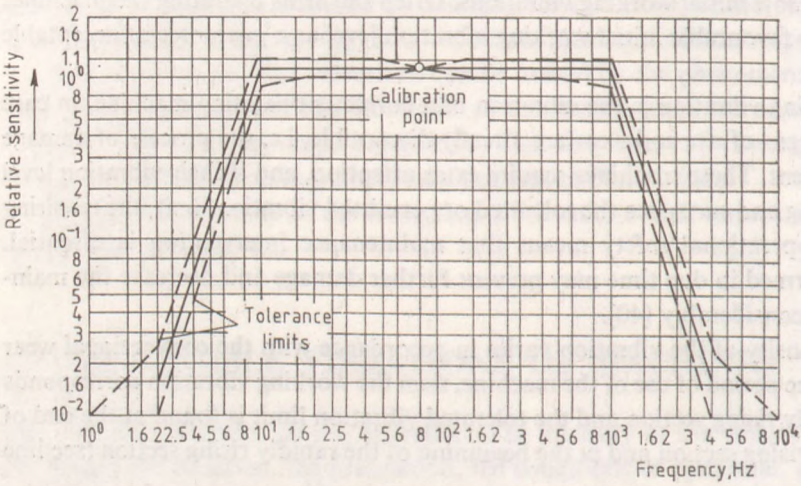


Fig. 4.7 Characteristic curve of the relative sensitivity of a vibrometer

The transfer factor is the quotient of the absolute values of the input and output signals in the case of sinusoidal vibrations at various frequencies. The relative transfer factor is the quotient of the value of transfer factors B and B_0 relative to 80 Hz:

$$b = \frac{B}{B_0}.$$

The requirements of the sensor and the indicating instrument to be used for the measurement are contained in the standard KGSZ 81 3120–73.

For carrying out a test, the machine may be either placed on soft springs or rigidly fixed to a baseplate.

In the former case the machine is placed on an elastic platform to ensure that the eigenfrequency of the machine, as a vibrating system, does not exceed 1/4 of the machine speed, i.e.

$$f_0 \leqq 0.25\, n,$$

where

f_0 is the eigenfrequency (Hz),

n is the machine speed (number of revolutions, s^{-1}).

The standard requires that the additional mass connected to the machine should not exceed 1/10 of the mass of the machine being tested. Generally, testing on soft springs is used only in the case of machines of mass less than 750 kg.

If the machine is rigidly fixed to a baseplate for the test, the machine and the baseplate must not move relative to each other before the working speed is attained, and the additional mass of the baseplate must not exceed 1/4 of the mass of the machine.

The machine is separated from other machines and tested, without load, at the nominal speed, driven by a flexible coupling. Machines of variable speed are tested over their whole speed range so as to permit the formation of the possible resonances.

The measuring positions are usually the bearing housings of rotating machines. The velocity of the vibration is measured in the horizontal, X, vertical, Y, and axial, Z, directions. The measuring directions X and Y should intersect the axis of the shaft, the direction Z should be parallel to it.

According to the standard MSZ 19 426–77, the relevant vibration intensity (the

Table 4.2

Vibration intensity ranges
(Hungarian standard, MSZ 19 426–77)

Symbol of the range	Relevant vibration intensity, mm s^{-1}	
	min.	max.
0.11	—	0.112
0.18	0.112	0.18
0.28	0.18	0.28
0.45	0.28	0.45
0.71	0.45	0.71
1.12	0.71	1.12
1.8	1.12	1.8
2.8	1.8	2.8
4.5	2.8	4.5
7.1	4.5	7.1
11.2	7.1	11.2
18	11.2	18
28	18	28
45	28	45
71	45	71

maximum vibration velocity measured at the measuring points) may be assigned to one one of the ranges listed in Table 4.2.

On the basis of this assignment, the extent to which the prescribed quality requirements are met can be determined, with the type of machine, and its conditions of use taken into account.

The test method prescribed in the standard was developed for quality control purposes associated with production and not for determining the extent of the wear during the operation of the machine.

The requirements of the international standards (e.g. ISO 2372–1974/E) are also similar. This standard permits the machine to be fixed to a foundation of at least twice the mass of the machine, in addition to putting it on soft springs or connecting it rigidly to a baseplate.

But, even in this case, the eigenfrequency of the whole system must not exceed 1/4 of the minimum excitation frequency of the machine. The best solution is putting the machine on soft springs; if this cannot be realized, it may be tested while fixed to the foundation in the plant, or to a structural base, but comparisons may be made

Table 4.3

Vibration intensity ranges for the qualification of machines (ISO 2372–1974/E).

Vibration intensity range	Effective value of the vibration velocity, mm s^{-1}	Qualification of the machine class			
		I.	II.	III.	IV.
0.28	0.28	A	A	A	A
0.45	0.45				
0.71	0.71				
1.12	1.12	B	B	B	B
1.8	1.8				
2.8	2.8	C	C	C	C
4.5	4.5				
7.1	7.1	D	D	D	D
11.2	11.2				
18	18				
28	28				
45	45				
71					

only if the characteristics of the machine foundations and the soil conditions are identical. Also, here the effective value of the vibration velocity is measured. The requirements concerning the masuring device, the selection of the measuring points, the mode of testing and the evaluation of the test results agree with those of the Hungarian standard, elaborated with due consideration to the prescriptions of the ISO. Table 4.3 offers an aid for determining the condition of various types of machine, and for assigning them to the categories A...D (from good to unacceptable). On the left-hand side of the table the recommended limit values of the vibration velocity corresponding to the individual types of machine are listed, as measured generally on the bearing housings in the horizontal or vertical axial plane.

Machines have been grouped into several classes according to the table.

Class I is the group of small machines; electric motors of less than 15 kW capacity belong to this group.

Class II is the group of machines of medium capacity; stationary machines performing only a rotary motion, pumps, fans, drive gears fixed to special foundations, with up to 300 kW transferred capacity, and electric motors of 15...75 kW belong to this group.

Class III contains large, heavy-duty machines performing only a rotary motion and power machines fixed to heavy foundations.

The machines of class IV are power and work machines of large mass, rotating at high speed (e.g. turbines, turbogenerators, turbo-electric machines).

Class V involves unbalanced machine systems which move to and fro in the direction of the measurement and stand on relatively soft foundations.

Class VI contains similar machines to those of class V, but having loosely connected parts (e.g. beating shafts in grinding mills, variably unbalanced machines such as centrifuges, vibrating sieves and vibrators).

The above table gives no instructions for the qualification of machines which may be assigned to class V and VI, as the vibration characteristics of machines which perform alternating motions may vary greatly. Effective velocity values of 50 m s^{-1} are rather frequent, but values which are ten times his may also occur temporarily.

Table 4.4 contains the vibration intensity values recommended by ISO 3945–1977/E for qualifying rotating machines of high value. Frequently, if the effective value of the vibration velocity of a machine is known we also wish to know the amplitude of the displacement. An accurate conversion can be made only in the case of harmonic vibrations of a given frequency. The standard does not recommend the determination of the effective value of the vibration velocity from the amplitude either. This is only possible for a spectrum consisting of discrete signals.

Figure 4.8 offers help in the conversion of the effective value of the velocity to the amplitude of the displacement at various frequencies.

If there are no available standards, other prescriptions or measurement data for determining the orientating value of the intitial or operational vibration intensity, the vibration diagrams found in the literature may be used for this purpose. Such a diagram is shown in Fig. 4.9 [7].

From a knowledge of the speed of the machine the magnitude of the corrected deflection may be determined for marking out a point *A* in the diagram. The position

Table 4.4

The qualification of machines based on the vibration intensity (ISO 3945–1977/E)

Vibration intensity v_{eff} mm s^{-1}	Erection of the machine	
	rigid	flexible
0.46		
0.71	good	good
1.12		
1.8		
2.8	satisfactory	
4.5		satisfactory
7.1	unsatisfactory	
11.2		unsatisfactory
18.0	unacceptable	
28.0		unacceptable
71.0		

of this point in the individual regions of the diagram supplies information concerning the condition of the machine and the range of its operation.

The individual regions indicate the following probable conditions of the machine:

I. Dangerous to operate, the machine must be stopped immediately.

II. The machine is in a condition which immediately precedes its failure. To prevent heavy damage it should not be operated for longer than two shifts (depending on the nature of the machine).

III. The machine is damaged, but it may be operated possibly for 7–10 more days without running the risk of more serious failures.

IV. Slight defects have occurred in the machine but repairs are not yet necessary.

V. The machine is perfectly faultless, like a new machine.

To use the diagram the corrected vibration value to be plotted in the diagram is obtained by multiplying the measured vibration value (peak value) by a modification factor. The magnitude of the modification factor depends on the type of machine involved, or its position in the production process. Table 4.5 gives some values of frequently used modification factors, related to the peak values, A_0 measured on bearing housings and screwed down fixed machines. If the machines are not fixed, the calculations are performed with multiplication factors which are 40% of the values.

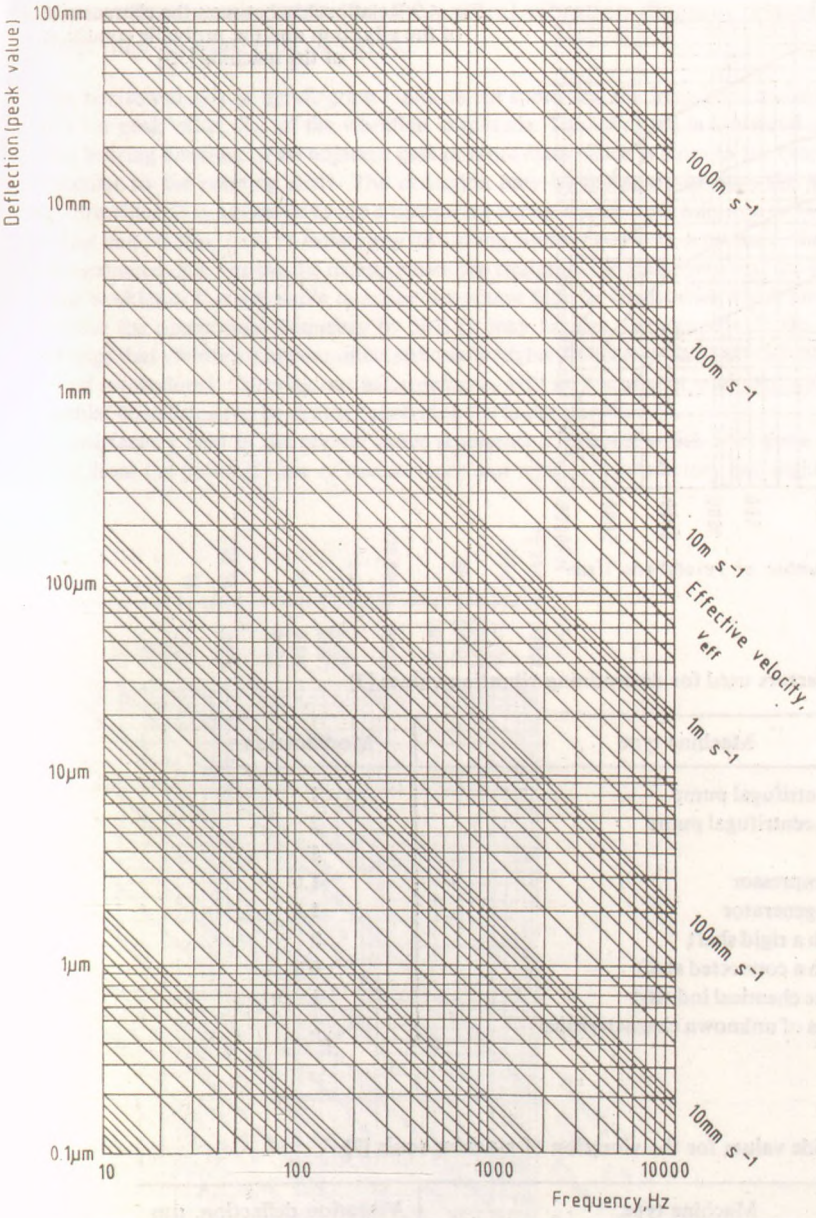


Fig. 4.8 Relationship between the amplitude of the deflection and the effective vibration velocity at various frequencies

The table contains no data for machine tools. In the case of machine tools the still tolerable deflection amplitudes are specified (Table 4.6) with regard to the function of these machines and the extra attention needed to meet the requirements of the machined surfaces.

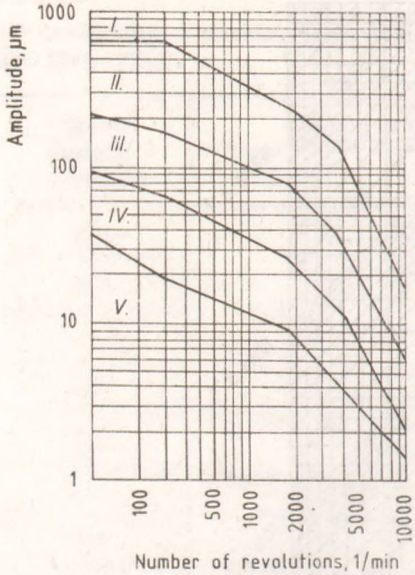


Fig. 4.9 Relationship between the characteristics of the vibration and the probable condition of the machine [30]

Table 4.5

Modification factors used for determining vibration values [7]

Machine type	Modification
Single-stage centrifugal pump	1
Multiple-stage centrifugal pump	2
Electric fan	1
Centrifugal compressor	1.6
Turbine, turbogenerator	1.6
Centrifuge with a rigid shaft	2
Centrifuge with a connected shaft	0.3
Machines of the chemical industry	1
Other machines of unknown characteristics	2

Table 4.6

Acceptable guide values for the vibration of machine tools [7]

Machine type	Vibration deflection, µm
Lathe	5.00...24.0
Drilling machine	1.5 ... 2.5
Mantle grinding machine	0.75... 2.5
Copying, or profile grinding machine	0.75... 2.0
Thread grinding machine	0.25... 1.5
Plane grinding machine	0.75... 5.0
Centreless grinding machine	1.0 ... 2.5

If no references are available, the general vibration diagram related to rotating machines may be used (Fig. 4.10) [4].

The horizontal axis of the diagram indicates the speed and the frequency, and the vertical axis indicates the peak value, A_0, of the vibration amplitude. The vibration is measured on the bearing bush, the bearing housing, or an adjacent part of large mass which is close to the bearing in a plane perpendicular to the rotating shaft. The deflection may be measured in both the horizontal and vertical directions. It is advisable to use a measuring apparatus which is adjusted to various frequencies and set bandwidths, so as to isolate any disturbing vibrations. From experience, the use of octave filters is recommended. During the measurement the equipment should operate at the working speed, under the workload. It is advisable to adjust the octave filter to frequencies which are less than and greater than the operational frequency by 50% (it may happen, for example, in the case of faulty ball bearings that vibrations appear of an essentially higher frequency than that corresponding to the number of revolutions). Carrying out measurements with an instrument equipped with a band-pass filter enables conclusions to be drawn concerning the faulty part as well.

A satisfactory field in the correct range represents a machine which is in good condition and is reliable from the point of view of maintenance and repair. A satisfactory but slightly coarse field

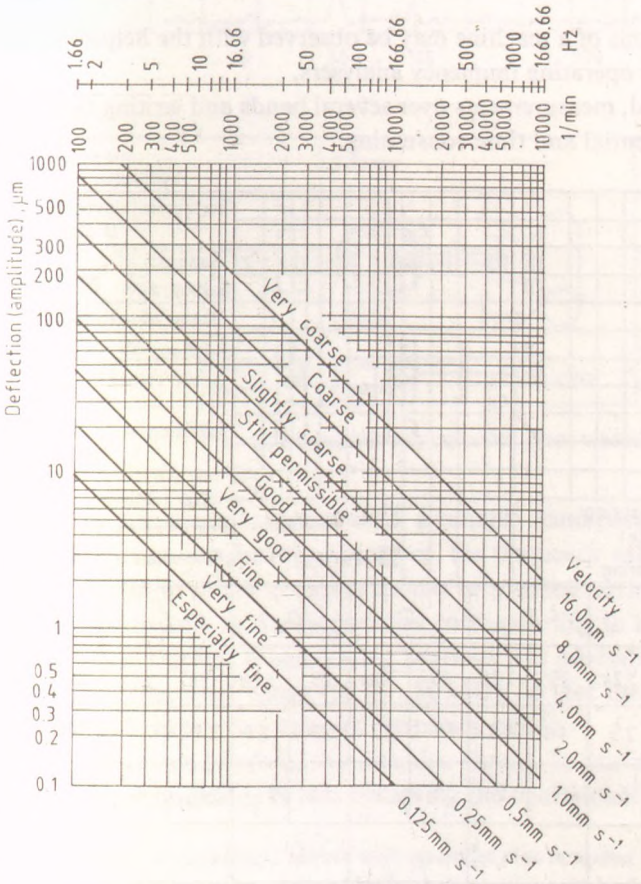


Fig. 4.10 Vibration diagram of machines [4]

calls attention to the development of damage in the near future, while a coarse field indicates that an immediate repair is necessary. The diagram should never be used under the illusion that the machine can withstand the accepted limit value of the vibration intensity without damage; the aim in using it is the early detection of a developing fault in order to prevent serious failures and unexpected breakdown of the machine in good time [4].

The diagrams were prepared on the basis of numerous tests, measurements and experience obtained during the examination of machines, yet they should be used only as aids to decision making, as, under different conditions, as much as a 25% deflection may appear without any observable difference in the condition of the machine. Therefore before accepting a decision the costs of stopping operations and the possible effects and losses caused by a reduced reliability of the machine and its sudden unexpected stopping should be considered carefully.

Determining the tolerable vibration of a machine is one of the most difficult tasks of vibrometry that can be achieved with only the data by the manufacturers, the vibration diagrams and prescriptions, as well as the plant measurement data being taken into account.

The operational vibrations of a machine may be observed with the help of band-pass filters and continuously operating frequency analysers.

If a tunable filter is used, measurements over several bands and writing the measurement report are non-essential and time consuming.

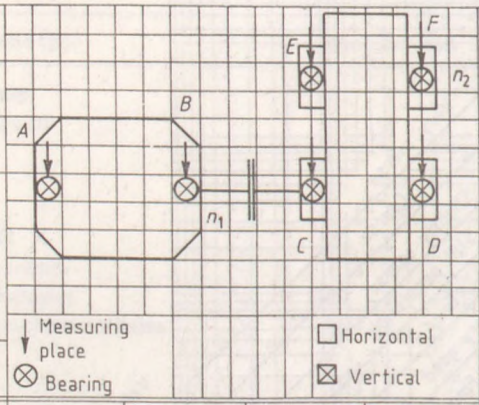
Machine: 22 Drive gear

Workshop: Rolling mill

Inventory No.:

Speed:
$n_1 = 2880$ $n_3 =$
$n_2 = 1440$ $n_4 =$

Date: 31.5.1985 Signature: [illegible]



Instrument: Inventory No. U 6742

Sensor: ☐ v, m s^{-1} ☒ a, m s^{-2}

Measured values: ☐ s ☒ v ☐ a

Scale values: (s)..... m
1 division ~ (v) 0.001 m s^{-1}
(a)..... m s^{-2}

Filter: octave

Remarks

Measuring position	A		B		C		D		E		F	
Nominal frequency, Hz (band centre)	25	3.2k	25	3.2k	25	3.2k	25	3.2k	50	3.2k	50	3.2k
Measured working vibration	1.8	2.5	1.6	2.3	1.9	2.2	2.0	2.6	2.1	2.4	2.4	2.7
Tolerable vibration limit	2.5	5	2.6	5	3.6	5	3.6	5	3.6	6	4	6
Remarks												

Fig. 4.11 Vibration level recorded in an industrial plant with a tunable band-pass filter

Essentially more information is supplied by the vibration spectrum prepared by the spectrum analyser and recorded by a level recorder or photography. Figures 4.11 and 4.12 show vibration spectra recorded in an industrial plant (on the printed forms used under plant conditions) using a tunable band-pass filter and a continuously recording spectrum analyser.

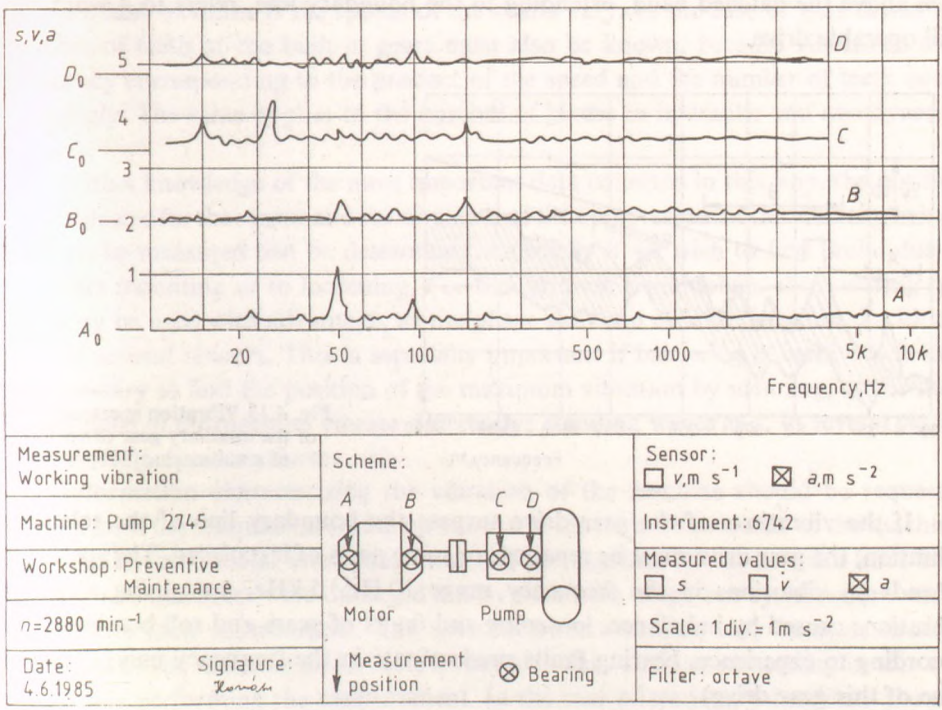


Fig. 4.12 Continuously recorded plant vibration spectrum

If an apparatus equipped with a tunable band-pass filter is used for measuring the operational or tolerable vibrations, the frequency values which are of importance to the condition of the machine should be scanned during the first test. These points are located usually at the frequencies corresponding to the number of revolutions or its integer multiples. When testing bearings it is advisable to select a band of a relatively wide frequency range (of 10...50 kHz), so that the following measurements are performed in only a few selected frequency bands.

As mentioned before, the machine should be put into a state corresponding to the working conditions before observing the operational vibration level.

Example. A centrifugal blower with no outlet pipe mounted on it showed violent aerodynamic vibrations, which led to misleading results on determining the basic spectrum of the vibrations. After mounting an outlet pipe the vibration of the machine agreed with the expected value [86].

The assessment of the technical condition based on vibration tests is applicable in every field of mechanical engineering. As an example, Fig. 4.13 shows the operational vibration spectrum of the auxiliary gear drive of a submarine [108]. The lower curve represents the vibration spectrum during the faultless operation of the gear drive; the vibration values of the hatched area express a warning that the wear, i.e. the deterioration of the technical condition, of the gear drive has already started. The area above the hatched band, extending to the boundary line, refers to a worn but still operable drive.

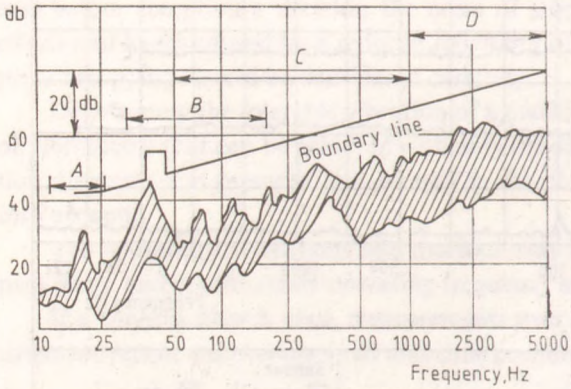


Fig. 4.13 Vibration spectrum of the auxiliary gear drive of a submarine [111]

If the vibrations of the gear drive surpass the boundary line of the tolerable vibration, the gear drive must be repaired, or some parts of it replaced. The spectrum extends to vibrations in the frequency range 10 Hz...5 kHz, containing also the vibrations caused by imbalance, loosening and faults of gears and roll bearings (e.g. according to experience, bearing faults predominate in the frequency band *C* in the case of this gear drive).

4.3 THE SELECTION OF MEASURING POSITIONS; THE ATTACHMENT OF SENSORS

The reliability of the measurement depends on correct marking out of the measuring positions as well as on the proper selection and attachment of the sensors. Before beginning the vibration testing of a machine it is worthwhile studying the design, the operation and the operational properties of the machine, in order to draw preliminary conclusions concerning the expected vibrations and their characteristics.

First of all, the working speed of the individual components of the machine should be assessed (in the case of machines with several shafts, for example gear drives, the input and output speeds are also determined), permitting recognition, during the analysis of the vibration, of the relatively simple imbalance faults and detection of the sources which excite the vibrations.

We examine where the bearings are located, if they are plain or rolling bearings, and which is the thrust bearing and the guide bearing. In the case of rolling bearings it is useful to know their type, their operating character (if the inner or outer ring is rotating), the directions in which the loads act, the number of rollers or balls, etc. All these data assist in the drawing of conclusions as to the characteristics of the vibrations resulting from any damaged parts (e.g. bearings).

We also examine if the speeds of the shafts vary. In the case of gear drives the number of teeth of the built-in gears must also be known, because vibrations of a frequency corresponding to the product of the speed and the number of teeth occur frequently. The same applies to the number of blades in hydraulic and aerodynamic devices.

With a knowledge of the most important data collected in this way, the position to be selected for the sensor and the direction of the components of the vibration which have to be measured can be determined, especially if we wish to find faults due to incorrect mounting or to loosening. For this prupose a multichannel measuring system may be used with advantage, as the phase shift can also be determined with the help of several sensors. This is especially important if balancing is involved. It may be necessary to find the position of the maximum vibration by scanning, as the individual parts of the machine vibrate differently; standing waves and, in several places, even nodal points may form.

Information characterizing the vibration of the machine should be requested in turn from the manufacturer, the operator and the maintenance staff of the machine.

The environmental vibrations must not be forgotten either, as there have been cases where machines were damaged not by their own usage but by vibrations which came from their environment. The environmental vibration may be even stronger than that of the machine under test; it may also vary in time, misleading in this way the people performing the measurement. In the case of stationary machines the disturbing background vibration may be eliminated by stopping the adjacent machines; alternatively by stopping the machine under test, the extent to which adjacent machines produce vibrations may be determined, with the possibility of determining if these environmental vibrations are capable of damaging the machine being tested. A general rule is that the environmental vibration must not exceed 1/3 of the intensity of the vibration of the machine under test, i.e. the signal to noise ratio must be a minimum of 10 dB.

A fault can even be detected with the help of a correctly installed sensor when the damage is just beginning; the efficiency of the test will then be higher and so less time need be devoted to fault detection.

Generally, marking out the measuring points close to the bearings on the bearing housings is most useful. If the bearing to be tested is not accessible, it is recommended that the technical drawing of the machine is used for finding the most favourable place for sensing, i.e. where the vibration waves reach the sensor after propagation in a straight line over as few contacting surfaces as possible.

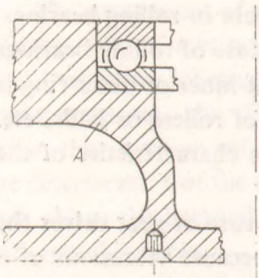


Fig. 4.14 The correct arrangement of the sensor

Figure 4.14 shows the best possible arrangement of the sensor for testing an axial bearing. It would have been better to place the sensor nearer to the bearing, at point *A*, but this point was inaccessible.

Bearing-housing covers should not be used as measuring points, because the vibrations would also have to penetrate an intermediate surface.

The intensity of the signal reaching the sensor depends, in the case of bearings, also on where the impact wave arises, whether between the inner or the outer ring and the rolling body. If the roller strikes the inner ring, the impact wave is transferred from the roller to the outer ring, from the outer ring to the bearing housing, and finally from the bearing housing to the sensor.

On passing through the three surfaces the loss may amount to 80% at each surface, so the intensity of the signal reaching the sensor will not be higher than 0.2^n, where n is the number of surfaces in contact (in our example $0.2^3=0.008$).

If the impact is against the outer ring, the intensity of the signal received by the sensor is $0.2^2=$ $=0.04$. Such large losses must be expected at high frequencies of $f>40$ kHz. Signals of a low frequency pass with smaller losses.

The difference is even more striking when an impact wave arising in one bearing is observed on the housing of another bearing, according to Fig. 4.15. The vibration caused by a fault of the outer ring which reaches measuring position *1* must traverse two surfaces, while the vibration wave reaches measuring position *2* through *7* contacting surfaces; hence $\frac{0.2^7}{0.2^2}=0.2^5$, i.e. the result is a 3000-fold reduction in the signal intensity.

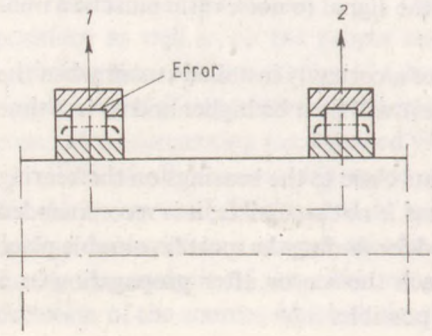


Fig. 4.15 The path of the vibration from its source to the sensor

A general guiding principle is that the sensor must be mounted on the machine to be tested as rigidly as possible. The best solution is to press or screw the sensor onto as large a surface area of the machine as possible with the greatest possible pressure, to enable the best possible transmission of the vibrations from the machine to the sensor.

Velocity and acceleration sensors of mass less than 50 g are often glued on, or brought into close contact with the machine with the help of a medium such as wax.

In practice the following fixing methods are widely used:

1. For laboratory test or for rapid changing of the position of the sensor (e.g. on searching for the most favourable place for taking measurements), the sensor is usually fixed to the machine to be tested with a wax supplied as an accessory with the piezo-electric acceleration sensors.

The frequency transfer of a sensor fixed in this way is relatively good (Fig. 4.16(a)) if the wax is not softened by a high environmental temperature. Resinous adhesives may be used up to 80 °C. Soft or rubber-like adhesives should not be used to fix

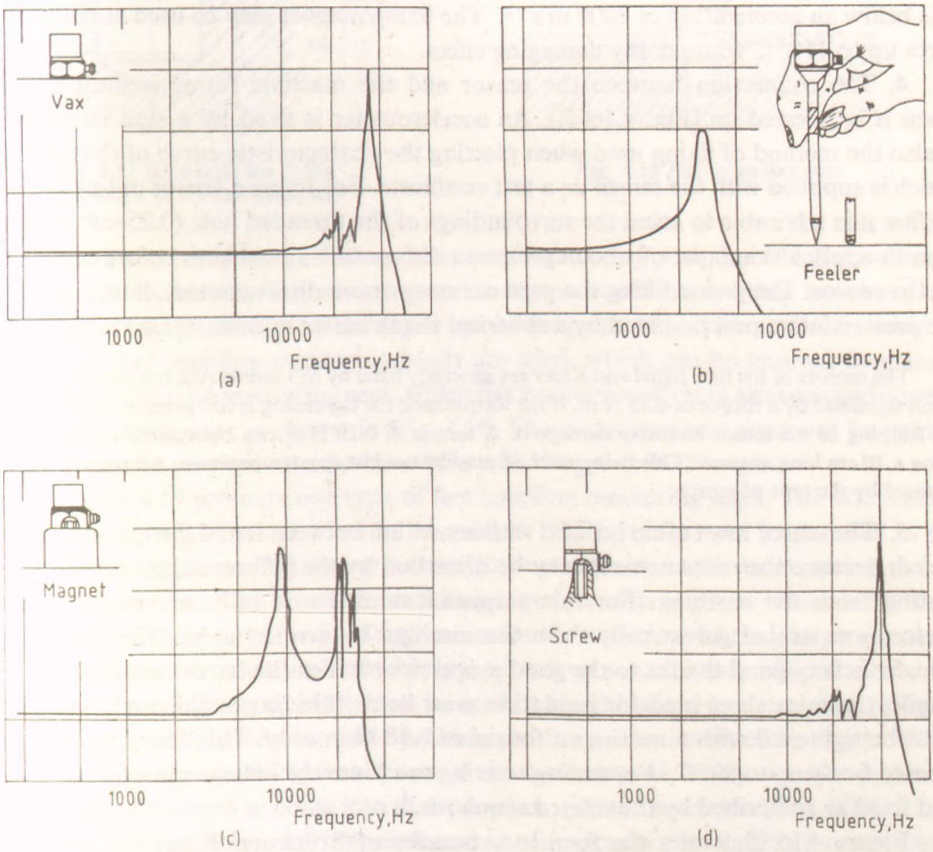


Fig. 4.16 Frequency transfer curves of feeler tips fixed by various means [19]

sensors as the vibration is damped by these substances and the frequency transfer will not be true. For permanent use the sensor is glued on with a two-component epoxy-resin, which is sufficiently hard and ensures inelastic adhesion.

2. For rapid or masswise testing with a portable instrument, a feeler tip is screwed into the sensor and pressed against the machine with a certain force — e.g. 2 N. The tip of the probe may be pointed or rounded off. Using such a feeler is comfortable, it can reach otherwise inaccessible places and the test is rapid. Its disadvantage is that its characteristic transfer curve is not good enough: it can transfer only relatively low frequencies accurately — over 1 kHz it gives false results (Fig. 4.16(b)).

3. The sensor can be fixed to ferromagnetic machine parts by an adhesive magnet (Fig. 4.16(c)). The magnetic circuit is closed when the sensor is fixed, so the diffuse magnetic field does not disturb the operation of the sensor. However, this method may be used only for fixing piezo-electric acceleration sensors, because electrodynamic sensors are disturbed by the magnetic field. Magnetic adhesion should be used only on smooth surfaces, possible only on ground surfaces, with careful observation to see whether or not it is thrown off by the fast acceleration. Generally there is no risk of this below an acceleration of 2000 m s^{-2}. The fixing magnet may be used at temperatures up to 150 °C without any damaging effect.

4. The connection between the sensor and the machine frame is most direct when it is screwed on (Fig. 4.16(d)). An accelerometer is fixed by a stud bolt. This is also the method of fixing used when plotting the characteristic curve of the sensor, which is supplied with the sensor as a test certificate. For fixing a sensor onto a rough surface it is advisable to grind the surroundings of the threaded hole ($0.25 < R_a < 1.6$), then to apply a thin layer of silicon grease to the contacting surfaces before screwing on the sensor. The grease filling the gaps ensures a more direct contact. Some sensors are pressed into a nest prepared by a spherical shank cutter of diameter 0.15 mm.

The sensors of the firm Brüel and Kjaer are generally fixed by M3 screws (the bore-hole is 4 mm deep) tightened by a torque of 0.18 N m. If the torque used for tightening is too great, it may deform the housing of the sensor and may damage it. A torque of 0.18 N m can be realized manually by using a 10 cm long spanner. This fixing method may be used at any temperature; the upper limit is dictated by the type of sensor.

5. The sensor must often be fixed with insulation between it and the machine to be tested, because the measurement may be disturbed by the differential potentials originating from the earthing. For this purpose insulated stud bolts are used and the sensor is separated galvanically from the machine by a mica sheet. The frequency transfer is very good thanks to the good properties of mica (its hardness), but, as it is fragile, the mica sheet used for insulation must be as thin as possible and the screw must be tightened with a maximum torque of 0.18 N m only. This fixing mode may be used freely up to 80 °C. For testing rotating machines the sensors must be arranged and fixed as prescribed by the related standards.

Figure 4.16 illustrates the frequency transfer of Brüel and Kjaer sensors fixed by different means [19].

Accordingly, the best frequency transfer is obtained when no screw is used for fixing the sensor. In some devices the sensor is pressed into a tapered nest and screwed down to attain a more efficient contact. Figure 4.17 shows the bore-hole required for this fixing mode.

The most favourable conditions for guiding the waves are ensured by a suitable taper, the width of the bearing surface and the correct pressure.

Some components of the waves are reflected by inaccurately matching surfaces or are unable to pass through the airgap.

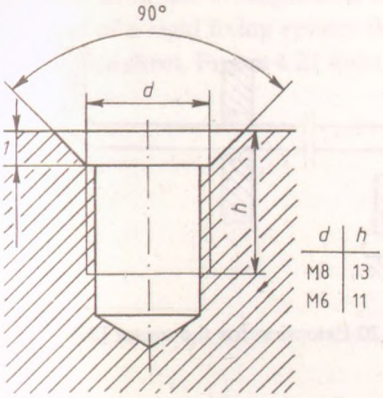


Fig. 4.17 Bore-hole for fixing the sensor or its seat (adapter)

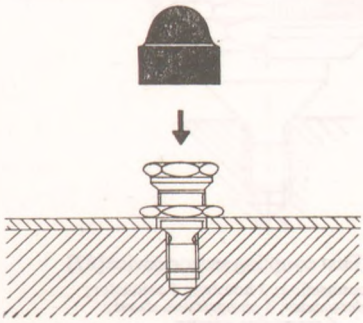


Fig. 4.18 Fast coupling seat

The sensor may have to be used in several measuring positions which are changed frequently. In this situation screwing on the sensor would involve time losses and also the measuring head may be damaged by the frequent remounting of the sensor. In this case fast coupling measuring heads are used, which can be inserted by a single movement into the measuring seat, which has been screwed on in advance and covered by a protective cap, as shown in Fig. 4.18. The measuring seat must not be painted and it must be screwed into the bore-hole with the prescribed torque.

Figure 4.19 presents one type of fast coupling measuring head. The safe connection and contact pressure must be controlled. The line must not be exposed to any force effect (e.g. drawing). Some instruments are delivered with an extension to connect to the sensor. This is useful for taking measurements in places which are difficult to reach, or where there is not enough room for the sensor, or where it cannot be fixed. Naturally such an extension is used only in an emergency, as it may affect the measurement results considerably. If it must be used, it should be led in a straight line out of the machine with as few bends as possible and it must be insulated by means of soft rubber rings, or suchlike, from parts of the machine which may transfer disturbing vibrations if allowed to come into direct contact with the extension (Fig. 4.20).

Disturbing effects may appear because of improper placement or mounting of the connecting cables for fixing the sensors. Frequently, disturbing voltages from

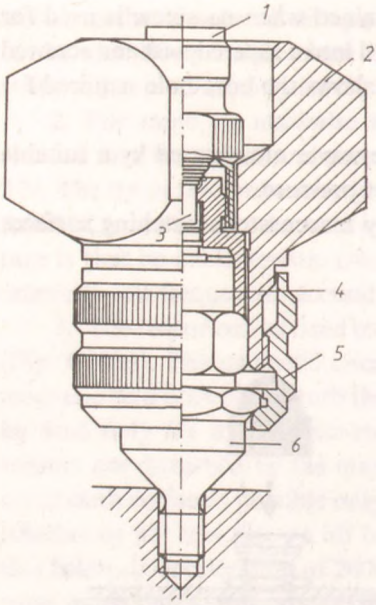


Fig. 4.19 Fast coupling sensor device [87]

1 cable connection; *2* housing; *3* contactor; *4* accelerometer; *5* compression sleeve; *6* connecting adapter

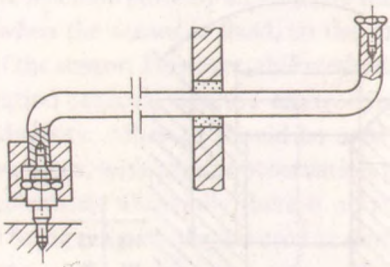


Fig. 4.20 Extension for the sensor [87]

various sources occur in the measuring cables because of electrostatic and frictional-electric effects, various stray currents, currents in earthing lines, etc. The electrostatic effects can be ascribed to moving or bending the cables, or allowing them to rub against other objects or each other. Therefore only special cables prepared for this purpose should be connected between the sensor and the preamplifier.

The cable should be led close to the machine frame or laid on it, and should leave

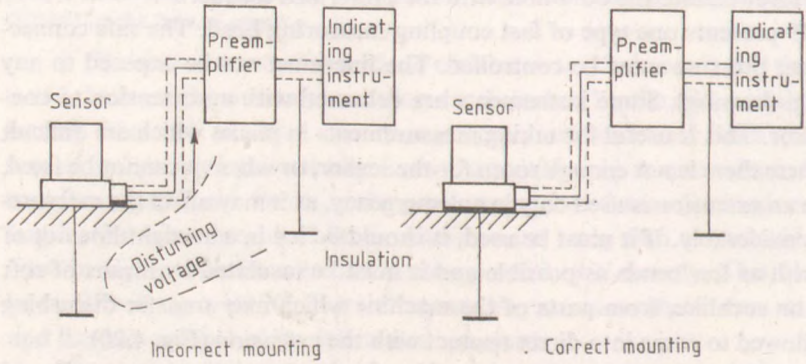


Fig. 4.21 The disturbing effect of earthing currents can be eliminated by insulating the sensor

it at a place where the vibration is slight. It should be fixed by means of adhesive wax or adhesive tape, or suchlike to the machine frame.

Measurements may be disturbed markedly by earthing lines, especially where several interconnected instruments are being used.

The currents flowing through the earthing lines induce disturbing voltages in the measuring cable. This effect can be avoided by earthing the measuring system at one place only, possibly at the indicating instrument. Therefore the sensor must be separated by insulation from the machine to which it is fixed, as the latter is in most cases earthed. In simple arrangements adhesive wax may be used for the insulation, but in the case of a rigid fixing system the sensor must be mounted with an insulated screw and a mica sheet. Figure 4.21 illustrates the incorrect and correct methods of mounting.

5 TESTING BEARINGS BY MEANS OF VIBROMETRY

5.1 TESTING ROLLING BEARINGS

The faults arising in rotating machines (e.g. electric motors, centrifugal pumps) are often bearing faults. The conventional means of monitoring the condition of bearings (e.g. their heating) by feeling or by measuring their temperature is insufficient for the early recognition of damage to the bearing. The vibrations sensed by feel and by the ear before the machine becomes inoperable call attention to the wear and to the irregular operation. In the past the operator of the machine listened to the vibrations of the machine by putting his ear to a screwdriver placed against the critical point or by feeling with his hand. These examinations and even the first instrumental tests were based on the observation of low-frequency vibrations, which are characteristic of the vibration of the whole machine, but mostly of an imbalance. Only very advanced bearing faults can be detected by sensing low frequencies. In this case the bearing fault is so serious already that the whole machine performs violent low-frequency vibrations and the bearing is ruined usually within a short time. On widening the observed frequency band it becomes clear that by observing higher frequencies ($f > 5$ kHz) the condition of the bearings can be characterized better. Observation of the vibrations by means of wide-band instruments or analysis of the vibrations enables bearing faults to be recognized at an early stage, so permitting maintenance work to be carried out in due time and thus avoiding unexpected failure and inoperability of the machine.

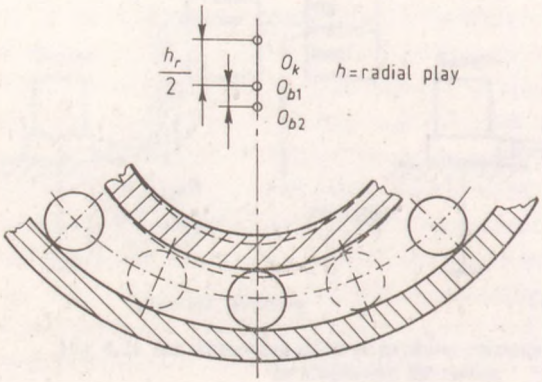


Fig. 5.1 Vibrational motion of the inner ring of a rolling bearing

A rolling bearing produces vibrations because of the bearing clearance, even if all its parts are of an ideal shape. Namely, the centre of the inner ring keeps changing its position, depending on the position of the ball bearings, as shown in Fig. 5.1. The centre of the outer ring is at O_k. The inner ring is displaced, because of the radial clearance, in the direction of the load and its centre moves to O_{b1}. This displacement equals half the radial play if the centre of the rolling body falls on the line of action of the load. If two adjacent rollers are positioned symmetrically with respect to the line of the acting force, then the inner ring, supported on both rollers, is displaced further and its centre moves to O_{b2}. The position of the centre of the inner ring will change periodically between O_{b1} and O_{b2}, depending on the position of the rolling bodies. Half the distance between O_{b1} and O_{b2} is the amplitude of the vibrational motion and its frequency is

$$f = \frac{1}{120}(1-\gamma)Z_n \text{ Hz},$$

where

$\gamma = d_g \cos\alpha / d_m$;
d_g = diameter of the rolling body (mm);
α = contact angle of the bearing;
d_m = mean diameter of the bearing (mm);
Z = number of rolling bodies running in one line;
n = number of revolutions (1/min).

The twofold theoretical amplitude of the vibration of the inner ring can be calculated from the geometrical data of the bearing by means of the following relationship [66]:

$$\Delta = \left(\frac{d_m}{2}+\frac{h_r}{4}\right)\cos\left(\frac{\pi}{Z}\right) - \sqrt{\left(\frac{d_m}{2}+\frac{h_r}{4}\right)^2 \cos^2\left(\frac{\pi}{Z}\right) - \frac{d_m h_r}{2}} - \frac{h_r}{2}.$$

If there were no radial play and the elements of the bearing were perfectly rigid bodies of ideal shape, then no vibration would be induced. But the vibration in the bearings may also have other causes. Vibration may be caused, for example, by movement of the rings and the rolling bodies — which are never of regular shape or of the prescribed dimensions — as they roll against each other, because their surfaces cannot be made perfectly smooth. Other causes are impacts between the rolling bodies, their deformation, discontinuities in their surfaces caused by chipping, and the effect of the abrasive materials found between them.

Rolling bearings are subject to wear during their operation, as are all machine parts, and as a result their theoretically prescribed geometrical dimensions, their trueness of form and surface quality change, causing a change, usually an increase, in the vibrations generated by the bearings. As the inner and outer rings and the rolling bodies in ball bearings and cylindrical roller bearings wear, the radial play increases.

As a consequence the rotating shaft is displaced from its original axis, and its rotation then no longer continues to be about the theoretical centre. The changes caused by wear lead to further displacement of the bearing components from each other and increased mutual impacts, resulting in enhanced vibrations. The irregularly worn and damaged rolling bodies will no longer move smoothly and regularly in the live rings which have irregular, worn and damaged shapes. They will impact against each other, raising vibrational waves depending on the amount of wear and damage and their motion relative to each other. Regular rolling is frustrated also by foreign solid material found in the bearings (e.g. dust in the grease, peelings, tiny solid particles), causing jerking and impacts, which induce shock waves.

Generally the vibrations raised by impacts are gradually damping, transient vibrational phenomena, but they recur periodically, depending on the relative speeds arising because of the rotation of the bearing. Therefore, the number of repeated impacts causing the vibration and hence the frequency of the shock waves are determined by the speed, the number of rolling bodies, and other factors.

The impact processes that occur in rolling bearings may be modelled as shown in Fig. 5.2, with the assumption that the rolling body rolls on an outer ring of infinite radius (a straightened outer ring) until it arrives at a damaged part of the track.

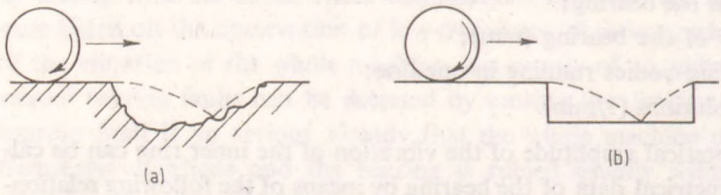


Fig. 5.2 The movement of a rolling body over a damaged track
(a) real defect; (b) idealized defect

If the track and the rolling body had perfectly ideal shapes and ideal surfaces, the latter would roll smoothly along the track without any interruption. However, the reality is different. If we remove the lubricating grease from a new bearing and rotate the bearing, we can feel and hear that the rolling is impeded — slight shocks and impacts occur due to the unevenness of the track, which can be observed on spinning the bearing. The rolling body falls into the steepest and deepest pits on the track and bumps against the protrusions. As the rolling body moves over the irregular track it is acted upon by inertial forces, which are proportional to its mass, causing accelerations, which can be observed over the whole bearing.

Local impact-force effects are also created at the points of contact, depending on the speed of the impact. Compression waves propagate in the material at the velocity of sound which is characteristic for the material. In the next phase of the impact the track and the rolling bodies are deformed and part of the energy of the impact is dissipated, inducing vibrations. Therefore the whole bearing performs a complex motion.

Also, the vibration caused by the impact is complex: it consists of vibrations of a relatively low frequency which are produced by deformations of the track and a high frequency vibration of about 40 kHz superimposed on this. The conditions may be appreciated easily if we regard the track as a beam with two supports (Fig. 5.3). The beam is bent by the ball falling onto it and performs a damped vibration with a relatively low frequency. The magnitude of the deflection depends on the energy of the ball falling on the beam. High frequency vibrations and shock waves emanate from the point of impact of the ball on the beam.

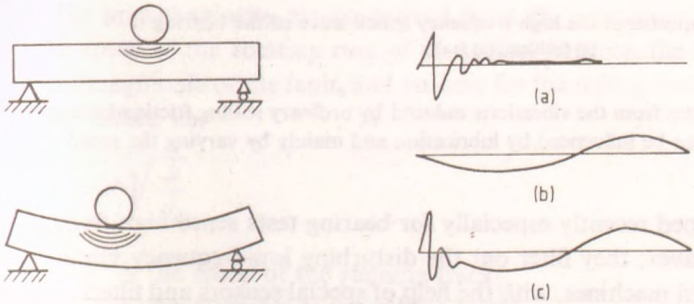


Fig. 5.3 Waves formed during an impact
(a) and (b) components; (c) resultant

The transient shock wave of the impact is formed in a very short time, causing very rapid vibrations in the body of the bearing.

The frequency of the resulting waves and their damping depend on the quality of the bearing and of the material, the size and design of the bearing — as the medium of wave propagation — and also on other factors.

When testing a bearing the mutually superimposed vibrations are separated. In the start-up phase, a bearing fault is hard to recognize from observations of the low-frequency vibrations, because of disturbing vibrations from other sources, while even quite insignificant bearing faults can be observed easily by studying the high frequencies.

The initial damage to a bearing may increase the shock-wave 10-fold, while a ruined bearing causes a 1000-fold increase compared to the shock-wave produced by a sound bearing [111].

For determining the magnitude of an observed fault, experiments were performed with double-row rolling bearings; a 0.076 mm wide and deep scratch was artificially created on the rolling track of the outer ring and the spectra of the vibrations produced by the sound and the faulty bearings were compared. Part of the low-frequency conducted sounds and part of the high frequency vibrations of the shock waves were analysed. During the conventional experiment there was hardly any difference between the spectra of the sound and the faulty bearings. The vibrations caused by the slight artificial scratch were swamped by the vibrations originating from other faults, the movements and elasticity of the bearing [93], but on examining the high-frequency vibrations recorded by sensors tuned to shock waves, significant deviations were observed between the vibration spectra of the faulty and the sound bearings, as shown in Fig. 5.4. The low peak (fault signal) seen in the vibration spectrum

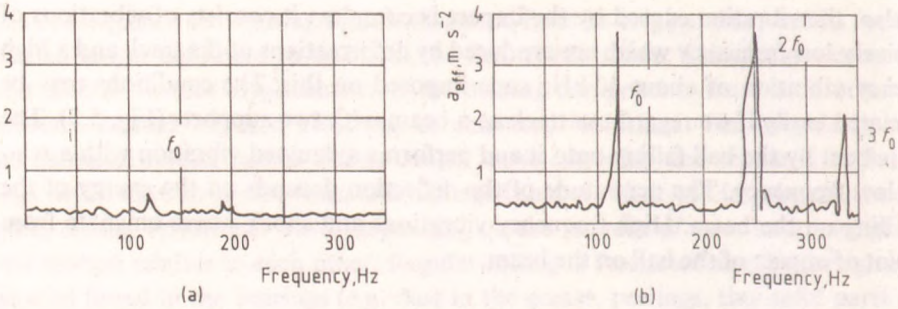


Fig. 5.4 Vibration spectra of the high frequency shock wave of the bearing
(a) faultless; (b) faulty

of the sound bearing originates from the vibrations induced by ordinary rolling friction between the bearings. This fault signal can be influenced by lubrication and mainly by varying the speed of the bearing.

The devices developed recently especially for bearing tests sense high-frequency vibrations and shock waves; they filter out the disturbing low-frequency vibrations, which appear generally in machines, with the help of special sensors and filters.

The test method is suitable for early fault detection in rolling bearings running at a speed of over 20 revs/min, with the exception of needle roller bearings.

These instruments measure the amplitude of high-frequency shock waves which are caused by impacts essentially at the position of the fault, and also indicate eventually the frequency of the shock waves. This test method is also called the Shock Pulse Method (SPM).

5.1.1 TESTING ROLLING BEARINGS BY THE SHOCK PULSE METHOD

A special vibration receiver, tuned for high-frequency vibrations and their observation decomposes the complex signal that arises from a complex vibration which contains also the low-frequency vibrations of the bearing and the machine, and uses only the high-frequency vibrations for the evaluation of the condition of the bearings. For this purpose a pulse generator transforms the vibrations which attain the set level to square signal pulses (Fig. 5.5). The height of the square pulses depends on the mass

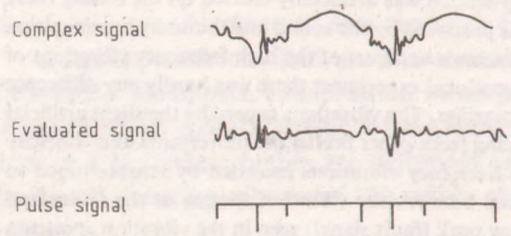


Fig. 5.5 The principle of the measurement of a shock pulse

of the bodies hitting the rings in the bearing, the elastic properties of their material and, naturally, on the velocity of the impact. In the case of a given bearing the elasticity characteristics may be regarded as constant. The mass can be determined form a knowledge of the size of the rolling bodies and the bearing rings, so the velocity of the impact in a given bearing is a function of the size of the fault.

The display instruments generally supply a signal which is proportional to the logarithm of the impact velocity v, so the so-called shock value is:

shock value $= \log(\alpha v^2)$,

where α is a constant characterizing the actual conditions.

The impact velocity can be derived from the geometrical conditions. It depends on the speed of the rotating ring of the rolling bearing, the size of the rolling body and the magnitude of the fault, and on how far the rolling body can penetrate into the surface defect, i.e.

$$v = v' \sqrt{\frac{h}{d_g}}$$

where

v' is the speed of the running track;

d_g is the diameter of the rolling body;

h is the penetration depth of the rolling bodies into the surface defect.

v' may be expressed in terms of the speed of the bearing; the diameter of the rolling body depends on the type of bearing. Therefore the shock value indicated by the instrument depends on the characteristics (the type) of the bearing and the magnitude of the defect.

shock value $= \log(\beta n^2 K h) = \log(C n^2 h)$,

i.e. the indicated shock value is proportional to the magnitude of the defect. The factor K is the so-called geometrical bearing constant, which is given in the manuals of the bearing-testing instruments, while C is a complex constant related to a given bearing and a given instrument. In the past the geometrical bearing constants were determined with the help of tables or nomograms, but now the displayed data (SV = shock value) of SPM devices are evaluated simply on the basis of the bore diameter and the speed of the bearing, no longer by means of the geometrical bearing factor. For the

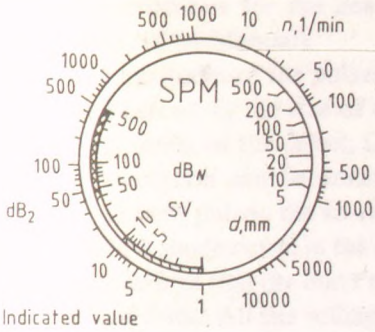


Fig. 5.6 Calculation dial of the 43A SPM instrument [62]

sake of simplicity in handling nomograms, calculation dials (Fig. 5.6) are also used for these devices [62].

The speed scale of the bearing is inscribed on the right, outside the dial; the scale showing the diameter of the bearing bore-hole is on the right-hand side of the internal rotatable dial.

The values corresponding to the size and speed of the tested bearing are set against each other by rotating the internal scale (e.g. the value corresponding to 50 mm bore diameter and 200 revs/min speed of the bearing).

The value indicated by the instrument is found on the outer left-hand scale (e.g. 20) and, by reading off the corresponding value on the inner left-hand dB_N-scale, we obtain the numerical value characterizing the condition of the bearing.

Its value is 20 in our example, i.e. it is in the band 10...40. A generally valid guide for the use of SPM-instruments is the following:

if the band 1...10 dB_N is obtained by the SPM instrument, the condition of the machine is good,

if the band 10...40 dB_N is obtained, the machine must be observed carefully, but it is still operable,

if the band 40...50 dB_N is obtained, initial damage has already appeared and repairs must be considered.

Bearings of different types and speeds and of various sizes will not give rise to identical shock pulses, even if their technical condition is identical; the larger and faster bearings rotate with greater noise and vibration. So, for assessing the condition of a bearing, the scale of the measuring instrument must be tuned to the size of the bearing to be tested. The value indicated by the instrument with the scale tuned to the bearing is not the absolute shock value but the (normalized) dB_N value, from which the technical condition of the bearing can be determined directly. Figure 5.7 shows the adjustable numerical dial of the type TMED 1 SKF-instrument, which serves this purpose. On setting the bore diameter of the bearing on the dial we obtain the dB_i (initial decibel) value indicated by the arrow for the value of the speed. By adjusting the initial dB_i value we tune (shift) the scale of the instrument to a value corresponding to the type of bearing (bore diameter) and the speed of the bearing, just as we did with the SPM 43A instrument shown in the preceding figure.

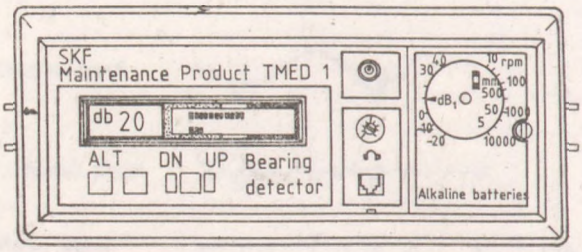


Fig. 5.7 Control panel of the instrument SKF TMED 1 [88]

Namely $dB_N = dB_{SV} - dB_i$, i.e. the shock value characterizing the condition of the bearing is the difference $dB_{SV} - dB_i$ (Fig. 5.8). The SKF instrument TMED 1 indicates, after setting the initial value the dB_C (decibel carpet value) and the dB_M (decibel maximum value) values.

The shock wave value, dB_C, can still be measured when the running surfaces of the rolling bearings are as smooth as when they were first machined, because relatively weak shock waves of high frequency are raised even then by the sliding and rolling motion. The intensity of the impact pulses depends mainly on the thickness of the oil film. If the lubrication, or the installation, or the adjustment of the bearing is incorrect, no oil film can be formed and the dB_C-value may increase.

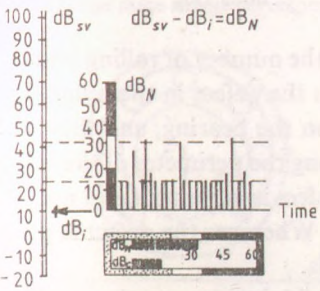


Fig. 5.8 Relationship between dB_{SV} and dB_N

The dB_M (maximum shock wave) value is high when the bearing elements are split, deformed, or damaged in any way. Generally the magnitude of these relatively rare low-frequency impact waves is characteristic of the damaged operational state of the bearing.

For bearings, the dB_C and the dB_M values, and the relationship between these values are measured after a certain running-in period has elapsed following their installation. Generally high dB_M values (over 35) and the low dB_C values (under 10) are characteristic of bearing faults or contaminated lubricating material.

dB_C values (over 10) and somewhat higher dB_M values indicate incorrect installation or unsatisfactory lubrication. Generally both values decrease after lubrication of the bearings, if no error was committed during their installation.

High dB_C values indicate dry running, or, in the case of pumps, cavitation. More detailed instructions for the analysis are given in the operational manuals of the instruments (SPM-Manual).

The magnitudes of the pulses measured by SPM equipment are proportional, as mentioned before, to the size of the defect. In numerous cases it is also desirable to know the position of the defect, i.e. which element of the bearing is defective. To this effect, a conclusion can be drawn from the frequency of the shock waves and the variation of their pulses, the so-called pulse-models. Let us examine the shock pulses produced by a single defect in the outer ring of a bearing (Fig. 5.9).

Let us assume that the outer ring is stationary; the inner ring rotates and is loaded by a vertical force. All the rolling bodies traverse the defect in the outer ring, so the

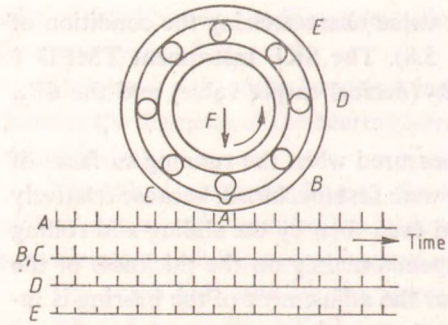


Fig. 5.9 Impacts in the case of a defective outer ring

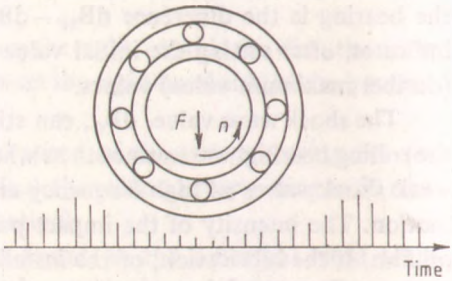


Fig. 5.10 Impacts in the case of a defective inner ring

frequency of hits depends on the speed of the bearing and the number of rolling bodies, It is also obvious that the impacts will be strongest when the defect in the outer ring is in line with, and in the direction of, the force acting on the bearing, and they will be weaker when the defect is shifted from this position along the perimeter of the outer ring in any direction. The shock wave is strongest if the defect is at point *A* and weaker at points *B* and *C* and weaker still at points *D*, *E* and *F*. Wherever the defect is positioned, the impacts follow each other with equal periods.

If the defect is in the inner ring, the magnitudes of the impacts — though they follow each other with equal periods — will not be constant, but will vary periodically. Clearly the impact will be the greatest when the defect in the inner ring is in the direction of the load and the rolling element rolls under it at just the same instant. Figure 5.10 illustrates the magnitude and the rate of impacts. The frequency of the recurrence of the maximum impact values equals the number of revolutions.

If a defect is formed on a rolling body, it impacts against both the inner and outer rings, i.e. two impacts during a revolution of the rolling body. These impacts will be strong when the defective roller or ball lies in the direction of the effective force and it just comes into contact (impact) with one of the rolling traces. When the defective rolling element is before, or behind, this position, the impact is weaker, as shown in Fig. 5.11. The frequency of the maximum shock pulses can be determined

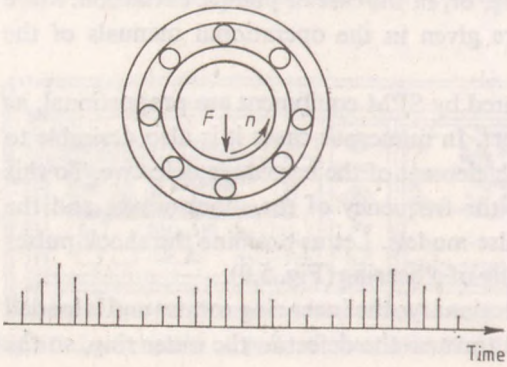


Fig. 5.11 Impacts in the case of a defective rolling body

from the number of revolutions of the roller along the outer ring, while the frequency of the subsequent impacts depends on the number of revolutions of the rolling body.

Naturally, similar phenomena are caused not only by split, but also by other defective (e.g. elliptic) forms of the rolling bodies and the rings. In reality several faults are often present simultaneously.

If two defects are found in the outer ring opposite each other, the nature of the series of pulses remains unchanged if the number of the rolling bodies is even; if it is odd, then both pulse series are superimposed on each other, as shown in Fig. 5.12. The effect of one or more defects of the inner ring can also be illustrated on the basis of similar considerations. In this case the groups of pulses appear with a frequency which is double, or some multiple of, the frequency of the pulses produced by a single defect. Defects which are positioned nearly opposite to each other on the rolling body amplify the pulse image corresponding to a single defect.

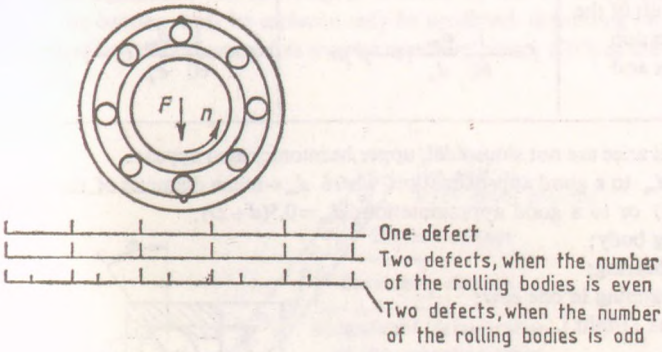


Fig. 5.12 Impacts in the case of two defects

The frequency of the impacts occurring in the bearings can be calculated from a knowledge of the size and the speed of the bearings using the relationships given in Table 5.1.

In practice only one of the faults appears in the phase of initial damage to the bearing with a high probability, so the frequency of the pulse enables a conclusion to be drawn concerning the position of the damage. Some instruments are therefore also supplied with a pulse counter (for measuring the frequency of the impacts or shocks) (the Shock Rate).

The measured data are integrated by the instrument within a time period depending on the frequency range, which must be taken into account when performing the measurement (e.g. several minutes in the case of 1...5 Hz, or a few seconds in the case of several hundred Hz).

According to experience an unlubricated bearing gives an impact signal of relatively low frequency, while very small impurities in the bearing grease give rise to weak shock signals of a relatively high frequency. Damaged bearings are characterized by strong impacts of medium frequency. Figure 5.13 illustrates the frequencies and shock values of characteristic faults.

Table 5.1

Important frequencies of the vibrations occurring in bearings [66]

Cause of the vibration	Stationary outer ring, rotating inner ring	Stationary inner ring, rotating outer ring
The passage of rolling bodies over a point in the stationary ring	$f=\frac{1}{120}(1-\gamma)Zn$	$f=\frac{1}{120}(1+\gamma)Zn$
Frequency arising when the number of defects on the ring is i	$f=\frac{1}{120}i(1+\gamma)Zn$	$f=\frac{1}{120}i(1-\gamma)Zn$
Radial striking of the rotating ring	$f=\frac{n}{60}$	$f=\frac{n}{60}$
Vibration caused by a radial fault of the rolling element, with consideration of its impacts against the inner and outer rings	$f=\frac{1}{60}\frac{d_m}{d_g}(1-\gamma^2)n$	$f=\frac{1}{60}\frac{d_m}{d_g}(1-\gamma^2)n$

Because the vibrations that arise are not sinusoidal, upper harmonics also appear.

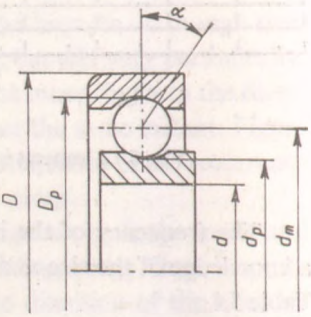
$\gamma=d_g\cos\alpha/d_m$, or $\gamma=d_s/d_m$ to a good approximation, where d_m=mean diameter of the bearing, $d_m=0.5\,(D_p+d_p)$ or to a good approximation: $d_m\approx 0.5(d+D)$;

d_g=diameter of the rolling body;

α =contact angle of the bearing;

Z =number of bearings running in one row;

n =number of revolutions (1/min.).



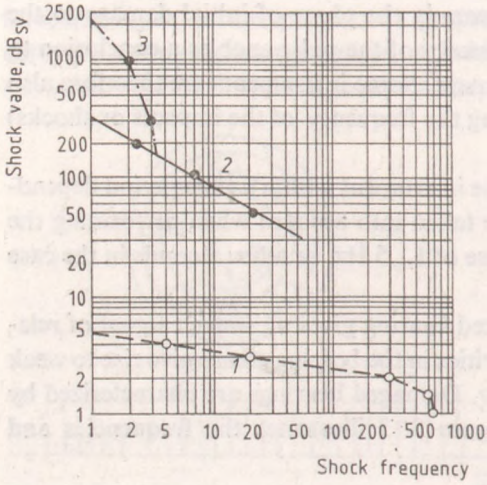


Fig. 5.13 Shock values and shock-frequency values of bearing faults [111]

1 contaminated lubricant; *2* bearing fault; *3* absence of lubrication

From experiences gained in measuring shock pulses, the value of the shock pulse shows a 8...10-fold increase when the first signs of deterioration appear, and may strengthen to several hundred times the original value, or even to 1000 times, as the deterioration progresses [111]. We must mention here that the shock pulse may decrease temporarily as well (e.g. when the rough surfaces of split parts are broken or rolled off), so the magnitude of the pulse may fluctuate with time.

Figure 5.14 presents the variation of the shock-pulse values of bearings versus the working period [33]. The value of the shock pulse is seen to rise by 50 dB after reaching half of the useful life, representing approximately a 300-fold increase in the shock pulse.

The condition of the bearing is still little changed and the damage still insignificant when the value of the shock impulse starts growing, according to the inscriptions in the figure. In the next stage damage develops rapidly, but is still not strikingly obvious. During subsequent operation, the damage becomes significant, and this stage is followed by that involving the risk of break-down. The date when the bearing must be replaced may be predicted, depending on the required safety. In the case of high safety requirements this may be necessary before 100% of the useful life of the equipment has elapsed.

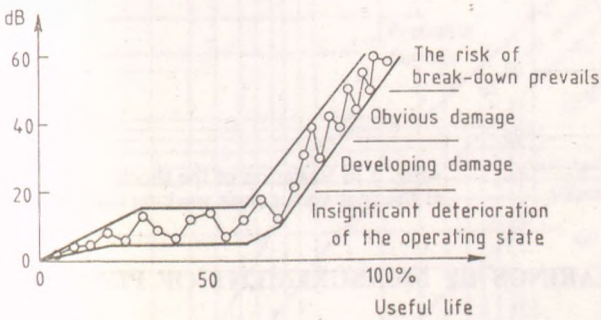


Fig. 5.14 Variation of the shock-pulse values of bearings versus their working time [3, 33, 91]

The manufacturers of bearings specify, in order to control the quality of the bearings, the expected initial vibration level (dB_i) of the correctly installed new bearings, depending on their size and speed. Such data are given in Fig. 5.15.

The initial vibration level of a new bearing of a given type may vary between wide limits, depending on the product, the production technology, etc. If we measure its vibration level during operation, we may judge approximately the measure of its usage on the basis of the deviation from Fig. 5.15, permitting, possibly, a check-up on the installation and the quality of the lubrication of the bearing as well.

Figure 5.16 shows the variation of the shock value of a bearing.

The advantage of the modern shock-pulse test method is that, even in the case of a relatively unfavourable signal to noise ratio, it is suited to the demonstration of the condition of a bearing and the contamination of the lubricants. Its high selectivity and sensitivity ensure its usefulness for testing the bearings of machines which give rise to complex vibrations as well [37].

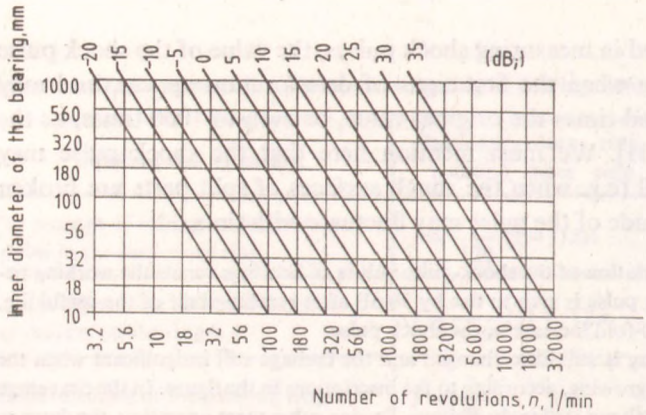


Fig. 5.15 Expected relative values of the vibration of new bearings, depending on the speed and the bore of the inner ring [13]

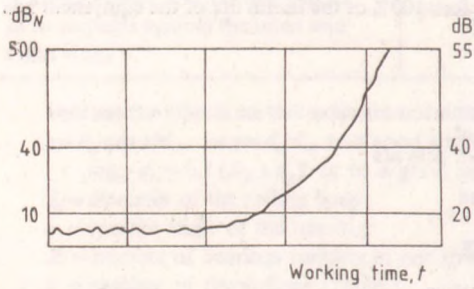


Fig. 5.16 Variation of the shock value of bearings versus their working time [85]

5.1.2 TESTING ROLLING BEARINGS BY MEASUREMENT OF PEAK VALUES

The peak-value test method was developed for determining the damage to bearings and the level of contamination of the lubricant by solid particles [37]. The essence of the method is the comparison of the measured peak values of the vibration velocity signal with the quadratic-mean value of the signal. The generally used instruments measure the effective velocity of vibrations with frequencies between 100...10 000 Hz, then they compare the adjusted multiple of the effective value with the peak values of the measured vibration. Whenever the peak value exceeds the level formed from the quadratic mean value, the instrument emits a pulse signal. The number of pulses observed in unit time indicates the frequency of the peak values which exceed the set value. A satisfactory result is obtained by this method only in the case of a suitable signal to noise ratio, i.e. when the bearings can be tested in an environment, which is as free as possible from background noise. The method is generally used for testing bearings of simple rotating machines, electric motors, etc.

Certain instruments are suitable for measuring the "Spike Energy". This method has been developed for measuring the condition of rolling bearings. A faultless bearing raises very weak vibrations of very high frequency (10–50 kHz) and of nearly

identical amplitude. When the bearing is already damaged, shocks of large amplitude appear, as shown in Fig. 5.4, with essentially a lower frequency (<10 kHz), than in the case of the original faultless bearing.

Therefore, the wear of the bearing is characterized by an increase in the amplitude and a decrease in the vibration frequency. Therefore, on testing the condition of the bearing, the peak value of the acceleration, the effective value of the acceleration and the frequency of the shock waves caused by the damage are measured. The measuring number SE ("Spike Energy") is formed from these three factors. According to experience this measurement method indicates any damage very early. Figure 5.17 shows SE-values corresponding to various states of the bearing versus the speed. The diagram was plotted with the help of a Reutlinger Model 970 apparatus, with a 220 mm long feeler placed directly on the bearing or close to it.

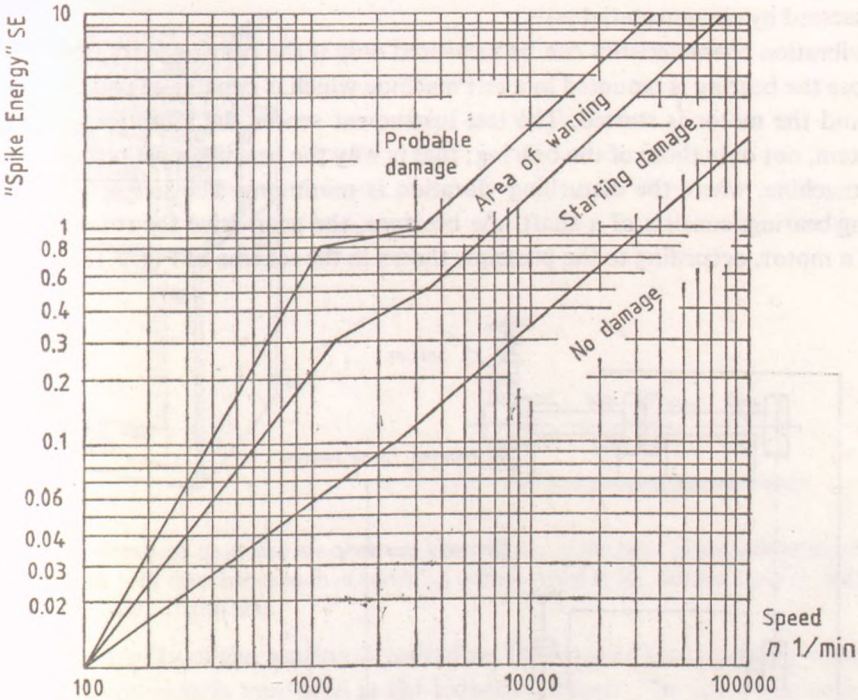


Fig. 5.17 Judging the condition of a bearing on the basis of Spike Energy (SE) measurement (with the kind permission of Reutlinger & GmbH) [111]

5.1.3 TESTING ROLLING BEARINGS BY VIBRATION ANALYSIS

The properties, the condition and the wear of rolling bearings may also be tested by conventional frequency analysis of the vibrations. The frequency spectrum taken during the test is compared with that of a new or "ideal" bearing. New, used, dismounted, and installed bearings can all be tested by this method.

5.1.3.1 TESTING BEARINGS BEFORE INSTALLATION

For testing new bearings their quality (noise) is checked and possible manufacturing faults are looked for.

In the case of used bearings their further usefulness is determined. Certain repair manuals prescribe periodic dismantling and inspection of the machine. If the plant carrying out the repairs or the renewal knows neither the date when the bearing was installed by the firm operating it nor its norm performance, then it can check the bearing by vibration measurement. If economic considerations are important, a bearing which qualifies as good is remounted (e.g. the main bearings of truck engines) or installed in a position where life and property may not be endangered if it fails. The checking of bearings by vibration analysis is especially advantageous when the rolling surfaces of the bearing cannot be inspected directly or when their initial defects can only be assessed by circumstantial tests.

The vibration characteristics can be measured only if the bearing is rotated. For this purpose the bearing is mounted in a test machine which is capable of rotating the bearing, and the motor is started. The test instrument senses the vibrations of the whole system, not only those of the bearing; that is why the bearing must be installed in a test machine, where the disturbing vibration is minimum. The simplest device for rotating bearings consists of a shaft, the bearings, the gear drive for rotating the atter, and a motor, according to the principle shown in the scheme of Fig. 5.18.

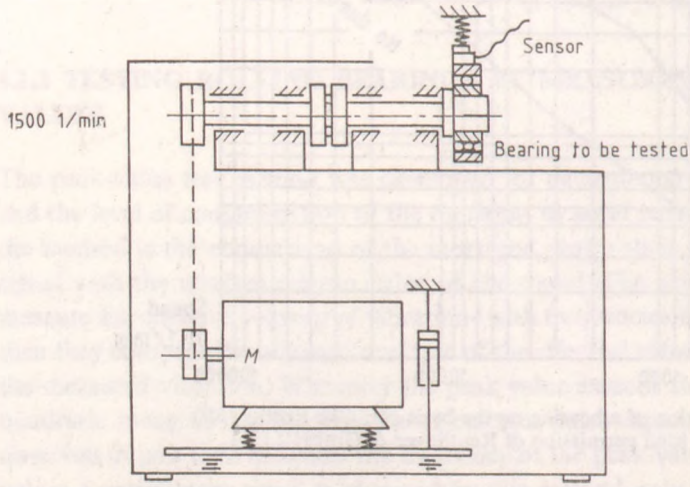


Fig. 5.18 The principle of bearing test equipment

To reduce the vibration caused by the drive mechanism of the equipment, sliding bearings and flat drive belts are used. The shafts of the Hungarian-made test devices rotate at speeds of 1500, 1600, or 1800 revs/min. The end of the shaft carries an expansion sleeve over which the various bearings to be tested are pushed manually. The

seat is quite tight, so on starting the machine the inner ring rotates and the stationary outer ring is loaded radially, or axially. In some devices the radial force is supplied by a hydraulic mechanism, which transfers the load, via a rubber insulation, to the outer ring of the bearing [75].

Tapered roller bearings are pressed against the shaft of the apparatus by the force of an axial spring. The acceleration or velocity sensor is fitted to the outer ring by an applied force. The magnitude of the disturbing vibrations is checked periodically on the block which supports the bearing. If the permitted value is exceeded, the measurement must be stopped until the cause of the vibration is eliminated. Vibrations exceeding the permissible value may be caused by overtightened belts, incorrectly adjusted couplings, a faulty vibration damper, or wear in the bearings of the test machine. Figure 5.19 shows such a bearing test device.

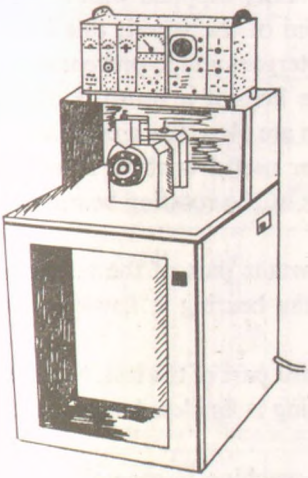


Fig. 5.19 Bearing test equipment

The vibrations produced are observed generally — on the basis of the related standard recommendations — in three bands each of width 2.5 octaves (band I: 50...300 Hz, band II: 300...1800 Hz, band III: 1800...10 000 Hz).

The quality of the bearing is judged on the basis of the quadratic mean value of the vibration signals measured in the individual bands. The cross connections of the band filters between the individual bands should be minimal. The measured results of the control instruments built into the production lines are displayed by light-indicators. The output signals of certain machines are used for controlling computers, which perform statistical data processing.

For laboratory research purposes complete spectra are generally plotted, i.e. not band by band. The scheme of the measuring device is illustrated in Fig. 5.20.

A basic requirement of comparative testing is cleanness of the bearings. Before beginning the tests, the bearings are generally washed in diesel fuel, taking care that no impurities remain on the rolling surfaces. The bearings are lifted out of the diesel fuel

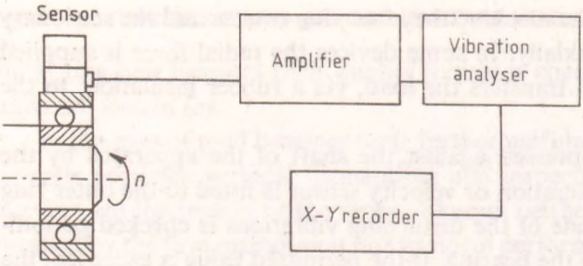


Fig. 5.20 Principal connection scheme of the instruments used to test bearings

immediately before the measurement, the front and mantle surfaces of the outer and inner rings are dried with cotton rags and then the bearing is put into the test machine. Used bearings must be washed before testing them in a vessel supplied with a perforated plate which separates the bearing from the bottom of the vessel; this is to avoid stirring up any impurities which have settled in the detergent and to prevent their repenetration of the bearing. Manual washing with the help of a brush is often replaced by use of a washing device, in which the bearings are placed on a perforated tray which performs vertical alternating motions. Another useful washing machine forces the washing liquid to flow over the rolling elements of the rotating bearing at a high pressure.

It must be emphasized that washing is a highly important part of the test, because very unfavourable vibrations may appear even if the bearing is flawless but contaminated.

It must be emphasized that washing is a highly important part of the test, because very unfavourable vibrations may appear even if the bearing is flawless but contaminated.

The vibration spectrum plotted by the bearing test machine is generally very rich in all the frequency bands. Figure 5.21 shows the vibration spectra of a type 6305 bearing recorded at various speeds [66].

Vibrations caused by deformation and surface defects of rolling bearings. Rolling bearings are rather complex mechanical systems from the point of view of their vibrations. They are generally inclined to self-oscillation, and discovering the relationships between the vibrations created and the mechanical characteristics is difficult.

Some vibrations are due to self-oscillation of the parts. Their amplitude depends on the frequency of the excitation, but they appear with a relatively constant frequency and their presence can always be relied upon. The most important of the eigenfrequencies observed on testing bearings are those of the outer ring. The eigenfrequencies of the outer ring may be calculated using the following relationship [66]:

$$f = 410 \frac{\sqrt{BD\left(1-\frac{D_0}{D}\right)}}{D^2} \frac{(m+1)\left[(m+1)^2-1\right]}{\sqrt{(m+1)^2+1}} \text{ kHz},$$

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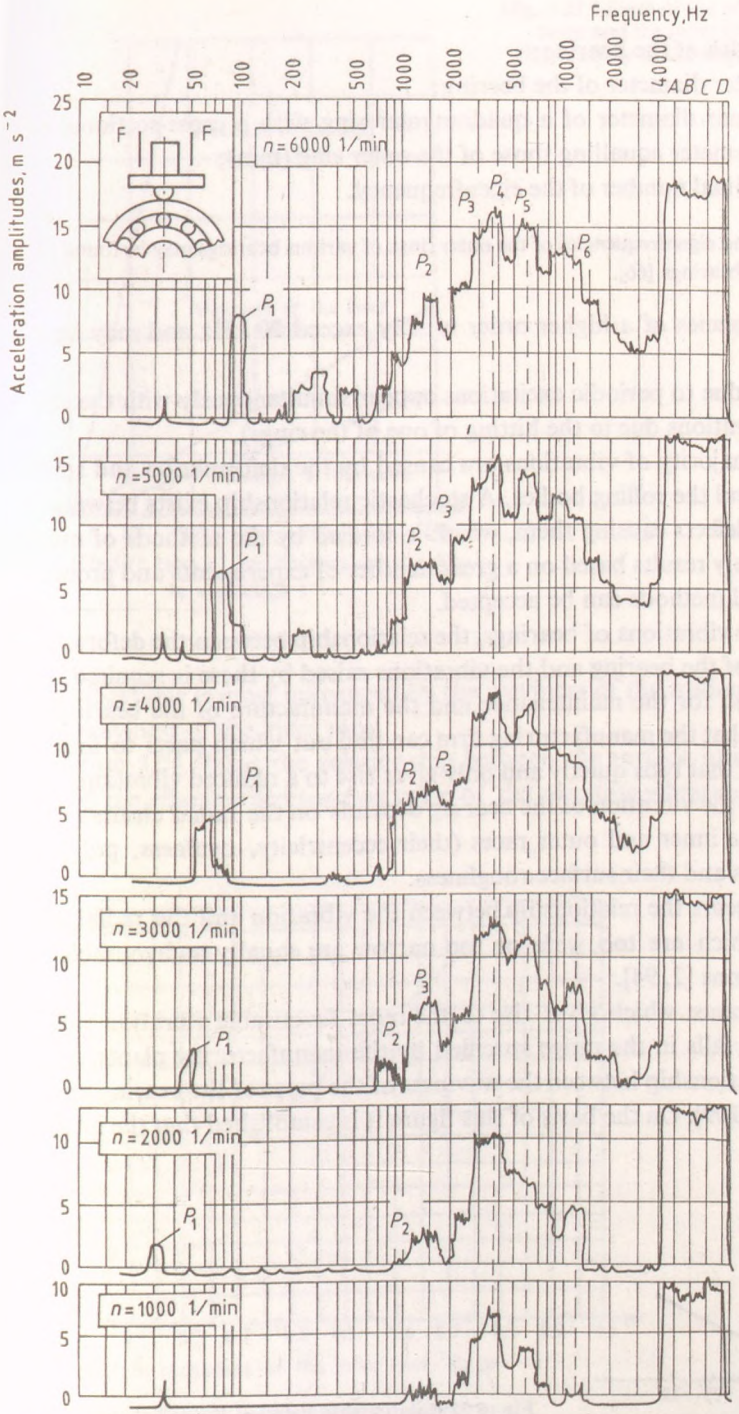


Fig. 5.21 Vibration spectra of a bearing of the type 6305 at various speeds [66]

where

B is the width of the bearing;

D is the outer diameter of the bearing;

D_0 is the inner diameter of a quadrangular ring with a cross-sectional area and a diameter equalling those of the outer ring (mm);

m is the ordinal number of the eigenfrequency.

The first and second eigenfrequencies of the outer rings of various bearings may be found in the literature dealing with bearings [66].

The eigenfrequencies of a higher order usually exceed 20 kHz and may be disregarded.

The vibrations due to periodic excitations appear simultaneously with the eigenfrequencies (e.g. vibrations due to the hitting of one of the rings).

However, the majority of vibrations are caused by the deformation and surface defects of the race and the rolling bodies. A stochastic relationship exists between the vibrations and the defects causing them, which is studied by the methods of mathematical statistics. Only results based on a great number of experiments and processed by modern statistical methods can be accepted.

On studying the vibrations of bearings, the relationship between the deformation and surface defects of the bearing and the vibrations raised by these is required. This is of importance both for the maintenance and the manufacture of the bearings, as this is the only way that the manufacturing firm can find out which parts to improve to produce a bearing that runs quietly and only gives rise to a reduced vibration.

The intensity of the vibration of the bearing depends on the radial clearance, the formal defects of the inner and outer races (their eccentricity, ovalness, polygonal form), their waviness and their surface roughness.

Figure 5.22 presents the relationship between the vibration and the radial clearance. Clearances which are too wide or too narrow are equally unfavourable and cause intense vibrations [2, 94].

The radial clearance which gives rise to the most favourable vibration value is probably that which falls in the range specified by the manufacturing plants. Figure 5.23 presents the relationship between the waviness of the parts of the bearing and the resulting vibrations [109]. On the basis of this figure it is established that the intensity

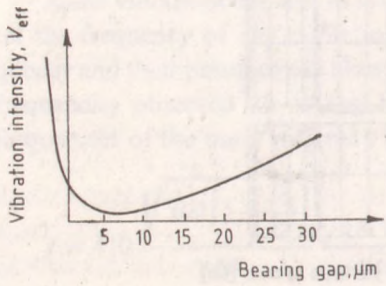


Fig. 5.22 Relationship between the radial clearance and the intensity of the vibration [94]

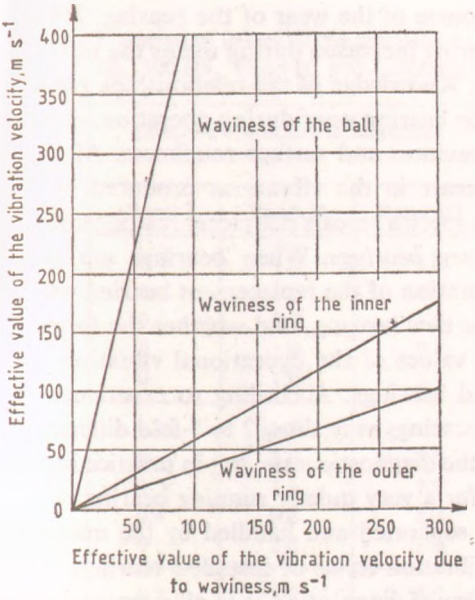


Fig. 5.23 Effects of the waviness of the rolling body and the inner and outer rings on the intensity of the bearing vibration [109]

of the vibrations is most influenced by the waviness of the rolling body and secondly by the waviness of the inner race.

Figure 5.24 shows the relationship between the surface roughness of the inner race and the effective acceleration value of the resulting vibration. The initial surface

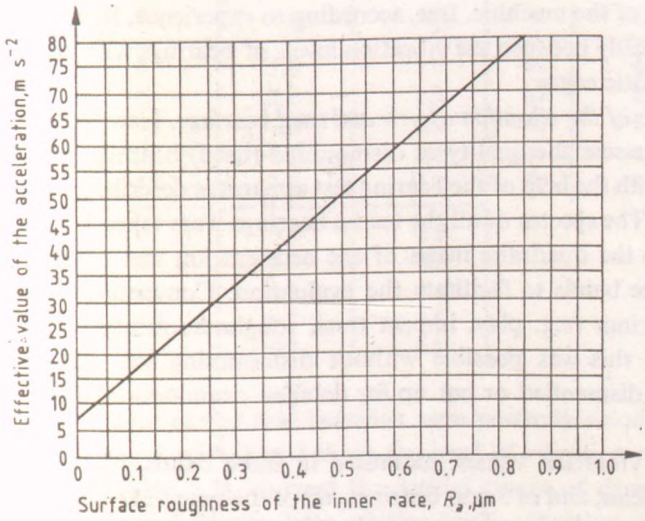


Fig. 5.24 Bearing vibration versus the surface roughness of the inner race

roughness may increase a 100-fold in the course of the wear of the bearing. It must be remembered that the vibration of the bearing increased during use as the waviness and surface roughness of the races increase. Knowledge of the relationships given in the manual is useful, because the races of the bearing wear during operation, causing seizure and flakes, thereby changing the waviness and surface roughness. All these changes contribute during wear to an increase in the vibrations produced by the bearings. A detailed analysis of the vibration spectra reveals still closer relationships.

The scatter of the vibration values of new bearings. When bearings are being replaced the question arises whether the vibration of the replacement bearing will be of the same intensity as that of the previous new bearing, and whether the formerly assessed vibration standards and the same values of the operational vibration may be applied when testing the newly installed bearings. According to experience the acceleration values of the vibration of new bearings may show 2 to 3-fold differences. This fact may cause some confusion during the diagnostic tests, but in practice installing a very noisy bearing as a replacement for a very quietly running bearing occurs rarely, especially as the quiet bearings are separated and handled by the manufacturers as special quality bearings. So the vibration levels of installed bearings show little scatter in practice. From the point of view of diagnostics, it is also reassuring to know that the vibration intensity of a worn, deteriorated bearing is more than three times that of a new bearing; it amounts to a 6 to 8-fold value, and on measuring shock waves it may even attain — as is well known — a value which is several hundred times higher.

On replacing bearings, it may occur that bearings of identical types have metallic or plastic ball cages. The latter may be thought to cause considerably weaker vibrations than the former, so the initial working vibration level must be controlled with a view to the good condition of the machine. But, according to experience, there is no essential difference diagnostically between the vibration levels of bearings with metallic cages from those with plastic cages.

The comparative analysis of the vibration of new and used bearings. New and used bearings were compared to assess the quality of dismounted (used) bearings on the basis of vibration analysis, with the help of the bearing test apparatus described above, at a speed of 1500 revs/min. The spectra of all the tested bearings were taken between 50 Hz and 15 kHz, and also the quadratic mean of the acceleration was measured in the usual manner in three bands to facilitate the evaluation. Conventional tests were performed on the bearings (e.g. play, impact tests, roughness measurements, etc.) in those cases where this was possible without dismounting the bearings. Finally a few bearings were dismantled or cut up for detailed examination after the vibration analysis.

Table 5.2 contains the vibration values, measured in three bands, of 50 used bearings, removed from vehicles, and of 5 new bearings, all of the type 6212 ZN.

Figure 5.25 illustrates the spectra of a few new bearings and Fig. 5.26 shows the spectra of the used bearings *25* and *38*.

Table 5.2

Effective acceleration values of the vibration of used and new bearings of the type 62.12 ZN (Band 1: 50...300 Hz; Band II: 300—1800 Hz; Band III: 1800—10.000 Hz)

Mark	Effective vibration acceleration $m\,s^{-2}$			Mark	Effective vibration acceleration $m\,s^{-2}$		
	Band I	Band II	Band III		Band I	Band II	Band III
1	0.4	1.0	1.8	26	0	0.9	4.0
2	0	1.5	5.0	27	0	0.5	1.8
3	0.4	2.0	11.0	28	0.5	1.0	9.0
4	0	1.0	2.5	29	Not evaluable		
5	0	1.0	5.0	30	Not evaluable		
6	9.0	13.0	11.0	31	0	3.0	6.0
7	0	2.0	8.0	32	0	0.5	8.0
8	0	1.5	7.0	33	0.5	1.5	7.0
9	0.4	0.8	2.2	34	0	0.9	0
10	0	2.0	7.0	35	0	0.9	1.8
11	0.2	1.0	4.0	36	0	1.0	8.0
12	0.1	1.0	4.0	37	1.5	1.5	3.5
13	0	0.8	2.0	38	17.0	5.0	8.0
14	0	1.8	0.9	39	0	1.5	2.5
15	1.0	2.0	3.0	40	1.0	1.8	14.0
16	0.2	0.8	4.5	41	0	0.8	1.8
17	0	0.8	6.0	42	2.5	5.0	10.0
18	0.6	2.0	15.0	43	0	1.0	3.5
19	0.1	2.5	8.0	44	0	0.8	2.5
20	0	1.8	4.0	45	1.0	1.0	2.0
21	0	1.0	3.5	46	0.2	1.0	3.0
22	0	2.0	2.5	47	0.5	1.0	5.0
23	0.1	1.0	6.0	48	0	1.0	5.0
24	0	0.7	2.5	49	0	1.0	4.0
25	4.0	3.0	8.0	50	0	0.8	2.5
1u	0	0.4	0.6				
2u	0	0.3	0.9				
3u	0	0.2	0.3				
4u	0.2	0.7	0.6				
5u	0	0.2	0.3				

The races of the new bearings were perfectly smooth and uninjured, while the used bearing *1*, with 2 to 3 times the level of vibration compared to that of the new bearings (Table 5.2), showed the intitial traces of damage. The vibration of both bearings *25* and *38* was large relative to the vibration of the new bearings, because their running surfaces were damaged by splits and peelings. Figures 5.27 and 5.28

Fig. 5.25 Vibration spectra of new bearings

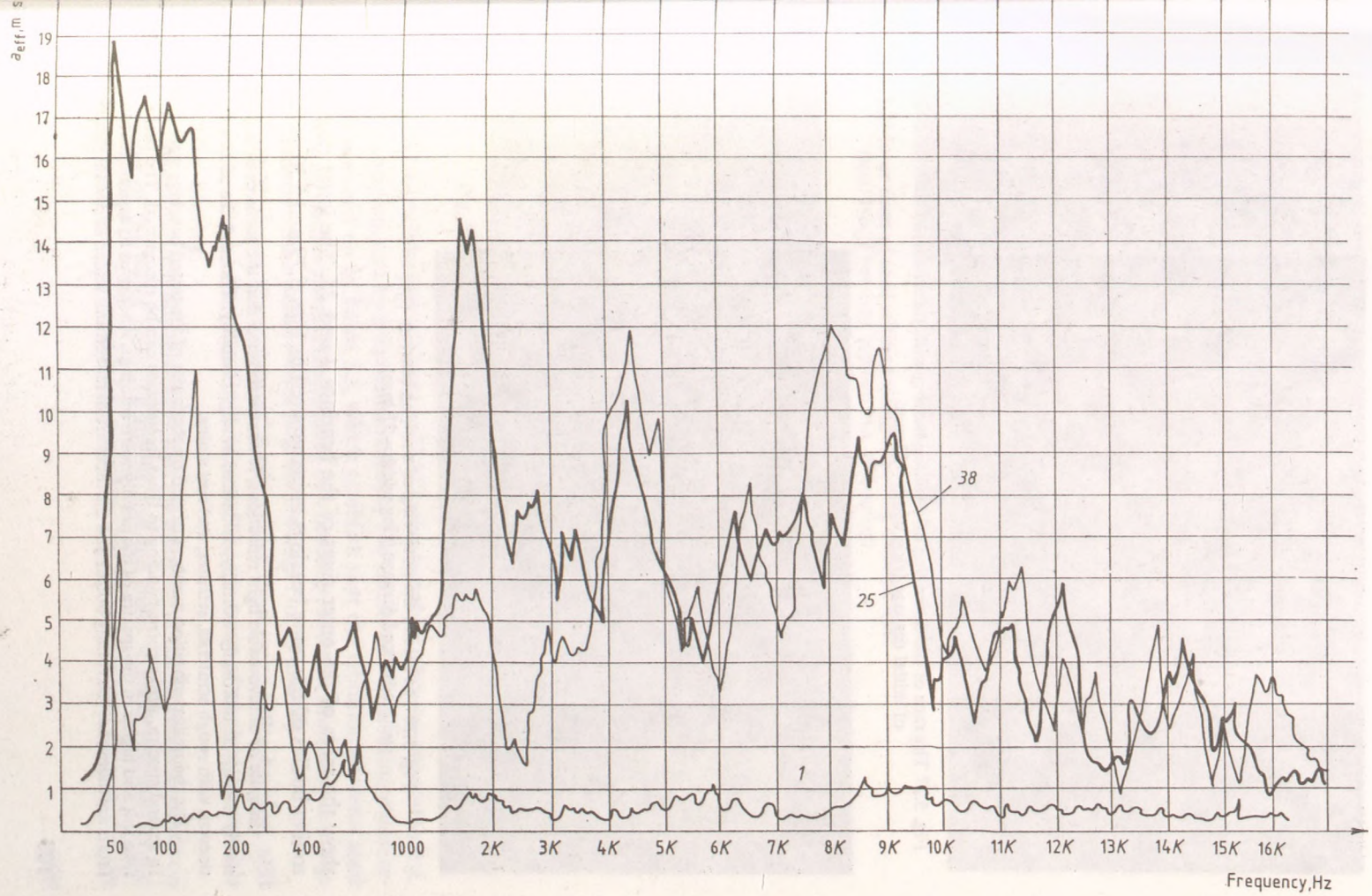


Fig. 5.26 Vibration spectra of used bearings

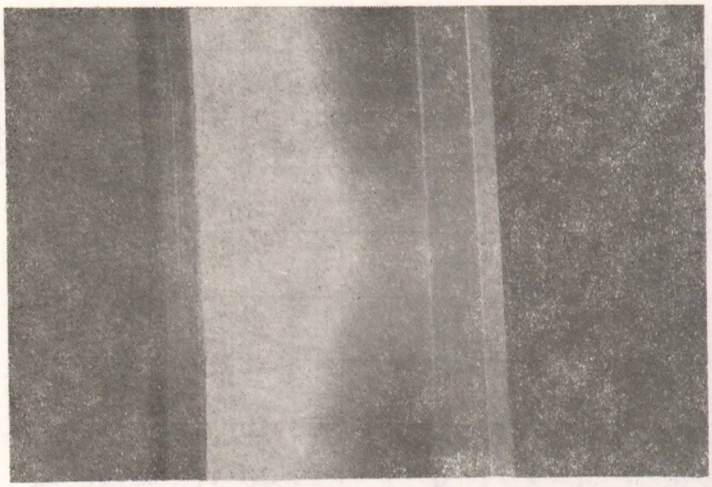


Fig. 5.27 The race of the outer ring of a rolling bearing with traces of initial damage (ring width = 22 mm)

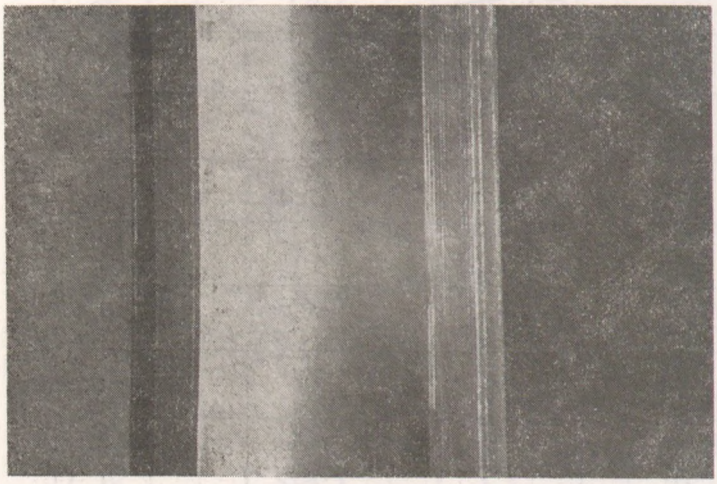


Fig. 5.28 The race of the outer ring of a rolling bearing with advanced damage (ring width = 22 mm)

show the races of the outer rings of the bearings *1* and *25*. The inner rings and the rolling bodies of the same bearings are compared in Figs 5.29 and 5.30, respectively.

In spite of unfavourable light reflections, it was established that the surfaces of bearing *1* were nearly sound or uniformly worn by oxidation, in all probability. The surfaces of bearing *25* were uneven, with rough scratches, seizures, and rust stains.

The theoretical vibration numbers caused by the faults of the tested bearings were determined. In several spectra, peaks corresponding to these frequencies can be recognized. The peaks are more frequent and higher in the spectra of the heavily worn bearings, leading us to assume that the exciting faults are many, i.e. the rolling bodies and the races are damaged and scaled in several places.

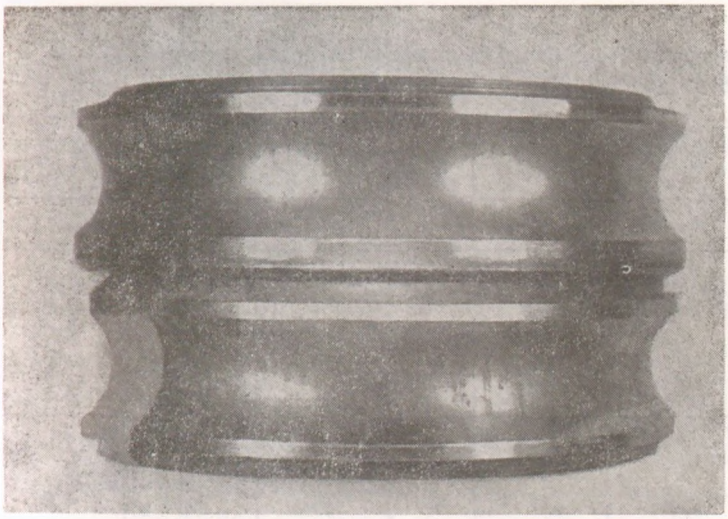


Fig. 5.29 Inner rings of used rolling bearings, showing different wear (top: No. *1* bearing; bottom: No. *25* bearing)

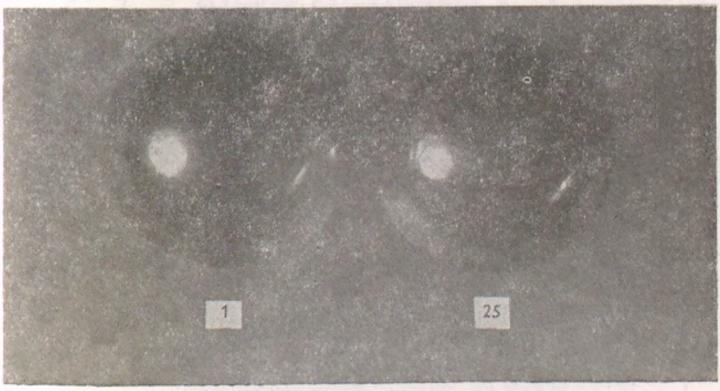


Fig. 5.30 Differently worn bearing balls

Tapered roller bearings of the type 32.217 were tested by vibration measurement, by visual inspection, and by a stereomicroscope. The results of the vibration measurements are summed up in Table 5.3, which establish that the vibration of some used bearings was no less favourable than that of new bearings, and that some used bearings ran more smoothly than the new ones (e.g. the bearings No. *59* and *60*).

Figure 5.31 shows the spectra of a new and a used taper roller bearing, still acceptable according to the visual inspection and the spectrum of an already unusable tapered roller bearing.

It can be established unequivocally that the intensity of the vibration depends on the condition of the outer race, the rollers and the cage. A good spectrum could never be assigned to a badly scaled race surface, but there have been cases where the

Table 5.3

Effective acceleration values of the vibration of used and new bearings of the type 32.217 (Band I: 50...300 Hz; Band II: 300...1800 Hz; Band III: 1800...10 000 Hz)

Mark	Effective vibration acceleration $m\,s^{-2}$			Mark	Effective vibration acceleration $m\,s^{-2}$		
	Band I	Band II	Band III		Band I	Band II	Band III
51	0.2	1.2	5	76	0.3	1	3.2
52	Overflows the measurement range			77	0.2	1.8	3.3
53	0.4	1.2	2.6	78	0.1	0.9	4.5
54	0.1	0.8	2.2	79	0.1	0.8	4.5
55	0.3	1.8	3.5	80	1.2	1.4	7.5
56	0.3	1.8	3.8	81	0	0.5	2.3
57	0.4	1	7.5	82	0.1	1.5	4
58	0.1	1.1	2.5	83	0.3	2	4.5
59	0	0.3	1	84	0.1	1.0	3
60	0	0.5	2	85	Overflows the measurement range		
61	0.2	0.9	4	86	1.8	3	6
62	0	0.5	5	87	0.1	0.9	3.5
63	0	1.0	10.0	88	0.1	0.8	4.3
64	0.2	0.8	2.2	89	0.1	0.9	3.2
65	0.2	1	3.6	90	0	1	2.5
66	0.1	0.8	5	91	1.0	2.0	10.0
67	1	1.2	5	92	0.8	1.9	9
68	1	3.5	9	93	0	0.7	3
69	1	1	5.5	94	0.1	1	2.4
70	0.1	0.8	4	95	0.1	0.8	2.5
71	0.3	1.1	4	96	0.1	0.8	3.2
72	0.1	0.9	3.6	97	11	3	15
73	0.1	1	3.5	98	Inner ring broken		
74	0.2	0.6	3.5	99	0.2	0.9	3.6
75	0.1	1.3	4	100	0.1	2	4.3
51μ	0.1	1.6	2	54u	0	1	2.5
52μ	0	1	2.5	55u	0	0.8	2
53μ	0.3	1.5	2.1				

race surface seemed to be flawless yet the vibration was unfavourable. In such cases any distortion of the form or any waviness would always be revealed by precise examinations.

Figure 5.32 shows the race of the outer ring of bearing *59*. The surface is sound and the vibration spectrum is better than those of new bearings. Also, the rolling bodies of the bearing are in very good condition (Fig. 5.33).

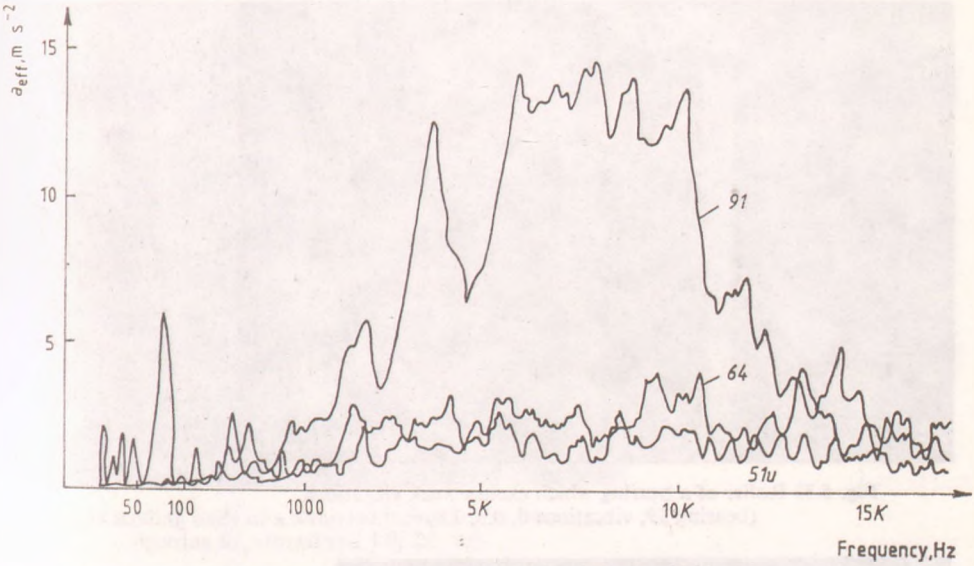


Fig. 5.31 Vibration diagrams of type 32.217 bearings
(*51u* new, *64* still applicable, *91* non-applicable bearings)

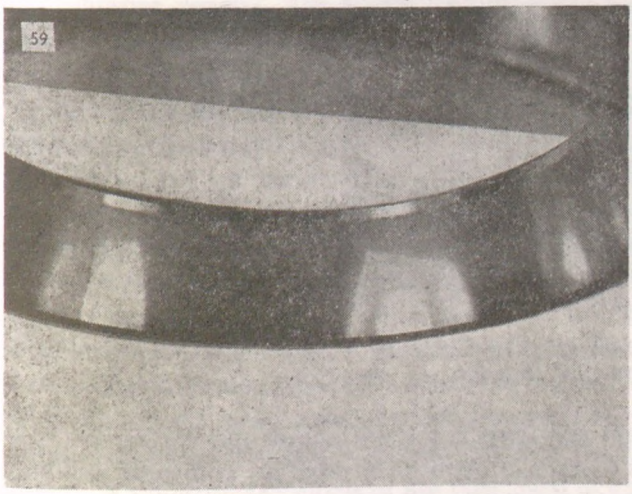


Fig. 5.32 Outer ring of a bearing which excites weak vibrations
(bearing *59*; vibrations: 0, 0.3, 1.0)

Figure 5.34 shows the deteriorated outer ring of bearing *86*, which was scrapped on the basis of the spectrum, and Fig. 5.35 shows the ruined rolling body of bearing *91*.

Figure 5.36 shows the influence of a 0.4 mm wide and 0.1 mm deep artificial defect etched with an electric pencil into the outer ring on the spectrum of a used tapered roller bearing.

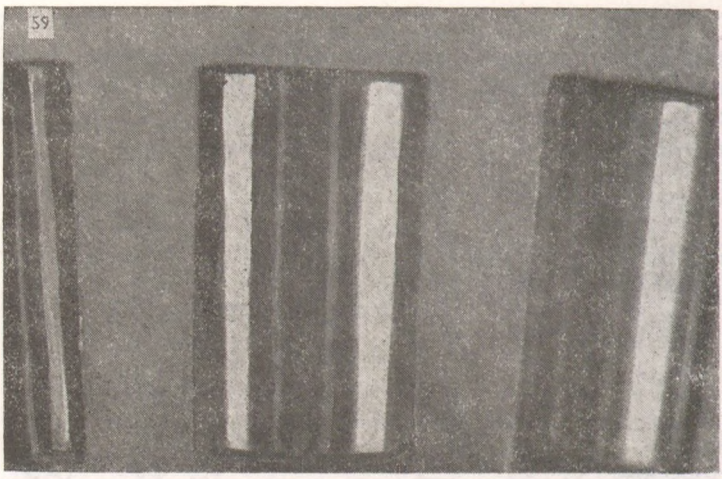


Fig. 5.33 Roller of a bearing which excites weak vibrations (bearing *59*, vibrations 0, 0.3, 1.0)

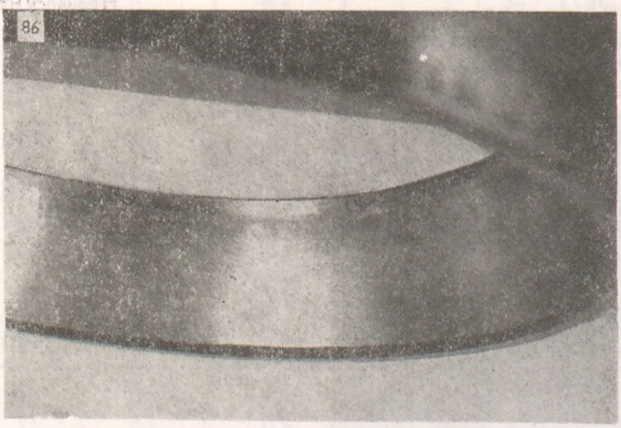


Fig. 5.34 Outer ring of a scrapped tapered roller bearing (bearing *86*; vibrations: 1.8, 3.0, 6.0)

An important preparatory operation is the washing of the bearing. In Figure 5.37, the spectra of bearings washed in petrol then dried are compared with those washed in gas oil and measured immediately and also with those of grease-lubricated bearings. The bearing that was washed in petrol and then dried perfectly gave rise to relatively strong vibrations, the spectrum of the bearing washed in gas oil reveals the lubricating, vibration-damping effect of the gas oil, while the vibrations of the grease-lubricated bearing are considerably damped. Anyway, such unequivocal results cannot be expected from the testing of built-in bearings which operate in a lubricated state and are subjected also to the disturbing vibrations of the other component parts.

Finally, the effect of the impurities (e.g. dust) found in the grease is presented in Fig. 5.38.

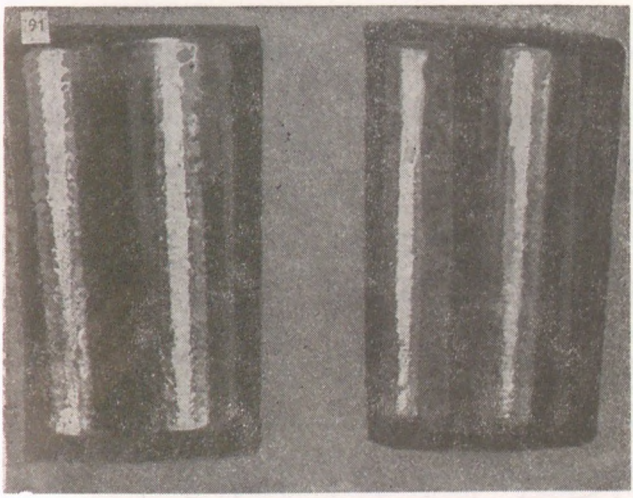


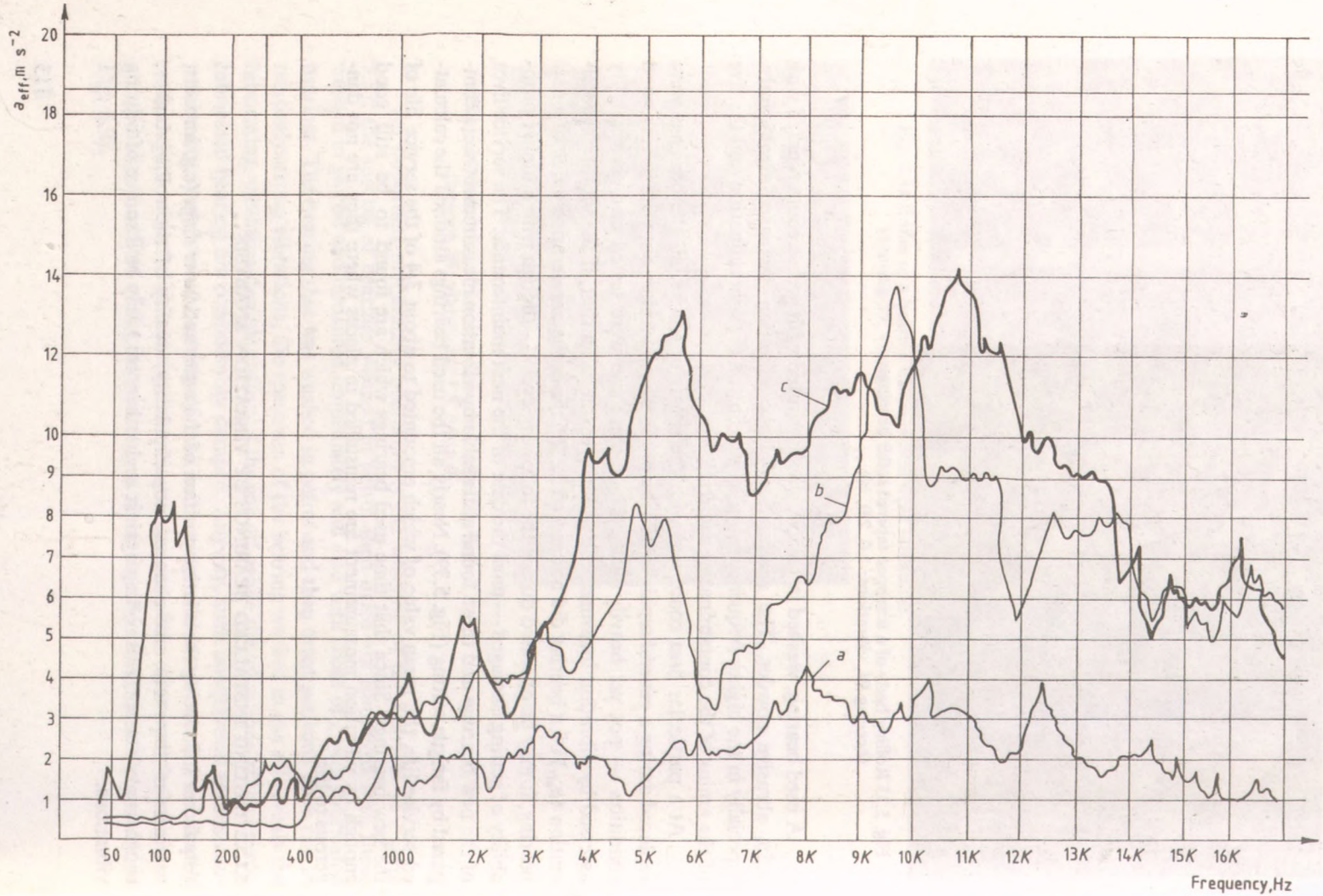
Fig. 5.35 Rolling body of a scrapped tapered roller bearing (bearing *91*, vibrations: 1.0, 2.0, 10)

A used bearing, washed in gas oil, was lubricated with grease contaminated with 0.5% abrasive powder. The acceleration of the vibration increased significantly, especially in the higher frequency range, depending on the contamination and the size of the grains of the contaminating material,

At a particular firm concerned with the repair of trucks, all the bearings were replaced during a general repair, but many of the bearings taken out were in very good condition — not yet heavily worn. Therefore these were tested and their quality assessed by vibration measurement with the assumption that, if the vibration acceleration (a_{eff}) of a bearing did not exceed 2...2.5 times the acceleration value of a new bearing, it may be operated during the following cycle — though with a higher probability of getting damaged — until the date of the next maintenance. The service lives of the new bearings and those found still usable by vibration measurement were compared by fatigue testing (Fig. 5.39). Nearly all the used bearings attained the calculated service life, the mean value of which amounted to about 3/4 of the service life of the new bearings. Since that time used bearings which are found to be still good enough by vibration measurement are reinstalled in places where they are not dangerous to life.

5.1.3.2 TESTING INSTALLED BEARINGS BY VIBRATION ANALYSIS

Any factor influencing the relative position of the outer and inner rings (e.g. impacts, waviness of the races, size changes, non-circularity, ovality of the rolling bodies, roughness of the surfaces rolling against each other, etc.) may be a source of bearing vibrations.


a_{eff}, m s^{-2}
20
18
16
14
12
10
9
8
7
6
5
4
3
2
1
c
b
a
50
100
200
400
1000
2K
3K
4K
5K
6K
7K
8K
9K
10K
11K
12K
13K
14K
15K
16K
Frequency, Hz

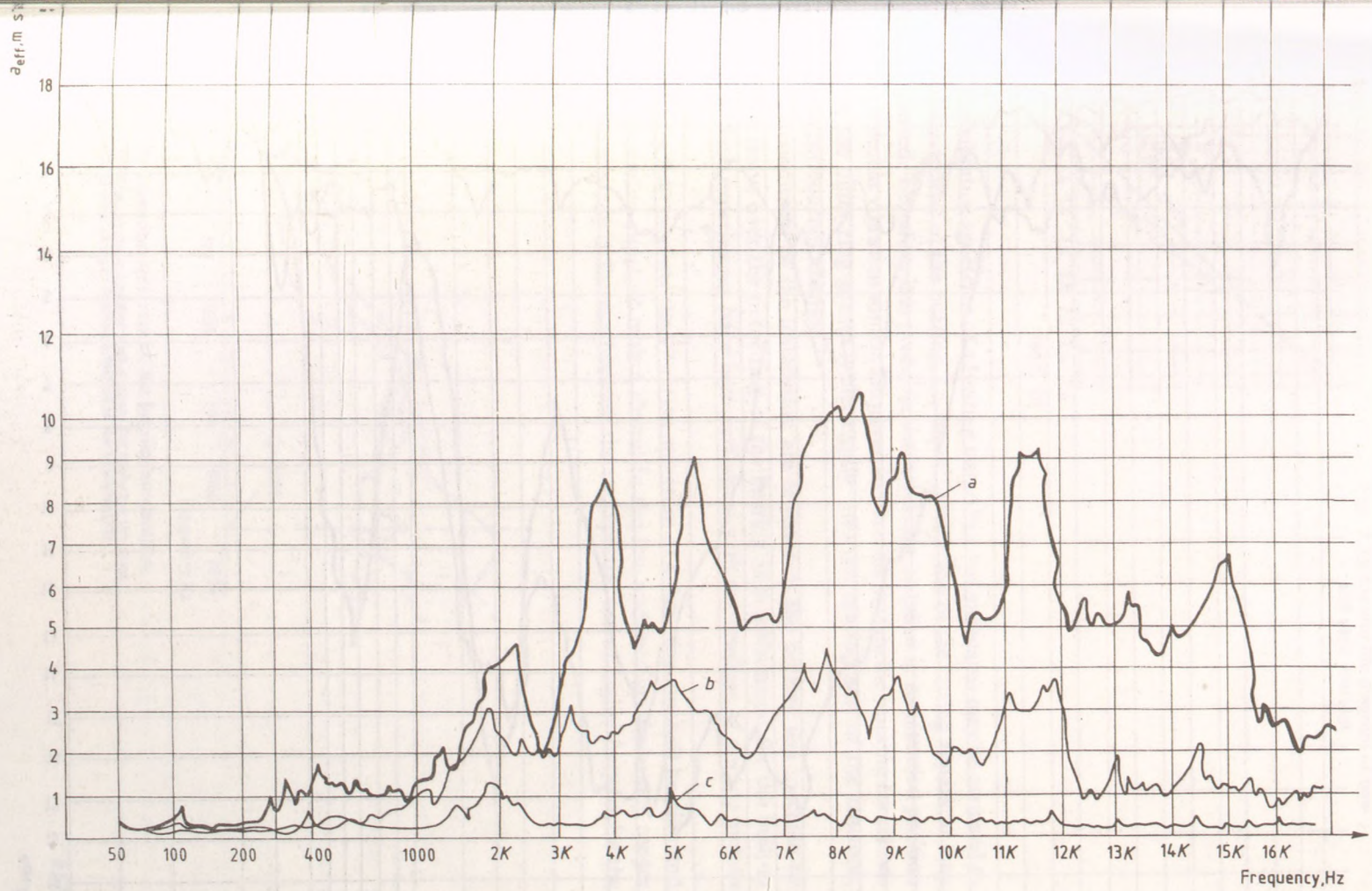
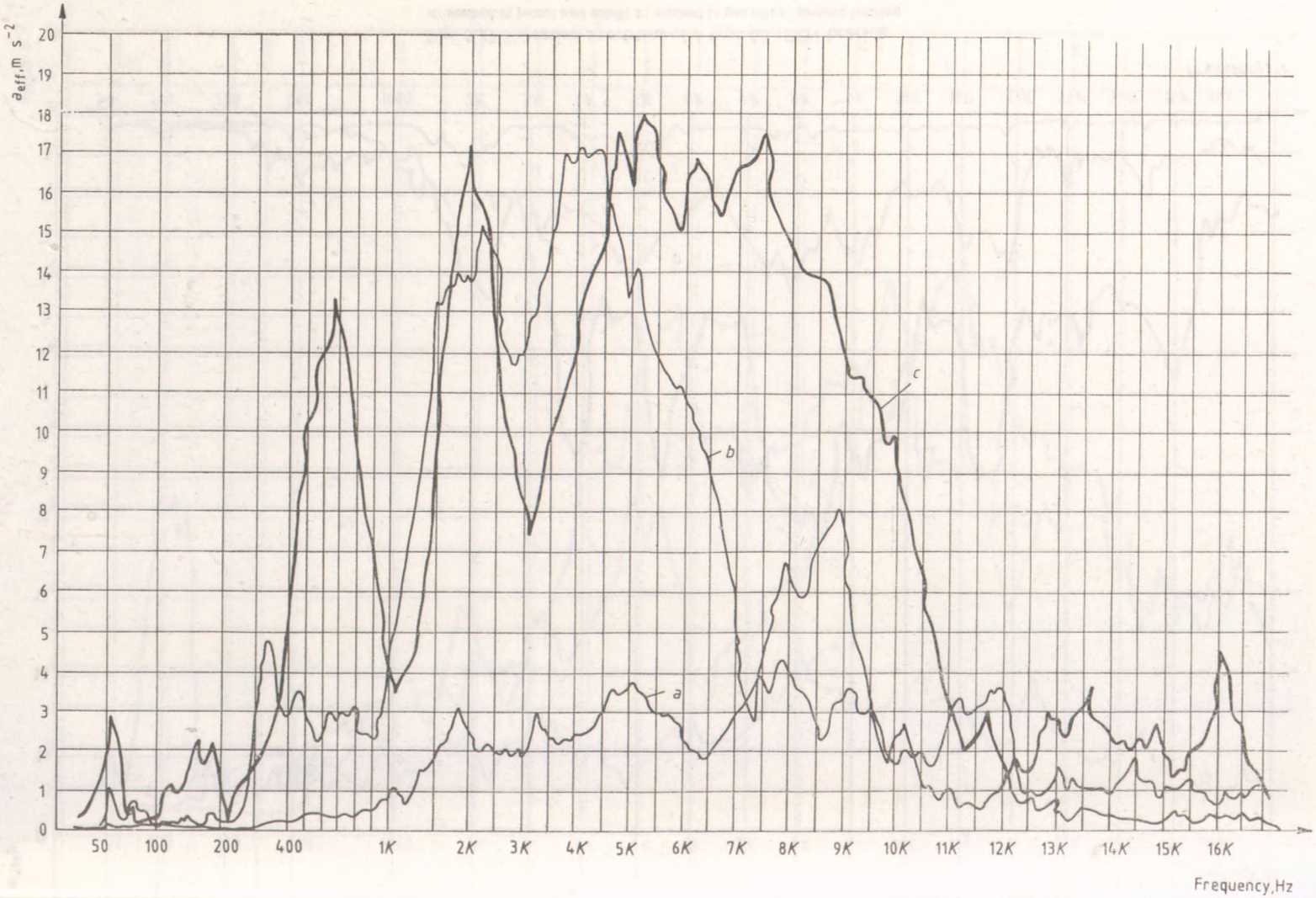


Fig. 5.37 Vibration spectrum of a type 62.12ZN bearing

a: washed in petrol and dried; *b*: washed in gas oil; *c*: greased bearing


a_{eff}, m s^{-2}
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
50
100
200
400
1K
2K
3K
4K
5K
6K
7K
8K
9K
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11K
12K
13K
14K
15K
16K
Frequency, Hz
a
b
c

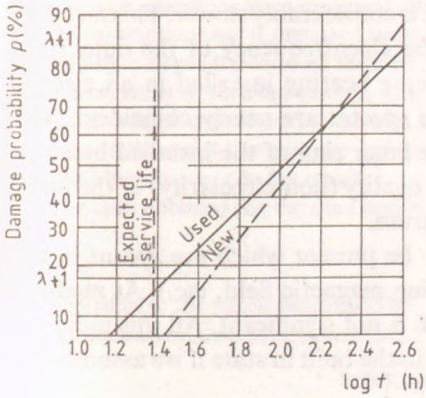


Fig. 5.39 Service life diagram of new and used bearings

The vibration spectrum of a bearing tested in a test apparatus may be assumed to be a characteristic of the bearing, i.e. the self-oscillation of the testing apparatus may be neglected. Therefore it is probable that a bearing installed in its intended position acts as a similar vibration source. However, in the built-in state the working conditions change; the outer ring generally fits tightly into the bearing nest of the machine, so its eigenfrequency changes.

The conditions of the installation are imitated in the bearing test machine by pulling a sleeve over the outer ring of the bearing as a replacement for the built-in masses. Such experiments have been performed with the bearings of electric motors [75].

Figure 5.40 presents the vibration spectrum of a free (dismounted) bearing and that of a bearing supplied with a sleeve. In the low and medium frequency ranges there is hardly any difference between the two spectra, but at higher frequencies the

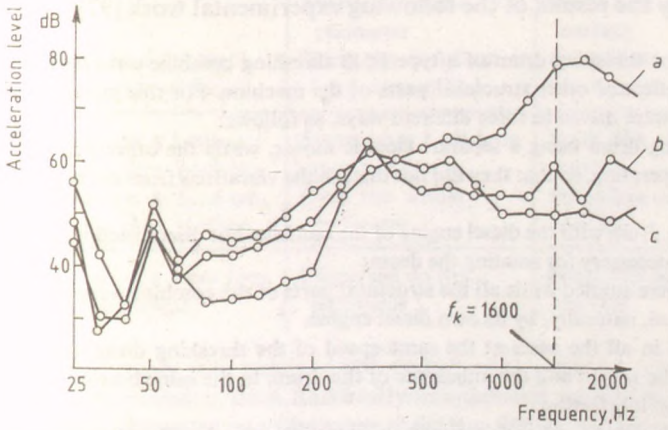


Fig. 5.40 Vibration spectrum of the bearing acceleration

a: free bearing; *b*: bearing supplied with a sleeve for replacing the mass of the bearing case; *c*: bearing installed in the machine [75]

acceleration of the bearing supplied with a sleeve is considerably slower. This fact may be observed well at around 1600 Hz, which is the eigenfrequency of the outer ring.

Curve *c* is the vibration spectrum of the same bearing installed in an electric motor. In the low-frequency range the vibration spectra are nearly coincident. The cause of the deviations is the deformation of the inner ring of the installed bearing; fitting the bearing to the motor shaft changes the ovality (non-circularity) of the inner ring, causing changes also in the excitation spectrum.

At the installation site exciting sources may be present which are absent in the test machine (e.g. an unbalanced rotor, a pulsating magnetic field, etc.). At medium and higher frequencies the deviation from curve *b* is not significant. All this indicates that a similar vibration spectrum is obtained also in the built in state if we assume that other disturbing vibration sources are absent.

The shaft of even the simplest machine is supported at least by two bearings. The vibrations of the second bearing — though they are weaker — are superimposed on the vibration of the bearing under test. Also disturbing vibrations appear, caused by imbalance, the belt drive, and numerous other sources, which aggravate the selection of those vibration signals which are valuable for diagnostic purposes from among the set of all the vibrations.

When testing installed bearings, their rotation and suitable radial and axial loading must be ensured.

Also the vibrations of the bearings must be insulated, as much as possible, from the vibrations of the environment, the so-called background noise. In the case of complex machines this may be solved by rotating only the shaft to be tested, while all other parts of the machine are in the resting state. The shaft involved, or bearing, may be driven by its own drive gear or power source if there is no other alternative, but in this case the signal to noise ratio will be worse.

The possibility of using conventional vibration analysis of bearings for diagnostic purposes is illustrated by the results of the following experimental work [97].

Two rolling bearings of the threshing drum of a type E512 threshing combine were tested to obtain data on the disturbing effect of other structural parts of the machine. For this purpose the bearings of the threshing drum were driven in three different ways, as follows:

1. by rotating the threshing drum using a separate electric motor, while the other structura units of the combine were not operating, so that they did not disturb the vibrations from the bearings of the drum;

2. by driving the threshing drum with the diesel engine of the combine but disconnecting all the structural units which were not necessary for rotating the drum;

3. finally, the vibrations were studied while all the structural parts of the combine were operating. The combine was now driven, naturally, by its own diesel engine.

The tests were performed in all the cases at the same speed of the threshing drum ($n=600$ revs/min), loaded similarly by the weight and the imbalance of the drum, in the same bearing cages, and under identical conditions of lubrication.

It was possible to replace the left-hand bearing — with respect to the running direction — of the threshing drum with less effort (adapter sleeve bearing) than the right-hand bearing, so the former was chosen for the test. Various quality bearings of different wear grades were installed in this posi-

tion and their vibrations were measured. On the other side a flawless counterbearing was installed with the assumption that it remained undamaged during the measurements and that the vibrations produced by it stayed the same all the time. Bearings of the type 6309 were used — some were completely new, some of medium quality, with local point-like pits in their outer and inner rings and rolling elements, and there were some with pot-holes over the whole race along the whole perimeter. The sizes of the point-like pits did not exceed 1 mm. These bearings were taken from machines which had been brought in for repair.

The data obtained for the used bearings installed during the tests are contained in Table 5.4.

Table 5.4

Characteristics of the tested bearings [97]

Serial number	Splitting in the			Radial play, μm	Symbol
	Outer ring	Inner ring	Rolling body		
1	A single point-like defect	—	—	30	*A*0
2	One defect on a 1.2 cm long race surface	—	—	30	*A*2
3	—	3 point-like defects	—	95	*B*0
4		One on a 1 cm long race surface	—	90	*B*1
5	—	One on a 1.5 cm long race surface	—	40	*B*2
6		Over the whole perimeter	Point-like on the surface	75	*C*1
7	—	Dispersed on the perimeter	Point-like on the surface	50	*C*2
8	Two on a 0.5 cm long race surface	Three on a 0.5 cm long race surface	Point-like on the surface	75	*D*1
9	Seven on a 1 cm long race surface	Seven on a 1 cm long race surface	Point-like on the surface	40	*D*2
10	Four on a 5...6 cm long race surface	Over the whole perimeter	Point-like on the surface	90	*D*3
11	Seven on a 1 cm long track surface	Dispersed on the perimeter	Point-like on the surface	45	*D*4

The vibrations were measured with the help of an apparatus produced in the GDR. Type KD 35a acceleration sensors (Metra, Radebeul) were attached by magnets to the bearing. The signals were amplified and integrated by a vibrometer of the type SM231, and the velocity signals were evaluated. The quadratic mean value of the whole vibration spectrum was measured and in some cases also third bandwitdh analyses were performed.

During the tests a microphone was placed close to the bearing housing, at a distance of 10 cm, and the sound vibrations were also measured.

In the case of the counterbearing, for structural reasons, the microphone could only be placed at a distance of 50 cm from it. The sounds characterizing the wear of a bearing cannot be sensed at such a distance because of the disturbing sounds arising from other parts of the machine and those produced by the airflow.

The test results obtained in the three modes of operation are presented in Fig. 5.41. The diagrams show the voltage values read off the instrument versus the radial play of the bearing; these values are proportional to the quadratic mean of the vibration velocity obtained by the integration of the vibration acceleration. If we consider the worn but not pitted bearings (continuous line), we can see that, when using an independent electric motor for driving the drum (Fig. 5.41(a)), the vibration intensity hardly increased at all with the radial play, while in the case of pitted bearings (*A*0–*D*4 in Table 5.4) important increases in the vibration were observed.

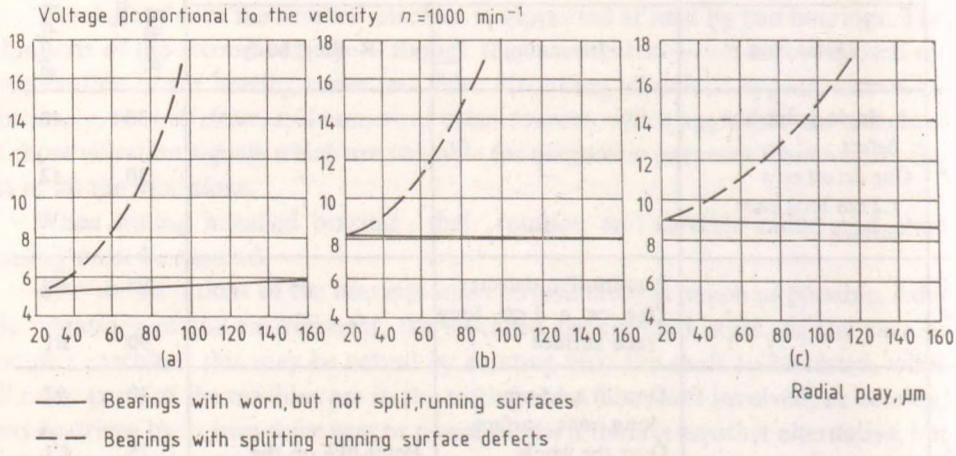


Fig. 5.41 Vibration velocity values versus the radial impact
(a) independent drive; (b) internal drive; (c) internal drive with the operation of the whole machine [97]

The deviation of the vibration intensities is especially great if filters that pass high frequencies are used (e.g. even a 20-fold growth of the vibration intensity was observed, depending on the pitting). A 2 to 3-fold increase of the vibration intensity was experienced even when splits were only present on the surface of the inner ring.

In the second mode of operation (Fig. 5.41(b)) the drum was driven by the diesel engine of the thresher combine. The speed of the diesel engine was selected on the basis of the following considerations:

— the vibrations of the diesel engine should be presented as much as possible from penetrating into the tested bearing; the possible resonance of the machine units participating in the drive should be avoided, i.e. the excited vibrations should be as weak as possible;

— the speed of the bearing to be tested should be suitable for the evaluation of the vibrations appearing;

— the tested drum and its bearings should be driven at a constant, steady speed by the diesel engine with the smallest possible irregularity.

On the basis of the above considerations, 1000 revs/min was adopted for the speed of the engine.

Fig. 5.41(b) shows the test results. The vibration increases slightly because of the disturbing vibrations of the diesel engine. The bearings with worn, but relatively good, race surface cannot be distinguished from the new bearings, so the vibration did not depend on the radial play of the bearing any more. A single split in the outer ring, *A*0, could hardly increase the total vibration at all. A 2...3-fold increase of the vibration was only caused by very extensive splitting. It is established that, under such measuring conditions, only the initial splitting of the bearings could be observed, because of the disturbing effect of the diesel engine.

In the third mode of operation (Fig. 5.41(c)) all the structural parts of the combine were functioning. The possibility of testing diminished further as the disturbing vibrations were so strong that neither defects of the outer running ring nor even slight splits in the inner ring (e.g. of the grade *B*0) were able to cause a significant increase in the total vibration intensity. Such an increase could be observed only when dispersed splitting appeared along the whole perimeter (bearings *C*1, *D*3, *D*4).

A better result could not be attained even by third bandwidth analysis of the so-called airsound vibrations received by the microphone, because of the disturbing soundwaves.

Figure 5.42 shows how the vibration intensity varied with the introduction of the disturbing vibrations produced by switching on the individual parts of the combine in turn.

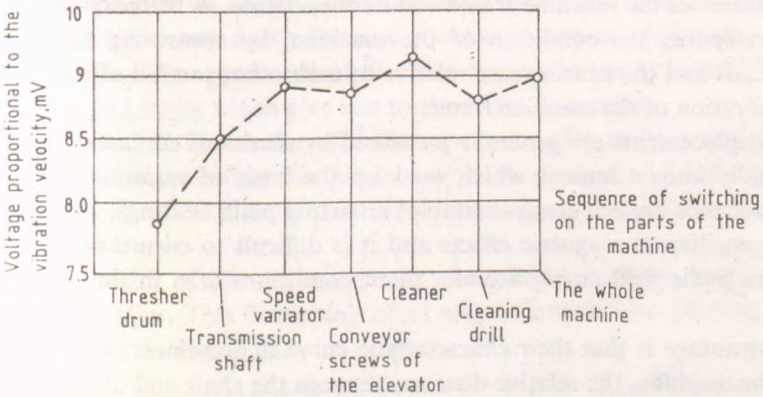


Fig. 5.42 Changes in the vibration intensity of an agricultural machine when its various structural parts are switched on [97]

This detailed description of the test illustrates the difficulties of testing a complex machine consisting of several structural units, as each unit is a source of vibration which usually masks the vibration from the machine part whose technical condition is being investigated. To enable the investigation to be carried out — as the example shows — the structural units which produce disturbing vibrations must be switched off or they must be driven independently. Machines driven by internal combustion engines should be tested by disconnecting their engine and then driving them with an independent electric motor. If the drive from the electric motor is transferred by means of a *V*-belt, the latter may cause considerable vibrations. This should be avoided by driving the unit to be tested with the help of an "infinite" flat belt.

5.2 TESTING PLAIN BEARINGS

5.2.1 TESTING PLAIN BEARINGS BY FREQUENCY ANALYSIS

The vibrations observed from plain bearings may be caused by imbalance of the rotating part, bending of the shaft, adjustment faults, or defects of the bearing and coupling. Often a self-excited oscillation caused by the lubricant film may also be observed. The shock waves produced by plain bearings are not so sharp as those in the case of rolling bearings, because a greater part of the vibration energy is absorbed by the damping effect of the oil film, especially the high-frequency vibrations. The magnitude of the absolute vibration (e.g. the absolute acceleration) depends on the ratio of the mass of the bearing case and the foundation of the machine relative to the mass of the rotating parts. A small rotating mass is unable to induce a strong vibration in the large masses of the machine frame and its foundation. A better result can be obtained, when testing the condition of the machine, by measuring the displacement of the shaft and the bearing case relative to each other instead of measuring the absolute vibration of the machine frame.

The relative displacements are generally measured by means of contactless converters. The generally known sensors which work on the basis of variations in the inductance or capacitance are not always suitable for testing plain bearings, as inductive receivers are sensitive to magnetic effects and it is difficult to ensure an undisturbed constant magnetic field strength under plant conditions (e.g. in the case of generators).

Another disadvantage is that their characteristic curve is non-linear, as, during the operation of the machine, the relative distance between the shaft and the bearing housing may vary considerably, depending on the load, as well as on processing faults and the amount of deformation. This variation in the distance may disturb the sensor, making it unable to detect the vibration signals of the shaft against this noise signal.

Sensors which function on the basis of variations in capacitance are sensitive to the magnitude of the dielectric constant of the medium (e.g. air) between the shaft and the sensor. Variations in the dielectric constant may cause a stronger disturbing signal than the vibration between the shaft and the bearing case.

When testing plain bearings the sensors are mounted according to the viewpoints described above. The vibrations are frequently measured in two mutually perpendicular directions, as in this way the so-called shaft-track curves, the Lissajous curves can be plotted. The sensed signal is evaluated by means of narrow-band vibration analysis for detecting the faults on the basis of the individual frequencies. The most frequently observed components of the vibration of plain bearings are as follows [51]:

f_0 — fundamental vibration, equalling the number of revolutions of the machine, caused generally by imbalance or the incorrect adjustment of the bearings. The

intensity of this vibration may be reduced by careful balancing and by ensuring that the bearing shells and the rotating part are coaxial.

$f_1=(0.4...0.6)f_0$: the self-excited vibrations raised by the oil film appear at the frequency corresponding to half the number of revolutions. In this the rotating shaft pivot does not slide uniformly on the oil film but keeps jumping on the oil pad. These vibrations exert a considerable load on the shaft, especially when the mass of the rotating part is large. These vibrations can be influenced by varying the viscosity or the temperature of the oil. The development of the self-excited vibrations is greatly influenced by the form and the arrangement of the bearing shells (e.g. the curvature of the supporting surface, the form, position, and size of the oil bags).

The experienced designer can prevent the appearance of hydraulic vibrations by the careful design of these details.

$f_2=2f_0$. The second harmonic of the basic vibration appears if the bearings or couplings are incorrectly adjusted, or in the case of loose connections, or when the rotating part or its shaft is not circularly symmetric (the second order moments acting on both main axes of the cross-sectional area are different).

$f_3=3f_0$. The appearance of the third harmonic can be caused by the same irregularities and faults which give rise to the second harmonic.

Figure 5.43 shows the vibration spectrum of a turbine rotating at a speed of 9500 revs/min, showing the fundamental vibration, f_0, of 158 Hz, the self-excited vibration, f_1, of 79 Hz, the second harmonic, f_2, of 316 Hz and the third harmonic, f_3, of 474 Hz.

The width of the peaks depends also on the damping gradient of the applied band-pass filter. This disturbing effect may be avoided by plotting the linear spectrum shown in Fig. 5.44.

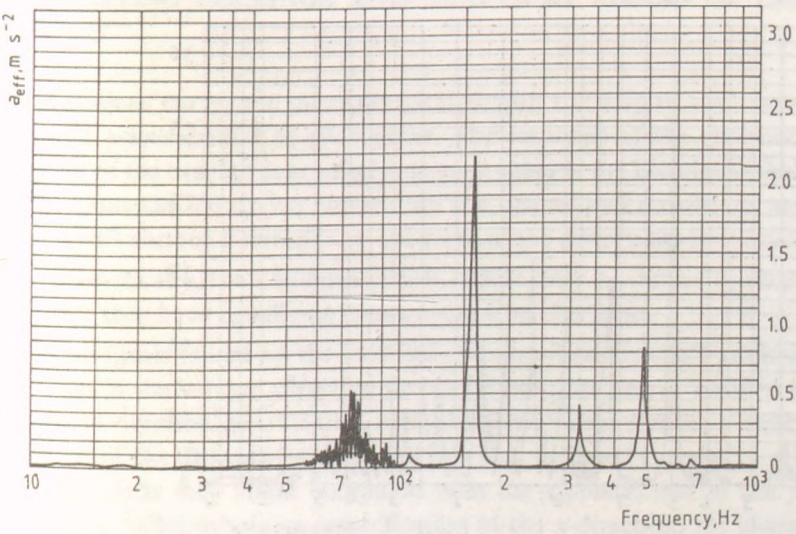


Fig. 5.43 Vibration spectrum of a turbine rotating at 9500 revs/min [52]

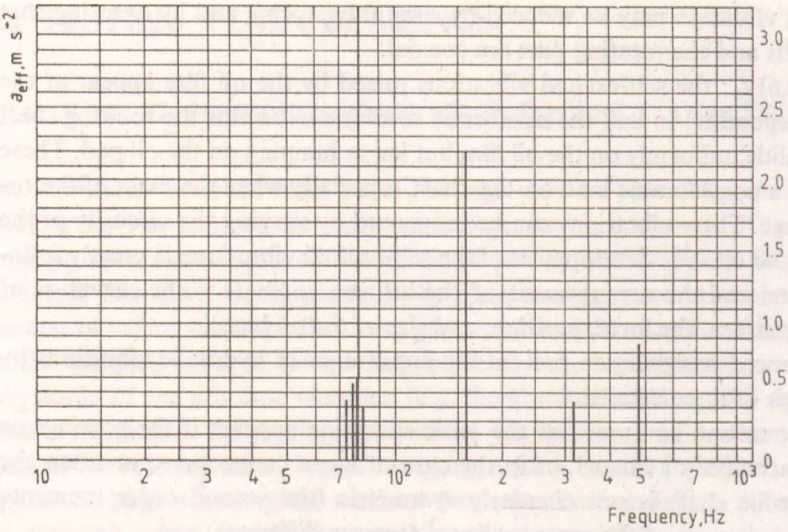


Fig. 5.44 As Fig. 5.43, but with a linear spectrum [52]

Contactless sensors supply signals which correspond to the variation in the distance between the bearing and the shaft. This signal is influenced by the deformation of the shaft, processing faults, surface roughness, the radial impact, etc. The disturbing signals due to these factors may be stronger than the measurable vibration signal. Therefore, if the disturbing (runout) signals are important, it is advisable to use an instrument which compensates for the disturbing signals. The compensation is realized in correspondence with the speed of the rotating shaft or its angular displacement (phase position) taking into account the actual defects — geometrical, surface, etc.

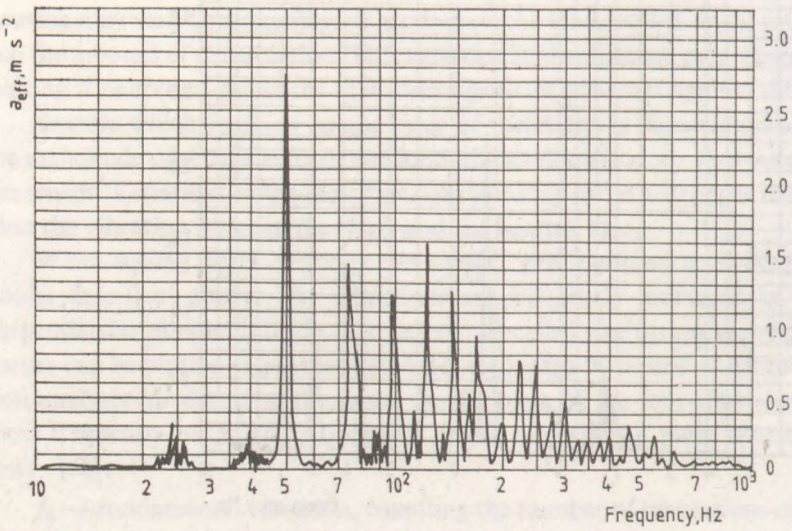


Fig. 5.45 Vibration spectrum of a turbine rotating at 3000 revs/min [52]

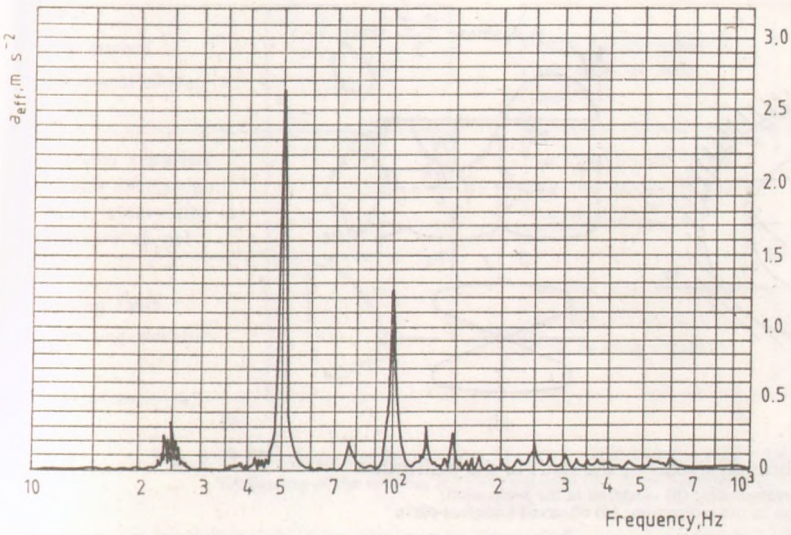


Fig. 5.46 As Fig. 5.45, but with compensation of disturbances [52]

Figure 5.45 shows the vibration spectrum of a turbine rotating at a speed of 3000 revs/min Figure 5.46 presents the vibration spectrum of the same turbine with compensation of the disturbing effects. The first spectrum is complicated and its evaluation is difficult; the second one is clearly understood permitting the easy recognition of the characteristic components with the frequencies f_0, f_1, f_2, f_3.

5.2.2 TESTING BEARINGS AND SHAFTS BY MEANS OF LISSAJOUS CURVES

The Lissajous curves are generally plotted with the help of two contactless sensors, arranged perpendicular to each other. The resultant of the vibrations of the shaft and that of the bearing cage relative to each other is the Lissajous (shaft-track) curve. Conclusions can be drawn concerning the operational conditions and any damage to the shaft and the bearing from the form of the distortions in the curve.

Theoretically, the Lissajous curves should have a circular or elliptical form, but in reality they have usually an interwoven, irregular form.

The signals issued by the installed sensors when the shaft performs a harmonic vibration in the vertical direction are as follows: the sensor fixed in the y direction senses the vibration $y = A \sin \omega t$, while the sensor found in the x-direction senses the variation of the distance between the bearings shell and the shaft in the x-direction. Though this is very slight compared with the displacement in the y-direction, on applying a sufficiently large magnification in the x-direction, the characteristic curve shown in Fig. 5.47 appears on the oscilloscope. It is obvious that the vibration fre-

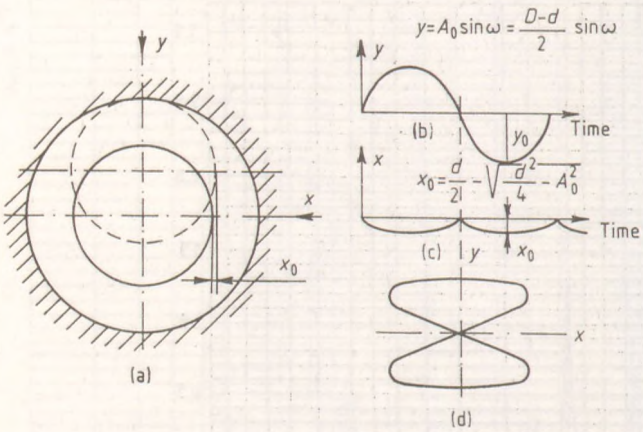


Fig. 5.47 Vibration signals of the shaft in the y-direction
(a) arrangement; (b) vibration in the y-direction;
(c) vibration in the x-direction; (d) observed Lissajous-curve

quency in the x-direction is twice as high as the frequency of the vibration in the y-direction.

If the shaft pivot performs a circular movement in the bearing, then the deflections in the x- and y-directions deviate from each other by a phase angle of $\frac{\pi}{2}$ and we obtain a circle on the oscilloscope.

The shapes of the curves were studied in the laboratory by performing experiments on a special test bench on which faults could be artificially created [106]. Examples of the curves obtained are shown in Fig. 5.48.

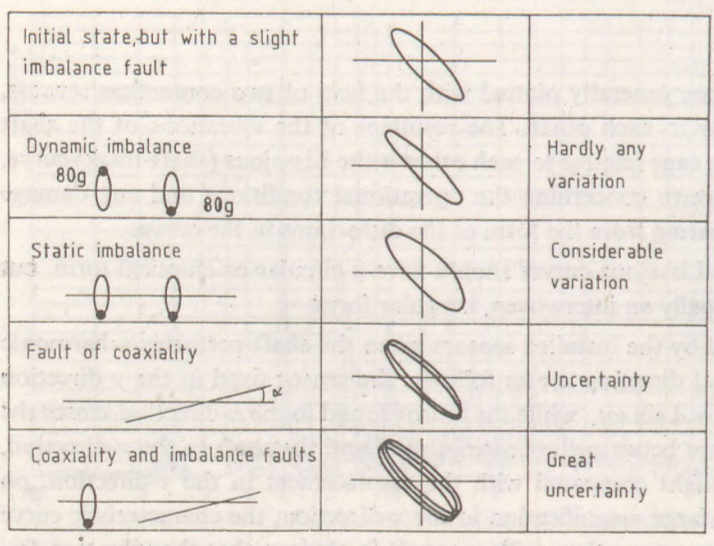


Fig. 5.48 Lissajous curves corresponding to various faults [106]

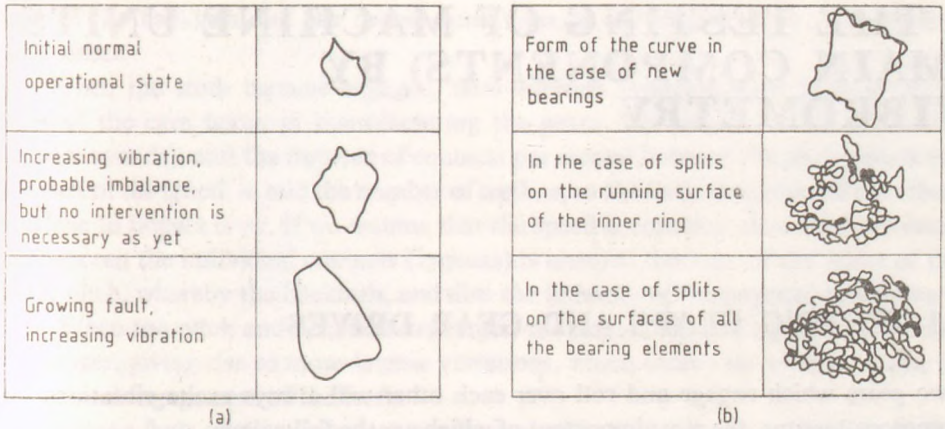


Fig. 5.49 Lissajous curves obtained for the plain bearings of a turbine
(a) bearing of the turbine; (b) shaft supported by roller bearings [40]

Figure 5.49(a) shows the Lissajous curves obtained for the plain bearing of a turbine and Fig. 5.49(b) the curves obtained for rolling bearings [110]. The Lissajous curves obtained by measurement are very different from the regular circle or ellipse mentioned before. Therefore efforts to find the cause of a fault of a rotating machine on the basis of the distortion are frequently unsuccessful, mainly because of the deformed shape of the rotating shaft, its surface quality and the inhomogeneity of its material, all having disturbing effects on the sensors. Therefore the current method of plotting these curves involves compensation of the disturbance signal [52]. Such curves are shown in Fig. 5.50.

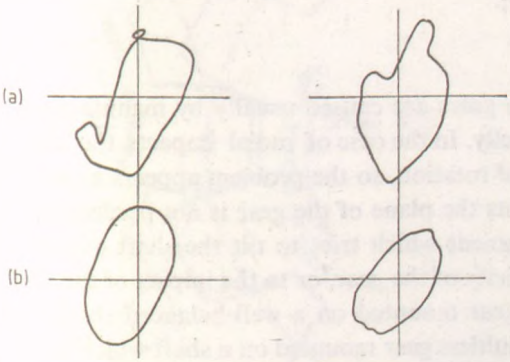


Fig. 5.50 Lissajous curves with disturbance-signal compensation

The observation of Lissajous curves is regarded only as a complementary method. If an ellipse is obtained, the compensation must be checked. If the image is irregular and unstable, a complex fault is probably present. More information may be obtained in this case by means of a detailed vibration analysis.

6 THE TESTING OF MACHINE UNITS (MAIN COMPONENTS) BY VIBROMETRY

6.1 TESTING GEARS AND GEAR DRIVES

Two gears which engage and roll over each other will always excite vibrations for numerous reasons, the most important of which are the following:

— the radial impact of the gears, owing to their eccentricity;
— the axial impact of the gears;
— a fault in the tooth pitch;
— the backlash of the teeth;
— the deviation of the tooth profile from the theoretical profile;
— the rolling and sliding conditions of the tooth profile, which depend also on the preceding factors;
— the surface quality of the tooth profile: its wear, deformation, cracks, and flakes developing on it during its operation;
— faults in the shafts carrying the gears.

The radial and axial impacts of the gears are caused usually by manufacturing faults, or by their being mounted incorrectly. In the case of radial impacts the centre of mass of the gears is outside the axis of rotation, so the problem appears as one of balance, while in the case of axial impacts the plane of the gear is not perpendicular to the axis of rotation, leading to a moment which tries to tilt the shaft out of its position. This may be due to the eccentricity of the gear, or to the impact of the shaft caused by its imbalance. An eccentric gear mounted on a well-balanced shaft may give rise to the same phenomenon as a faultless gear mounted on a shaft which excites vibration. In this case it is difficult to distinguish between the individual faults and to identify them; the only fact that can be established is that there is some trouble with the shaft outfit, the consequence of which is the transmission of vibrations from the shaft to the bearing housing or the whole gear box. The vibrations caused by radial and axial impacts can be identified, as their frequency, or its upper harmonics, correspond to the number of revolutions of the gear or the shaft. Naturally these frequencies may also originate — as mentioned above — in the eccentricity and imbalance of the gearshaft, or a nonuniform deformation (shaft deflection) caused by a load. But whatever is the cause of the radial or axial impact, it influences the magni-

tude of the backlash and the rolling conditions of the teeth as they mesh against each other.

When the teeth become engaged, their surfaces impact against each other, in spite of the care taken in manufacturing the gears. The impacts cause vibrations (impact sounds), and the number of contacts per second between the teeth equals the product of the speed, n, and the number of teeth, z, so the basic frequency of the vibration due to impact is nz. If we assume that the speed is constant, then the time interval between the individual contacts (impacts) is unequal, because of the faults of the tooth pitch, whereby the backlash, and also the intensity of the impacts, will be variable. When the pitch and backlash are larger, the impact of teeth against each other is stronger, giving rise to more intense vibrations, which exceed the vibrations due to the normal engagement of the teeth.

Hence a fault of the tooth pitch can be proved in principle, but its demonstration as a manufacturing fault is not our task here. We wish to utilize the above principle rather for detecting other damages that occur during the operation — e.g. cracked teeth. If a pair of gears is run under load, the teeth are deformed by the effect of the forces acting on them. The teeth of the driving gear are bent backwards, those of the driven gear forwards, when they mesh with each other (Fig. 6.1).

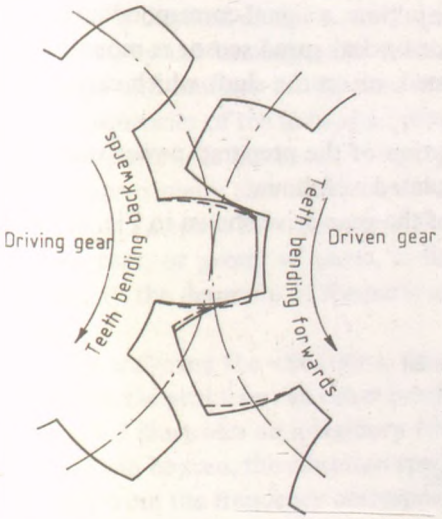


Fig. 6.1 Deformation of the meshing gears

A tooth which is bent backwards moves closer to the following tooth, which is loaded differently, so the pitch of the driving gear is decreased, while the pitch of the driven gear is increased. The following pair of teeth are engaged accordingly and excite impact vibrations at a rate corresponding to this situation. If the tooth pitch and the backlash are constant, then all the teeth will be deformed uniformly and the magnitude and the period of the impact waves raised when the teeth mesh will be constant. But if one of the teeth is cracked, its second order moment is lower and its

deformation will be larger. The larger deformation causes a larger pitch and an impact wave of a different magnitude. Hence a cracked faulty tooth can be identified by vibrometry.

If the rolling and sliding conditions of the tooth profile vary, the related vibration varies along with it. The lowered surface quality of the tooth profile and the contamination of the lubricant increases the high-frequency components (>15 kHz) of the vibration. Splits and flakes appearing on the tooth profile lead to harder impacts between the contacting teeth, permitting recognition of the faults.

For testing the drive gear it may be driven either by means of an independent motor or by actuating the whole mechanical device. The advantage of employing an independent drive is that its vibrations will not disturb the vibrations to be measured as much as, for example, a built-in diesel engine. An important requirement is that the drive should have a uniform, constant speed; this can be ensured more easily by the use of an independent drive, even though in this case the test must be carried out on a test bench. The same applies to the load testing of the power transmission gears of the power transmission system. The load is usually applied on test benches equipped with brake-dynamos or airblades. Closed-chain test benches are generally unsuitable for vibration tests because of the presence of disturbing effects.

The testing speed should be as close as possible to the working speed.

For studying the course of the impact wave in time, a signal corresponding to the speed is also necessary. Therefore a magnetic or optical speed sensor is mounted on the gear drive, generally on the shaft to be tested, or on the shaft which carries the part to be tested.

The characteristic frequencies of the vibration of the prepared power transmission gears, driven at a constant speed, are calculated as follows.

The fundamental frequencies in the case of the gear drive shown in Fig. 6.2 are:

Shaft I: $n_1 = 3000$/min, 50 Hz;
Shaft II: $n_2 = 1500$/min, 25 Hz;
Shaft III: $n_3 = 1000$/min, 16.66 Hz.

The tooth impact frequency of the small bevel gear is:

$$\frac{n_1 z}{60} = \frac{3000 \times 15}{60} = 750 \text{ Hz}$$

The tooth-impact frequency of the crown wheel is:

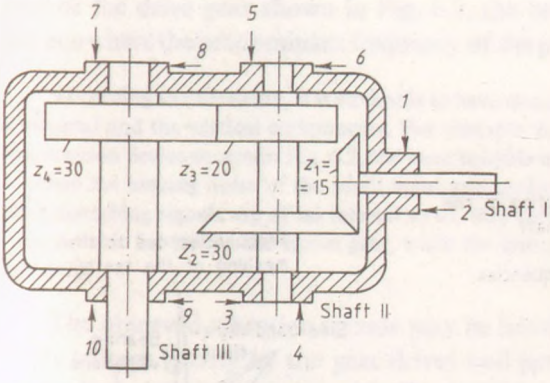
$$\frac{n_2 z}{60} = \frac{1500 \times 30}{60} = 750 \text{ Hz.}$$

The frequency of the small spur gear is:

$$\frac{n_2 z}{60} = \frac{1500 \times 20}{60} = 500 \text{ Hz.}$$

The frequency of the large spur gear is:

$$\frac{n_1 z}{60} = \frac{1000 \times 30}{60} = 500 \text{ Hz.}$$



Denomination	Hz
Shaft I	50
Shaft II	25
Shaft III	16.66
Gear z_1	750
Gear z_2	750
Gear z_3	500
Gear z_4	500
Bearings	-

Fig. 6.2 Testing a gear drive

Naturally, the upper harmonics of these frequencies appear as well, and as modulations of the amplitude and the frequency are also possible, the following additional frequencies are expected: $\frac{(z-1)n}{60}$ and $\frac{(z+1)n}{60}$. The characteristic vibrations of the bearing must be added to all these.

For a detailed analysis, the expected fundamental frequencies of the vibrations produced by the bearings, and their upper harmonics, are also calculated, knowing the type and number of the balls or rollers of the bearing. Often the approximate eigenfrequencies of the individual parts are also determined. The simplest metod used for this purpose is known as "ringing out the parts", though the results obtained are only approximate; this involves mounting a sensor of slight mass on the part to be tested (e.g. a tooth of a gear) and hitting the part. The approximate eigenfrequency of the part, or group of parts, is determined by recording the damped vibration. Naturally, the detuning of the parts after their assembly is to be taken into consideration.

On analysing the vibrations, these various high frequencies appear by necessity, and from the peaks we can draw conclusions as to the source of the stronger vibration. Figure 6.3 illustrates an imaginary vibration spectra of a gear drive.

As can be seen, the vibration spectrum of a gear drive is very wide. The spectrum extends from the frequency corresponding to the vibration of the shaft rotating at its lowest speed up to the frequencies, and their upper harmonics, of the vibrations due to the meshing of the teeth and the bearing noises. If this wide frequency band cannot be spanned over its whole width, it is advisable to perform the analysis in three stages.

The first band should extend from half the frequency of the vibrations produced by the shaft rotating at the lowest speed up to 4...5-times the rotational frequency of the fastest shaft.

The second band should range from 1 kHz to 2 kHz. The eigenfrequencies of the individual parts appear here.

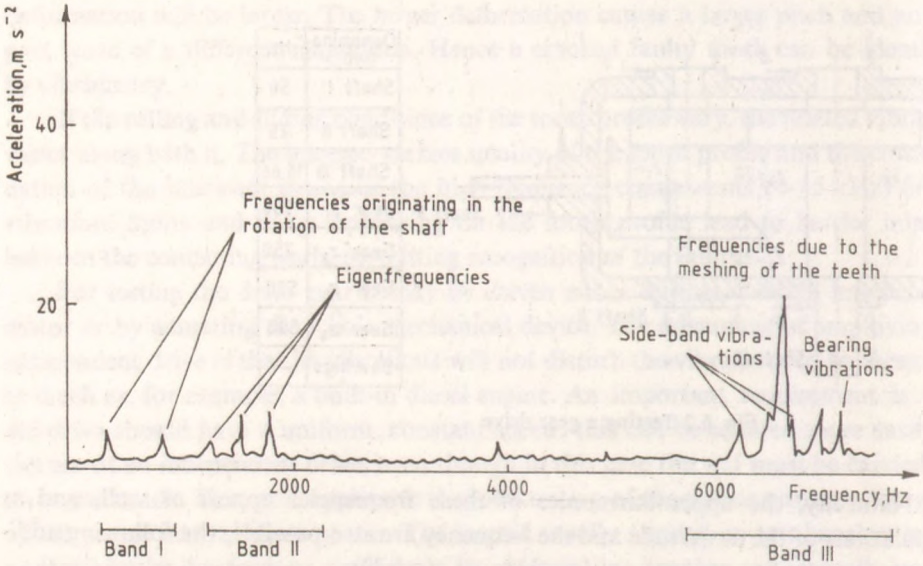


Fig. 6.3 Principal vibration spectrum of a gear drive

The third band contains the frequencies which are due to the meshing of the teeth and possibly those originating from the bearings, including also the side-band frequencies and possibly multiples of the tooth meshing frequencies as well. According to experiences the upper harmonics of the frequencies due to the meshing of the teeth are characteristic of the condition of the gear [65]. In the case of helical gears the axial vibrations are also measured, with the help of contactless sensors, as these vibrations supply information not only on the eccentricity and the lateral impact but also on the magnitude of the backlash.

An important preparatory task in the testing of a gear drive or a speed-change box is finding the most suitable measuring point by mapping, possibly, the whole gear drive. It is surprising that the vibration signals are very strong at some points, while at other points hardly any vibration can be obtained at all. This is because a gear-box behaves in the excited state like a box made from metal plates, where, at some places, there are nodal points, and at others there are areas which perform the maximum deflection. The places where the vibration intensity is a maximum are not always the best positions for testing the shafts and the bearings of the gear drive, but it may occur that, at some positions, the signals of interest are amplified by the effect of resonance.

On selecting the final testing position, the fact that the vibrations propagate faster in the material of the structure than in air must also be taken into account. So it is probable that the most favourable measuring positions are somewhere near the bearing supports of the shafts or at the shaft outlets; also the noise of the gears reaches these places most easily. The point of measurement must be selected with consideration for the requested signal frequencies (e.g. for detecting the faults of the large spur

gear of the drive gear shown in Fig. 6.2, the best measuring point will probably be the one where the predominant frequency of the plotted vibration spectrum is 500 Hz).

According to experience, it is advisable to have two sensors at the critical position measuring the horizontal and the vertical components. For example, for testing the small bevel gear of the power transmission device shown in Fig. 6.2, the most suitable measuring points are those marked by *1* and *2*, where the bearing noise of the small bevel gear and the imbalance of its shaft is greatest. But if these disturbing signals are of no interest to us, they may be filtered out eventually. Points *3* and *4* are suitable for testing the crown gear, while the spur gears are tested best at the measuring points *5–10*.

The observed vibration signals may be heavily influenced by fluctuations of the mains voltage (partly by the gear drives and partly by the instrumental evaluation), the quality of the lubricant used in the tested gear drive and the operational temperature, especially if the viscosity of the lubricant varies greatly as a function of the temperature.

When testing gear drives, the signals received by the sensors are evaluated in two ways: using conventional frequency analysis and by examining the course of the shock wave with time.

6.1.1 TESTING GEAR DRIVES AND SPEED CHANGERS BY FREQUENCY ANALYSIS

During maintenance tests the vibration intensities and spectra belonging to sound and worn gear drives are compared, as in the vibrational testing of any other machine. The spectra are taken at various measuring points at various frequencies, and the appearance of the fundamental frequencies, which were determined by preliminary calculations, is looked for.

During plant measurements, generally it is the peak values of the vibration frequency spectra that are measured, while laboratory measurements frequently extend to the variations of the Curtosis-parameter (the ratio of the peak value to the effective value), which contains — according to the experiences — much information concerning the wear. In the case of speed changers the measurements are performed in the various speed ranges as well. In numerous cases the characteristics that appear permit conclusions to be drawn as to the faulty component group even without employing a vibration analysis.

Example. Figure 6.4 illustrates the kinematic scheme and the speed diagram of a 6-stage gear drive, with the vibration intensities measured at the individual speed ranges.

From the different values of the vibration intensity corresponding to the individual speed values, conclusions may be drawn as to the faulty group of components. For example, the increase of the vibration intensity at the speeds n_3 and n_4 leads to the guess that it is caused by a group of parts which is in operation at both speeds, but which transfers no momentum at any other gear engagement.

The gears connecting shafts II and III are certainly faultless, as they also operate in other stages without causing any intense vibration. On the other hand, at the speed n_2 the pair of gears with 49

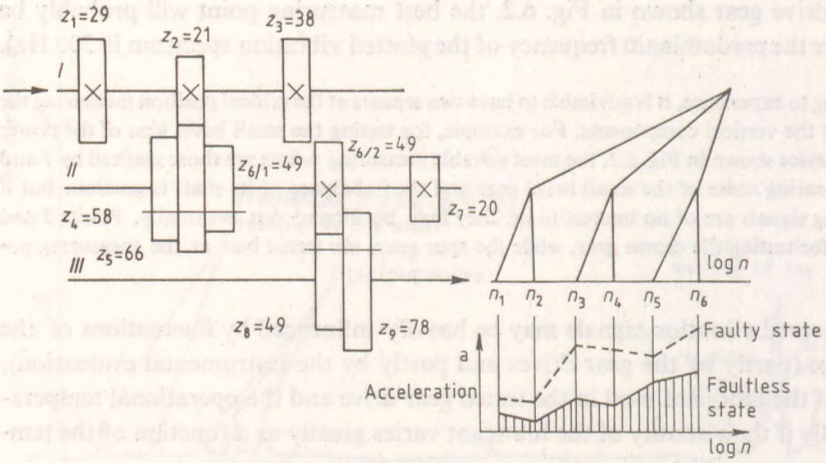


Fig. 6.4 Vibration testing of a gear drive

teeth each is operating, with a transmission ratio of 1:1, while at the speed n_5 the pair of gears with 20 and 78 teeth, respectively, is in operation. Therefore, the pair of gears found between shafts I and II is the one giving rise to the excessive vibrations. At the speeds n_3 and n_4 the gears of $z_1=29$ and $z_4=$ $=58$ are engaged and operated, so these gears must be examined thoroughly. This is carried out usually by vibration analysis, looking for the appearance in the spectrum of those frequencies which were assumed on the basis of perliminary calculations or obtained in the case of the faultless standard gear drive.

If, in the above example, the pair of gears of the transmission 29/58 is faulty, then the frequency of the vibrations resulting from the fault, or its upper harmonics, will probably appear more strongly.

If the suspected pair of gears can be bypassed by speed changing and the intensity of the frequency band belonging to it decreases, then this confirms that this faulty component is the source of the vibration. (The vibrations arising from the bearings can be identified usually by sensing close to the bearing housings directly, or the state of the bearing can be determined by comparing its vibration with the accepted standard vibration.)

In this case the origin of the observed vibration from a given bearing or gear can be found by ruling out alternative possibilities by trying out various speeds and various connections.

One of the strongest sources of vibration in gear drives is the bevel gear; this type of gear may cause strong vibrations even if it is almost faultless, has no radial or axial eccentricity, and has faultless shafts; the vibration is caused only by reason of inaccurate mounting. The intensity of the created vibration depends on the position of the bevel gear with respect to the crown gear. The vibrations are stronger if the bevel gear is set with too wide or too narrow gap. In the latter case it is the high-frequency signals that are the stronger, while in the former case the low-frequency signals are stronger, similar to the situation that occurs with spur gears. Figure 6.5 shows how the intensity of the vibration produced by a pair of bevel gears varies with the mounting distance.

As we have seen, the eccentricity of the gears may lead to modulations of the amplitude. Some experts recommend analysis of the spectrum — by methods and

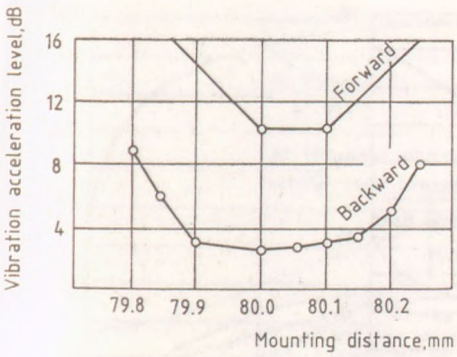


Fig. 6.5 Effect of the mounting distance of a pair of bevel gears on the excited vibration [2]

instruments described in the literature [15, 19] — to examine the vibrations appearing in this case.

Figure 6.6 presents the variation of the vibrations produced by a series gear drive of a machine tool when operated without oil or with oils of various viscosities. Without oil the spectrum of the vibration is greatly articulated. The oil, as a vibration damping medium, suppresses the spectrum and reduces its articulation as well as the unevenness of its intensity.

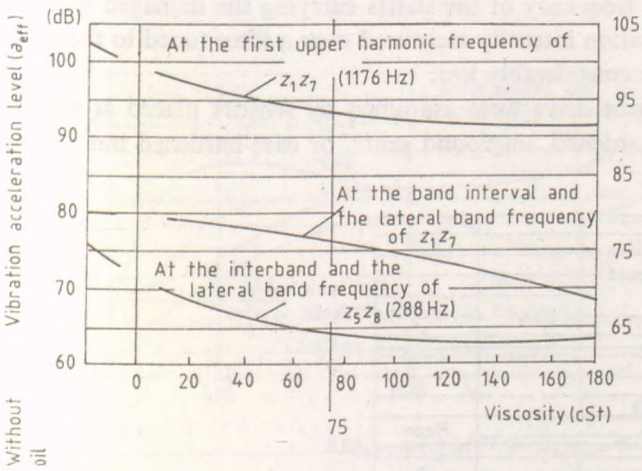


Fig. 6.6 The effective acceleration value measured on a gear drive versus the viscosity of the oil [104]

The measurement was made with the help of a band-pass filter set to the tooth frequencies of various gear pairs, or their upper harmonics. The result of the measurement enables the quality, the contamination and the quantity of the oil to be assessed.

Also, the influence of an artificial deformation of a tooth in one of the gears in a gear-pair of the same gear drive on the excited vibrations was examined. Figure 6.7 shows how the vibration intensity varies with the deformation of a single tooth,

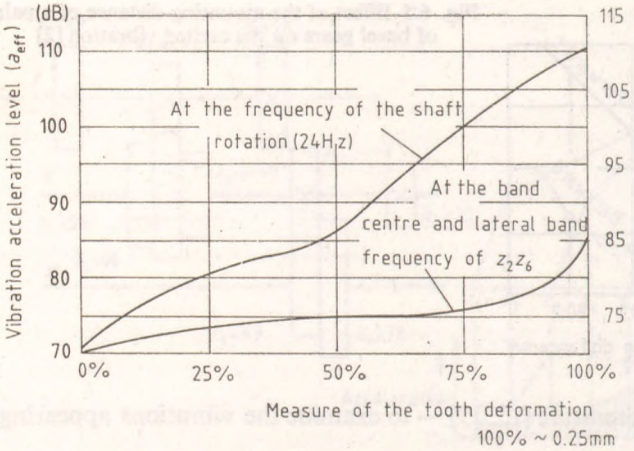


Fig. 6.7 Effect of tooth deformation on the intensity of the excited vibrations [104]

expressed as a percentage of the prescribed tooth gap of 0.25 mm. The vibration intensity is seen to rise especially steeply if measured over a band-pass filter tuned to the frequency of the shaft rotation. This means that the frequency distinguished for diagnostics is the rotational frequency of the shafts carrying the damaged machine part. The growth of the vibration intensity measured over a filter tuned to the tooth meshing frequency $(n \times z)$ is considerably less.

The vibrations of the gear drive were examined by sensors placed at various points $(A...F)$, with case-hardened, unground gears, or case-hardened but ground

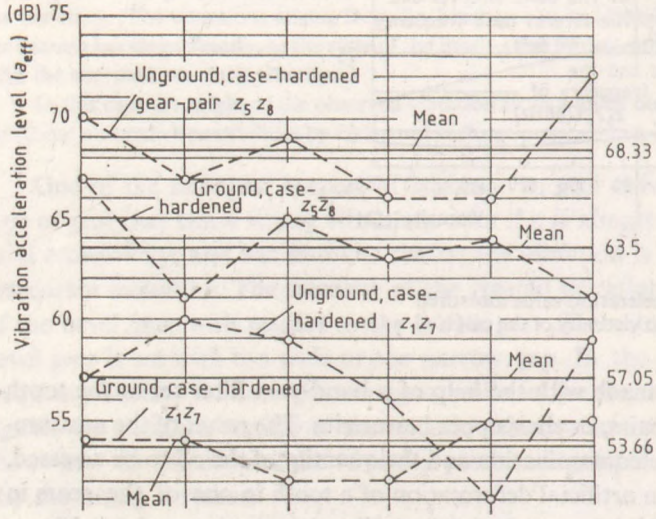


Fig. 6.8 Variation of the vibration intensity of a gear drive with ground and unground gears [104]

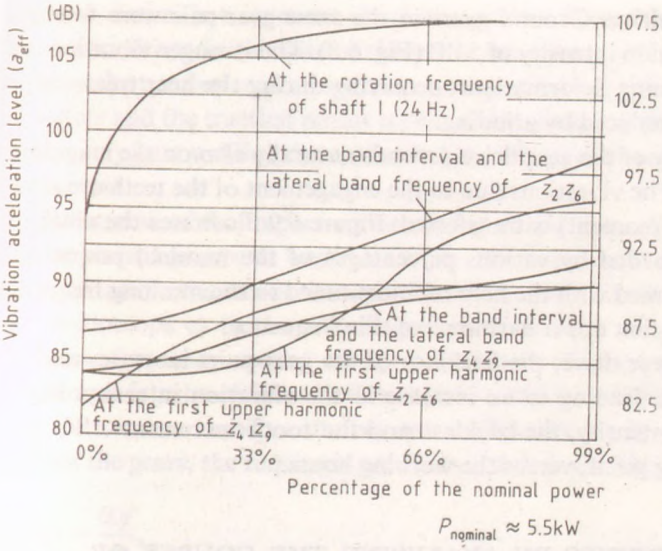


Fig. 6.9 Effect of the transferred power on the variation of the intensity of the excited vibrations [104]

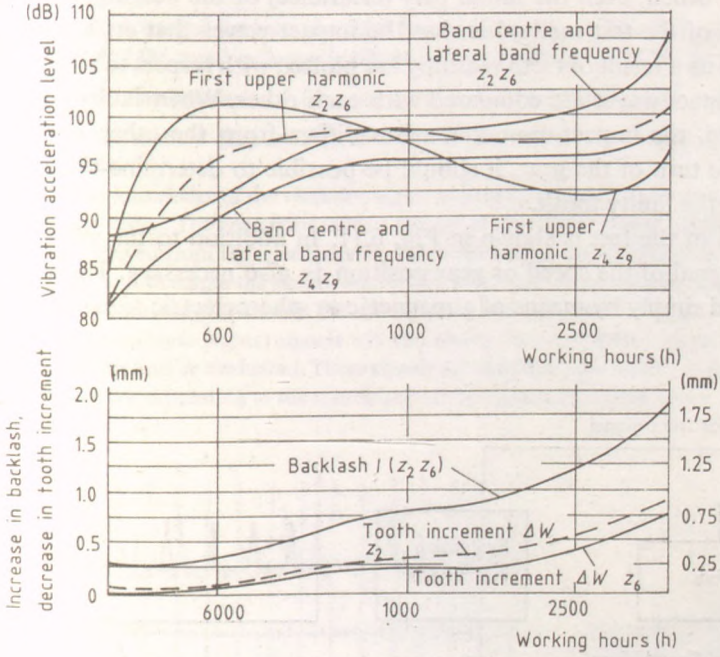


Fig. 6.10 The vibration intensity, the backlash (I), and the tooth increment (ΔW) versus the working hours [104]

gears, built into the gear drive. Ground gears in the same gear-pair were found to operate with a lower vibration intensity of 5 dB (Fig. 6.8). The stronger vibrations are probably caused by hardening deformations, occurring during the heat treatment of the gears, which were not reduced by grinding.

The vibration intensity of the gear drive depends naturally also on the magnitude of the transferred power. The vibrations due to the engagement of the teeth are stronger when a higher power (moment) is transferred. Figure 6.9 illustrates the vibration intensity of gear driven loaded by various percentages of the nominal power. The measurements were performed with the help of filters tuned to the meshing frequency of the gear-pair and to the first upper harmonic of this frequency.

During wear of the gear drive, the backlash of the gear-pairs increases and the tooth increment decreases, leading to an increase in the vibration intensity. Figure 6.10 shows the vibration intensity, the backlash and the tooth increment ΔW, measured at a given measuring point, versus the working hours.

6.1.2 TESTING GEAR DRIVES BY ANALYSING THE COURSE OF IMPACT WAVES WITH RESPECT TO TIME

The method is suitable for detecting the faulty bearings of gears and gear drives. In the course of the test, splits, indentations, broken teeth, cracked bearings and gears, and, on a special test bench, even the radial play (clearance) of the bearings can be detected. The essence of the test method is that the impact waves that arise during rolling of the bearings as a result of teeth meshing are plotted with respect to time and then the individual impact waves are compared with each other. When faulty, damaged teeth are meshed, the impact wave that arises differs from the other ones. If a plot is made for one turn of the gear, it should be possible to determine the exact point of the fault, i.e. the faulty tooth.

A block diagram of the test is shown in Fig. 6.11. In addition to the vibration sensor, an accurate signal of the speed or gear position is also necessary. The speed signal can be provided simply by means of a magnetic or photoelectric signal transmitter.

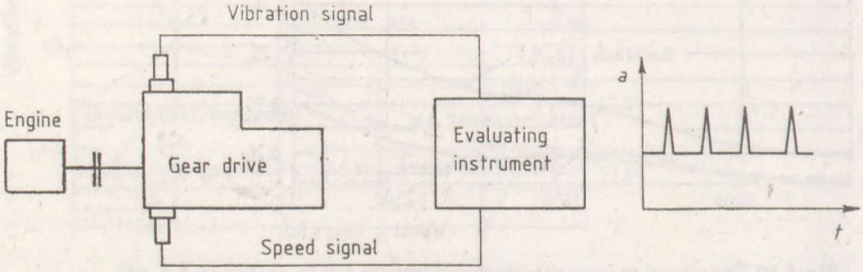


Fig. 6.11 Testing gear drives by analysing the course of impact waves with respect to time

It is desirable to prevent the driving motor from transferring vibrations to the tested gear drive. In the case of electric locomotives (e.g. when testing the gear drive between the traction motors and the railway wheel set) the wheel set in question is put on rollers and the traction motor is powered by an external d.c. source [32]. The power source may be a welding dynamo, allowing adjustment of the speed of the gear drive during test.

Diagrams of the impact waves arising during bearing faults and tooth meshing can be plotted in several ways. A simple way to produce the wave pattern is to pass the signals from the sensor through a bandfilter and then transmit them to the evaluating oscilloscope or to the recording instrument. The motor speed is set so that the vibration frequency excited by the tested pair of gears corresponds to the middle frequency of the band filter. In this way the vibrations excited by the tested equipment are more or less tuned to the bandpass filter. With a knowledge of the number of teeth of the gears, the required speed is calculated with the following formula:

$$n = \frac{60f}{z},$$

where:

z is the number of teeth of the tested gear;
f is the mean frequency of the band filter;
n is the speed (r.p.m.).

In the course of the tests, the most favourable position of the acceleration sensor is determined by trials, and the speed indicator is placed in the oil filling pipe of the gearbox. The clearance of the bearing and the backlash of the gear can be determined reasonably accurately $\left(\text{a few } \frac{1}{100} \text{ mm}\right)$ with this test method [32]. In Figure 6.12 the diagram of the vibration intensity of a gear drive can be seen.

From knowledge of the vibration signal and speed signal, the impact waves can also be evaluated with a correlator. The correlator takes a sample of the vibration signal from the moment the starting pulse is received from the speed sensor; the period of signal sampling can be set on the instrument. The correlator averages the signal, and in this way those signals whose frequency corresponds to the speed pulse become visible, and the other signals are suppressed. Thus, in the course of the test, the speed-proportionate impact signals will rise above the complicated signal consisting of many vibrations and can thus be evaluated. These signals are equally characteristic of the axial and radial impacts (frequency corresponding to the speed) and of the tooth meshing as well.

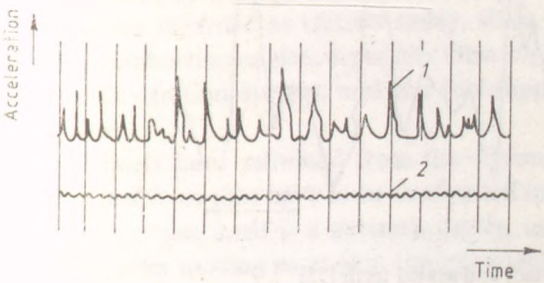


Fig. 6.12 Diagram of the vibration intensity versus time of a gear drive
1 acceleration values; *2* synchronizing mark of the speed (a complete wave is $2\pi/Z$, where Z = number of teeth or rollers [32])

The advantage of this test method is that only the vibrations of the structural elements situated on the tested shaft are sensed, while the signals participating in the flux of force but arising from other shafts are disregarded by the vibrometer.

A plot of the impact waves arising on the ring gear of a transmission system versus the time is shown in Fig. 6.13. The diversity of the impact waves points to a deviation in the tooth meshing. Cracks, tooth deformations, scaling, and other faults can all be recognized with this test method. Faults can also be observed in both the loaded and unloaded operational modes of the gear drives.

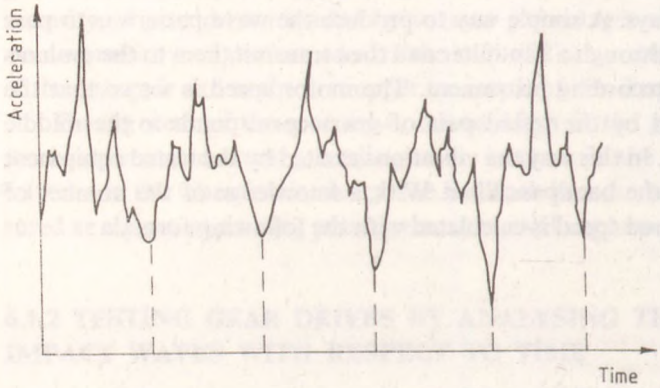


Fig. 6.13 Impact diagrams arising in tooth engagement or a ring gear [55]

The diagram in Fig. 6.14 illustrates the impacts of the gears in the transmission system of a motor vehicle. During the test, the transmission machinery of the motor vehicle was rotated by its engine on a roller test bench; the signals were recorded by a tape recorder, then evaluated in the laboratory. The impact waves arising at the meshing of each tooth show great differences in the diagram. Upon dismantling the gear drive, splits and scaling of the tooth surfaces were found on the tested teeth.

When testing gear drives, it frequently occurs that the magnitudes of the impact waves that arise owing to the meshing of the teeth vary periodically. In this case the wave frequency agrees with the frequency corresponding to the speed of the gears. The waves generally appear equally whether the gear drive is loaded or unloaded. Upon dismantling the gear drive, it can be clearly established

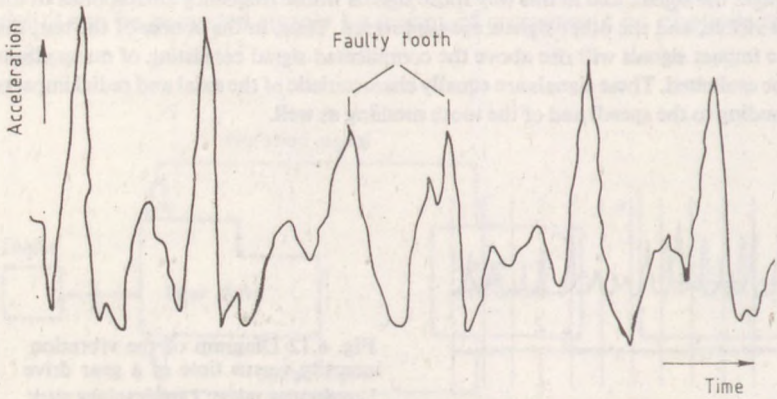


Fig. 6.14 Impact pattern of intact and scaled teeth [55]

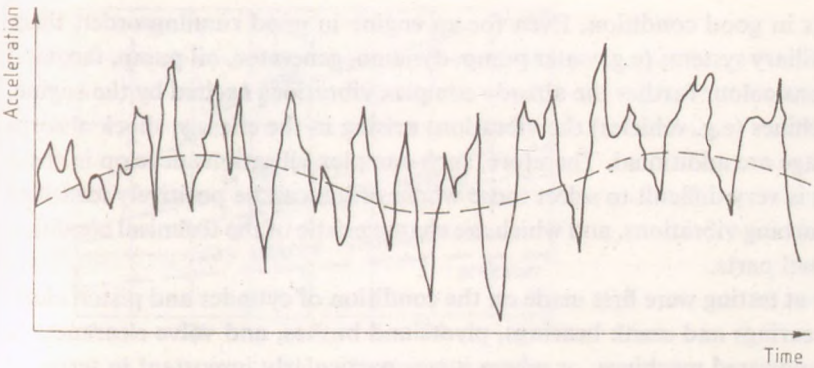


Fig. 6.15 Effect of gear impact (eccentricity) on the impact pattern [55]

from experience, that the waves are due to impacts or eccentricity of the gear. Eccentricity can be diagnosed in the impact-wave diagram shown in Fig. 6.15. [55].

Theoretically there is nothing to prevent the tests being conducted under laboratory conditions. In working out the methods of application under operating conditions, significant results have already been achieved [55].

The vibration analysis of gear drives and gearboxes is, as yet, not a simple routine task. A great deal of engineering research work and experiments are needed before a reliable model of the vibration patterns and vibration changes caused by various types of wear can be successfully established.

Here it must be stressed that almost all mechanisms, even those that are mass produced, have individual vibrational spectra. The wall thickness of the cast cases of gear drives varies; hence the rigidities are different, which frequently causes difficulty in the evaluation. Time and again, dirt and layers of paint disturb the tests.

6.2 TESTING INTERNAL COMBUSTION ENGINES

Trials were conducted recently for the development of vibrometry-based diagnostic testing methods for internal combustion engines. Up to now, no emphasis has been placed on such investigations, because the condition of internal combustion engines can be relatively simply, and more reliably, diagnosed with other methods. These methods are regarded as classics today, since they were first developed just for the internal combustion engine, especially Otto engines. The methods of testing cylinders, valves, the ignition system, and the fuel-supply system are considered to be fully developed.

Scientists have refrained from the vibrometry-based test because of the many difficult problems that have to be confronted in the case of internal combustion engines. The engine itself is a dynamic device, in which, besides the crank mechanism, there are other moving parts (e.g. timing) which may excite intensive vibrations, even

if the engine is in good condition. Even for an engine in good running order, there are several auxiliary systems (e.g. water pump, dynamo, generator, oil pump, fan, etc.) whose vibrations colour further the already complex vibrations excited by the engine. In mobile machines (e.g. vehicles) the vibrations arising in the chassis, shock absorption and carriage are additional. Therefore, such complex vibrations develop in these engines that it is very difficult to select those whose origin can be positively identified out of the disturbing vibrations, and which are characteristic of the technical condition of the individual parts.

Attempts at testing were first made on the condition of cylinder and piston clearances, main bearings and crank bearings, pivots and bushes, and valve clearances in valuable high-powered machines, or where it was particularly important in terms of personal safety and material security (e.g. aircraft).

The following is a description of a test method, which is based on frequency analysis, and its result, picked out as an example from the technical literature, which characterizes the results achieved so far in testing engines.

In the Soviet Union the engines of tractors were tested by means of vibrometry [112]. The trials were conducted on JAMZ—238NB and JAMZ—240B type engines and included determination of the clearance between the cylinder and piston. The most difficult part of the task was the selection of the most suitable measuring points. The most favourable measuring point appears to be on the centreline of the cylinders in the engine block, about 100 mm from the plane of the engine block and cylinder head. The test was not conducted on different engines in different conditions, but on a single engine, which was subjected to accelerated wear (with the use of quartz sand) to find out how the vibration parameters changed with the changing condition of the engine; of particular interest was the clearance between the cylinder and piston. The vibrations were measured with an EMDR—2 type instrument at 5 hour intervals while the engine idled and also when it was under load. The engine was rotated by an external torque in the operational mode with and without compression (valves open). According to the test result, the values of the vibration (the voltage proportional to the acceleration) for the different cylinders can be expressed approximately by the following equations, depending on the time that the engine has been exposed to wear (working hours):

idle run	under load (65 kW)
cylinder 1 $u=16.4+0.135t$	$u=19.5+6.8 \log t$ (mV)
cylinder 2 $u=20.6+0.090t$	$u=18.3+8.9 \log t$
cylinder 3 $u=19.0+0.185t$	$u=17.6+10.9 \log t$
cylinder 4 $u=21.0+0.040t$	$u=21.0+6.1 \log t$
cylinder 5 $u=21.6+0.050t$	$u=21.3+7.9 \log t$
cylinder 6 $u=23.3+0.011t$	$u=24.0+8.3 \log t$
cylinder 7 $u=20.3+0.120t$	$u=18.5+10.6 \log t$
cylinder 8 $u=18.8+0.180t$	$u=25.7+5.4 \log t$

As a matter of interest, the acceleration varied linearly with the working time, and logarithmically when under load.

Under various running conditions, the acceleration of the vibration (with 1.8... 2.57 kHz band filter) versus the clearance, h, between the cylinder and piston (running clearance) varied as shown in Fig. 6.16. It was found that, as a result of changes in the running clearance, not only the intensity of the shock pulse varied in relation to the

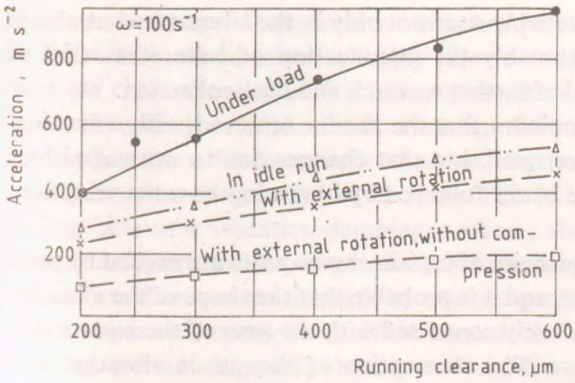


Fig. 6.16 Running clearance versus vibration acceleration [112]

angular position of the crankshaft but also its phase. A greater running clearance h entails a greater phase displacement (Fig. 6.17).

The results obtained were mathematically processed and the functions $a=f(h)$ and $\varphi=f(h)$ were calculated, which supported the results of the theoretical analysis.

Trials were conducted to test the condition of crank bearings. For this purpose, the sensor was mounted onto the end-face of the crankshaft or on the side of the crankcase. According to the trials, clearances in the crank bearings were observed mostly as the engine accelerated. The period of acceleration can be increased by disconnecting certain cylinders, by braking or throttling down the engine.

Similar results were obtained in diagnosing faults in the control mechanism and the magnitude of the valve clearance. Tests were conducted on four engines; thus 32 sets of data were obtained on the running clearance, and connecting-rod bearing, and 64 on the valve mechanism. During tests, the engines were dismantled and the clearances were determined by conventional linear measurement. According to the purpose

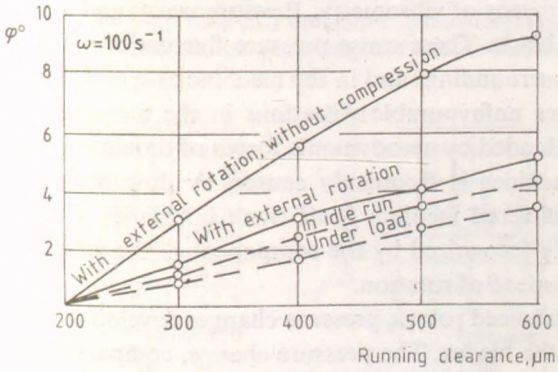


Fig. 6.17 Running clearance versus phase displacement [112]

of the trial, the test method can be reliably used not only in the laboratory but also in running conditions. However, presumably the introduction of tests of a serial or routine type will require a great deal of further research and data collection.

Other authorities are of the opinion that the results achieved with vibration-spectrum analysis should not be overrated, but that changes due to wear of pistons and especially diesel-engines can be better followed by observing how the vibrations change with time [78].

For example, the resonant frequencies of certain engine parts are excited by pulse forces from the pistons of the engine, and it is probable that the shape of the vibration signal with respect to time is more closely connected with the wear of these parts than the shape of the frequency spectrum. The observation of changes in vibration with time naturally requires a synchronizing signal to indicate the position of the main shaft or camshaft; this is given by a magnetic or photoelectric signal transmitter, or, alternatively, by a certain point of the vibration curve (e.g. its maximum). Generally the vibration signals of several cycles, from the same part of each cycle, are averaged, thereby raising those details of the signal that occur in each cycle above the parts of the signal that are due to randomly occurring processes. From the shape of the resulting signal, conclusions can be drawn as to the functioning or condition of the individual parts of the structure. This principle is used to test the engagement between the gear drive and the gears. One of the most developed testing methods involves the observation of the pressure wave of the injection system with an oscilloscope [15, 19, 28, 77, 107].

6.3 TESTING BLADED MACHINES (HYDRAULIC AND AERODYNAMIC)

The most frequent failures occurring in bladed machines (pumps, compressors, turbines) are faults in the bearing and shaft, imbalance and blade faults and breakage. These can be diagnosed generally by means of vibrometry. Pressure waves and vortices arise in the medium around the blade. These cause pressure fluctuations, which induce vibrations in the blade, in its surroundings and in the machine case and rotary shaft. Aerodynamic imbalance excites unfavourable vibrations in the propellers of aircraft. In such cases the blades are loaded by aerodynamic forces of different intensities [25]. Imbalance in bladed machines is frequently caused by dust deposited between impeller blades or by the delivered medium itself. Such imbalance — as is well known — can be relatively easily recognized by the component of the vibration with frequency corresponding to the speed of rotation.

Even in bladed machines with balanced rotors, pressure changes develop around the blades and rotate together with the blades. The pressure change, compared with a stationary pump or compressor, excites a pressure wave of frequency corresponding to the speed multiplied by the number of blades. The pressure wave — according to

the pressure change — causes the whole body of the machine to vibrate and forces the delivered medium into the supply line at a rate corresponding to the frequency. From the characteristics of this induced vibration of relatively high frequency, conclusions may be drawn as to the functioning of the blades and any faults in them.

Frequently, a blade-excited vibration can be better observed on the impeller housing than on the bearing blocks [65]. This is because the oil film on the sliding bearings acts as a vibration-damping medium, absorbing the high-frequency vibrations. For this reason the so-called blade frequencies — especially in the case of gas and steam turbines — are detected by a sensor positioned at the middle of the housing instead of on the bearing block. If the mass of the machine case (impeller case) is not great (e.g. steam turbines, fans) compared with the mass of the rotor, then accelaration sensors are used for observing the lower frequencies. They are particularly useful in this respect, for normally they span the whole frequency range and are not sensitive to high temperatures.

When using acceleration sensors, in order to detect the low-frequency vibrations, the speed of the vibration is determined by integration and then evaluated. In the case of machine bodies with large mass, where the fastening to the pedestal is rigid (e.g. steam turbines), non-contact relative sensors are used to detect the rotor's movement. If the axial vibration is also detected by the relative sensor, then it is necessary to consider the change in length of large machines caused by changes in temperature. The sensing system can be used to maintain uniform heating, for example, of the rotor and the machine case (turbine case) in compliance with the specifications.

The form of the pressure versus time curve for the blade-induced pressure wave (assuming steady flow conditions) can be regarded to remain the same provided the shape of the blade is constant. On the other hand, if the shape of the blade varies, or the blade bends, becomes loose, or breaks, then the shape of the curve will change. When the pressure is analysed, its harmonic components appear in the vibration spectrum.

Figure 6.18(a) shows the vibration spectrum of a faultless, bladed air-turbine, with the peak values of the blade frequency and upper harmonics [111]. If a blade develops a fault, the frequency spectrum will also change. For example, if a blade becomes shorter, then the amplitude of the vibra-

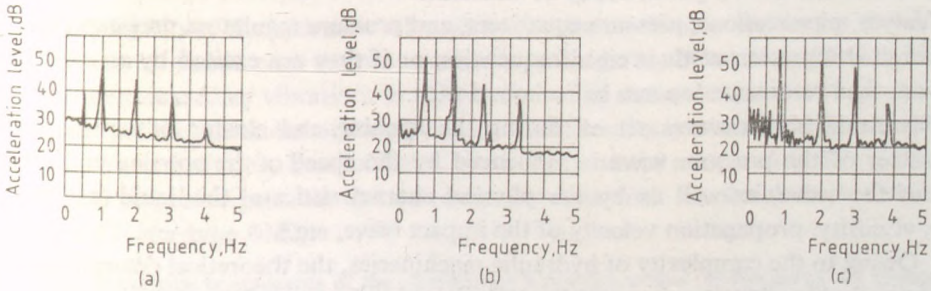


Fig. 6.18 Changes in the frequency spectrum due to faulty blades
(a) faultless blades; (b) change of blade length; (c) missing blade [111]

tion will pulsate, i.e. amplitude modulation takes place. In this case side-band frequencies always appear in the vibration spectrum (Fig. 6.18(b)). If a blade were to be lost, then the time between two consecutive pressure waves will increase, and accordingly the components with frequency lower than that of the blade appear in the vibration spectrum. Such a spectrum is shown in Fig. 6.18(c).

The own frequency vibrations of the blades may also modulate the blade frequencies, and thus side-band frequencies may also appear. However, any damage to the blades changes the self-frequencies consequent upon changes in the side-band tifrequencies [78].

Thus analysing the side-band changes is an important method of detecting failures of the blades. According to experience, even slight blade deformations are noticeable, causing significant changes in the spectrum.

Generally the excited vibrations are checked in two bands. The low-frequency band begins at a minimum of 1/5th of the fundamental frequency corresponding to the speed, and extends to five times the fundamental frequency. The upper band spans the blade frequencies and its side-band frequencies [65].

Cavitation phenomena in pumps may excite vibrations of frequency 40...60 kHz. If the test includes these too, then the upper frequency limit should be selected accordingly.

Not only hydraulic and aerodynamic machines vibrate, but so do the pipeline systems which are supplied by them. During delivery of a liquid, the spreading pressure wave governs the flow velocity of the liquid. These pressure changes occur not only as transient phenomenon but also owing to continuous excitation, the vibration may become permanent. Sudden pressure impulses can frequently overload machineries and induce mechanical vibrations in other parts as well. The hydraulic vibrations (pressure changes) in networks are sensed mostly with piezo-electric pressure gauges, and the mechanical vibrations in machineries are tested conventionally (e.g. with acceleration sensors and speed sensors).

The most frequent reasons for hydraulic vibrations in pipelines and machineries are the following:

— Pulsation of liquid delivery. The phenomenon occurs in all piston pumps. The delivery of bladed pumps also pulsates: this increases if the impeller is unbalanced, or if the blades are irregular, faulty or deficient.

— Valves, pipe sections, pressure equalizers, and pressure regulators may experience self-excited vibration at their eigenfrequencies, or if they are excited by an external source, then resonance too can be reckoned with.

— Hydraulic shock waves set off during the opening and closing of valves. The character of the pressure wave is influenced by the speed of the opening and closing of the valve, as well as by the physical characteristics of the liquid (e.g. density, viscosity, propagation velocity of the impact wave, etc.).

Owing to the complexity of hydraulic machineries, the theoretical determination of the probable vibrations is generally difficult and only approximate calculations can be performed. Namely, the components of the vibrational waves cannot be taken

into account exactly, the various modes of damping in the system are not known either, etc. Intense, so-called resonant vibrations, which are detrimental to the machineries, can be avoided if the eigenfrequencies of the individual hydraulic units (e.g. supply pump, pipeline, safety valves, pressure reducers, etc.) differ widely from each other and are unable to mutually excite one another.

The eigenfrequencies of the individual units can be calculated approximately.

The liquid is delivered by the pump, piston pump, or gear pump with a pulsation frequency proportional to the product of the speed, n, and the blade number, z, the piston number, z, or the teeth number, z, respectively.

The theoretical eigenfrequency of the pipeline, in terms of the propagation velocity, c, of the pressure wave and the length, l, of the pipeline, is

$$f = \frac{c}{2l} \text{ Hz},$$

if the velocity is expressed in the units m s^{-1} and the length is in m.

The eigenfrequency of spring-actuated safety and pressure reducing valves, disregarding any liquid or other damping is

$$f = 0.16 \sqrt{\frac{s}{m}} \text{ Hz},$$

where s is the stiffness of the spring (N/m);

m is the moving mass, kg.

A hydraulic system is generally excited by a pump. If the vibration of a unit is intense, it can be reduced by changing, detuning, the eigenfrequency of this unit. This can be achieved generally with the correct selection of spring constants and masses.

Overlarge pressure waves are avoidable if:

— presure limiters are built into the system;
— the flexibility of the system is increased (e.g. elastic elements, hoses);
— storage units are connected to the system (this is usually a pressure tank, making the delivery of liquid more stable).

The majority of the hydraulic vibrations that cause problems for the maintenance engineer can be traced back to construction, manufacturing or mounting faults. Most problems are caused by vibrations in the pipelines. The antinodes (points of maximum oscillation) of the vibrations of the pipe can be found with a conventional feeler probe. On the occasion of more detailed tests, the deviations are measured in the different directions. It is also possible to take measurements with a triaxial converter (e.g. Brüel and Kjaer type 4340).

The rectangular components of the vibration of a pipeline versus the frequency are shown in Fig. 6.19. By detuning the pipelines, or fixing them to a suitable spot, the moderation or, possibly, the elimination of the vibrations can be achieved.

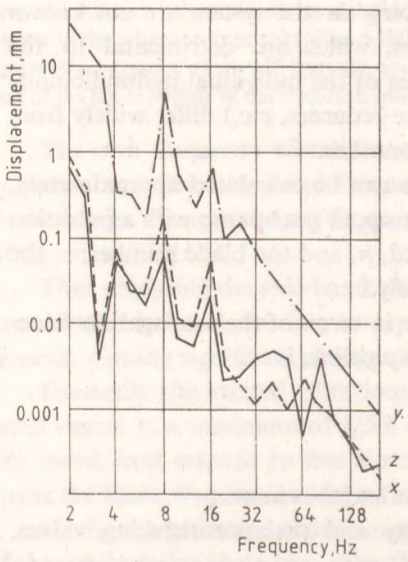


Fig. 6.19 Vibrational displacement of pipeline [63]

6.4 TESTING ELECTRICALLY DRIVEN ROTATING MACHINES

The testing of electrically driven rotating machines (e.g. electric motors, generators) is similar to the testing of other rotating machines. The most frequent failures are bearing faults and those due to imbalance. These can be diagnosed using vibrometry. The vibration or noise testing of low-power electric motors is a routine job nowadays. Industry requires quiet, silently running electric motors, such as the motors of tape recorders, household machines, fans, etc. Here we are concerned with the testing of high-power motors, the sudden breakdown of which may represent loss of production as well.

Electric motors transmit their torque to the machines which they drive generally through a coupling or a belt drive. Undoubtedly, testing the motors when they are not connected to other machines would be the ideal; this however is only possible in the case of new machines, or when connection is via a belt drive. When connection is by means of a coupling drive — since the motor and the associated machine cannot be separated without disconnection — the vibrations due to the associated machine must also be taken into account. However, the advantage of such a test is that it also includes testing for faults and faulty assembly of the coupling. Naturally, if the tests are conducted under operating conditions with the required load, then disconnection or separation of the associated machine from the motor is not permissible even in the case of a belt drive.

The method of vibration testing of independent electric rotating motors, not connected with other machines, is standardized. The effective vibration speed is

measured and the parameters of the instrument used for the test are also specified in the standards. The specifications and requirements are similar to the parameters of the instruments used for the rotating machines described earlier. The sensor must not be sensitive to changes in magnetic fields. If, under the influence of a magnetic field of field strength $H = 100$ A/m and frequency 50 Hz, the pointer of the instrument deflects by more than 5% of the scale length (in the most sensitive measuring range), then the actual level of interference arising from the magnetic field has to be taken into account during evaluation of the test results. The sensor is rigidly connected to the electric rotating machine.

The methods of testing and classification of electric rotating machines are regulated by the specifications of the standards.

Electric rotating machines heavier than 0.25 kg and running at 600...6000 r.p.m. should be tested in a flexible, rigid and operative or final mounting. In a flexible mounting only those machines lighter than 750 kg, and in an operative mounting only those heavier than 750 kg, can be tested.

The vibration tests are only permissible on a self-supported, fully equipped (fan, cover, closed terminal block) motor which is ready for operation.

The machines of a machinery consisting of several rotating machines are tested individually. If rigidly coupled auxiliary machines (generators) cannot be detached, or tested with an independent drive, they must not be classified with large machines.

Very large machines and machineries (e.g. turbogenerators) can naturally be tested only in operative mountings. Machines with a key-grooved shaft are tested with a mounted and secured key-groove. The a.c. supply voltage may be sinusoidal, or in the case of multi-phase supplies it may be symmetrical. In the case of a d.c. or pulsating supply, the supply voltage should be the same as the voltage applied in normal use. Shunt machines are tested in idle run (supplied with the rated voltage and without a load), and machines with multiple-series circuits are tested without a load at the rated speed (supplied with a reduced voltage). In the case of variable-speed machines, the vibration intensity is measured over the whole speed range.

The vibration components in the perpendicular directions x and y are measured with sensors. In the case of pedestal-mounted machines the direction y is perpendicular to the pedestal, and for flanged machines the axis x is horizontal. The sensors are generally positioned on the bearing blocks, but, by separate agreements, they may be anywhere. In the case of rigidly or operatively mounted machines, if the rigid fixing of the sensor is not possible, the use of feeler-pointed sensors is also permissible.

Table 6.1 sums up the vibrometry method and classifies machines with different shaft lengths running at 60...3600 r.p.m.

In the international specification ISO 2373—1974(E) the measuring conditions and classification are similar to those above. The standard specifications apply generally to the method of classifying new or reconditioned electric rotating machines. Naturally, diagnostic tests can be conducted individually under different circumstances, with different conditions, instruments and sensors as well.

Table 6.1

Classification of electric rotating machines based on vibrometry (KGSZ 400 653—71)

Symbol	Speed	Length of shaft		
		$H \leqq 132$	$132 < H < 225$	$225 < H < 400$
		v_{eff} mm s^{-1}		
N	1800—3600	2.8	4.5	7.1
	600—1800	1.8	2.8	4.5
R	1800—3600	1.1	1.8	2.8
	600—1800	0.71	1.1	1.8
S	1800—3600	0.71	1.1	1.8
	600—1800	0.45	0.71	1.1
P	1800—3600	0.45	0.71	1.1
	600—1800	0.28	0.45	0.71

Note;

N — normal machines;
R — machines with reduced vibration;
S — low-vibration machines;
P — precision machines.

Noted, however, that there is no point in experimenting with relative displacement sensors. This is because the mass of the rotor in an electric machine is relatively heavy compared with the masses of the case and shields of the electric motor, and, furthermore, the case is generally flexible. As a result of this, the whole electric machine may vibrate, and therefore measuring the relative displacement of the sahft and motor case does not lead to a meaningful result [65].

If a machinery consisting of several machines is tested as a whole, the axial vibrations and their phase-displacements call attention to the faulty mounting of clutches and to uniaxial faults [87].

Besides the imbalance and bearing faults, the most serious problem during testing is caused by the pulsating magnetic field of the electric motor. This induces mechanical vibrations and may be misleading during the test. The reasons for these vibrations are that the axis of the rotating magnetic field does not coincide with the axis of the electric motor, the clearance is not constant, and the coiling is faulty and asymmetric. The vibrations are frequently more intense on the motor case than on the bearing block or shields. For this reason, a measuring point is frequently arranged also on the motor case (on motors with cooling fans, if there is no room for the sensor between the fans, it can be fixed onto the fan.). If more intense vibrations are detected on the motor case than on the shields in the vicinity of the bearings, they are probably

of magnetic origin. This can be easily verified by disconnecting the motor from the mains. If there are no longer any vibrations, then they were due to magnetic effects.

The disturbing effect of the vibrations originating from changes in the magnetic-field strength can be reducted if the motor is driven at a voltage lower than the rated voltage during the time of the test. Motors which are not connected to other machines are driven with an endless flat belt during testing for faulty bearings and for imbalance. In the case of commutator machines, the test can be performed with lifted brushes, although vibrations from the commutator may supply valuable information on its condition.

7 TESTING THE CONDITION OF MACHINES IN THE PLANT

7.1 ORGANIZATIONAL TASKS OF THE PLANT TEST

Prior to introducing a maintenance system based on vibrometry, careful technical and organizational tasks must be worked out and an economic analysis performed. The main taks are the following:

1. The machines to be included in the test system must be selected from the machines of the plant. This usually depends on the value or complexity of the machines and their role in the production.

Generally, simple machines of low value which are readily replaced when they fail, and consequently do not cause significant loss of production, are not subjected to vibrometry. Also omitted are those machines which are unlikely to fail accidentally, and whose wear and life expectancy can be relatively accurately estimated in advance. Similarly, those machines the diagnosis of which by means of vibrometry is difficult and complicated, or can only be accomplished in the laboratory, are not tested.

Generally, it is the permanently running rotating machines and those which are costly to repair and would lead to significant losses in production if they broke down which are subjected to testing.

2. The measuring positions must be found and marked on the selected machines. The main goal in the selection is to detect those vibrations which are characteristic of the technical condition of the machine and any changes in these vibrations. If possible, the positions suitable for picking up the horizontal, vertical and axial vibrations are marked, generally on the bearing blocks, where the maximum amplitude of the vibrations can be measured. These measuring positions should be marked out on the machine without fail and noted in the sketch shown on the record card of the machine.

3. Furthermore, the operational vibration at the measuring positions on the tested machines and the tolerated vibration intensities at these positions have to be determined. If no data are available from the manufacturing firm, these can be determined by the method described in Chapter 4.2, based on vibration diagrams, or from tables. In the case of a new or reconditioned machine in good condition, the measured vibration value is acceptable as a value of the initial operational vibration. If several machines of the same type, and functioning under similar conditions, are in the plant, the measured results have to be compared with each other. This is because an incor-

rectly set machine may accidentally function in an unbalanced state among those believed to be good. Upon comparison of the data, this will usually be discovered, enabling the faulty machines to be repaired; at the same time the opportunity is there for a more reliable diagnosis of the machine's operational vibration.

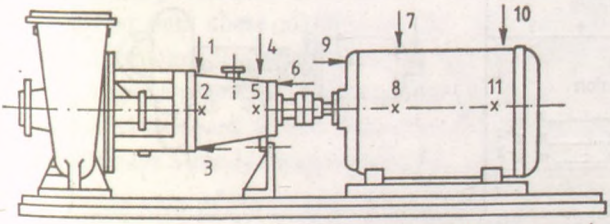
It is stressed that no strict limit can be set for the tolerated vibration, i.e. it is not possible to define a value the overstepping of which would result in the instantaneous breakdown of the machine. The changes occurring in the operation of the various machines and items of equipment (wear, break-down, etc.) and their mechanical failure, are phenomena which are too complicated to enable such limit values to be set with complete assurance. The selected limits of vibration simply indicate deterioration of the machine's technical condition, its expected failure, and that further monitoring is necessary. Thus, concerning the vibrations, each machine has its individual characteristics, to be considered carefully when the operational vibrations and limit values are determined.

4. The next step is to determine the period of the test measurements. Theoretically, as short a period as possible is required to enable the smallest changes in the condition of the machine to be noticed. However, testing too frequently is costly. From a knowledge of the machine's "habits" and the tangential direction of the wear-curve, the time interval can be determined during which any changes are noticeable, or

Borsod Chemical Works
Technical Control and
Co-ordination Branch

Test record

Subject: Vibrometry
Location of measuring ..... plant
Measured machine .....
Date of measurement 19 .....
The measuring instrument is a VP 102 type indicator
Outline drawing of the measuring points



Vibration speed, v_{peak}, mm s^{-1}

Symbol \ Working place	1	2	3	4	5	6	7	8	9	10	11	Category

Note
Kazincbarcika, ...
..... plant technician

Fig. 7.1 Test record

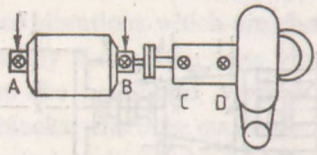
during which no excessive damage will develop. The test period varies according to the type of machine. If possible, the measurements should be carried out in the course of the periodic control and maintenance of the machine.

When determining the interval between measurements, the character of the machine and the requirements connected with its precise operation should be considered. The highly valuable, high-speed machines (e.g. gas and steam turbines) must be checked more frequently than the general machineries (e.g. electric and diesel-engines, compressors, pumps, etc.). In the case of a precision grinding machine, vibration testing is required after each tool replacement and occasionally after adjustment of the stone in order to avoid undesirable vibrations.

5. To monitor the condition of the machines and to determine the accurate vibration levels, recording, registration and evaluation of the measured data are necessary. Suitable data recording systems should be instituted for this purpose. The basic recording document is a data sheet containing an outline drawing of the machine, the measuring points on the machine, the main data and settings of the measuring instruments, vibrometry data, measuring units and the measured vibration values.

Figure 7.1 shows a standard test sheet for plant use [10]. Frequently, a standard machine data sheet is used that contains all perioadical determined measuring data. Such a sheet is shown in Fig. 7.2.

To simplify the work of the measuring personnel, the results are frequently recorded in the order of measuring. In this case, for a better overview of the recording and further processing, the results are transcribed to the data sheet containing the vibration values.

Machine: Pump	Instrument: H 11025	
Workshop: Forge	Sensor: ⓐ v	
Inventory No.11-643	Measured value ⓐ v s	
Speed n=1140 r.p.m.	Scale value 1 scale division s.....m v.....m s^{-1} a.....m s^{-2}	
Note:	Filter:	

Date	Measuring point				Signature
	A	B	C	D	
	Measured value				
1.12.1980	0.85	1.1	1.2	1.30	
8.12.1980	0.80	1.15	1.2	1.35	
15.12.1980	0.85	1.2	1.2	1.35	
23.12.1980	0.85	1.15	1.2	1.40	
30.12.1980	0.85	1.15	1.25	1.45	

Fig. 7.2 Data sheet of machine, containing vibration data

Today, with the aid of instrument provided with memories, the vibration data of machines can be collected and evaluated without any administrative work. Such instruments were described in Chapter 3.

The results of the measured vibration of the group of machines forming the pressure-intensifying compressor of a plant is shown in Fig. 7.3. For the sake of easier inspection, the data are also represented in a diagram. A diagram showing the changing speed values measured at point 9 of the first turbine machine-group is shown in Fig. 7.4. When processing a great number of data, they are recorded on punch tape, magnetic tape, etc., and then fed into a computer, which, with the aid of the relevant evaluating programs, performs the evaluation.

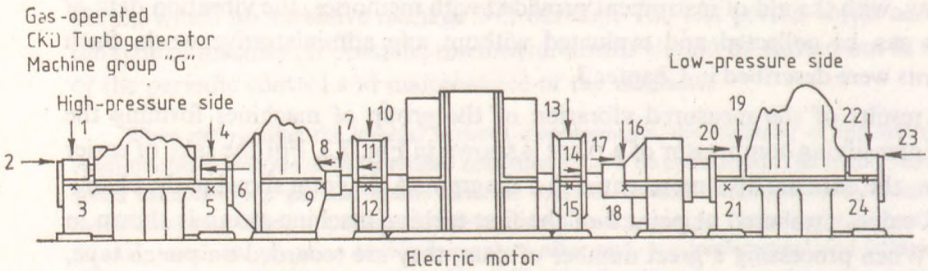
The introduction of a control system includes the selection and procurement of suitable instruments and the training of the control personnel. If feasible, the same machines are always controlled by the same person. No special qualifications are required of the measuring personnel. It is sufficient for them to know the measuring points, the measurement conditions (speed, load on the machine), how to handle the instruments, the method of measuring and the required administrative activities.

7.2 THE PERIODIC MANUAL AND AUTOMATIC TESTING OF MACHINES

Manual testing means the periodic sensing of the intensity of the vibrations at the measuring points, usually with portable instruments. Generally the peak or effective value of the acceleration or speed of the vibration is measured. In certain cases it may be necessary to observe the vibrations in a certain frequency band. For this purpose, instruments provided with filters are used. The measured data are usually written on the test or data sheet of the machine. In doubtful cases, in addition to the simple manual instruments, portable analysers can also be used to determine the faulty spot or part more precisely. During manual testing, the form of recording required is decided with regard to the characteristics of the plant and the machines. Examples of these have already been described.

In the case of several machines or measuring points, the manual testing and the evaluation of the measured results require considerable time; hence the machines are frequently checked by automatic test instruments, the so-called vibration-guard instruments. Automatic testing systems process the signals from built-in sensors. The installation of an automatic system is generally justified in the following cases:

— the testing of machines located in remote, inaccessible place (e.g. pumps in shafts, wells, underwater equipment);
— machine in the chemical industry, in explosive surroundings or exposed to radiation hazards, where personnel are prohibited from the vicinity of the machine;



No. of measuring points \ Date month day hour	1978 12 01 10^{30}	12 04 11^{00}	12 06 18^{00}	12 07 8^{00}	12 09 12^{00}	12 15 10^{30}	12 18 15^{10}	12 19 12^{30}	12 20 11^{30}	12 23 10^{20}	12 27 10^{15}	12 29 10^{40}	1979 01 02 10^{30}	01 04 10^{40}
1	7.6	7.3	6.2	6.5	6.5	7.5	5.7	6.2	5.8	4.4	4.3	3.7	5.2	4.5
2	9.3	3.2	3.1	3.3	6.7	2.2	2.5	1.8	2.9	2.6	1.7	1.4	3.0	2.9
3	4.4	4.4	5.0	4.1	2.4	6.0	4.0	5.0	3.7	3.8	3.8	3.2	5.2	4.6
4	6.5	6.8	5.3	4.7	5.3	6.0	5.3	4.9	5.0	4.7	3.5	3.3	4.4	4.3
5	1.9	1.8	2.5	1.9	2.4	1.9	2.0	2.1	2.5	2.0	1.5	1.6	2.5	2.7
6	3.5	4.1	4.0	3.6	3.4	5.8	4.5	4.9	3.4	3.8	4.1	3.7	4.9	4.5
7	6.2	5.7	5.2	5.0	5.6	5.1	5.2	5.9	6.2	5.5	5.8	5.7	5.7	6.5
8	9.0	8.0	8.2	7.5	7.5	8.5	8.5	8.0	5.2	5.2	7.3	7.5	7.6	6.8
9	9.2	10.5	7.6	7.5	8.0	11.0	10.5	10.5	6.7	9.6	10.0	8.2	9.0	7.5
10	2.9	2.8	1.9	2.0	2.0	2.4	2.5	2.5	2.3	2.4	2.2	2.3	2.3	2.2
11	3.0	3.0	2.8	2.5	2.5	3.8	3.1	2.7	2.6	2.7	3.1	2.3	2.8	2.7
12	5.8	6.2	6.0	4.0	4.5	6.5	6.1	6.1	4.5	5 6	5.8	2.7	6.1	5.7
13	1.4	2.0	1.8	1.9	1.9	2.2	2.5	2.0	2.0	2.0	1.8	2.1	2.0	1.5
14	2.1	2.1	1.5	1.8	1.8	3.1	1.9	1.7	1.9	2.1	1.7	1.9	1.4	2.1
15	3.2	3.0	3.1	3.0	3.2	2.9	3.3	3.6	2.9	2.7	9.0	3.1	2.8	3.4
16	3.1	3.2	2.9	2.3	2.4	3.5	3.0	2.9	2.9	2.9	3.1	3.3	2.3	2.9
17	3.2	3.8	2.2	2.5	2.8	2.1	2.6	3.0	3.1	1.8	4.9	3.7	4.1	3.5
18	4.3	4.6	4.0	3.8	4.7	3.5	4.8	4.9	4.3	4.9	4.7	5.6	5.5	5.4
19	1.9	2.0	1.9	2.0	1.9	1.8	2.1	1.9	1.9	2.2	2.2	2.1	2.0	2.4
20	1 5	1.4	1.3	1.2	1.2	1.3	1.5	1.4	1.3	1.3	1.2	1.2	1.3	1.5
21	2.7	3.2	5.4	3.7	3.8	4.0	3.5	3.4	4.9	3.0	2.1	3.2	4.2	4.4
22	1.4	1.3	1.5	1.5	1.4	1.1	1.4	1.2	1.6	1.3	1.3	1.2	1.4	1.3
23	1.2	1.2	0.6	1.4	1.0	1.2	1.3	1.1	1.0	1.2	1.3	1.3	1.2	1.0
24	3.6	3.5	1.7	2.8	2.1	3.3	3.2	2.9	1.4	3.0	3.6	1.7	3.9	2.6

Fig. 7.3

— highly valuable, high-powered machines (e.g. steam and gas-turbines, high-capacity pumps, water turbines), where unexpected break-down of the machine would entail high costs or serious damage;

— machines where periodic manual testing is not feasible, because the faults develop relatively quickly, almost without any preliminary indication (e.g. material becomes detached from the rotor of a turbine, or breakage of a blade occurs).

When installing the automatic monitors, vibration sensors are mounted at the critical points of the machine.

If the vibration of the machine exceeds the present limit value, the monitor emits a warning or emergency signal; then it stops the machine. A block diagram of a monitor is shown in Fig. 7.5.

01 10 11^{20}	01 13 6^{30}	01 15 8^{15}	01 18 7^{30}	01 20 8^{15}	01 22 7^{30}	01 23 8^{15}	01 24 12^{00}	01 25 8^{15}	01 26 8^{00}	01 29 8^{40}	01 31 9^{50}	02 01 8^{30}	02 05 7^{20}	02 05 6^{10}	02 06 7^{10}	02 07 7^{20}	02 09 7^{20}	02 12 10^{40}	02 13 16^{00}
12.0	9.0	3.8	2.7	2.8	2.9	2.9	3.1	3.2	3.4	3.3	2.7	2.6	2.9	3.2	3.4	3.0	2.8	3.3	3.4
4.0	9.2	1.4	1.8	1.5	1.4	1.3	1.2	1.8	2.0	1.9	3.1	1.5	2.2	2.0	2.0	1.8	3.1	1.8	1.9
6.5	7.4	7.0	8.0	7.5	7.5	7.2	6.8	6.2	7.5	7.2	7.3	6.8	8.0	5.4	7.2	8.0	6.8	6.5	6.5
12.0	4.0	3.2	3.7	3.8	3.8	3.3	3.5	3.4	3.6	3.8	2.7	2.7	3.8	2.9	2.8	3.5	2.8	4.0	3.5
5.0	2.0	2.2	1.6	1.6	6.0	1.8	1.6	1.7	2.2	1.0	2.8	1.6	2.2	3.4	2.2	2.2	3.3	2.5	2.2
8.5	6.5	6.2	5.7	5.8	6.2	6.0	6.0	5.8	6.5	6.5	5.2	4.8	8.1	7.0	7.5	8.8	5.8	4.9	5.8
6.0	5.5	6.5	6.7	7.2	8.3	7.5	7.3	8.0	8.5	8.2	4.0	1.2	4.3	5.1	4.2	4.4	4.1	4.2	4.5
9.5	7.0	7.2	6.0	7.5	7.0	10.5	9.0	9.2	9.8	8.5	4.5	5.4	6.5	5.8	7.0	5.8	5.1	4.8	4.1
8.5	8.2	9.0	9.5	10.2	9.8	10.0	9.2	9.5	10.5	11.5	10.2	8.2	11.5	11.2	10.2	13.0	11.0	8.8	8.5
2.8	4.1	4.5	3.8	4.2	4.1	3.9	3.7	5.0	5.0	5.2	3.1	3.9	3.4	2.7	3.4	3.2	2.2	2.7	2.9
3.4	5.3	5.8	4.9	5.5	5.6	5.9	5.4	5.6	6.0	6.5	5.1	6.5	8.2	4.3	7.5	7.8	3.4	4.6	4.7
5.2	6.2	6.5	6.8	7.0	6.8	7.0	7.5	6.1	7.5	6.5	8.0	6.2	7.3	6.8	6.8	8.0	8.0	5.9	5.8
2.4	3.7	4.2	3.6	3.9	4.3	4.5	4.2	4.0	4.3	4.3	2.6	3.3	4.1	2.3	3.4	3.6	3.0	4.1	3.9
2.6	4.9	5.5	5.5	5.3	5.5	5.7	5.4	5.5	5.6	5.4	3.1	4.2	4.3	2.7	3.6	4.2	4.0	4.9	4.8
4.0	4.4	4.0	4.9	4.9	5.4	5.4	4.9	5.2	5.4	5.6	4.1	4.5	6.2	3.7	5.1	5.9	4.0	4.7	4.6
3.0	3.1	3.3	2.8	3.1	3.1	3.5	3.1	3.0	3.3	3.4	2.9	3.0	3.4	2.4	3.2	4.4	3.1	3.1	3.3
3.5	2.9	3.6	3.5	3.6	3.7	3.7	3.6	3.9	3.9	3.8	3.8	3.0	4.0	4.3	3.8	4.8	3.4	4.6	2.4
5.8	3.8	4.8	4.3	4.2	4.5	4.6	5.1	4.2	4.6	5.0	3.7	5.0	4.8	3.8	4.2	5.2	3.1	4.5	4.5
2.4	2.0	2.7	2.5	2.4	2.3	2.3	2.2	2.5	2.7	2.8	2.6	2.3	2.5	2.7	2.3	2.4	2.2	2.3	2.3
1.6	1.5	1.6	1.7	2.0	1.6	1.9	1.7	2.0	2.7	2.0	1.2	1.6	2.0	1.4	2.2	1.8	1.9	1.6	1.8
4.7	4.0	2.8	3.5	2.8	2.5	2.3	2.4	2.4	2.8	2.6	4.1	3.6	6.2	6.8	4.1	6.2	3.9	2.9	2.8
1.5	1.7	1.2	1.4	1 4	1.1	1.2	1.1	1.2	1.2	1.2	1.5	1.5	1 4	1.4	1.3	1.2	1.5	1.5	1.5
1.4	1.6	1.4	1.8	2.3	1.6	1.5	1.7	2.0	2.0	1.8	0.8	1.2	1.8	1.2	2.2	1.9	1.4	1.9	2.0
2.8	5.0	5.6	5.3	5.2	5.5	5.4	5.2	5.3	6.5	6.0	3.9	5.4	9.0	6.5	7.2	8.8	4.3	5.4	5.4

Vibration data of a pressure-intensifying compressor

The monitors — independently from their system — are capable of checking certain vibrational parameters at the measuring point in a specific frequency range (Fig. 7.6(a)). Such a parameter may be the peak or effective value of the acceleration or speed of the vibration. The drawback of this system is that it is sensitive only to very rough, serious faults (e.g. the high-frequency vibrations of a bearing need to be very intense to substantially increase the level of the whole frequency range). Such a monitor system does not give any information about the frequency of the increased vibration, and does not present a basis for pinpointing the faulty machine part.

A much more sensitive instrument is that which closely follows the whole spectrum. These instruments divide the spectrum into relatively narrow zones (e.g. third zones), and the signal received is compared with the input signal corresponding to the

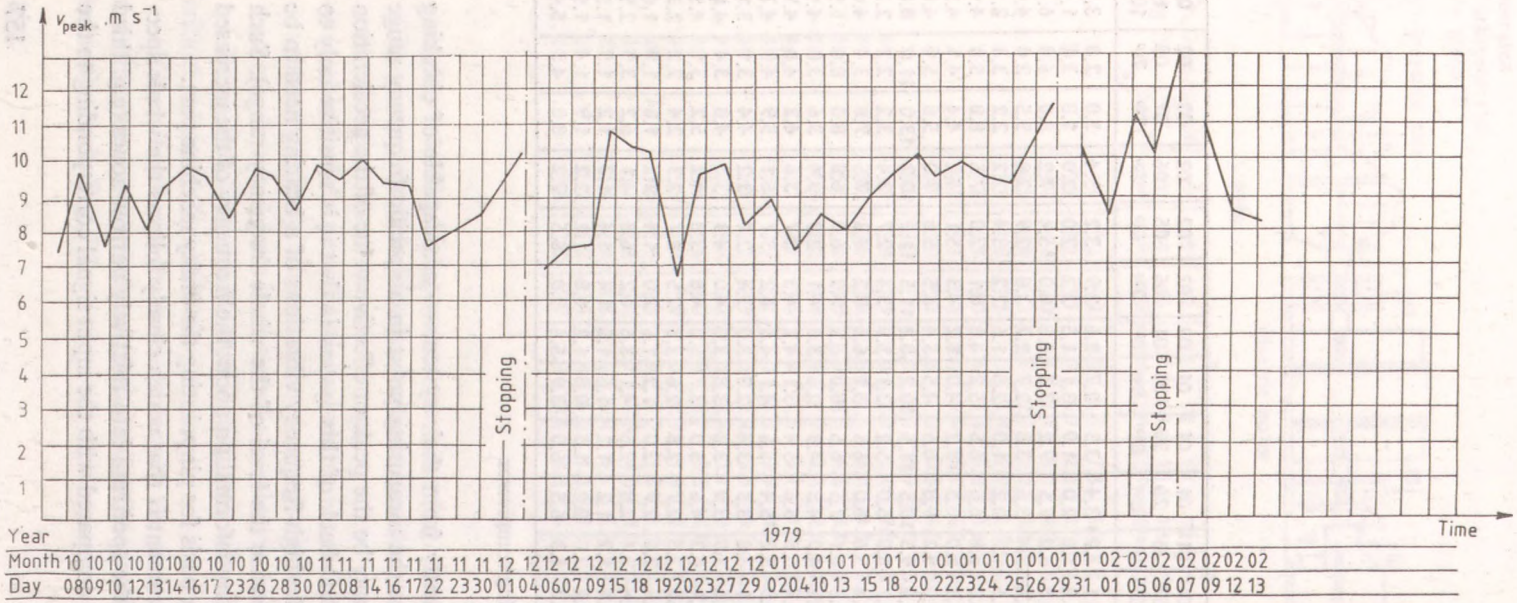


Fig. 7.4 Vibration diagram of compressor [10]

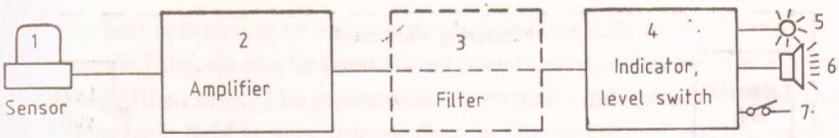


Fig. 7.5 Block diagram of simple monitor

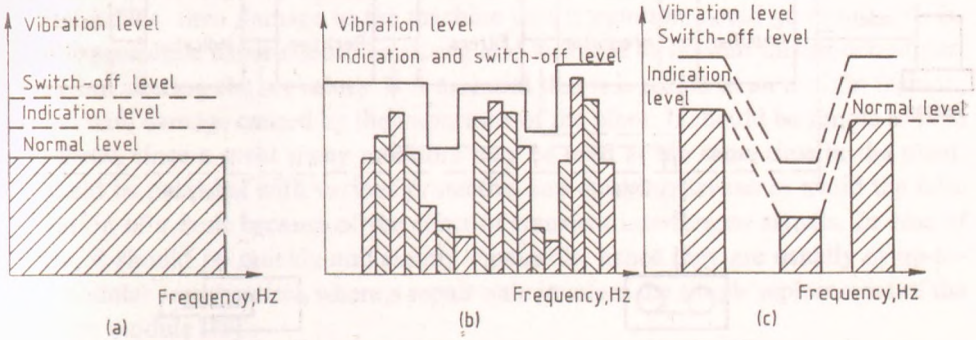


Fig. 7.6 Control mode of vibration signals

limit value in each zone (Fig. 7.6(b)). To simplify the instruments, in many cases not the whole spectrum is monitored, but only the critical zones (e.g. those parts corresponding to the frequencies of the vibrations due to imbalance, tooth engagement, and roller bearings) (Fig. 7.6(c)).

A block diagram of a simple monitor is shown in Fig. 7.7. The instrument includes a preamplifier, and in the case of an accelerometer, also electronic integrator units for measuring the deflection and speed, various band filters, a rectifier to form the peak or effective values, an indicator, level switches set to different limit values and a calibrating or control unit. Different outputs allow for the further analysis and recording of the signals received.

The most important element of a monitor is the sensor. These may be acceleration, speed or contactless sensors. The aspects of their selection have already been described. A sensor is fixed with a screw to the measuring point, and is provided with a protective case, which is relatively massive, and a threaded cable attachment with a packing gland.

To meet the different measuring tasks and conditions, various types of sensors are produced commercially. Some of them can be used up to 400 °C (e.g. in the power sources of aircraft), and there are also instruments which function faultlessly under water or in explosive and agressive media [19, 29, 39].

A sensor suitable for permanent installation is shown in Fig. 7.8. The signal emitted from the sensor is transmitted through a properly clamped electric cable, possibly concealed in a sleeve pipe, to the evaluating instrument. In the case of a long cable, its capacity, its incidental vibration and the dispersed magnetic field of the surroundings may considerably reduce the signal to noise ratio. Therefore it is advis-

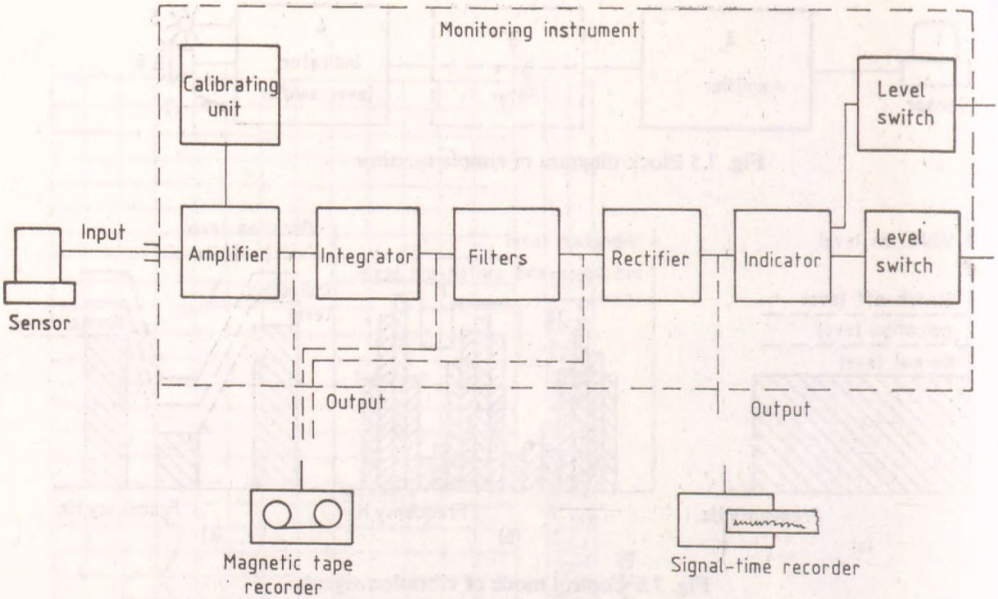


Fig. 7.7 Block diagram of machine-monitoring instrument [19]

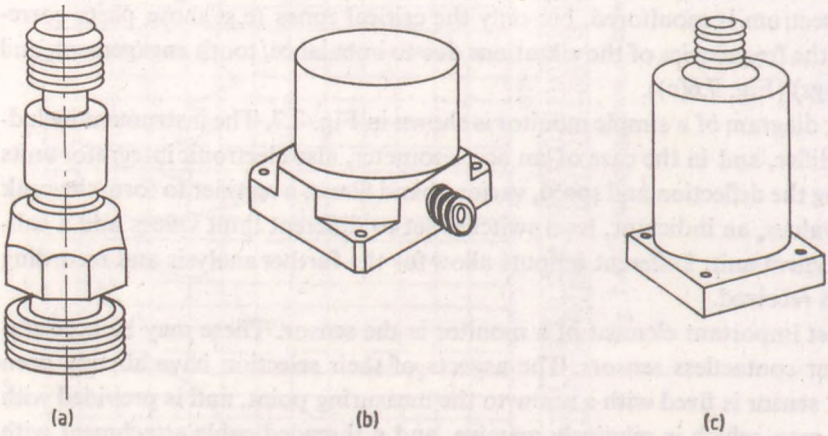


Fig. 7.8 Sensors suitable for permanent installation
(a) sensor of SMP 32 V instrument; (b) B & K sensor; (c) IRD Mechanalysis sensor

able to lead the cable from the sensor to the measuring instrument in the shortest possible way. If the distance between the sensor and the monitor is short, special so-called low-noise-level standard cables are recommended by the manufacturers. If the distance between the sensor and monitor is long, the special low-noise-level cable is used only as far as a junction box (usually to the machine base), to which the cables are connected from the other measuring points. The signals are transmitted from the junction box to the monitor, usually through multicore, PVC-insulated cable [19].

The best solution is to arrange a small preamplifier in the vicinity of the sensor. The amplified signals can be transmitted (free from interference) for a greater distance than unamplified ones. The preamplifier is virtually indispensable when the disturbing electromagnetic field is very intense (e.g. in the vicinity of electric machines) or the cable is exposed to intense vibrations.

The monitor is a highly reliable electronic instrument, constructed for long life, since if it fails, then damage to the machine that it monitors could go unnoticed. Its case or protective cover should be strong and lockable to prevent unauthorized persons from altering the set values. It is essential that it is sealed in an airtight fashion, to eliminate damage caused by the impure air of the plant. It should be simple to read and check, since a great many monitors may be used at the same time in the plant. It should be provided with various protective and delaying devices to avoid the false indication of a fault because of the effect of transient interference signals. In case of failure, it should be quickly and simply repairable, hence they are usually of up-to-date modular construction, where a repair only involves the simple replacement of the defective module [19].

The signals from the vibration sensor can be observed continuously or intermittently. The choice between continuous or intermittent monitoring is decided according to the character and type of the machines to be protected.

For example, in the case of high-speed compressors and turbines, where there exists the risk of the sudden detachment of materials deposited on the rotor or breakage of the blade, continuous control is required, even if it is more expensive. On the other hand, in the case of fans — besides the bearing faults — the most probable fault is increasing imbalance. The imbalance increases continuously and at a relatively slow rate as a result of the abrasive effect of or deposition from, the delivered medium. For such machines, intermittent control is perfectly satisfactory.

Concerning continuous control, if the vibrations of the machine are monitored without a band filter, or in a wide band, the principle of the instrument is shown in Fig. 7.5.

If, as according to Fig. 7.6(b) and (c), different bands are monitored, the instrument should be provided with band filters.

Figure 7.9(a) shows the block diagram of an instrument in which the bands are monitored with paralelly connected band filters. The signal in the filters can be amplified or damped according to the importance of the individual bands. The level switch senses the sum of the three bands.

The instrument shown in Fig. 7.9(b) is superior to the above, since it indicates the band in which the vibration level is higher than the permitted level. The location of the fault can be determined instantly from such information.

In machines where a failure occurs over a relatively long time, the permanent monitoring of the observed vibrations is unnecessary, but intermittent control is sufficient. In this instance, the control instrument "queries" or "scans" the sensors,

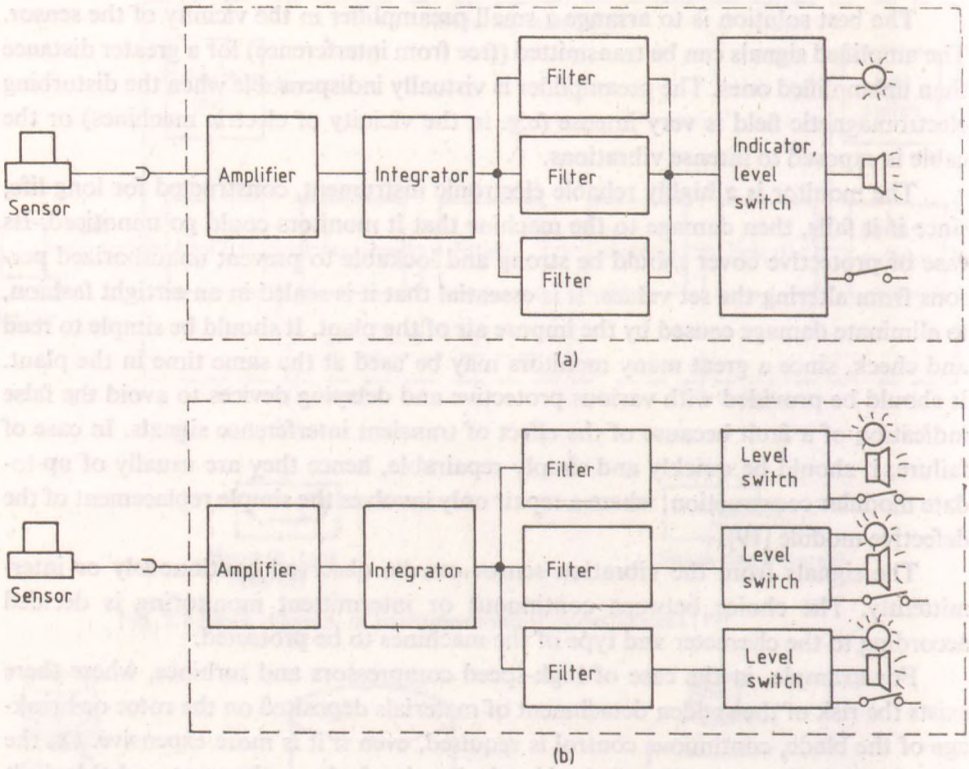


Fig. 7.9 Instrument monitoring several bands

and the periodically received signals are sensed and evaluated. This is known as a multiplex system.

In most instruments the value of the set vibration level must not be different for each sensor; hence an adjustable preamplifier is connected between the sensors and the monitor [19]. In this way the monitoring system will indicate at the specific level of each sensor.

A block diagram of a monitor of a multiplex system is shown in Fig. 7.10.

A drawback of the multiplex system is that the protection is not continuous. Only as many sensors can be connected to the instrument so that the measuring time per channel is longer than the set delay time, and this should be sufficient to characterize the condition of the machine. The time between queries shall not be longer than the time taken to check the machine to be protected, i.e. during the time between two measurements no significant failures should occur. Generally, the maximum number of channels per instrument is 30.

The monitoring instruments are provided with filter cycles or band filters, which make it impossible for an instrument to indicate an emergency due to a random increase of the background noise [86]. The function of these filters is based on the fact

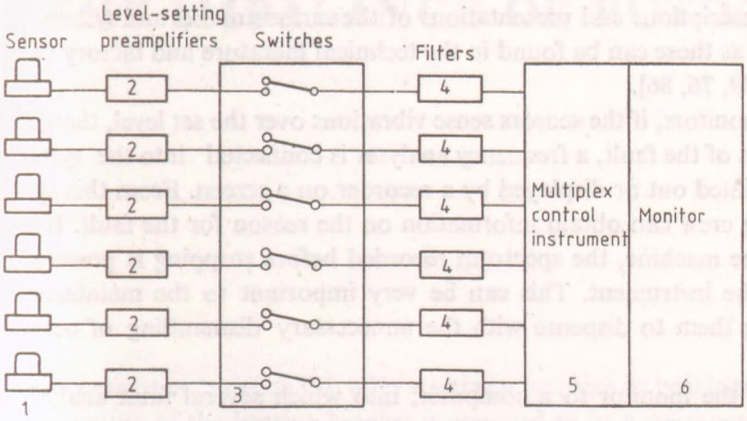


Fig. 7.10 Monitor with multiplex system

that the frequency of the background noise usually differs from the frequencies induced by the machine.

It is necessary to provide each monitor with a delay device that prevents the sensors from indicating an emergency because of the occurrence of sudden external "transitional" vibrations. Such vibrations may arise from vehicles or cranes moving in the vicinity of the machine, or from machines started nearby, etc. The instrument disregards for a time such external disturbing vibrations, even if the frequency is the same as the monitored frequency. Naturally, the delay time should not be longer than the time during which a change in the vibration might occur in case of the failure of the monitored machine.

The delay devices stop the unnecessary warnings (alarms) caused by external interferences. In certain cases it may be necessary to reduce the time delay (e.g. if the instrument observes sudden extremely intense vibrations). Such vibrations may arise as a result of a sudden failure and the disconnection of the delay device protects the monitored machine against catastrophic damage and accidents.

During the starting and stopping of certain machines, the vibrations may be extremely intense (e.g. as a rotor moves through a critical turn). Without special devices, the instruments would indicate an emergency. In order to prevent this, use is made of high-pass filters, which do not let through the low-frequency, large amplitude vibrations. Another solution to this problem is for the monitor to raise the level of the emergency signal about a factor of four for the periods of starting and stopping. In this case, however, the instruments must receive starting and stopping information [86].

In the interest of improved operational safety, the instrument checks its own cable network and also the reliability of the sensors. In the case of a fault the operating crew is warned by the instrument. The latest instruments even emit warning signals of the voltage level if the signal drops below the normal. Naturally the monitor switches off when the machine stops.

No detailed descriptions and presentations of the various makes and systems of monitors are given as these can be found in the technical literature and factory catalogues [19, 29, 38, 39, 76, 86].

With certain monitors, if the sensors sense vibrations over the set level, then, for the detailed analysis of the fault, a frequency analyser is connected into the system. The spectrum is printed out or displayed by a recorder on a screen. From this spectrum, the operating crew can obtain information on the reason for the fault. If the monitor stopped the machine, the spectrum recorded before stopping is preserved, i.e. it is stored by the instrument. This can be very important to the maintenance personnel, enabling them to dispense with the unnecessary dismantling of certain machine parts.

By connecting the monitor to a computer, into which several fault analysing programs have been fed, the computer can be used to perform the comparisons which would otherwise be the task of a qualified operating crew. The computer indicates the observed fault and, moreover, the method of its elimination. A block diagram of such a multiplex system is shown in Fig. 7.11 [86]. The multiplex instrument transmits the signals from the sensors — in the event of an emergency — into the frequency analyser, where the machine compares the received spectrum with the separately entered operational spectrum of the tested machine. On the basis of the differences, the instrument indicates the probable mechanical faults. Instruments provided with memory units are already available. These instruments considerably facilitate the work of the operating crew, and administrative work is unnecessary. Such an instrument is described in Chapter 3.

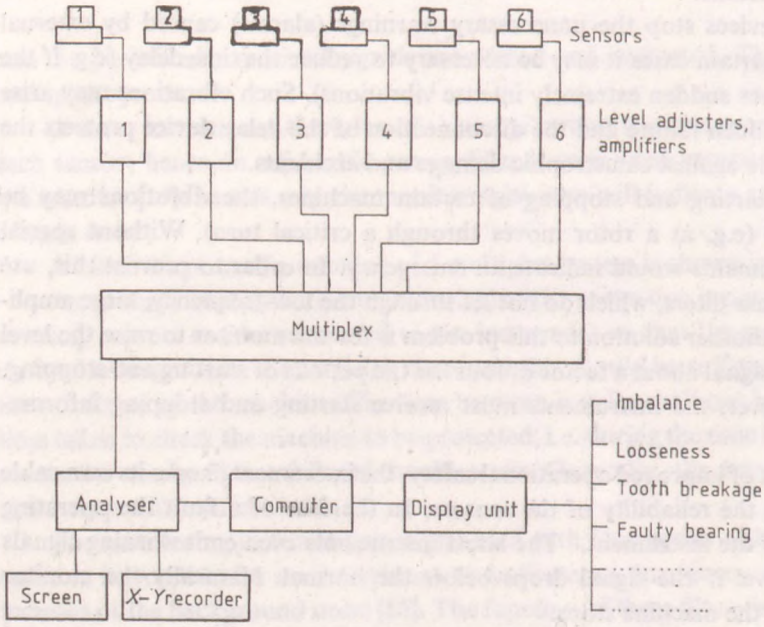


Fig. 7.11 Multiplex monitors with computer evaluation [86]

8 THE IMBALANCE OF ROTORS

It is a common experience that after starting a machine its bearings vibrate. Obviously the vibration of the bearing bracket is assumed to be a consequence of the rotation of the rotor, whereby a supporting force of variable direction is added to the stationary supporting forces. The vibrations are different even in machines of identical type (e.g. in two identical electric motors). Let us examine the relationship between the characteristics of the rotor and the supporting forces arising in the bearings due to the rotation of a rigid rotor. Let us model the rotor and its bearings as shown in Fig. 8.1.

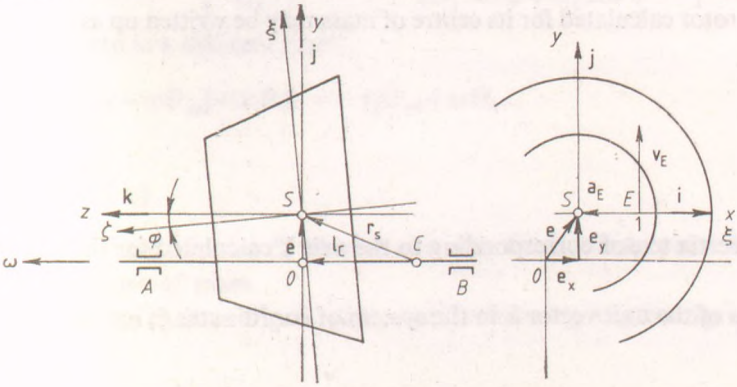


Fig. 8.1 Model of the rotor and its bearings

Let the origin of the system of coordinates xyz be the centre of mass of the rotor and let the axis z be parallel to the axis of rotation AB. The unit vectors **i**, **j**, and **k** correspond to the axes x, y and z; ξ, η, ζ are the principial axes of inertia belonging to the centre of mass of the rotor, and Θ_ξ, Θ_η, Θ_ζ are the moments of inertia calculated for the principal axes. Accordingly, the matrix of the moment of inertia in the system of ξ, η, ζ coordinates is

$$\Theta_S = \begin{bmatrix} \Theta_\xi & 0 & 0 \\ 0 & \Theta_\eta & 0 \\ 0 & 0 & \Theta_\zeta \end{bmatrix}$$

Generally $\Theta_\xi \neq \Theta_\eta \neq \Theta_\zeta$, but for rotating bodies $\Theta_\xi = \Theta_\eta \neq \Theta_\zeta$, apart from special cases. Let the axis ζ make an angle φ with the axis z in the yz plane and let the rotor rotate with the angular velocity $\boldsymbol{\omega}$ about the axis of rotation AB. The spin of the system at any arbitrary point C is

$$\pi_C = \pi_S + \mathbf{r}_S \times m\mathbf{v}_S, \tag{1}$$

where

π_S is the spin calculated for the centre of mass;
m is the mass of the rotor;
$\mathbf{v}_S$ is the velocity of the centre of mass.

The moment of the external supporting forces about the stationary point C equals — according to the theory of spin — the derivative with respect to time of the rotor spin calculated for point C:

$$\mathbf{M}_C = \frac{\mathrm{d}\pi_C}{\mathrm{d}t}, \tag{2}$$

where

$\mathbf{M}_C$ is the moment of the external forces about point C;
π_C is the spin of the rotor about point C.

The spin of the rotor calculated for its centre of mass may be written up as

$$\pi_S = \omega \mathbf{T}_S(\mathbf{k})$$

and

$$\mathbf{T}_S(\mathbf{k}) = \boldsymbol{\Theta}\mathbf{k},$$

where $\mathbf{T}_S(\mathbf{k})$ is the inertia tensor corresponding to the axis $\mathbf{k}$ calculated for the centre of mass.

The coordinates of the unit vector $\mathbf{k}$ in the system of coordinates ξ, η, ζ are

$$\mathbf{k} = \begin{bmatrix} 0 \\ \sin\varphi \\ \cos\varphi \end{bmatrix},$$

so

$$\mathbf{T}_S(\mathbf{k}) = \boldsymbol{\Theta}\mathbf{k} = \begin{bmatrix} \Theta_\xi & 0 & 0 \\ 0 & \Theta_\eta & 0 \\ 0 & 0 & \Theta_\zeta \end{bmatrix} \times \begin{bmatrix} 0 \\ \sin\varphi \\ \cos\varphi \end{bmatrix} = \begin{bmatrix} 0 \\ \Theta_\eta \sin\varphi \\ \Theta_\zeta \cos\varphi \end{bmatrix}.$$

$\mathbf{T}_S(\mathbf{k})$ may be generated with the help of the components T_{xz}, T_{yz}, T_{zz}:

$$\mathbf{T}_S(\mathbf{k}) = \begin{bmatrix} T_{xz} \\ T_{yz} \\ T_{zz} \end{bmatrix},$$

where

$$T_{xz} = \mathbf{i}^*\mathbf{T}_S(\mathbf{k}) = [1\ 0\ 0]\begin{bmatrix} 0 \\ \Theta_\eta \sin\varphi \\ \Theta_\zeta \cos\varphi \end{bmatrix} = 0;$$

$$T_{yz} = \mathbf{j}^*\mathbf{T}_S(\mathbf{k}) = [0\ \cos\varphi\ -\sin\varphi]\begin{bmatrix} 0 \\ \Theta_\eta \sin\varphi \\ \Theta_\zeta \cos\varphi \end{bmatrix} =$$

$$= -(\Theta_\zeta - \Theta_\eta)\sin\varphi\cos\varphi = -\Theta_{yz};$$

$$T_{zz} = \mathbf{k}^*\mathbf{T}_S(\mathbf{k}) = [0\ \sin\varphi\ \cos\varphi]\begin{bmatrix} 0 \\ \Theta_\eta \sin\varphi \\ \Theta_\zeta \cos\varphi \end{bmatrix} =$$

$$= \Theta_\eta \sin^2\varphi + \Theta_\zeta \cos^2\varphi = \Theta_z.$$

Using the above expressions, the spin vector calculated for the centre of mass of the rotor is

$$\pi_S = \omega[\mathbf{i}\,\mathbf{j}\,\mathbf{k}]\begin{bmatrix} 0 \\ -\Theta_{yz} \\ \Theta_z \end{bmatrix} = \omega[\mathbf{i}\,\mathbf{j}\,\mathbf{k}]\begin{bmatrix} 0 \\ -(\Theta_\zeta - \Theta_\eta)\sin\varphi\cos\varphi \\ \Theta_\eta \sin^2\varphi + \Theta_\zeta \cos^2\varphi \end{bmatrix},$$

or, written in a different form,

$$\pi_S = -\omega\Theta_{yz}\mathbf{j} + \omega\Theta_z\mathbf{k} = -\mathbf{v}_E\Theta_{yz} + \boldsymbol{\omega}\Theta_z,$$

as

$$\mathbf{v}_E = \omega\mathbf{j}$$

is the velocity of point E, which is at a unit distance from point S along axis x relative to its centre of mass.

On the basis of Fig. 8.1 the position-vector $\mathbf{r}_S$ may be expressed as the sum of $\mathbf{l}_C$ and $\mathbf{e}$:

$$\mathbf{r}_S = \mathbf{l}_C + \mathbf{e}.$$

Accordingly the spin vector for point C is

$$\pi_C = -\mathbf{v}_E\Theta_{yz} + \boldsymbol{\omega}\Theta_z + \mathbf{l}_C \times m\mathbf{v}_S + \mathbf{e} \times m\mathbf{v}_S$$

and the moment of the supporting forces is

$$\mathbf{M}_C = \frac{\mathrm{d}\pi_C}{\mathrm{d}t} = -\mathbf{a}_E\Theta_{yz} + \dot{\boldsymbol{\omega}}\Theta_z + (\mathbf{l}_C \times m\mathbf{a}_x) + (\mathbf{e} \times m\mathbf{a}_S),$$

where

$\mathbf{a}_E$ is the acceleration of point E in axis x relative to its centre of mass;
$\mathbf{a}_S$ is the acceleration of point S.

In the case of uniform rotation $\boldsymbol{\omega}=\text{const.}$, so $\dot{\boldsymbol{\omega}}=0$, whereby

$$\mathbf{a}_E = -\omega^2\mathbf{i}$$

and

$$\mathbf{e}\times m\mathbf{a}_S = 0,$$

as the directions of the vectors $\mathbf{a}_S$ and $\mathbf{e}$ are identical.

The acceleration of the centre of mass is

$$\mathbf{a}_S = -\omega^2\mathbf{e} = -\omega^2 e\mathbf{e}_1,$$

where

$\mathbf{e}_1$ is the unit vector pointing in the direction of $\mathbf{e}$.

Accordingly the moment of the supporting forces about point C is

$$\mathbf{M}_C = \omega^2[(\Theta_\zeta-\Theta_\eta)\sin\varphi\cos\varphi\mathbf{i}-l_C me(\mathbf{k}\times\mathbf{e}_1)], \tag{3}$$

i.e. the moment of the supporting forces changes as compared with the stationary position, owing to the rotation of the rotor. As the geometry of the rotor has not changed, it is obvious that the supporting forces have changed.

The moment of the supporting forces was brought about by a moment of opposite sense created by the rotation of the rotor, the moment of the so-called centrifugal forces.

The moment of the centrifugal forces about point C is

$$\mathbf{M}_{FC}=-\mathbf{M}_C, \tag{4}$$

or

$$\mathbf{M}_{FC}=\omega^2[(\Theta_\eta-\Theta_\zeta)\sin\varphi\cos\varphi\mathbf{i}+\mathbf{l}_C me(\mathbf{k}\times\mathbf{e}_1)]. \tag{5}$$

The rotor is said to be imbalanced when the moment of the centrifugal forces, arising as a consequence of the rotation, produces a vibrating force and vibratory motions of variable magnitude in the bearings, depending on the dynamic properties of the bearings.

We distinguish between the following fundamental imbalanced states of the rotor in terms of the parameters appearing in relationship (5):

— statically imbalanced state;
— imbalanced moment;
— quasi-statically imbalanced state;
— dynamically imbalanced state.

8.1 THE STATICALLY IMBALANCED ROTOR

The rotor is statically imbalanced if one of its principal axes of inertia is parallel to the axis of rotation.

In relationship (3) $\varphi=0°$ or $\varphi=90°$.

The moment of the supporting forces about point C is

$$\mathbf{M}_C = -\omega^2 l_C me(\mathbf{k}\times\mathbf{e}_1). \tag{6}$$

The moment of the supporting forces is generated by the moment of the centrifugal force appearing as a consequence of the rotation of the rotor.

The moment of the centrifugal forces about point C is

$$\mathbf{M}_{FC} = \omega^2 l_C(\mathbf{k}\times me). \tag{7}$$

The product $U=me$ is called the magnitude of the imbalance. In practice it is given in units of mm g. The vector of the imbalance is the product of the mass generating the imbalance and the position vector of the centre of mass measured from the axis of rotation.

The magnitude of the imbalance vector $\mathbf{U}$ is the imbalance U. Its direction is determined by its angular position measured from some origin determined in advance, and its sense is pointing outwards from the axis of rotation. The vector of the imbalance is generally distributed over the whole rotor, but it can be reduced. In the case of static imbalance the resultant vector of imbalance is the only vector passing through the centre of mass perpendicular to the axis.

Let us place a mass m_1 in the perpendicular plane passing through the centre of mass at a distance r_1 from the axis of rotation of the grinding wheel, whose mass, m, is homogeneously distributed. The centre of mass, S, of the body was initially in the axis of rotation, but the centre of mass shifts, after introducing the mass m_1, to a distance e in the direction of m_1. The shift of centre of mass is

$$e = \frac{m_1}{m+m_1} r_1. \tag{8}$$

Generally the mass of the wheel exceeds the introduced mass, m_1, by several orders of magnitude ($m \gg m_1$). Therefore the shift in the centre of mass is given, to a good approximation, by

$$e = \frac{m_1}{m} r. \tag{9}$$

The shift in the centre of mass is given mostly in μm units (10^{-6} m):

$$\frac{1 \text{ mm g}}{1 \text{ kg}} = 1\ \mu\text{m}.$$

Note that a mass of 1 g placed at a distance of 1 mm from the axis of rotation of a rotor of mass 1 kg in a plane passing perpendicularly to the axis of rotation through its centre of mass causes a shift in the centre of mass of 1 μm.

Example. A component part of mass $m_1 = 150$ g is fixed at a distance $r_1 = 200$ mm in the plane which passes, perpendicular to the axis of rotation, through the centre of mass of a rotor, whose mass is $m = 100$ kg and has a homogeneous mass distribution — as shown in Fig. 8.2.

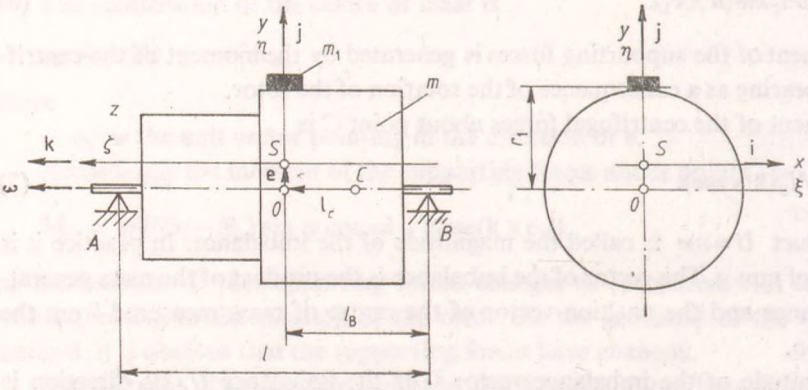


Fig. 8.2 A statically imbalanced rotor

Determine the magnitude of the imbalance of the rotor and the shift of the centre of mass, and also the magnitude of the supporting forces arising in the bearings due to the rotation, if the angular velocity is $\omega = 150$ rad/s ($n = 1432.4$/min), $l = 650$ mm, $l_B = 310$ mm.

Solution.

The magnitude of the imbalance is

$$U = m_1 r_1 = 150 \times 200 \text{ mm g} = 30\,000 \text{ mm g}.$$

The shift of the centre of mass is exactly

$$e = \frac{m_1}{m + m_1} r_1 = \frac{150}{100 \times 10^3 + 150} 200 \text{ mm} = 0.29955 \text{ mm} = 299.55 \text{ μm}.$$

The shift of the centre of mass, using the approximate formula, is

$$e = \frac{U}{m} = \frac{30\,000}{100 \times 10^3} \text{ mm} = 0.3 \text{ mm} = 300 \text{ μm}.$$

The moment of the supporting forces arising in the bearings is obtained with the help of relationship (3).

Let us place point C in bearing B; in this case

$$\mathbf{M}_B = -\omega^2 l_B m e (\mathbf{k} \times \mathbf{e}_1) = -150^2 \times 310 \times 10^{-3} \times 100 \times 0.3 \times 10^{-3} \times$$

$$\times (\mathbf{k} \times \mathbf{e}_1) \text{N m} = -209.25 (\mathbf{k} \times \mathbf{e}_1) \text{ N m}.$$

The moment of the supporting force A for point B is

$$lA(\mathbf{k} \times \mathbf{j}) = \mathbf{M}_B,$$

from which (as $\mathbf{e}=\mathbf{j}$)

$$A = \frac{\mathbf{M}_B}{l} = \frac{209.25}{650\times 10^{-3}}\,\mathrm{N} = -321.92\,\mathrm{N};$$

$$\mathbf{A} = -341.92\mathbf{j}\,\mathrm{N}\ \text{(pointing down)}.$$

To determine the reaction force **B**, let us place point C in bearing. A. The moment of the supporting force **B** is then

$$\mathbf{M}_A = -\omega^2(l-l_B)\,U\,(-\mathbf{k}\times\mathbf{e}_1) = +150^2(650-310)\,10^{-3}\times$$

$$\times 30\,000\times 10^{-6}(\mathbf{k}\times\mathbf{e}_1)\,\mathrm{N\,m} = 229.5\,(\mathbf{k}\times\mathbf{e}_1)\,\mathrm{N\,m}.$$

The moment of the supporting force **B** for point A is

$$\mathbf{M}_A = l_B\,(-\mathbf{k}\times\mathbf{j}).$$

On comparing the equations we have

$$B = -\frac{M_A}{l} = -\frac{229.5}{650\times 10^{-3}}\,\mathrm{N} = 353.08\,\mathrm{N}$$

and

$$\mathbf{B} = -353.08\mathbf{j}\,\mathrm{N}\ \text{(pointing down)}.$$

8.2 THE MOMENT-IMBALANCED ROTOR

A rotor is moment-imbalanced if its principal axis of inertia and its axis of rotation intersect each other at the centre of mass.

In expression (3)

$$0 < \varphi < \frac{\pi}{2} \quad \text{and} \quad \Theta_\xi = \Theta_\eta \neq \Theta_\zeta$$

and

$$e = 0.$$

Substituting these values we obtain the moment of the supporting forces in the form

$$\mathbf{M}_m = \omega^2(\Theta_\zeta - \Theta_\eta)\sin\varphi\cos\varphi\mathbf{i}. \tag{10}$$

The moment of the supporting forces is generated by the moment of the centrifugal forces brought about by the rotation of the rotor.

The moment of the centrifugal forces is

$$\mathbf{M}_{Fm} = \omega^2(\Theta_\eta - \Theta_\zeta)\sin\varphi\cos\varphi\,\mathbf{i}. \tag{11}$$

The vector of the moment-imbalance is

$$\mathbf{U}_m = (\Theta_\eta - \Theta_\zeta)\sin\varphi\cos\varphi\,\mathbf{i}. \tag{12}$$

The moment-imbalance is caused by a couple of forces. The moment-imbalance is the same at all points of the plane of the couple of forces. The vector of moment-

imbalance is equivalent — on the plane perpendicular to it — to two parallel, but opposing imbalance vectors of the same size.

Figure 8.3 shows two imbalances in the plane lying on the axis of the rotor, both of an identical magnitude, but of opposite senses. The magnitude of the moment-imbalance is

$$U_m = l_1 U \text{ mm}^2 \text{ g}. \tag{13}$$

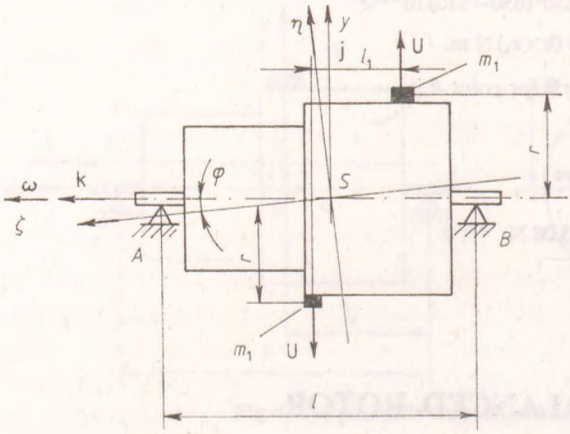


Fig. 8.3 Moment-imbalanced rotor

Knowing the moment-imbalance and the moments of inertia the angle enclosed by the axis of rotation and the principal axis of inertia may be determined:

$$U_m = (\Theta_\eta - \Theta_\zeta) \sin\varphi \cos\varphi = \frac{1}{2}(\Theta_\eta - \Theta_\zeta) \sin 2\varphi, \tag{14}$$

from which

$$\varphi = \frac{1}{2} \arcsin \frac{2U_m}{\Theta_\eta - \Theta_\xi}. \tag{15}$$

As this angle is generally small, $\sin\varphi \approx \varphi$, so in the case of small angles and for bodies of rotation,

$$\varphi = \frac{U_m}{\Theta_\eta - \Theta_\zeta}. \tag{16}$$

If the rotor is not a body of rotation, i.e. $\Theta_\eta \neq \Theta_\xi$, then the moment-imbalance must be decomposed into components in the directions of the axes x and y and the corresponding moments of inertia must be taken into account:

$$\varphi_{xz} = \frac{U_{mx}}{\Theta_\xi - \Theta_\zeta}, \tag{17}$$

$$\varphi_{yz} = \frac{U_{my}}{\Theta_\eta - \Theta_\zeta}. \tag{18}$$

Example. Two fittings, each of mass $m_1 = 150$ g, are threaded into the mantle of a rotor in a plane lying on the axis of rotation; they are positioned opposite each other, at a distance of $r = 200$ mm from the axis of rotation and at a distance of $l_1 = 350$ mm from each other, as shown in Fig. 8.3. Determine the moment-imbalance vector of the rotor and the reaction force arising in the bearings, if $l = 650$ mm is the distance between the bearings and $\omega = 150$ rad/s is the angular velocity of the rotor.

Solution.

The vector of the moment-imbalance is

$$\mathbf{U}_m = l_1 r m_1 (\mathbf{k} \times -\mathbf{j}) = +l_1 r m \mathbf{i} = 350 \times 200 \times 150 \mathbf{i} \text{ g mm}^2 = 150 \times 10^5 \mathbf{i} \text{ g mm}^2.$$

The moment of the supporting forces is

$$\mathbf{M}_m = -\omega^2 \mathbf{U}_m = -150^2 \times 105 \times 10^5 \mathbf{i} \frac{\text{g mm}^2}{\text{s}^2} = -236.25 \mathbf{i} \text{ N m}.$$

If the supporting force is assumed to point upwards, then

$$\mathbf{A} = A\mathbf{j};$$

$$lA(\mathbf{k} \times \mathbf{j}) = -0.65 A\mathbf{i} = 236.25\mathbf{i};$$

$$\mathbf{A} = \frac{236.25}{0.65} = 363.46 \text{ N};$$

$$\mathbf{A} = 363.46\mathbf{j} \text{ N (pointing up)},$$

and so

$$\mathbf{B} = -363.46\mathbf{j} \text{ N (pointing down)}.$$

Example. Determine the angle enclosed between the principal axis of inertia and the axis of rotation of a rotor whose moment-imbalance is $U_m = 0.0105$ kg m², $\Theta_\eta = 3.37$ kg m² and $\Theta_\zeta = 1.65$ kg m².

Solution.

$$\varphi = \frac{U_m}{\theta_\eta - \theta_\zeta} = \frac{0.0105}{3.37 - 1.65} \text{ rad} = 6.105 \times 10^{-3} \text{ rad} = 0.35° = 20.98''.$$

8.3 THE QUASI-STATICALLY IMBALANCED ROTOR

A rotor is quasi-statically imbalanced if its principal axis of inertia and its axis of rotation intersect each other at a point lying outside the centre of mass. This occurs in the case where the static imbalance vector and the moment-imbalance vector are perpendicular to each other. Let

$$\mathbf{e}_1 = \mathbf{j}; \quad e \neq 0; \quad 0 < \varphi < \frac{\pi}{2} \quad \text{and} \quad \Theta_\zeta \neq \Theta_\eta.$$

The moment of the supporting forces for any C point is, according to relationship (3)

$$\mathbf{M}_C = \omega^2[(\Theta_\zeta - \Theta_\eta) \sin\varphi \cos\varphi + l_C m e]\,\mathbf{i}, \tag{19}$$

because

$$\mathbf{k}\times\mathbf{j} = -\mathbf{i}.$$

The moment of the centrifugal forces generating the moment of the supporting forces, given by

$$\mathbf{M}_{FC} = \omega^2[(\Theta_\eta - \Theta_\zeta)\sin\varphi\cos\varphi - l_C me]\,\mathbf{i}, \tag{20}$$

consists of two parts, namely the vector of the moment-imbalance,

$$\mathbf{U}_m = (\Theta_\eta - \Theta_\zeta)\sin\varphi\cos\varphi\mathbf{i}, \tag{21}$$

and the vector-moment of the static imbalance,

$$\mathbf{l}_C\times\mathbf{U}_S = l_C me\mathbf{i}. \tag{22}$$

The moment-imbalance may be conceived as a couple, whose imbalance vectors have the same direction as that of the static-imbalance vector. If we determine the distance between the imbalance vectors forming the couple in such a way that the magnitude of the vectors equals that of the static-imbalance vector, then the vector of the quasi-static imbalance may be characterized by a single imbalance vector lying outside of the centre of gravity. Namely, according to Fig. 8.4,

$$U_m\mathbf{i} = -l_Q\mathbf{k}\times U_S\mathbf{j} = l_Q U_S\mathbf{i},$$

from which

$$l_Q = \frac{U_m}{U_S}. \tag{23}$$

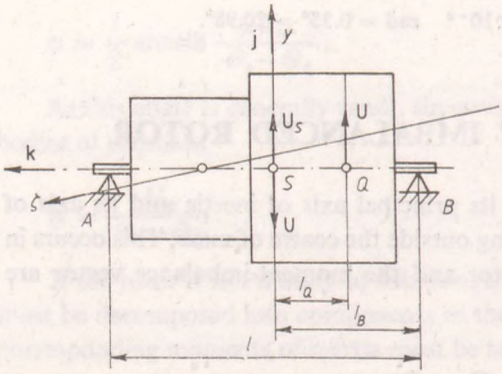


Fig. 8.4 The quasi-statically imbalanced rotor

Therefore, the moment-imbalance vector of the quasi-static imbalance and the vector of the imbalance arising at the centre of mass are equivalent to the single imbalance vector $U = U_S\mathbf{j}$ acting at the point Q and marked by the position vector $-l_Q\mathbf{k}$.

Example. Determine the magnitude of the supporting forces arising in the bearings as a result of the imbalance of a rotor of mass $m=100$ kg, whose centre of mass shifting vector is $\mathbf{e}=300\,\mathbf{j}\,\mu$m; its moment of inertia is $\Theta_\zeta=1.65$ kg m², $\Theta_\xi=\Theta_\eta=3.37$ kg m²; the principal axis of inertia makes an angle with the axis of rotation of $\varphi_{\xi\zeta}=0.35°$; the angular velocity is $\omega=150$ rad/s; $l=650$ mm; $l_B=310$ mm (see Fig. 8.4).

Determine the position and the magnitude of the imbalance vector.

Solution.

As the shifting vector of the centre of mass, the principal axis of inertia and the axis of rotation are all in the same plane, the imbalance of the rotor is quasi-static.

The moment-imbalance component of the vector is, according to relationship (21),

$$\mathbf{U}_m(\Theta_\eta-\Theta_\zeta)\sin\varphi\cos\varphi\mathbf{i}=$$
$$=(3.37-1.65)\sin 0.35°\cos 0.35°\,\mathbf{i}\text{ kg m}^2=0.015\mathbf{i}\text{ kg m}^2.$$

The static-imbalance vector is

$$\mathbf{U}_S=me\mathbf{j}=100\times300\times10^{-6}\,\mathbf{j}\text{ kg m}=0.03\mathbf{j}\text{ kg m}.$$

The moment of the supporting forces for point B is

$$\mathbf{M}_B=\omega^2(-U_m+U_S l_b)\,\mathbf{i}=150^2(-0.0105+0.03\times310\times10^{-3})\,\mathbf{i}=$$
$$=-27.14\mathbf{i}\text{ N m}.$$

On the other hand, the moment of the supporting force $\mathbf{A}=A\mathbf{j}$ arising in the bearing A may be written as

$$\mathbf{M}_B=lA(\mathbf{k}\times\mathbf{j})=-lA\mathbf{i}=-650\times10^{-3}A\mathbf{i}.$$

On comparing the above we obtain

$$A=\frac{-27.14}{-650\times10^{-3}}=41.77\text{ N}.$$

The vector of the supporting force is

$$\mathbf{A}=41.77\mathbf{j}\text{ N}.$$

Similarly the moment of the supporting force B is

$$\mathbf{M}_A=\omega^2[-U_m-U_S(l-l_B)]\mathbf{i}=$$
$$=150^2[-0.0105-0.03(650-310)10^{-3}]\,\mathbf{i}=-465.90\text{ N m}.$$

The moment of the supporting force $\mathbf{B}=B\mathbf{j}$ arising in the bearing B is, for point A,

$$\mathbf{M}_A=lB(-\mathbf{k}\times\mathbf{j})=0.65B\mathbf{i}.$$

Accordingly

$$B=\frac{-465.90}{0.65}=-716.76\text{ N}.$$

The vector of the supporting force is

$$\mathbf{B}=-714.74\mathbf{j}\text{ N}.$$

The point of application, Q, of the imbalance, $\mathbf{U}_S = 0.03\mathbf{j}$ kg m, causing the quasi-static imbalance is to the right of the centre of mass at a point determined by the vector $\mathbf{l}_Q = -l_Q\mathbf{k}$:

$$U_m\mathbf{i} = -l_Q\mathbf{k}\times U_S\mathbf{j} = l_Q\mathbf{U}_S\mathbf{i}.$$

Accordingly,

$$l_Q = \frac{U_m}{U_S} = \frac{0.0105}{0.03}\,\text{m} = 0.35\,\text{m}.$$

So the position-vector of point Q from the centre of mass S is

$$\mathbf{l}_Q = -0.35\mathbf{k}\ \text{m}.$$

Point Q is outside the span between the bearing supports, at a distance of 400 mm from bearing B.

8.4 THE DYNAMICALLY IMBALANCED ROTOR

A rotor is dynamically imbalanced if its principal axis of inertia through its centre of gravity and its axis of rotation are divergent (see Fig. 8.4). In relationship (3) the shift of the centre of mass is

$$\mathbf{e}_1 = \cos\alpha\mathbf{i} + \sin\alpha\mathbf{j}. \tag{24}$$

The moment of the supporting forces for point C is

$$\mathbf{M}_C = \omega^2[(\Theta_\zeta - \Theta_\eta)\sin\varphi\cos\varphi\mathbf{i} - l_C me(\mathbf{k}\times\mathbf{e}_1)]. \tag{25}$$

The moment of the supporting forces is generated by the centrifugal forces appearing as a consequence of the rotation of the rotor.

The moment of the centrifugal forces is

$$\mathbf{M}_{FD} = \omega^2[(\Theta_\eta - \Theta_\zeta)\sin\varphi\cos\varphi\mathbf{i} + l_C me(\mathbf{k}\times\mathbf{e}_1)]. \qquad 26)$$

The vector

$$\mathbf{U}_D = (\Theta_\eta - \Theta_\zeta)\sin\varphi\cos\varphi\mathbf{i} + me(\mathbf{k}\times\mathbf{e}_1) \tag{27}$$

is the dynamic imbalance vector, composed of a moment-imbalance vector and a static-imbalance vector positioned relative to each other so that they are irreducible:

$$\mathbf{U}_D = U_m\mathbf{i} + U_S(\mathbf{k}\times\mathbf{e}).$$

Example. Determine the magnitude of the supporting forces arising in the bearings of a rotor of mass $m = 100$ kg, if the shift of the centre of mass is $e = 300\ \mu$m, the angle enclosed between the direction of the shift of the centre of mass and the axis x is $\alpha = 30°$, the angle between the principal axis of inertia of the rotor and its axis of rotation is $\varphi = 0.35°$, $\Theta_\zeta = 1.65$ kg m², $\Theta_\xi = \Theta_\eta = 3.37$ kg m², $\omega = 150$ rad/s, $l = 650$ mm, $l_B = 310$ mm (Fig. 8.5).

Solution.

The imbalance of the rotor is composed of two parts:

— the moment-imbalance and

— the static-imbalance.

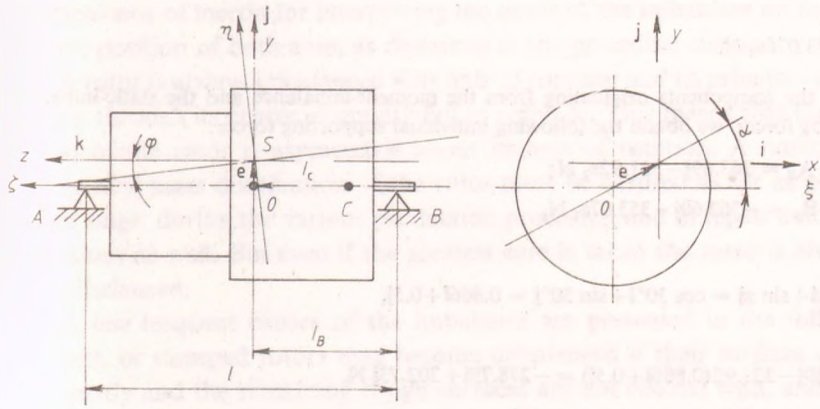


Fig. 8.5 The dynamically imbalanced rotor

The moment-imbalance vector is given in the preceding example. Accordingly,

$\mathbf{U}_m = (\Theta_\eta - \Theta_\xi) \sin \varphi \cos \varphi \mathbf{i} = 0.0105\mathbf{i}$ kg m².

So the moment of the supporting force is

$\mathbf{M}_m = -\omega^2 \mathbf{U}_m = -150^2 \times 0.0105\mathbf{i} = -236.398\mathbf{i}$ N m.

The vectors of the supporting forces obtained from the above are

$\mathbf{A}_m = 363.69\mathbf{j}$ N (pointing up),

$\mathbf{B}_m = -363.69\mathbf{j}$ N (pointing down).

The moments of the supporting forces generated by the static-imbalance are

$\mathbf{M}_{BS} = -l_B me(\mathbf{k} \times \mathbf{e}_1) = -209.25(\mathbf{k} \times \mathbf{e}_1)$ N m

and

$\mathbf{M}_{AS} = -(l - l_B)\, U\, (-\mathbf{k} \times \mathbf{e}_1) = 229.5(\mathbf{k} \times \mathbf{e}_1)$ N m.

The direction of the supporting forces generated by the static-imbalance agrees with that of the imbalance. The moments $A_S = A_S\mathbf{e}_1$ and $\mathbf{B}_S\mathbf{e}_1$ of the supporting forces may be written as

$\mathbf{M}_{BS} = lA_S(\mathbf{k} \times \mathbf{e}_1) = 0.65\mathbf{A}_S(\mathbf{k} \times \mathbf{e}_1)$

and

$\mathbf{M}_{AS} = -lB_S(\mathbf{k} \times \mathbf{e}_1) = -0.65\mathbf{B}_S(\mathbf{k} \times \mathbf{e}_1)$.

By comparing these equations we obtain

$$A_a = -\frac{209.25}{0.65}\,\mathrm{N} = 321.92\ \mathrm{N},$$

and

$\mathbf{A}_S = -321.92\mathbf{e}_1$ N;

further,

$$B_S = \frac{229.5}{-0.65} = -353.076\ \mathrm{N}.$$

and

$$\mathbf{B}_S = -353.076\mathbf{e}_1 \text{ N}.$$

Summing the components originating from the moment-imbalance and the static-imbalance of the supporting forces, we obtain the following individual supporting forces:

$$\mathbf{A} = \mathbf{A}_m + \mathbf{A}_S = 363.69\mathbf{j} - 321.92\mathbf{e}_1 \text{ N};$$

$$\mathbf{B} = \mathbf{B}_m + \mathbf{B}_S = -363.69\mathbf{j} - 353.07\mathbf{e}_1 \text{ N}.$$

As

$$\mathbf{e}_1 = \cos\alpha\mathbf{i} + \sin\alpha\mathbf{j} = \cos 30°\mathbf{i} + \sin 30°\mathbf{j} = 0.866\mathbf{i} + 0.5\mathbf{j},$$

then

$$\mathbf{A} = 363.69\mathbf{j} - 321.92(0.866\mathbf{i} + 0.5\mathbf{j}) = -278.79\mathbf{i} + 202.73\mathbf{j} \text{ N}.$$

The magnitude of the supporting force **A** is

$$A = (279.79^2 + 202.73^2)^{1/2} = 344.70 \text{ N},$$

and the angle enclosed between its direction and the axis x is

$$\alpha_A = \text{arc tg}\,\frac{202.73}{-278.79} = 143.97°.$$

The supporting force **B** is

$$\mathbf{B} = -363.69\mathbf{j} - 353.076(0.866\mathbf{i} + 0.5\mathbf{j}) = -305.77\mathbf{i} - 540.22\mathbf{j} \text{ N}.$$

The magnitude of the supporting force **B** is

$$B = (305.77^2 + 540.22^2)^{1/2} = 620.76 \text{ N},$$

and the angle enclosed between its direction and the axis x is

$$\alpha_B = \text{arc tg}\,\frac{-540.22}{-305.77} = 240.49°.$$

8.5 THE CAUSE OF THE IMBALANCE

What is the cause of the imbalance of the rotors? If we place the end of the pointed shaft of a gyroscope on a horizontal plane and rotate it without supporting the shaft radially, the gyroscope will rotate about one of the principle axes of inertia that pass through its centre of gravity, as about a free axis. In this case we cannot speak of imbalance, even if we fix a mass at a point on the circumference of the gyroscope and rotate the rotor again. The rotor will rotate as before, but around its new principal axis of inertia, as this axis has shifted under the effect of the applied mass. The rotor in the second case can be said to be neither imbalanced nor balanced. We cannot speak of the imbalanced state of a rotor unless its axis of rotation is specified unequivocally. The axis of rotation is determined by holding the rotor in the bearings. The imbalance of the rotor presupposes a knowledge of both the axis of rotation and the

principal axis of inertia for interpreting the mode of the imbalance on the basis of the relative position of both axes, as described in the preceding chapter.

A rotor is always imbalanced if its axis of rotation and its principal axis of inertia passing through its centre of gravity fail to coincide. This means that the mass distribution of the rotor is asymmetric about its axis of rotation. A homogeneous and symmetrical mass distribution of the rotor must be ensured as far as possible in the design stage, during the various production processes, and in repair and maintenance operations as well. But even if the greatest care is taken the rotor is always more or less imbalanced.

A few frequent causes of the imbalance are presented in the following. Cast, wrought, or stamped rotors may become unbalanced if their surfaces are machined only partly and the remaining rough surfaces are not coaxial with, and possibly not symmetric with respect to, the axis of rotation of the rotor (Fig. 8.6).

Cast, wrought, drawn, or rolled material may contain cavities, cracks, or unevennesses causing a nonuniform mass distribution relative to the axis of rotation (Fig. 8.7).

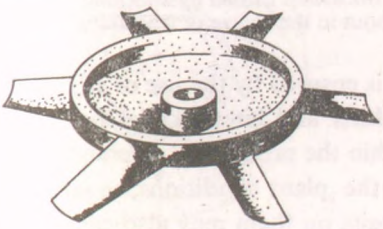


Fig. 8.6 Unmachined cast component

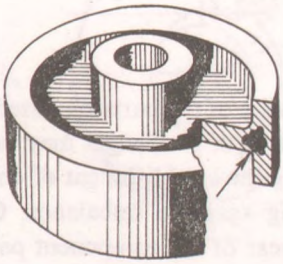


Fig. 8.7 Cavity in a rotating component

In some cases a symmetric mass distribution with respect to the axis of rotation cannot be ensured by the design, or during production (e.g. in the case of crankshafts, or electric motors with wound rotors, whose closing coil is more distant from the axis of rotation than its opposite counterpart) (Fig. 8.8).

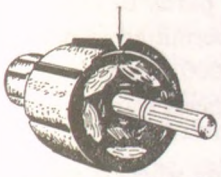


Fig. 8.8 Imbalance caused by the mass of the closing coil

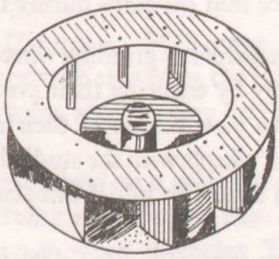


Fig. 8.9 Easily deformable components

Some elements of elastic rotors change their radial position at the working speed — e.g. the impeller of fans prepared from plate material (see Fig. 8.9), or the rubber tyres of the wheels of motor vehicles. Rotors with thin shafts may also become imbalanced during operation.

Rotors made of welded or riveted plates are imbalanced even if they are produced with the greatest care. Imbalance is caused also by machined mantle surfaces and lateral surfaces of various machine parts, which may shift relative to each other within the tolerance limits of the production process, resulting in radial and lateral knocking (Fig. 8.10).

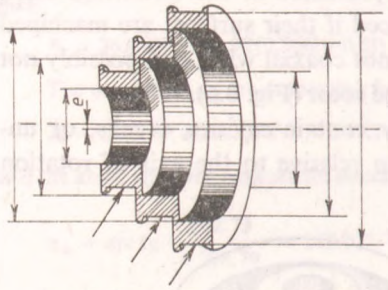


Fig. 8.10 Imbalance caused by knocking brought about in the course of machining

The coaxiality of the parts mounted on a shaft is ensured by the use of adjusting flanges. The fitting of the flanges may cause radial shifts and lateral knocking of the parts, impairing the establishment of coaxiality within the production tolerance limits, and leading again to imbalance. Changes in the plant conditions, usage, the non-uniform wear of the component parts, or deposits on them may also cause imbalance.

In the following the results of plant measurements performed to reveal various causes and variations of the imbalance of smooth grinding wheels of 400× ×60×203 mm, used in centre grinding machines, are described. According to experience, the imbalance of a grinding wheel is caused mainly by

— production faults,
— eccentric mounting and
— the imbalance that develops during the grinding operations.

The manufacturers of grinding wheels try to produce them with as little imbalance as possible, partly by ensuring their precise geometrical form and partly by ensuring the homogeneous distribution of their material. The imbalance permitted in manufactured grinding wheels with the studied dimensions is 12 000...20 000 mm g, depending on the grain size. The initial imbalance of all the examined wheels was less than 10 000 mm g.

Any imbalance resulting from the manufacture of the grinding wheel is mainly due to inhomogeneity. This imbalance may be eliminated during the mounting of the wheel. When the wheel has suffered some wear and after the control truing of the

wheel, its mass distribution changes, so the balancing masses introduced into the wheel hub cause a secondary imbalance (Fig. 8.11).

On determining the imbalance caused by eccentric mounting of the grinding wheel, we assume that the form of the grinding wheel is regular and that its mass distribution is homogeneous, so its centre of mass coincides with its geometrical centre. The bore of the wheel must always be greater than the fitting flange of the receiving hub. The eccentricity of the studied wheel size — with the specified tolerance limits taken into account — amounts in the least favourable case to

$$e = \frac{1}{2}(d_k - d_a) = 0.65 \text{ mm}$$

(see Fig. 8.12). With the mass of the wheel ($m = 11.70$ kg) taken into account, the imbalance due to eccentric mounting is merely $U = em = 7605$ mm g.

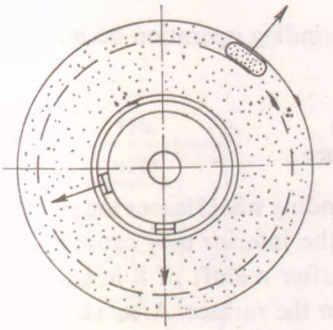


Fig. 8.11 Imbalance arising in the manufacture of a grinding wheel

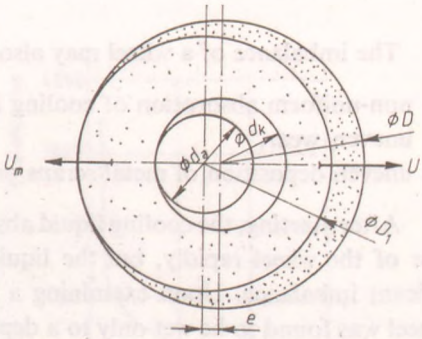


Fig. 8.12 Eccentric mounting of the grinding wheel

After eliminating the imbalance of the wheel in its eccentric position, then by truing it in the machine (by removing the dotted part in Fig. 8.12), the rotor becomes imbalanced again. The centre of mass shifts, on removing the dotted part, by

$$e_t = -e\frac{d_k^2}{D_1^2 - d_k^2} = -0.225 \text{ mm}.$$

This corresponds to an imbalance of $U_t = e_t m = -2638$ mm g. The negative sign means that the imbalance shifted to the opposite side from the initial imbalance. The magnitude of the imbalance does not vary any more during further truing if the mass distribution of the wheel is homogeneous.

Let us imagine that we have mounted a new wheel on a grinding machine, eliminated its initial imbalance and then started truing it gradually. Fig. 8.13 shows the variation of the imbalance versus the reduction in wheel diameter. The mass distribution of the studied wheel was not homogeneous. After truing the wheel to the minimum permitted diameter, an imbalance of 2520 mm g was obtained.

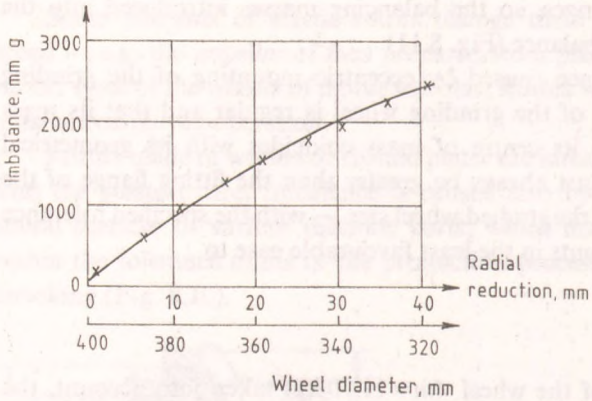


Fig. 8.13 Variation of the imbalance of a grinding wheel versus the reduction of its diameter

The imbalance of a wheel may also vary during a grinding operation because of

— non-uniform absorption of cooling liquid,
— uneven wear,
— uneven deposition of metal scraps on the wheel surface.

After starting, the cooling liquid absorbed by the grinding wheel leaves the perimeter of the wheel rapidly, but the liquid remaining in the interior may cause a significant imbalance. Upon examining a grinding wheel after a shift of 8 hours, the wheel was found to be wet only to a depth of 6 mm under the surface. After stopping the wheel the liquid was absorbed to a depth of 30 mm, causing an imbalance of 10 mm g.

We have also studied the variation of the imbalance in the case of grinding wheels saturated with cooling liquid. The mass of a saturated grinding wheel increases by 2.4 kg. Figure 8.14 shows the variation of the imbalance of the rotating wheel with time.

Owing to gravity, the liquid absorbed by the wheel accumulates after grinding in the lower part of the wheel.

The wheel was saturated with cooling liquid as shown in Fig. 8.15. Figure 8.16 illustrates how the imbalance of the saturated wheel varies with time. The major part of the liquid left the wheel after the start, but the initial imbalance of the wheel could only be obtained 2 hours later.

The wear of the grinding wheel is a function of the cutting depth, if it is not influenced by other factors. If the wheel is imbalanced, the path of its centre is almost elliptical in relation to the workpiece; when the centre of the wheel runs over the section of the elliptical path which is closest to the workpiece, the cutting depth and the wear of the wheel both exceed the values corresponding over the section of the elliptical path which is furthest from the workpiece.

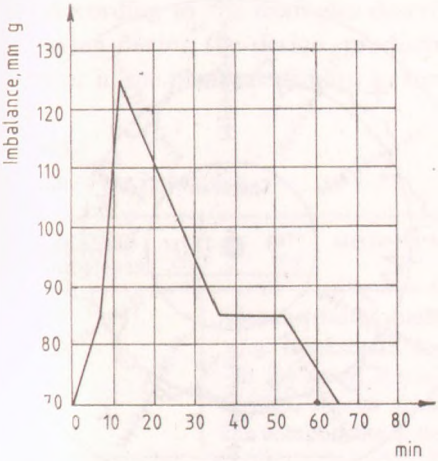


Fig. 8.14 Variation of the imbalance of a wheel saturated with cooling liquid

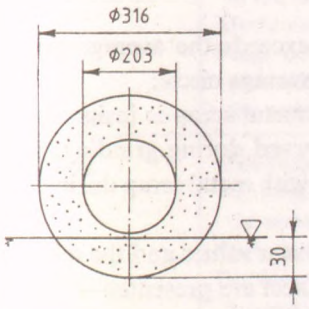


Fig. 8.15 Saturation of part of the wheel with cooling liquid

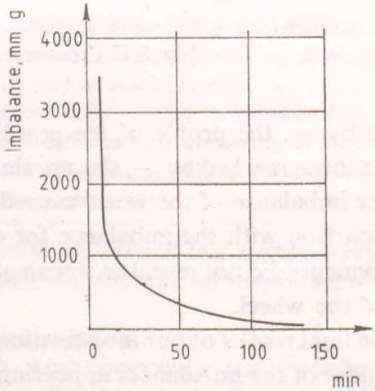


Fig 8.16 Variation of the imbalance versus time

The workpiece may be regarded as a circular truing tool, which shapes — though slowly — the form of the wheel in correspondence with the relative motions, and so the form of the grinding wheel deviates from a circle.

Figure 8.17 illustrates the circularity diagrams of two grinding wheels.

Figure 8.17(a) shows the circularity of a wheel, after truing in the balanced state, at 5000× magnification. The maximum fault in the wheel form is 2.5 μm.

Figure 8.17(b) presents the form of a wheel after grinding 32 workpieces, at a magnification of 500×. We have increased the imbalance of the grinding wheel gradually to 5000 mm g. The mass causing the imbalance was always in the same direction on the wheel. The maximum observed fault in circularity amounted to 40 μm. The imbalance due to uneven wear amounted to 1820 mm g. The circularity diagram also shows the ovality of the wheel in addition to the local wear. At the places

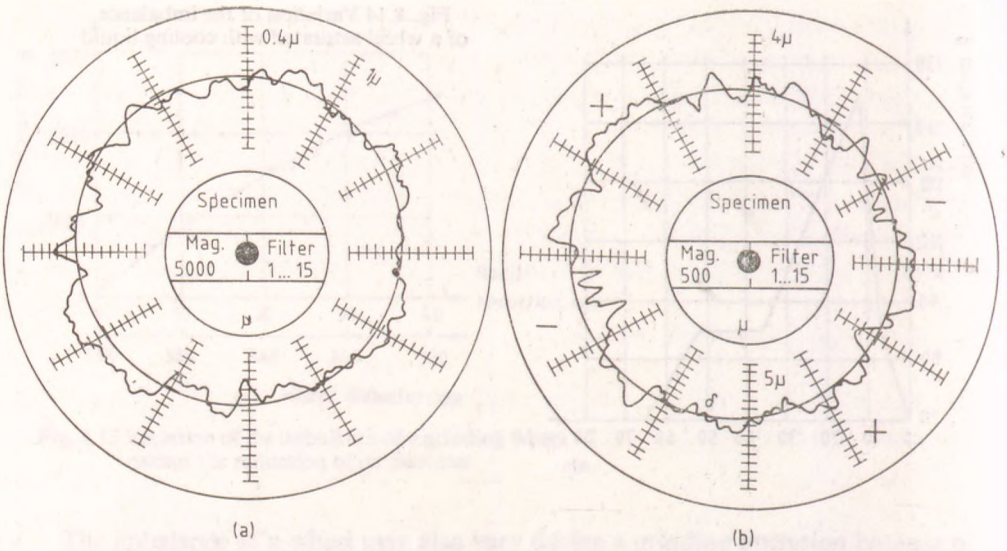


Fig. 8.17 Cross-sections of the grinding wheels

marked by +, the profile of the grinding wheel always exceeds the average circle, while at those marked by −, the profile falls short of the average circle.

The imbalance of the wheel caused by clogging with metal scrap is insignificant in comparison with the imbalance for other reasons observed during grinding. Our measurements did not reveal an imbalance due to clogging with metal scrap during the wear of the wheel.

The final results of our investigations in terms of the mean values and the relative magnitudes of the imbalances appearing in the grinding wheel are presented in Table 8.1, with the effect causing the least imbalance being taken as unity, to which the effects of the other factors are compared. The maximum imbalance is caused by eccentric mounting of the wheel.

Table 8.1

Causes of the imbalance of the grinding wheel

Serial number	Origin of the imbalance of the grinding wheel	Imbalance U mm g	Relative number
1	Absorption of cooling liquid (after 10 minutes rotation)	910	1
2	Uneven wear and clogging	1820	2
3	Total wear of the wheel	2520	2.78
4	Imbalance of the trued wheel after mounting	2650	2.91
5	Eccentric mounting of the wheel	7605	8

According to the examples described above, the imbalance is caused by faults occurring during the design, production and mounting of the components, and by changes in the plant conditions, as surveyed individually in Table 8.2.

Table 8.2

Causes of the imbalance

Sources of imbalance	Causes of the imbalance
Design	Asymmetrically rotating parts, arising because of the working conditions (e.g. crankshafts, asymmetric parts of machines). Unmachined surfaces on the rotor. Radial and lateral knocking due to loosely fitting joints. Circlips, shorter than the grooves. The components of the bearing fitted with too much play.
Production of the components	Inhomogeneity of materials (e.g. cavities, rolled-in inclusions, variations in thickness). Mass shifting caused by the production process (e.g. core shifting, welded/soldered junctions, stamped parts, shrinkage, or knocking of the cylinders in the case of calendered parts, play in bearings, etc.). Machining inaccuracies (e.g. eccentricity, ovality, non-coaxiality). Form defects due to permanent deformations appearing in the course of machining (e.g. deformation of components caused by chucking, or by the cutting force, which may be permanent).
Mounting	Mounting of complementary retaining elements (e.g. circlips, wedges, cotter pins, bolts of various lengths, washers, nuts, asymmetric adjustment of the same, etc.). Stresses due to the uneven tightening of bolts. Displacement of inadequately fixed components (e.g. "wandering" of incorrectly fitted inner rings of rolling bearnigs, leading to variable imbalances). Shifting of the mass centres of component groups (e.g. coiling). Inaccurate running of component groups (e.g. radial and axial knocking of couplings).
Plant conditions	Usage, wear of plain bearings, fatigue wear of roller bearings, uneven wear of grinding wheels and vehicle tyres, abrasion (e.g. of mud pumps), etc. Cavitation in hydrodynamic machines, mass shifting due to deposition (e.g. of limestone or coal dust on the rotors in mines). Deformation of the rotor (e.g. by shifting of coils). Overloads causing elastic or plastic deformation (e.g. of drive shafts), thermal deformations (e.g. of fans, turbines and turbo-chargers). Changes of the load (e.g. uncertainty in the mass distribution in washing machines). Variation of driving or loading moments for mechanical, electrical, hydraulic, or tribological reasons.

8.5.1 EFFECTS OF THE IMBALANCE

The bearings of an imbalanced rotor are loaded — in addition to their static load — also by a periodically varying force, which is proportional to the square of the speed of the rotor. Frequently the dynamic load may exceed the static load of the bearing. The bearings — not dimensioned for this excess load — may suffer untimely failure because of overloading.

The bearings transmit their load via the bearing housing to the machine frame; moreover the vibrations are transferred by the ground to other adjacent machines. The vibrating machine components are fatigued and may break. The vibrations transferred to the environment may impede the operation of high precision machines or measuring devices in the vicinity. The operation of certain machines may even be prevented by the vibrations.

In the case of grinding and fine-boring machines the position of the tool with respect to the workpiece may be changed by the vibration, so the form-trueness of the machined part will be unsatisfactory.

The vibrations due to imbalance may cause noise, which may affect the operators physiologically and reduce their working capacity.

The force and the vibration of the bearings caused by an imbalanced rotor depends on the dynamic ratio of the machine, the rigidity of the bearing, the foundation of the machine, the ratio between the masses of the stator and the rotor, the speed of the machine, etc. So the imbalanced state of a rotor cannot be judged directly from the amplitude of the induced force or vibration. Generally the effect of the static imbalance exceeds that of the moment-imbalance.

Instead of studying the dynamic conditions of a rotor supported by bearings in two places, which is more complex, we shall examine them for the simpler case of a vibrating system with one degree of freedom, whose results may also be extended in principle to rotors with two supports.

Figure 8.18 shows a model of the rotor and the stator of the machine.

The mass q, situated at a distance r from the axis of rotation of a rotor supported by a bearing in the machine frame of mass m_a, causes a static imbalance of $U=rq$. The spring constant of the machine frame is denoted by c, its damping factor by k.

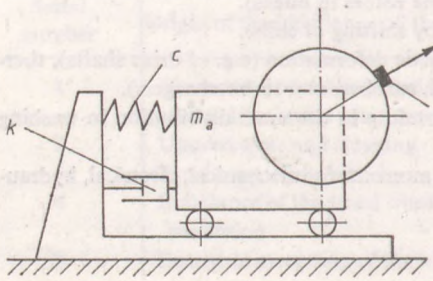


Fig. 8.18 The model of the machine

The imbalance **U** rotating with the rotor keeps the machine frame vibrating. The equation of motion, written in terms of complex quantities, is

$$m_a\ddot{\mathbf{z}}+k\dot{\mathbf{z}}+\frac{1}{c}\mathbf{z} = U\omega^2 e^{j\omega t}. \tag{28}$$

The particular solution of the inhomogeneous differential equation, i.e. the complex vector of the deflection of the mass, is sought in the form

$$\mathbf{z} = ze^{j\omega t}, \tag{29}$$

by which

$$j\omega\left(jm_a\omega+k+\frac{1}{jc\omega}\right)\mathbf{z} = U\omega^2 e^{j\omega t}. \tag{30}$$

Introducing the expression

$$\mathbf{Z} = k+j\left(m_a\omega-\frac{1}{c\omega}\right) = Z_e^{j\psi} \tag{31}$$

to denote the complex resistance, the mechanical impedance of the vibrating system, we obtain the solution

$$\mathbf{z} = \frac{U\omega}{j\mathbf{Z}}e^{j\omega t}. \tag{32}$$

For the argument ψ of **Z** we may write

$$\tan\psi = \frac{m_a\omega-\dfrac{1}{c\omega}}{k}. \tag{33}$$

As

$$j\mathbf{Z} = Ze^{j\left(\psi+\frac{\pi}{2}\right)} = Ze^{j\varphi}, \tag{34}$$

so

$$\tan\varphi = \frac{k}{\dfrac{1}{c\omega}-m_a\omega}. \tag{35}$$

The projecting angular velocity of the natural vibration of the undamped system is

$$\alpha^2 = \frac{1}{m_a c}. \tag{36}$$

Let us introduce the notation

$$D = \frac{k}{2m_a\alpha} \tag{37}$$

to characterize the damping of the system. So the solution of the equation of motion is

$$z = \frac{U\omega}{Z} e^{j(\omega t - \varphi)}, \tag{38}$$

where the phase angle is

$$\varphi = \arctan \frac{2D\left(\frac{\omega}{\alpha}\right)}{1-\left(\frac{\omega}{\alpha}\right)^2}. \tag{39}$$

The amplitude of the forced vibration of the machine frame is

$$z = \frac{U\omega}{Z} = \frac{U}{m_a} \frac{\left(\frac{\omega}{\alpha}\right)^2}{\left[1-\left(\frac{\omega}{\alpha}\right)^2\right]^2 + 4D^2\left(\frac{\omega}{\alpha}\right)^2} \tag{40}$$

Figure 8.19 and Fig. 8.20 show the amplitude and the phase angle of the vibration, respectively, versus ω/α, while the polar diagram in Fig. 8.21 illustrates the variation of the vibration amplitude and the phase angle for two different values of damping. The numbers indicated beside the curves are values of ω/α.

The distance between the individual points and the zero point of the curve is the amplitude of the forced vibration of the machine frame, and the angle φ made with the zero line is the phase angle, as the exciting force is on the zero line. The relationships obtained in this way enable the behaviour of the machine frames to be judged in principle and are suitable for classifying balancing machines whose machine-frame bearings are supplied with sensing devices enabling the supporting force to be

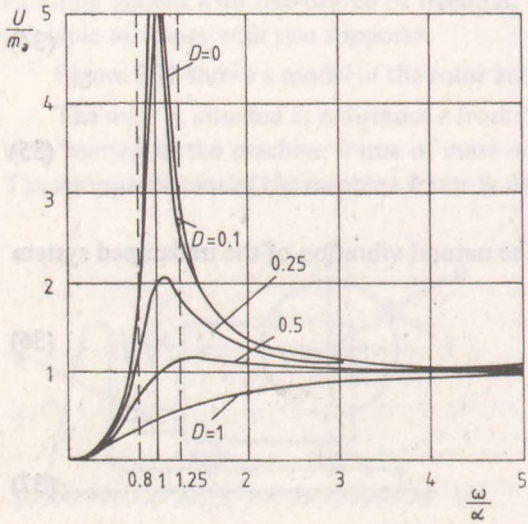


Fig. 8.19 Variation of the amplitude versus ω/α

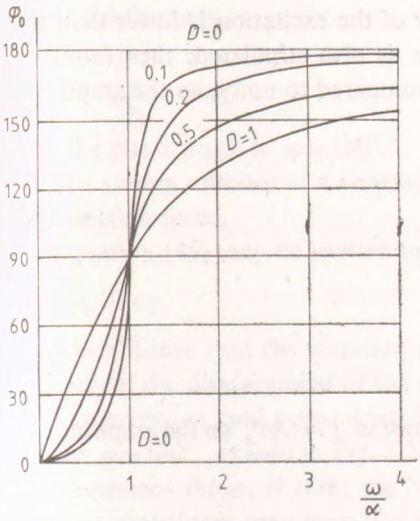


Fig. 8.20 Variation of the phase angle versus ω/α

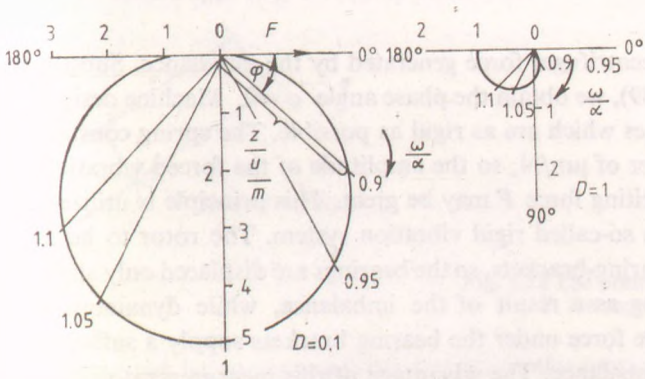


Fig. 8.21 Locus curves at two different damping values

measured, or the vibration of the bearing caused by the imbalanced rotor. The frame may be elastic or relatively rigid.

For the sake of simplicity, damping is neglected, and so $D=0$. Accordingly, on the basis of relationship (40), the amplitude of the forced vibration is

$$z = \frac{U}{m_a} \frac{\left(\frac{\omega}{\alpha}\right)^2}{1-\left(\frac{\omega}{\alpha}\right)^2}. \tag{41}$$

Three ranges may be distinguished depending on the ratio ω/α:

(a) the subresonance range,
(b) the resonance range and
(c) the supraresonance range.

(a) In the subresonance range the frequency of the excitation is lower than the eigenfrequency of the machine frame. If $\omega \ll \alpha$ is also stipulated, then $(\omega/\alpha)^2$ in the denominator of expression (41) is negligible compared to unity, so the amplitude of the vibration is

$$z \cong \frac{U}{m_a}\left(\frac{\omega}{\alpha}\right)^2. \tag{42}$$

With relationship (36) taken into account,

$$z = U\omega^2 c. \tag{43}$$

As the centrifugal force caused by the imbalance is $F = U\omega^2$, so the amplitude of the vibration is

$$z = Fc, \tag{44}$$

i.e. it is proportional to the centrifugal force generated by the imbalance. Substituting $D \cong 0$ into relationship (39), we obtain the phase angle $\varphi \approx 0$. Machine designers strive to attain machine frames which are as rigid as possible. The spring constant c of such a frame is of the order of μm/N; so the amplitude of the forced vibration is relatively low, though the exciting force F may be great. This principle is utilized in the balancing machines of the so-called rigid vibration system. The rotor to be balanced is supported in rigid bearing-brackets, so the bearings are displaced only slightly by the excitation force arising as a result of the imbalance, while dynamometers introduced into the flux of the force under the bearing brackets supply a sufficiently large signal for sensing the imbalance. The advantage of this measurement principle is that neither the mass of the rotor nor that of the machine frame appears in the relationship, while its disadvantage is that it reacts noticeably to variations in the speed.

(b) The resonance range. If the frequency of the excitation force agrees with the eigenfrequency of the system, i.e. $\omega = \alpha$, then the denominator of equation (41) tends to zero and the amplitude of the deflection increases beyond all limits. The phase angle is $\varphi = 90°$. Resonance must be intentionally avoided in the machines, with regard to the risk of breakage. Formerly, the effect of the increasing resonance amplitude was utilized in balancing machines that operate on a mechanical principle; but in practice the 5–7-fold gain attainable in this way is negligible compared with the amplification possible with electronic circuits, so the resonance principle is no longer applied in balancing machines.

(c) The supraresonance range. In this range the frequency of the excitation force exceeds the eigenfrequency. If $\omega \gg \alpha$ is stipulated, then in the denominator of equation (41) 1 is negligible compared with $(\omega/\alpha)^2$, and the amplitude of the forced vibra-

tion of the frame is

$$z = \frac{U}{m_a}, \tag{45}$$

while the phase angle is $\varphi \cong 180°$.

The system vibrates at a constant amplitude, independently of the frequency of the excitation force.

Equation (45) may be written in the form

$$zm_a = rq, \tag{46}$$

which establishes that the common centre of mass of the masses m_a and q remains stationary if the displacement of the mass m_a is unimpeded in any direction.

The masses m_a and q are situated at opposite sides of the common centre of mass because $\varphi = 180°$, as seen in Fig. 8.22. An elastic machine frame, i.e. operation in the supraresonance range, is rare; the bearing blocks of rotors in certain washing machines or centrifuges may have this behaviour.

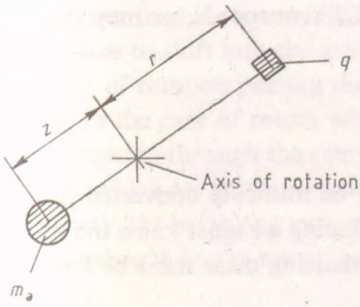


Fig. 8.22 The position of the centre of mass in the supraresonance range

Numerous balancing machines operate in the supraresonance range. The advantage of such a system is that the amplitude of the vibration is independent of the speed and may be sensed without difficulty. Its disadvantage is that the mass m_a appears in the relationship, so the balancing machine requires calibration depending on the mass of the rotor (its moment of inertia).

8.5.2 THE UNIT OF MEASUREMENT OF IMBALANCE

The magnitude of the static imbalance can be given as the product of the mass causing the imbalance and the distance measured between the centre of mass and the axis of rotation:

$$U = qr.$$

The unit of mass is g, or kg, that of the radius is mm, or m, so the unit of the imbalance is mm g, g m, kg m (1 kg m = 1000 g m = 10^6 mm g). The imbalance specified in this way is called the absolute imbalance.

The numerical value of the absolute imbalance supplies no information whatever on the effect of the imbalance unless the mass of the rotor is known. Obviously an imbalance of $U = 1000$ mm g exerts quite a different effect on the rotor of an electric motor of mass 1 kg than on the rotor of a turbine whose mass is 10 000 kg. For ensuring the comparability of the imbalances of rotors with different masses, the concept of the specific imbalance has been introduced.

The specific imbalance is the absolute imbalance related to the mass of the rotor.

In the case of a static imbalance the specific imbalance agrees with the shift of the centre of mass of the rotor, i.e.

$$e = \frac{U}{m}.$$

In practice the specific imbalance is measured in units of length (μm and mm). Although the quality of the imbalance of two rotors, whose masses are different, may be compared by specifying the specific imbalance, this is not satisfactory, because it disregards the different possible working speeds of the rotors. For comparing the imbalances of rotors with different masses running at different speeds, we may specify the speed of the centre of mass:

$$v_e = e\omega,$$

measured usually in mm s^{-1}.

The units of measurement of the imbalance may be mutually converted in the knowledge of the actual data. For carrying out the balancing we must know the value in absolute units, or if the radius is given, then the balancing mass must be known.

8.5.3 BALANCING THE ROTORS

Balancing is the operation which aims to change the mass distribution in a rotor so as to prevent the development of a force in the bearing brackets which rotates with the angular velocity of the rotation during rotation or displacement of the rotor. Generally this is not attainable, so we must content ourselves with reducing it to within permissible limit values.

The rotor is balanced, in mechanical terms, if its axis of rotation coincides with the principal axis of inertia through its centre of gravity, or if the deviation of both does not exceed a permitted tolerance limit. In practice, a rotor without any imbalance after its manufacture is rare. To eliminate by balancing the effects of the dynamic forces loading the bearing brackets of the rotor, we must apply a moment whose magnitude equals the resultant moment of the centrifugal forces arising in the imbalanced state and dispersed in the rigid rotor and which acts in the opposite direction, i.e. the condition

$$\mathbf{M}_{FC} - \mathbf{M}_{qC} = 0 \tag{47}$$

must be met at any point C on the axis of rotation, where

$\mathbf{M}_{qC}$ is the moment of the applied masses about point C during the balancing;
$\mathbf{M}_{FC}$ is the moment of the force effects arising about point C due to the rotation of the rotor.

8.5.4 BALANCING PLANES

We have seen that in the case of a static imbalance the force arising due to the imbalance of the rotor acts through the centre of mass. If the imbalance is quasi-static, this single force acts somewhere outside the centre of mass. The moment-imbalance causes a moment, while in the most general case we must consider a force acting through the centre of mass in addition to a moment.

The force can be suppressed by a force of identical magnitude and direction but of opposite sense, while the moment can be balanced by a couple applied in the same plane.

Accordingly a mass of sufficient magnitude to cause the centre of mass of the workpiece to shift into the axis of rotation must be fixed in a plane perpendicular to the axis of rotation passing through the centre of mass of the statically imbalanced rotor. In the case of rotors which may be regarded as rigid, the selection of a single plane passing through the centre of mass is not an absolute requirement. At least two planes, which are perpendicular to the axis of rotation, may be marked out arbitrarily to contain the balancing masses. The common centre of mass of the applied balancing masses should be the mirror image of the centre of mass of the rotor with respect to the axis.

The quasi-static imbalance of a rotor can also be eliminated by a single balancing force applied in a suitably selected single plane which is perpendicular to the axis of rotation.

At the start of the balancing operation it is not possible to know if the imbalance of the rotor is quasi-static or not. Therefore it is advisable — also for measurement and technical reasons — to select two planes perpendicular to the axis of rotation for balancing purposes.

At least two planes, running perpendicularly to the axis of rotation, must be selected, in which are fixed — in suitable angular positions — masses of magnitudes which are suitable for balancing the initial moment imbalances and dynamic imbalances of the rotor. The planes selected for balancing are called balancing planes.

Narrow discs (e.g. coupling discs) are usually balanced in a single plane. A disc balanced in a single plane runs steadily if the distance between the bearing brackets of its shaft is sufficiently great. Balancing in a single plane is justified when the time available for the operation is short, even where it could be done in two planes. The effect of the static component of the imbalances found in practice is generally more important than that of the moment-imbalance.

In numerous cases it is advisable to balance even narrow discs (e.g. of motor vehicle wheels) in two planes, selected to be as far as possible from each other, usually near both ends of the rotor. The imbalances dispersed in the body may be integrated in both balancing planes as a single imbalance in each.

The vectors of the imbalances reduced in the balancing plane may be given in polar coordinates (Fig. 8.23), or by components interpreted in orthogonal, or oblique systems of coordinates specified in advance (Fig. 8.24).

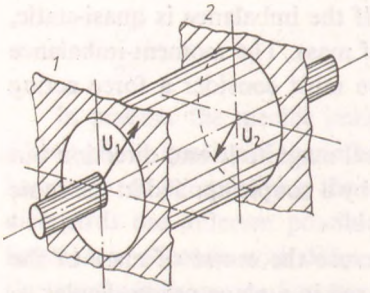


Fig. 8.23 Balancing in polar coordinates (polar balancing)

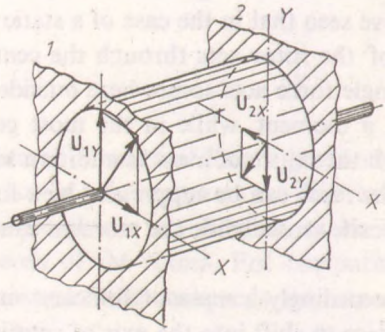


Fig. 8.24 In-place balancing with orthogonal components

This balancing mode is called in-place balancing. In this case mass is either removed from or added to the rotor.

Frequently, more than two balancing planes are used for the purpose of in-place balancing (e.g. of multicylinder crankshafts, see Fig. 8.25).

The crankshaft may also be balanced by machining away part of the counter weights off.

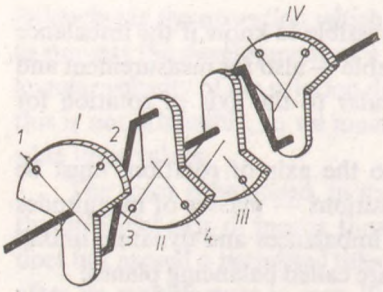


Fig. 8.25 Multiple-plane in-place balancing in oblique coordinates

In the balancing plane *I* part of the mass can be removed only at points *1* and *2*. If the position of the imbalance does not permit the crankshaft to be balanced by removing suitable amounts of mass at points *1* and *2* (e.g. when the direction of the imbalance-vector is in the angular ranges *1* and *3*), then mass may be removed from point *1* of plane *I* and point *3* of plane *II*.

Modern balancing machines permit the performance of both multiple-plane and in-place balancing by determining directly the amount of mass to be removed from each individual balancing point, with the help of their electronic measuring systems.

After balancing, the equilibrium with the imbalance vectors dispersed in the rotor is maintained by the balancing vectors found in the balancing planes. During the rotation of a balanced rotor the centrifugal forces generated by the imbalances existing in the diverging planes strive to bend the rotor. If the rotor is relatively rigid, the bending moment will not deform it significantly, an elastic body may suffer a deleterious bending. Threfore the balancing of elastic rotors is a special task. The elastic deformation can be compensated only by multiple-plane balancing.

On balancing drive shafts of motor vehicles their working speed may attain as much as 75% of the critical speed. The drive shafts are first prebalanced at a low speed, placing the masses q_1 and q_2 in the balancing planes selected close to both ends of the shaft (Fig. 8.26).

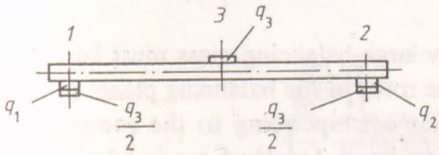


Fig. 8.26 Balancing an elastic shaft

Then, at nearly the critical speed, a sufficiently large mass q_3 is fixed in the central balancing plane *3* and simultaneously the masses $q_3/2$ are arranged in the balancing planes *1* and *2* opposite the mass q_3 so as not to spoil the result of the preliminary low-speed balancing.

The working speed of some rotors may be also to their second or third critical speed. In this case additional balancing planes must be selected to attain a correct

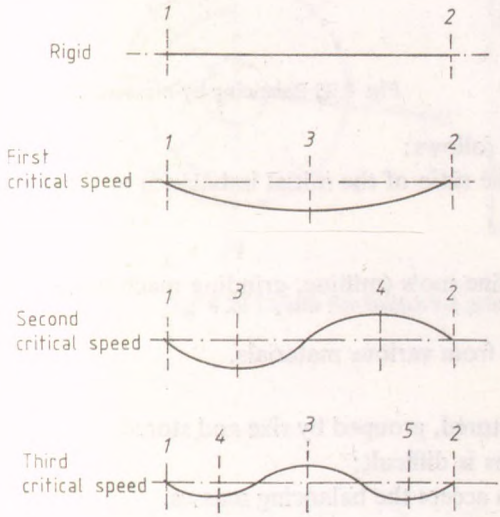


Fig. 8.27 The selection of balancing planes for shafts rotating at speeds close to the resonance speed

balance. While two balancing planes were sufficient for rigid rotors, here at least three are required to avoid the effect of the first critical speed, four at least in the vicinity of the second critical speed and a minimum of five near the third critical speed. Figure 8.27 illustrates the number of the balancing planes, their position and the deformed shape of the workpiece.

In the course of the balancing operation the mass distribution of the rotor may be changed in the balancing planes by

— mass addition,
— mass removal and,
— by rearranging the mass of the rotor.

8.5.5 BALANCING BY MASS ADDITION

This method is often applied to the balancing of rotors for both in-place and polar balancing.

In the case of polar balancing a sufficiently large balancing mass must be fixed in line with the imbalance vector opposite to the rotor in the balancing plane. In the case of in-place balancing two balancing masses corresponding to the component parts of the imbalance must be fixed in directions *1* and *2* marked out in advance in the balancing plane (Fig. 8.28).

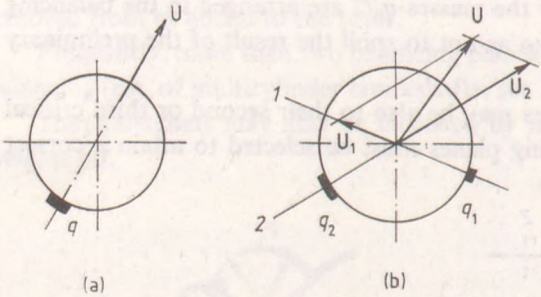


Fig. 8.28 Balancing by mass addition

The advantages of the method are as follows:

— the balancing time is independent of the ratio of the initial imbalance to the permanent imbalance;
— the work process is simple;
— no chip removal is necessary, so machine tools (milling, grinding machines, etc.) are not required;
— the balancing masses may be prepared from various materials.

Disadvantages of method:

— the balancing masses must be manufactured, grouped by size and stored;
— automation of the balancing operations is difficult;
— the rotors must be suitably designed to accept the balancing masses.

A few frequently used solutions are presented in Table 8.3.

Figure 8.29 shows a device for balancing grinding wheels by mass addition, where the cooling liquid of the grinding machine is used as the balancing mass. A hollow cylinder (*1*) rotating with the grinding wheel is mounted on the main spindle beside the front face of the grinding wheel. The cavity in the cylinder is divided into four equal chambers. Four circular grooves (*2*) are sunk in the front face of the cylinder; each is connected to a chamber. On opening the corresponding valves the jets (*3*) facing the grooves shoot cooling liquid into the grooves, from where the liquid enters the appropriate chamber. While the grinding wheel is rotating, the liquid entering the chambers is driven onto the mantle surface and, depending on the quantity, is capable of balancing the initial imbalance U of the wheel. The quantity of liquid flowing through the valves may be controlled either manually or automatically.

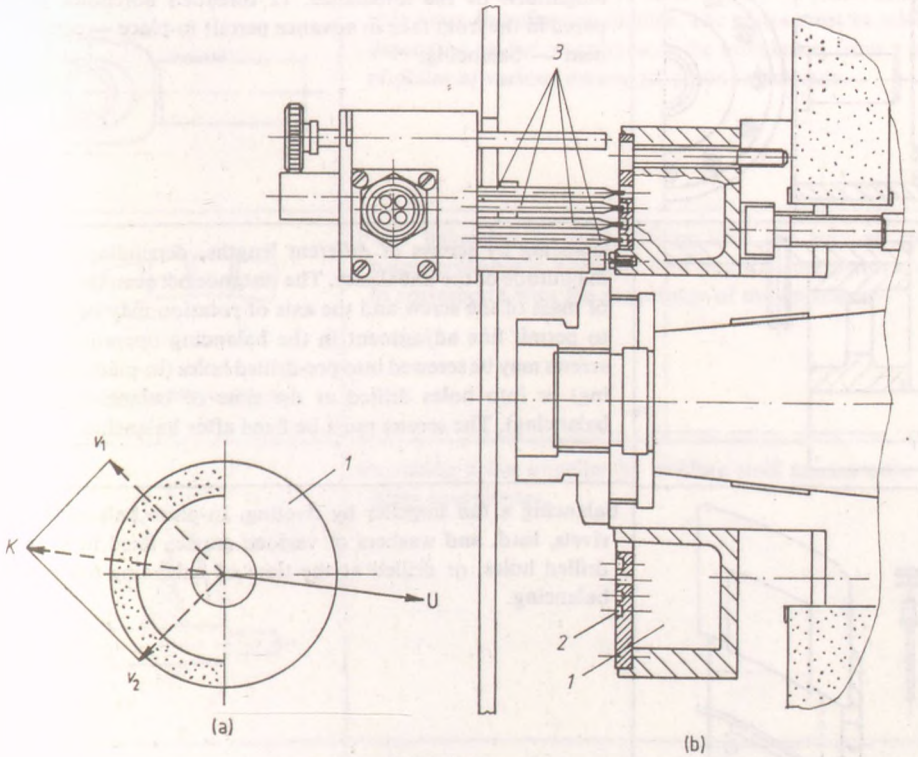


Fig. 8.29 Device for balancing grinding wheels by mass addition

Table 8.3

Balancing with mass addition

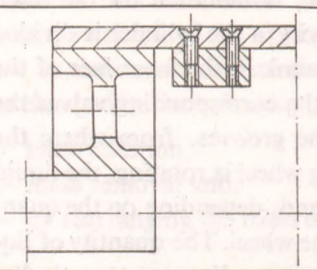
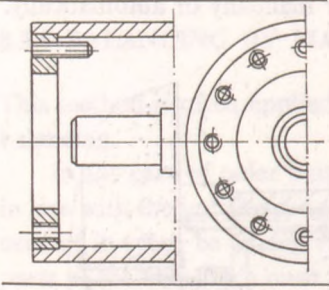
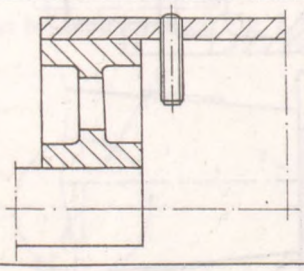
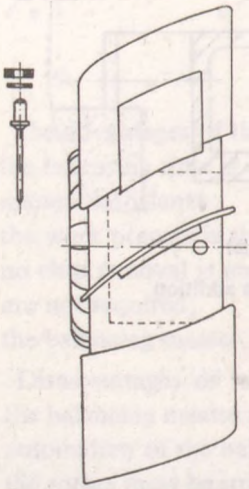
Example	Remark
	Screwing the balancing mass into the inner mantle surface or the front face. Polar balancing.
	Balancing by screws of different lengths, depending on the magnitude of the imbalance. 12 threaded boreholes prepared in the front face in advance permit in-place — component — balancing.
	Balancing by screws of different lengths, depending on the magnitude of the imbalance. The distance between the centre of mass of the screw and the axis of rotation may be varied to permit fine adjustment in the balancing operation. The screws may be screwed into pre-drilled holes (in-place balancing) or into holes drilled at the time of balancing (polar balancing). The screws must be fixed after balancing.
	Balancing a fan impeller by riveting. In-place balancing by rivets, lead, and washers of various masses fixed in 4 pre-drilled holes, or drilled at the time of balancing for polar balancing.

Table 8.3 (cont.)

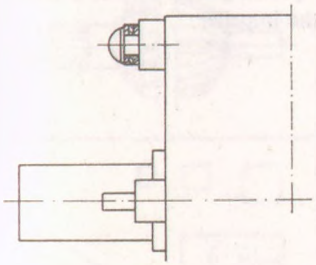
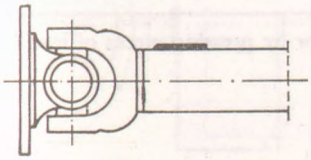
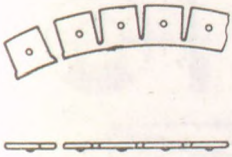
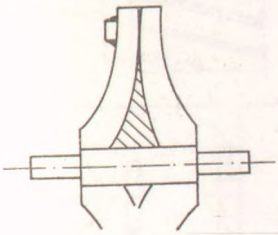
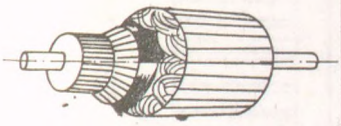
Example	Remark
	Balancing by varying the number of washers and by riveting in predetermined positions (e.g. on pins prepared in advance at orthogonal coordinates).
	Balancing of drive shafts by spot-welding on plates of a suitable mass in polar coordinates. The plates must be selected, depending on the magnitude of the imbalance, from a series of plates of various masses prepared in advance.
	Polar balancing by spot welding with the help of special plates prepared in advance. The length of the plates are cut to correspond with the magnitude of the imbalance.
	Balancing a fan impeller by welding steel masses onto it in polar coordinates.
	Balancing the armature of an electric motor by the application of solder onto the surface of the insulated wire wound on the coil head, in polar coordinates.

Table 8.3 (cont.)

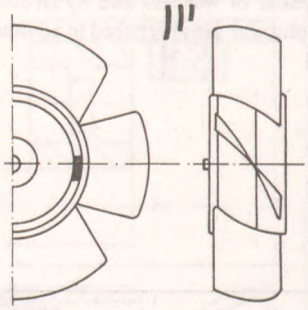
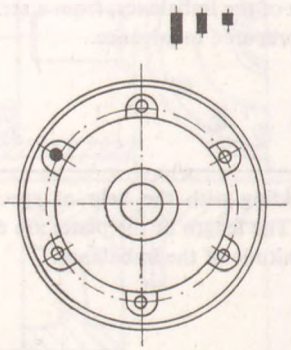
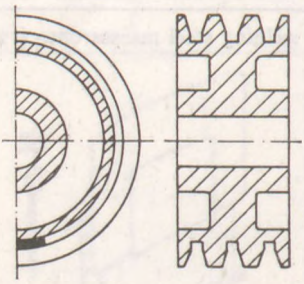
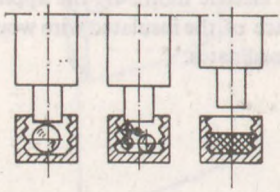
Example	Remark
	Balancing a fan impeller by pressing lead wire of a certain length into the groove of the impeller.
	In-place balancing of a rotor by pressing pieces of lead into pre-drilled holes.
	Polar balancing of a belt pulley by pressing a piece of lead into the groove found on the mantle surface of the pulley.
	Forming a groove or bore-hole into which the balancing mass is pressed.

Table 8.3 (cont.)

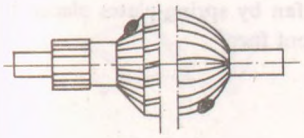
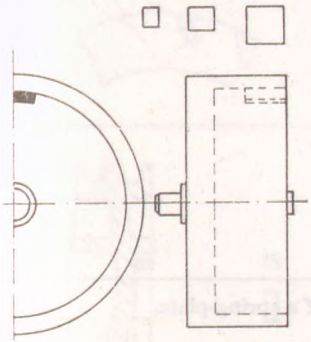
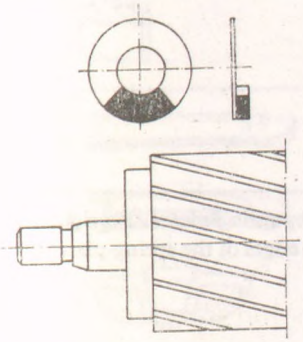
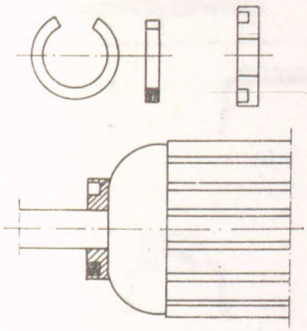
Example	Remark
	Polar balancing of the armature of a motor with the help of a high density, single-component or two-component paste. The mass and the position of the paste can be varied easily during the balancing operation. When it solidifies it ensures the final balancing.
	Polar balancing by sticking plates onto the inner mantle surface. The plates are pressed onto the surface by the centrifugal force, so sticking isn't loaded significantly.
	Balancing a rotor by fixing washers with various imbalances onto the shaft in polar coordinates.
	Polar balancing by placing circular rings of various opening angles into grooves prepared in advance.

Table 8.3 (cont.)

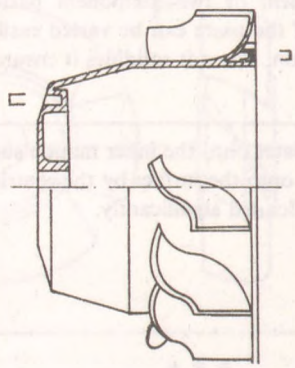
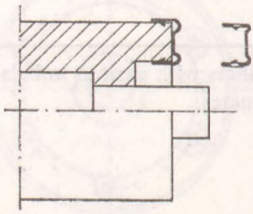
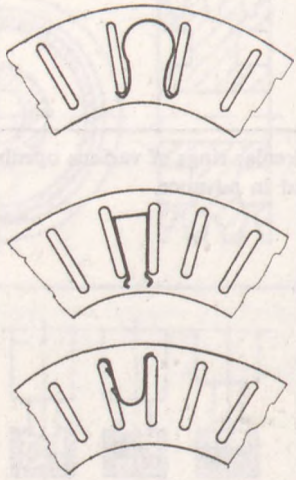
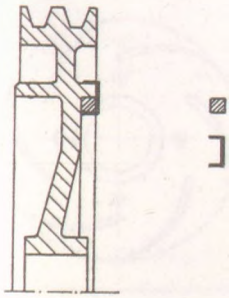
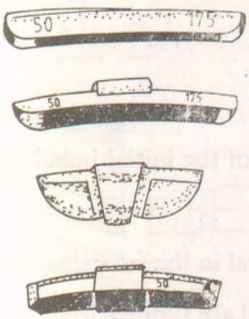
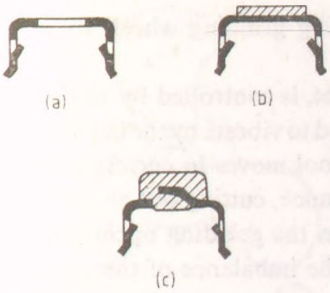
Example	Remark
	Polar balancing of a small fan by spring-plates placed into grooves prepared in the front face.
	Polar balancing by means of a spring-plate.
	Near-polar balancing of fans by spring-plates of various forms. The form, the hooks and the edges of the spring-plates secure them against flying off.

Table 8.3 (cont.)

Example	Remark
	Polar balancing of a belt pulley with the help of a balancing mass held fast by a spring-plate.
	Masses supplied with spring-plates for the polar balancing of motor vehicle wheels.
	Spring-plates of various forms for fixing balancing masses: (a) without a mass, (b) with a mass welded or glued onto it, (c) with a cast mass.

8.5.6 BALANCING BY MASS REMOVAL

This is a very frequently used balancing method, which is applicable to both polar and in-place balancing.

In the case of polar balancing a suitable quantity of material is removed from the rotor from the influence line of the single imbalance vector reduced in the balancing plane, or, for in-place balancing, from the influence line of two components of the imbalance in a given direction, on the side corresponding to the sense of the imbalance. The circles marked in Fig. 8.30 on the influence lines of the vectors indicate the bore-holes required for the balancing operation.

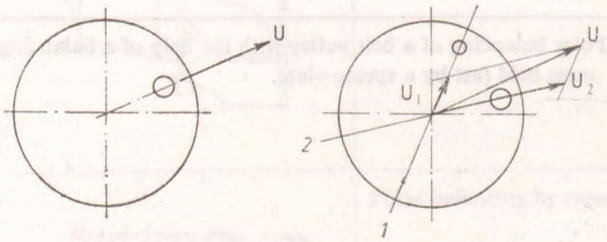


Fig. 8.30 Balancing by mass removal

Advantages of the method:

— easy automation, without any restriction on the size;
— the established cutting machine-tools may be used.

Disadvantages of the method:

— the time taken for the machining depends on the ratio of the initial imbalance to the permissible permanent imbalance,
— machine tools must be used,
— the rotors must be designed so as to permit mass removal in the balancing planes.

A few practical solutions to balancing by mass removal are summarized in Table 8.4.

Figure 8.31 shows a specific apparatus for balancing grinding wheels by mass removal.

The diamond tool (*1*) supplied with an electromagnet, is controlled by the vibration sensor (*2*) — placed on the machine frame and caused to vibrate by the imbalanced grinding wheel — via the amplifier (*3*). The diamond tool moves in correspondence with the frequency of the vibration caused by the imbalance, cutting out an arc from the surface of the wheel where it does not participate in the grinding operation (i.e. from the front face of the mantle grinders) to reduce the imbalance of the wheel as required.

Table 8.4

Balancing by mass removal

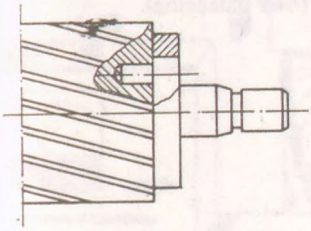
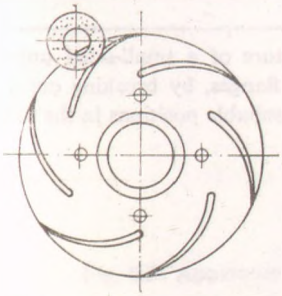
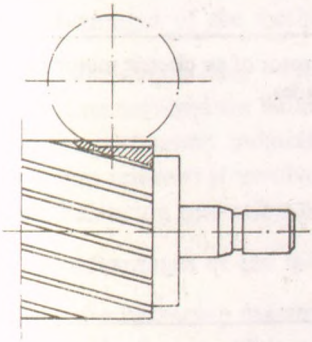
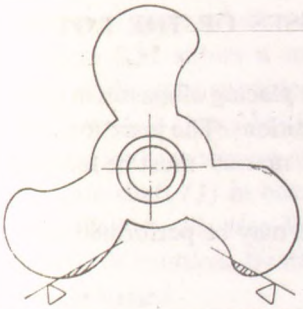
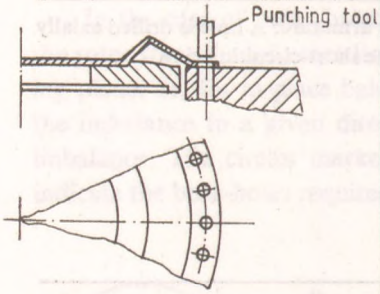
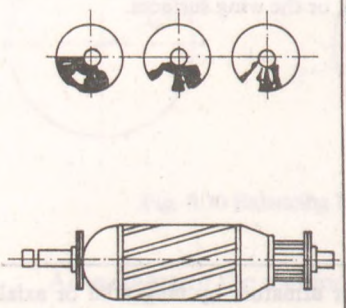
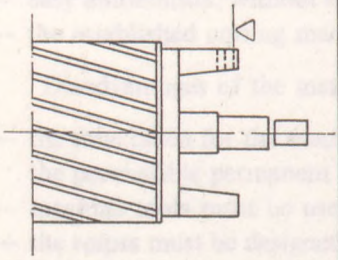
Example	Remark
	Polar balancing of a motor armature. A hole is drilled axially in the sheet pack within the short-circuiting ring.
	Polar balancing of the impeller of a turbo-charger by grinding the front face, the mantle, or the wing surfaces.
	Polar balancing of a motor armature by tangential or axial milling of the mantle surface of the sheet pack.
	In-place balancing of a fan blade by cutting off parts of the blade.

Table 8.4 (cont.)

Example	Remark
	Balancing a wheel supplied with a sheet flange by varying the number of punched holes (polar balancing).
	Polar balancing of the armature of a small-sized universal motor, supplied with special flanges, by breaking out a suit able number of spokes from suitable positions in the flanges.
	Balancing a short-circuited rotor of an electric motor by cutting off parts of the fan blades.

8.5.7 BALANCING BY REARRANGING THE MASSES OF THE ROTOR

This is a rarely applied balancing method, involving the placing of masses in advance in the balancing planes of the rotor and varying their positions. The insertion and the possibility of adjusting the positions of the balancing masses must be foreseen in the design.

Both polar balancing and balancing by components may be performed with the help of suitable apparatus.

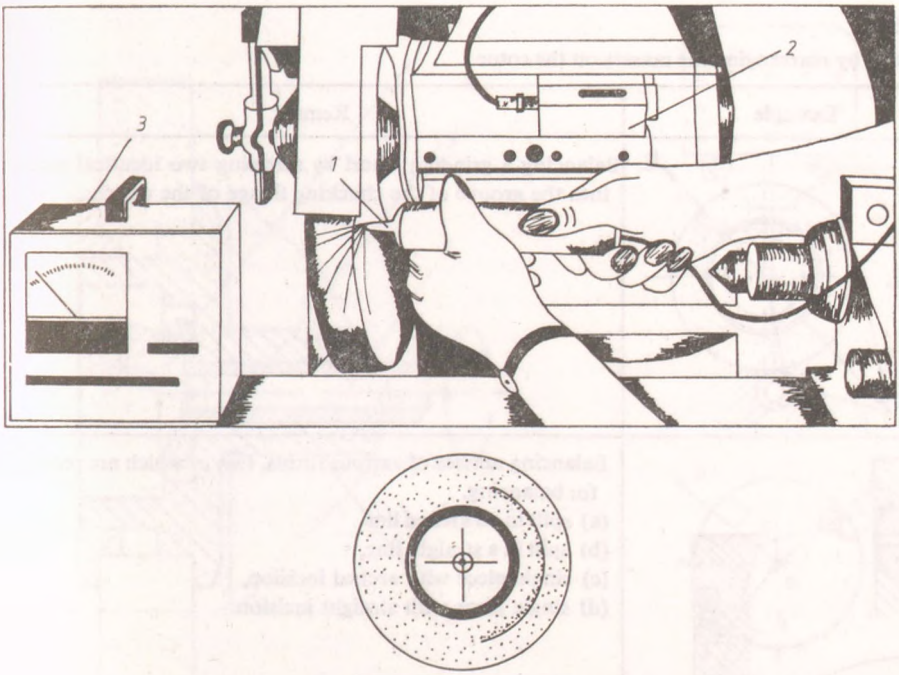


Fig. 8.31 Apparatus for balancing grinding wheels by mass removal

Advantages of the method:

— the balancing is easy — it amounts to an adjustment;
— the time required for balancing is independent of the ratio of the initial imbalance to the permanent imbalance;
— no chip removal is involved, so cutting machine tools are not necessary;
— the balancing operation may be easily automated.

Disadvantages of the method:

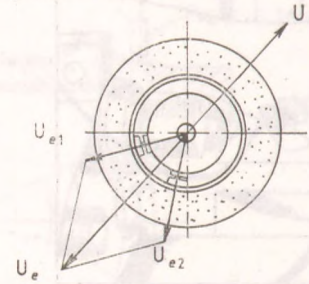
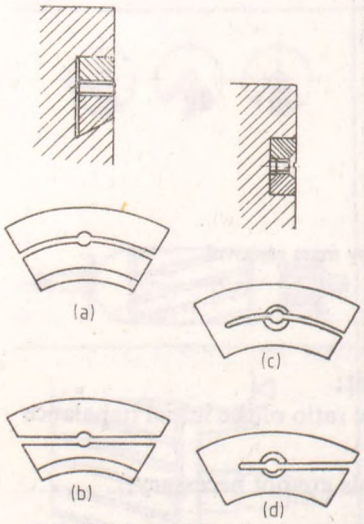
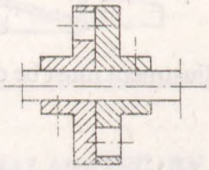
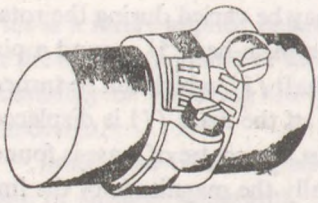
— both the balancing masses and the parts ensuring their adjustment must be designed in advance, together with the rotor.

A few of these solutions are summarized in Table 8.5.

Figure 8.32 shows a special device for balancing grinding wheels, where the position of the mass mounted in the hub of the wheel may be varied during the rotation. The mass *(1)* can be tilted by the axial setting of a push-bar *(5)* around a pin *(4)*, found in the extension of the front face of the tube shaft *(3)*, which can be turned in the main shaft *(2)* in bearings. The centre of gravity of the mass *(1)* is displaced nearly radially in this case. When the push-bar is drawn in, the centre of mass is found in the axis of rotation. By moving the push-bar *(5)* axially the magnitude of the imbalance is varied.

Table 8.5

Balancing by rearranging the masses on the rotor

Example	Remark
	Balancing a grinding wheel by screwing two identical masses into the groove of the chucking flange of the wheel.
	Balancing masses of various forms, two of which are required for balancing. (a) split in an arched line, (b) split in a straight line, (c) single piece with arched incision, (d) single piece with straight incision.
	Balancing flanges. The balancing is realized by turning the flanges with respect to each other.
	Balancing by rearranging the mass of the rotor with the help of two ABA clamps. The balancing masses are supplied by the eccentric tightening bolts and nuts of the clamps. The balancing is realized by adjusting both masses. The advantage is the applicability of the method without any preparation to all cylindrical surfaces. The disadvantage is that only relatively slight imbalances can be eliminated in this way.

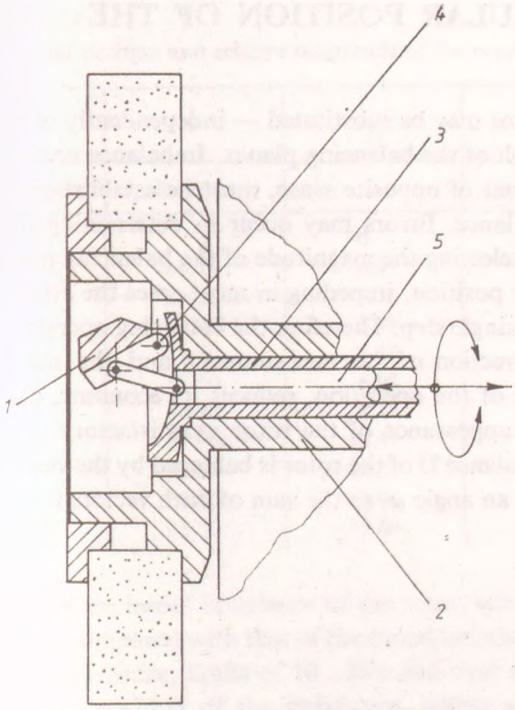


Fig. 8.32 Balancing device for grinding wheels by varying the position of the mass inserted into the wheel hub

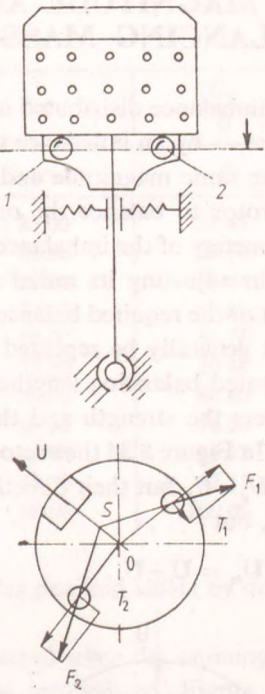


Fig. 8.33 Balancing a centrifuge by the automatic rearrangement of built-in masses

The angular position of the imbalance is varied by turning the tube-shaft *(3)* relative to the main shaft *(2)*. The required angular and radial position of the mass *(1)* is adjusted by the gear drive.

Figure 8.33 illustrates the balancing of the rotor of a laundry centrifuge by means of balls *(2)* which move over a positive guide *(1)* arranged coaxially with the shaft of the centrifuge supported in elastic bearings.

In the stationary position the balls are at the bottom of the arched surface *(1)* directly beside the shaft. When the shaft is rotated, the centrifugal force drives the balls up the arched surface and they come to rest as shown in Fig. 8.33. This occurs when the shaft runs at a working speed that exceeds the critical speed and the rotor of the centrifuge rotates about the axis that passes through the common centre of mass S, i.e. not about the zero axis. The tangential components $\mathbf{T}_1$ and $\mathbf{T}_2$ of the centrifugal forces $\mathbf{F}_1$ and $\mathbf{F}_2$ acting on the balls keep them moving until the common centre of mass reaches 0; the rotor may then be regarded as balanced.

This automatic balancing of the rotor is only applicable for machines operating above the critical speed. At low speeds the imbalance of the system is not increased significantly by the balls, as they are close to the shaft in this case.

8.6 MAGNITUDE AND ANGULAR POSITION OF THE BALANCING MASS

The imbalance distributed in a rigid rotor may be substituted — independently of its nature — by an imbalance vector in each of the balancing planes. Imbalance vectors of the same magnitude and direction, but of opposite sense, must be established in the rotor to balance the original imbalance. Errors may occur in determining the parameters of the imbalance vector, in selecting the magnitude of the balancing mass and in adjusting its radial and angular position, impeding in most cases the attainment of the required balanced state in a single step. Therefore the balancing operation must generally be repeated for the correction of the measurement and the mass. Repeated balancing lengthens the time of the operation, reduces its economy, and renders the strength and the aesthetic appearance of the rotor unsatisfactory.

In Figure 8.34 the vector of the imbalance **U** of the rotor is balanced by the vector $\mathbf{U}_e$. $U_e = U$, but their directions enclose an angle φ, so the sum of both vectors is not zero, but

$$\mathbf{U}_m = \mathbf{U} + \mathbf{U}_e.$$

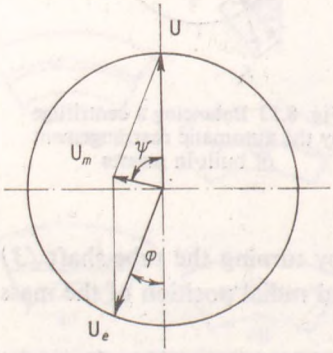


Fig. 8.34 Permanent imbalance in the erroneous determination of the angular position

The permanent imbalance U_m can be calculated by the following simple geometrical relationship:

$$U_m = 2U \sin \frac{\varphi}{2}. \tag{48}$$

The permanent imbalance makes an angle ψ with the initial imbalance U:

$$\psi = 90° - \frac{\varphi}{2}. \tag{49}$$

Table 8.6 contains the angular position of the permanent imbalance versus φ and its magnitude relative to the initial imbalance.

It may be stated on the basis of Table 8.6 that an error of $\pm 6°$ in the angular position of a correctly chosen balancing mass leads to a permanent imbalance of over

Table 8.6

Angular position and relative magnitude of the permanent imbalance

φ	ψ	$\frac{U_m}{U}$	φ	ψ	$\frac{U_m}{U}$
Degrees	Degrees	%	Degrees	Degrees	%
0		0	10	85.00	17.43
0.5	89.75	0.87	15	82.50	26.11
1	89.50	1.75	20	80.00	34.73
2	89.00	3.49	25	77.50	43.29
3	88.50	5.24	30	75.00	51.76
4	88.00	6.98	35	72.50	60.44
5	87.50	8.72	40	70.00	68.40
6	87.00	10.47	45	67.50	76.50
7	86.50	12.21	50	65.00	84.52
8	86.00	13.95	55	62.50	92.35
9	85.50	15.69	60	60.00	100.00

10% of the initial imbalance of the rotor, while the angular position shifts by nearly 90° as compared with that of the initial imbalance.

In practice, faults of 10...20% and over must be expected when determining the angular position of the imbalance, unless special means are utilized. Figure 8.35 shows the variation of the permanent imbalance, assuming an identical angular fault during the repeated balancing.

We have balanced the initial imbalance **U** by adding the mass q_1 with an angular fault φ. The permanent imbalance $\mathbf{U}_{m1}$ was eliminated by the mass q_2 during the second balancing, again with the angular fault φ. The angle between q_2 and q_1 was nearly 90°, because we have regarded the angular fault as having a relatively low value.

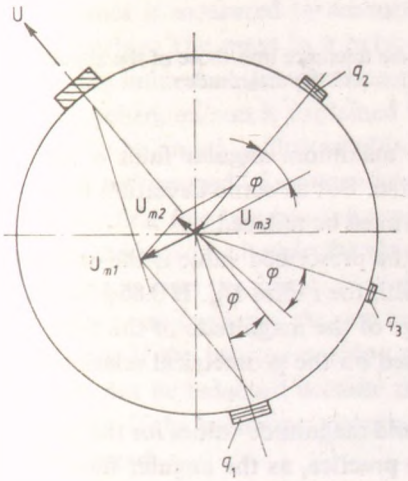


Fig 8.35 Variation of the permanent imbalance on balancing with an angular fault

Under the effect of mass q_2 the permanent imbalance changed to $\mathbf{U}_{m2}$. We reduced $\mathbf{U}_{m2}$ by adding the mass q_3 with the previous angular fault . The resulting imbalance $\mathbf{U}_{m3}$ was less than the permitted imbalance, so we then discontinued the balancing operation.

According to the example, the determination of the angular fault of the imbalance requires the addition, or removal of several masses in angular positions well separated from each other along the perimeter, as occurs frequently in practice. This can be avoided by correcting the position of the balancing masses, if the permanent imbalance after the first balancing operation seems to be nearly perpendicular to the initial imbalance.

So far we have studied the case where, in spite of angular faults made in the balancing, the mass used for balancing agrees with its theoretically determined value. But in reality the magnitude of the balancing mass is also generally inaccurate; the magnitude of the permitted permanent imbalance $\mathbf{U}_m$ is specified for the actual balancing cases, and previous experience gives information about the maximum initial imbalance U as well, which may be present in the given workpiece. As seen in Fig. 8.36, the sum of the maximum initial imbalance $\mathbf{U}$ and the balancing vector $\mathbf{U}_e$ gives the permitted permanent imbalance $\mathbf{U}_m$.

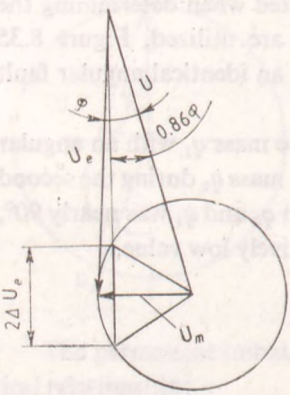


Fig. 8.36 Grouping the tolerance into those of the angular position and the magnitude

With a knowledge of the ratio U_m/U, the maximum angular fault φ may be found from Table 8.6, if U_e is the theoretical value. But as some deviation from the theoretical value is also permitted, the value of φ must be reduced.

The imbalance of the rotor will not exceed the prescribed value if the end point of the vector $\mathbf{U}+\mathbf{U}_e$ is within the circle drawn with the radius U_m. If 0.86φ is chosen for the angular fault $\Delta\varphi$, then the tolerance range of the magnitude of the balancing vector $\mathbf{U}_e$ is, according to an approximation based on the geometrical relationships, $2\Delta U_e \cong U_m$.

Grouping the tolerance ranges into angular and magnitude values for the purpose of balancing is arbitrary, but has proved well in practice, as the angular fault is not

much less than the possible values that occur, while the tolerance range of the magnitude is greatly extended in this way.

Example. The balancing of motor-vehicle wheels. The permanent imbalance to be attained is $U_m = 5$ g. The maximum permissible imbalance in the wheels is $U = 100$ g. Determine the accurate balancing masses required and the permissible angular fault for balancing in a single step.

Remark. The imbalance of motor-vehicle wheels is normally given in g units, as the balancing mass can be fixed on the rim of the wheel disc and the rim diameter of the individual types of wheel is constant.

Solution.

The maximum theoretical angular fault corresponding to

$$\frac{U_m}{U} = \frac{5}{100} 100 = 5\%$$

is $\varphi \approx 3°$ in Table 8.6; therefore

$$\Delta\varphi = \pm 0.86\ \varphi = \pm 0.86 \times 3 = \pm 2.6°$$

is the permitted angular fault.

The tolerance of the balancing mass is

$$\Delta U_e = \pm \frac{1}{2} U_m = \pm \frac{1}{2} 5 = \pm 2.5 \text{ g.}$$

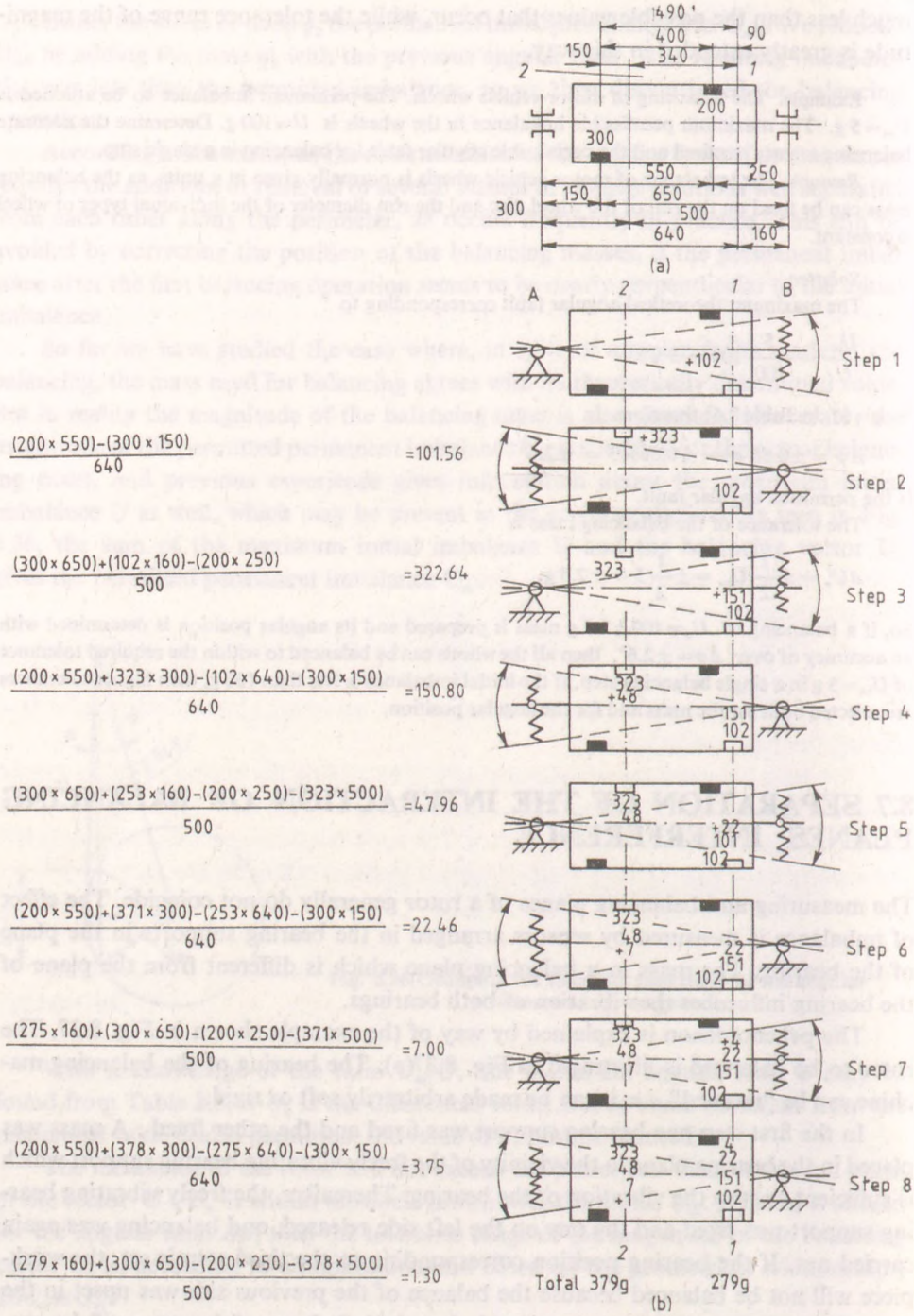
So, if a balancing of $U_e = 100 \pm 2.5$ g mass is prepared and its angular position is determined with an accuracy of over $\Delta\varphi = \pm 2.6°$, then all the wheels can be balanced to within the required tolerance of $U_m = 5$ g in a single balancing step. If the initial imbalance is less than 100 g, then higher tolerances are selected both for the mass and for the angular position.

8.7 SEPARATION OF THE INTERACTION OF BALANCING PLANES; INTERFERENCE

The measuring and balancing planes of a rotor generally do not coincide. The effect of imbalance is measured by sensors arranged in the bearing supports in the plane of the bearing. The mass in a balancing plane which is different from the plane of the bearing influences the vibration of both bearings.

The pehenomenon is explained by way of the example shown in Fig. 8.37. The rotor to be balanced is illustrated in Fig. 8.37(a). The bearing of the balancing machine can be "retuned", i.e. it can be made arbitrarily soft or rigid.

In the first step one bearing support was fixed and the other freed . A mass was placed in the bearing plane in the vicinity of the freely vibrating bearing support which is sufficient to stop the vibration of the bearing. Thereafter, the freely vibrating bearing support was fixed and the one on the left side released, and balancing was again carried out. If the bearing position corresponding to the third step is set, the workpiece will not be balanced because the balance of the previous side was upset in the second step. If the balancing is continued, then the imbalance of the rotor will be gradually reduced.



$$\frac{(200\times550)-(300\times150)}{640}=101.56$$

$$\frac{(300\times650)+(102\times160)-(200\times250)}{500}=322.64$$

$$\frac{(200\times550)+(323\times300)-(102\times640)-(300\times150)}{640}=150.80$$

$$\frac{(300\times650)+(253\times160)-(200\times250)-(323\times500)}{500}=47.96$$

$$\frac{(200\times550)+(371\times300)-(253\times640)-(300\times150)}{640}=22.46$$

$$\frac{(275\times160)+(300\times650)-(200\times250)-(371\times500)}{500}=7.00$$

$$\frac{(200\times550)+(378\times300)-(275\times640)-(300\times150)}{640}=3.75$$

$$\frac{(279\times160)+(300\times650)-(200\times250)-(378\times500)}{500}=1.30$$

Fig. 8.37 Balancing with a gradual approach to the balanced state

With the numerical example in Fig. 8.37, the balanced state was gradually approached in eight steps. The number of steps is determined by the distance between the balancing and measuring planes and by the magnitude of the permissible imbalance.

This lengthy procedure was necessary because the mass placed in one of the balancing planes brought about such a change in the sensing of the other balancing plane that it appeared as if the imbalance of the rotor had had changed in this plane too. Interference is the phenomenon where the mass in one of the balancing planes can also be sensed as a fault of the other balancing plane.

The interference is expressed in terms of the interference ratio. This ratio of the rotor *1* and balancing plane *2* is given as a percentage:

$$I_{1,2} = 100 \frac{U_{1,2}}{U_{2,2}} \%, \tag{50}$$

where $U_{1,2}$ and $U_{2,2}$ represent the values indicated by the instrument sensing the imbalance in the balancing planes *1* and *2* due to the effect of the imbalance in balancing plane *2*.

Similarly the interference ratio of plane *1* to plane *2* is

$$I_{2,1} = 100 \frac{U_{2,1}}{U_{1,1}} \%, \tag{51}$$

where $U_{2,1}$ and $U_{1,1}$ represent the values indicated by the instrument sensing the imbalance due to the effect of the imbalance in balancing plane *1*.

The balancing is economical if the permissible value of the residual imbalance can be approached with the minimum possible number of steps, i.e. the interference ratio is as small as possible.

The rotor in the preceding example can be balanced in two steps if the supporting parts are alternately placed in the balancing planes.

The bearings of the rotor in Fig. 8.38 were fitted onto a frame. This was supported first in balancing plane *1* and counterbalanced with a mass in plane *2*; then it was supported in balancing plane *2* and counterbalanced with a mass in balancing plane *1*. Since the influence line of the centrifugal force brought about by the masses placed in the balancing planes passes through the supporting points, there are no moments about these points, and thus the previous balanced state of the rotor is not affected. If the balancing machine is provided with such a frame structure, the separation of the balancing planes is accomplished and the interference ratio is zero.

This method of plane separation is not applicable to on-site balancing because the machine is generally not provided with a frame system with adjustable supports. The practical solution is shown in Fig. 8.39 can be followed for the example of a rotor fitted with a relatively rigid bearing.

Let us assume that the imbalance of the rotor is equivalent to masses q_1 and q_2 on a given radius in balancing planes *1* and *2*. Furthermore let us assume that the

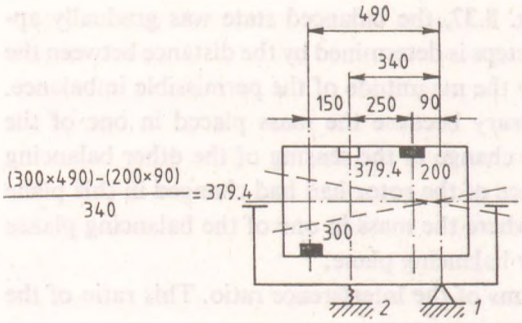
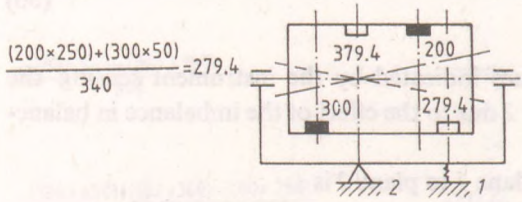


Fig. 8.38 Balancing with a frame



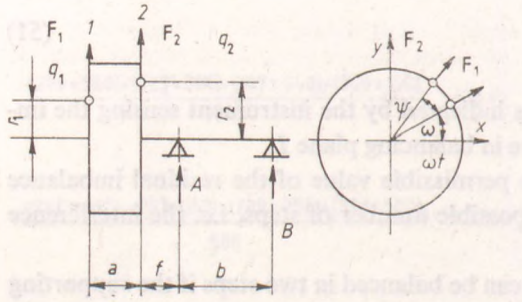


Fig. 8.39 Rotor fitted with a rigid bearing

supporting forces arising in bearings A and B during balancing, or the proportional physical quantities (e.g. amplitude of bearing deflection), are measured.

The complex forces applied to masses q_1 and q_2 in a system rotating at angular velocity ω are

$$\mathbf{F}_1 = F_1 e^{j\omega t} = q_1 r_1 \omega^2 e^{j\omega t} \tag{52}$$

and

$$\mathbf{F}_2 = F_2 e^{j(\omega t+\psi)} = q_2 r_2 \omega^2 e^{j(\omega t+\psi)}, \tag{53}$$

where

$$\mathrm{j} = \sqrt{-1}.$$

Introducing the concept of complex mass with respect to q_1 and q_2:

$$\mathbf{q}_1 = q_1 e^{j\omega t} \tag{54}$$

$$\mathbf{q}_2 = q_2 e^{j(\omega t-\psi)}. \tag{55}$$

The question is what kind of relationship exists between the mechanical parameters of the workpiece and the masses $\mathbf{q}_1$ and $\mathbf{q}_2$ causing the imbalance?

Complex reaction forces **A** and **B** arise in the bearings during dynamic balancing. These can be expressed in matrix form with the aid of the moment equations of balance:

$$\begin{bmatrix}\mathbf{A}\\ \mathbf{B}\end{bmatrix} = \frac{1}{b}\begin{bmatrix}-(a+b+f) & -(b+f)\\ a+f & f\end{bmatrix}\begin{bmatrix}\mathbf{F}_1\\ \mathbf{F}_2\end{bmatrix} \tag{56}$$

Using relations (53), (54), (55) and (56), as well as assuming that $r_1=r_2=r$, then

$$\begin{bmatrix}\mathbf{A}\\ \mathbf{B}\end{bmatrix} = \frac{\omega^2}{b}r\begin{bmatrix}-(a+b+f) & -(b+f)\\ a+f & f\end{bmatrix}\begin{bmatrix}\mathbf{q}_1\\ \mathbf{q}_2\end{bmatrix}. \tag{57}$$

Denote the elements of the matrix as follows:

$$b_{11} = -(a+b+f) \tag{58}$$

$$b_{12} = -(b+f) \tag{59}$$

$$b_{21} = a+f \tag{60}$$

$$b_{22} = f. \tag{61}$$

Accordingly,

$$\mathbf{B} = \begin{bmatrix}b_{11} & b_{12}\\ b_{21} & b_{22}\end{bmatrix}. \tag{62}$$

The column matrices of complex element are denoted separately as follows:

$$\mathbf{Q} = \begin{bmatrix}\mathbf{q}_1\\ \mathbf{q}_2\end{bmatrix} \text{ vector of mass,} \tag{63}$$

$$\mathbf{R} = \begin{bmatrix}\mathbf{A}\\ \mathbf{B}\end{bmatrix} \text{ vector of reactions.} \tag{64}$$

With the use of these relations, the following matrix equation is obtained:

$$\mathbf{R} = \frac{\omega^2}{b}r\mathbf{BQ}. \tag{65}$$

In the course of balancing, the supporting forces are measured and the position and magnitude of the balancing masses have to be found. For this purpose equation (65) is solved for **Q**:

$$\mathbf{Q} = \frac{b}{r\omega^2}\mathbf{B}^{-1}\mathbf{R} = \mathbf{K}_k\mathbf{R}; \tag{66}$$

$$\mathbf{K}_k = \frac{b}{r\omega^2}\mathbf{B}^{-1}. \tag{67}$$

The elements of the intermediary matrix $\mathbf{K}_k$ are real and depend, besides the constants, on the geometrical dimensions of the workpiece to be balanced. With a knowledge of these, the complex masses can be determined on the basis of the measurement of the complex reaction forces.

Equation (65) expressed in detail is

$$\begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \end{bmatrix} = \frac{b}{\omega^2} \frac{1}{r(b_{11}b_{22}-b_{12}b_{21})} \begin{bmatrix} b_{22}-b_{12} \\ -b_{21}+b_{11} \end{bmatrix} \begin{bmatrix} \mathbf{A} \\ \mathbf{B} \end{bmatrix}. \tag{68}$$

According to the matrix equation, an electric-circuit analogue can be devised which gives directly the vector of the balancing masses. Assuming that **A** and **B** are the voltages given by dynamometers arranged in the bearings, then the voltages corresponding to complex masses $\mathbf{q}_1$ and $\mathbf{q}_2$ are obtained at the output terminals of the electric network shown in Fig. 8.40. Naturally, the potentiometers have to be set in accordance with the elements of the intermediary matrix.

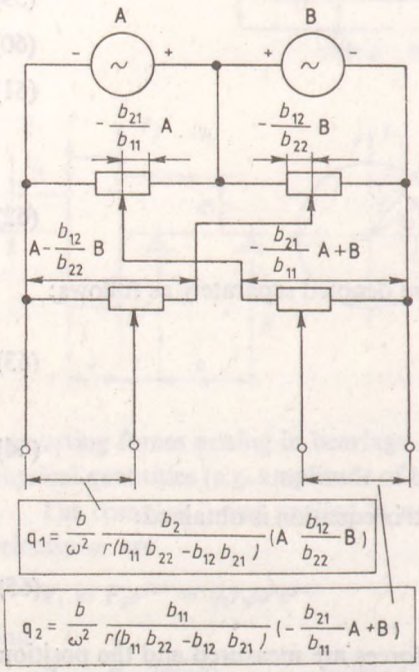


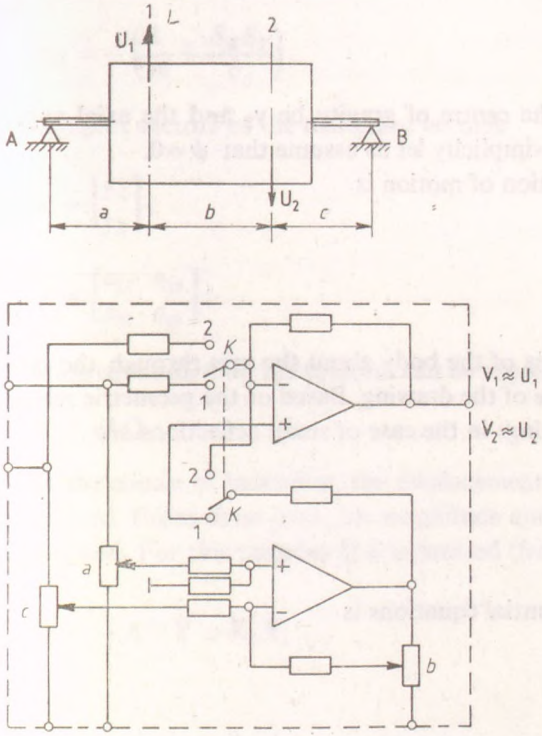
Fig. 8.40 Electric frame for a rigid bearing

If the voltages proportional to values $\mathbf{q}_1=q_1e^{j\omega t}$ and $\mathbf{q}_2=q_2^{j(\omega t+\psi)}$ can be successfully produced with the method described, then the magnitude of the balancing mass (q_1 and q_2) and its angular position (ωt and $\omega t+\psi$) can be determined with some kind of electric measuring method.

The circuit shown in Fig. 8.40 eliminates the effect of masses arranged in the individual balancing planes upon the other balancing plane. In this way it solves a

similar problem to that solved by the balancing frame shown in Fig. 8.38. Because of the similarity, this circuit is called an electric frame.

Figure 8.41 illustrates the solution of the same problem with the use of active elements (analogue operational amplifiers). This analogue calculating unit produces from the input voltages **A** and **B**, proportional to the reaction forces of the bearing brought about by the effect of the imbalanced rotor, a voltage $\mathbf{V}_1$ and, by a change-over of switch K, a voltage $\mathbf{V}_2$ proportional to complex masses $\mathbf{q}_1$ and $\mathbf{q}_2$, according to the set geometric data a, b, c.



$$V_1 = -A + \frac{aA - cB}{b}$$

$$V_2 = -B = \frac{aA - cB}{b}$$

Fig. 8.41 Analogue calculating unit

So far it has been assumed that the bearing of the rotor, i.e. the frame of the machine, is relatively rigid. In the following the case is examined for which the bearings are very soft, i.e. the spring constants $c_A = c_B \approx \infty$ (Fig. 8.42).

As a result of the excitation effect of the projections of the rotary forces $\mathbf{F}_1$ and $\mathbf{F}_2$ the bivariant vibrational system performs partly progressive and partly rotary

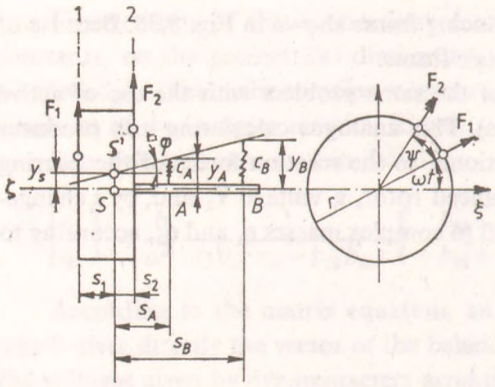


Fig. 8.42 Rotor with bivariant, soft dynamic characteristic

motion. Let the displacement of the centre of gravity be $\mathbf{y}_S$ and the axial angular displacement be ψ. For the sake of simplicity let us assume that $\psi=0$.

The complex form of the equation of motion is

$$m\ddot{\mathbf{y}}_S = \mathbf{F}_1+\mathbf{F}_2; \tag{69}$$

$$\theta_\xi\ddot{\boldsymbol{\varphi}} = +S_2\mathbf{F}_2-S_1\mathbf{F}_1. \tag{70}$$

Here θ_ξ is the moment of inertia of the body about the axis through the centre of gravity perpendicular to the plane of the drawing. Based on the geometric relationships, the displacements of the bearings in the case of small deflections are

$$\mathbf{y}_A = \mathbf{y}_S+S_A\boldsymbol{\varphi} \tag{71}$$

$$\mathbf{y}_B = \mathbf{y}_S+S_B\boldsymbol{\varphi}. \tag{72}$$

The solution of the system of differential equations is

$$\mathbf{y}_S = -\frac{1}{m\omega^2}(\mathbf{F}_1+\mathbf{F}_2); \tag{73}$$

$$\varphi = \frac{1}{\theta_\xi\omega^2}(S_1\mathbf{F}_1-S_2\mathbf{F}_2). \tag{74}$$

On the basis of equations (73) and (74), using the expressions (52), (53), (54) and (55) and if $r_1=r_2=r$, a short calculation gives

$$\mathbf{y}_A = -\left(\frac{1}{m}-\frac{S_AS_1}{\theta_\xi}\right)r\mathbf{q}_1-\left(\frac{1}{m}+\frac{S_AS_2}{\theta_\xi}\right)r\mathbf{q}_2; \tag{75}$$

$$\mathbf{y}_B = \left(\frac{1}{m}-\frac{S_BS_1}{\theta_\xi}\right)r\mathbf{q}_1-\left(\frac{1}{m}-\frac{S_BS_2}{\theta_\xi}\right)r\mathbf{q}_2. \tag{76}$$

Using the symbols

$$a_{11} = -\left(\frac{1}{m} - \frac{S_A S_1}{\theta_\xi}\right), \tag{77}$$

$$a_{12} = -\left(\frac{1}{m} + \frac{S_A S_2}{\theta_\xi}\right), \tag{78}$$

$$a_{21} = -\left(\frac{1}{m} - \frac{S_B S_1}{\theta_\xi}\right) \tag{79}$$

and

$$a_{22} = -\left(\frac{1}{m} + \frac{S_B S_2}{\theta_\xi}\right). \tag{80}$$

The complex vectors of the deflection become

$$\mathbf{Y} = \begin{bmatrix} \mathbf{y}_A \\ \mathbf{y}_B \end{bmatrix}, \tag{81}$$

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}; \tag{82}$$

and using equation (63), the solution can be expressed as the matrix equation

$$\mathbf{Y} = r\mathbf{A}\mathbf{Q}. \tag{83}$$

In the course of balancing, the displacement of both points A and B of the axis is measured. From these data, the magnitude and angular position of the imbalance are obtained. For this purpose $\mathbf{Q}$ is expressed (from equation (83)) as

$$\mathbf{Q} = \frac{1}{r}\mathbf{A}^{-1}\mathbf{Y} = \mathbf{K}_L\mathbf{Y}, \tag{84}$$

where

$$\mathbf{K}_L = \frac{1}{r}\mathbf{A}^{-1}. \tag{85}$$

In the case of a concentric rotor, the elements of matrix $\mathbf{K}_L$ are real and depend on the mass, moment of inertia and geometric dimensions of the piece to be balanced. The value of $\mathbf{K}_L$ is the same for a uniform rotor. From a knowledge of matrix $\mathbf{K}_L$ for a given type of part — based on the measured deflections — the complex vector of the balancing masses (84) can be calculated.

Expressing equation (84) in detail gives

$$\begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \end{bmatrix} = \frac{1}{r(a_{11}a_{22} - a_{12}a_{21})} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} \begin{bmatrix} \mathbf{y}_A \\ \mathbf{y}_B \end{bmatrix}; \tag{86}$$

from which

$$\mathbf{q}_1 = \frac{1}{r}\,\frac{a_{22}}{a_{11}a_{22}-a_{12}a_{21}}\left(\mathbf{y}_A - \frac{a_{12}}{a_{22}}\mathbf{y}_B\right), \tag{87}$$

$$\mathbf{q}_2 = \frac{1}{r}\,\frac{a_{11}}{a_{11}a_{22}-a_{12}a_{21}}\left(-\frac{a_{12}}{a_{11}}\mathbf{y}_A + \mathbf{y}_B\right). \tag{88}$$

The last two equations are already suitable for obtaining the magnitude of the balancing masses directly by means of an appropriate electric circuit. If electric sensors are arranged at points A and B on the axis, which produce voltages proportional to the displacements, then the values $\mathbf{y}_A$ and $\mathbf{y}_B$ can be regarded as voltages. In the appropriate electric circuits, the values proportional to the elements of the intermediary matrix can be set; thus they produce the output voltages $\mathbf{q}_1$ and $\mathbf{q}_2$ according to equations (78) and (88).

The circuit diagram of an electric frame suitable for solving equations (87) and (88) is shown in Fig. 8.43. The four potentiometers in the circuit should be set accord-

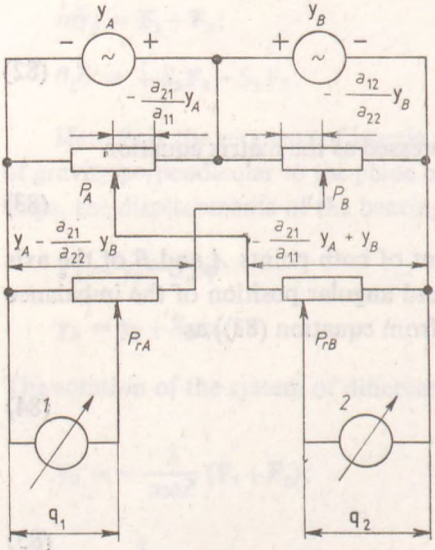


Fig. 8.43 Electric frame for machine fitted with soft bearing

ing to the elements of the intermediary matrix. Since these elements depend, besides the easily measurable geometric data, on the mass and moment of inertia of the rotor — which is generally not known — then consequently the potentiometers are set with the aid of a preliminary trial. For setting purposes a mass of known magnitude is placed in the balancing plane *1* of a well-balanced rotor, then the potentiometer P_A is adjusted until the instrument *2* indicates zero. This process is repeated with the setting mass arranged in the balancing plane *2*. Thereafter, with a knowledge of the distance between the mass and axis, the potentiometers P_{rA} and P_{rB} are adjusted according to the magnitude of the setting mass. Following the setting procedure, the

balancing masses of pieces having the same size, mass and moment of inertia can be determined in both balancing planes during a single turn.

The interaction of the balancing planes can be eliminated in the two previously mentioned limiting cases (completely rigid and completely soft spring-supported bearings). The solution and the electric frame are similar in both cases. The difference is between the physical senses of the intermediary matrix elements. These are influenced in the case of a rigid bearing only by the geometric distance between the planes, while in the case of a soft bearing they depend on the mass of the rotor and the moment of inertia calculated about the axis perpendicular to the axis of rotation.

Equations (57), (75) and (76) can be expressed generally in the following form:

$$\begin{bmatrix} \mathbf{A} \\ \mathbf{B} \end{bmatrix} = \begin{bmatrix} \varkappa_{11} & \varkappa_{12} \\ \varkappa_{21} & \varkappa_{22} \end{bmatrix} \begin{bmatrix} \mathbf{U}_1 \\ \mathbf{U}_2 \end{bmatrix}, \tag{89}$$

where

the complex quantities **A** and **B** are the supporting forces in the plane of the bearing, or other proportional physical parameters;

$\mathbf{U}_1$, $\mathbf{U}_2$ are vectors of imbalance resolved in the individual balancing planes and, similarly, are in complex form.

Generally $\varkappa_{11}$, $\varkappa_{12}$, $\varkappa_{21}$ and $\varkappa_{22}$ are complex factors characteristic of the rotor and bearing.

On the other hand, if the damping is negligible, as in the two examples, then the imaginary parts of these factors are also negligible.

Such is the vibrational system of a balancing machine. Here, to simplify the measuring procedure, the number of self-vibrations and the damping of the vibrating part are determined according to the specific conditions.

The bearing frames of the various machines are generally not dimensioned from the point of view of vibrometry; hence the ratio of the number of self-vibrations of the machine frame to the speed of the rotor may be as high as possible — it may fall

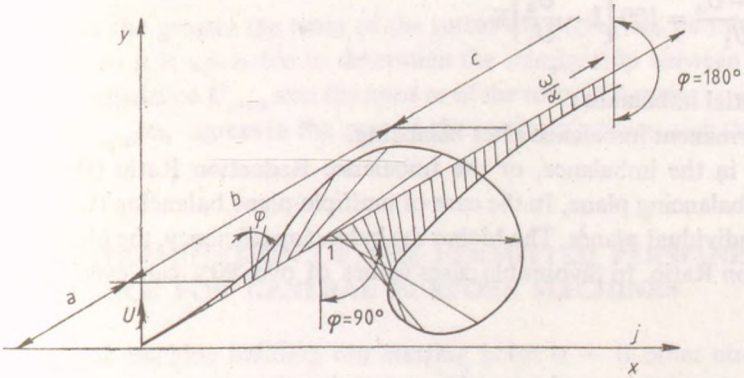


Fig. 8.44 Vibration amplitude and phase diagram versus frequency ratio of bearing excited by imbalance

into the resonance range resulting in greater amplitude and phase displacement (Fig. 8.44).

Here the resonance curve of a univariant vibrational system is shown in a spatial coordinate system, because in this way the phase angle of the excited vibration can also be observed. The phase angle, measured from the axis y, far below the resonance (range a) and far above the resonance (range c) is nearly 0 and 180° respectively, in which case the factors are practically real numbers, while in the vicinity of the resonance (range b) they are complex vectors. The factors in range a can be calculated on the basis of geometric data, and they are measurable in ranges b and c.

8.8 REDUCTION OF THE IMBALANCE

By balancing the rotor we strive to reduce its imbalance economically, in the shortest possible time, to below the permissible value of the permanent imbalance. The required balancing time is influenced considerably by the number of starting operations. In the most favourable case the initial imbalance of the rotor can be reduced to the desired value by means of a single measurement and mass correction.

Success in this operation is influenced by the measurement errors of the balancing instruments, by the balancing mass, the geometrical dimensions, the distance between the balancing planes, the variation in the radius, the measuring error in the angular position, as well as the inaccuracies in the measurements and adjustments during the separation of the planes. Frequently the balancing procedure must be repeated, for various reasons, until the required balanced state of the rotor is attained.

The efficiency of the balancing procedure is characterized by the reduction in the imbalance attained by the balancing of the rotor (rotation, measurement and mass correction).

The reduction ratio of the imbalance (IRR) is

$$\mathrm{IRR} = 100\frac{U_1 - U_2}{U_1} = 100\left[1 - \frac{U_2}{U_1}\right]\%, \tag{90}$$

where

U_1 is the initial imbalance;

U_2 is the permanent imbalance after balancing.

The reduction in the imbalance, or the Imbalance Reduction Ratio (IRR), is always related to a balancing plane. In the case of multiple-plane balancing this value is different in the individual planes. The higher the balancing efficiency, the higher the Imbalance Reduction Ratio. In favourable cases values of over 90% can even be attained.

8.9 PERMISSIBLE VALUE OF THE PERMANENT IMBALANCE OF RIGID ROTORS

The aim of balancing is to alter the mass distribution of the rotor in order to avoid the generation of supporting forces in the bearings as a result of the rotary motion. This aim can only be approximated, as a certain imbalance always remains in the rotor. This permanent imbalance may be reduced to a very low value by modern instruments and a due time expenditure. Yet, exaggerated quality requirements must not be prescribed. The optimum minimum value of the permanent imbalance which is justified technically and economically can be determined by laboratory experiments or numerous on-site measurements. The value of the imbalance belonging to the individual balancing planes at which the balanced state of the rotor is already acceptable is called the "permitted permanent imbalance".

Frequently a simple relationship between the permissible permanent imbalance and the vibrating state of a machine cannot be found. Numerous factors influence the amplitude of the vibrations (e.g. the masses of the rotor, of the stator and of the machine frame, the rigidity of the bearings and bearing brackets, the proximity of the resonance, the damping, etc.). The excitation effect caused by the imbalance depends also on the type of imbalance (static, moment and dynamic imbalance), and also it is undeniable that the vibration of a machine is caused only partly by the imbalance of the rotor.

Various recommendations and methods, based on experience and theoretical considerations, exist for establishing the permitted permanent imbalance of the rotor; these are generally acceptable, but in certain cases this value must be determined by individual methods.

8.9.1 THE MASS OF THE ROTOR AND THE PERMITTED PERMANENT IMBALANCE

Generally the greater the mass of the rotor, the higher the permitted permanent imbalance; so it is advisable to determine the relationship between the permitted permanent imbalance U_{perm} and the mass m of the rotor. The permitted specific imbalance $e_{perm} = U_{perm}/m$, agrees in the case of the static imbalance with the shift in the centre of mass.

8.9.2 DETERMINATION OF THE PERMITTED PERMANENT IMBALANCE FOR GENERAL PURPOSE MACHINES

In general machine building our starting point is — if other circumstances do not demand an extraordinarily steady run of the machine — that according to experience a vibration having a maximum acceleration amounting to 1/10 part of the gravita-

tional acceleration involves no risk to the machine of a failure due to fatigue. The acceleration of the whole machine if it is assumed to experience free displacement would be

$$a = \frac{F}{m+m_a} = \frac{m}{m+m_a} e_{\text{perm}} \left(\frac{\pi}{30}\right)^2 n^2 \leqq 0.1\,\text{g}, \tag{91}$$

where

a is the maximum acceleration of the whole machine unit;

m is the mass of the rotor (kg);

m_a is the mass of the stator (kg);

n is the number of revolutions per minute of the rotor (1/min).

The permitted specific imbalance, obtained from relationship (91) is

$$e_{\text{perm}} \cong \frac{10^8}{n^2} \times \frac{m+m_a}{m}\,\mu\text{m}. \tag{92}$$

The value of e_{perm} in the function of the speed may be directly read off the nomogram shown in Fig. 8.45 as the value corresponding to the ratio of the mass of the rotor to that of the stator.

Example. Let us determine the permitted specific imbalance of the rotor of an agricultural machine, if $m = 70$ kg is the mass of the rotor and $m_a = 100$ kg is the mass of the stator. The working speed of the rotor is $n = 1600$/min.

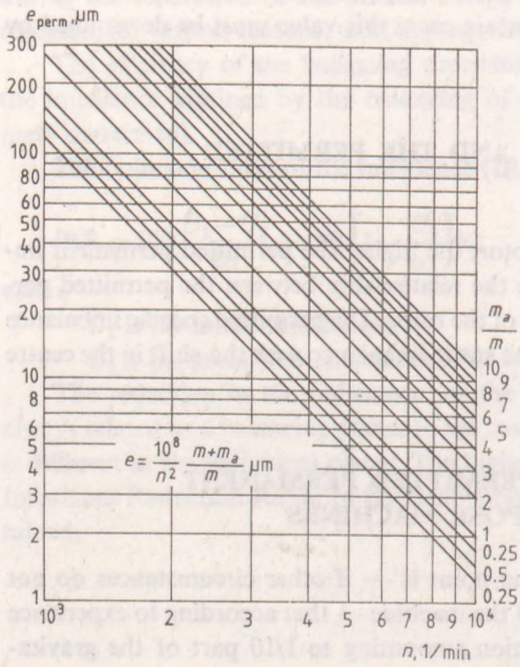


Fig. 8.45 Permitted specific imbalance in general machine building

Solution.
On the basis of relationship (92),

$$e_{\text{perm}} = \frac{10^8}{n^2} \times \frac{m+m_a}{m} = \frac{10^8}{1600^2} \times \frac{70+100}{70} \cong 95\ \mu\text{m}.$$

The speed of the centre of mass of the rotor is $e_{\text{perm}}\omega = 15.9\ \text{mm}^{-1}\ \text{s}$.

8.9.3 DETERMINATION OF THE PERMITTED PERMANENT IMBALANCE BASED ON THE SERVICE LIFE OF THE BEARING

It is advisable to determine the permitted value of the permanent imbalance for fast rotating parts and to satisfy requirements of a long working life on the basis of the service life of the bearings, if no other requirements prevail.

The manufacturers of rolling bearings specify in their catalogues the value of C, the basic dynamic load rating, for every bearing, at which the bearing is capable of sustaining 10^6 revolutions.

The following relationship exists between the given equivalent bearing load, P, the rated service life, specified in L million revolutions and the basic load:

$$P = \frac{C}{\sqrt[p]{L}}, \tag{93}$$

where

$p=3$ for ball bearings and
$p=10/3$ for roller bearings.

In the case of bearings rotating at constant speed the following relationship prevails between the speed n (1/min.), the service life L_h (in working hours) and the number of revolutions, L, completed until fatigue failure occurs

$$10^6 L = 60 L_h h. \tag{94}$$

The loading force caused by the imbalance gives, on comparing relationships (93) and (94) the permitted specific imbalance based on the service life of the bearings.

For ball bearings it is

$$e_{\text{perm}} = 2.32 \times 10^9 \frac{C}{m^3 \sqrt{L_h n^7}}\ \mu\text{m}; \tag{95}$$

and for roller bearings it is

$$e_{\text{perm}} = 1.68 \times 10^9 \frac{C}{m^{10/3} \sqrt{L_h n^{23/3}}}\ \mu\text{m}. \tag{96}$$

If we express the basic load C in newtons, the mass m in kg, the value of L_h in hours and n in 1/min., we obtain the permitted specific imbalance in μm.

Example. Let us determine the permissible specific imbalance on the basis of the life of the bearing in a rotor supported at both ends by type SKF 6004 ball bearings.

These bearings are not loaded by radial reaction forces. The mass of the rotor is $m=15$ kg, its working speed is $n=12\,000$/min. and $L_h=30\,000$ is the service life required.

Solution.

According to the SKF catalogue the basic dynamic load of the type 6004 bearing is $C=735\times\times 9.81=7210$ N. According to eq. (95),

$$e_{perm}=2.32\times 10^9\frac{7210}{15^3\sqrt{30\,000\times 12\,000^7}}=10\,\mu\text{m}.$$

8.9.4 DETERMINATION OF THE PERMITTED PERMANENT IMBALANCE FOR MACHINE TOOLS

The permitted imbalance for rotors of machine tools is to be considered first of all in terms of its effect on the form-trueness and the surface-roughness of the workpiece produced by the tool.

The machine frame, the tool and the workpiece holder form an elastic system. Under the effect of a periodically varying excitation force, this elastic system, or parts of it, will deform in correspondence with the dynamic effects, thereby changing the distance between the tool and the workpiece. The position of the tool relative to the workpiece influences the form-trueness and the surface-roughness of the machined part.

If we specify in advance the permitted relative displacement, x, required to attain the form-trueness of the machined part, then, knowing the rigidity conditions of the machine, the excitation force and thereby the permissible imbalance of the rotor can be determined.

The dynamic rigidity of the machine may be determined experimentally. A curve of the dynamic rigidity is plotted versus the frequency of the excitation force. The maximum value of the dynamic rigidity, i.e. the periodically varying force corresponding to unit relative displacement, is

$$R_{dyn}=\frac{P}{x}, \tag{97}$$

where

R_{dyn} is the dynamic rigidity (MN/m);

x is the relative displacement between the workpiece and the tool (μm);

P is the maximum value of the periodically varying excitation force (N).

Figure 8.46 illustrates the dynamic-rigidity curve of a centre grinder. According to experience the eigenfrequency of the vibration of the workpiece holder is relatively low in these machines. The dynamic conditions without the workpiece are illustrated in Fig. 8.46(a) and with the workpiece in Fig. 8.46(b). Without the workpiece, the eigenfrequency of the vibration of the workpiece holder is 3500/min. The minimum dynamic rigidity is 2 MN/m in this case.

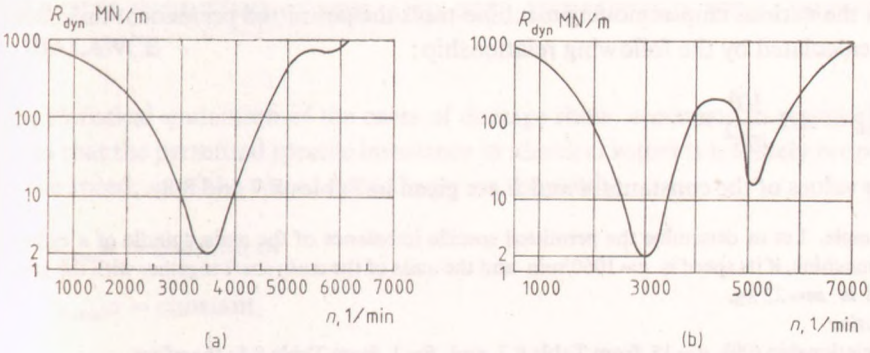


Fig. 8.46 Dynamic-rigidity curves of a centre grinder

Figure 8.46(b) shows the dynamic-rigidity curve of the grinder with the workpiece. Two local minima are seen in the curve. The eigenfrequency of the vibration of the workpiece holder decreases from 3500/min to 3000/min as a result of the effect of the workpiece, while the second local minimum represents the eigenfrequency of the vibration of the workpiece.

On machining workpieces of a relatively low mass, the form-trueness is satisfactory, as the eigenfrequency of the workpiece is not influenced considerably by such workpieces. This is because, at a higher rigidity of the workpiece holder ($R_{dyn}=10$ MN/m), the mean imbalance of an electric motor of $n=2880$/min does not essentially affect the relative displacement of the tool and the workpiece, while if the mass of the workpiece is large, the balancing value mentioned above is insufficient, because a minimum rigidity of $R_{dyn}=2$ MN/m corresponds to a speed of 2880/min. Accordingly the curve must be plotted for the machine both with and without workpieces of various masses.

The rigidity curve can be plotted for finished machines, but the designer has to specify the permitted permanent imbalance of the individual rotating parts still in the design stage. Experience shows that the minimum dynamic rigidity of machine tools is 2...3 MN/m, whether they are of welded or cast construction. Also the minimum relative distance between the tool and the workpiece, which does not yet affect the form of the workpiece, must be taken into account (e.g. the value of the relative displacement for grinding machines is 0.1 μm).

Substituting these values into eq. (97) and taking the force generated by the imbalance into account, the following permissible specific imbalance is obtained:

$$e_{perm} = 17.8 \frac{10^6}{mn^2} \mu\text{m}. \tag{98}$$

According to observations, the minimum rigidity range is between 1000... ...10 000/min. Parts rotating at a lower or a higher speed compared with this range may be manufactured with a larger permanent imbalance.

For the various chip-removing machine-tools the permitted permanent imbalance may be calculated by the following relationship:

$$e_{\text{perm}} = AB\frac{10^6}{mn^2}\,\mu\text{m}. \tag{99}$$

The values of the constants A and B are given in Tables 8.7 and 8.8.

Example. Let us determine the permitted specific imbalance of the main spindle of a cylinder grinding machine, if its speed is $n=1060$/min and the mass of the main shaft together with the grinding wheel is $m=22$ kg.

Solution

In relationship (99) $A=15$ from Table 8.7 and $B=1$ from Table 8.8; therefore

$$e_{\text{perm}} = AB\frac{10^6}{mn^2} = 15\times 1\,\frac{10^6}{22\times 1060^2} = 0.6\,\mu\text{m}.$$

Table 8.7

The proportionality factor A depending on the field of application of the machine

Field of application	A	
	for rotors found close to the workpiece and the tool (e.g. main spindles etc.)	for more remote rotors
Grinding machines. High precision lathes. Internal (bore) grinding spindles of $n<5000$/min.	15	30
Fine boring machines. Lathes. Internal (bore) grinding spindles of $n=5000\ldots10\,000$/min.	30	50
Other machine tools	50	100
Internal (bore) grinding spindles of $n>10.000$/min.	100–200	—

Table 8.8

The proportionality factor B depending on the speed of the rotor

Speed (1/min)	B
Under 1000	3
1000...10 000	1
Over 10 000	3

8.9.5 THE WORKING SPEED AND THE PERMITTED PERMANENT IMBALANCE

The statistical evaluation of the cases of damage show, according to practical experiences that the permitted specific imbalance in identical rotors is inversely proportional to the speed, n, of the rotor. Accordingly we may write

$$e_{\text{perm}} n = \text{constant or}$$

$$e_{\text{perm}} \omega = \text{constant},$$

where

$e\omega$ is the speed of the centre of mass, given usually in units of mm s^{-1}.

The aspect of similarity also gives the same relationship. If we assume rigid bearings, then identical stresses and surface pressures arise in geometrically similar rotors, if the product $e\omega$=constant.

The correctness of this statement is easily admitted, considering that the tangential and radial stresses in geometrically similar rotors are proportional to the square of the peripheral speed and have similar distributions, so identical stresses and bearing loads arise in the case of identical peripheral speeds.

The recommendation ISO 1940, "Balance Quality of Rotating Rigid Bodies", classifying the permitted permanent imbalance in rotors of various designations on the basis of the peripheral speed, v_s, of the shift in the centre of mass, is based on this principle.

8.9.6 QUALITY GRADES

Any value of the product $v_{sm} = e_{\text{perm}}\omega$ can be calculated. For simplicity's sake the recommendation specifies a standard series of numbers with the gradation factor 2.5. The individual quality grades are designated by the letter G and the numerical value of the product $e_{\text{perm}}\omega$, expressed in mm s^{-1} units (e.g. G 16). The quality grades may be formed from each other with the help of the factor 2.5, but in certain cases (e.g. for high-precision balancing) a finer gradation may be necessary.

Every quality grade G consists of a range whose lower limit value is zero and upper limit value is e_{perm}. Figure 8.47 illustrates the upper limit values of the permitted specific imbalances versus the maximum working speed.

Example Let us determine the value of the permitted specific imbalance corresponding to the quality grade G 16, if the working speed is n=6000/min.

Solution.

We find on the horizontal axis the value corresponding to the specified n=6000/min and draw from it a vertical line intersecting the oblique line marked G 16. The point of intersection projected to the vertical axis gives the permitted specific imbalance e_{perm}=25 μm.

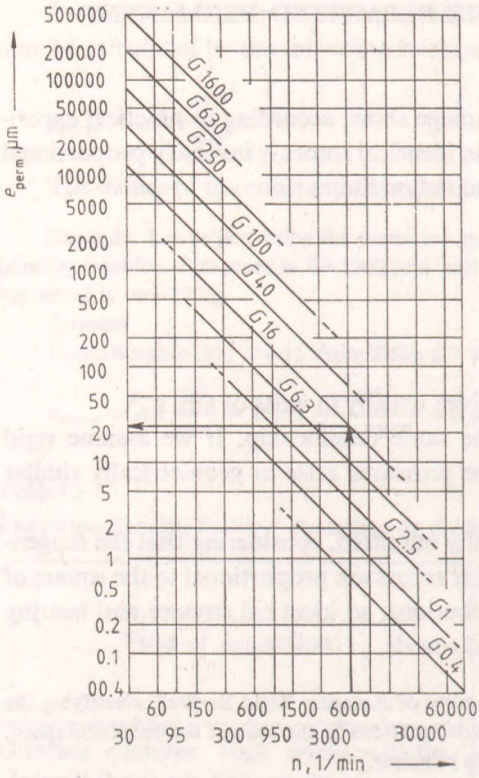


Fig. 8.47 Permitted specific permanent imbalances versus the maximum working speed

The same result is obtained by calculation as follows: The quality grade $G\,16$ means that the permitted speed of the centre of mass is $v_{s\,\mathrm{perm}} = 16$ mm/s. Accordingly

$$e_{\mathrm{perm}} = \frac{v_{s\,\mathrm{perm}}}{\omega} = \frac{16}{\frac{\pi}{30}\,6000} = 0.0254 = 25.4\ \mu\mathrm{m}.$$

8.9.7 CLASSIFICATION OF RIGID ROTORS

Table 8.9 contains the classification of rotors used in practice, according to the different grades of quality. The classification of rotors was based on actual experience. If the values are adhered to, there is a high probability that the rotor will run quietly. Obviously the table cannot be regarded as complete, since rotors for new purposes and further aspects may be devised in the future, thus leading to a revaluation of the permissible value of the residual imbalance and altering the classification.

Certain rotors — according to their purpose — may appear in several grades of quality (e.g. crankshafts and rotors of electric motors).

The lines in Fig. 8.47. do not pass through the whole speed range. The limits in the fine-quality grade shift in the high-speed range. This is understandable, considering that the speed of the diesel

Table 8.9
Classification of rigid rotors

Quality grade	$v_{s\,\mathrm{perm}} = e_{\mathrm{perm}}$ mm s^{-1}	Rotors or machines (examples)
	>1600	Stable crank drives; low-speed diesel engines with odd number of cylinders for ships
G 1600	1600	Stable crank drives with large two-stroke engines
G 630	630	Stable crank drives; high-speed, four-cylinder diesel engines Crank drives; spring-mounted diesel engines for ships
G 100	100	Stable crank drives; high-speed, six-cylinder and multi-cylinder diesel engines Motorcars, trucks, locomotive engines
G 40	40	Motorcar wheels, wheel discs, compound gears, drive shafts. Spring-mounted crank drives, high-speed, six-cylinder and multi-cylinder four-stroke engines. Crank drives of motorcars, trucks and locomotives.
G 16	16	Drive shafts with special requirements; parts of crushers and agricultural machines; certain parts of crank drives of motorcars, trucks and tractors; crank drives of six-cylinder and multi-cylinder engines with special requirements.
G 6.3	6.3	Parts of production-technological apparatus: centrifuge drums, fans, flywheels, centrifuge pumps, machine parts and machine-tool parts, rotors of general electric motors. Crank drives with special requirements.
G 2.5	2.5	Rotors of jet-drives, drives of gas and steam turbines, turbine blowers, turbogenerators, machine tools. Rotors of medium and large electric motors with special requirements. Rotors of small motors, turbine pumps.
G 1 Fine balancing	1	Drives of tape recorders and record players. Drives of grinding machines, rotors of small motors with special requirements.
G 0.4 Precision balancing	0.4	Rotors, shafts and wheels of precision grinding machines, gyroscopes.

engine of a low-speed ship is not greater than 400 r.p.m., and it is in the coarsest quality grade, while the speed of a gyrocompass is not less than 1000 r. p. m. and it is found in the finest quality grade.

The low values of the quality grades are G 1 and G 0.4. If these quality grades are specified, in many cases it is necessary to compromise between the technical requirements and the actual conditions realizable.

The limit of the balancing capacity is characterized by repeatedly attaining the minimum residual imbalance of the rotor. The strict quality grades can be attained in practice only by providing appropriate conditions (e.g. if the mounting of the rotor is perfect, if the bearing pins are concentric, if the bearings are particularly accurate, etc.). Already, in the quality grade G 1, the rotor must be balanced on its operational bearing, and a drive shaft cannot be used for balancing. Rotors in the quality grade G 0.4 are balanced on their bearings, assembled with their own drives.

The quality grades are not compulsory specifications, but recommendations. The quality grades assist in avoiding gross errors and avert unrealizable requirements. The values can be used as initial data for experimenting with the balancing quality individually specified for the actual type of rotor.

Determination of balancing quality by means of experiment, The limit value of the permissible residual imbalance of a specific rotor can be determined with a carefully balanced rotor. The imbalance must be a maximum of 1/10...1/20th of the presumed permissible value. Next the imbalance of the rotor is increased by mounting an experimental mass on the rotor — meanwhile studying its effect in the assembled condition — until the imbalance contributes considerably in increasing the vibration amplitude of the machine unit, or influences the quiet running of the machine and other functions of the machine incidentally.

In the case of balancing in two planes, the various effects, such as the effect of the static imbalance and moment imbalance, have to be analysed separately.

The value of the permissible imbalance is determined so that a further slight imbalance arising during assembly or operation should still be permissible.

8.10 DISTRIBUTION OF THE PERMISSIBLE VALUE OF THE PERMANENT IMBALANCE

8.10.1 ROTOR WITH A SINGLE BALANCING PLANE

In the case of disc-shaped rotors (Fig. 8.48) selecting a single balancing plane suffices if the distance between the bearings is great enough and the axial knocking of the disc is slight. The realization of these conditions must be examined in every case.

We balance several of these rotors in a single plane, then measure their maximum permanent moment-imbalance U_m, divide this by the distance, l, between the bearings, and obtain the imbalances U_{Am} and U_{Bm} caused by the moment-imbalance existing

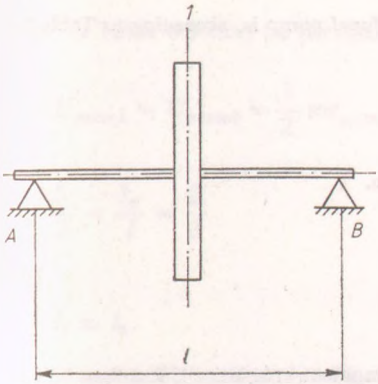


Fig. 8.48 Rotor with a single balancing plane

in the plane of the bearings. If this imbalance does not exceed — even in the least favourable case — half the value of the permitted permanent imbalance of the rotor, as determined from Table 8.9, then single-plane balancing is satisfactory, apart from certain special cases. So the total value of $v_{s\ \mathrm{perm}}$ is permitted in this case.

The measured moment-imbalance generates in the plane of the bearings A and B the following imbalance:

$$U_{Am} = U_{Bm} = \frac{U_m}{l}. \tag{100}$$

The permitted specific imbalance is, on the basis of the speed of the centre of mass $v_{s\ \mathrm{perm}}$ given in Table 8.9,

$$e_{\mathrm{perm}} = \frac{v_{s\ \mathrm{perm}}}{\omega}. \tag{101}$$

The permitted imbalance in the single balancing plane is

$$U_{\mathrm{perm}} = me_{\mathrm{perm}}, \tag{102}$$

if the following condition is met:

$$U_{Am} = U_{Bm} \leqq \frac{U_{\mathrm{perm}}}{2}. \tag{103}$$

If the condition (103) is not met, the rotor must be balanced in two planes.

Example Let the mass of the impeller of a centrifugal pump be $m=32$ kg, its speed $n=$ $n=1500$/min and the distance between the bearings $l=650$ mm. After balancing a great number of impellers in a single plane the maximum moment-imbalance is found to be $U_m=212\,000$ mm² g. How is the impeller to be balanced and what is the value of its permitted absolute imbalance?

Solution.

The maximum moment-imbalance in the plane of the bearings is, according to the measurements (on the basis of eq. (100),

$$U_{Am} = U_{Bm} = \frac{U_m}{l} = \frac{212\,000}{650}\ 326\ \mathrm{mm\ g}.$$

The permitted speed of the centre of mass of a centrifugal pump is, according to Table 8.9,

$v_{s\,\mathrm{perm}} = e_{\mathrm{perm}}\omega = 4.3\ \mathrm{mm\,s^{-1}}$.

Accordingly,

$$e_{\mathrm{perm}} = \frac{6.3}{\omega} = \frac{6.3}{\frac{\pi}{30}n} = \frac{6.3}{\frac{\pi}{30}1500} = 0.04\ \mathrm{mm} = 40\ \mu\mathrm{m}.$$

The permitted absolute unbalance is:

$U_{\mathrm{perm}} = me_{\mathrm{perm}} = 32\times 40 = 1283\ \mathrm{mm\,g}.$

As the imbalance $U = 326$ mm g originating from the moment-imbalance is less than half the permitted imbalance, $U_{\mathrm{perm}}/2 = 641.5$ mm g, then balancing the rotor of a centrifugal pump in a single plane is sufficient. The absolute value of the permitted permanent imbalance in the balancing plane is

$U_{\mathrm{perm}} = 1283\ \mathrm{mm\,g}.$

The magnitude of the moment-imbalance depends on the distance between the bearing and the selected balancing plane. If one of several balancing planes can be selected, the plane causing the minimum moment-imbalance must be determined experimentally.

8.10.2 ROTOR WITH TWO BALANCING PLANES

If a rotor fails to meet the conditions described in the preceding section, it must be balanced in two planes (Fig. 8.49). When the centre of mass of the rotor is in the central third of the distance between the bearing brackets, and the distances of the balancing planes from the centre of mass of the rotor are nearly identical, then half the value

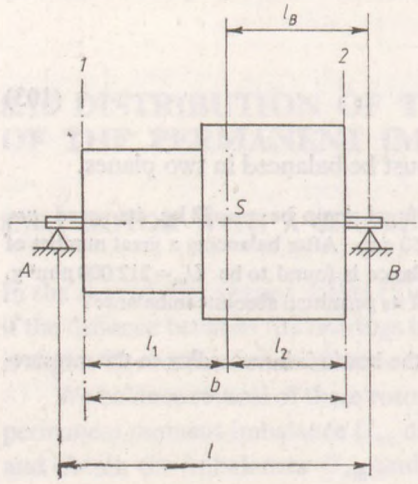


Fig. 8.49 Rotor with two balancing planes

given in Table 8.9 may be permitted in the individual planes, i.e.

$$U_{\text{perm}\,1} = U_{\text{perm}\,2} = \frac{1}{2} m e_{\text{perm}}, \tag{104}$$

if

$$\frac{1}{3} < \frac{l_B}{l} < \frac{2}{3} \tag{105}$$

and

$$l_1 \approx l_2. \tag{106}$$

Example. With reference to Fig. 8.49, let $l = 650$ mm be the distance between the bearings of the rotor, $l_B = 310$ mm the distance between the centre of mass and the bearing B, $l_1 = 300$ mm and $l_2 = 250$ mm the distances between the balancing planes *1* and *2*, respectively, and the centre of mass, $m = 100$ kg the mass of the rotor and $e_{\text{perm}} = 22\,\mu$m the permitted specific imbalance. Let us determine the permitted imbalance for each balancing plane.

Solution.

(a) Let us examine the position of the centre of mass of the rotor between the bearing brackets:

$$\frac{l_B}{l} = \frac{310}{650} = 0.47,$$

so $\frac{1}{2} < 0.47 < \frac{2}{3}2$, i.e. the centre of mass is found in the central third of the distance between the bearing brackets.

(b) Let us control the distances between the balancing planes and the centre of mass: $l_1 = 300 \approx \approx l_2 = 250$ mm. Both distances are nearly equal, so the permitted imbalance will be divided equally in both balancing planes.

(c) The permitted permanent imbalance in each balancing plane is, according to eq. (104),

$$U_{\text{perm}\,1} = U_{\text{perm}\,2} = \frac{1}{2} m e_{\text{perm}} = \frac{100}{2} 22 = 1100 \text{ mm g}.$$

If, in the case of any rotor, the axial distances of the balancing planes from the centre of mass are not equal, the permanent imbalance must be distributed between them according to the mass ratio of the rotor, assuming that the major part of the mass of the rotor is found between the bearing brackets.

In special cases the value of the permitted permanent imbalance must be determined with consideration of the actual circumstances, including the loads acting on the bearings.

In the case of asymmetric rotors the permitted value of the imbalance is distributed — according to the ISO 1940 recommendation — in proportion to the mass etween the individual balancing planes.

Let

$$m_1 + m_2 = m. \tag{107}$$

According to the above condition, imbalances proportional to the partial masses may

be permitted in the individual balancing planes, i.e.

$$U_{\text{perm 1}} = \frac{m_1}{m} U_{\text{perm}} \tag{108}$$

and

$$U_{\text{perm 2}} = \frac{m_2}{m} U_{\text{perm}}, \tag{109}$$

respectively, where

$$U_{\text{perm}} = me_{\text{perm}}.$$

The ratios of the masses to the proportional distances are determined from the moment equation, written up for the balancing planes, i.e.

$$m_1 b = ml_2$$

and

$$m_2 b = ml_1.$$

Therefore

$$\frac{m_1}{m} = \frac{l_2}{b} \tag{110}$$

and

$$\frac{m_2}{m} = \frac{l_1}{b}. \tag{111}$$

The imbalance permitted in the individual planes is obtained by combining eqs (108)–(111):

$$U_{\text{perm 1}} = \frac{l_2}{b} me_{\text{perm}} \tag{112}$$

$$U_{\text{perm 2}} = \frac{l_1}{2} me_{\text{perm}}. \tag{113}$$

If $l_1 = l_2$, then (112) and (113) agree with relationship (104).

Example. Let us determine the permitted maximum imbalance of the rotor described in the preceding example in the individual balancing planes, when the distance of balancing plane *2* from the centre of mass is $l_2 = 100$ mm. The other data are unchanged.

The permitted permanent imbalance in the individual balancing planes are, with the application of eqs (112) and (113):

$$U_{\text{perm 1}} = \frac{l_2}{l_1 + l_2} me_{\text{perm}} = \frac{100}{300 + 100} 100 \times 22 = 550 \text{ mm g}$$

and

$$U_{\text{perm 2}} = \frac{300}{300 + 100} 100 \times 22 = 1450 \text{ mm g}.$$

In the examples discussed so far we have determined the permissible imbalance on the basis of the static imbalances, as the latter always raise greater reaction forces in the bearings than the moment-imbalance, if the distance between the latter exceeds the distance between the balancing planes. If this is greater than the distance between the bearings, then the permitted permanent imbalance must be determined as desrcibed below.

8.10.3 ROTOR WITH WELL-SEPARATED BALANCING PLANES

If the distance betewen the balancing planes exceeds that between the bearings, the effect of the moment-imbalance exerted on the bearings is greater than that of the static imbalance.

The imbalance in each of the balancing planes of the symmetrical rotor shown in Fig. 8.50 is $U_1(=U_2)$ causing, in the case of a moment-imbalance,

$$U_{Am}-U_{Bm}=U_1\frac{b}{l} \tag{114}$$

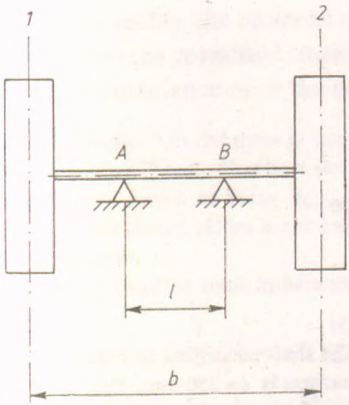


Fig. 8.50 Rotor with distant balancing planes

in bearings A and B, while in the case of the static imbalance

$$U_{As}=U_{Bs}=U_1. \tag{115}$$

As $b>l$, the load originating from the moment-imbalance is greater than that caused by the static imbalance; therefore the permanent imbalance permitted in the balancing planes must be determined on the basis of the moment-imbalance. The bearings should be loaded only by the load corresponding to the static load according to eq. (115), i.e.

$$U_{Am}=U_{Bm}=\frac{1}{2}me_{\text{perm}}. \tag{116}$$

From the comparison of eqs (114) and (115) it follows that

$$U_{\text{perm 1}} = U_{\text{perm 2}} = \frac{l}{2b} me_{\text{perm}}. \tag{117}$$

Example. Let the permitted permanent imbalance of the symmetrical rotor shown in Fig. 8.50 be $U_{\text{perm}} = 3250$ mm g, the distance between the bearings $l = 750$ mm and that between the balancing planes $b = 1000$ mm. Let us determine the permissible permanent imbalance in the individual balancing planes.

Solution.

According to eq. (117),

$$U_{\text{perm 1,2}} = \frac{l}{2b} U_{\text{perm}} = \frac{750}{2 \times 1000} 3250 = 1218.75 \text{ mm g}.$$

If the distance between the balancing planes is much bigger than that between the bearings $(b/l > 2)$, it is advisable to determine the permitted values of the static imbalance and the moment-imbalance separately and to assign half the permitted shifting of the centre of mass to the static imbalance and half of it to the moment-imbalance, Thus the permitted static imbalance of the rotor is

$$U_{\text{perm s}} = \frac{1}{2} me_{\text{perm}},$$

and in the individual balancing planes

$$U_{\text{perm s 1,2}} = \frac{U_{\text{perm s}}}{2} = \frac{1}{4} me_{\text{perm}}. \tag{118}$$

According to eq. (117) for the moment imbalance with

$$e_{\text{perm}} m/2, \quad U_{\text{perm 1,2}} = \frac{l}{4b} me_{\text{perm}}. \tag{119}$$

Example. With the belt pulleys mounted on both ends of the shaft according to Fig. 8.50, the total mass of the rotor is $m = 52$ kg, the distance between the bearings is $l = 250$ mm, that between the balancing planes is $b = 670$ mm and the permitted shift of the centre of mass is $e_{\text{perm}} = 16\ \mu$m. Let us determine the permitted permanent static imbalance and moment-imbalance values in the individual balancing planes.

Solution.

As $b/l = 670/250 = 2.68 > 2$, so the permitted static imbalance in both balancing planes is, according to eq. (118),

$$U_{\text{perm s 1,2}} = \frac{1}{4} me_{\text{perm}} = \frac{16}{4} 52 = 208 \text{ mm g}.$$

The permitted moment-imbalance in both planes is, according to eq. (119),

$$U_{\text{perm m 1,2}} = \frac{l}{4b} me_{\text{perm}} = \frac{250 \times 52}{4 \times 670} 14 = 77.61 \text{ mm g}.$$

8.10.4 ROTOR WITH CLOSE BALANCING PLANES

The relationship (118) and (119) are valid, but the ratio of the permitted moment-imbalance to the static imbalance is different owing to the different ratio of the distance between the balancing planes to the distance between the bearings (Fig. 8.51). The permitted moment-imbalance may exceed the static imbalance considerably.

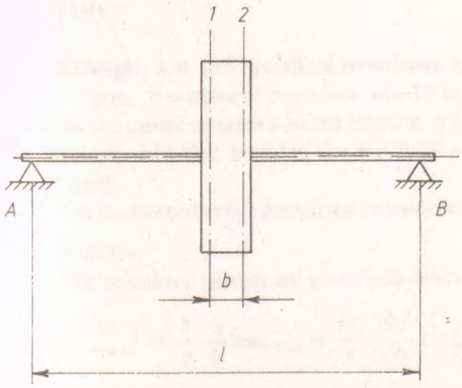


Fig. 8.51 Rotor with close balancing planes

If in reality the moment-imbalance arising in the rotor is less than its permitted value, then the permitted static imbalance may be increased at the cost of the permitted moment-imbalance until the conditions of single-plane balancing are attained.

Example. Let the mass of the rotor shown in Fig. 8.51 be $m=52$ kg, the distance between the bearings $l=670$ mm, that between the balancing planes $b=250$ mm, and the permitted maximum shift of the centre of mass $e_{perm}=16\,\mu$m. Let us determine the permitted permanent static and moment imbalance values in the individual balancing planes.

Solution.

The permitted static imbalance in both balancing planes is, according to eq. (118),

$$U_{perm\,s\,1,2} = \frac{1}{4} m e_{perm} = \frac{16}{4} \times 52 = 208 \text{ mm g.}$$

The permitted moment-imbalance in both planes is, according to eq. (119),

$$U_{perm\,m\,1,2} = \frac{l}{4b} m e_{perm} = \frac{670 \times 52}{4 \times 250} \times 16 = 557.44 \text{ mm g.}$$

8.10.5 ROTOR WITH CANTILEVERED BEARINGS

The recommendation ISO 1940 does not deal with the permitted imbalance in the balancing planes of rotors with cantilevered bearings; therefore we describe the practically applicable solutions for three starting conditions (Fig. 8.52).

(a) *The case where both bearings have an equal load capacity.*

The ratio of the loads on both bearings due to the imbalance can be identical with the ratio of the loads exerted by the mass of the rotor. The static loads of the bearings

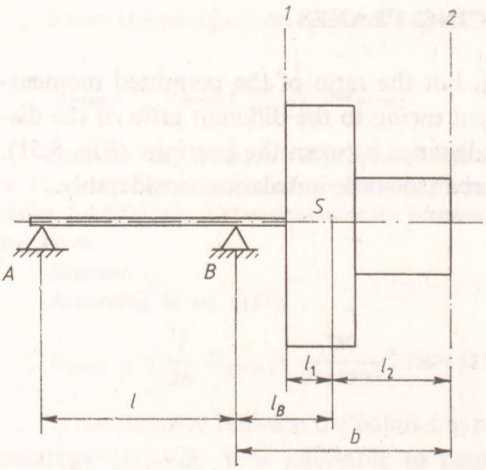


Fig. 8.52 Rotor with cantilevered bearings

are

$$A = mg\frac{l_B}{l} \tag{120}$$

and

$$B = -mg\frac{l+l_B}{l}. \tag{121}$$

If the bearing B can support the load increased by the ratio $(l+l_B)/l$, then the permitted imbalance may be increased by a similar ratio. Considering the static imbalance and the moment-imbalance separately again, the permitted static imbalance in bearing B may be written in the following form:

$$U_{\text{perm}\,s\,B} = \frac{l+l_B}{l}\frac{m}{2}e_{\text{perm}}. \tag{122}$$

On comparing eqs (121) and (122) we find that the static imbalance permitted in the centre of mass of the rotor is

$$U_{\text{perm}\,s} = \frac{1}{2}me_{\text{perm}}. \tag{123}$$

The value calculated by eq. (123) is distributed in correspondence with the ratio of the masses between the individual balancing planes to obtain the permitted static imbalance. Accordingly

$$U_{\text{perm}\,s\,1} = \frac{1}{2}\frac{l_2}{b}me_{\text{perm}} \tag{124}$$

and

$$U_{\text{perm}\,s\,2} = \frac{1}{2}\frac{l_1}{b}me_{\text{perm}}. \tag{125}$$

If the distances of the balancing planes from the centre of mass are equal $(l_1 - l_2)$, then the permitted static imbalance in both balancing planes is

$$U_{\text{perm}\,s\,1,2} = \frac{1}{4} m e_{\text{perm}}. \tag{126}$$

The permitted value of the moment-imbalance may be determined by means of eq. (119).

Example. Let the permitted imbalance for a rotor with cantilevered bearings (Fig. 8.52) be $e_{\text{perm}} = 2\,\mu\text{m}$, the mass of the rotor $m = 15$ kg, the distance between the bearings $l = 250$ mm, that between the centre of mass and the bearing B $l_B = 80$ mm, the distance between the balancing planes $b = 200$ mm, and that between the centre of mass and balancing plane 1 $l_1 = 60$ mm $(l_2 = b - l_1 = 140$ mm).

Let us determine the permitted imbalance if both bearings of the rotor are identical.

Solution.

The permitted permanent static imbalance in balancing plane *1* is, according to eq. (124),

$$U_{\text{perm}\,s\,1} = \frac{1}{2}\frac{l_2}{b} m e_{\text{perm}} = \frac{1}{2}\frac{140}{200} 15 \times 2 = 10.5 \text{ mm g}.$$

and in balancing plane *2* it is, according to eq. (125),

$$U_{\text{perm}\,s\,2} = \frac{1}{2}\frac{l_1}{b} m e_{\text{perm}} = \frac{1}{2}\frac{60}{200} 15 \times 2 = 4.5 \text{ mm g}.$$

The permitted permanent moment-imbalance in both balancing planes is, according to eq. (119),

$$U_{\text{perm}\,m\,1,2} = \frac{l}{4b} m e_{\text{perm}} = \frac{250}{4 \times 200} 15 \times 2 = 9.37 \text{ mm g}.$$

If $l_1 = l_2$ in this example, then the permitted permanent static imbalance in both balancing planes is

$$U_{\text{perm}\,s\,1,2} = \frac{1}{4} m e_{\text{perm}} = \frac{1}{4} 15 \times 2 = 7.5 \text{ mm g}.$$

The value of the moment-imbalance stays unchanged.

(b) *The load capacity of the bearings corresponds with the static load*

In this case the static imbalance must be distributed accordingly. Both bearings are loaded identically due to the moment-imbalance. As bearing A is weaker, the moment-imbalance must be low enough to prevent it being overloaded; therefore the permitted imbalance cannot be divided into equal static-imbalance and moment-imbalance components. The simplest way is to distribute the permitted imbalance according to the ratio of the distances from the centre of mass as expressed in eqs (112) and (113), taking no account of whether it is due to static imbalance or moment-imbalance.

Example. The bearings of the rotor in the previous example were designed with respect to the static load. Let us determine the permitted imbalance in the individual balancing planes.

Solution.

The permitted permanent imbalance is distributed according to relationships (112) and (113):

$$U_{\text{perm}1} = \frac{l_2}{b} me_{\text{perm}} = \frac{140}{200} 15 \times 2 = 21 \text{ mm g}$$

and

$$U_{\text{perm}2} = \frac{l_1}{b} me_{\text{perm}} = \frac{60}{200} 15 \times 2 = 9 \text{ mm g.}$$

If the imbalance vectors generate static imbalances in both balancing planes, then the maximum imbalance values calculated for the planes of the individual bearings are as follows:

$$U_A = \frac{l_B}{l} me_{\text{perm}} = \frac{80}{250} 15 \times 2 = 9.6 \text{ mm g}$$

and

$$U_B = \frac{l + l_B}{l} me_{\text{perm}} = \frac{250 + 80}{250} 15 \times 2 = 39.6 \text{ mm g.}$$

If the imbalance vectors are of opposite sense in both balancing planes, then the imbalances calculated for the planes of the individual bearings are as follows:

$$U_A = \frac{(l_B - l_1) U_{\text{perm}1} - (b + l_B - l_1) U_{\text{perm}2}}{l} =$$

$$= \frac{(80 - 60) 10.5 - (200 + 80 - 60) 4.5}{250} = -3.12 \text{ mm g;}$$

$$U_B = \frac{(l + l_B - l_1) U_{\text{perm}1} - (l + b + l_B - l_1) U_{\text{perm}2}}{l} =$$

$$= \frac{(250 + 80 - 60) 10.5 - (250 + 200 + 80 - 60) 4.5}{250} = 2.88 \text{ mm g.}$$

As the example shows, the static imbalance exerts a greater load on bearing A than does the moment-imbalance.

(c) *The observation of special requirements*

In addition to the conditions described other restrictions may also exist (e.g. a permitted vibration amplitude etc.). In this case the permitted permanent imbalance in the individual balancing planes must be determined experimentally, taking the given restrictions into account.

The relationships to be applied for calculating the permitted permanent imbalance for various types of rotor are summarized in Table 8.10.

8.10.6 BALANCING THE ROTOR ON A PROVISORY SHAFT

Frequently rotors must be balanced without their own shaft (e.g. motor-vehicle wheels, belt pulleys, flywheels, fan impellers, etc.). Such rotors are mounted on provisory shafts, of clamped in a special apparatus for balancing them. Most rotors have

Table 8.10

Distribution of the permissible residual imbalance in the balancing planes

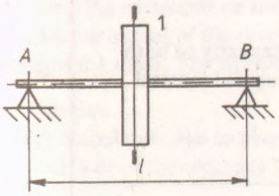
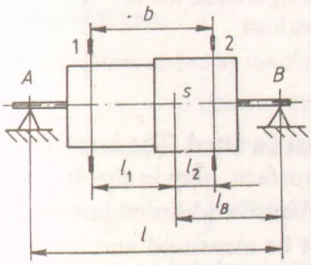
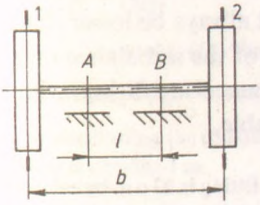
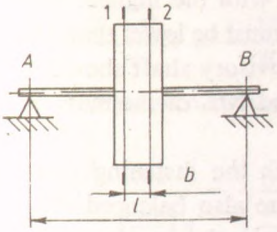
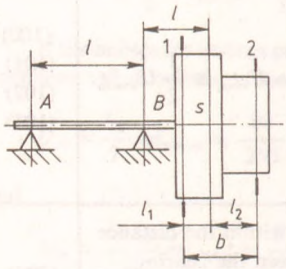
Number of figures	Permissible residual imbalance in the individual balancing planes	Condition of application	Number of formulae
	$U_{\text{perm}\,1} = me_{\text{perm}}$	$U_{\text{Am}} = U_{\text{Bm}} \leqq \frac{1}{2} U_{\text{perm}}$	(100) (101) (102) (103)
	$U_{\text{perm}\,1} = \frac{1}{2} me_{\text{perm}}$ $U_{\text{perm}\,2} = \frac{1}{2} me_{\text{perm}}$	Mass within the distance between the bearing supports $\frac{1}{3} < \frac{l_B}{l} < \frac{2}{3}$ $l_1 \approx l_2$	(104) (105) (106)
	$U_{\text{perm}\,1} = \frac{l_2}{b} me_{\text{perm}}$ $U_{\text{perm}\,2} = \frac{l_1}{b} me_{\text{perm}}$	Mass within the distance between the bearing supports	(112) (113)
	$U_{\text{perm}\,1} = \frac{l}{2b} me_{\text{perm}}$ $U_{\text{perm}\,2} = \frac{l}{2b} me_{\text{perm}}$	$b/l \leqq 2$	(117)
	$U_{\text{perm}\,s\,1,2} = \frac{1}{4} me_{\text{perm}}$ $U_{\text{perm}\,m\,1,2} = \frac{l}{4b} me_{\text{perm}}$	$b/l > 2$	(118) (119)
	$U_{\text{perm}\,s\,1,2} = \frac{1}{4} me_{\text{perm}}$ $U_{\text{perm}\,m\,1,2} = \frac{l}{4b} me_{\text{perm}}$	$b \ll 1$	(118) (119)

Table 8.10 (cont.)

Number of figures	Permissible residual imbalance in the individual balancing planes	Condition of application	Number of formulae
	$U_{\text{perm}\,s\,1} = \frac{l_2}{2b} me_{\text{perm}}$	Load capacity of both bearings is identical	(124)
	$U_{\text{perm}\,s\,2} = \frac{l_1}{2b} me_{\text{perm}}$		(125)
	$U_{\text{perm}\,m\,1,2} = \frac{l}{4b} me_{\text{perm}}$		(119)
	$U_{\text{perm}\,1} = \frac{l_2}{b} me_{\text{perm}}$	Load capacity of the bearing is equal to the static load	(112)
	$U_{\text{perm}\,2} = \frac{l_1}{b} me_{\text{perm}}$		(113)

a central cylindrical bore, to which the provisory shaft must be fitted. The imbalances of the provisory shaft (radial knocking on the contacting surface, play in the fit, axial knocking due to inaccurate mounting of the rotating part) cause additional imbalance of the rotor to be balanced. The resultant imbalance must be measured and lowered to the permitted value in the course of balancing. If a rotor balanced in this way is removed from the provisory shaft and mounted on its final shaft, its imbalance may prove to be greater than the permitted value. The same result may also be observed when a clamping apparatus is used for balancing a rotor without its own shaft.

The imbalance of an assembled rotor in a machine must always be lower than the permitted value. For attaining this aim the permitted value of the imbalance must be divided into that of the rotor and the imbalance caused by inaccurate fitting, with due consideration as to whether or not this divisioning is realizable.

Example. If the permitted imbalance is 25 μm and that due to the fitting is 23 μm by calculation, the rotor should be balanced to 2 μm, which is actually an exaggerated requirement. In this case a tighter fit, or a fit with no play is employed (e.g. by means of conical joints or circlips, etc.).

The provisory shaft, or apparatus, must be balanced with the highest possible precision. Its permanent imbalance in the balancing planes must be lower than 10% of the permitted imbalance of the rotor. The mass of the provisory shaft should be as small as possible; even in the worst case it must not exceed 10% of the mass of the rotor to be balanced.

The provisory shaft must be balanced together with the fastening elements (wedges, locks, etc.). In numerous cases the final shafts are also balanced with the fastening elements (e.g. electric motors, as prescribed in the recommendation

ISO 2373). In this case the provisory shaft must also be balanced together with the locks.

Example. Let us determine the imbalance, which may arise on fitting the impeller of a centrifugal pump, if $m=40$ kg is the mass of the impeller, $\theta_\xi=0.8$ kg m² and $\theta_\zeta=\theta_\eta=0.6$ kg m² are its moments of inertia, $d=50$ mm is the diameter of the bore and the shaft fits with a tolerance of H7/h6 in the bore-hole, $\delta_r=0.04$ mm is the radial knocking of the shaft and $\delta_a=0.03$ mm is the axial knocking measured on the diameter $D=300$ mm of the assembled rotor. The distance between the balancing planes of the rotor is $b=120$ mm and the planes are at equal distances from the centre of mass of the rotor. The working speed of the rotor is $n=450$/min.

Solution.

(1) Imbalance due to play in the fit:

With a fit of the accuracy 50 H7/h6 the play of the shaft in the bore is

$$J = 25+16 = 41\ \mu\text{m}.$$

The maximum eccentricity equals half the play:

$$e_j = 20.5\ \mu\text{m}.$$

Imbalance caused by the play in the fit is

$$U_j = me_j = 40\times 20.5 = 820\ \text{mm g}.$$

(2) Imbalance due to the radial knocking of the shaft:

$$U_r = \frac{1}{2}m\delta_r = \frac{1}{2}40\times 0.04 = 0.8\ \text{mm kg} = 800\ \text{mm g}.$$

(3) Moment-imbalance due to the lateral knocking of the rotor:

The angle enclosed between the plane of the impeller and the plane perpendicular to the axis of rotation is

$$\varphi \cong \frac{\delta_a}{D} = \frac{0.03}{300} = 1\times 10^{-3}\ \text{rad}.$$

According to eq. (16) the moment-imbalance is

$$U_m = \varphi(\theta_\zeta-\theta_\eta) = 1\times 10^{-3}(0.8-0.6) = 2\times 10^{-4}\ \text{kg m}^2 = 2\times 10^5\ \text{mm}^2\ \text{g}.$$

On distributing the imbalances due to various reasons over both balancing planes, the maximum imbalance is obtained as

$$U_{v1} = U_{v2} = \frac{U_j}{2}+\frac{U_r}{2}+\frac{U_m}{2} = \frac{820}{2}+\frac{800}{2}+\frac{2\times 10^5}{120} = 2476.6\ \text{mm g}.$$

Accordingly the shift of the centre of mass of the impeller is:

$$e_v = \frac{U_{v1}+U_{v2}}{m} = \frac{2\times 2476.6}{40} = 124\ \mu\text{m}.$$

In Table 8.9 the centrifugal pump is assigned to the quality grade G 6.3. The permitted specific imbalance is

$$e_{\text{perm}} = \frac{6.3}{\left(\frac{\pi}{30}\right)n} = \frac{6.3}{\left(\frac{\pi}{30}\right)450} = 0.133\ \text{mm} = 133\ \mu\text{m}.$$

The permitted imbalance of the impeller is

$$e_c = e_{perm} - e_v = 133 - 124 = 9\ \mu m.$$

The above example shows that the greater part of the permitted imbalance ($e_v = 124\ \mu m$) is due to the fit and the elimination of the fault in the provisory shaft, while only a small part is due to the balancing of the impeller. In the interest of a more favourable ratio it is advisable to fit the impeller more tightly, with little play (e.g. by means of conical joints, or by balancing it on its own shaft). In this way the balancing is easier and more economical.

8.10.7 BALANCING FOR DEFLECTION

The imbalance $\mathbf{U}_v$ of the provisory shaft and the imbalance $\mathbf{U}$ of the part to be balanced may be determined by two subsequent measurements, also in the assembled state. Figure 8.53 shows a belt pulley and a provisory shaft, both statically imbalanced. Figure. 8.53(a) illustrates the unkown imbalance vector $\mathbf{U}$ and Fig. 8.53(b) the unknown imbalance vector $\mathbf{U}_v$ in the system of coordinates related to the provisory shaft. Figure 8.53(c) shows the belt pulley mounted on the provisory shaft. We assume that the fit and the provisory shaft are not eccentric.

The resultant imbalance of the assembled rotor is

$$\mathbf{U}_1 = \mathbf{U}_v + \mathbf{U}_A. \tag{127}$$

Let us turn the belt pulley about the provisory shaft by 180 degrees as shown in Fig. 8.53(d). The imbalance of the rotor changes to

$$\mathbf{U}_2 = \mathbf{U}_v + \mathbf{U}_B. \tag{128}$$

Subtracting $\mathbf{U}_2$ from $\mathbf{U}_1$, we have

$$\mathbf{U}_1 - \mathbf{U}_2 = \mathbf{U}_A - \mathbf{U}_B = 2\mathbf{U}, \tag{129}$$

as

$$\mathbf{U} = \mathbf{U}_A = -\mathbf{U}_B.$$

Figure 8.54 illustrates the vector triangles of Figs 8.53(c) and (d) drawn together

On the basis of the above discussion, a part mounted on a provisory shaft or clamped in a balancing apparatus can be balanced as described in the following example.

Example. Let us balance a belt pulley without its own shaft.

Solution.

We have measured the magnitude $U_1 = 78$ mm g and the angular position $\alpha_1 = 70°$ for the imbalance of the assembled unit. Let us turn the belt pulley about the provisory shaft by 180° and determine the imbalance also in this position. Now $U_2 = 70$ mm g and $\alpha_2 = 310°$ are the readings of the instruments.

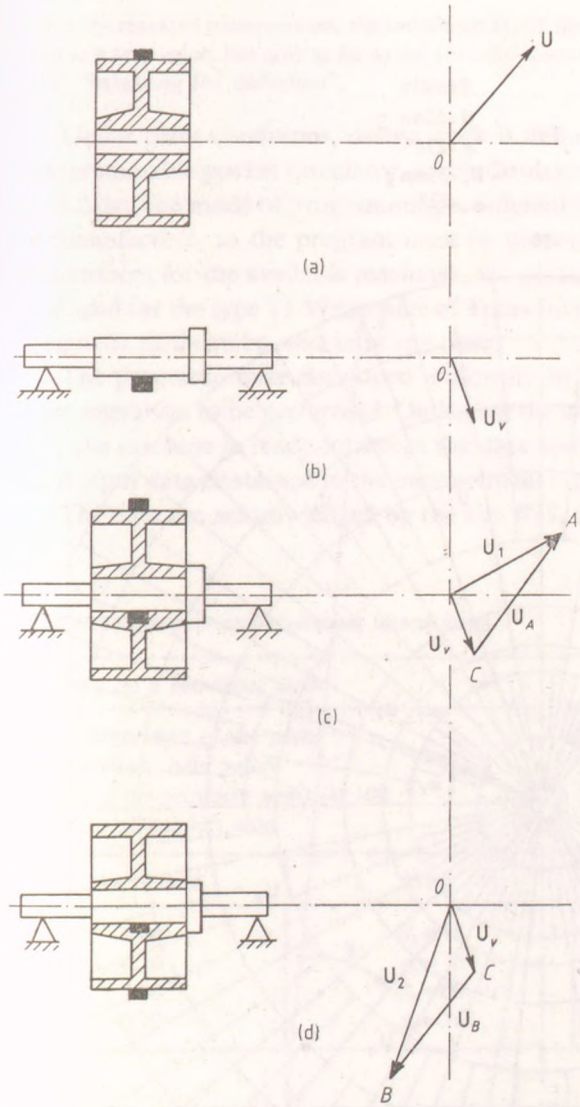


Fig. 8.53 Separating the imbalance of the belt pulley from that of the provisory shaft

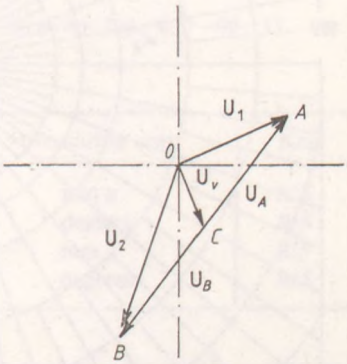


Fig. 8.54 Vector diagram of the belt pulley and the provisory shaft in the 0 and the 180° positions

Let us draw the measured vectors $\mathbf{U}_1$ and $\mathbf{U}_2$ as shown in Fig. 8.55. (It is advisable to use a preprinted polar coordinate system.) The letters A and B indicate the end points of the vectors. Let us connect these points by a straight line and then bisect this line. Hence we obtain point C, which represents the imbalance of the provisory shaft ($U_v = 35$ mm g, its angular position is $\alpha_v = 25°$). Let us draw a line parallel to the straight line AB, passing through the point 0, then intersect it with a straight line drawn from point B parallel to the straight line $0C$. In this way we obtain the imbalance of the belt pulley ($U = 60$ mm g, $\alpha = 276°$). Accordingly, at the angular position $\alpha = 276°$, the imbalance $U = 60$ mm g must be removed from the belt pulley. After balancing the belt pulley in this way, we

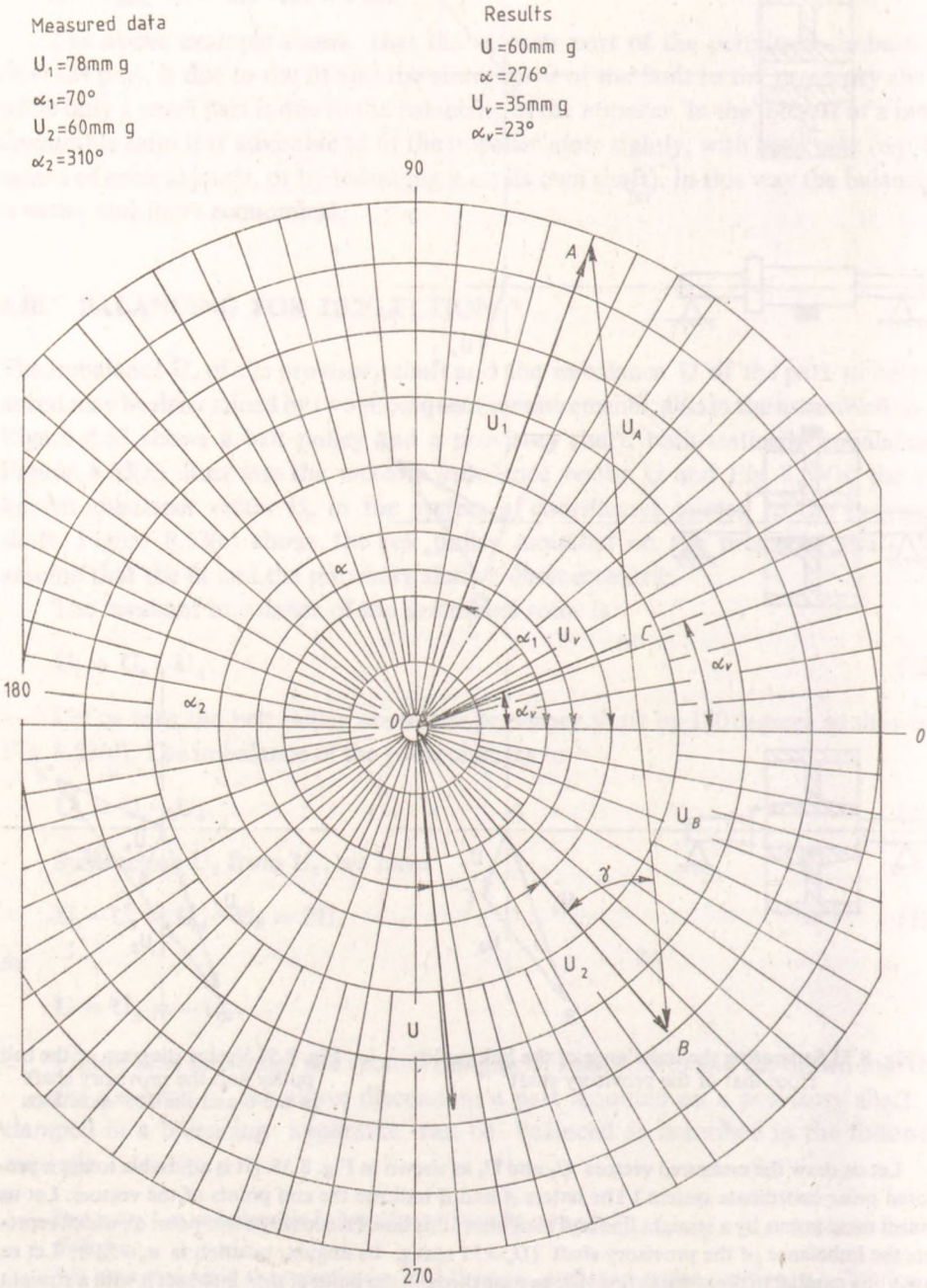


Fig. 8.55 Balancing on a provisory shaft

obtain, by repeated measurement, the imbalance U_v of the provisory shaft. As the balancing is not taken to a zero value, but only as far as the given imbalance U_v of the provisory shaft, the method is called "balancing for deflection".

Under plant conditions, design work is difficult; therefore it is advisable to use a programmable pocket calculator, which displays the results after inputting the measured data. The mode of programming is different for the various calculators of different manufacture, so the program must be prepared on the basis of the geometrical relationships for the available machines. We present, along with other programs, one developed for the type TI 59 machine of Texas Instruments. The program recorded on a magnetic card can be read-in at any time.

The programs were developed uniformly by assigning an address to the name of the operation to be performed. On keying the address after reading in the magnetic card, the machine is ready to accept the data and displays the number of the subsequent input data contained in the measurement report and the operation instructions.

The data are acknowledged by the key *R/S*, then the display indicates the serial

Table 8.11

Measurement report and operation instructions

Balancing on a provisory shaft	Address A

Identification data of the parts
Denomination: belt pulley — Drawing No.: 630—00—11
Number of the provisory shaft: K 145
Balancing plane: left side

No. of data	Data	Measuring unit	Key
1.00	$U_1=78$	mm g	*R/S*
0.10	$\alpha_1=78$	degrees	*R/S*
2.00	$U_2=60$	mm g	*R/S*
0.20	$\alpha_2=310$	degrees	*R/S*

Results

$U=59.92$	mm g	*R/S*	$\alpha=275.69°$	*R/S*
$U_v=35.37$	mm g	*R/S*	$\alpha_v=22.73°$	*R/S*

Control	Data		
3.00	$U_3=29$	mm g	*R/S*
0.30	$\alpha_3=10$	degrees	*R/S*

Result: Permanent imbalance of the part

$U_m=8.37$	mm g	*R/S*	$\alpha_m=238.57°$	*R/S*

Date: 20.7.1984. Signature: Fejes

number of the following item of data to be written in. After acknowledgement of the last piece of data, the results are displayed in the sequence given in the Table. If we press the key R/S after the last result, the program returns into the starting position and the calculation may be repeated by the input of new data. It is advisable to use Table 8.11 for facilitating the measurement and for the documentation of the data. We have included in the table the initial data of the preceding example and the results calculated by the programmed calculators.

After eliminating the imbalance **U** from the part, $\mathbf{U}_v$ must be measured during the testing procedure. The measured value $\mathbf{U}_3$ may deviate from $\mathbf{U}_v$; reading $\mathbf{U}_3$ into the computer, we obtain the permanent imbalance U_m, α_m of the part (the belt pulley in our example). The program is given in Fig. 8.56.

Table 8.11 and the corresponding program were prepared on the basis of the following relationships:

$$U = \frac{1}{2}[U_1^2 + U_2^2 - 2U_1U_2\cos(\alpha_1 - \alpha_2)]^{1/2}; \tag{130}$$

$$\gamma = \arcsin\frac{U_1}{2U}\sin(\alpha_1 - \alpha_2); \tag{131}$$

$$\alpha = \alpha_2 - \gamma; \tag{132}$$

$$U_v = (U^2 + U_2^2 - 2UU_2\cos\gamma)^{1/2}; \tag{133}$$

$$\beta = \arcsin\frac{U}{U_v}\sin\gamma; \tag{134}$$

$$\alpha_v = \alpha_2 - \beta; \tag{135}$$

$$U_m = [U_v^2 + U_3^2 - 2U_vU_3\cos(\alpha_v - \alpha_3)^{1/2}; \tag{136}$$

$$\varepsilon = \arcsin\frac{U_3}{U_m}\sin(\alpha_v - \alpha_3); \tag{137}$$

$$\Gamma = U_3\cos(\alpha_v - \alpha_3) - U_v; \tag{138}$$

$$\alpha_m = \alpha_v - \varepsilon \quad \text{if} \quad \varepsilon < 0 \quad \text{and} \quad \Gamma > 0;$$

$$\alpha_m = \alpha_v + \varepsilon + 180^\circ \quad \text{if} \quad \Gamma < 0;$$

$$\alpha_m = \alpha_v - \varepsilon + 360^\circ \quad \text{if} \quad \varepsilon > 0 \quad \text{and} \quad \Gamma > 0.$$

The question arises whether balancing the provisory shaft without the part and then balancing the part with the provisory shaft to "zero" equilibrium would not be simpler.

If the provisory shaft has no radial knocking and the part fits without play — as we have assumed — then it is indeed simpler. But this is a general method and the part can be balanced even in the presence of the faults described.

```
000 76 LBL
001 25 CLR
002 32 X⇄T
003 00 0
004 77 GE
005 22 INV
006 03 3
007 06 6
008 00 0
009 77 GE
010 23 LNX
011 32 X⇄T
012 75 —
013 03 3
014 06 6
015 00 0
016 95 =
017 92 RTN
018 76 LBL
019 22 INV
020 32 X⇄T
021 85 +
022 03 3
023 06 6
024 00 0
025 95 =
026 92 RTN
027 76 LBL
028 23 LNX
029 22 X⇄T
030 92 RTN
031 76 LBL
032 11 A
033 47 CMS
034 58 FIX
035 02 02
036 01 1
037 50 |x|
038 91 R/S
039 42 STO
040 01 01
041 42 STO
042 02 02
043 42 STO
044 03 03
045 39 PRD
046 01 01
047 93 .
048 01 1
049 50 |x|
050 91 R/S
051 22 INV
052 44 SUM
053 04 4
054 02 2
055 50 |x|
056 91 R/S
057 49 PRD
058 02 02
059 42 STO
060 07 07
061 33 X²
062 44 SUM
063 01 01
064 93 .
065 02 2
066 50 |x|
067 91 R/S
068 42 STO
069 06 06
070 44 SUM
071 04 04
072 43 RCL
073 01 01
074 75 —
075 02 2
076 65 ×
077 43 RCL
078 02 02
079 65 ×
080 43 RCL
081 04 04
082 39 COS
083 95 =
084 34 √X
085 55 ÷
086 02 2
087 95 =
088 42 STO
089 08 08
090 91 R/S
091 65 ×
092 02 2
093 95 =
094 35 1/X
095 65 ×
096 43 RCL
097 03 03
098 65 ×
099 43 RCL
100 04 04
101 38 SIN
102 95 =
103 22 INV
104 38 SIN
105 42 STO
106 05 05
107 75 +
108 43 RCL
109 06 06
110 95 =
111 71 SBR
112 25 CLR
113 91 R/S
114 43 RCL
115 08 08
116 33 X²
117 85 +
118 43 RCL
119 07 07
120 33 X²
121 75 —
122 02 2
123 65 ×
124 43 RCL
125 08 08
126 65 ×
127 43 RCL
128 07 07
129 65 ×
130 43 RCL
131 05 05
132 39 COS
133 95 =
134 34 √X
135 42 STO
136 09 09
137 92 R/S
138 43 RCL
139 05 05
140 38 SIN
141 65 ×
142 43 RCL
143 08 08
144 55 ÷
145 43 RCL
146 09 09
147 95 =
148 22 INV
149 38 SIN
150 94 +/—
151 85 +
152 43 RCL
153 06 06
154 95 =
155 71 SBR
156 26 CLR
157 42 STO
158 10 10
159 42 STO
160 13 13
161 42 STO
162 14 14
163 91 R/S
164 03 3
165 50 |x|
166 91 R/S
167 42 STO
168 11 11
169 93 .
170 03 3
171 50 |x|
172 91 R/S
173 22 INV
174 44 SUM
175 10 10
176 43 RCL
177 10 10
178 39 COS
179 65 ×
180 43 RCL
181 11 11
182 65 ×
183 43 RCL
184 09 09
185 65 ×
186 02 2
187 95 =
188 94 +/—
189 85 +
190 43 RCL
191 09 09
192 33 X²
193 85 +
194 43 RCL
195 11 11
196 33 X²
197 95 =
198 34 √X
199 42 STO
200 12 12
201 91 R/S
202 35 1/X
203 65 ×
204 43 RCL
205 11 11
206 65 X
207 43 RCL
208 10 10
209 38 SIN
210 95 =
211 22 INV
212 38 SIN
213 44 SUM
214 14 14
215 22 INV
216 44 SUM
217 13 13
218 42 STO
219 16 16
220 43 RCL
221 10 10
222 39 COS
223 65 ×
224 43 RCL
225 11 11
226 75 —
227 43 RCL
228 09 09
229 95 =
230 42 STO
231 15 15
232 29 CP
233 77 GE
234 33 X²
235 43 RCL
236 14 14
237 85 +
238 01 1
239 08 8
240 00 0
241 95 =
242 71 SBR
243 25 CLR
244 91 R/S
245 61 GTO
246 11 A
247 76 LBL
248 33 X²
249 43 RCL
250 16 16
251 29 CP
252 77 GE
253 35 1/X
254 43 RCL
255 13 13
256 71 SBR
257 25 CLR
258 91 R/S
259 61 GTO
260 11 A
261 76 LBL
262 33 1/X
263 43 RCL
264 13 13
265 85 +
266 03 3
267 06 6
268 00 0
269 95 =
270 71 SBR
271 25 CLR
272 91 R/S
273 61 GTO
274 11 A
```

Fig. 8.56 Calculation program for balancing on a provisory shaft (for the Type TI-59 pocket calculator)

We have assumed so far that the provisory shaft has no eccentricity and that the rotor fits onto it without any play. If these conditions are not met, then the vector diagram is modified according to Fig. 8.57, where $\mathbf{U}_v$ is the imbalance of the provisory shaft, $\mathbf{U}_e$ is the imbalance due to the eccentricity of the shaft (the product of the eccentricity and the mass of the part to be balanced) and $\mathbf{U}_j$ is the imbalance caused by the play existing in the fit. If we add the vector of the initial imbalance $\mathbf{U}_A$ of the part to the sum of these three vectors, we obtain the vector $\mathbf{U}_1$.

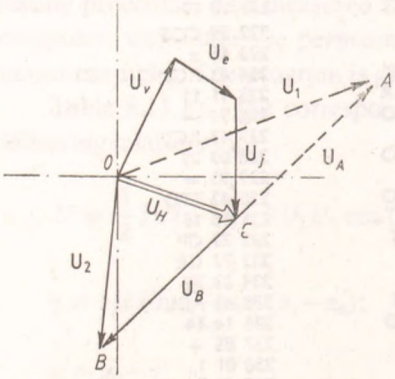


Fig. 8.57 Balancing for deflection

The imbalances $\mathbf{U}_v$ and $\mathbf{U}_e$ do not change if we turn the part to be balanced about the provisory shaft. Similarly the imbalance $\mathbf{U}_j$ stays unchanged if we ensure that the clearance is in the same position on the provisory shaft after turning the part about it. If this condition is met, then, on turning the part about the provisory shaft, the imbalance vector turns about point C. On the basis of both the measurements described above, the rotor can in principle be balanced perfectly, in spite of the presence of various faults.

We have presented the method for the static-imbalance case. If the part must be balanced in several planes, the process described must be repeated in each one of the planes involved.

8.11 BALANCING ASSEMBLED ROTORS

Most rotors consist of several parts (e.g. shaft, rolling bearings, belt pulley, fan, impeller, etc.). The parts may be balanced individually, but after assembly their residual imbalances are added vectorially. In the least favourable case we must consider the sum of the residual imbalances of all the individual parts plus the imbalance due to their combination, which must also be added. So the balancing quality of the assembled rotor is expected to be worse than that of its individual parts. Therefore if the assembled rotor must meet strict balancing quality requirements, the parts should not be balanced individually and then assembled, but the whole rotor must be bal-

anced in its own rolling bearings. In some cases balancing the assembled main component may suffice. The assembled and balanced rotor must not be dismantled again; if this is unavoidable, the parts must be carefully matched and marked together, then reassembled in the same position as marked.

Fitting the parts of large mass and the inner rings of rolling bearings into the same positions as they were before is an especially important task during balancing.

Example. Let the permissible radial knocking of the inner ring of a bearing of 30 mm bore diameter and normal accuracy be 13 μm. Let us balance the rotor together with these bearings to a permitted shift of the centre of mass of $e_{perm}=2$ μm. If we remove the bearings, then reassemble them in a different angular position, we spoil the balance of the rotor, because the eccentricity of the inner ring will exceed the permitted shift of the centre of mass.

It is impossible to reassemble the bearings precisely in the same position as formerly even if the parts are match-marked, so a certain deterioration of the balancing quality after removing and then reassembling the rotor is to be expected.

8.11.1 QUALITY CONTROL OF THE BALANCING

Rotors are balanced in the manufacturing plant with the help of the instruments or machines available. The customer checks the balancing quality using another instrument and sometimes using a different method. Therefore the errors due to the differences between the applied measuring instruments and methods must be taken into account. The residual imbalance must be somewhat lower at the manufacturer and somewhat higher at the customer. Table 8.12 contains the permitted values of the residual imbalance at the manufaturer and the user according to the recommendation ISO 1940.

Table 8.12
Permitted deviations of the quality grade

Quality grade	Permitted deviation	
	at the manufacturer	at the user
G 2.5–*G* 16	−15%	+15%
G 1	−30%	+30%
G 0.4	−50%	+50%

If the accuracy of the balancing machine is doubtful, the residual imbalance of the rotor may be determined with the least possible error in the following way.

Let us take a test mass causing a 5...10 times higher imbalance than the assumed residual imbalance of the rotor. It is advisable to use artists' plastiline for the test mass, as it can be easily formed, its quantity may be easily altered, it is sufficiently adhesive and easily removable from steel and metal surfaces. Let us divide the perimeter of the

rotor into, for example, 12 equal parts in the balancing plane. Put the test mass on the rotor, always using the same radius for each spacing, and take the reading of instrument deflection proportionate to imbalance of the rotor. In order to minimize the measuring errors, it is advisable to put the test mass on the rotor by leaving out some positions, for example using each fifth spacing, as shown in Fig. 8.58. Table 8.13 contains the measurement results.

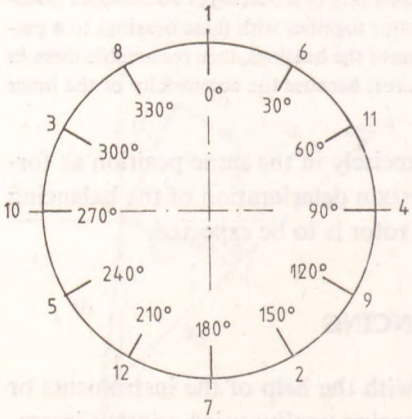


Fig. 8.58 Arrangement of the test mass for determining the residual imbalance of the rotor

Table 8.13

Variation of the imbalance depending on the position of the test mass

Serial No. of the test mass positions	1	2	3	4	5	6	7	8	9	10	11	12
Angular position of the test mass, °	0	150	300	90	240	30	180	330	120	270	60	210
Instrument deflection, divisions of the scale	12.9	11.3	14.4	10	14.4	11	12.9	14.1	10	15	11	13.1

We plot the measurement results versus the angular position as shown in Fig. 8.59. On connecting the individual points, an approximate sine curve is obtained. The distance k between the centreline of the curve and the axis φ, i.e. the arithmetic mean of the measurement results, is proportional to the imbalance caused by the test mass, while the amplitude of the sine curve corresponds to the residual imbalance of the rotor.

In our example the mean value of the measurements is $k=12.5$ scale divisions, the residual imbalance is $a=2.5$ scale divisions. From a knowledge of the test mass and the radius we can determine the relationship between the imbalance and the deflection of the instrument and so also the residual imbalance of the rotor. The angular position is given by φ_0 corresponding to the peak of the sine curve.

In our example a mass $q=5$ g was placed on the radius $r=50$ mm; the imbalance caused by the test mass was $U=5\times50=250$ mm g, corresponding to $k=12.5$ divisions of the scale, so 1 scale

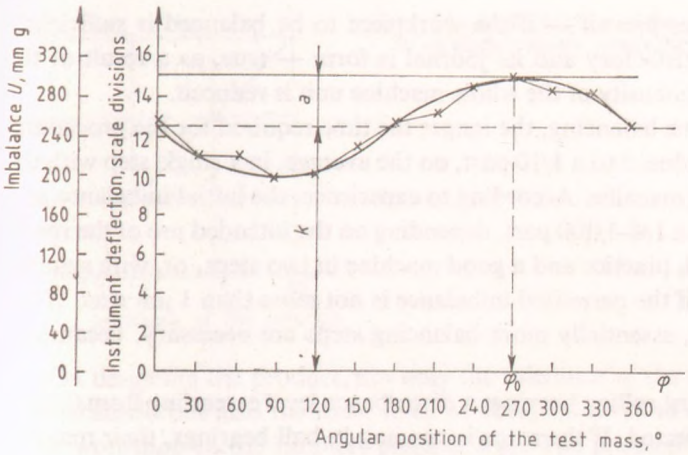


Fig. 8.59 Imbalance caused by the test mass

division equals 20 mm g and the residual imbalance of the rotor is $a=2.5$ scale divisions (50 mm g). The angular position of the residual imbalance of the rotor is $\varphi_0 \approx 270°$.

If the positions of the measurement points fail to follow the sine-curve, the residual imbalance is at the limit of reproducibility.

If the linearity of the system is doubtful, the series of measurements must be repeated with smaller, or bigger test masses. The measurements must be performed independently in each individual balancing plane.

8.12 GUIDING PRINCIPLES FOR BALANCING

It may occur that, although we observe the balancing limits determined by calculation when balancing the workpiece, we yet obtain an unsatisfactory result after assembly. The irregular running may have numerous causes, such as:

— incorrect mounting, seizure;
— insufficient accuracy, impurity of the ball bearings;
— erroneous selection of the clearance of the plain bearings;
— incorrect mounting, play and tolerance of flanged fittings;
— inclination of the rotor, seizure of the couplings;
— aerodynamic and hydrodynamic forces and the force of gravity;
— circularly asymmetric bending-rigidity of the workpiece;
— imbalanced magnetic forces.

Reducing the magnitude of the residual imbalance rarely gives the desired result; attempting to determine the real causes of the irregular running is more promising.

The magnitude of the permissible imbalance of a workpiece is not only a technical problem but also an economic one. Balancing has a real worth, especially where

strict balancing tolerances prevail — if the workpiece to be balanced is sufficiently rigid, its bearings are satisfactory and its journal is form — true, as a result of the balancing, the vibration intensity of the whole machine unit is reduced.

The more accurate the balancing, the longer the time required for the procedure. The imbalance may be reduced to a 1/10 part, on the average, in a single step with the help of a good balancing machine. According to experience, the initial imbalance of a rotor must be reduced to a 1/4–1/100 part, depending on the intended use of the rotor. This can be achieved with practice and a good machine in two steps, or, with suitable checking, in three steps, if the permitted imbalance is not more than 1 μm max. If the prescriptions are stricter, essentially more balancing steps are necessary, because of the disturbing effects.

In the case of standard rolling bearings a disturbance level extending from 0.5 μm to several μm can be expected. If the rotor is running in ball bearings, their removal and reassembly in a different position from the original one may cause a residual imbalance of about 5 μm. Therefore, if the mounting makes it necessary to remove the bearing, the shaft and the inner ring of the bearing must be match-marked, or else there is no sense in limiting the residual imbalance of the rotor to less than 5 μm.

If the specified imbalance is between 0.5 and 2 μm, the rotors must be balanced in their own ball bearings.

For attaining a residual imbalance of under 0.5 μm, the rotor must be balanced when installed in its casing, and bearings of a suitable quality class must be used. Rolling bearings must not be used where complete freedom from vibration and noise is required.

The accuracy of the balancing procedure is limited by the circularity of the journal. If the cross-section of the journal is not circular, the axis of rotation of the workpiece during balancing is not well defined, whether or not it runs in prismatic supports or in rolling bearings, and the balancing becomes uncertain.

A slight ovality of the journal does not greatly affect the vibration of the bearings, owing to the oil film which is present between the journal and the bearing shell. The frequency of the bearing vibration caused by a significant ovality of the journal is twice as high as that caused by an imbalance.

Example. The following case illustrates some exaggerated accuracy requirements. Consider the balancing of rotors of asynchronous electric motors. As these rotors are rigid, we may prescribe in principle a residual imbalance of under 1 μm. After fitting a rotor balanced to this precision into a motor, the measured vibration amplitude was 6 μm.

When the imbalance of the rotor was gradually increased, the vibration amplitude of the motor started increasing only after a certain imbalance was attained.

Figure 8.60 shows the vibration amplitude of the assembled motor versus the imbalance of the rotor. As can be seen, the vibration amplitude of the assembled motor is still 6 μm even when the imbalance of the rotor is less than 12 μm. So it is not worthwhile balancing the rotor to a higher accuracy than 12 μm. If, nevertheless, we wish to reduce the vibration amplitude then, along with specifying a lower residual imbalance, we must install bearings of a higher accuracy grade and limit the vibration caused by the imbalanced magnetic fields.

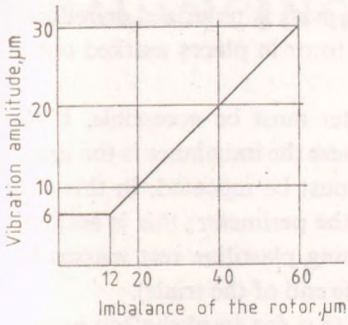


Fig. 8.60 Variation of the vibration amplitude of an electric motor versus the imbalance of the rotor

On designing the product, not only the tolerance of the residual imbalance must be determined but also the rotor must be designed so as to enable it to be balanced in the workshop in the simplest possible way. The position of the balancing planes must be determined. The balancing planes of rigid bodies must be as distant from each other as possible, to permit the balancing to be performed in these planes over the biggest possible radius. Where there exists no such balancing plane, it must be created by the addition of suitable flanges. The possibility of arranging the mass required for balancing in the individual balancing planes and of removing it must be ensured. The magnitude and the choice of the masses required for balancing depend on the intended destination of the workpiece, its mass and the balancing radius.

Table 8.14 contains orienting values for determining the masses required for the balancing of rotors of various destinations giving the product of the necessary balancing mass with the radius per kg of the workpiece.

Post-working after balancing may last for a considerable time (2...50 times the measurement period). Therefore the mass distribution must be foreseen in the design stage with a view to limiting the corrections to the unavoidable unevennesses.

Table 8.14

Masses required for balancing

Destination of the rotor	Product of the mass required for balancing with the radius per kg of the rotor, mm g/kg
Finely processed symmetrical rotor to be used in precision mechanics	5...20
Rotor of a small-sized electric motor	10...50
Shorted and wound rotors of large electric motors	20...100
Partly machined die-forged shafts and rotors (e.g. crankshafts)	100...5000
Die-forged shafts and rotors, weight-centred before machining	20...100
Cast and welded rotors (e.g. flywheels)	200...1000
Hollow, cast rotors	500...2000

It is necessary to consider beforehand whether the mass is polarized or reduced to components when removed from or added to the rotor in places marked out in advance.

In the case of polar balancing the whole perimeter must be accessible. If the required accuracy cannot be achieved in a single step, where the imbalance is too great, or because of inaccurate measurement, the operation must be repeated. In this case two, or even three, masses, or bore-holes, may fall on the perimeter; this is aesthetically unsatisfactory and should be avoided (e.g. by using plastiline test masses for balancing, and performing the final correction only at the end of the trials).

Polar balancing cannot be prescribed in every case (e.g. for crankshafts); in such cases the component parts must be balanced in situ one by one. The most favourable solution is to mark out the places at 90 degree angular distance on the perimeter. For balancing in situ, generally two places are chosen in each balancing plane for application of the balancing masses, and the correction after balancing is also restricted to two places. In the case of rectangular components 40% more material must be added or removed at the most than for polar balancing (if the balancing masses are arranged at 120° angular distances, this surplus may even amount to 100%).

The balancing is the more economic the more accurately the magnitude of the mass to be removed is determined; thus grinding and filing off are unfavourable. A mass removed by drilling can be measured the best, as its magnitude depends on the depth of the bore-hole.

If drilling is used for balancing, a scatter of 5...10% is to be expected, taking into account the volume of the conical end of the drill and the distance of the centre of mass of the drilled-out mass from the axis of rotation.

Much more material can be removed by milling than by drilling. It is advisable to prepare moulds of a defined shape to measure the material removed conveniently.

Balancing by mass addition involves no chip formation, so it may be carried out with the help of a balancing machine.

In the most favourable case the designed structure of the component ensures that balancing is possible. A circular groove containing two movable balancing masses is prepared in the component.

For balancing in situ, four threaded bore-holes for inserting the balancing screws are pre-drilled.

Riveting, welding, or soldering the mass onto the component to be balanced may be carried out without any preparatory construction, but it is more time-consuming. Motor vehicle wheels are balanced by fixing balancing masses onto the rim of the wheel disc by means of leaf springs.

The constructive solutions which may be applied for balancing are summarized in Tables 8.3, 8.4 and 8.5.

9 BALANCING MACHINES

The moment imbalance and dynamic imbalance of a rotor cannot be determined by simple means (e.g. rolling along horizontal, parallel edges). Such an imbalance is perceptible only in the case of rotation, on the basis of the effect exerted on the bearing supports. A diagram of a balancing machine for taking measurements is shown in Fig. 9.1.

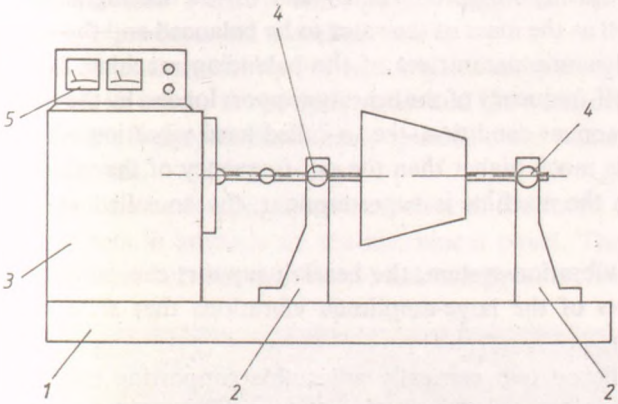


Fig. 9.1 Outline drawing of a universal balancing machine
1 engine base; *2* bearing brackets; *3* drive; *4* sensor; *5* measuring unit

According to where they are to be used, balancing machines are either universal or single-purpose machines.

The shaft of a universal machine is horizontal, while that of a single-purpose machine is either vertical or horizontal, which ever is best suited to the purpose.

The machine may be manually operated, semi-automatic, or automatic.

With a manually operated machine, all functions are carried out by the operator, who writes down the measured parameters of the imbalance, or keeps them in mind until the rotor is balanced.

The semi-automatic machine indicates directly the parameters of the imbalance and stores them until the machine is restarted. The operator only carries out the setting and the correction of the rotor's mass.

An automatic balancing machine consists of a measuring machine, which determines the parameters of the imbalance, and a balancing device. The measuring machine measures fully automatically and controls the balancing device according to the measurement results.

Manual and semi-automatic balancing machines are suitable for balancing individual components and rotors not produced in large series, while automatic balancing machines are suitable for balancing mass-produced rotors.

The sensing and measuring unit of the machine may be mechanical, electrical, or a combination of both. Owing to the rapid development of electronics, only machines equipped with electronic measuring systems are produced today.

The driving and measuring units are mounted on the base of the balancing machine. It is possible to slide the two bearing brackets along the machine base, and they can be set and then fixed to the bearing distance of the rotor. The bearing supports, which are spring-suspended on the brackets, can be easily shifted horizontally. The height of the bearing supports connected to the bearing brackets is adjustable according to the pivot diameter of the rotor. Vibration sensors, and occasionally vibration dampers, are attached to the bearing supports. The rigidity of the bearing support and the vibration sensor, as well as the mass of the rotor to be balanced and the speed of balancing, determine the dynamic parameters of the balancing machine. If this speed is much lower than the self-frequency of the bearing support loaded by the rotor, then the machine is in a subresonant condition, the so-called hard vibration system; if, however, the rotor's speed is much higher than the self-frequency of the vibrating part loaded by the rotor, then the machine is superresonant, the so-called soft vibration system.

On machines of the soft vibration system, the bearing support can be fixed to prevent the detrimental effects of the large-amplitude vibrations that arise upon starting.

The pivots of the rotor lie on two vertically adjustable supporting rollers or prismatic plastic bearings for each bracket. The surface of the rollers is slightly convex to prevent the rotor from wedging-up in case of opposite vibrations. A plastic prism is used for balancing rotors that have small mass or are provided with rolling bearings.

On machines of the soft vibration system, generally electrodynamic sensors are used, while on machines of the hard vibration system, mostly piezo-electric sensors are used.

The rotor to be balanced is rotated by means of a drive shaft or belt.

In the latter case, drive shafts of different sizes can be connected between the main spindle and the workpiece. The shaft-end of the rotor to be balanced is provided with a carrier fixed to the drive shaft.

In the case of a belt drive, the rotor is rotated by means of a belt passing over the mantle, with the aid of a stepped belt-pulley from an electric motor situated below the rotor, which enables the speed of the rotor to be set in the required range.

Sometimes a special drive is used (e.g. compressed air, rotary magnetic field, friction drive, etc.).

The electronic measuring unit of a balancing machine displays the signal from the sensor — the voltage being proportional to the magnitude of the imbalance — on an analogue or a digital instrument.

The angular position of the imbalance is sensed by an electric phase-meter. The magnitude of the angle is determined by means of a mark that rotates together with the rotor (numerical scale or white and black markings).

Such a measuring system comprises a pre-tuned filter or a filter that automatically follows the speed of the rotor. This filter selects the signals from the spectrum given by the sensor, the frequency of which corresponds with the speed of the rotor.

The electric frame described earlier is an important part of the measuring system, which converts the information received in the plane of the sensor to the balancing planes.

The development and production of balancing machines commenced in Hungary at the Machinetool Development Institute. At the present time the Telecommunication Machine Factory develops and produces these machines. Examples of the factory's products are described below.

The machine shown in Fig. 9.2 is driven by a drive shaft and its vibration system is rigid; its sensor is electrodynamic and the measuring unit includes a tuned filter, an electric frame, an electronic amplitude-meter and a phase-meter, as well as a signal storage unit.

On the basis of the geometric data of the rotor, the machine can be set with the parameters in advance on the instrument panel. The bearing distance of the rotor, the distance between the selected balancing planes, and the radius of the selected mass

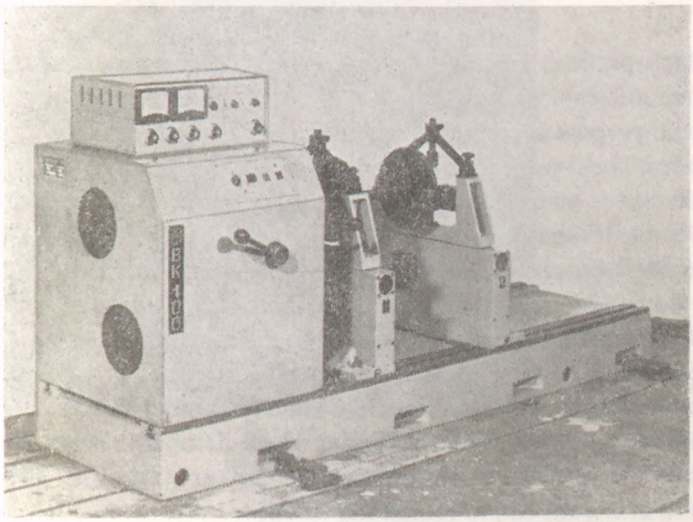


Fig. 9.2 BK 100 type universal balancing machine driven by drive-shaft

correction in the balancing planes are required for setting the machine. After setting these data, the instrument indicates the size of the corrective masses required in g.

The arms on the two brackets used for fixing the bearing supports are clearly seen on the BH 10 type machine of the soft vibration system (Fig. 9.3). The sensor of the machine is electrodynamic; its measuring unit consists of an electric frame, electronic filter and electronic amplitude-meter and phase-meter. The begining of the angular position is given by the borderline of white and black marks arranged on the rotor, sensed by a photoelectric signal transmitter.

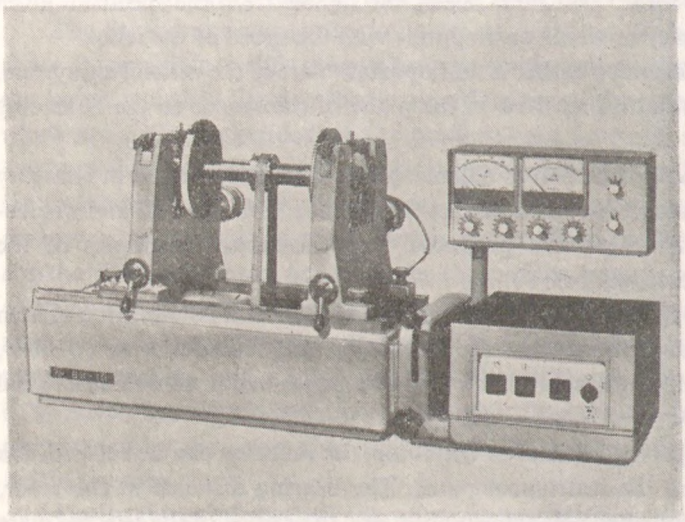


Fig. 9.3 BH 10 type belt-driven universal balancing machine

The BV 50 type, single-purpose machine shown in Fig. 9.4 was produced for balancing the rotors of fans and cabledrums.

The BA 44 type, single-purpose machine shown in Fig. 9.5 is used for balancing the wheels of motor vehicles. The construction of the two machines is similar.

The rotor to be balanced is mounted on the machine by means of a cantilever. This arrangement is preferable if the rotor does not have its own shaft.

The sensors of these single-purpose machines are piezo-ceramics. The measuring units also consist of an electric frame, as well as an electronic amplitude-meter and a phase-meter. The BV 50 type machine can be driven at two different speeds.

In order to increase productivity, the BA 44 type machine measures and indicates simultaneously, in an automatic cycle, the magnitude and angular position of the imbalance in two balancing planes. Owing to a special wheel-clamping device, the time required for balancing the wheel of a medium-sized motor car is less than 60 s, including the time taken to dismount and re-mount the wheel.

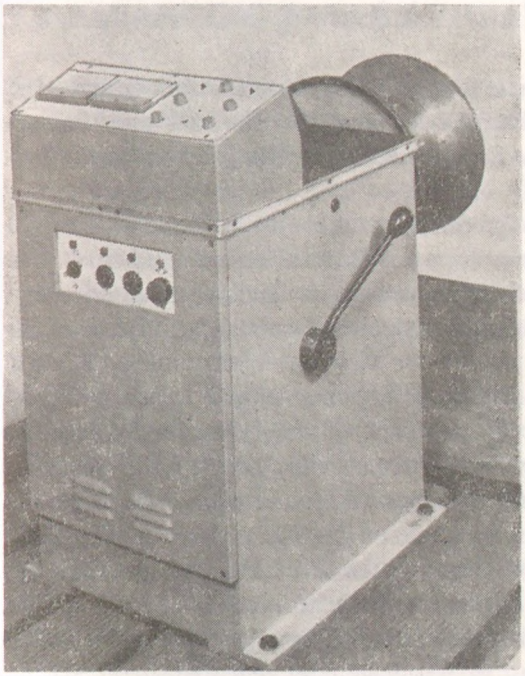


Fig. 9.4 BV 50 type fan and cabledrum balancing machine

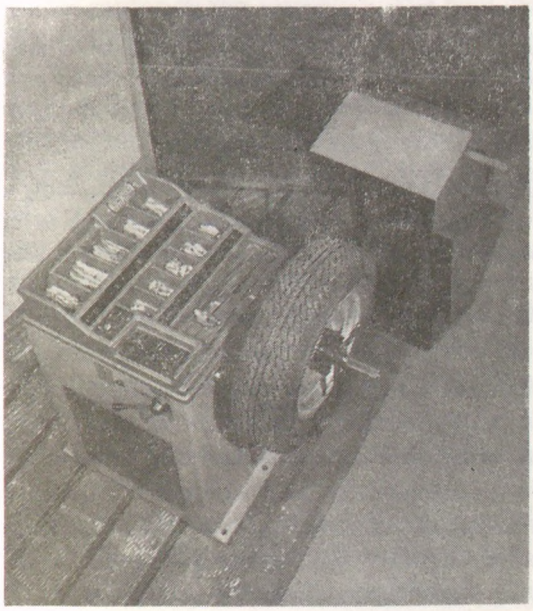


Fig. 9.5 BA 44 type wheel-balancing machine

9.1 USE OF BALANCING MACHINES

The activities and individual characteristic features of the machines are described in detail in the instructions for handling each balancing machine. The setting and balancing method of the machines are different for the machines of the hard and soft vibration systems.

A machine of the hard vibration system can be set in advance, based on the geometric dimensions of the rotor, and an instrument of the machine indicates directly, in g, the magnitude of the rotor's imbalance in the selected balancing planes related to the radius set. Other parameters proportional to the imbalance (e.g. the depth of the hole at the selected drill diameter) can also be set by means of the radius-adjusting switch on the machine. Following the setting, which takes only a few seconds, the balancing procedure can be commenced. Machines of the hard vibration system measure the supporting force caused by the imbalance, which depends on the speed of balancing, and consequently such machines generally run at a constant speed and are mostly drive-shaft driven.

Machines of the soft vibration system measure the deflection of the bearing supports. The deflection depends not only on the geometric dimensions of the rotor but also on its mass and moment of inertia. Hence the setting of these machines takes longer.

The process of setting is as follows:

One rotor is selected from those to be balanced, then — by way of trial — this is balanced with plastiline first in one balancing plane then in the other. The balancing is continued alternately until the residual imbalance of the rotor falls well below the permissible value. Next, plastiline of known mass, nearly equal to the expected maximum imbalance, is placed in one (e.g. left) of the balancing planes, then the deflection of the instrument is set to zero by turning the knob corresponding to the balancing plane on the right-hand side. Then the sensitivity adjusting knob on the left-hand side is turned until the instrument pertaining to the left balancing plane indicates a deflection equal to the mass of plastiline. This is followed by transferring the plastiline to the right balancing plane at the distance from the axis of rotation where the mass correction will be performed. The method of adjustment is the same as before. In this way the machine is made ready for balancing the given types of rotors.

The imbalance presented by machines of the soft vibration system does not depend on the speed, because the deflection of the bearing supports is measured, which is nearly constant well above the resonance. For this reason a belt drive may be used.

To sum up, the machine of the hard vibration system is preferable equally in repair and maintenance plants and in mass production, while the machine of the soft vibration system is mostly used in mass production. This explains why machines of the hard vibration system are gradually replacing machines of the soft vibration system.

The advantage of the machines of the soft vibration system is that they are more sensitive in the case of rotors with small mass.

9.2 BALANCING FAULTS

The quality of the balancing of a rotor is restricted by the potentialities of the balancing machine. With a perfectly functioning balancing machine driven by a drive shaft a centre of mass displacement of 1...2 μm can be attained. If balancing corresponding to the quality grade *G* 1 is to be attained, the drive-shaft balancing machine is no longer suitable. In the even finer quality grade *G* 0.4, the rotor must be assembled with its own drive and balanced under operating conditions. For more accurate balancing than 1 μm, the rotor must be rigid and the ambient conditions constant (e.g. upon changes in temperature, the imbalance of a rotor balanced to 1 μm may increase to 2...3 μm).

Some faults that frequently occur in balancing are listed below:

1. Faults due to the drive

— the motor, the main spindle of the balancing machine, or the guide rollers of the belt may be imbalanced, and through the machine frame they induce very disturbing signals in the sensors. Such faults may prevent fine balancing. For detecting the fault, the machine is switched on in its most sensitive position without the workpiece. If the machine is faultless, the instrument must not indicate a non-zero value. A reference signal must be provided for the test in the case of a machine with a belt drive.

— The drive-shaft may be imbalanced, or mounted eccentrically on the machine or on the carrier of the workpiece. The fault can be balanced together with the rotor. The fault is revealed if the carrier is turned by 180° with respect to the workpiece.

— The joints of the drive-shaft may be jammed, making it impossible to attain more accurate quality grades.

— The joints of the drive-shaft may be fitted with excessive play. As a result, the pointer of the instrument moves backwards and forwards.

— An unstable belt may also cause a backwards and forwards motion of the instrument's pointer.

2. Faults in other parts of the machine

— Knocking of the rollers supporting the rotor may be excessive, owing to wear of the bearing. The pointer of the instrument moves backwards and forwards. A similar phenomenon occurs if the diameter of the rollers is nearly equal to the pivot diameter of the rotor.

— Mainly in machines of the soft vibration system, it may occur that a worn plastic bearing prism inhibits the free swing of the bearing supports. To detect the fault, a moment imbalance of different magnitude is brought about by mounting additional masses on a balanced rotor. The instrument must indicate a deflection proportional to the imbalance; if the deflection is less, it is caused by tightness of the bearings.

— The bearing support of a machine of the soft vibration system may impact upon one or both limiters. This fault can be recognized by the sound of the impact, but mainly it is revealed by the fact that the angular position is shown erroneously by the machine. If the impact occurs only upon one of the limiters, the machine is not level-

led. If the swinging part impacts upon both limiters, the imbalance of the rotor is greater than the upper measuring limit of the machine, or the damper of the bearing support is faulty. Upon impact, the machine mainly measures the angular position of the imbalance wrongly. (In the former case, the imbalance of the rotor is to be reduced (e.g. balancing by trial); in the latter case the damper has to be filled up with oil of a specific quality.)

3. Faults of setting

— The reduction ratio of the imbalance may not be satisfactory, and the required balance can be achieved only in several steps. This may be because of the inaccurate setting of the electric frame, but it is also possible that the correcting masses are not in the set balancing plane or radius. This is experienced mainly when a mass is removed by drilling. The centre of gravity of the removed mass changes with the drilling depth and it may deviate from the balancing plane. Even a displacement of 1 mm can result in a significant deviation if the balancing planes are close to each other.

— In machines driven by a drive shaft it may occur that the driving spindle and rotor are not coaxial; this is most frequenctly caused by the incorrect vertical setting of the bearing brackets. In this case, the pointer of the instrument moves backwards and forwards.

— In belt-driven machines an excessively tight belt causes a hovering performance. The belt should be tightened only to such an extent as to just transmit the torque required for rotation. Tightness of the belt may result in deformation of the workpiece, and this introduces errors into the measurement.

4. Faults caused by the workpiece

— If the imbalance of the workpiece is excessive, the centrifugal force lifts the pivot of the rotor on the bearing bracket. The machine then measures erroneously. The bearing or pivot of the rotor has to be supported from the top with an accident-preventing supporting roller until the imbalance of the rotor is reduced so that the "jumping" stops. Next, the upper supporting roller is removed to a distance of 0.5–1 mm and the balancing is continued. If the speed of the machine can be reduced, the balancing can take place at a lower speed.

— The bearing ring may travel along the centreline of the workpiece. The extent of the imbalance gradually increases, then decreases. The bearing ring has to be fixed.

— Upon starting or braking the rotor, the imbalance may alter. This means that a certain part of the rotor is not properly fixed.

— Upon changing the speed of balancing, the imbalance of the rotor may alter. This means that there is some kind of elastic element in the rotor, or the rotor itself is not rigid. In this case the balancing takes place at the operating speed.

— When balancing the rotary parts of a fan, the air is stirred by the blades, and the unsteady air delivery, similarly to the imbalance, alters the reaction forces, and the instrument indicates a hovering performance. In order to reduce this effect, the wheel is turned counter to the operational rotation, or if no satisfactory result is achieved in this way either, the rotor must be balanced in the cover.

10 IN-SITU BALANCING OF ROTORS

During the operation of machines the imbalance of their rotors is variable in most cases, owing to wear of the parts, the relative displacement of the matched elements and the deposition of impurities on some of the surfaces, thereby changing the amplitude of the vibration of the bearings in the machine. The vibrational state may be evaluated according to the recommendation of the international standard ISO 2372. If it exceeds the value given in the recommendation and it is caused by imbalance, then the rotor must be balanced again. In numerous cases power and money may be saved by balancing the machine in situ, without dismantling it.

Several processes for balancing in situ have been developed, differing from each other by the instruments used and the number of starting operations required for balancing. A balancing procedure with more starting operations and less calculations may be chosen; the decision depends on the type of balancing task. The acceleration and stopping of rotors of small mass takes only a few seconds, while each starting and stopping of large machines (e.g. turbine rotors) may take hours. In this case a process involving fewer starting operations and more calculation work is more advantageous.

The calculations may be facilitated by using a programmable calculator.

One of the methods is the constructive calculation process. For the construction it is advisable to use a pre-printed polar diagram.

For balancing in situ, a vibrometer, or a phase angle meter with built-in vibrometer, is used. Vibrometers with tunable-frequency band-pass filters are very useful, as they permit a higher balancing accuracy. If the vibration of the machine due to imbalance is greater than that originating from other sources, good results can be obtained also with the help of a simple measuring meter.

The meter is attached to a rigid construction, while its feeler rests against the bearing case of the machine, i.e. it is a relative vibrometer. The difference between the maximum and minimum deflections of the meter is twice the value of the vibration amplitude. The amplitude of the vibration may also be measured by fixing the meter to a heavy handle, which is held in the hand while pressing the feeler of the meter against the moving part of the machine (Fig. 10.1).

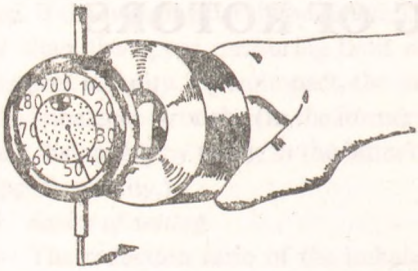


Fig. 10.1 Measuring meter converted for vibration measurements

The mass of the handle is usually about 1.0 kg, to which 1.2 kg may be added. The aforementioned vibrometer may be used in the frequency range 5...50 Hz for measuring vibrations of 0.01... 0.5 μm.

When balancing in situ, the vibration amplitude, velocity, or acceleration of the bearings may be measured. As the qualification is based on the velocity of the vibration, we shall discuss the velocity $\mathbf{v}$ of the vibration.

The balancing is performed in arbitrarily selected balancing planes which are as distant as possible from each other and do not coincide with the planes of the bearings. After selection of the balancing planes and radii, the imbalance may be characterized in these planes by the mass vectors $\mathbf{q}_1$ and $\mathbf{q}_2$. On rewriting eq. (89) with these notations, the following basic relationship for balancing in situ is obtained:

$$\begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix} = \begin{bmatrix} \varkappa_{11} & \varkappa_{12} \\ \varkappa_{21} & \varkappa_{22} \end{bmatrix} \times \begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \end{bmatrix}. \tag{139}$$

We measure the vibration vectors $\mathbf{v}_1$, $\mathbf{v}_2$ of the bearings and require to determine the mass vectors $\mathbf{q}_1$, $\mathbf{q}_2$ of the balancing planes. At the start of the balancing six unknowns appear in eq. (139) together with the factors $\varkappa_{11}, \varkappa_{12}, \varkappa_{21}, \varkappa_{22}$. The processes described below serve to determine the unknowns.

During balancing in situ the following conditions must be observed:

— the dynamic state of the machine must not change in the course of balancing. This condition must be ensured by maintaining a constant speed and direction of rotation, as well as ensuring a stable foundation;
— the measuring points on the bearing housing must be close to the bearing; the direction of the measurement should be the one supplying the maximum value; the measurement position and its direction must not change in the course of balancing;
— the measurements and the measured values must be reproducible;
— good results can be obtained if the vibrating system is linear, i.e. the vibration of the bearing is proportional to the imbalance and is correctly inphase, meaning that the angular shift α of the imbalance causes an angular displacement α of the velocity vector.

10.1 IN-SITU BALANCING IN A SINGLE PLANE

10.1.1 TRIPLE-POINT APPROXIMATION METHOD

The triple-point approximation method described below serves to determine the approximate angular position of the imbalance. It is popular because it is simple and it supplies a satisfactory result in 5–6 starts and requires only a simple vibrometer. The steps of the balancing procedure are as follows:

1. Start the machine and, when the rotor attains its working speed, measure v_0, the velocity of the bearing vibration in the direction normal to its axis.
2. Mark the numbers *1*, *2*, *3* at angular separations of 120° on the front surface of the rotor.
3. For the following measurements a test mass of known magnitude is needed. It is advisable to calculate the magnitude of the test mass by means of the following relationships:

$$q_p = \frac{v_0 m}{\omega r} \tag{140}$$

or

$$q_p = \frac{a_0 m}{r},$$

where

q_p is the test mass (g);
v_0 is the velocity amplitude of the bearing vibration at the initial imbalance of the rotor (μm s^{-1});
m is the mass of the rotor (kg);
ω is the angular velocity of the rotor (s^{-1});
r is the radius of the position of the test mass placed on the rotor (mm);
a_0 is the vibration amplitude of the bearing (μm).

It is advisable to use plastiline test masses which have been weighed precisely.

4. Place the test mass on the number *1* marked on the rotor and at the selected radius. Start the machine, then measure the amplitude v_1 of the vibration velocity on the machine frame.
5. Relocate the test mass from position *1* to position *2* on the rotor and measure the vibration v_2 of the bearing.
6. Relocate the test mass from position *2* to position *3* and measure the vibration v_3 of the bearing.
7. Plot, with a suitably selected scale, the measured values v_1, v_2, v_3 on the three axes of a 120° system of coordinates numbered identically in the rotational sense with the numbering on the workpiece (Fig. 10.2). Construct the centre of the circle which passes through the end points of the vectors. On connecting point *A* with

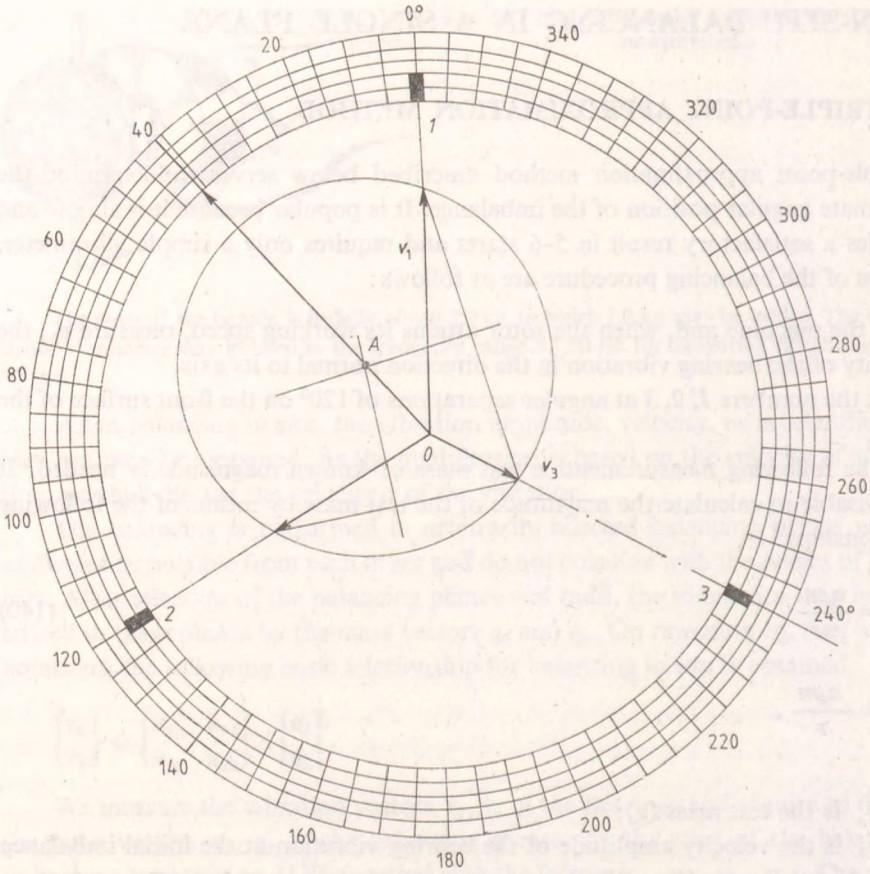


Fig. 10.2 In-situ balancing by the triple-point approximation

the origin O of the system of coordinates we obtain the direction of the imbalance. Indicate this direction on the rotor.

8. Now the magnitude of the imbalance must be determined. For this purpose remove a known mass of material from the direction of the imbalance, or add a known mass on the opposite side (by 180°) of the rotor. This mass should not be larger than the test mass calculated for step 3. Now start the machine again and after attaining the working speed measure the vibration v_4 of the bearing.
9. The magnitude of the imbalance and that of the mass required for balancing can be calculated from the measured data by means of the following relationship:

$$q = \frac{v_0}{v_0 - v_4} q_p, \tag{141}$$

or if the amplitude of the vibration is measured, then from

$$q = \frac{a_0}{a_0 - a_4} q_p,$$

where

q is the mass required for balancing (g);

q_p is the test mass placed in step 8 in the direction of the imbalance (g).

10. In step 9 the mass q is removed from the direction determined in step 7 in the sense $O \rightarrow A$ on the selected radius, or it is added to the rotor in the sense $A \rightarrow O$. This is the means of balancing the rotor. If during the test the residual imbalance exceeds the permitted value, then first of all the angular position of the mass must be corrected.

Example. Let us balance in-situ the imbalance of a rotor in a machine by the triple-point approximation process in one plane. The mass of the rotor is $m = 178$ kg. The balancing is to be performed on a radius of $r = 300$ mm.

Solution.

The available instrument is a distance sensor, so we measure the deflection amplitude of the vibration of the bearing. We measured the amplitude of the vibration of the bearing case perpendicular to the axis, horizontally and vertically. As the horizontally measured amplitude was the greater, the following measurements were performed horizontally, at a defined point on the bearing case. The maximum vibration amplitude of the bearing case is $a_0 = 42$ μm. We calculate the test mass by means of eq. (140):

$$q_q = \frac{a_0 m}{r} = \frac{42 \times 178}{300} = 24.9 \approx 25 \text{ g}.$$

On placing the test mass at the marks 1, 2, and 3 on the circle of radius $r = 300$ mm of the rotor we measured the following deflection amplitudes:

$a_1 = 47.3$ μm

$a_2 = 35.5$ μm

$a_3 = 18.7$ μm.

We have plotted these values on the axes of a 120° system of coordinates in Fig. 10.2, then constructed the centre, A, of the circle passing through the three points.

The straight line $0A$ gives the angular position of the imbalance, which is at $\alpha = 40°$ from axis 1 towards the direction 2, as can be seen in the diagram. The mass $q'_p = 14$ g placed at $40 + 180 = 220°$ causes a deflection of amplitude $a_4 = 31$ μm in the bearing. So the magnitude of the imbalance of the bearing is, according to eq. (141),

$$q = \frac{a_0}{a_0 - a_4} q'_p = \frac{42}{42 - 31} 14 \cong 5.35 \text{ g},$$

i.e. a mass of $q = 53.5$ g must be attached in the angular position $\alpha = 220°$ on the radius $r = 300$ μm to achieve an approximate balance.

10.1.2 TRIPLE-POINT METHOD ACCORDING TO DEN HARTOG

By this method the angular position of the imbalance is determined accurately, and the magnitude of the imbalance may also be obtained by proportioning. Altogether, four rotations, a known test mass and a vibrometer are required for the operation.

In the first step we measure the vibration v_0 of the bearing caused by the initial imbalance of the rotor. The known test mass q_p is put into position *1* on the radius *r* in the selected balancing plane of the rotor, according to Fig. 10.3(a), then the vibration v_1 of the bearing is measured. Now the test mass is put in position *2*, then in position *3* of the rotor, and the vibrations v_2 and v_3 of the bearing are measured. From a knowledge of v_0, v_1, v_2, v_3 and q_p the imbalance of the rotor can be constructed (Fig. 10.3(b)). To obtain the points *A* and *B*, the value of v_0 is measured twice true to scale in arbitrary direction from the origin *O*. A circular arc of length v_1 is drawn around point *O* intersected by circular arc of radius v_2 drawn from point *B*, whereby the position of point *C* is obtained. Construct a parallelogram, one diagonal of which is *OB*, while half of its other diagonal is *AC*. In this way point *D* is also determined. As $OA = v_0$ and $OC = v_1$, so $AC = v_{p1}$; this bearing vibration would be measured if only the imbalance q_p were present in the rotor. Namely

$$\mathbf{v}_0 + \mathbf{v}_{q1} = \mathbf{v}_1$$

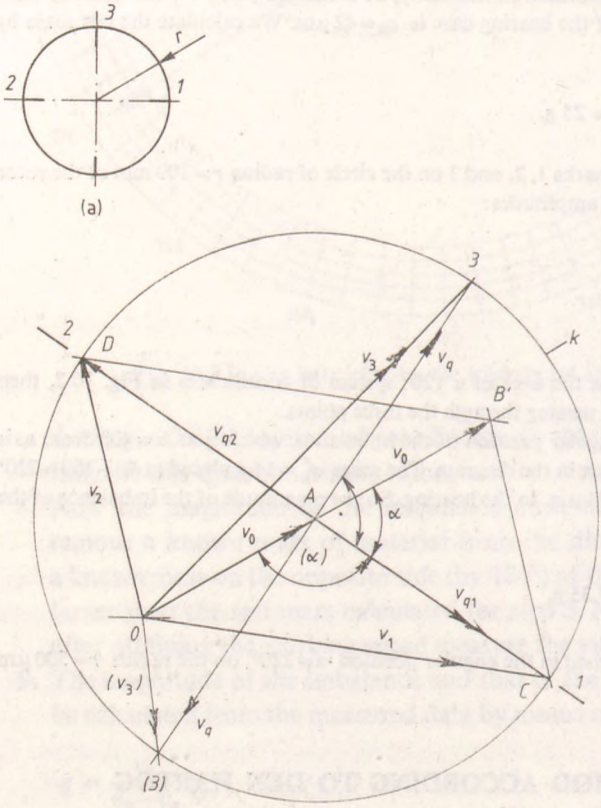


Fig. 10.3 Balancing by means of the construction according to Den Hartog
(a) position of the test mass; (b) vector diagram

and

$$\mathbf{v}_0 + \mathbf{v}_{q2} = \mathbf{v}_2,$$

where

$$v_{q1} = v_{q2} = v_q.$$

The angle of the imbalance is that enclosed by the straight lines AC and OB. One of two possibilities may be selected with the help of v_3. A straight line is drawn normally to the line CD at point A, and a circle k with the radius v_p is drawn around the point A. A circular arc is drawn from point O with the radius v_3 and the circle k is intersected by it. If the measurement and the construction is accurate, the intersection will be at point *3*, or point *(3)*. If it is at point *3*, then the position of the imbalance is at an angle α from the position marked *1*.

The magnitude of the imbalance may be calculated by the following relationship:

$$q = \frac{v_0}{v_q} q_p, \tag{142}$$

where v_q can be measured from the figure.

Example. Let us determine the angular position and the magnitude of the imbalance by means of Den Hartog's construction, if the measured data are as follows:

$v_0 = 5.2$ mm s^{-1}, $v_1 = 10.8$ mm s^{-1}, $v_2 = 6.8$ mm s^{-1}, $v_3 = 12.2$ mm s^{-1}, $q_p = 50$ g.

Solution.

The scale selected for the construction is: 1 mm$\approx$0.1 mm s^{-1}. Figure 10.3 shows the construction prepared with the measured data. The angular position of the imbalance is $\alpha = 63°$ measured from position *1* towards *B* in the figure.

The magnitude of the imbalance is, according to eq. (142):

$$q = \frac{v_0}{v_q} q_p = \frac{5.2}{7.4} 50 = 35.14 \text{ g}.$$

10.1.3 TRIPLE-POINT CALCULATIONAL METHOD

The imbalance of the rotor can be determined also by calculation with the measurement data described above. The relationships of the calculation may be followed with reference to Fig. 10.4. The vibration of the bearing obtained under the effect of the test mass attached to positions *1*, *2* and *3* of the balanced rotor may be represented by the vectors $\mathbf{v}_{q1}$, $\mathbf{v}_{q2}$, $\mathbf{v}_{q3}$, drawn from point O of the system of coordinates. If the rotor is imbalanced, the vibration v_0 of the bearing caused by the initial imbalance is added to these three vectors. The starting point of the vectors is shifted to point. A. For the measured vectors $\mathbf{v}_0$, $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ the following relationships prevail:

$$\mathbf{v}_1 = \mathbf{v}_0 + \mathbf{v}_{q1};$$

$$\mathbf{v}_2 = \mathbf{v}_0 + \mathbf{v}_{q2};$$

$$\mathbf{v}_3 = \mathbf{v}_0 + \mathbf{v}_{q3};$$

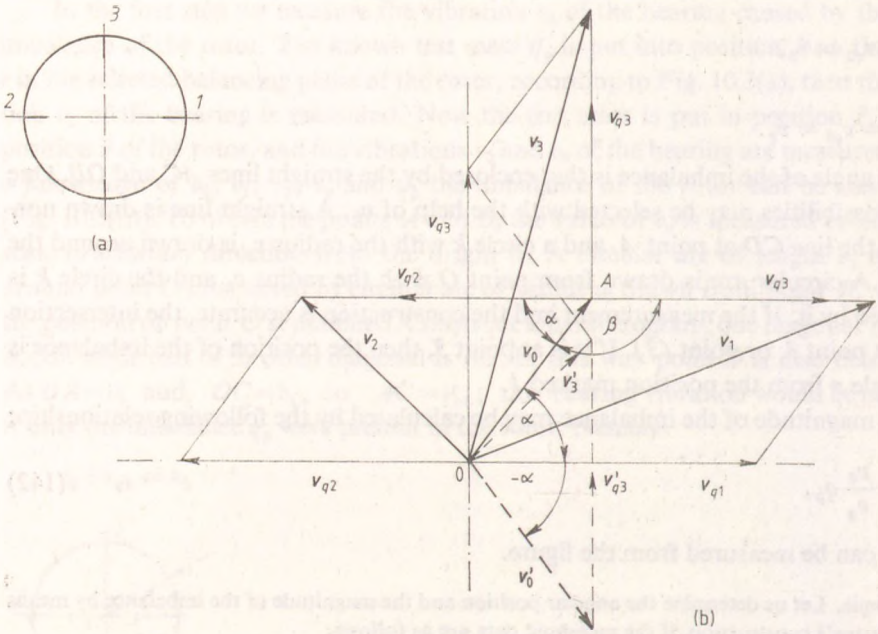


Fig. 10.4 Vector diagram for the calculation of the imbalance

or expressed in terms of geometrical relationships:

$$v_1^2 = v_0^2 + v_{q1}^2 + 2v_0 v_{q1} \cos \alpha; \tag{143}$$

$$v_2^2 = v_0^2 + v_{q2}^2 - 2v_0 v_{q2} \cos \alpha; \tag{144}$$

As

$$v_{q1} = v_{q2} = v_{q3} = v_q,$$

$$v_q = \sqrt{\frac{v_1^2 + v_2^2 - 2v_0^2}{2}} \tag{145}$$

and

$$\alpha = \arccos \frac{v_1^2 - v_2^2}{4v_0 v_q}, \tag{146}$$

where α is the angular position of the imbalance measured from position *1*.

α may have both positive and negative values, so v_3^3 may also have two values. These, expressed as geometrical relationships, are as follows:

$$v_{3c} = (v_0^2 + v_q^2 + 2v_0 v_q \sin \alpha)^{1/2} \tag{147}$$

$$v_{3c}' = (v_0^2 + v_q^2 - 2v_0 v_q \sin \alpha)^{1/2}. \tag{148}$$

```
000 76 LBL
001 38 SIN
002 43 RCL
003 07 07
004 42 STO
005 09 09
006 91 R/S
007 61 GTO
008 39 COS
009 76 LBL
010 12 B
011 58 FIX
012 02 02
013 02 2
014 42 STO
015 01 01
016 04 4
017 42 STO
018 02 02
019 00 0
020 50 |×|
021 91 R/S
022 42 STO
023 00 00
024 42 STO
025 03 03
026 49 PRD
027 01 01
028 49 PRD
029 02 02
030 49 PRD
031 03 03
032 01 1
033 50 |×|
034 91 R/S
035 33 X²
036 42 STO
037 04 04
038 42 STO
039 05 05
040 02 2
041 50 |×|
042 91 R/S
043 33 X²
044 44 SUM
045 04 04
046 22 INV
047 44 SUM
048 05 05
049 43 RCL
050 04 04
051 65 X
052 93 .
053 05 5
054 75 —
055 43 RCL
056 03 03
057 95 =
058 44 SUM
059 03 03
060 34 √X
061 49 PRD
062 01 01
063 49 PRD
064 02 02
065 35 1/X
066 49 PRD
067 00 00
068 42 STO
069 08 08
070 03 3
071 50 |×|
072 91 R/S
073 33 X²
074 42 STO
075 06 06
076 43 RCL
077 05 05
078 55 ÷
079 43 RCL
080 02 02
081 95 =
082 22 INV
083 39 COS
084 42 STO
085 07 07
086 38 SIN
087 49 PRD
088 01 01
089 04 4
090 00 0
091 50 |×|
092 91 R/S
093 49 PRD
094 08 08
095 49 PRD
096 00 00
097 43 RCL
098 00 00
099 91 R/S
100 43 RCL
101 03 03
102 85 +
103 43 RCL
104 01 01
105 75 —
106 43 RCL
107 06 06
108 95 =
109 50 |×|
110 32 X⇌T
111 43 RCL
112 03 03
113 75 —
114 43 RCL
115 01 01
116 75 —
117 43 RCL
118 06 06
119 95 =
120 50 |×|
121 77 GE
122 38 SIN
123 03 3
124 06 6
125 00 0
126 75 —
127 43 RCL
128 07 07
129 95 =
130 42 STO
131 09 09
132 91 R/S
133 76 LBL
134 39 COS
135 43 RCL
136 09 09
137 39 COS
138 65 X
139 43 RCL
140 00 00
141 95 =
142 91 R/S
143 43 RCL
144 00 00
145 65 x
146 43 RCL
147 09 09
148 38 SIN
149 95 =
150 91 R/S
151 04 4
152 50 |×|
153 91 R/S
154 65 x
155 43 RCL
156 08 08
157 95 =
158 91 R/S
159 12 B
```

Fig. 10.5 Program for the triple-point calculational method of balancing, for use with a type TI–59 pocket calculator

Table 10.1

Measurement report and operation instructions

Balancing by the triple-point method	Address: B
Identification data of the rotor: Denomination: Flywheel Drawing No.: 42—01—05 Balancing plane: left Permitted imbalance: 1 mm s^{-1}	Arrangement of the test mass in the balancing plane

Position of the test mass	No. of the data	Measured values	Measuring unit	Key
Without test mass	0.00	$v_0=5.2$	mm s^{-1}	*R/S*
1	1.00	$v_1=10.8$	mm s^{-1}	*R/S*
2	2.00	$v_2=6.8$	mm s^{-1}	*R/S*
3	3.00	$v_3=12.2$	mm s^{-1}	*R/S*
	40.00	Test mass $q_p=50$	g	*R/S*

Results

In polar coordinates

$q=35.25$	g	*R/S*	$\alpha=62.68$	°	*R/S*

With rectangular components

$q_x=16.18$	g	*R/S*	$q_y=31.32$	g	*R/S*

Control: Data and measured value after balancing

Data No.: 4.00	$v_4=0.8$	mm s^{-1}	*R/S*

Result: Residual imbalance of the rotor

$q_m=5.42$	g	*R/S*	Qualification: satisfactory unsatisfactory

Date: 15.2.1984. Signature: Fejes

We compare the measured and the calculated v_3-values and if

$$|v_3 - v_{3c}| < |v_3 - v_{3c}|, \tag{149}$$

then $\alpha > 0$; in the opposite case $\alpha < 0$.

The magnitude of the imbalance can be determined by means of eq. (142). We have prepared a program employing the relationships shown in Fig. 10.5 for the type TI 59 pocket calculator. The characteristics of the imbalance can be calculated rapidly with the help of this program (see the instructions in Table 10.1); the rectangular components q_x, q_y of the balancing mass can also determined.

After fixing the mass q onto the rotor in the angular position α, we control the quality of the balancing, then obtain after the input of the measured value of v_4 the residual imbalance in g units.

For an example, we have written the starting data of the preceding example into Table 10.1 and calculated the imbalance of the rotor.

10.1.4 BALANCING BY PHASE-ANGLE MEASUREMENT AND CONSTRUCTION

So far we have measured the vibration of the bearing caused by imbalance of the rotor. In linear systems the imbalance of the rotor may be determined by two starts and by measurement with the help of a vibrometer supplied with an adjustable filter and a phase-angle meter. The balancing steps are as follows:

1. Accelerate the rotor to the working speed, adjust the tunable filter of the instrument, then measure the vibration v_0 of the bearing and its angular position α_0 and identify the angle on the 360° scale of the paper disc glued to the end on the rotor shaft.
2. Attach the test mass q_p, calculated by means of eq. (140), in the angular position 0° to the radius marked out in the selected balancing plane of the rotor, then start the machine and measure the vibration v_1 of the bearing and the angular position α_1.
3. Plot the vectors obtained, with a suitably chosen scale, as in Fig. 10.6, connect the end points of the vectors $\mathbf{v}_0$ and $\mathbf{v}_1$ and draw a line parallel to this line passing through the O point; $\mathbf{v}_p$ is the vector of the bearing vibration that would appear if the imbalance of the rotor were caused by the test mass q_p alone. The angular position of the imbalance is the angle enclosed by the vectors $\mathbf{v}_0$ and $\mathbf{v}_q$ and can be read off the diagram. The balancing mass must be attached to the rotor in the following angular position:

$$\alpha = \alpha_0 - \alpha_q. \tag{150}$$

The mass required for the elimination of the imbalance can be calculated by means of eq. (142). By balancing in this way about 90% of the imbalance can

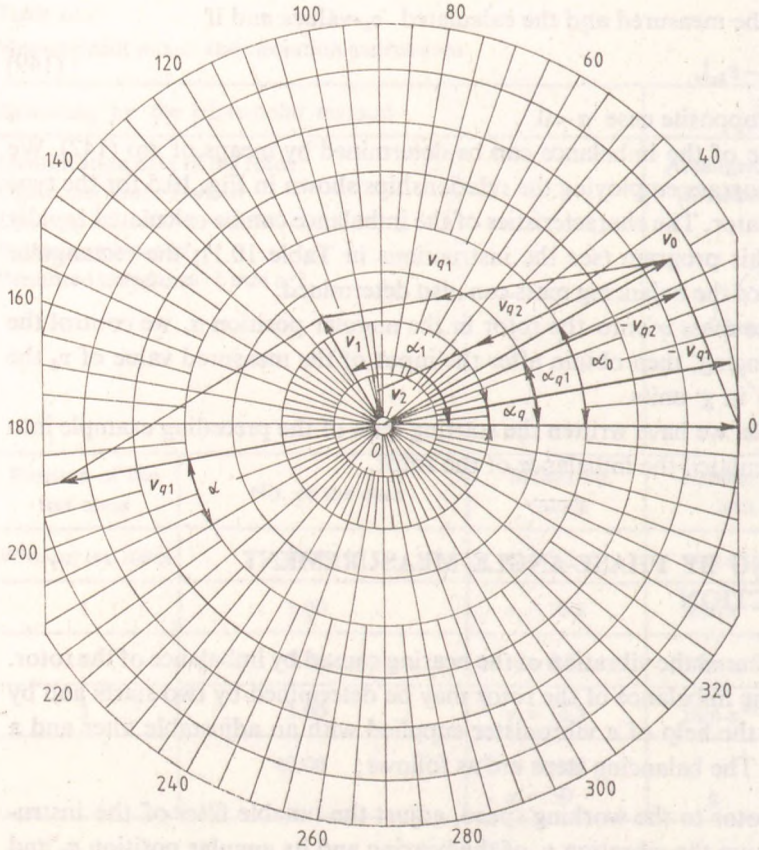


Fig. 10.6 Balancing in a single plane with a phase-angle meter

be eliminated. If, after applying the balancing mass, the vibration v_2 of the bearing is still unsatisfactory, we measure the angular position α_2 and continue refining the imbalance of the rotor.

4. Draw $\mathbf{v}_2$ in the angular position α_2 in the properly scaled vector diagram, then interconnect the end points of $\mathbf{v}_2$ and vector $\mathbf{v}_0$ to obtain $\mathbf{v}_{q1}$. Also draw the vector $\mathbf{v}_{q1}$ from the centre O. From a knowledge of these values the corrected balancing mass, q_h, and the corrected angular position may be calculated by means of the following relationships:

$$q_h = \frac{v_0}{v_{q1}} q; \tag{151}$$

$$\alpha_h = \alpha_0 + \alpha - \alpha_{q1}. \tag{152}$$

Example. Let us determine the magnitude and the angular position of the mass to be attached to the rotor of a machine to ensure its regular running.

Solution.

A vibrometer and a phase-angle meter are available. After starting the machine, an effective vibration velocity $v_0 = 6.6$ mm s^{-1} and $\alpha_0 = 30°$ were measured. The test mass calculated by means of eq. (140) is $q_p = 12$ g.

We attach the test mass to the rotor in the angular position 0° and measure again the vibration of the bearing: $v_1 = 2.4$ mm s^{-1}, $\alpha_1 = 120°$.

We select the scale 1 division = 0.1 mm s^{-1}, and plot accordingly the vectors $\mathbf{v}_0$ and $\mathbf{v}_1$ as shown in Fig. 10.6. We construct the vector $\mathbf{v}_q$, measure its magnitude and its angular position: $v_q = 7.0$ mm s^{-1}, and $\alpha_q = 10°$. The balancing mass must be fixed, according to eq. (150) at the following angle:

$$\alpha = \alpha_0 - \alpha = 30 - 10 = 20°.$$

The balancing mass is, according to eq. (142):

$$q = \frac{v_0}{v_q} q_p = \frac{6.6}{7.0} 12 = 11.31 \text{ g}.$$

After fixing this mass we measured $v_2 = 0.64$ mm s^{-1} and $\alpha_2 = 100°$.

To correct the balancing, we plot the vector $\mathbf{v}_2$, connect its end point with the end point of $\mathbf{v}_0$, and obtain $\alpha_{q1} = 24°$ and $v_{q1} = 6.4$ mm s^{-1}, as measured in the diagram. Substituting these values into eqs (151) and (152) we obtain the corrected balancing mass:

$$q_h = \frac{v_0}{v_{q1}} q = \frac{6.6}{6.4} 11.31 = 11.66 \text{ g}.$$

The corrected angular position, according to eq. (152), is

$$\alpha_h = \alpha_0 + \alpha - \alpha_{q1} = 30° + 20° - 24° = 26°.$$

10.1.5 BALANCING BY MEANS OF A PHASE-ANGLE METER AND CALCULATION

It is advisable to evaluate the balancing in situ by means of calculation, as in this way the imbalance of the rotor can be solved accurately with a few starting operations. For measuring the vibration of the bearing an adjustable (tunable) vibrometer and a phase-angle meter are required. A paper disc with 360° divisions is glued onto a suitable position on the rotor (e.g. on the front face of its shaft). The evaluation is greatly facilitated and made faster by use of a programmable pocket calculator.

On balancing in a single plane the system of qeuations (139) is reduced to the following simple form:

$$\mathbf{v}_0 = \varkappa \mathbf{q}, \tag{153}$$

where

- $\mathbf{v}_0$ is the measured complex vector of the bearing vibration induced by the imbalance of the rotor,
- $\varkappa$ is the complex factor characterizing the rotor and its bearing,
- $\mathbf{q}$ is the unknown complex vector of the imbalance of the rotor related to a given radius.

Equation (153) contains two unknowns, so one more equation is needed. We place a known test mass q_p in the angular position 0° of the rotor on the selected radius in the balancing plane and measure the vibration $\mathbf{v}_1$ of the bearing:

$$\mathbf{v}_1 = \varkappa(\mathbf{q}+\mathbf{q}_p). \tag{154}$$

Subtracting relationship (153) and after arrangement, we obtain

$$\frac{\mathbf{v}_1-\mathbf{v}_0}{\mathbf{q}_p} = \varkappa. \tag{155}$$

It is advisable to use the reciprocal form of this relation:

$$\boldsymbol{\beta} = \frac{1}{\varkappa} = \frac{\mathbf{q}_p}{\mathbf{v}_1-\mathbf{v}_0}. \tag{156}$$

Accordingly the required complex vector of the imbalance is

$$\mathbf{q} = \boldsymbol{\beta}\mathbf{v}. \tag{157}$$

The calculation and the programming are performed with the components of the complex vectors:

$$\mathbf{v}_0 = v_{0x}+jv_{0y}; \tag{158}$$

$$\mathbf{v}_1 = v_{1x}+jv_{1y}; \tag{159}$$

$$\mathbf{q} = q_x+jq_y; \tag{160}$$

$$\boldsymbol{\beta} = \frac{\mathbf{q}_p}{\mathbf{v}_1-\mathbf{v}_0} = \beta_x+j\beta_y, \tag{161}$$

where

$$\beta_x = \frac{(v_{1x}-v_{0x})q_p}{(v_{1x}-v_{0x})^2+(v_{1y}-v_{0y})^2} \tag{162}$$

$$\beta_y = -\frac{(v_{1y}-v_{0y})q_p}{(v_{1x}-v_{0x})^2-(v_{1y}-v_{0y})^2}. \tag{163}$$

Here we have taken into account the fact that the vector $\mathbf{q}_p$ has no imaginary component, because we have mounted the test mass in the angular position 0°. Using the above relationships, the components of the imbalance vector are as follows:

$$q_x = v_{0x}\beta_x-v_{0y}\beta_y; \tag{164}$$

$$q_y = v_{0x}\beta_y+v_{0y}\beta_x. \tag{165}$$

The angular position of the imbalance is

$$\alpha = \arctan\frac{q_y}{q_x}. \tag{166}$$

```
000 76 LBL
001 18 C
002 25 CLR
003 47 CMS
004 58 FIX
005 02 02
006 02 2
007 42 STO
008 05 05
009 02 2
010 93 .
011 01 1
012 42 STO
013 06 06
014 01 1
015 50 I×I
016 91 R/S
017 42 STO
018 00 00
019 93 .
020 01 1
021 50 I×I
022 91 R/S
023 42 STO
024 01 01
025 25 CLR
026 43 RCL
027 00 00
028 32 X⇌T
029 42 RCL
030 01 01
031 37 P/R
032 42 STO
033 13 13
034 22 INV
035 44 SUM
036 10 10
037 32 X⇌T
038 42 STO
039 12 12
040 22 INV
041 44 SUM
042 11 11
043 01 1
044 01 1
045 50 I×I
046 91 R/S
047 42 STO
048 02 02
049 93 .
050 01 1
051 01 1
052 50 I×I
053 91 R/S
054 42 STO
055 03 03
056 25 CLR
057 43 RCL
058 02 02
059 32 X⇌T
060 43 RCL
061 03 03
062 37 P/R
063 44 SUM
064 10 10
065 32 X⇌T
066 44 SUM
067 11 11
068 01 1
069 00 0
070 00 0
071 50 I×I
072 91 R/S
073 42 STO
074 04 04
075 43 RCL
076 11 11
077 33 X²
078 85 +
079 43 RCL
080 10 10
081 33 X²
082 95 =
083 22 INV
084 49 PRD
085 11 11
086 94 +/−
087 22 INV
088 49 PRD
089 10 10
090 43 RCL
091 04 04
092 49 PRD
093 10 10
094 49 PRD
095 11 11
096 76 LBL
097 33 X²
098 43 RCL
099 12 12
100 65 ×
101 43 RCL
102 11 11
103 75 −
104 43 RCL
105 13 13
106 65 ×
107 43 RCL
108 10 10
109 95 =
110 42 STO
111 14 14
112 43 RCL
113 12 12
114 65 ×
115 43 RCL
116 10 10
117 85 +
118 43 RCL
119 13 13
120 65 ×
121 43 RCL
122 11 11
123 95 =
124 42 STO
125 15 15
126 43 RCL
127 14 14
128 32 X⇌T
129 43 RCL
130 15 15
131 22 INV
132 37 P/R
133 42 STO
134 16 16
135 32 X⇌T
136 91 R/S
137 29 CP
138 43 RCL
139 16 16
140 77 GE
141 25 CLR
142 85 +
143 03 3
144 06 6
145 00 0
146 95 =
147 91 R/S
148 61 GTO
149 34 √X
150 76 LBL
151 25 CLR
152 43 RCL
153 16 16
154 91 R/S
155 61 GTO
156 34 √X
157 76 LBL
158 34 √X
159 43 RCL
160 14 14
161 91 R/S
162 43 RCL
163 15 15
164 91 R/S
165 43 RCL
166 05 05
167 50 I×I
168 91 R/S
169 42 STO
170 17 17
171 43 RCL
172 06 06
173 50 I×I
174 91 R/S
175 42 STO
176 18 18
177 25 CLR
178 32 X⇌T
179 43 RCL
180 17 17
181 32 X⇌T
182 43 RCL
183 18 18
184 37 P/R
185 42 STO
186 13 13
187 32 X⇌T
188 42 STO
189 12 12
190 01 1
191 44 SUM
192 05 05
193 44 SUM
194 06 06
195 61 GTO
196 33 X²
```

Fig. 10.7 Program of balancing in a single plane (for the type TI–59 pocket calculator)

Table 10.2

Measurement report and operation instructions

Balancing in a single plane by calculation	Address **C**
Identification data of the machine: denomination: pump machine No.: 101—43/75 Balancing plane: left Permitted residual imbalance: $v = 1$ mm s^{-1}	Arrangement of the test mass in the balancing plane:

Position of the test mass	Serial No of the data	Data: Measured results	Measuring unit	Pushbutton
Without test mass	1.00	$v_0 = 8.3$	mm s^{-1}	*R/S*
	0.10	$\alpha_0 = 30$	°	*R/S*
	11.00	$v_1 = 3.0$	mm s^{-1}	*R/S*
	0.11	$\alpha_1 = 120$	°	*R/S*
	100.00	Test mass: $q_p = 12$	g	*R/S*

Results:

in polar coordinates:

$q = 11.29$	g	*R/S*	$\alpha = 199.87$	°	*R/S*

in rectangular coordinates:

$q_x = -10.61$	g	*R/S*	$q_y = -3.84$	°	*R/S*

Control: Data: values measured after balancing

	2.00	$v_2 = 0.8$	mm s^{-1}	*R/S*
	2.10	$\alpha_2 = 100$	°	*R/S*

Results: for post-balancing

in polar coordinates

$q_m = 1.09$	g	*R/S*	$\alpha_m = 269.87$	°	*R/S*

in rectangular coordinates

$q_{mx} = 0$	g	*R/S*	$q_{mr} = -1.09$	g	*R/S/C*

Date: Signature:

We have prepared a programme for the type T–59 pocket calculator on the basis of these relationships (Fig. 10.7), which may be recorded — by keying it into the machine — on a magnetic card, so it is always readily available.

The measurements and the calculations must be performed in the sequence given in Table 10.2. After reading in the magnetic card and calling the address *C*, the calculator is ready to accept the data. After the data number we input the measured results and acknowledge them by keying *R/S*.

After the input of the last item of data and keying *R/S*, the results appear in the sequence of the operation instructions. After balancing the rotor, a test is performed and its data are also recorded in the measurement report. The calculator handles these data as new measurement results and supplies data for the eventual post-balancing procedure. The post-balancing procedure may be repeated several times. In every post-balancing the serial number of the data increases.

If we wish to balance the rotor of another machine, the new calculation may be started, after the previous calculation has finished, by keying *C*.

If the test mass read-in was given in g units, the result is obtained in g units. The imbalance caused by the test mass may also be given in mm g units and then the end result is obtained in mm g units.

If the test mass is mounted on the rotor, the calculation gives the angular position of the imbalance, i.e. the overweight position on the rotor from which mass is to be removed, while if the imbalance is caused by the removal of the test mass, the calculation gives the direction of the underweight position on the rotor where mass is to be added.

If the balancing is carried out carefully and the system is linear, a reduction ratio exceeding 90% of the imbalance may be expected.

Table 10.2 shows, as an example, the measurements involved in the balancing of a machine.

10.2 BALANCING IN SITU IN TWO PLANES

In most cases rotors must be balanced in two planes. Any single plane balancing method may be used for balancing the rotor in both planes in turn. First the balancing plane closest to a bearing showing pronounced vibration is chosen , where attempts are made to reduce the vibration of the bearing to 1/2 or 1/3 of its intensity; then the balancing in the second plane is performed. 5–6 such alternate balancing operations are required (with 3 starts in each plane in the least favourable case), so 12–18 measurements must be performed altogether for the satisfactory balancing of the rotor by the single-plane balancing method.

If a rotor has only a moment-imbalance (e.g. it has been statically balanced previously), the number of the starting operations may be reduced to 5–6 by means

of the triple-point method, with the test masses applied in pairs in both balancing planes.

The number of starting operations may be reduced further by solving the system of equations (139) on balancing in situ in two balancing planes.

We shall describe a calculation program and a construction method for the purpose of determining the solution.

If the measurements specified in Table 10.3 are performed in both methods, the machine has to be started only three times. For measuring the vibrations of the bearing, a vibrometer equipped with a tunable filter and an apparatus for sensing the phase angle of the bearing vibration are necessary. The quantity v in the table can represent the velocity, the acceleration, or the deflection of the vibration. For the arrangement of the vibration sensor no special condition is required. The only important requirement is that it should sense the vibration of the bearing perpendicularly to the axial direction of the machine, but this direction need not be necessarily the same at both bearings.

Table 10.3

Measurements required for balancing in two planes

	Test mass		Measured quantity			
	In balancing plane 1	In balancing plane 2	on bearing 1		on bearing 2	
			amplitude	phase	amplitude	phase
1	None	None	v_{10} $\mathbf{v}_{10}=v_{10}e^{j\alpha_{10}}$	α_{10}	v_{20} $\mathbf{v}_{20}=v_{20}e^{j\alpha_{20}}$	α_{20}
2	q_{p1} (at 0 degree)	None	v_{11} $\mathbf{v}_{11}=v_{11}e^{j\alpha_{11}}$	α_{11}	v_{21} $\mathbf{v}_{21}=v_{21}e^{j\alpha_{21}}$	α_{21}
3	None	q_{p2} (at 0 degree)	v_{12} $\mathbf{v}_{12}=v_{12}e^{j\alpha_{12}}$	α_{12}	v_{22} $\mathbf{v}_{22}=v_{22}e^{j\alpha_{22}}$	α_{22}

Two complex amplitudes each may be expressed in a column vector; so in the case of the first start,

$$\mathbf{V}_0=\begin{bmatrix}v_{10}\\ v_{20}\end{bmatrix}=\begin{bmatrix}v_{10}e^{j\alpha_{10}}\\ v_{20}e^{j\alpha_{20}}\end{bmatrix}. \tag{167}$$

The imbalance of the rotor to be determined may be denoted similarly:

$$\mathbf{Q}=\begin{bmatrix}\mathbf{q}_1\\ \mathbf{q}_2\end{bmatrix}=\begin{bmatrix}q_1e^{j\alpha_1}\\ q_2e^{j\alpha_2}\end{bmatrix}. \tag{168}$$

If the mass q_1 or q_2 is put into each balancing plane contrary to the angular positions α_1 and α_2, then the rotor is theoretically balanced.

In the case of a linear system the relationship between the vectors can be solved by means of a matrix equation:

$$\mathbf{V}_0 = \mathbf{LQ}. \tag{169}$$

The elements of matrix **L** characterizing the rotor are generally complex; they do not depend on the magnitude of the imbalance but on the speed, and therefore the speed of the rotor must be the same during all three measurements. We denote the complex elements of the matrix by

$$\mathbf{L} = \begin{bmatrix} \varkappa_{11} & \varkappa_{12} \\ \varkappa_{21} & \varkappa_{22} \end{bmatrix}. \tag{170}$$

The measurements performed with both test masses serve to determine the elements of the matrix. We arrange the test masses in the angular position 0 marked out on the rotor, so that the corresponding column vectors will contain only real elements:

$$\mathbf{Q}_{p1} = \begin{bmatrix} \mathbf{q}_{p1} \\ 0 \end{bmatrix} = \begin{bmatrix} q_{p1} \\ 0 \end{bmatrix}; \tag{171}$$

$$\mathbf{Q}_{p2} = \begin{bmatrix} 0 \\ \mathbf{q}_{p2} \end{bmatrix} = \begin{bmatrix} 0 \\ q_{p2} \end{bmatrix}. \tag{172}$$

The imbalance caused by the test masses is added to the initial imbalance of the rotor. Accordingly the following matrix equations may be written:

$$\mathbf{V}_1 = (\mathbf{Q}+\mathbf{Q}_{p1}); \tag{173}$$

$$\mathbf{V}_2 = (\mathbf{Q}+\mathbf{Q}_{p2}). \tag{174}$$

After carrying out the multiplication and with the equality $\mathbf{V}_0 = \mathbf{LQ}$ taken into account, we have

$$\mathbf{V}_1 - \mathbf{V}_0 = \mathbf{LQ}_{p1} = \begin{bmatrix} \varkappa_{11} & \varkappa_{12} \\ \varkappa_{21} & \varkappa_{22} \end{bmatrix} \begin{bmatrix} q_{p1} \\ 0 \end{bmatrix} = \begin{bmatrix} \varkappa_{11} q_{p1} \\ \varkappa_{21} q_{p1} \end{bmatrix} = \begin{bmatrix} \mathbf{v}_{11} - \mathbf{v}_{10} \\ \mathbf{v}_{21} - \mathbf{v}_{20} \end{bmatrix}; \tag{175}$$

$$\mathbf{V}_2 - \mathbf{V}_0 = \mathbf{LQ}_{p2} = \begin{bmatrix} \varkappa_{11} & \varkappa_{12} \\ \varkappa_{21} & \varkappa_{22} \end{bmatrix} \begin{bmatrix} 0 \\ q_{p2} \end{bmatrix} = \begin{bmatrix} \varkappa_{12} q_{p2} \\ \varkappa_{22} q_{p2} \end{bmatrix} = \begin{bmatrix} \mathbf{v}_{12} - \mathbf{v}_{10} \\ \mathbf{v}_{22} - \mathbf{v}_{20} \end{bmatrix}. \tag{176}$$

Accordingly the matrix characterizing the machine unit is

$$\mathbf{L} = \begin{bmatrix} \dfrac{\mathbf{v}_{11} - \mathbf{v}_{10}}{q_{p1}} & \dfrac{\mathbf{v}_{12} - \mathbf{v}_{10}}{q_{p2}} \\ \dfrac{\mathbf{v}_{21} - \mathbf{v}_{20}}{q_{p1}} & \dfrac{\mathbf{v}_{22} - \mathbf{v}_{20}}{q_{p2}} \end{bmatrix}. \tag{177}$$

With this knowledge, the vector of the initial imbalance of the rotor is

$$\mathbf{Q} = \mathbf{L}^{-1}\mathbf{V}_0 = \mathbf{KV}_0, \tag{178}$$

where the intermediate matrix

$$\mathbf{K} = \mathbf{L}^{-1} \tag{179}$$

means the reciprocal of matrix $\mathbf{L}$, which is the quotient of the determinant formed from the elements of the adjunct matrix and the matrix.

10.2.1 BALANCING IN TWO PLANES ACCORDING TO WAHRHEIT

The calculation of the matrix equation of complex elements is laborious and subject to easily committed errors. For facilitating the calculation Wahrheit has developed the simple calculation and construction scheme shown in Table 10.4 for the case of the observation of a realizable condition.

If we assume that

$$q_{p1} = q_{p2} = q_p,$$

then

$$\mathbf{K} = \frac{\operatorname{adj} \mathbf{L}}{\det \mathbf{L}} = \frac{q_p \begin{bmatrix} (\mathbf{v}_{22}-\mathbf{v}_{20}) & -(\mathbf{v}_{12}-\mathbf{v}_{10}) \\ -(\mathbf{v}_{21}-\mathbf{v}_{20}) & (\mathbf{v}_{11}-\mathbf{v}_{10}) \end{bmatrix}}{(\mathbf{v}_{11}-\mathbf{v}_{10})(\mathbf{v}_{22}-\mathbf{v}_{20})-(\mathbf{v}_{12}-\mathbf{v}_{10})(\mathbf{v}_{21}-\mathbf{v}_{20})}. \tag{180}$$

Accordingly the complex elements of the vector of the imbalance are

$$\mathbf{q}_1 = q_1 e^{j\alpha_1} = \frac{(\mathbf{v}_{22}-\mathbf{v}_{20})\mathbf{v}_{10}-(\mathbf{v}_{12}-\mathbf{v}_{10})\mathbf{v}_{20}}{(\mathbf{v}_{11}-\mathbf{v}_{10})(\mathbf{v}_{22}-\mathbf{v}_{20})-(\mathbf{v}_{12}-\mathbf{v}_{10})(\mathbf{v}_{21}-\mathbf{v}_{20})} q_p, \tag{181}$$

$$\mathbf{q}_2 = q_2 e^{j\alpha_2} = \frac{(\mathbf{v}_{11}-\mathbf{v}_{10})\mathbf{v}_{20}-(\mathbf{v}_{21}-\mathbf{v}_{20})\mathbf{v}_{10}}{(\mathbf{v}_{11}-\mathbf{v}_{10})(\mathbf{v}_{22}-\mathbf{v}_{20})-(\mathbf{v}_{12}-\mathbf{v}_{10})(\mathbf{v}_{21}-\mathbf{v}_{20})} q_p. \tag{182}$$

In the above expressions all the v elements are complex quantities, so the prescribed operations (substraction, multiplications, divisions) must be carried out according to the algebraic rules of complex numbers.

The calculations prescribed in Table 10.4 can be performed on the basis of these relationships. The maximum value obtained as a result of the left-hand side multiplications is accepted as 100%, and the other values required for the construction are calculated and written up in the central left-hand column of the table as proportional ratios of the same.

The angular values must not exceed 360°. If a higher value is obtained by calculation, then 360° is subtracted from it and this value is written in the central right-hand column of the table.

We plot the points A, B, C, D, E and F, properly scaled in a polar diagram, as shown in Fig. 10.8. The distances AB, CD and EF are halved to obtain the points K, L and M. We construct the centre of gravity, S, of the triangle defined by these three points. We determine from the diagram the coordinates of the points K, L and S

Table 10.4

Measurement report

Type of the machine: generator	Working speed: 1500/min.
Balanced by: Fejes Date: 10.3. 1984.	Balancing speed: 1500/min.

Measured data	In plane 1		In plane 2	
	vibration, μm	angle, degree	vibration, μm	angle, degree
Measurement 1: imbalanced rotor	$v_{10}=100$	$\alpha_{10}=127$	$v_{20}=98$	$\alpha_{20}=114$
Measurement 2: test mass in plane 1	$v_{11}=62$	$\alpha_{11}=41$	$v_{21}=67$	$\alpha_{21}=179$
Measurement 3: test mass in plane 2	$v_{12}=28$	$\alpha_{12}=100$	$v_{22}=79$	$\alpha_{22}=331$

Test mass (in both planes at 0°): $q_p=40$ g

	Calculation of the quantity	%	Angle, degree	Calculation of the angle
A	$v_{12}v_{20}=28\times98=2744$	34.7	214	$\alpha_{12}+\alpha_{20}=100+114=214$
B	$v_{10}v_{22}=100\times79=7900$	100	278	$180+\alpha_{10}+\alpha_{22}=180+127+331=638$
C	$v_{10}v_{21}=100\times67=6700$	84.8	306	$\alpha_{10}+\alpha_{21}=127+179=306$
D	$v_{11}v_{20}=62\times98=6076$	76.9	335	$180+\alpha_{11}+\alpha_{20}=180+41+114=335$
E	$v_{11}v_{22}=62\times79=4898$	62.0	12	$\alpha_{11}+\alpha_{22}=41+331=372$
F	$v_{12}v_{21}=28\times67=1876$	23.8	99	$180+\alpha_{12}+\alpha_{21}=180+100+179=459$

Data measured in Fig. 10.8	K	L	S
Length, mm	$v_K=60$	$v_L=78.3$	$v_S=40.8$
Angle, degree	$\alpha_K=263$	$\alpha_L=320$	$\alpha_S=331$

Calculation with the data of the diagram

$v_v=\dfrac{q_p}{3v_S}=\dfrac{40}{3\times40.8}=0.327$	$v_\alpha=360-\alpha_S=360-311=49$
$q_1=v_vv_K=0.327\times60=19.6$ g	$q_2=v_vv_L=0.327\times78.3=25.6$ g
$\alpha_1=v_\alpha+\alpha_K=49+263=312°$	$\alpha_2=v_\alpha+\alpha_L=49+320=(369)\ 9°$

Testing after balancing

Plane 1: $v_m=8$ μm	Plane 2: $v_m=7$ μm

Table 10.5

Measurement report and operation instruction

Balancing in two planes by calculation	Address: **D**

Identification data of the machine: Name: generator Machine No 44 321 Speed: 3000/min.
Permitted residual imbalance in plane 1: 0.4 μm Made out by: Fejes
in plane 2: 0.15 μm Date 30.7. 1984

Measured data

Serial No. of start	Mode of measurement	Plane 1						Plane 2					
		Data No.	Vibration (μm)	Key	Date No.	Angle °	Key	Date No.	Vibration (μm)	Key	Date No.	Angle °	Key
1.	Without test mass	1.00	$v_{10}=100$	*R/S*	2.00	$\alpha_{10}=127$	*R/S*	3.00	$v_{20}=98$	*R/S*	4.00	$\alpha_{20}=114$	*R/S*
2.	With test mass in plane 1	5.00	$v_{11}=62$	*R/S*	6.00	$\alpha_{11}=41$	*R/S*	7.00	$v_{21}=67$	*R/S*	8.00	$\alpha_{21}=179$	*R/S*
3.	With test mass in plane 2	9.00	$v_{12}=28$	*R/S*	10.00	$\alpha_{12}=100$	*R/S*	11.00	$v_{22}=79$	*R/S*	12.00	$\alpha_{22}=331$	*R/S*
Test mass		13.00	$q_{p1}=$	40		g	*R/S*	14.00	$q_{p2}=$	40		g	*R/S*

Result for balancing

In polar coordinates	$q_1=19.55$ g	*R/S*	$\alpha_1=131.43°$	*R/S*	$q_2=25.64$ g	*R/S*	$\alpha_2=188.36°$	*R/S*
In rectangular coordinates	$q_{1x}=-12.93$ g	*R/S*	$q_{1y}=14.66$ g	*R/S*	$q_{2x}=-25.36$ g	*R/S*	$q_{2y}=-3.73$ g	*R/S*

Measured data after balancing

Residual imbalance	15.00	$v_1=9$	*R/S*	16.00	$\alpha_1=187$	*R/S*	17.00	$v_2=10$	*R/S*	18.00	$\alpha_2=250$	*R/S*

Results for the 1st post-balancing

In polar coordinates	$q_1=2.96$ g	*R/S*	$\alpha_1=156.20°$	*R/S*	$q_2=3.95$ g	*R/S*	$\alpha_2=291.10°$	*R/S*
In rectangular coordinates	$q_{1x}=-2.71$ g	*R/S*	$q_{1y}=1.20$ g	*R/S*	$q_{2x}=1.42$ g	*R/S*	$q_{2y}=-3.68$ g	*R/S*

Measured data after the 1st post-balancing

Residual imbalance after the 1st post-balancing	19.00	$v_1=0.48$	R/S	20.00	$\alpha_1=248.50$	R/S	21.00	$v_2=0.17$	R/S	22.00	$\alpha_2=160.90$	R/S

Result for the 2nd post-balancing

In polar coordinates	$q_1=0.17$ g	R/S	$\alpha_1=256.77°$	R/S	$q_2=0.05$ g	R/S	$\alpha_2=10.52°$	R/S
In rectangular coordinates	$q_{1x}=-0.04$ g	R/S	$q_{1y}=-0.96$ g	R/S	$q_{2x}=0.05$ g	R/S	$q_{2y}=0.01$ g	R/S

Measured data after the 2nd post-balancing

Residual imbalance after the 2nd post-balancing	23.00	$v_1\approx 0$	R/S	24.00	$\alpha_1=$	R/S	25.00	$v_2\approx 0$	R/S	26.00	$\alpha_2=$	R/S

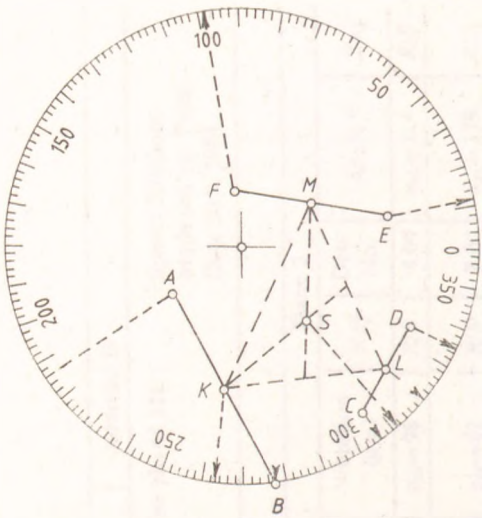


Fig. 10.8 Balancing in two planes by means of construction

and introduce them into the table, then continue the calculation according to the instructions.

The results of the calculation are the magnitude of the balancing mass and the angular positions of the underweight places in both planes.

10.2.2 BALANCING IN TWO PLANES WITH THE HELP OF A PROGRAMMABLE POCKET CALCULATOR

If a programmable calculator is used, it is not necessary to choose test pieces of an identical magnitude, hence the intermediate matrix is

$$\mathbf{K} = \frac{1}{\det \mathbf{L}} \begin{bmatrix} \dfrac{\mathbf{v}_{22}-\mathbf{v}_{20}}{q_{p2}} & -\dfrac{\mathbf{v}_{21}-\mathbf{v}_{20}}{q_{p1}} \\ -\dfrac{\mathbf{v}_{12}-\mathbf{v}_{10}}{q_{p2}} & \dfrac{\mathbf{v}_{11}-\mathbf{v}_{10}}{q_{p1}} \end{bmatrix} = \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix}, \tag{183}$$

and the requested imbalance is

$$\begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \end{bmatrix} = \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \begin{bmatrix} \mathbf{v}_{10} \\ \mathbf{v}_{20} \end{bmatrix}. \tag{184}$$

It is advisable to construct the program so that the calculator calculates the elements of the intermediate matrix so as to have them accessible, in order to avoid repeating the calculation with new data, if post balancing is necessary after the first balancing.

Figure 10.9 shows the program developed for the type TI–59 programmable pocket calculator. If the program is keyed into the machine, then recorded on a mag-

```
000 76 LBL
001 12 B
002 73 RC*
003 00 00
004 32 X⇄T
005 73 RC*
006 01 01
007 22 INV
008 37 P/R
009 42 STO
010 02 02
011 32 X⇄T
012 91 R/S
013 25 CLR
014 32 X⇄T
015 77 GE
016 77 GE
017 85 +
018 03 3
019 06 6
020 00 0
021 95 =
022 92 RTN
023 76 LBL
024 77 GE
025 43 RCL
026 02 02
027 92 RTN
028 76 LBL
029 10 E′
030 73 RC*
031 00 00
032 65 ×
033 43 RCL
034 34 34
035 85 +
036 73 RC*
037 01 01
038 65 ×
039 43 RCL
040 35 35
041 95 =
042 72 ST ✱
043 02 02
044 69 OP
045 22 22
046 73 RC ✱
047 00 00
048 65 ×
049 43 RCL
050 35 35
051 75 −
052 73 RC ✱
053 01 01
054 65 ×
055 43 RCL
056 34 34
057 95 =
058 72 ST ✱
059 02 02
060 02 2
061 44 SUM
062 00 00
063 44 SUM
064 01 01
065 69 OP
066 22 22
067 92 RTN
068 76 LBL
069 19 D′
070 73 RC ✱
071 00 00
072 75 −
073 73 RC ✱
074 01 01
075 95 =
076 72 ST ✱
077 00 00
078 69 OP
079 20 20
080 69 OP
081 21 21
082 92 RTN
083 76 LBL
084 18 C′
085 73 RC ✱
086 00 00
087 32 X⇄T
088 69 OP
089 20 20
090 73 RC ✱
091 00 00
092 37 P/R
093 44 SUM
094 34 34
095 32 X⇄T
096 44 SUM
097 35 35
098 69 OP
099 20 20
100 92 RTN
101 76 LBL
102 11 A
103 37 P/R
104 72 ST ✱
105 00 00
106 69 OP
107 20 20
108 32 X⇄T
109 72 ST ✱
110 00 00
111 69 OP
112 20 20
113 92 RTN
114 76 LBL
115 13 C
116 69 OP
117 26 26
118 43 RCL
119 06 06
120 91 R/S
121 64 PD ✱
122 07 07
123 64 PD ✱
124 08 08
125 32 X⇄T
126 69 OP
127 26 26
128 43 RCL
129 06 06
130 91 R/S
131 69 OP
132 27 27
133 69 OP
134 28 28
135 74 SM ✱
136 07 07
137 74 SM ✱
138 08 08
139 92 RTN
140 76 LBL
141 14 D
142 47 CMS
143 01 1
144 00 0
145 42 STO
146 00 00
147 58 FIX
148 02 02
149 01 1
150 44 SUM
151 30 30
152 44 SUM
153 32 32
154 44 SUM
155 22 22
156 94 +/−
157 44 SUM
158 24 24
159 46 SUM
160 26 26
161 44 SUM
162 28 28
163 03 3
164 00 0
165 44 SUM
166 07 07
167 02 2
168 04 4
169 44 SUM
170 08 08
171 13 C
172 11 A
173 69 OP
174 27 27
175 69 OP
176 28 28
177 13 C
178 11 A
179 02 2
180 02 2
181 42 STO
182 07 07
183 69 OP
184 38 38
185 13 C
186 11 A
187 05 5
188 44 SUM
189 07 07
190 03 3
191 44 SUM
192 08 08
193 13 C
194 11 A
195 69 OP
196 37 37
197 69 OP
198 28 28
199 13 C
200 11 A
201 02 2
202 02 2
203 42 STO
204 07 07
205 02 2
206 04 4
207 42 STO
208 08 08
209 13 C
210 11 A
211 69 OP
212 26 26
213 43 RCL
214 06 06
215 91 R/S
216 42 STO
217 50 50
218 69 OP
219 26 26
220 43 RCL
221 06 06
222 91 R/S
223 42 STO
224 51 51
225 18 C.
226 18 C.
227 18 C.
228 18 C.
229 18 C.
230 18 C.
231 43 RCL
232 34 34
233 33 X²
234 85 +
235 43 RCL
236 35 35
237 33 X²
238 95 =
239 35 1/X
240 49 PRD
241 34 34
242 49 PRD
243 35 35
244 01 1
245 04 4
246 42 STO
247 00 00
248 01 1
249 00 0
250 42 STO
251 01 01
```

```
252 19 D'
253 19 D'
254 19 D'
255 19 D'
256 01 1
257 00 0
258 42 STO
259 01 01
260 19 D'
261 19 D'
262 19 D'
263 19 D'
264 43 RCL
265 50 50
266 49 PRD
267 20 20
268 49 PRD
269 21 21
270 94 +/−
271 49 PRD
272 18 18
273 49 PRD
274 19 19
275 43 RCL
276 51 51
277 49 PRD
278 14 14
279 49 PRD
280 15 15
281 94 +/−
282 49 PRD
283 16 16
284 49 PRD
285 17 17
286 01 1
287 04 4
288 42 STO
289 00 00
290 01 1
291 05 5
292 42 STO
293 01 01
294 05 5
295 02 2
296 42 STO
297 02 02
298 10 E,
299 10 E,
300 10 E,
301 10 E,
302 43 RCL
303 56 56
304 65 x
305 43 RCL
306 13 13
307 85 +
308 43 RCL
309 58 58
310 65 x
311 43 RCL
312 11 11
313 75 −
314 43 RCL
315 57 57
316 65 x
317 43 RCL
318 12 12
319 75 −
320 43 RCL
321 59 59
322 65 x
323 43 RCL
324 10 10
325 95 =
326 42 STO
327 46 46
328 42 RCL
329 52 52
330 65 x
331 43 RCL
332 13 13
333 85 +
334 43 RCL
335 54 54
336 65 x
337 43 RCL
338 11 11
339 75 −
340 43 RCL
341 53 53
342 65 x
343 43 RCL
344 12 12
345 75 −
346 43 RCL
347 55 55
348 65 x
349 43 RCL
350 10 10
351 95 =
352 42 STO
353 47 47
354 43 RCL
355 59 59
356 65 x
357 43 RCL
358 11 11
359 85 +
360 43 RCL
361 58 58
362 65 x
363 43 RCL
364 10 10
365 85 +
366 43 RCL
367 57 57
368 65 x
369 43 RCL
370 13 13
371 85 +
372 43 RCL
373 56 56
374 65 x
375 43 RCL
376 12 12
377 95 =
378 42 STO
379 48 48
380 43 RCL
381 55 55
382 65 x
383 43 RCL
384 11 11
385 85 +
386 43 RCL
387 54 54
388 65 x
389 43 RCL
390 10 10
391 85 +
392 43 RCL
393 53 53
394 65 x
395 43 RCL
396 13 13
397 85 +
398 43 RCL
399 52 52
400 65 x
401 43 RCL
402 12 12
403 95 =
404 42 STO
405 49 49
406 04 4
407 06 6
408 42 STO
409 00 00
410 04 4
411 08 8
412 42 STO
413 01 01
414 12 B
415 91 R/S
416 69 OP
417 20 20
418 69 OP
419 21 21
420 12 B
421 91 R/S
422 43 RCL
423 46 46
424 91 R/S
425 43 RCL
426 48 48
427 91 R/S
428 43 RCL
429 47 47
430 91 R/S
431 43 RCL
432 49 49
433 91 R/S
434 01 1
435 04 4
436 42 STO
437 07 07
438 42 STO
439 08 08
440 01 1
441 00 0
442 42 STO
443 00 00
444 13 C
445 11 A
446 13 C
447 11 A
448 61 GTO
449 03 03
450 02 02
```

Fig. 10.9 Program for balancing in two planes (for the type TI–59 pocket calculator)

netic card, the balancing in situ in two planes may be performed in a short time by using this record of the program.

Table 10.5 contains the measurement report corresponding to the program, and the operation instructions, enabling the course of the balancing procedure to be followed on the basis of a numerical example.

After balancing the rotor according to the calculated values we test the result. If the system is linear, then for a correct arrangement of the mass and correct measurements, a reduction of over 90% in the imbalance should be expected.

If we find, during the test, that the result is still unsatisfactory, then a post-balancing operation must be performed. Our program also supplies the necessary calculation for the post-balancing by keying R/S.

The program is reset to the beginning by keying D after finishing the calculation and is ready to accept the data for balancing the next rotor.

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In this book, the authors deal with vibration tests and their applications to machinery. Like the use of other technical tests for machine operating conditions, these have been developing rapidly in recent years alongside conventional maintenance procedures. Not only are the direct costs of repair and maintenance reduced by applying the new methods but unexpected failures, with possible dire consequences, can be avoided.

The purpose of the book is to alert specialists engaged in the maintenance of machines to the technical diagnostic methods available, with particular regard to tests based on vibration measurements. The authors demonstrate the importance and timeliness of this method by way of practical examples. The book should be of interest to specialists with intermediate and university qualifications, and may be useful as a handbook in intermediate and university teaching.

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