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A Toast

Here's to Wolfgang Pauli
Who made a funny joke
Here's to the great Enrico
who then of weakness spoke

Here's to all those present
To celebrate the fruits
Of all the patient workers
Who followed these astutes.

Here's to the proposition
that we shall meet again
And here's to the fond hope
the sun will shine till then.

F. Reines
June 15, 1972

VOLUME I

VOLUME II

SCALING PROPERTIES IN WEAK AND ELECTROMAGNETIC PROCESSES*

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In this talk, I shall first review the nature of the scaling property that has been recently discovered in various high energy weak and electromagnetic processes, then examine its theoretical basis, and finally comment on its implications for the future of high energy physics.

1. Scaling Hypothesis

The scaling property is the consequence of the scaling hypothesis which was first suggested[1] by Bjorken and others. Here, we wish to state the scaling hypothesis in a form somewhat different from its original formulation, one that is perhaps more directly related to experimental results, and appears to be symmetric with respect to leptons and hadrons. For definiteness, we consider a purely leptonic or semi-leptonic reaction which can be either a second order electromagnetic process or a first order weak interaction process, e.g.,

$$e^+ + e^- \rightarrow \mu^+ + \mu^- ,$$

$$e^+ + e^- \rightarrow \text{hadrons} ,$$

$$\nu_\mu + n \rightarrow \mu^- + \text{hadrons, etc.}$$

Furthermore, for the semi-leptonic reaction we shall always sum over <u>all final hadronic channels</u>. Let $d\sigma$ be the appropriate differential cross section, which can be, in general, written as

$$d\sigma = f(s, q^2, m_\ell, m_N) \times \begin{cases} \alpha^2 \\ G^2 \end{cases} , \qquad (1)$$

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where the factors α^2 and G^2 are, respectively, the squares of the fine structure constant and of the Fermi constant, depending on whether the process is electromagnetic or weak,

$$s = (\text{center-of-mass energy})^2 \quad ,$$

q^2 represents the various relevant $(\text{4-momentum transfer})^2$, m_ℓ denotes the various lepton masses (m_e or m_μ), and m_N denotes the various hadron masses (which can be either the nucleon mass m_N itself, or the ρ and pion masses, etc.).

The scaling hypothesis states that (i) if s and $|q^2|$ are much larger than m_ℓ^2 then it is a good approximation to set $m_\ell = 0$ in the expression for $d\sigma$ and (ii) if s and $|q^2|$ are much larger than m_N^2 , then it is a good approximation to set $m_N = 0$ in the expression for $d\sigma$, provided that all final hadronic channels are summed over. We emphasize that if one does not sum over different final hadron channels, then there would obviously be cases in which one cannot neglect hadron masses. For example, in $e^+e^- \to \rho^o$, the physical mass and the width of ρ^o clearly cannot be neglected. One notes further that even in case (i) for the leptons, though not explicitly stated, it is understood that all different final channels of infrared photons are being summed over; otherwise $d\sigma$ would be zero.

According to the scaling hypothesis, for s and $|q^2|$ larger than a few $(\text{GeV})^2$, one may set as a good approximation $m_\ell = m_N = 0$; therefore, (1) becomes simply

$$d\sigma = f(s, q^2) \times \begin{cases} \alpha^2 \\ G^2 \end{cases} \quad . \qquad (2)$$

Apart from the coupling constant α^2 or G^2, the differential cross section $d\sigma$ now depends only on s and the various q^2. These quantities represent (in the natural units $\hbar = c = 1$) the only physical observables with the dimension $(\text{length})^{-2}$. All the consequences of the scaling hypothesis can then be easily derived by a pure and simple dimensional analysis[2]. The scaling hypothesis means simply the absence of any basic physical energy scale, such as m_ℓ and m_N. As we shall see, this enables us to connect various cross sections at a relatively low energy range to those at a much higher energy range.

2. Applications

(i) To illustrate the use of the scaling hypothesis, we shall first consider the following two electromagnetic processes, one purely leptonic and the other semi-leptonic:

$$e^+ + e^- \rightarrow \mu^+ + \mu^- \tag{3}$$

and

$$e^+ + e^- \rightarrow \text{hadrons}\,. \tag{4}$$

It follows from the scaling hypothesis that for

$$s = (\text{center-of-mass energy})^2 \gg m_\mu^2$$

one may set $m_e = m_\mu = 0$. The total cross section for the purely leptonic reaction (3) depends then only on α^2 and s. From simple dimensional considerations, one sees that

$$\sigma(e^+e^- \rightarrow \mu^+\mu^-) = \text{constant} \cdot \alpha^2/s\,.$$

The constant can be evaluated by using quantum electrodynamics, which is consistent with the scaling hypothesis provided that radiative corrections are neglected; one finds then

$$\sigma(e^+e^- \rightarrow \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s} .$$

Similarly, according to the scaling hypothesis, if one sums over all final hadronic channels in the semi-leptonic reaction (4), for $s >$ a few $(\mathrm{GeV})^2$ one may set $m_N = m_\ell = 0$. A simple dimensional analysis leads to

$$\sigma(e^+e^- \rightarrow \text{hadrons}) = \text{constant} \cdot \alpha^2 / s$$

where the constant may be determined by a relatively low energy experiment, which then enables one to predict the cross section in a much higher energy region. The present colliding beam results from Frascati[3] are in agreement with the predicted s^{-1} dependence.

(ii) Next, we consider the following weak processes:

$$\nu_e + e^\pm \rightarrow \nu_e + e^\pm \qquad (5)$$

and

$$\nu_\mu + N \rightarrow \mu^- + \text{hadrons} . \qquad (6)$$

Let q^2 denote the (4-momentum transfer)^2 between the incident neutrino and the target, and s be the (center-of-mass energy)^2 , as before. For the purely leptonic reaction (5), if s and q^2 are $\gg m_e^2$, on account of the scaling hypothesis, one may set $m_e = 0$ in the expression for $d\sigma$. Similarly, for the semi-leptonic reaction

(6), if s and q^2 are greater than a few $(GeV)^2$, one may set $m_\mu = m_N = 0$, provided all hadron channels are summed over. In either case, the differential cross section is proportional to G^2 and the proportionality factor depends only on q^2 and s. Recalling that the dimension of G is $(length)^2$, one finds from simple dimensional considerations that the differential cross sections of both reactions must be of the form

$$\frac{d\sigma}{dq^2} = G^2 \cdot f\left(\frac{q^2}{s}\right) \tag{7}$$

where f is a dimensionless function depending only on the ratio (q^2/s), which varies from 0 to 1 since $s = q^2_{max}$, the maximum value of q^2. The corresponding total cross sections are of the form

$$\sigma = \text{constant} \cdot G^2 s . \tag{8}$$

According to the usual (Current X Current) theory of the weak interaction, one can readily show that

$$\frac{d\sigma}{dq^2}(\nu_e e^- \to \nu_e e^-) = \frac{G^2}{\pi}$$

and

$$\frac{d\sigma}{dq}(\nu_e e^+ \to \nu_e e^+) = \frac{G^2}{\pi}\left(1 - \frac{q^2}{s}\right)^2$$

which agree with (7). In Figure 1, we reproduce the results from the CERN neutrino experiment[4], which gives, after averaging over $N = p$ and n,

$$\sigma(\nu_\mu + N \to \mu^- + \text{hadrons}) \cong 0.6 \times 10^{-38} \, (\text{cm}^2/\text{nucleon}) \times \left[(E_\nu)_{lab} \text{ in GeV}\right]$$

in good agreement with (8).

(iii) As a further example, one may consider the following two electromagnetic processes:

$$e^{\pm} + \mu^{\pm} \rightarrow e^{\pm} + \mu^{\pm} \tag{9}$$

and

$$e^{\pm} + p \rightarrow e^{\pm} + \text{hadrons} \tag{10}$$

in which one sums over all final hadron channels, as is done in the "deep" inelastic experiments of SLAC[5].

In this problem, there are three independent invariant variables: s, q^2 and $p \cdot q$ where s is the center-of-mass-energy squared, q is the virtual photon momentum, and p denotes either the 4-momentum of the initial proton [in (10)] or that of the initial muon [in (9)]. It is customary to introduce a dimensionless variable, called the scaling variable

$$\omega \equiv -2p \cdot q / q^2 \quad .$$

According to the scaling hypothesis, for s and q^2 greater than a few $(\text{GeV})^2$, one may set $m_\ell = m_N = 0$; therefore, the differential cross section depends only on q^2, $s = q^2_{max}$ and ω. From simple dimensional considerations, one deduces that for the deep inelastic ep scattering

$$\frac{d^2\sigma}{dq^2\, d\omega} = \frac{\alpha^2}{(q^2)^2} F\left(\frac{q^2}{s}, \omega\right) \quad . \tag{11}$$

For the purely leptonic reaction (9), because $e\mu$ scattering is an elastic process, one has $p^2 = (p+q)^2 = -m_\mu^2$. Consequently, the scaling variable ω

equals 1 and the corresponding function F is, therefore, proportional to $\delta(\omega - 1)$. One may write, instead of (11),

$$\frac{d\sigma}{dq^2}\,(e^{\pm}\mu^{\pm} \to e^{\pm}\mu^{\pm}) = \frac{\alpha^2}{(q^2)^2}\, f\left(\frac{q^2}{s}\right) . \qquad (12)$$

The $\frac{q^2}{s}$ dependence in both (11) and (12) can be explicitly evaluated by using quantum electrodynamics, since it involves only lepton variables. One finds (for $m_\ell = m_N = 0$)

$$\frac{d\sigma}{dq^2}\,(e^{\pm}\mu^{\pm} \to e^{\pm}\mu^{\pm}) = \frac{4\pi\alpha^2}{(q^2)^2}\left[1 - \frac{q^2}{s} + \frac{1}{2}\left(\frac{q^2}{s}\right)^2\right] \qquad (13)$$

and

$$\frac{d^2\sigma}{dq^2\, d\omega}\,(e^{\pm}p \to e^{\pm} + \text{hadrons}) = \frac{4\pi\alpha^2}{(q^2)^2}\left[\left(\frac{1}{\omega} - \frac{q^2}{s}\right)(\nu W_2) + \left(\frac{q^2}{s}\right)^2 W_1\right] \qquad (14)$$

where W_1 and νW_2 are called structure functions; both are dimensionless and depend on ω only. As shown in Figure 2, the validity of the scaling hypothesis has been verified by the recent SLAC data[5].

3. Theoretical Difficulties

While the statement of the scaling property is simple, it turns out to be rather difficult to find a solid theoretical basis of such a property for the hadrons. This difficulty is connected with the so-called "mass singularities" of local field theories[6]. In the case of quantum electrodynamics, such mass singularities are well known. They are connected with the high degeneracy between all states consisting of any number of pairs and photons moving along the same direction and with the same total momentum. The approximation $m_\ell = 0$ can only be made in the first Born term; it leads to logarithmic divergences in higher order radiative corrections. Experimentally, the mass singularity is supported by the fact that all zero-mass particles, such as the neutrino, the photon and the graviton, are found to be neutral. Fortunately, in quantum electrodynamics, the coupling constant is small, and therefore the zero mass approximation is a good one provided the energy ν is not extraordinarily high, so that $\alpha \ln \frac{\nu}{m_\ell}$ is $\ll 1$. For the strong interaction of hadrons, the problem becomes more serious because of the large coupling constant associated with strong interactions. Let me briefly review various previous theoretical attempts and their difficulties:

(i) Parton model

While the original parton idea of Feynman[1] has important heuristic values, it nevertheless puts a special emphasis on the infinite-momentum frame of reference. It is suggested that in the infinite-momentum frame, the electromagnetic property of the assumed pointlike constituents of the physical nucleon can be treated as that of an assembly of independent free particles. The "infinite momentum frame", by itself, is clearly not a Lorentz invariant concept. Furthermore, one can easily show[7] that, in general, the direction of the infinite momentum cannot be arbitrary. It must be limited to a certain restrictive set of directions, depending on the virtual photon momentum; otherwise, the mass of each of the pointlike constituents has to be lighter than that of the physical proton, and that would be too unphysical. Naturally, this leads to questions of whether the parton model, especially of a spin $\frac{1}{2}$ particle such as the physical proton, can be derived from a relativistically invariant theory.

(ii) Perturbation theory

In the literature, there have been several attempts to try to derive the scaling property from the usual relativistic local field theory. So far, the only success has been limited to either the trivial case of free particles (free except for their electromagnetic interaction), or the unphysical case of a super-renormalizable ϕ^3-type theory[8] in which all particles must be of zero spin. For the physically interesting case of spin $\frac{1}{2}$ charged particles with some non-electromagnetic interaction, straightforward calculation in perturbative expansions leads to logarithmic deviations from scaling behavior[9].

(iii) Perturbation theory with cut-offs

Efforts have been made to introduce a transverse momentum cut-off[10] to the perturbation theory. However, the transverse momentum cut-off in the field theoretical derivation of scaling leads to a formalism and a scattering amplitude that are current-conserving only in the infinite momentum frame and in the scaling region. Therefore, it is difficult to see how one may derive such an *ad hoc* cut-off procedure in a *bona fide* relativistic and gauge invariant field theory.

(iv) Light cone commutator

A straightforward application of field equations for interacting spin $\frac{1}{2}$ particles leads to a current commutator that is more singular than that for free fields[11]; therefore, it does not seem to yield the desired scaling properties. Efforts have been made to introduce the so-called "formal manipulation" of current operators[12]. However, at present the theoretical foundation of such rules appears to be quite uncertain. In particular, if one identifies such a "formal manipulation" with the usual Feynman regulator with a negative metric, then it is possible to show that while one may regularize the light-cone commutator, it is not possible to regularize the deep inelastic cross section unless such negative-metric particles are indeed produced asymptotically; this would violate unitarity and is certainly unrelated to the SLAC experiments.

Remarks:

We note that in both ep and νp type experiments the magnitudes of the deep inelastic cross sections are comparable to those of the corresponding purely leptonic ones. This strongly suggests that we may characterize the electromagnetic and weak interactions of hadrons, like those of the leptons, by some local interactions. On the other hand, as mentioned before, in a local field theory there is this difficulty of the "mass-singularity". Remembering that for leptons, while the "mass-singularity" becomes important in the infinite energy range, the approximation $m_\ell = 0$ remains a good one over quite an extensive intermediate energy range

$$m_\ell < \nu < m_\ell \exp(\alpha^{-1}) \quad .$$

For the hadrons, the theoretical difficulties discussed above are all associated with the mathematical infinite energy limit; only in such a limit are the deep inelastic scaling phenomena determined by the current commutator on the light cone. Comparison with the similar situation for leptons at least raises the question that perhaps for hadrons one should also regard the scaling property to be valid not necessarily in the infinite energy limit, but only in an intermediate energy range, one that includes all presently available machine energies. This view seems all the more reasonable, since at infinite energy and infinite 4-momentum transfer very likely the higher order electromagnetic and weak effects would become comparable in magnitude to the so-called strong interaction effects. In such a case, the observed deep inelastic cross sections are no longer simply related to the appropriate light-cone commutator. Experimentally, the physical "scale" for scaling is known to be only $\sim 0(m_N)$ which is also of the same order of magnitude as the physical scale for the elastic electromagnetic form factors of the nucleon. Thus, to explain the observed scaling phenomena it is certainly not necessary to require the relevant current commutator to have the desired behavior as an operator equation valid on the mathematical light cone. All that is needed is to construct theories in which the zero-mass approximation is a good one for the matrix element of the current commutator after it has been averaged over the physical nucleon state and at values of s and $|q^2|$ that are large but need not be infinite.

As we shall see, it is indeed possible to develop such a bona fide local field theory, provided one regards the physical nucleon as a composite, not represented by a single elementary local field.

4. Bound-State Model

Next, I wish to discuss some recent theoretical progress[13] made in this direction; this work was done in collaboration with S. D. Drell. Our basic view is to regard the physical proton p as a bound state of some local fields. To illustrate this bound-state concept, let me first discuss a simple model[14].

The model that we shall discuss consists of only three fields: a spin $\frac{1}{2}$ charged field $\psi(x)$, a pseudoscalar neutral meson field $\pi(x)$ and a scalar neutral gluon field $\phi(x)$. [The model can, of course, be easily extended to include charged meson fields.] The interaction Lagrangian density is assumed to be

$$\mathcal{L}_\kappa + \mathcal{L}_f \tag{15}$$

where

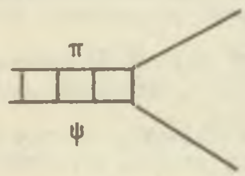
$$\mathcal{L}_\kappa = \kappa_o \pi^2 \phi \quad ,$$

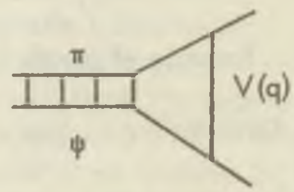
$$\mathcal{L}_f = f_o \psi^\dagger \gamma_4 \psi \phi \quad ,$$

f_o and κ_o are unrenormalized coupling constants. Since $\mathcal{L}_\kappa$ is a superrenormalizable interaction and $\mathcal{L}_f$ is a renormalizable one, the usual renormalization process can be easily carried out. The physical proton p is assumed to be a bound system $(\psi\pi)$ in the $s_{\frac{1}{2}}$ state. Therefore, the field ψ is of opposite parity from p. In order for p to be the lowest baryon state, the renormalized coupling constants f and κ must satisfy

$$\frac{f\kappa}{4\pi} \sim 0(m_\pi) \tag{16}$$

To describe the bound state, one can easily set up the Bethe-Salpeter equation. In the ladder approximation (which will be removed in later discussions), the wave function $\Phi(k)$ is determined by

$\Phi(k) =$ 

$=$ 

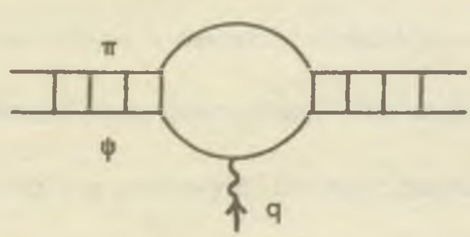
where k is the relative momentum between ψ and π, and V(q) is the covariant potential generated by the gluon field ϕ. Since $V(q) \sim 0(\frac{1}{q^2})$ at large momentum transfer q, one can easily establish that

$$\Phi(k) \sim 0(\frac{1}{k^2}) \qquad \text{as} \quad k^2 \rightarrow \infty . \qquad (17)$$

In the simple model in which only ψ is charged, the standard minimal electromagnetic interaction is

$$i\, \psi^{\dagger} \gamma_4 \gamma_\lambda \psi A_\lambda$$

where A_λ denotes the electromagnetic field. In the "parton" language[1], one would then say that the charged constituent (or parton) ψ has a "point-like" electromagnetic structure. The evaluation of the electromagnetic form factors of the physical nucleon can be readily carried out. By using the diagram

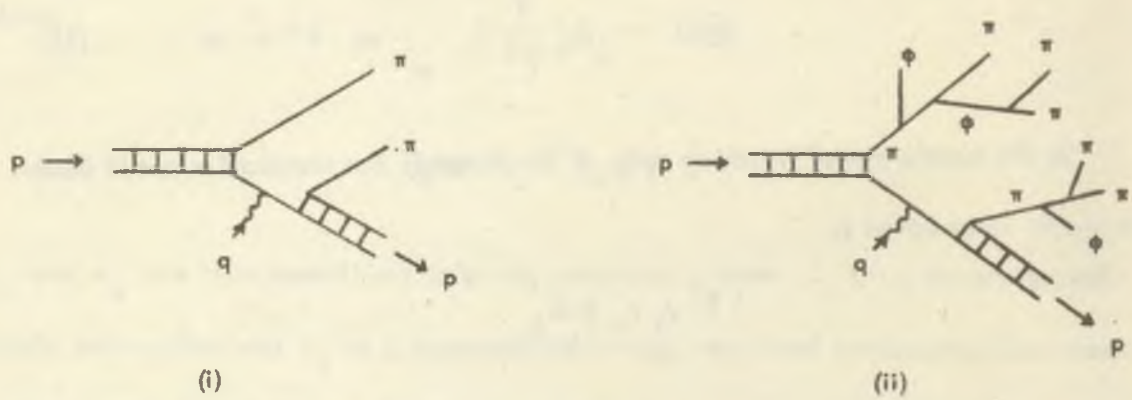


one finds the familiar result[15] that both $F_1(q^2)$ and $F_2(q^2)$, being proportional to the square of the wave function, decrease as q^{-4} (apart from $\ln q^2$ factors) at large q^2.

In this theory, the field ψ represents the interpolating field of the continuum $(p\pi)$. Because of parity conservation, $\psi \not\to p$; ψ is consequently not an interpolating field for any single stable particle. Therefore, in any collision process ψ cannot appear in the asymptotic states. The following are two typical diagrams for

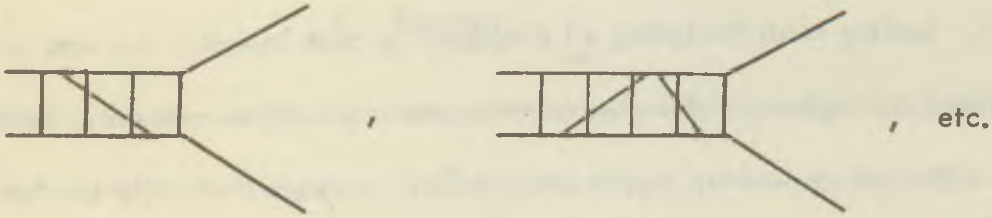
$$e + p \to e + p + \cdots,$$

where the final hadron state "..." can consist of any number of mesons, π or ϕ:

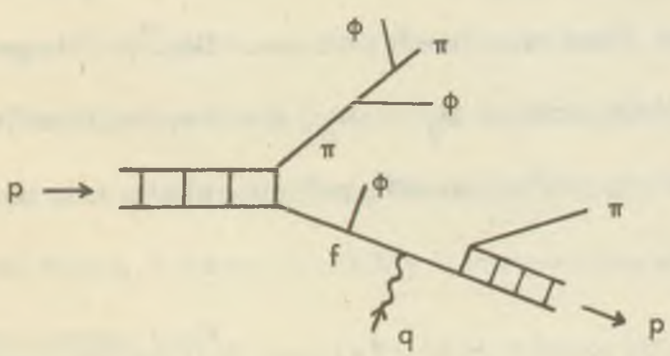


Because of the convergence property of the wave function $\Phi(k)$, one derives the desired scaling property.

The ladder approximation is not necessary, since the inclusion of any crossed diagrams such as



leads only to additional convergent integrals. The asymptotic behavior of $\Phi(k) \sim \frac{1}{k^2}$ is therefore unaltered. All the above conclusions on the elastic form factors and the deep-inelastic scaling property remain valid if we include (at least iteratively) all crossed diagrams in the description of the bound state p. The only diagrams that may lead to non-scaling results are diagrams in which hard mesons (i.e., mesons with large transverse momentum) are emitted with a large probability, such as



To limit the probability of such hard meson emission we insist that

$$\epsilon \equiv \frac{f^2}{4\pi} \quad \text{should be small} , \tag{17}$$

say $\lesssim 0(10^{-1})$. This condition is certainly compatible with the coupling constant constraint (16) for the bound-state description. The details of the model are given in Ref. 14. The following is a summary of the main features of this model:

1. Scaling holds (including all diagrams) for deep inelastic ep and en reactions, provided one neglects high order electromagnetic corrections and provided the laboratory virtual photon energy ν, in units of GeV, is larger than $0(1)$ but less than $0(e^{1/\epsilon})$. Since ϵ is a free (though non-zero) parameter, one may approach the light cone as nearly as possible by taking the limit $\epsilon \to 0+$.

2. Because of (17), as $\epsilon \to 0+$, there are only soft meson emissions; the transverse momentum distribution of the mesons is $\sim k_\perp^{-6} \, d^2 k_\perp$ at large transverse momentum $k_\perp$.

3. As the scaling variable $x \equiv \omega^{-1} \to 1$, both W_1 and νW_2 approach $(1-x)^3$. This is rather encouraging, since it is in good agreement with the present experimental result, unlike most spin $\frac{1}{2}$ parton models[16] with an ad hoc transverse momentum cut-off which lead naturally to a linear $(1-x)$ dependence as $x \to 1$.

4. As mentioned before, the elastic form factors are $\sim 0(q^{-4})$ at large q^2. In this model, all masses are of the order of m_π or m_p; this then "explains" why the physical scale for deep inelastic scaling, as well as that for elastic form factors, are both of the order of 1 GeV.

The idea that p is the bound state of $(\psi\pi)$ and ψ is the interpolating field of the $(p\pi)$ continuum differs from the bootstrap idea in a fundamental way. Here, ψ is a local field; its interactions with the photon and with intermediate bosons (if they exist) must be of local character. If one wishes, one may also view this bound-state description as a Lorentz-invariant, gauge-invariant formulation of the parton model, in which the usual "point-like" assumption for the electromagnetic vertex of the constituent emerges simply as the standard minimal electromagnetic interaction in a local field theory.

5. Remarks

Assuming that the bound-state concept is correct, the theoretical basis of scaling can therefore be understood at least qualitatively. While many of the details remain to be worked out, for most of the experimental applications, as we have discussed, all that is necessary is simply to use dimensional analysis, and that, after a little while, could become quite dull. Fortunately, there are good reasons to believe that the scaling hypothesis may not be an exact law of nature in the extremely high-energy limit. Besides the problem of mass singularities, mentioned earlier, we shall show that there must exist a new basic high energy scale, hitherto undiscovered. Since scaling means the absence of a basic physical energy scale, the presence of such a new basic energy scale therefore means the breakdown of scaling. It is well-known that at a center-of-mass energy higher than 300 GeV, the present Fermi theory of weak interactions would violate its unitarity limit. A new scale may also set in due to strong interactions, e.g., if real quarks do exist. For the Fermi theory, a natural possibility is to regard the basic scale to be given by the Fermi constant itself,

$$G^{-\frac{1}{2}} \sim 300 \text{ GeV} .$$

On the other hand, if one assumes the weak interaction is governed by the same dimensionless constant α as that in the electromagnetic interaction, then the relevant scale can be much lower,

$$(\alpha/G)^{\frac{1}{2}} \sim 30 \text{ GeV} .$$

[A more careful consideration may lead[17, 18] to 37.3 GeV, or higher values[19].]

Throughout the study of microscopic physics, new frontiers are opened whenever a new basic energy scale is reached. We have the case of atomic and molecular physics with the electron-volt energy scale, nuclear physics with the MeV energy scale and the present strong-interaction physics with the GeV energy scale. In each case, at the energy scale of interest, we encounter a vastly rich structure of multiple energy levels and detailed dynamics; yet, when viewed against a much larger energy scale, this superstructure simply dissolves into the continuum. The recent discovery of the scaling property strongly indicates that we are now again in a transition region: The familiar GeV scale is no longer significant, but the still higher new high-energy scale is, as yet, unreached. While scaling is important, the future discovery of its violation should be of even greater significance.

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7. For simplicity, let us assume that in the infinite momentum frame the 3-momentum $\mathbf{k}$ of the parton is parallel to the 3-momentum $\mathbf{p}$ of the physical proton; i.e., in the frame $\mathbf{p} \to \infty$, $\mathbf{k} = x\mathbf{p}$. Therefore, one may write for the 4-momentum $k_\lambda = x p_\lambda + 0(|\mathbf{p}|^{-1})$. In the evaluation of the νW_2 function, in order to replace $\delta(q^2 + 2k \cdot q)$ by $\delta(q^2 + 2xp \cdot q)$, one must have either the time component q_o of the virtual photon to remain finite as $\mathbf{p} \to \infty$, or the differenc $k_\lambda - x p_\lambda$ to be zero identically $[\text{not just } 0(|\mathbf{p}|^{-1})]$. The former restricts the direction $\mathbf{p}$ of the infinite momentum frame, and the latter implies that the parton has no strong interaction and, furthermore, it is of mass $x m_p$ where m_p denotes the proton mass.

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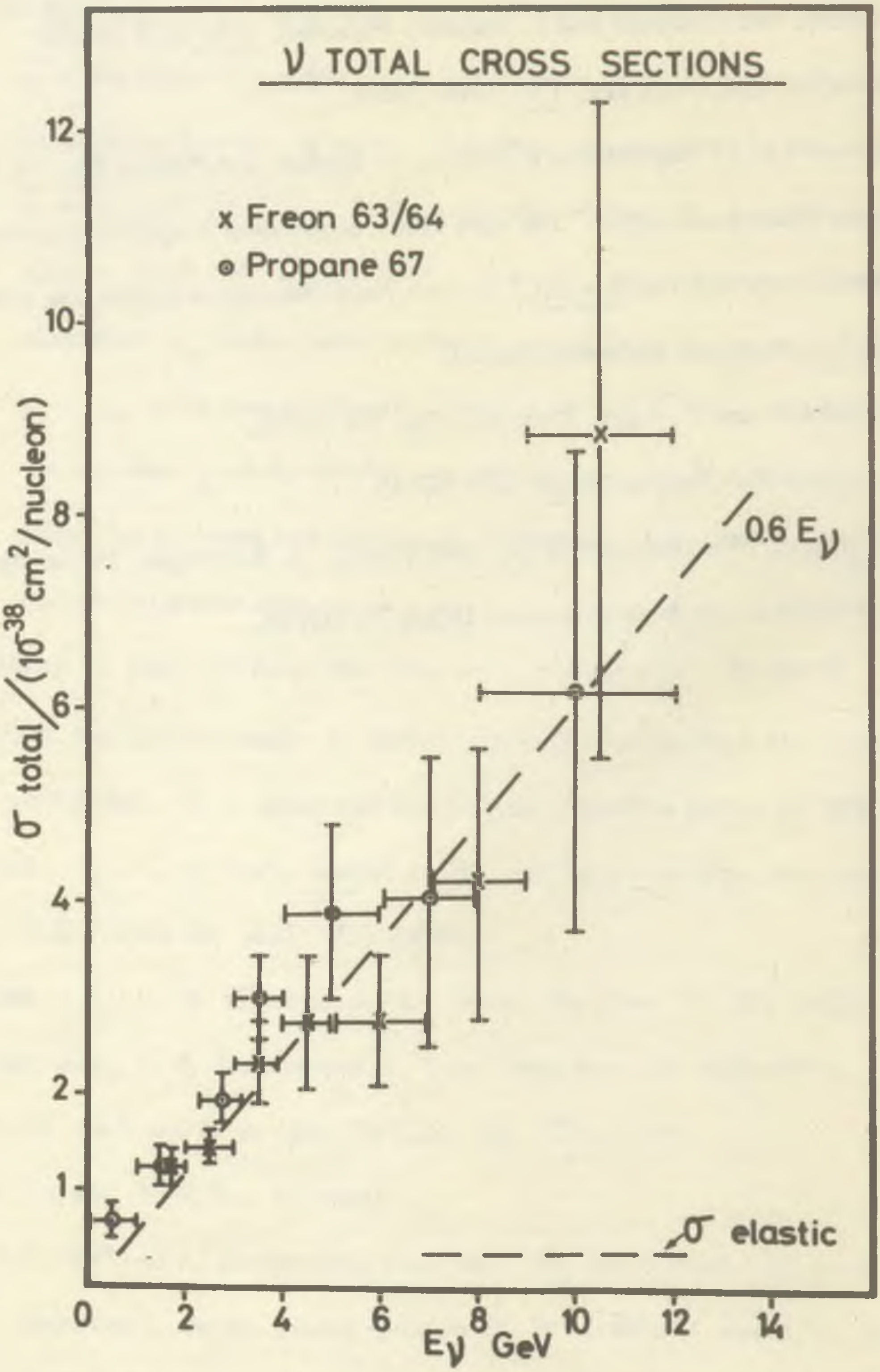

Figure 1. Experimental data from CERN on total neutrino cross sections as a function of laboratory neutrino energy (Ref. 4).

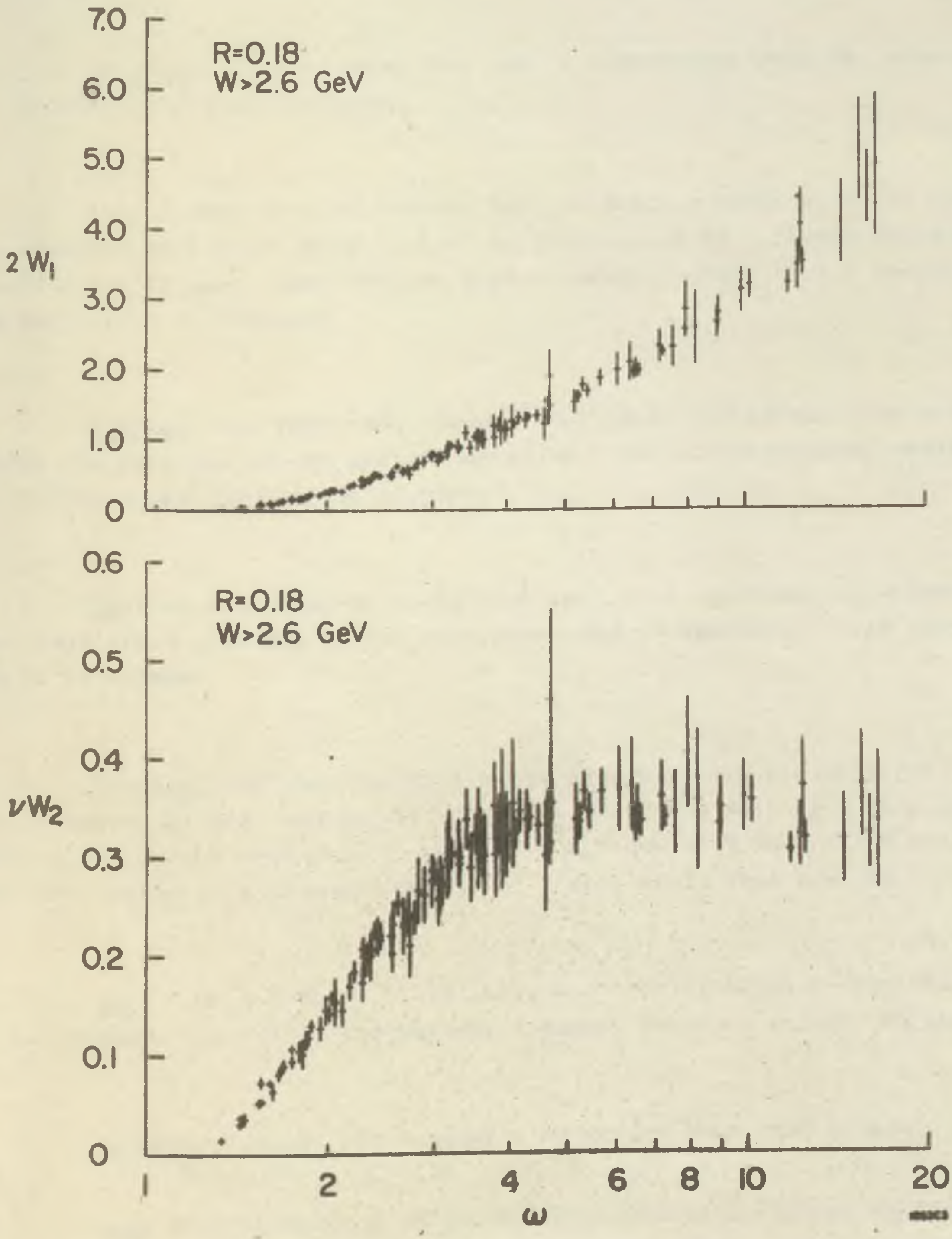

Figure 2. Experimental data from SLAC on structure functions in deep inelastic ep scattering (Ref. 5).

DISCUSSION

Kögerler: In your model you have a dimensional coupling constant G. Doesn't it disturb scaling?

Lee: G does have dimension, but the scaling applies to the remaining hadronic part only. This kind of application is of a phenomenological nature only. It must break down at higher energy, namely at the energy of the scale $1/\sqrt{G}$ in distance.

Frenkel: Can your model assure that after the interaction with the photon the physical proton doesn't break up into particles corresponding to the fields which compose the proton?

Lee: In this type of field theories it is necessary to assume that the local field ψ belongs to the continuum, and no stable particle corresponding to it exists.

Telegdi: Are you expecting a new energy range, and what you are really saying is that scaling is wonderful but let's wait till it's broken? That is really the next step in physics O.K.? But if I understand correctly the ISR results at a thousand GeV^2 c.m. energy still show scaling.

Lee: I do not think it is known what really happens with scaling at the highest ISR energy and perhaps somebody wants to comment on that.

Telegdi: I think the inclusive reactions show good scaling.

Lee: Oh, the scaling in strong interactions is rather different from the scaling as we have discussed in the electromagnetic and weak interactions. I think the analysis is not the same; but Dick, may be you would like to comment...

Feynman: I dont know how to comment because I dont know what you have added to the idea that many people have to separate strong interaction from weak interactions. For example in all of the discussion that we usually make of this leptic hadron interaction we separate the piece that has to do with the weak interaction from the strong interaction. Now let's suppose we can still do that. Then the 37 GeV that you were talking about might be a failure of scaling at the weak interaction end, and it could still be that the strong interaction system, if it could be separated, would scale ad infinitum and that the difficulty is only in the weak interactions or the electrodynamics. So I'd like to ask whether you feel that the weak interaction scaling failure implies a failure of the scaling of strong interaction?

Lee: I think it probably does imply a failure. Not immediately, but in the distant future such a failure is almost inevitable because the separation between the weak, electromagnetic and strong interactions is only of a temporary nature. Let us consider reactions where all q^2's are very, very large. Theoretically the separation would be difficult because it can be made only after one is able to calculate everything. Experimentally it would be difficult because hadrons produced with very large transverse momenta may be due to strong or electromagnetic interactions or to a mixture of the two. Thus I think a 30 GeV c.m.s. energy we may still separate these interactions, but at 300 GeV it would be much harder.

Marshak : Rough calculations due to several groups have indicated that a critical length in the weak interaction may correspond to 10 GeV rather than to 37 GeV. If it turned out that the mass of the W boson were down around 10 GeV would this alter your statement?

Lee: No, I think I would be delighted. I think most physicists would be very happy and certainly the goverments would be even happier.

Moffat: Is there a possibility to establish the critical length just working with the weak interactions?

Lee: This of course depends on when does the weak interaction length come in. If it comes in around 300 GeV, then the separation is nearly impossible because it reaches the unitarity limit. If it sets in at 37 GeV then it probably may be carried out; this will be just like with the electromagnetic interaction. If it comes in at your 10 GeV then I think the weak interaction is a very different entity.

Bell: I would like to clarify the role played by the bound state of the proton. If you were to work out deep inelastic scattering on your pion would it show a scaling regime?

Lee: The answer is yes. While the physical proton must be a bound state because it has spin 1/2, the physical pion may be described either by a canonical field as in the model discussed here, or as a bound state, but in the latter case you have to redo the whole game one step further.

Bell: Does the Drell-Levy-Yan model, where the proton is not a bound state, show a scaling regime?

Lee: No. If you assume that there is a single local field for the physical proton, you will run into difficulties. The electromagnetic form factor will not go down like q^{-4}. Scaling will not be true. Conversely, the q^{-4} behavior of the form factor is a strong argument for a compound proton, as Zachariasen and Amati stated before, and the scaling in deep-inelastic scattering indicates again that the proton cannot be described by a single canonical field.

Radicati: Two of the remarkable features of the quark model are that the three quarks predict the baryon spectrum so well and that they haven't been seen. Now how does your proton fit into SU/3/ because it consists of a ψ and a π ?

Lee: The introduction of SU/3/ into the model presents no difficulty. This, however, does not necessarily imply the existence of quarks. The present evidence on the mass spectrum, due to Gell-Mann, is of course a very strong argument for the validity of SU/3/. The experimental evidence on sum rules also supports the validity of SU/3/, but only indirectly does it suggest the possible existence of the quarks, since these are connected only with low--energy phenomena of the order of a few GeV. To prove the existence of quarks we have to create them, and we have to go to very high energies.

Filippov: I have two remarks. The first one is that in nonrenormalizable models /e.g. in a model with the exchange of ρ-mesons/ a powerlike asymptotic behaviour for formactors is possible. This was shown some time by ago B.Arbuzov and myself for the model in which ρ-meson was considered as "bound state" of π-mesons. The crucial point is vanishing of the bound state wave function at the origin and this is quite possible for nonrenormalizable interaction as well as for renormalizable one. The second remark is that the applications of the dimensional analysis mentioned by professor Lee were investigated in details by the Dubna group and reviewed in the paper by Matveev, Muradian and Tavkhelidze. /JINR preprint E2-6036, Dubna 1971/

PRELIMINARY RESULTS ON THE RATIO OF ANTINEUTRINO TO NEUTRINO TOTAL CORSS-SECTIONS

Aachen, Brussels, CERN, Paris (E.P.), Milan, Orsay, London (UCL) Collaboration (presented by B. Degrange, LPNHE, Ecole Polytechnique Paris).

We present here the very preliminary results of the CERN 1971 neutrino experiment, using the large heavy liquid bubble chamber "Gargamelle", exposed to the CERN neutrino beam. 500 000 photographs, equally divided among neutrino and antineutrino exposures, are being analyzed in the laboratories at Aachen, Brussels, CERN, Paris (E.P.), Milan, Orsay and London (UCL

I.- EXPERIMENTAL CONDITIONS

a) Rate of ν and $\bar{\nu}$ events

Table 1 and table 2 show the comparison between the present experiment and the previous ν and $\bar{\nu}$ experiments using the 1.2 m heavy liquid bubble chamber at CERN (1). One can see that the total statistics expected in the 1971 runs will be about 10 times the previous available statistics of ν events, and 30 times the previous statistics of $\bar{\nu}$ events. This is due, first of all, to the large visible volume of "Gargamelle" (about 7 m^3), which we have however restricted to a fiducial volume of 3.14 m^3 for neutrino interaction vertices, in order to avoid measurement problems.

The remaining factor in the increase of the statistics is due to a higher intensity of the CERN proton synchrotron and to improvements in the neutrino beam.

The preliminary results presented here are based on the analysis of 90% of the antineutrino film and only 3% of the neutrino film.

b) The neutrino beam

The neutrino and antineutrino energy spectra allow cross-section measurements between 1 GeV and about 8 GeV. Under 1 GeV, the neutrino flux is badly known due to large uncertainties in the meson production spectrum ; neutrinos from pion decays contribute essentially under 5 GeV, whereas neutrinos from kaon decays contribute at higher energies.

The neutrino (or antineutrino) flux is calculated by fitting the measured muon flux in the shielding to the results of a Monte Carlo program using as input the production spectra of pions and kaons measured experimentally at the proton energy of 24 GeV, and extrapolated at 26 GeV, and the currents in the focusing horns.

Our present result only concerns the ratio of anti-neutrino to neutrino cross sections ; the errors on the neutrino and antineutrino flux normalizations are thus minimized.

c) Facilities offered by "Gargamelle".

"Gargamelle" is a cylindrical chamber, with a length of 4.8 m and a diameter of 2 m. The visible volume of 7 m^3 allows a good identification of the muons, since pions have a high probability of interacting before leaving the chamber ; (the pion interaction length, about 60 cm, has to be compared to the average potential length of 1.5 m).

Another advantage is the detection of neutral pions. The γ-rays from π° decays are most of the time converted into electron pairs inside the visible volume, due to the short radiation length of CF_3Br ($X_\circ$ = 11 cm). The loss in γ-rays is thus small and has a negligible effect when one compares the energies of missing γ-rays and the total visible energy of the neutrino or antineutrino event.

Moreover, neutrons produced by a neutrino or antineutrino interaction are often detected by the observation of a neutral star in the visible volume.

II.- ANALYSIS OF EVENTS

a) Selection of ν and $\bar{\nu}$ candidates

Events having at least one muon candidate (i.e. having a track, leaving the chamber or decaying in the chamber, or a negative track stopping in the chamber) have been retained as neutrino or antineutrino candidates. They have been measured when the vertex was located in the 3.14 m^3 fiducial volume. We have requested that the total visible momentum in the beam direction, p_x, be greater than 0.6 GeV/c in order to remove the interactions of incoming pions in the chamber simulating a neutrino (or antineutrino) event. Candidates have been retained only if their total visible energy was greater than 1 GeV since the flux is unknown at lower energies. Moreover this cut removes most of the background of neutron interactions.

The accuracy on the total visible energy is of the order of 10 %, and practically always better than 30 %.

b) Analysis of a part of the neutrino film

This preliminary result is based on the analysis of 11000 photographs only which leads to a sample of 214 events satisfying the cuts defined previously. This sample is practically free of background of neutral stars and antineutrino events. Although the statistics are still poor, the results shown in figure 1 are compatible with the cross-section $\sigma(\nu) = (0.8 + 0.2)\ 10^{-38} cm^2 x E_{GeV}$ par nucleon obtained in the previous CERN bubble chamber experiments [(2)].

c) Analysis of the antineutrino film

About 90% of the antineutrino film has been scanned and measured.

A first correction has to be applied, since the scanning efficiency for the antineutrino elastic interactions ($\bar{\nu} + p \rightarrow \mu^+ + n$) is lower than the efficiency for other interactions, due to the special topology of these events, (one single muon), which can easily remain unnoticed by the scanner.

Another correction is related to the background of neutrino events in the antineutrino film which is bigger than the background of antineutrino events in the neutrino film.

In order to estimate the neutrino contamination in events having both a μ^+ and a μ^- candidate, (called ambiguous events), we have used the fact that ambiguous events in the neutrino film are practically all actual neutrino events ; the probability for a neutrino event to be ambigous can thus be estimated in several energy regions ; on the basis of these data from the neutrino film, and of the number of unambiguous neutrino events in the antineutrino film, we find that the background is of the order of 2% under 4 GeV, and of the order of 10% at higher energy.

III.- RESULTS

Figure 2 shows the values found for the ratio R of antineutrino to neutrino cross-sections in five energy regions from 1 GeV to 6 GeV ; the loss of elastic antineutrino events at the scanning stage has been accounted for . One can conclude that this ratio is surely less than one.

Various theoretical models [3] predict values of this ratio, extending from 1/3 to 1, at least for high energies. Our result is in contradiction with the diffractive models which predict the value R=1; it is in agreement with the parton model which predicts $1/3 \leq R < 0.89$; it is even compatible with the lower limit $R = \frac{1}{3}$.

In order to show a more precise comparison with the parton model, we follow the notations defined by O. NACHTMANN in a recent theoretical paper [4]. The variables Z and $\bar{Z}$, respectively proportional to the neutrino and antineutrino total cross-sections, averaged on protons and neutrons, are defined by the following formulae :

$$\frac{1}{2}\,(\sigma^{\nu p} + \sigma^{\nu n}) = \frac{G^2}{\pi}\ M.E.Z.$$

$$\frac{1}{2}\,(\sigma^{\bar{\nu} p} + \sigma^{\bar{\nu} n}) = \frac{G^2}{\pi}\ M.E.\bar{Z}.$$

where G is the Fermi constant, M the nucleon mass, and E the neutrino (or antineutrino) energy.

Using the data from the SLAC electroproduction experiment [5], NACHTMANN shows that Z and $\bar{Z}$ should obey several inequalities in the framework of the parton model. The allowed region in the (Z, $\bar{Z}$) plane is inside the contour shown in figure 3. In the same figure, we have indicated the previous CERN result [2] on the neutrino total cross-section and our present result on the ratio $R = \frac{\bar{Z}}{Z}$. In order to reduce the contribution of elastic events we have only used here our data at energies greater than 3 GeV, which leads to the result :

$$R = 0.42 \pm 0.08$$

One can see that the experimental data, available at the present time, favour a restricted area in the (Z, $\bar{Z}$) plane, compatible with the constraints of the parton model. The complete analysis of our neutrino film will allow to define this area with a higher accuracy.

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FIGURE CAPTIONS

Figure 1 : Preliminary results on ν total cross-section using only 214 events. Comparison with the value obtained in the CERN 1967 experiment.

Figure 2 : Antineutrino to neutrino cross-section ratio.

Figure 3 : Comparison with the parton model ; the dotted lines indicate the one standard deviation interval for experimental results.

	NUMBER OF PULSES	ν-DETEC^N VOLUME	CHAMBER LIQUID (DENSITY)	NUMBER OF PROTONS/ PULSE	PROTON BEAM ENERGY	TOTAL NUMBER OF ν INTERACTIONS	REFERENCE
CERN ν 63-64	1.28×10^6	$0.22\ m^3$	CF_3Br (1.5)	$\sim 0.6 \times 10^{12}$	24.5 GeV	~ 400	1(a)
CERN ν 67	1.08×10^6	$0.51\ m^3$	C_3H_8 (0.4)	$\sim 0.6 \times 10^{12}$	20.6 GeV	~ 400	1 (c) 2
CERN ν 71	0.25×10^6	$3.14\ m^3$	CF_3Br (1.5)	$\sim 1.2 \times 10^{12}$	26 GeV	~ 7000 (expected)	this experiment

TABLE 1 : CERN heavy liquid chamber ν experiments

	NUMBER OF PULSES	ν-DETEC^N VOLUME	CHAMBER LIQUID (DENSITY)	NUMBER OF PROTONS/ PULSE	PROTON BEAM ENERGY	TOTAL NUMBER OF $\overline{\nu}$ INTERACTIONS	REFERENCE
CERN $\overline{\nu}$ 63-65	0.38×10^6	$0.64\ m^3$	CF_3Br (1.5)	$\sim 0.6 \times 10^{12}$	22 and 24.5 GeV	34	1 (b)
CERN $\overline{\nu}$ 71	0.25×10^6	$3.14\ m^3$	CF_3Br (1.5)	$\sim 1.2 \times 10^{12}$	26 GeV	~ 1000	this experiment

TABLE 2 : CERN heavy liquid chamber $\overline{\nu}$ experiments.

FIGURE 1

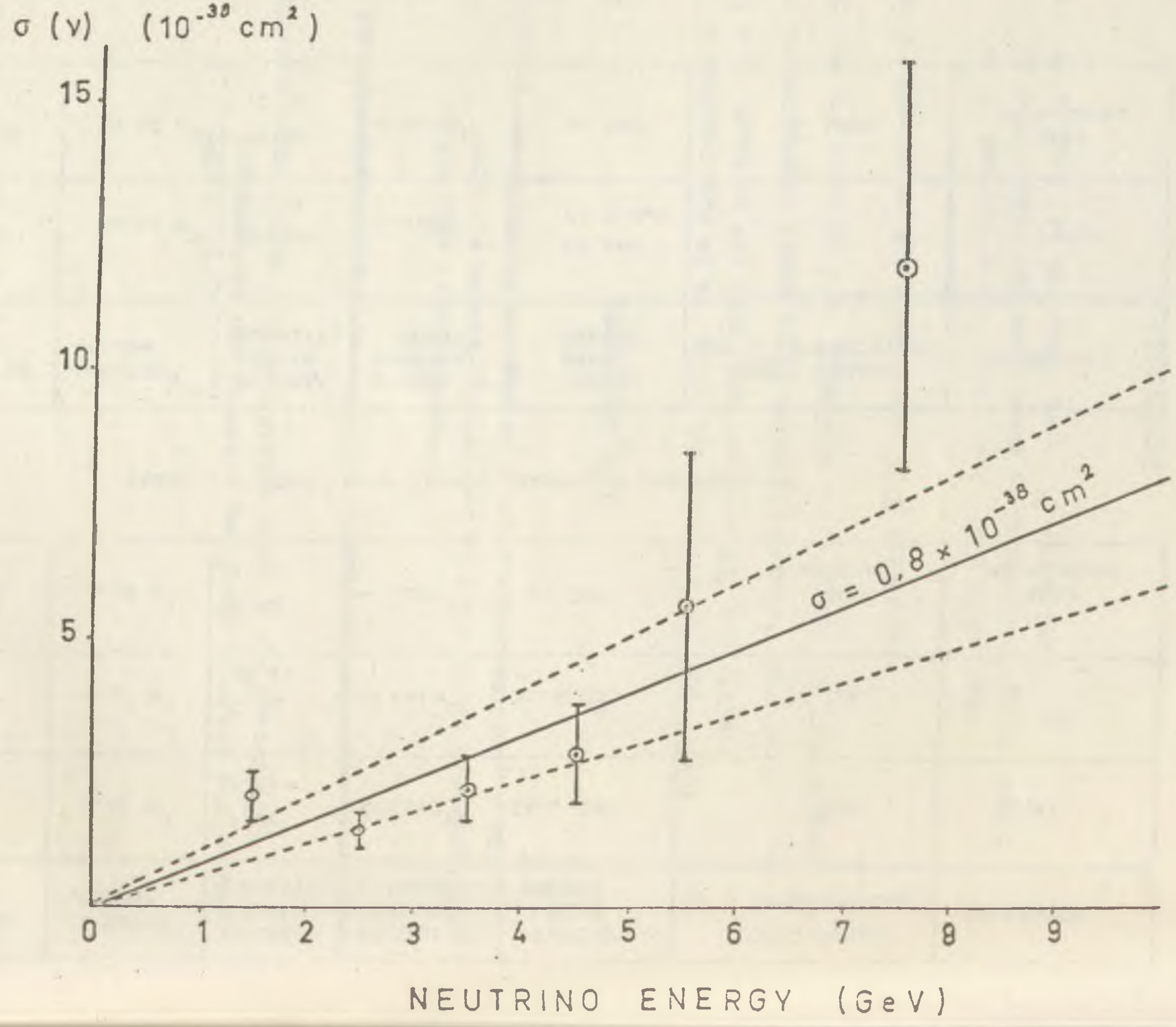

σ (ν) (10^-38 cm²)
15
10
5
0
1
2
3
4
5
6
7
8
9
σ = 0,8 × 10^-38 cm²
NEUTRINO ENERGY (GeV)

FIGURE 2

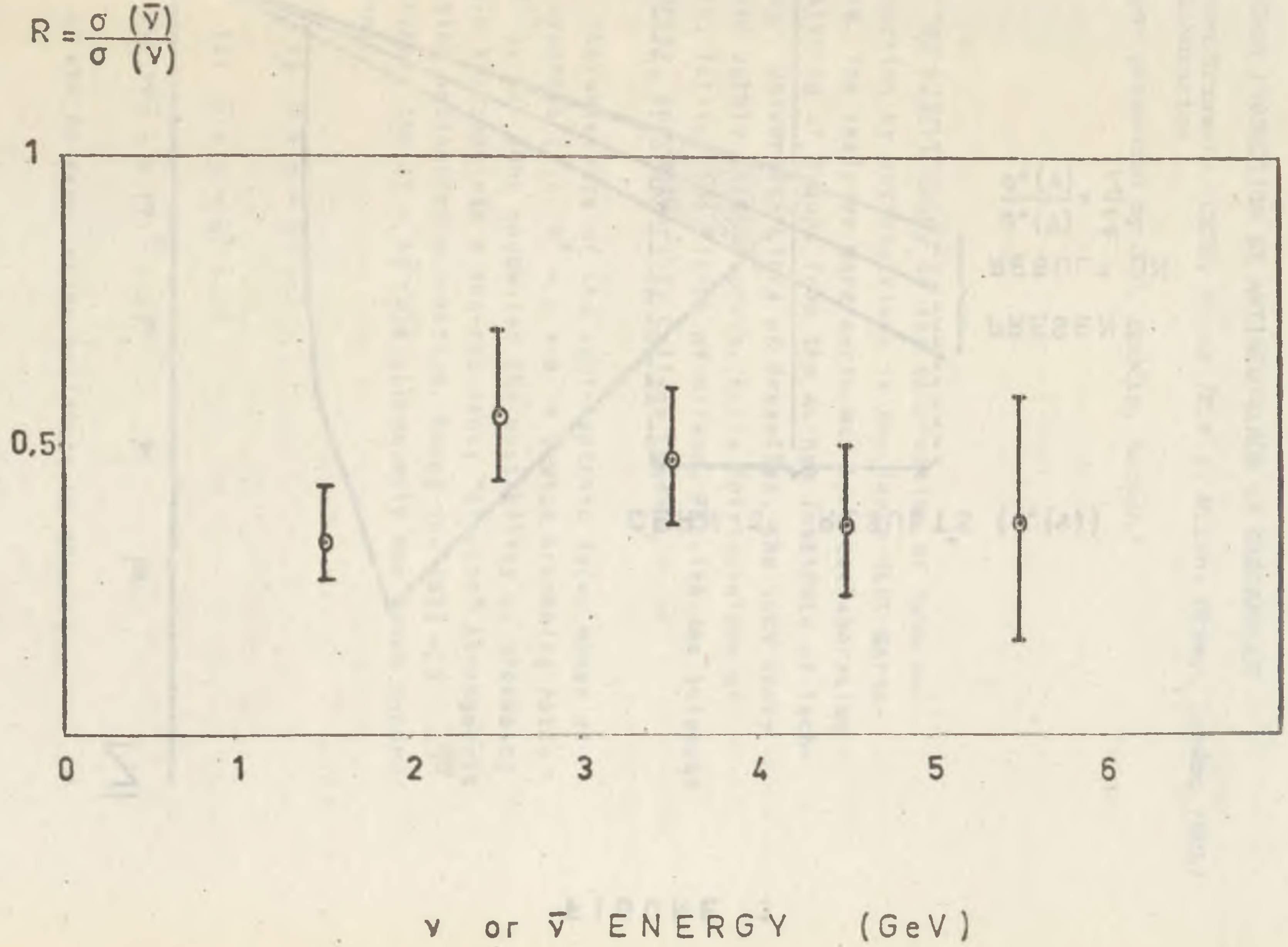
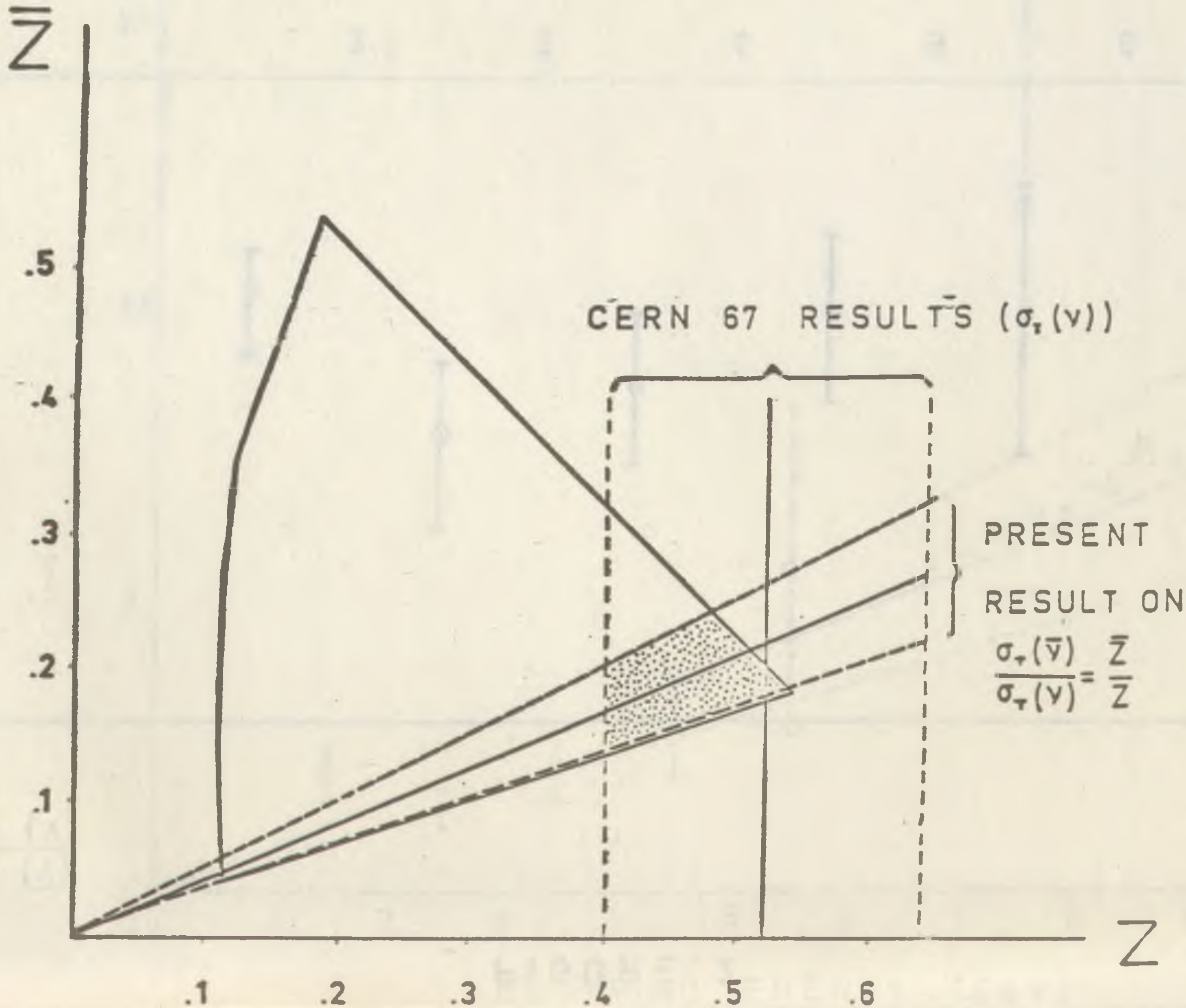


FIGURE 3



$\bar{Z}$
.5
.4
.3
.2
.1
CERN 67 RESULTS ($\sigma_T(\nu)$)
PRESENT
RESULT ON
$\frac{\sigma_T(\bar{\nu})}{\sigma_T(\nu)} = \frac{\bar{Z}}{Z}$
Z
.1
.2
.3
.4
.5
.6

HYPERON PRODUCTION BY ANTINEUTRINOS IN GARGAMELLE

Aachen,Brussels, CERN, Paris /E.P./, Milan, Orsay, London /UCL/ Collaboration

/Paper presented by J.G. Morfin, Aachen/

We report today on the observation of hyperon production by antineutrinos in the large HLBC Gargamelle. The analyses were performed by a collaboration consisting of groups from the Aachen Institute of Technology, Université Libre of Bruxelles, the CERN heavy liquid bubble chamber group, Ecole Polytechnique of Paris, Istituto di Fisica of Milano, Faculté des Sciences of Orsay, and University College London.

The existence of the semi-leptonic decay modes of the hyperons (i.e. $\Lambda^0 \rightarrow p + \mu^- + \bar{\nu}$ with branching ratio $\sim 10^{-4}$), has long suggested the possibility of producing single hyperons via a non-resonant, "elastic" strangeness changing antineutrino reaction. Among the $|\Delta Y| = 1$ reactions, the $\Delta Y = \Delta Q$ rule allows only the three interactions:

i) $\bar{\nu} + p \rightarrow \mu^+ + \Lambda$

ii) $\bar{\nu} + p \rightarrow \mu^+ + \Sigma^0$

iii) $\bar{\nu} + n \rightarrow \mu^+ + \Sigma^-$

All, as can be seen, using antineutrino primaries.

All neutrino induced hyperon production is prohibited by $\Delta Y = \Delta Q$. Furthermore, should the $\Delta Y = -\Delta Q$ transition be at all permitted, there is still only one quasi-elastic interaction of neutrinos off nucleon targets possible:

$$\text{iv)} \quad \nu + n \rightarrow \mu^- + \Sigma^+$$

It was not until an attempt was made to predict the decay rates of the $|\Delta Y| = 1$ processes that the original V-A model [1,2] encountered any major difficulties. While treating the $\Delta Y = 0$ decays, the discrepancies between the predictions of the theory and the experimental results could easily be explained by strong-interaction effects. However, for the $|\Delta Y| = 1$ reactions, the observed decay rates were always at least an order of magnitude lower [3] and as much as a factor of 50 lower for the Σ^- leptonic decays.

At this point Cabibbo introduced his formulation of the universal current-current interaction based on a unified SU_3 treatment of both the $|\Delta Y| = 1$ and $\Delta Y = 0$ hadron currents. This theory was first applied by Cabibbo and Chilton [4] and M.M. Block [5] to antineutrino reactions during the course of the first CERN $\nu/\bar{\nu}$ experiment [6]. The general expression for the vector and axial-vector matrix elements involves six form factors all of which are functions of q^2 and, assuming T-invariance, real. For the treatment of the hyperon decays the small four-momenta transfers involved (the highest q^2 is for Σ^- decay and is only $\sim .06\ GeV^2$) allowed one to use the $q^2 = 0$ approximation and reduced the number of form factors to two; one vector and one axial-vector factor. For the study of antineutrino produced hyperons, on the other hand, considerable q^2 values are obtainable and, of the sample we presently report on, 80 % have q^2

higher than the upper limit for decays. We must therefore examine the q^2 dependence of the three vector and three axial-vector form factors.

The initial questions in determining the q^2 dependence of the form factors were whether a monopole or dipole form more closely agreed with the experimental results and what mass value should the pole itself have.

The general form then is:

$$F(q^2) = (1 + q^2/M^2)^{-n}$$

where n is one or two. Fig. 1 demonstrates the effect of the parameters M & n on the cross section for Λ production [9]. As is evident, the values derived from the single pole model are uniformly higher than those of the double pole model. Furthermore, the monopole model predicts a much more rapid rise with $\bar{\nu}$ energy than the dipole model. The results of the last $\bar{\nu}$-experiment tend to support the dipole model and, in light of this, table 1 presents the expected average cross section, using the CERN antineutrino spectrum and the dipole formula, as a function of the mass M.

This experiment is the first carried out in the large heavy liquid bubble chamber Gargamelle [7]. The chamber and the beam have been fully described earlier in this conference so we only repeat the essential characteristics that Gargamelle is a cylinder 4.8 m long and 1.85 m in diameter with a visible volume of 7 m^3 and a magnetic field of 20 kg. Heavy freon CF_3Br with radiation length 11 cm was used as the target giving γ-ray detection efficiency greater than 90 % and facilitating the identification of muon secondaries.

The beam used 26 GeV primary protons and the chamber was exposed to the antineutrino beam for 250,000 pictures.

The scan rules were such that all events were accepted which had a non-interacting positive track -- μ^+ candidate -- and at least 1 "Vee" configuration associated with the primary vertex. After double scanning all film a total of 30 events were found to exhibit possible hyperon production. Of these 30, two are candidates for single Σ^- production, but because of the difficulties in recognition and unambiguous identification we have chosen not to consider this category at present. The remaining 28 events can be distributed in the following four categories: .

a) 10 events with $\mu^+\Lambda$ ($\Lambda \to p\,\pi^-$)

b) 8 events with $\mu^+\Lambda$ + π's, p's etc...

c) 2 events with $\mu^+\Sigma^0$ ($\Sigma^0 \to \gamma\Lambda$)

d) 8 events associated production

The ten type a) events are clear candidates for quasi-elastic Λ production. Of the eight type b) events, initially candidates for Y^* production, three have only a very low energy ($T < 30$ MeV) nucleon in addition to the Λ and can reasonably be included with the type a) events. The third category, weak production of the Σ^0, emphasizes the benefit of doing this experiment in heavy liquid since we were able to detect the γ and reconstruct the Σ^0. The eight associated production events were, naturally, some combination of K^+ or K^0 with an $S = -1$ state such as Λ, $\overline{K^0}$ or K^-.

Of our sample then, 13 events are possible candidates for single Λ production. It is this category that we concentrate on today.

There are several factors that we must consider before we may arrive at a cross section with which to compare the theory. To get the actual number of events among the observed candidates we examine possible background sources. Next, to obtain the production rate in freon from these observed events involves the application of several experimental corrections. And finally to obtain the cross section with respect to a single nucleon involves examining nuclear effects.

The purging of background events, our first step, leads us to examine two possible sources. The first is associated production of ΛK° where the K° is not detected. Only two of our eight associated production events consist of a Λ and K° and of these one had pions and would not have been confused with quasi-elastic hyperon production if the K° had not been detected. If one takes into account the long lived K° and the neutral K_S decays we can estimate the background from associated production to be 1.5 ± 1.5 events.

The other possible background source is neutron stars having two short tracks in a Vee configuration pointing toward the interaction vertex. These can only contribute since the Λ's are mostly low momentum ($P^{\Lambda}_{avg.} \sim 0.5$ GeV) and the track lengths of the decay products are usually quite short. If such a neutron star background exists it would simulate a particle with decay length equal to the interaction length in CF_3Br which is ~ 60 cm, whereas the true Λ's within our sample would have decay lengths of only several centimeters. One would also expect a broad invariant mass distribution from neutron stars whereas true Λ's should cluster at the Λ mass. Fig. 2 shows a scatter plot of momentum versus decay-length for the quasi-elastic Λ sample. The decay length distribution is certainly normal within statistics, and we may safely conclude that the background due to neutron stars is much less than one event out of the 13 original candidates. Thus, after background considerations we are left with 11.5 ± 1.5 events.

We now have the corrected number of <u>observed</u> events and to get the actual production rate in freon, our next step, we must apply three further corrections. These may be tabulated as follows:

1) Since the neutral decay of the Λ involves γ's which might be confused with π^0's produced at the origin, we have only counted those Λ's that decay via the charged mode. We must therefore apply a correction factor of 1.5 to account for these neutral decays.

2) To avoid, at this point, the possibly inaccurate task of determining the invariant mass of all $\pi^- p$ combinations coming from the main origin we have demanded a minimum decay length. The loss of these very short decay length events gives a factor of 1.2.

3) To correct for scanning loss we apply a correction of $1.1^{+0.5}_{-0.1}$.

The final corrected number of $\mu^+\Lambda$ events produced in freon is 23^{+12}_{-5} . If we now take the calculated $\bar{\nu}$ flux passing through the visible volume of Gargamelle we obtain an observed cross section for $\mu^+\Lambda$ production in freon, averaged over the spectrum, of

$$\sigma_{\Lambda(\text{freon})} = 1.3^{+.6}_{-.3} \times 10^{-40} \text{ cm}^2/\text{proton}$$

where the error includes a 15% uncertainty in the $\bar{\nu}$ flux.

Thus, we have at this point the cross section for freon, a fairly complex configuration of nuclei, whereas theoretical values of the cross sections are with respect to single nucleons. To get our cross section in terms suitable for comparison to the thoretical predictions we must consider nuclear effects, and for freon this is a non-trivial exercise. Fortunately, we may reduce the number of effects studied to nuclear absorption and Σ to Λ conversion during passage through matter. The earlier $\nu/\bar{\nu}$ experiment used studies by Jastrow[8] and Franzinetti and obtained total absorption values in the vicinity of 40%. Subsequent studies[10] of this problem have resulted in reduced values for this absorption factor and, most recently, Monte Carlo calculations performed at Bruxelles indicate a 15% Λ absorption and a 20% $\Sigma \rightarrow \Lambda$ conversion factor. If we now also take into account the prediction, as shown in Table 1, that the Σ production is approximately 75% of the Λ production, essentially regardless of form factor, then the nuclear effects imply that the cross section on free protons is 1.0 ± 0.5 of the cross section in freon. The large error in this factor certainly accommodates the uncertainty in the nuclear effects.

We conclude then that we have definitely observed single Λ's produced by antineutrinos, inverse Λ decay, and the experimental cross section is

$$\sigma_{\Lambda(P)} = 1.3 {}^{+0.9}_{-0.7} \times 10^{-40} \text{ cm}^2/\text{proton}$$

Again referring to Table 1 we see that although this value of σ favors a mass of .6 GeV in the dipole form factor, the experimental errors make the result not incompatible with the assumption that $M = M_A = M_V = 0.84$ GeV.

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3) R.E. Marshak, Riazuddin and C.P. Ryan, Theory of Weak Interactions in Particle Physics, Wiley-Interscience, p. 390, New York (1969)

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9) E.C.M. Young, CERN Rep. 67-12 (1967). (We have used the cross-sections as calculated in this report which are corrected for the error in sign of Ref. 4)

10) See for example W.L. Knight, F.R. Stannard, F. Oppenheimer, B. Rickey, and R. Wilson, Il Nuovo Cimento 32, 598 (1964)

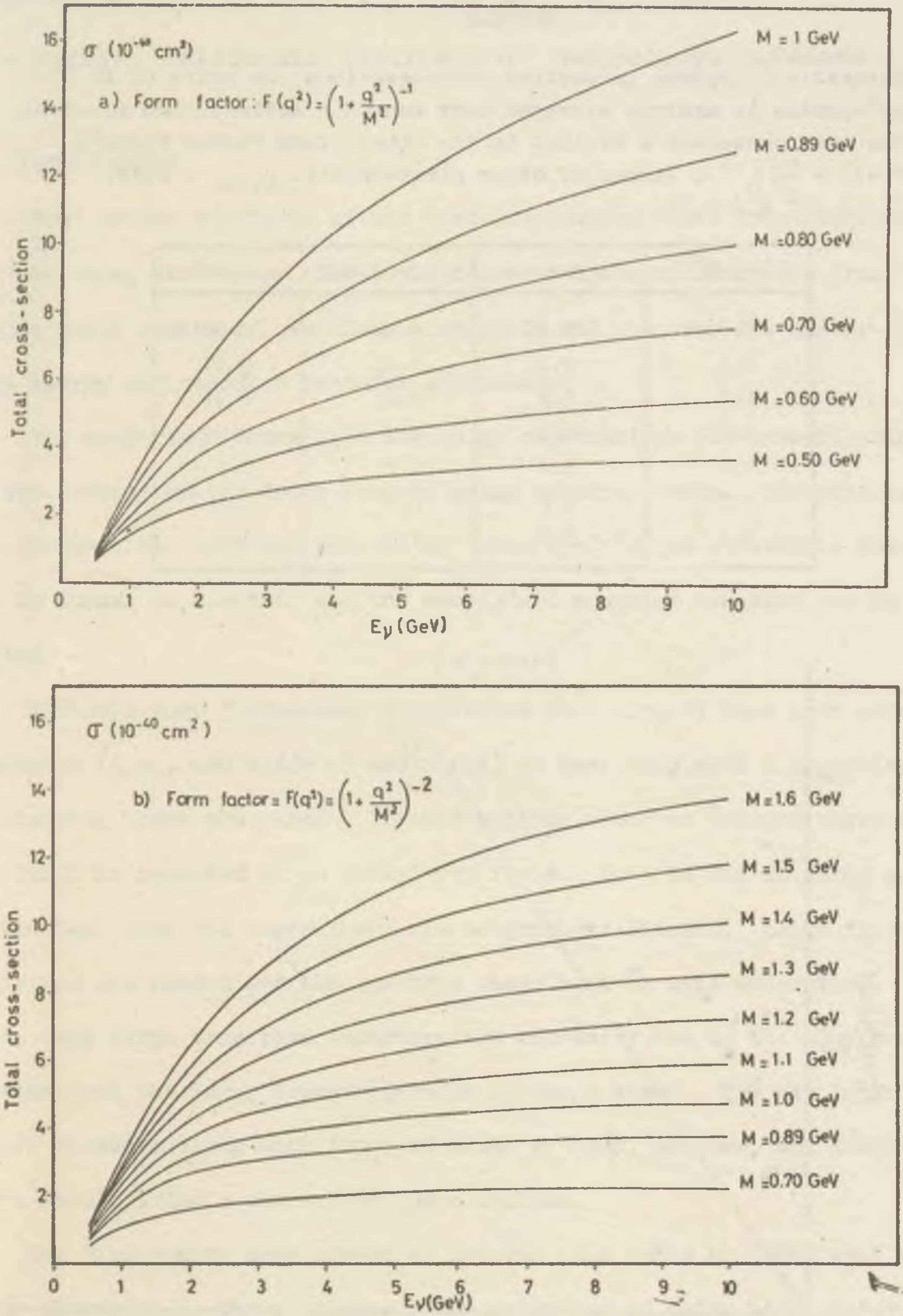


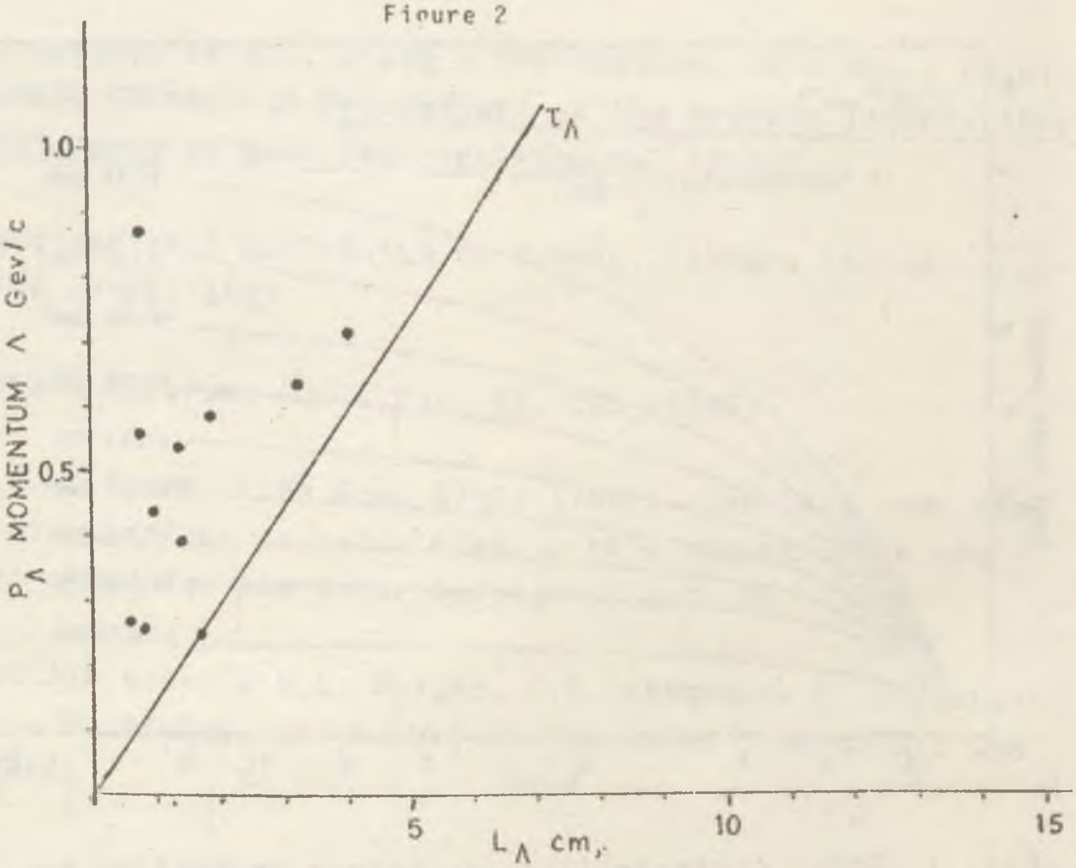
Fig. 1 Total cross-section for Lambda production vs. E

Table 1

Theoretical hyperon production cross-sections, in units of 10^{-40} cm^2/proton or neutron averaged over the CERN antineutrino spectrum. The mass parameter M is that in the dipole form factor formula $F = (1 + \frac{q^2}{M^2})^{-2}$. Values of other parameters: $\theta_{Cabibbo} = 0.24$, $f = 0.45$, $d = 0.78$.

$M(GeV/c^2)$	σ_Λ/proton	σ_{Σ^-}/neutron	σ_{Σ^0}/proton
0.3	0.44	0.19	0.09
0.4	0.66	0.29	0.15
0.6	1.3	0.63	0.31
0.8	2.2	1.1	0.55
1.0	3.1	1.7	0.8
1.2	4.2	2.3	1.15
1.4	5.2	3.0	1.6

Figure 2



MOMENTUM VERSUS DECAY LENGTH FOR Λ's OF $\mu^+ \Lambda$ CANDIDATES

NEUTRINO PHYSICS AT BATAVIA: PROSPECTS AND PROGRESS

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I. INTRODUCTION

Most of our knowledge of the weak interaction comes from measurements of weak decay processes. The limitations in these studies come from the rather small number of reactions accessible and also the low center-of-mass energy and momentum transfer available.

For many years there have been high expectations for investigations of the inverse of the decay process using neutrino beams. The advantages are obvious: the neutrino becomes an "observable"; the accessible reactions are no longer so limited; and the energy and momentum transfer can be varied.

Although some fundamental discoveries have already been made using neutrinos (i.e., two kinds of neutrinos) we have only made a beginning on studying these processes. In most regards neutrino induced physics can still be regarded as an unexplored field. This is due in large part to the fact that the experiments are extremely difficult. Large fluxes of neutrinos are needed and the spectrum shape must be well understood. Also, very large sensitive detectors are necessary due to the tiny cross sections and the large transverse size of the ν-beams. The new large bubble chambers along with improved beams at CERN, Argonne, and Brookhaven should bring a new era to these studies.

The high energy accelerator at Batavia (and later at CERN) will add a new dimension to these studies. The prospects of using high energy neutrinos opens up the possibility of seeing modifications to the simple current x current theory of the weak interaction.

In this theory the effective Lagrangian for decay is written:

$$L = \frac{G}{\sqrt{2}} J_{\mu}^{+} J_{\mu}$$

where the currents J_{μ}^{+} are:

$$J_{\mu} = \bar{\psi}_{\ell}\gamma_{\mu}(1 + \gamma_5)\psi_{\nu_{\ell}}$$

This represents the interaction of four fermions at a single space-time point. The same form is used to describe hadron decays, where the Lagrangian is then the product of a hadron current x lepton current.

This same theory that describes muon decay

$$\mu^{+} \rightarrow e^{+} + \nu_{e} + \bar{\nu}_{\mu}$$

also predicts cross sections for the inverse reaction,

$$\nu_{\mu} + e^{-} \rightarrow \mu^{-} + \nu_{e}$$

which do not make sense at high energy.

In particular, the cross section is

$$\sigma = \frac{G^2}{\pi} \epsilon^2$$

where $G = \frac{10^{-5}}{m_p^2}$ = weak interaction coupling constant and ϵ = total center-of-mass energy. However, we know that for s-wave scattering

$$\sigma \leq \sigma_{max} \simeq \frac{1}{2} \pi ƛ^2$$

This gives the well-known result that $\sigma = \sigma_{max}$ at about $\epsilon \approx 300$ GeV. This means we have a required cutoff to the weak interaction from unitarity

$$\epsilon_c < 300 \text{ GeV}$$

and therefore, the Lagrangian written down for muon decay must break down and is valid only at very low energies.

There are reasons to expect that effects will be observable at somewhat lower energies. For example, second order calculations are divergent and require a cutoff to remain finite. This cutoff energy is another indicator of where modifications might be expected. For example, using second order calculations:

(1) The mass difference $M_{K_L^0} - M_{K_S^0}$ has been calculated by Mohapatra[1] et al. They require a cutoff $\epsilon_c \approx$ 3-4 GeV in order to get the experimental value $\Delta M = 0.36 \times 10^{-11}$ MeV.

(2) The experimental limit on the rate $K_L^0 \rightarrow \mu^+ + \mu^-$ has been measured to be $\Gamma < 1.8\ 10^{-9}\ \Gamma_T$. The rate has been calculated to vary as $\Gamma \sim K\epsilon_c^4$ and gives a cutoff energy $\epsilon_c \leq$ 5-15 GeV.[1]

The difficulties with the theory have prompted various proposed modifications to the theory of weak interaction. A massive Intermediate Vector Boson that mediates the weak interaction and is responsible for the force was proposed many years ago. The existence of such a particle does not in itself solve the problems in the weak interaction. However, its existence would certainly bring us some way in our understanding of the weak force. Prospects for detecting such a particle at NAL will be discussed later in this report.

Various models that might lead to a renormalizable theory have been proposed. Some possible consequences might include existence of heavy leptons, neutral currents, or scaler bosons in addition to the vector boson.

Anyway, it is obvious that it will be extremely important to start obtaining information of the weak interaction at high energies.

In addition to the study of the weak interaction, high energy neutrinos also will allow a study of deep inelastic scattering $\nu + N \rightarrow \mu^- +$ hadrons for very large q^2 (momentum transfer) and ν (energy transfer to nucleon system).

I will discuss in the following sections some aspects of the neutrino program at NAL. I emphasize the use of high energy neutrinos and in particular the most likely early results.

II. PHYSICS

A. Neutrino Scattering

Ideally one would like to study lepton-lepton scattering at high energies.

$$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$$

We have indicated that it is interesting to investigate the scattering process at very high COM energies. The energies given by the second order cutoff energy are of the order of $\epsilon = 10$ GeV, and other estimates are higher.

For E_ν large,

$$E_\nu = \frac{\epsilon^2}{2m_e}$$

and for $\epsilon = 10$ GeV we need $E_\nu = 10^5$ GeV.

We can conclude from this that in order to study the weak interaction at reasonable values of ϵ, we need a heavier target.

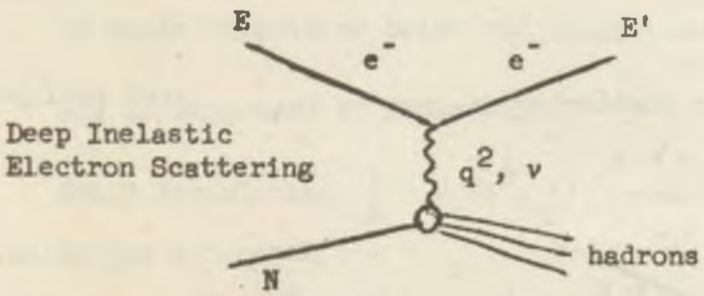
If a nucleon target is used, we have the "elastic reaction"

$$\nu_\mu + n \rightarrow \mu^- + p.$$

The structure of the nucleon damps this cross section at high momentum transfers.

It appears then that the most promising way to study high center-of-mass energies soon will be to study the behavior of the total cross section on nucleons with an eventual goal of understanding the detailed dependence of the inelastic scattering.

The inelastic neutrino scattering problem on nucleons can be written in analogy to the by now familiar deep inelastic electron scattering picture:



Kinematic variables:

$q^2 = 2\,EE'\,(1 - \cos\theta')$

$\nu = E - E'$

q^2 and ν are the four momentum transfer and the energy transfer to the nucleon system.

The cross section for this process can be written in terms of the laboratory variables as:

$$\frac{d^2\sigma}{dE'\,d\cos\theta'} = \frac{8\pi\alpha^2}{q^4}(E')^2\left[W_2^{e}(q^2,\nu)\cos^2\frac{\theta}{2} + 2\,W_1^{e}(q^2,\nu)\sin^2\frac{\theta}{2}\right]$$

where W_1 and W_2 are the two structure function for inelastic scattering. The very interesting result which at least seems to hold at SLAC energies is the principle of Scale Invariance. This is simply that

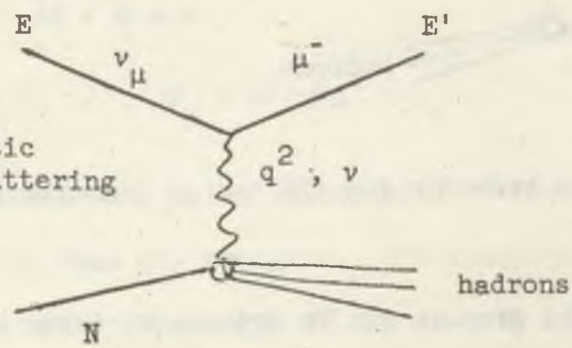
$$\frac{1}{M} \nu W_2(q^2, \nu) \xrightarrow{E \to \infty} F_2(x)$$

$$W_1(q^2, \nu) \xrightarrow{E \to \infty} F_1(x)$$

where $x = q^2/2M\nu$. This means that the structure functions written this way are only a function of x and not a function of q^2, ν, E, or θ independently.

These results have been interpreted in many ways, but for example, in a "parton picture" it says that the constituents that make up the hadronic structure are point like. This is a very important point when we study neutrino scattering.

For neutrino scattering the scattering diagram is analogous to the electron case:



Deep Inelastic Neutrino Scattering

Kinematic variables:

$q^2 = 2\,EE'\,(1 - \cos\theta')$

$\nu = E - E'$

The expression for the scattering cross section is written

$$\frac{d^2\sigma^{\nu,\bar{\nu}}}{d\,E'\,d\cos\theta'} = \frac{G^2E'^2}{\pi}\left[W_2^{\nu}\cos^2\frac{\theta}{2} + 2\,W_1^{\nu}\sin^2\frac{\theta}{2} \pm W_3\,\frac{E+E'}{M}\sin^2\frac{\theta}{2}\right]$$

This formula is similar to the electron (or muon) case with some important differences. First, the coefficients is now the weak instead of electromagnetic coupling constant. Second, a very important practical difference arises since neutrino scattering does not have a $1/q^4$ dependence. This is because in the electron case the propagator is zero mass and for neutrinos it is either infinite mass or very heavy. In practice this means that a much larger fraction of neutrino scattering is at large q^2, which is, of course, the very interesting region. The third difference is in the structure functions. In the neutrino case W_1 and W_2 have both vector and axial vector parts and therefore a third structure function exists that represents the interference (or the parity violating part). Also, note that this extra structure function changes sign in changing from a neutrino to antineutrino beam.

If scale invariance holds the constituents are point-like and this requires that

$$\text{Scale invariance:} \quad \frac{1}{M} \nu W_2(q^2, \nu) \xrightarrow{E \to \infty} F_2(x)$$

$$\frac{1}{M} \nu W_3(q^2, \nu) \xrightarrow{E \to \infty} F_3(x)$$

$$W_1(q^2, \nu) \xrightarrow{E \to \infty} F_1(x)$$

where $x = q^2/2m\nu$ as before. If we integrate the expression for the differential cross section to obtain the total cross section we get

$$\sigma^{\nu, \bar{\nu}}(E) = \frac{G^2 M E_\nu}{\pi} \int_0^1 dx \left[\frac{1}{2} F_2(x) + \frac{x}{3} \left(F_1(x) \pm F_3(x) \right) \right]$$

so in the limit of the hadron structure behaving point-like the cross section will rise linearly with neutrino energy E_ν .

The present data from CERN up to about 10 GeV appears to be consistent with a linearly rising cross section. That data is shown in figure 1.

It will be extremely interesting to see whether this behavior continues at NAL energies. It appears quite feasable to make accurate total cross section measurements up to about $E_\nu \sim 300$ GeV at NAL. We have given considerable emphasis to this in the early program as I will explain later.

It is interesting to see how this linearly rising cross section might be affected, for example, by the existence of an Intermediate Boson. If a boson exists then in the cross section formula $G \to \dfrac{G}{1 + q^2/M_W^2}$ which will damp the cross section at high q^2 causing the total cross section to turn over. This is illustrated in figure 2.

Of course, if a non-linear rise was observed further studies of the damping mechanism would have to be studied to distinguish a breakdown of scaling from the effect of a W-propagator.

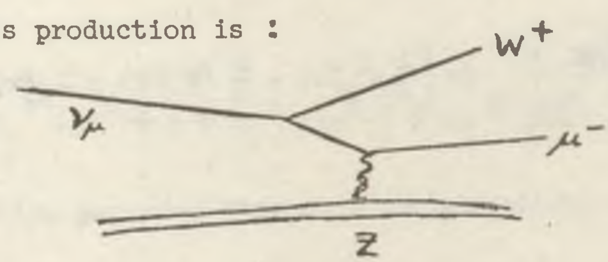
W-Boson Search

A second thrust of the initial experiments will be to search for direct production of W-bosons. Just as ordinary neutrino interactions ($\nu_\mu + N \to \mu^- +$ hadrons) is analogous to ($e^- + N \to e^- +$ hadrons), W-production is analogous to Bremsstrahlung.

Bremsstrahlung $e^- + Z \to e^- + \gamma + Z$

"W-Bremsstrahlung" $\nu_\mu + Z \to \mu^- + W^+ + Z$.

The main diagram for this production is :



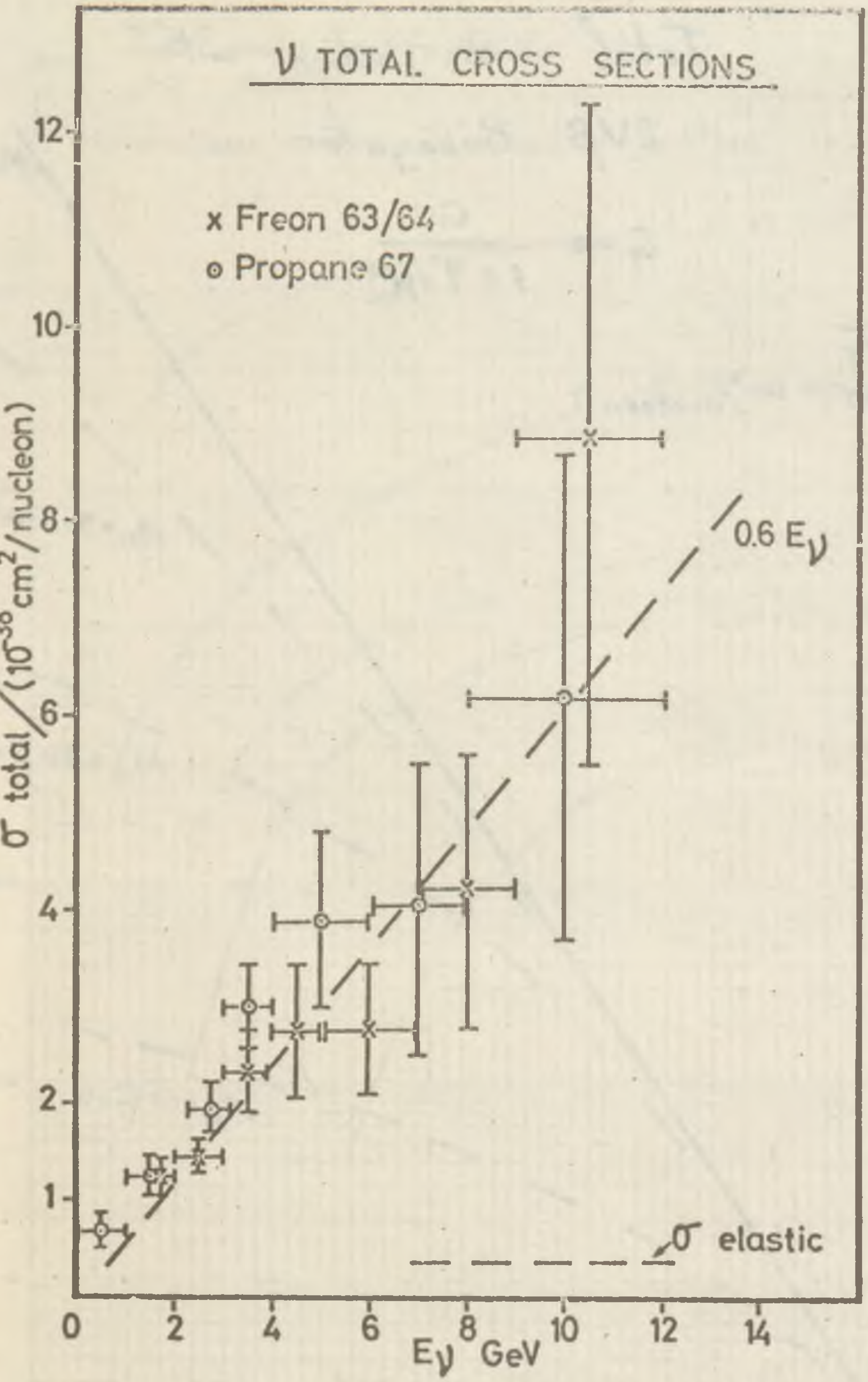
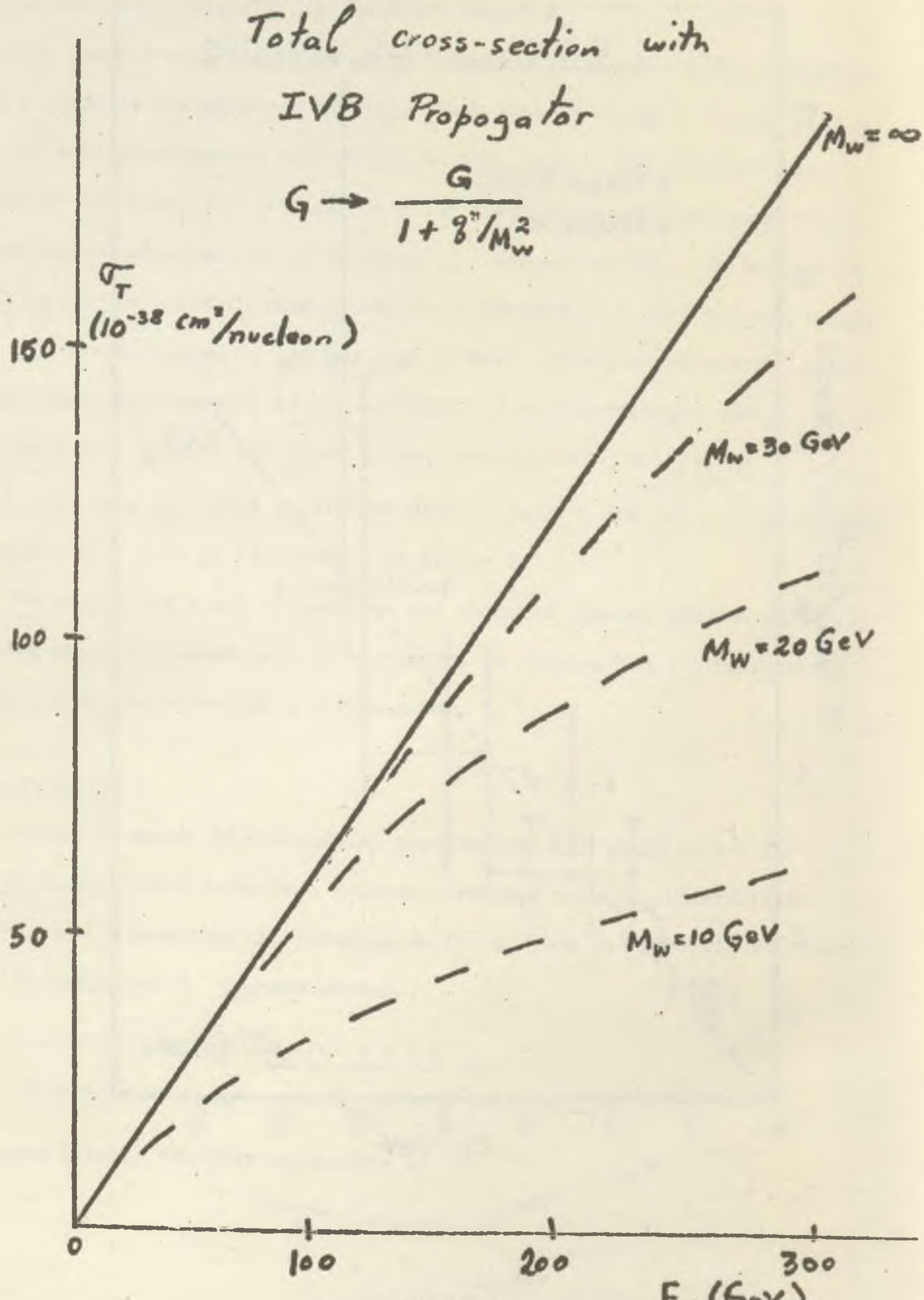

ν TOTAL CROSS SECTIONS
x Freon 63/64
⊙ Propane 67
σ total/(10^{-38}cm^{2}/nucleon)
0.6 E$_ν$
σ elastic
E$_ν$ GeV
0
2
4
6
8
10
12
14
1
2
4
6
8
10
12

Figure 1



Total cross-section with
IVB Propogator
G → G / (1 + q²/M_w²)
σ_T
(10^-38 cm²/nucleon)
150
100
50
0
100
200
300
E_ν (GeV)
M_W = ∞
M_W = 30 GeV
M_W = 20 GeV
M_W = 10 GeV

Figure 2

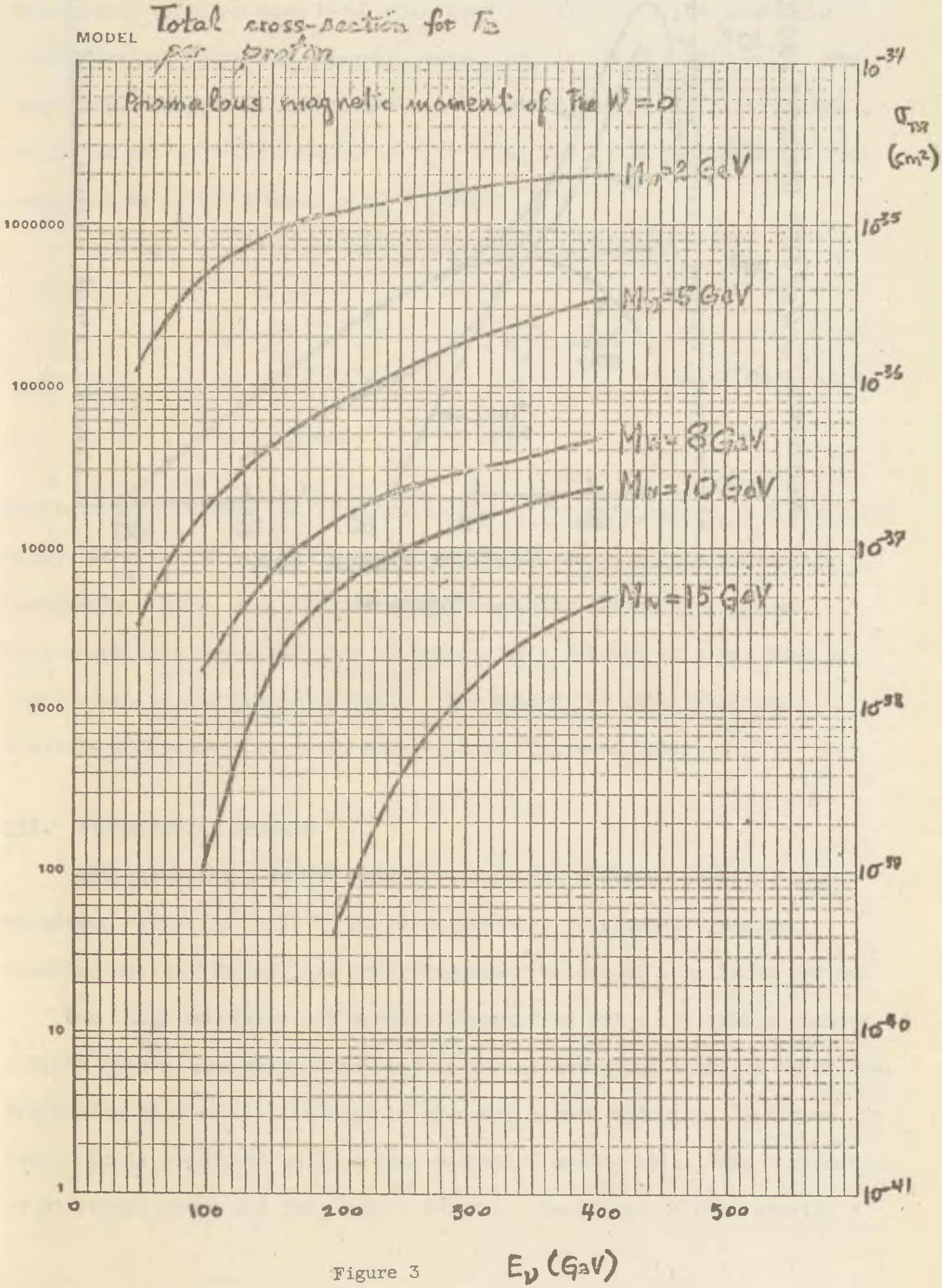
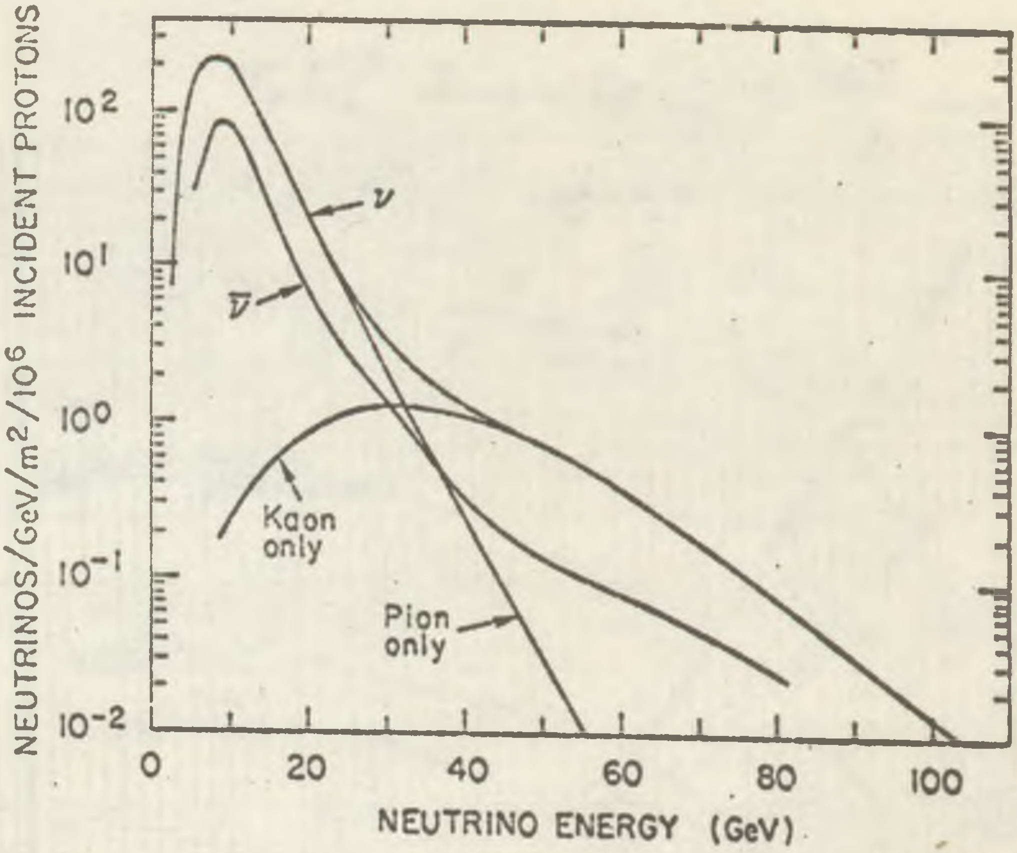

MODEL
Total cross-section for
per proton
Anomalous magnetic moment of the W = 0
M_W = 2 GeV
M_W = 5 GeV
M_W = 8 GeV
M_W = 10 GeV
M_W = 15 GeV
1000000
100000
10000
1000
100
10
1
0
100
200
300
400
500
10^{-34}
σ (cm^2)
10^{-35}
10^{-36}
10^{-37}
10^{-38}
10^{-39}
10^{-40}
10^{-41}
E_ν (GeV)

Figure 3



NEUTRINOS/GeV/m²/10⁶ INCIDENT PROTONS
10²
10¹
10⁰
10⁻¹
10⁻²
ν
ν̄
Kaon only
Pion only
0
20
40
60
80
100
NEUTRINO ENERGY (GeV)

Figure 4

These cross sections have been calculated in detail and the production cross section, including coherent scattering, is shown in figure 3. The very steep energy dependence makes it clear that extremely high energy neutrinos are needed to search to $M_W \sim 10\text{-}15$ GeV. The present limit from ν-production of the W mass is $M_W < 2$ GeV.

The W can decay either leptonically or into hadrons.

$$W^+ \begin{array}{l} \nearrow \mu^+ + \nu_\mu \\ \searrow e + \nu_e \end{array} \qquad \Gamma \propto M_W^3$$

$$W \rightarrow \text{hadrons} \qquad \Gamma = ?$$

The rate into leptons can be calculated and, for example, a $M_W \sim 2$ GeV decays at a rate $\Gamma \sim 5\ 10^{18}\ \text{sec}^{-1}$. However, the rate into hadrons is completely unknown and, therefore, the branching ratio is not known. This means that one would like to search for a W-boson in a way that is independent of the branching ratio. The scheme for distinguishing W-events independent of decay mode will be discussed later.

III. EXPERIMENTAL PROGRAM

NAL has built a rather extensive area for neutrino physics. The targeting region is built to be very flexible. Different types of focusing devices (beams) are interchangeable by use of a railroad system.

Two large counter-spark chamber experiments and a 15' bubble chamber (30,000 liters) are being assembled in series along the neutrino beam line. The bubble chamber will be capable of using either hydrogen, deuterium, or neon. It is scheduled to come into operation during 1973. The combination of different beams and the variety detection techniques should provide a powerful attack on neutrino physics.

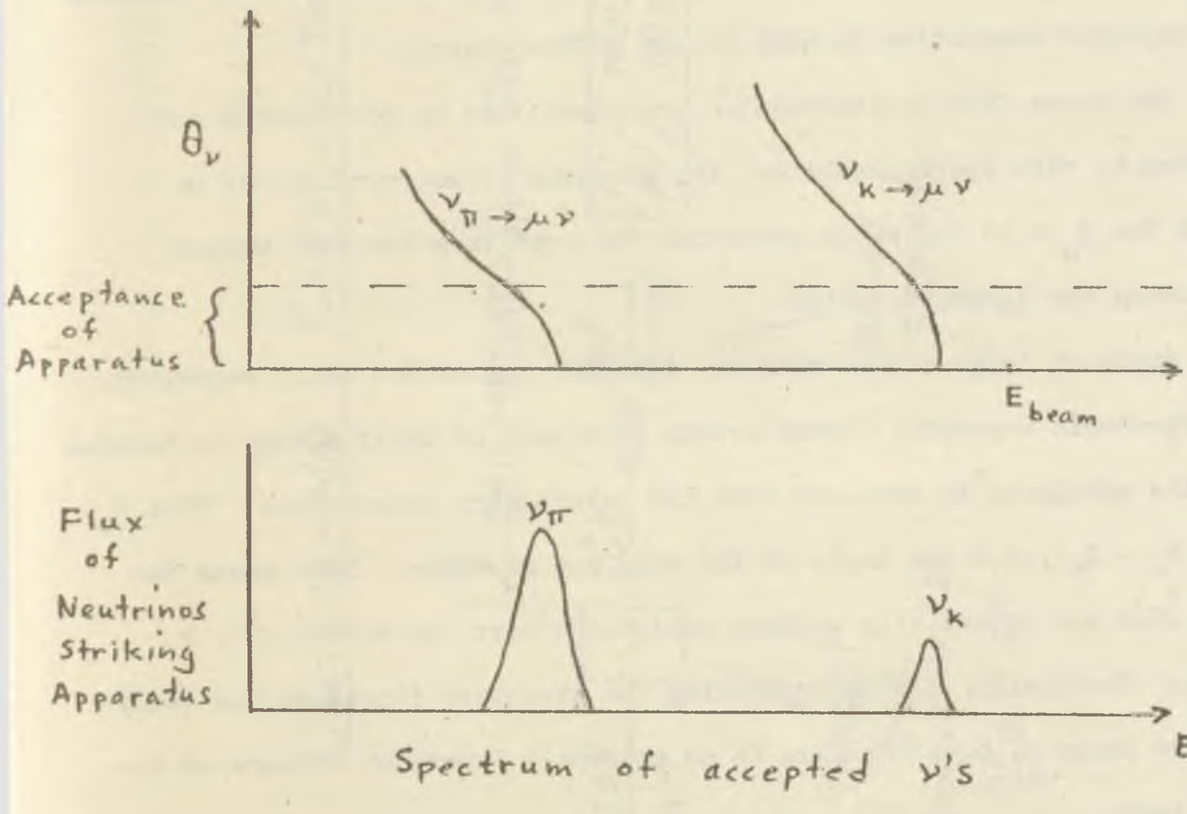
The early priorities of the laboratory are to emphasize exploration of very high energy neutrino interactions. The Caltech-NAL experiment, which I will discuss, has been optimized for this study.

The "traditional" method for producing a neutrino beam has been to strike a target with protons from the accelerator, follow it with a broad band focusing device (i.e., a horn), and send the resulting pions and kaons down a decay region. This decay region is then followed by a shield, which first removes the hadrons by absorption, and then muons by range. The exact shape of the resulting ν-spectrum depends on the properties of the focusing devices and the actual pion and kaon yields at high energy, which are unknown. One estimate of this spectrum at NAL energies is shown in figure 4.

The two most immediate goals at high energies are to determine whether the total cross section continues to rise linearly and to search for direct production of W-bosons. The rapidly falling neutrino spectrum with a preponderance of low energy neutrinos create severe problems in making these measurements. The steeply falling spectrum makes determination of σ_T vs E_ν extremely difficult. The large flux of low energy neutrinos plus the presence of antineutrinos present formidable background problems for a W-boson search.

For these reasons, and also because of the obvious advantages of knowing the neutrino energy, the initial beam that has been installed at NAL is a "dichromatic" beam. The scheme for making this type of neutrino beam is rather simple. The protons are extracted from the accelerator and strike a target. The resulting pions and kaons are focussed, point to parallel, and momentum selected ($\Delta p = \pm 5\%$) by a simple beam transport system. The hadron beam is then directed down a decay tube 400 meters long which is followed by a shield to remove the remaining pions and kaons by absorption and decay muons by range.

The important point is that for two body decays $K \to \mu + \nu$ or $\pi \to \mu + \nu$ the neutrino energy is correlated with the laboratory decay angle. The detection apparatus is placed such that it subtends only small angle decays and therefore only accepts neutrinos within two small momentum bands near the respective end points for pion and kaon decays:



The two bands of neutrinos differ in energy by more than a factor of 2! Therefore, only a rough measurement (~25-30%) of the total final state energy in a neutrino interaction is necessary to determine whether an event comes from the ν_π or ν_K peak. Once this ambiguity is resolved the incident neutrino energy is determined to about ±6%.

The general experimental layout at NAL is shown in figure 5. An estimate of the final spectrum of neutrinos, including the momentum spread of the beam, K_{e3} and $K_{\mu 3}$ contamination, wide band contamination, and angular resolution of the apparatus is shown in figure 6.

The detection apparatus for the Caltech-NAL experiment consists of a large target (170 tons of steel) which also serves as an ionization calorimeter for measuring E_{hadron}. The hadron energy is measuring by sampling the ionization in Fe every 10 cm following the interaction. The apparatus can be restacked for finer resolution having 5 cam sampling. The expected resolution is ~20% on the hadron energy.

The muons from an interaction are identified by penetration and tracked by wire spark chambers. The momentum is measured either by range for $E_\mu < 10$ GeV or by measuring the bend in a toroidal magnet following the target-detector.

Muons at large angles miss the toroidal magnet but still represent interpretable ν-events. These events give most of their energy to hadrons and the ambiguity is resolved from the calorimetry measurement. Then E_ν, $E_\mu = E_\nu - E_h$, and the angle of the muon are measured. This means the apparatus has essentially uniform acceptance over the entire q^2, ν plane. Eventually, for understanding the structure functions and studying the large q^2 behavior this is an extremely important feature of the experiment.

The events of interest will produce two main topologies in our apparatus:

(a) Inelastic scattering. These events are characterized by the

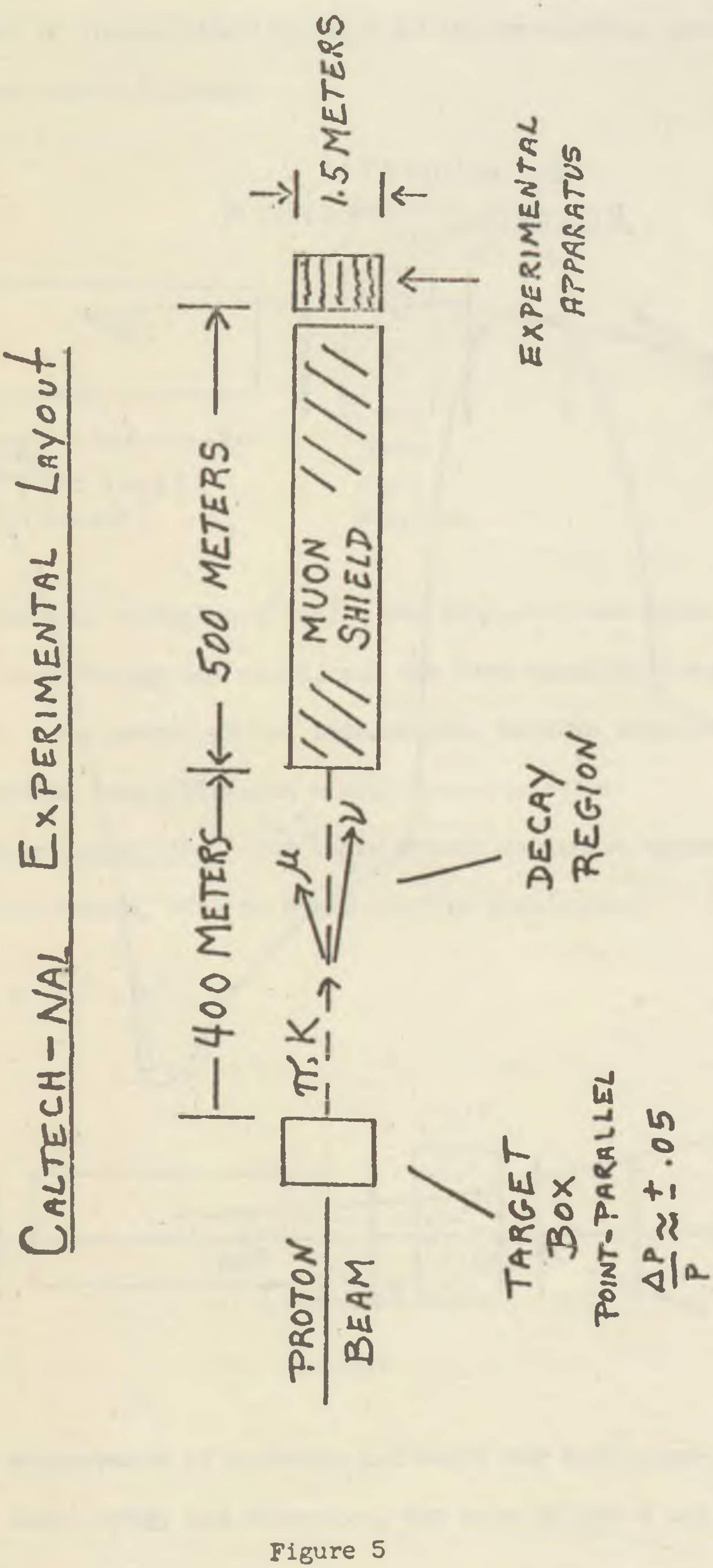

CALTECH-NAL EXPERIMENTAL LAYOUT
PROTON BEAM
TARGET BOX
POINT-PARALLEL
$\frac{\Delta P}{P} \approx \pm .05$
π, K
μ
ν
400 METERS
500 METERS
DECAY REGION
MUON SHIELD
EXPERIMENTAL APPARATUS
1.5 METERS

Figure 5

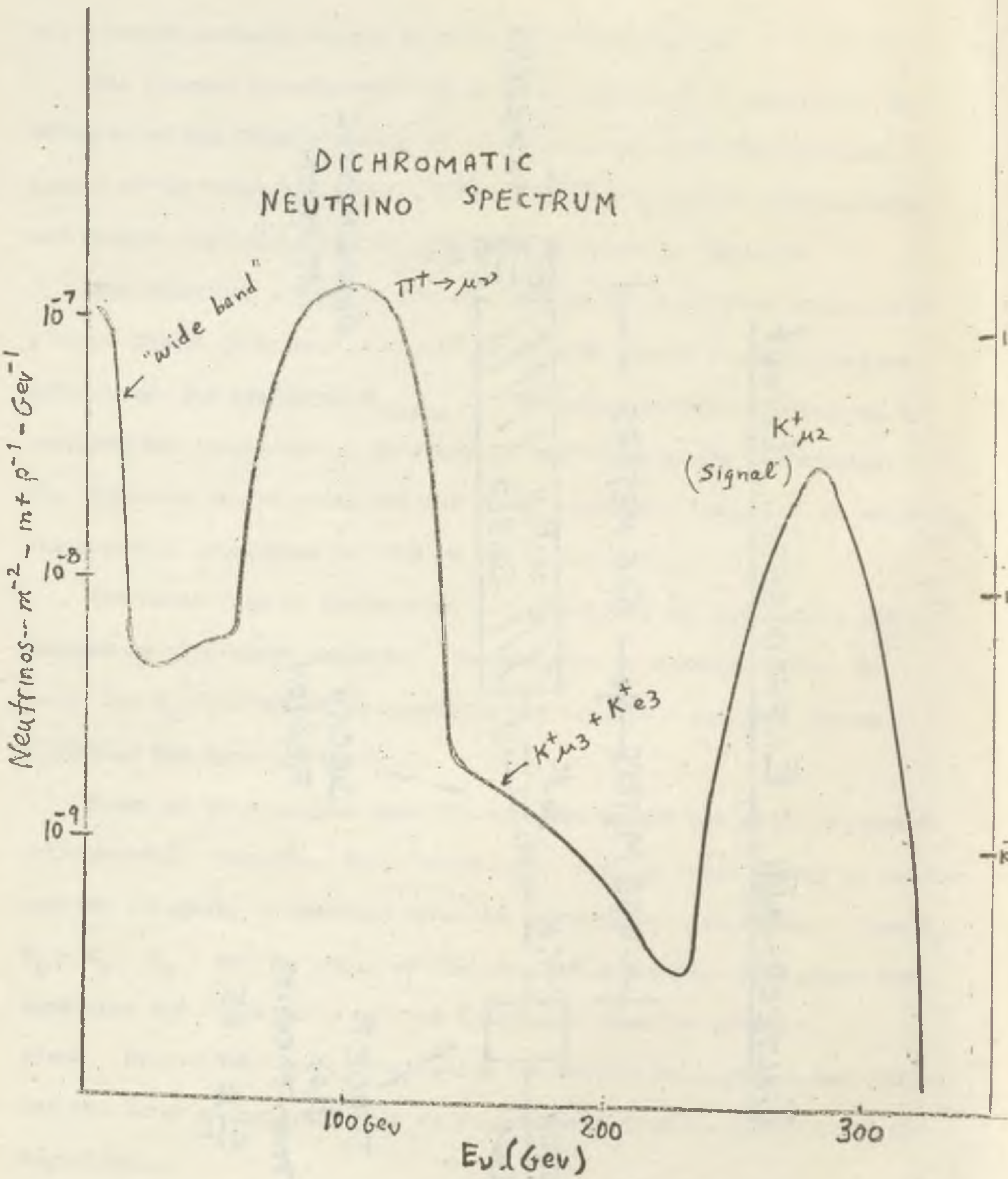
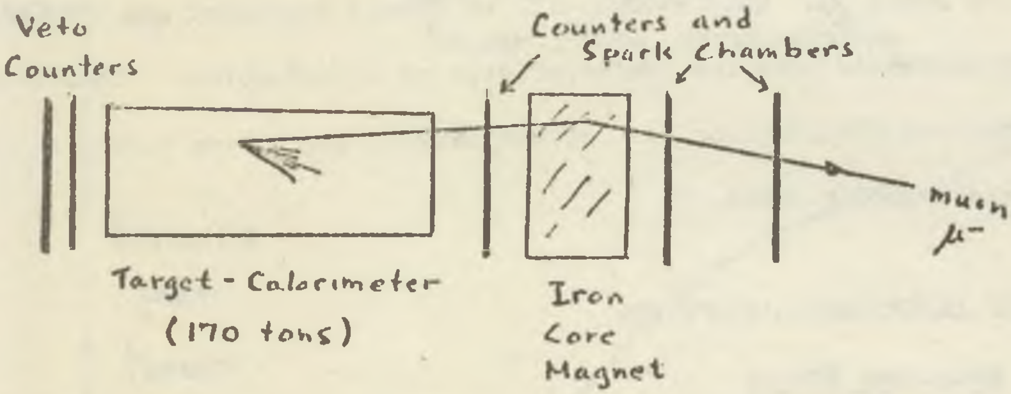

DICHROMATIC
NEUTRINO SPECTRUM
"wide band"
$\pi^+ \to \mu\nu$
$K^+_{\mu 2}$
(Signal)
$K^+_{\mu 3} + K^+_{e3}$
10^{-7}
10^{-8}
10^{-9}
Neutrinos -- m^{-2} - int p^{-1} - Gev^{-1}
100 Gev
200
300
E_ν (Gev)

Figure 6

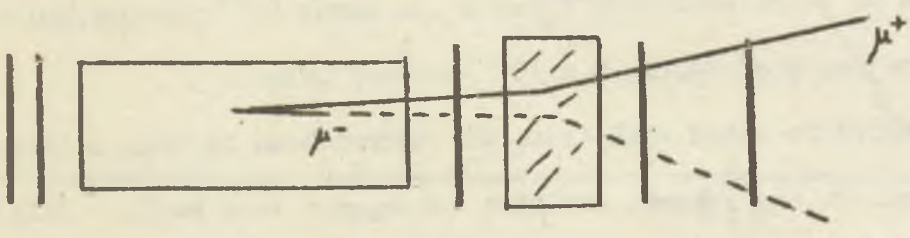
appearance of a muon (distinguished by its penetration accompanied by a hadron induced shower:



The kinematical variables q^2 and ν are reconstructed from the measured muon energy and angle, and the know neutrino energy and direction. The hadron shower information, besides separating the E_ν peaks, further constrains the energy reconstruction.

(b) W-boson production. For these events two muons emerge from the interaction vertex, with no other visible particles.

$$\nu + Z \rightarrow \mu^- + W^+ + Z'$$
$$W^+ \hookrightarrow \mu^+ + \nu$$



From the measurement of momentum and angle for both muons, together with the beam energy and direction, the mass of the W can be reconstructed.

W-production with decay other than $\mu + \nu$ will have topologies similar to inelastic scatters. However, the distribution of the μ^- will contain a large excess at both small angle and low energy (most of the ν energy goes to the heavy W). This excess will be energy dependent and will show the steep threshold behavior characteristic of W-production. Therefore, if an anamolous distribution is observed, varying the ν beam energy should provide a convincing check.

IV. EARLY EXPERIMENTAL PROSPECTS

A. Anamolous Events

The initial neutrino events will be studied at the few percent level for lepton conservation (μ^+ events/μ^- events), neutral currents (no - μ events/μ^- events), etc. These tests should come rather early and represent checks at much larger momentum transfers than in the past.

B. Total Cross Sections: Comparison σ_ν vs $\sigma_{\bar{\nu}}$

The expected event rate for high energy neutrino interactions for a linearly rising cross section is shown in figure 7. This is with the dichromatic beam and only includes the kaon neutrino events. The pion neutrino events are expectdd to be more plentiful. Therefore, it appears reasonable to start measuring σ_T vs E_ν at about 10^{12} protons/pulse (design intensity of NAL is 5×10^{13} protons/pulse).

It should be noted that since the hadron beam is sign selected, the $\bar{\nu}$ component in the ν beam, and visa versa, are very small. This means that by just reversing the sign of the hadron beam σ_ν vs $\sigma_{\bar{\nu}}$ will be measured.

Measurements of the details of the inelastic spectrum will require high energy (400 GeV) and more intensity. Alternately, as the yields of

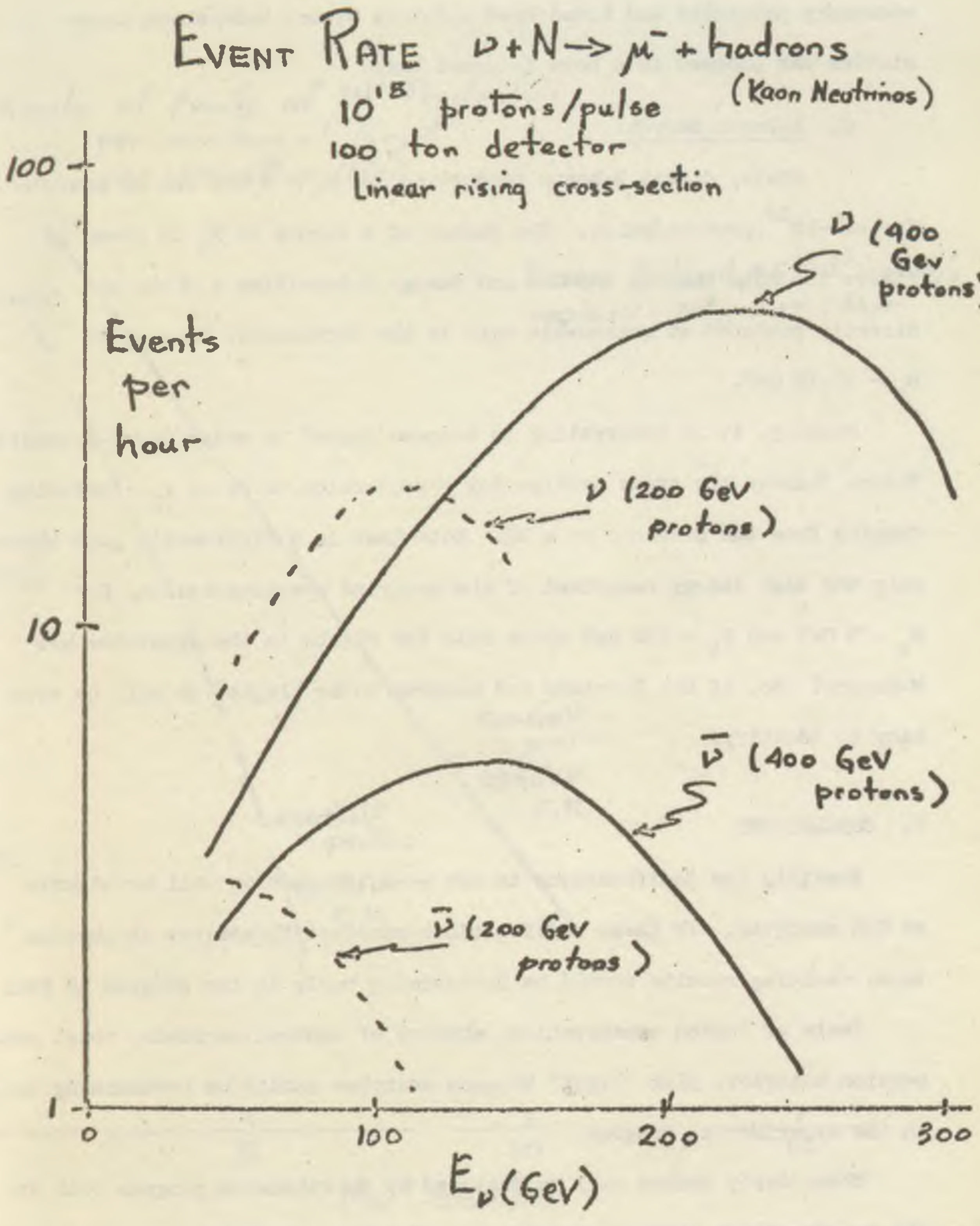

EVENT RATE ν + N → μ⁻ + hadrons
(Kaon Neutrinos)
10^18 protons/pulse
100 ton detector
Linear rising cross-section
Events per hour
100
10
1
ν (400 Gev protons)
ν (200 GeV protons)
ν̄ (400 GeV protons)
ν̄ (200 GeV protons)
0
100
200
300
E_ν (GeV)

Figure 7

secondary particles and broad band spectrum become understood these studies can proceed in a horn focussed beam.

C. W-Boson Search.

Again, direct W-boson production for $M_W < 5$ GeV can be searched for at $\sim 10^{12}$ protons/pulse. The number of W events vs M_W is shown in figure 8. Note that at 400 GeV and design intensities a W can be directly produced at reasonable rate in the dichromatic beam up to $M_W \sim 10$-12 GeV.

Finally, it is interesting to compare signal vs noise for W-production. Figure 9 shows the cross section for W-production on Fe vs $\sigma_T \cdot$(including damping from the presence of a W). Note that in a dichromatic beam where only the high energy component of the neutrino spectrum exists, for $M_W = 5$ GeV and $E_\nu \approx 150$ GeV about half the events in the apparatus are W-bosons! So, if the W exists and happens to be "light" it will be very easy to identify.

V. CONCLUSIONS

Possibly the modifications to the weak interaction will be visable at NAL energies. If these modifications manifest themselves in obvious ways, exciting results should be forthcoming early in the program at NAL.

Tests of lepton conservation, absence of neutral currents, total cross section behavior, plus "light" W-boson searches should be forthcoming early in the experimental program.

These early probes will be followed by an extensive program both in the counter-spark chamber experiments and bubble chamber to search for heavier W-bosons, measure the inelastic structure functions, search for

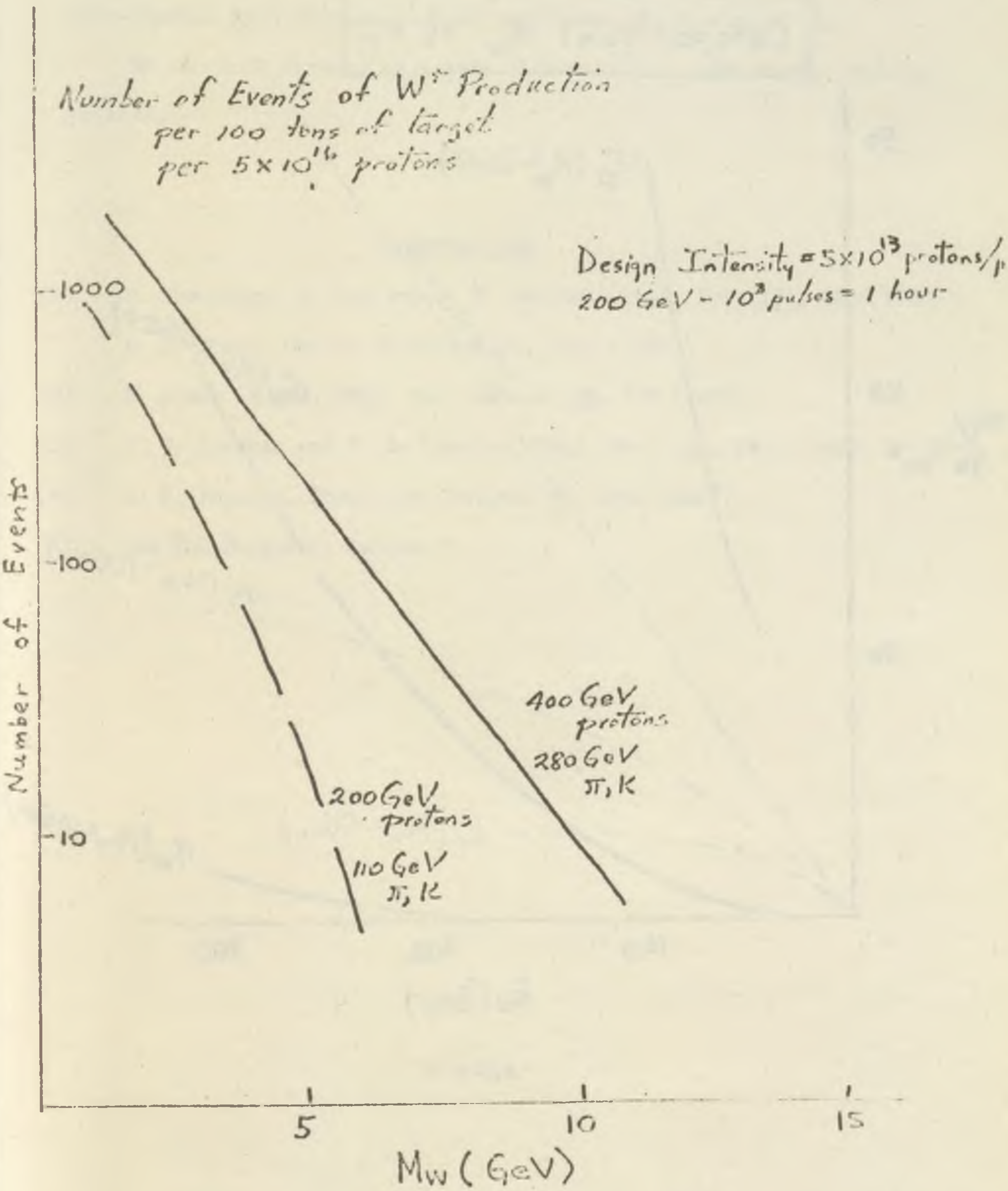

Number of Events of W± Production
per 100 tons of target
per 5×10^16 protons
Design Intensity = 5×10^13 protons/p
200 GeV - 10^3 pulses = 1 hour
Number of Events
1000
100
10
400 GeV protons
280 GeV π, K
200 GeV protons
110 GeV π, K
5
10
15
Mw (GeV)

Figure 8

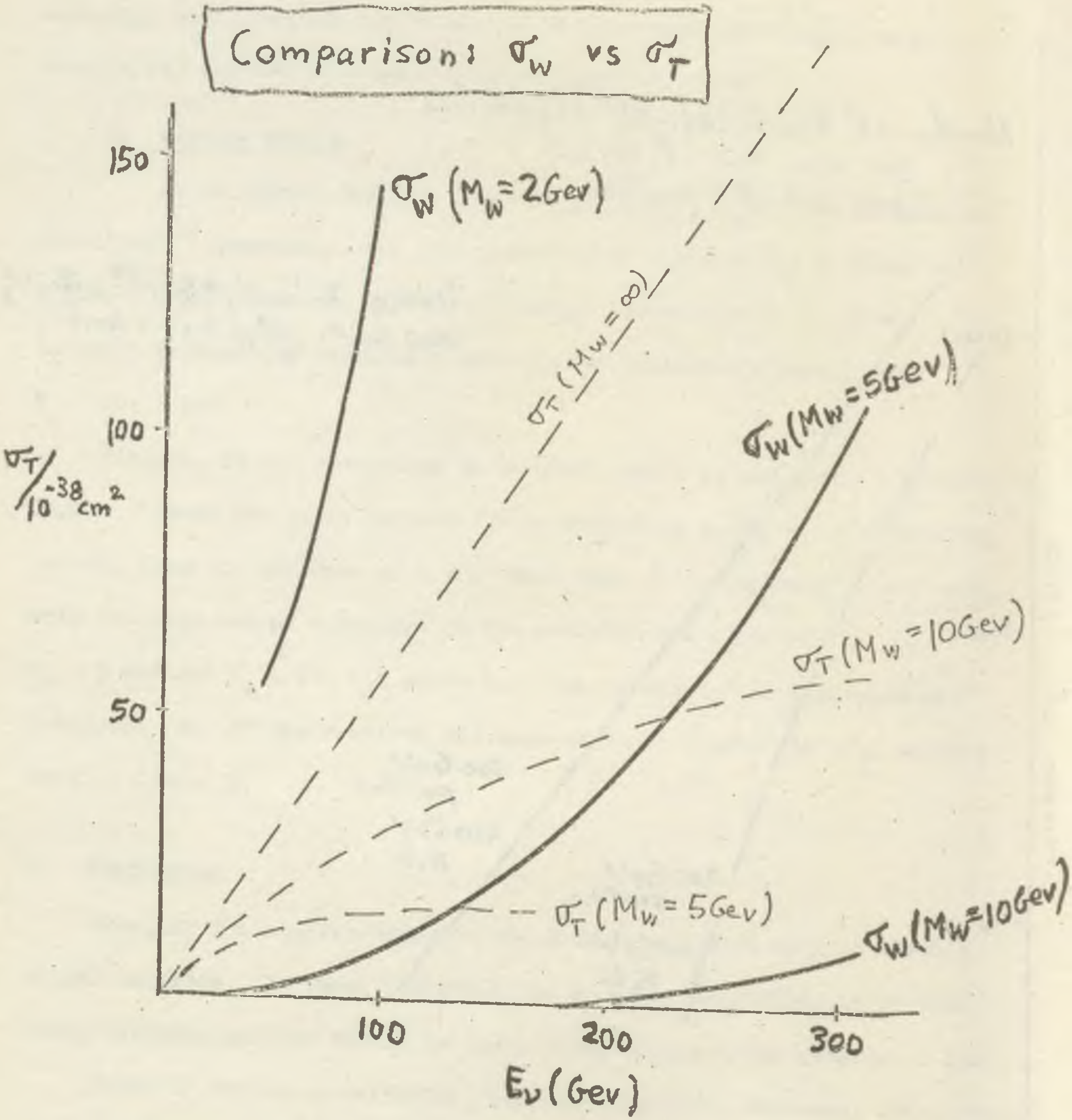

Comparison: σ_W vs σ_T
150
100
50
$\sigma_T / 10^{-38} cm^2$
σ_W (M_W=2Gev)
σ_T (M_W = ∞)
σ_W(M_W=5Gev)
σ_T (M_W =10Gev)
σ_T (M_W = 5Gev)
σ_W(M_W=10Gev)
100
200
300
E_ν (Gev)

Figure 9

four-fermion type events, analyze individual final states, etc.

We can look forward to a very rich future in high energy neutrino physics.

References

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(2) E. Bloom et al., Phys. Rev. Letters 23, 930 (1969).

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(5) See NAL Proposals 1A and 21.

WHAT NEUTRINOS CAN TELL US ABOUT PARTONS

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The parton model has been useful in guessing at regularities expected at high energy both for hadron collisions and deep inelastic electron or lepton scattering. The model gets its idea from field theory. According to a field theory a hadron state wave function (an eigenstate of the field Hamiltonian), could be described by giving the amplitude to find various numbers of particles of the basic fields of the theory at various momenta in the state. A quantum of the basic field theory, whose specific properties we do not know, of course, is called a parton. In our present knowledge of these things we shall have to guess both at the kinds of partons there might be as well as the way they are distributed in the hadron state. Both of these things can, of course, ultimately be determined by experiment - and this suggests a program for the future - (provided the framework is correct). It is the purpose of this paper to show how this might be done. The method I shall use is to take, as an example, very specific assumptions on what partons are (quarks) and how they are distributed and to show, by this example, how predictions might be made and how the specific assumptions could be tested by experiment. It should become obvious by analogy how to make testable predictions for some other choice of what partons are and how they act.

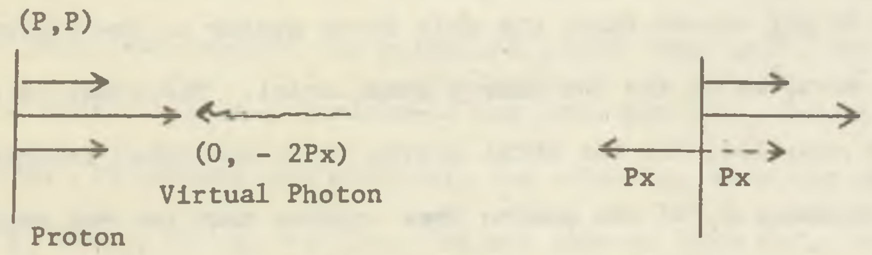
First we give a brief review of the parton idea and show how they have been applied to inclusive electron scattering ep → e + anything. Then the applications are described to inclusive neutrino scattering. Our assumptions to this point will be consistent with the assumptions of light cone algebra so all the conclusions of that theory can be obtained here too. Finally in the last part of the paper we discuss how, by making additional hypotheses, predictions about the distribution of hadrons in the final state resulting from the collision can be made. In all cases we restrict ourselves to the deep inelastic region.

THE PARTON PICTURE OF DEEP INELASTIC ELECTRON SCATTERING

The wave function for a proton moving with a large momentum P to the right along the z axis, is supposed to be large only when the transverse momenta of the partons are finite (e.g. of order GeV). This is suggested by energetic hadron-hadron collisions. The longitudinal momenta may be finite, or large of order P. In the latter case we write the parton longitudinal momentum as xP and then suppose that as P approaches infinity the amplitude to find partons with various values of x (from 0 to 1) is independent of P. Reasons for these assumptions are discussed more fully in reference 1. These assumptions may or may not be consistent with the quantum field theory which inspired the model in the first place, but we make them anyway, disregarding to some extent the original motivation of the model.

When we scatter a high energy lepton from such a proton it scatters from a particular parton and, from the conservation of energy and momentum we can determine the momentum, or x, of the parton that did the scattering. Thus the spectrum of the scattered lepton determines the distribution in x of the parts inside, in a manner analogous to the way the frequency distribution of radar scattered from a swarm of bees determines the velocity distribution of the bees inside the swarm. To use the conservation of energy, however, we use, to sufficient approximation, the energy of the parton as if free, whereas it is in fact in interaction with the other partons of the proton. We explicitly assume that the interaction between two partons of large relative momentum is not similarly large, so if we do this experiment with sufficient momentum, this interaction makes relatively little error.

Let q represent the change in momentum of the lepton - therefore the momentum of the virtual photon in electron scattering. We shall review very briefly electron scattering and then go on to our main subject, neutrino scattering. Let p be the momentum of the proton, $p \cdot q = M\nu$ where ν is the energy loss of the lepton in the lab. Now we take $-q^2 = 2M\nu x$ and let $\nu \to \infty$ keeping x fixed. A good system in which to visualize things is the one in which the virtual photon momentum q is spacelike say (0, - 2Px) and the proton has large momentum P, energy practically P also p = (P,P). Hence $P^2 = M^2\nu^2/(-q^2)$ and the deep inelastic region corresponds to $P \to \infty$. We picture the proton as a group of partons of which a typical one has momentum $P\xi$ (Figure 1)



(a) Before Interaction

(b) Immediately After Interaction

FIGURE 1

When the virtual photon interacts with one parton its momentum is changed by -2Px but the magnitude of the momentum must not be changed (the approximate conservation of energy we mentioned above) so that only the parton with momentum Px can interact - it gets its momentum reversed to -Px. The state of the partons just after the event is illustrated in Figure 1(b); there is one parton moving to the left with momentum -Px and the original set that were in the proton, less the parton at Px that was scattered away, going to the right. The total momentum of this set is now P(1-x).

The probability that something happens is proportional to the probability of finding a parton with momentum x times its charge (the coupling to the photon) squared.

PARTONS AS QUARKS IN ELECTRON SCATTERING

To illustrate this we shall take as an example (as we will throughout this paper) the case that the charged partons are quarks or antiquarks. We then may characterize the proton by six functions. Let u(x) be the number of up quarks (charge + 2/3, isospin + 1/2, zero strangeness) with momentum fraction x per dx in the proton. Let d(x), s(x) be the corresponding number of down (isospin - 1/2) and strange (isospin 0) quarks respectively; and $\bar{u}(x)$, $\bar{d}(x)$, $\bar{s}(x)$ the corresponding number of antiquarks.

We do not assume there are only three quarks in the proton, two ups and one down, as in the low energy quark model. There may be many quark pairs in addition, but the total charge + 1, and total isospin + 1/2, and strangeness 0, of the proton does require that the net number of up quarks be 2, net down be 1, and net strange be 0;

$$\int_0^1 \left(u(x) - \bar{u}(x)\right) dx = 2$$

$$\int_0^1 \left(d(x) - \bar{d}(x)\right) dx = 1 \qquad (1)$$

$$\int_0^1 \left(s(x) - \bar{s}(x)\right) dx = 0$$

The probability it was an up quark that scattered the photon is then 4/9 (the square of the charge 2/3) times u(x), the number of up quarks available. Thus the total probability of interaction with virtual photons is expressed in terms of our six functions by

$$f^{ep}(x) = \frac{4}{9}\left(u(x) + \bar{u}(x)\right) + \frac{1}{9}\left(d(x) + \bar{d}(x)\right) + \frac{1}{9}\left(s(x) + \bar{s}(x)\right) \quad (2)$$

(This $f^{ep}(x)$ is related to experimental quantities in a way described in Kutis' talk[2)] at this meeting. It is the function $2MW_1$ in the scaling limit.) We have been able to disregard the mass and transverse momenta of the partons as we are here only dealing with the leading terms in the results at high momentum.

Properly we should deal separately with three directions of polarization e_μ of the virtual photon which we can take as two transverse; positive helicity, negative helicity, and one "longitudinal" in the t direction (it must be perpendicular to q which is in the z direction). For unpolarized protons the two helicities give the same result of course. For brevity we shall not analyze the polarized proton case here, but it is discussed by Kuti[2)], where a remarkable sum rule for G_A/G_V due to Bjorken, results. If partons are spin zero the coupling, when one changes momentum from p_1 to p_2, is $(p_1 + p_2)\cdot e$. We see this is zero (i.e. of lower order in P) for transverse e_μ, and large for e_μ in the t direction. On the other hand if partons were spin 1/2 the coupling $(\bar{u}_2 \not{e}\, u_1)$ is large for transverse e_μ, but zero for e in the direction of $p_1 + p_2$ (the t direction). In a given experiment with q, ν fixed, as we vary the angle of the electron scattering by which the virtual photon is produced, we vary the proportion of longitudinal and transverse polarization produced by the electron. Thus we can separate the contribution of the longitudinal and transverse polarization. (Hence there are two structure functions which scale, they are combinations of νW_2 and $2MW_1$). Experiments indicate that the longitudinal scattering is small, near 20% of the transverse. We shall, therefore, guess that the charged partons have spin 1/2, the remaining 20% would presumably be the result of not having large enough P in the experiments.

(Thus there should at high energy be only one independent function, we expect $\nu W_2 = x \cdot 2MW_1$ in the limit.) This is our first example of how lepton scattering can say something about the character of the charged partons.

The function $f^{ep}(x)$ is, of course, known from experiment. For small x it goes roughly as .32/x, near x = 1/2 it is about .3 and as x approaches 1 it falls away possibly as $(1-x)^3$ (as suggested by Drell and Yan who relate it to the proton form factor). A fall off near x = 1 is expected for if one parton has nearly all the momentum all the others must be restricted to low momentum and the probability of that is small. Now we must discuss in more detail the region of small x.

WEE PARTON REGION

For a given P if x gets small enough, of order 1 GeV/P, which we call "wee," the momentum of the partons becomes finite and many of our approximations fail there. The formula dx/x probably fails there. We expect that it does not continue to rise in this region, for it must eventually fall toward zero in the x negative wee region. The total number of wee partons is then always finite and the total number of partons grows logarithmically with P (a wave function is not a covariant idea). This is best appreciated by describing things in rapidity space, $y = 1/2 \ln \frac{\varepsilon + p_z}{\varepsilon - p_z}$ where p_z is the longitudinal momentum in GeV and $\varepsilon = \sqrt{p_z^2 + p_\perp^2 + m^2}$ where m is some convenient mass (say .3 GeV). Then for finite x, $p_z = Px$ and $y \approx \ln x + \ln 2P$, whereas wee partons correspond to finite y. The $\frac{dx}{x}$ behavior for small x becomes a long plateau dy from finite y to the region of $y = \ln 2P$. In this variable the distribution of some parton looks like Figure 2.

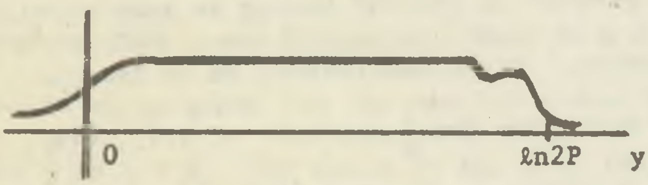


FIGURE 2

Scaling implies that as we increase ℓn2P the upper end moves out, only the plateau region stretches. We shall suppose that the tail near y = 0 stays the same.

This distribution arises from the interactions among partons given by the field Hamiltonian and we shall suppose that it happens in the following way:. Interactions are only important over a finite range of y - the entire range of y gets filled by a cascading produced by the Hamiltonian. Finally, because the scaling character of the equation changes near x = 0, the cascade is terminated in a characteristic way. It is analogous to the cascade of cosmic ray showers. No matter how they start they develop similarly and end (when ionization losses become important and change the equation) in a characteristic way. It is like the wave function for a liquid layer (with y replaced by space) between two surfaces (except the number of molecules is not fixed). At each surface there is a characteristic behavior, and there is a uniform density region in between although interactions are always local. It is the spirit of this paper to make as strong assumptions as possible to generate targets for experiments to shoot down. We shall try the assumption that the wee region (y near 0) and, by continuity, the plateau region (x small) are the same for every hadron, only the y - ℓn2P (larger x) region varies from case to case.

That implies the plateau is neutral having as many particles of a kind as antiparticles. It is indifferent as to isospin. For example near $x = 0$ we must have the $\frac{1}{x}$ behavior of $u(x)$, $\bar{u}(x)$, $d(x)$, $\bar{d}(x)$ to have the same coefficient a/x, and $s(x)$, $\bar{s}(x)$ to go as $\beta a/x$. SU_3 would imply $\beta = 1$ but we do not have nerve enough to assume that - for after all of the interactions, the SU_3 breaking could produce differences. We know from experiment $a = .24/(1 + \frac{1}{6}(\beta-1))$.

ELECTRON-NEUTRON SCATTERING

For scattering from neutrons we obtain the same formula as (2) except that $u(x)$ would be replaced by the number of up quarks in the neutron etc. However by isospin reflection this is the number of down quarks in the proton, which we have called $d(x)$. So if we do not change the definition of our six functions, so they still refer to the proton, we find

$$f^{en} = \frac{4}{9}\left(d(x) + \bar{d}(x)\right) + \frac{1}{9}\left(u(x) + \bar{u}(x)\right) + \frac{1}{9}\left(s(x) + \bar{s}(x)\right) \qquad (3)$$

Thus we have available two experimental functions to determine our six. Data on the neutron shows that f^{en}/f^{ep} starts at 1 at $x = 0$ (as implied by our assumption of a universal plateau) and falls gradually to perhaps .4 at $x = .8$, data is not available above $x = .8$. Since all the functions $u(x)$, etc., are positive, we see immediately that should the ratio fall below 1/4, the partons could not be quarks. There is no such difficulty as yet.

For x near 1 all the quarks but one must be pushed to low x. Perhaps the difficulty of doing this depends on the total quantum number of the state of these low quarks, e.g., whether $I = 1$ or $I = 0$. If this is so one case would be easiest and dominate. Only the $I = 0$ (and quark number two) state gives f^{en}/f^{ep} below 1, for $x \sim 1$, so we explicitly assume this state dominates. Then as $x \to 1$, the last quark in the proton is an up quark (in the neutron a down quark) so f^{en}/f^{ep} must approach $\frac{1}{4}$ as $x \to 1$. The ratio of the form factors G_M^P/G_M^N would go to -2 as $q^2 \to -\infty$. We shall also assume the total angular momentum of the slow quarks is zero, so the u spins as does the proton. Otherwise G_M^P would change sign as $-q^2$ varies from zero to large values.

Finally we notice that the total fractions of the proton momentum carried by the charged quarks is $k = \int_0^1 x(u + \bar{u} + d + \bar{d} + s + \bar{s})dx$ while $\int_0^1 x\ f^{ep}dx = .18, \int_0^1 x\ f^{en}dx = .13$. From this we find that unless the strange quarks carry 70% of the momentum of the proton, which is probably absurd, k is less than 1. Hence there must be some kind of neutral partons in addition to quarks (that carry perhaps 40% of the momentum of the proton).

So far no experiment has definitely proved or disproved the reality of the peculiar quantum numbers of quarks. It should be possible soon, either by using polarized protons or by neutrino scattering. We discuss this next.

NEUTRINO SCATTERING

We now turn to neutrino scattering experiments. We use the usual theory of the lepton current coupling with a hadron current J_μ^h. Tests of whether scaling works directly as for point interaction or only after a suitable q^2 dependent factor for "intermediate W-meson propagation" are, of course, of first importance. They do not ultimately effect what we shall say here for we are studying the hadron current factor.

The other factor (from the leptons) can be, without implying anything physical, represented as an external virtual vector meson field W_μ coupled to J_μ^h. Following Cabibbo we shall take this coupling to be

$(\bar{Q}_2 \gamma_\mu (1+i\gamma_5)Q_1)W_\mu$ (plus its complex conjugate) where Q_2 is an up quark and Q_1 is a "Cabibbo quark" that is one which has amplitude $\cos\theta_c$ to be down and $\sin\theta_c$ to be strange. Experimentally $\sin^2\theta_c$ is small (.06) and for clarity and simplicity I shall give all the discussion here neglecting it. It will be obvious how to rederive all the formulas to allow for this generally small correction. Thus a positive virtual W, produced by incoming neutrinos going to μ^-, can convert a d parton in the proton to a u or a $\bar{u}$ to a $\bar{d}$.

This "virtual W meson" will have momentum q and three polarizations helicity +, say, W_+; helicity −, W_-; and longitudinal, W_t. The proportion of these produced by the neutrino depends on the angle of ν,μ^- scattering, therefore three structure functions are needed to describe the data now, (they are νW_2, $2MW_1$ and νW_3). As before the W_t shouldn't couple in the deep region $P \to \infty$, (so νW_2 should be equal to $x \cdot 2MW_1$). Conservation of z-component spin requires that the positive helicity W_+ couple only with a positive helicity parton, sending it back with + helicity.(We neglect mass and transverse momentum.) But the $1 + i\gamma_5$ says that energetic quarks interact in the weak interaction only if these have negative helicity, antiquarks interact only if they have positive helicity. Thus W_+ couples only with antiquarks and hence only with $\bar{u}$, converting it to $\bar{d}$. W_- sends $d \to u$. Using antineutrinos generates anti-W which if they have positive helicity $\bar{W}_+$ convert $\bar{d}$ to $\bar{u}$; $\bar{W}_-$ converts $u \to d$. Thus neutrino and antineutrino scattering can permit us to select the action on one type of quark at a time and permits, for example, separate determination of u(x), d(x), $\bar{u}$(x) and $\bar{d}$(x).(Because of the smallness of $\sin^2\theta_c$ our handle on s(x) or $\bar{s}$(x) is too weak to be useful.) We describe below the one arm structure functions (e.g. $f_+^{\nu p}$ is for scattering of neutrinos on protons positive helicity part, hence done via W_+ etc.) for each case, give their expressions in terms of the conventional structure functions ($f_1 = 2MW_1$, $f_3 = \nu W_3$) and give their theoretical expression in terms of u(x) etc., for $\sin^2 \theta_c = 0$:

$$W_+, \quad \bar{u} \to \bar{d} : \quad f_+^{\nu p} = \frac{1}{4} (f_1^{\nu p} + f_3^{\nu p}) = \bar{u}(x)$$

$$W_-, \quad d \to u : \quad f_-^{\nu p} = \frac{1}{4} (f_1^{\nu p} - f_3^{\nu p}) = d(x)$$

$$\bar{W}_+, \quad \bar{d} \to \bar{u} : \quad f_+^{\bar{\nu} p} = \frac{1}{4} (f_1^{\bar{\nu} p} + f_3^{\bar{\nu} p}) = \bar{d}(x)$$

$$\bar{W}_-, \quad u \to d : \quad f_-^{\bar{\nu} p} = \frac{1}{4} (f_1^{\bar{\nu} p} - f_3^{\bar{\nu} p}) = u(x) \qquad (4)$$

Having the possibility of determining these four functions individually now leads to a lot of predictions obtained upon substituting these into our previous formulas. (We note that f_1 will be much easier to measure than f_3.) First we get two sum rules by substituting into equation (1); we write them separating the f_1 and f_3 parts:

$$\int_0^1 (f_1^{\bar{\nu} p} - f_1^{\nu p})dx = 2$$

$$\int_0^1 (f_3^{\nu p} + f_3^{\bar{\nu} p})dx = -6 \qquad (5)$$

The first was discovered by Adler, derived from the equal time commutation rules of Gell-Mann. It is of central importance to check it. It does not check the specific assumptions of the quark model as well as would the second relation in (5), due to Llewellyn Smith. The -6 here is a special consequence of quark quantum numbers.

The difference of f^{ep} (Eq.(2)) and f^{en}(Eq.(3)) is $\frac{1}{3}(u+\bar{u}) - \frac{1}{3}(d+\bar{d})$ and can be expressed via (4) as

$$f_3^{\nu p} - f_3^{\bar{\nu} p} = 6\,(f^{ep} - f^{en}) \tag{6}$$

This relation, also due to Llewellyn Smith, is to be valid at every value of x and is definitely quark dependent. It's verification would represent a fundamental demonstration of the reality of quark quantum numbers. Unfortunately its verification requires measuring the difficult quantities f_3. If only f_1 is available, we note

$$f^{ep} + f^{en} = \frac{5}{9}(u + \bar{u} + d + \bar{d}) + \frac{2}{9}(s + \bar{s}) = \frac{10}{9}(f_1^{\nu p} + f_1^{\bar{\nu} p}) + \frac{2}{9}(s + \bar{s}) \tag{7}$$

and we do have the possibility of seeing the theory is wrong right away because the $s + \bar{s}$ term must be positive (an inequality) and probably a fairly small fraction (e.g. less than 20%) of the term preceding it.[5]

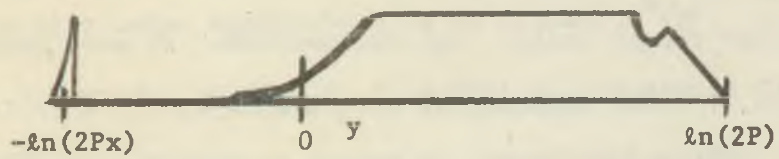
For small x all four functions in (4) should become equal to a/x, so f_3 should not go as 1/x for small x.

For $x \to 1$ where we expect only u(x) to survive, $f_-^{\bar{\nu} p}$ is the largest, and should ultimately become equal to (9/4) $f^{ep}(x)$.

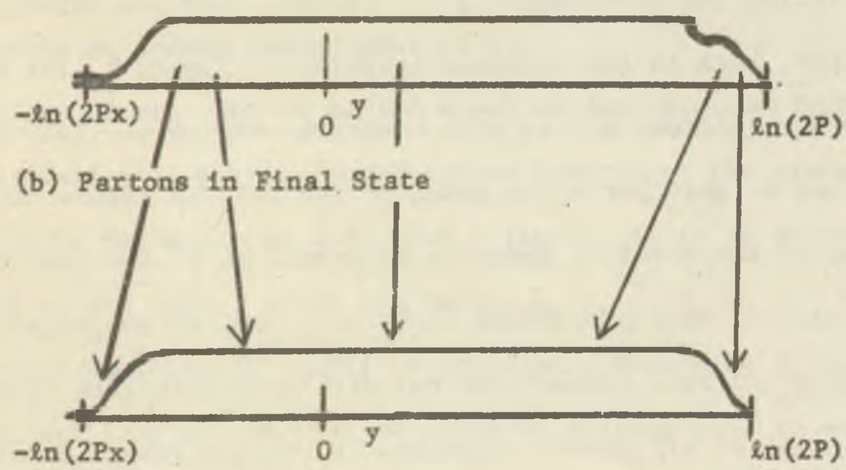
For scattering from neutrons, for example $f_-^{\bar{\nu} n}$ should give scattering from the up quarks in the neutron, equal in number to the down quarks of the proton or $f_-^{\nu p}$. We find in general antineutrino on neutron is neutrino on proton and vice versa. This should work only if $\sin^2 \theta_c = 0$. If data ever becomes accurate enough, neutron data can be used along with proton to eliminate some $\sin^2 \theta_c$ uncertain terms in other tests. One experiment, neutrino on deuterium, would directly measure $f_1^{\nu p} + f_1^{\bar{\nu} p}$ needed to test inequality (7) where it is compared to electron on deuterium.[6]

FINAL STATE HADRONS FROM DEEP INELASTIC COLLISIONS

We now come to discuss two arm experiments on $\nu + p \rightarrow \mu^- +$ one hadron + other hadrons in which some attempt is made to study the hadron products of the collision. To do this we shall have to make further theoretical suggestions. We shall suppose that immediately after a collision at a given x the partons in rapidity space appear as in Figure 3a (corresponding to Figure 1b).



(a) Partons Immediately After Interaction



(b) Partons in Final State

(c) Hadrons in Final State

FIGURE 3

This is like our original distribution of Figure 2 with an additional parton at $p_z = -Px$ and one missing at Px. Now this is an initial state and, with time, the Hamiltonian operator forms a cascade of partons producing a distribution like Figure 3b. There is a long plateau reaching from $-2Px$ to 0. Whether this really fits smoothly at 0 to the previous pleateau or there is a jog in level there I do not now know. It will not effect what I am going to say, for I will again suppose that the plateau (to the left of zero at least) is universal and does not depend on the character of the parton which produced it. This final state is then realized in nature as a set of outgoing hadron particles with finite transverse momenta $p_\perp$ and with rapidities ($y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}$ with $E^2 = p_z^2 + p_\perp^2 + M^2$ where M is the hadron mass) spread over the former range. But the character of each hadron will be determined, I shall suppose, by the partons within a finite range Δy of its own rapidity. (This may not be physically directly true, but things farther away on the plateau are universal, so product hadrons in two experiments will differ only if nearby partons are expected to be different in the two experiments.) It is seen that the thrust of these ideas is that, back in the coordinate system of Figure 1, for a fixed x as P increases, hadrons moving with momentum $-Pxz$ to the left will depend only on z and on what parton is going to the left in Figure 1b; whereas those going to the right at momentum $P\eta$ depend on η, the fact it was a proton collision, what parton was taken out, and, of course, x. Particles with finite p_z in this system come out with a distribution which is universal and independent of all these variables. We should remark at once that this may only apply in detail at very high energy indeed - scaling for the total one arm experiment may set in much before the products scale precisely - if experience with corresponding non-relativistic models is any guide. But at least the appearance of jets with finite transverse momenta should become clear early.

We shall study here, primarily the products to the left - the fragmentation products of the parton, of the quark. Neutrino experiments permit us to choose an especially simple case for consideration - the case where the left moving quark is purely of one kind. For example consider the products expected from a pure W_-, a W meson of negative helicity, which plucks a d quark from the proton and kicks it out to the left at momentum - Px as a pure up quark. The probability it does anything is d(x) but having done it the probability that a particular hadron type i has longitudinal momentum - Pxz,(i.e. a fraction z of the quark momentum)is a function of z only, appropriate to up quarks, $D_u^i(z)$. If p_i is the four momentum of the product hadron i, p is that of the proton then $z = (p \cdot p_i)/(p \cdot q)$, simply the laboratory energy of the product divided by ν, the energy loss of the lepton. The probability we get such a product using a W_- meson with $q^2 = -2M\nu x$ is $D_u^i(z)d(x)$ This could be tested by seeing if the result was indeed a product - that the z distribution aside from normalization is indeed independent of x.

Theoretically we are led to define a set of distribution functions $D_\alpha^i(z)$ which we could call parton fragmentation functions; the probability that a product i is found to the left with z (in dz) if it is known that a parton α goes to the left. α has six values u, d, $\bar{u}$, $\bar{d}$, s, $\bar{s}$. For four of these, neutrino experiments could, in principle, determine $D_\alpha^i(z)$.

If this is all true we see that we would be getting near to measuring fundamental properties of the hadron system - a limited number of distribution functions having to do with kinds of partons (which in our example are quarks). There are many obvious ways to test these ideas and I shall not attempt to choose among them to find the most easily analyzed for the experiments to be done soonest. Instead I will just give a number of theoretical examples.

For small z the universal plateau idea suggests that $D^i_\alpha(z)$ varies inversly with z like C_i/z with a constant that depends on the product hadron i but not on α.

Relations can be derived from I-spin symmetry like $D^{\pi^+}_u = D^{\pi^-}_d$, or charge conjugation like $D^{\pi^+}_u = D^{\pi^-}_{\bar{u}}$, thus there are only three independent distribution functions for pions:

$$D^{\pi^+}_u = D^{\pi^-}_d = D^{\pi^+}_{\bar{d}} = D^{\pi^-}_{\bar{u}}$$

$$D^{\pi^+}_{\bar{u}} = D^{\pi^-}_{\bar{d}} = D^{\pi^+}_d = D^{\pi^-}_u$$

$$D^{\pi^+}_s = D^{\pi^-}_s = D^{\pi^+}_{\bar{s}} = D^{\pi^-}_{\bar{s}} \quad (8)$$

Neutrino experiments have the theoretical advantage that the functions can be separated for pure quarks and hence as a product of a function of z and of x. But if less is measured, for example only the products in νp corresponding to $f_1^{\nu p}$ we have for the number of left-going hadrons i of momentum z in an experiment done at fixed x,

$$N^i(z,x) = 2\, D^i_{\bar{d}}(z)\bar{u}(x) + 2\, D^i_u(z)d(x)$$

normalized to $f_1^{\nu p}(x)$. This is a little more complicated to test for it is in general a combination of two functions depending on z. If certain special questions are asked it will factor again. For example if we ask for the number of π^+ (so $i = \pi^+$) by (8) the functions are equal and we have

$$N^{\pi^+}(z,x) = D^{\pi^+}_u(z)\, f_1^{\nu p}(x)$$

a factorization which should be easy to test, and to determine $D^{\pi^+}_u$. In the same way from $N^{\pi^-}(z,x)$ we can get $D^{\pi^-}_u(z)$.

For electron proton scattering the expected number of hadron i is a considerable tangle in general,

$$N^{i}(z,x) = \frac{4}{9}\left(u(x)\ D_{u}^{i}(z) + \bar{u}(x)\ D_{\bar{u}}^{i}(z)\right) + \frac{1}{9}\left(d(x)\ D_{d}^{i}(z) + \bar{d}(x)\ D_{\bar{d}}^{i}(z)\right) +$$

$$+ \frac{1}{9}\left(s(x)\ D_{s}^{i}(z) + \bar{s}(x)\ D_{\bar{s}}^{i}(z)\right) ,$$

but even here we may do a lot to simplify it. For example if we measure the excess of π^+ over π^-, we find again a simple product in virtue of (8)

$$N^{\pi^+}(z,x) - N^{\pi^-}(z,x) = \left[D_{u}^{\pi^+}(z) - D_{u}^{\pi^-}(z)\right]\left[\frac{4}{9}\left(u(x) - \bar{u}(x)\right) - \frac{1}{9}\left(d(x) - \bar{d}(x)\right)\right]$$

The last x dependent factor has some obvious sum rule properties in view of (1); its integral over x from 0 to 1 should be 7/9. As another example it turns out that, if we measure the number of charged k's minus the number of neutral k's, it should be a function of z times the function of x, $\frac{4}{9}(u + \bar{u}) - \frac{1}{9}(d + \bar{d})$. Thus measurements on final hadrons in electron scattering could also help in isolating the functions u(x) etc.

We have noted that a proton, when nearly a pure quark (x near 1), is a u quark. This leads us to guess that of all quarks which produce protons near $z \to 1$, u quarks do it most easily. Thus whereas all $D_{\alpha}^{p}(z)$ probably fall as a power of (1-z) as $z \to 1$ that power is probably least for $\alpha = u$. Arturo Cisneros (private communication) has suggested by analogy that for a pion or kaon near $z \to 1$, $D_{\alpha}^{i}(z)$ is largest for the quark and antiquark that make them up according to the low energy quark model. Thus $D_{u}^{\pi^+}(z) >> D_{\bar{u}}^{\pi^+}(z)$ as $z \to 1$ (although they both go to zero). If this is true there are still further ways of finding the u(x) etc., from electron experiments, because by going to z near 1 fewer $D_{\alpha}^{i}(z)$ functions are involved.

PARTON QUANTUM NUMBERS

There is one point of considerable theoretical interest that should be made. Suppose Q is some additive conserved quantum number like charge, or 3-component isospin or baryon number and Q_i is that number for a particular hadron species i that appears to the left in relative mean number $N_i(z)$. The total $\int_0^1 \sum_i Q_i N_i(z)dz$ we shall call the mean total quantum number Q for all the left-moving particles. We refer not to the value for one particular event, of course, but to the statistical average over events. Near the lower limit z = 0 various $N_i(z)$ are going to infinity as 1/z but in a neutral manner, as many of positive charge as of negative charge, for example, for the y plateau is neutral - so the integral converges, and we do not have to specify precisely where in the y plateau we cut the integral off (near z = 0) in defining the distinction of left and right momenta. Suppose for example we know the reaction is via W_- so we know we have a u quark initially to the left. Then the parton cascading and the eventual conversion to hadrons cannot change the total quantum number provided we have a sufficiently long plateau. For then all kinds of hadrons, strange and baryon, had a reasonable chance to be formed, so that the plateau is fully neutral to all quantum numbers. Under these circumstances the left mean quantum numbers $\int \sum_i Q_i D_u^i(z)dz$ must be those of the quark Q_u, or in general

$$Q_\alpha = \int_0^1 \sum_i Q_i D_\alpha^i(z)dz \qquad (9)$$

Thus, in principle, quantum numbers of the partons can be defined directly in terms of experimental quantities. The charge of a parton thus defined need not be integral, like the hadrons, for it is statistically defined. Even were it to turn out that a field theory with parton quarks does not exist, it might still come out that experiments to determine the sums in (9) from neutrino scattering to select "pure quark" states could give the characteristic third-integral quantum numbers.

However, how high in energy would we have to go to verify this? Probably pions are easier to make than kaons as products, and these are still easier than baryons. Therefore I think the required mean will be approximated most rapidly (lower Px will be sufficient) for 3-isospin, less rapidly for hyperon charge and least easily for baryon number (+ 1/3 for a quark, - 1/3 for antiquark). Therefore charge (which is 3-isospin plus 1/2 hypercharge) will only work when we have enough energy to balance hypercharge. Strangeness requires baryon number be averaged by the plateau.

The easiest to check is isospin, but that is less interesting as the characteristic one-thirds do not yet come in. The easiest place is $f_1^{\nu p}$ neutrino-proton scattering. (There is no need to isolate $f_+^{\nu p}$ from $f_-^{\nu p}$ for they both produce quarks of the same isospin, + 1/2; $\bar{d}$ and u respectively.) We measure; (Number of π^+ - Number of π^-) + 1/2 (Number of k^+ - Number of k^-) - 1/2 (Number of k^o - Number of $\bar{k}^o$) + etc., integrated over all positive z greater than some small number. The total should be + 1/2. Probably the first term alone gives the bulk of the sum.

It goes without saying that the mean quantum number of the left and the rights together is just that expected from conservation of total quantum number because that is so even for each event alone. For example $\sin^2 \theta_c$ could be measured, in principle, by a W_+ experiment, by the mean total strangeness per collision of all the products right and left at high energy.

We have said little about the products to the right; those moving in the system of Figure 1 to the right with momentum + Pξ for fixed ξ as P goes to ∞.

They are more complicated and less fundamental than those going to the left,, as they depend on all the quarks in the proton less one at x. In the special case x is very small, we can say something. Only the low x partons are disturbed, those at larger x are distributed just as in the proton in hadron-proton collisions where only the wee partons interact. Thus for sufficiently small x and for ξ not too small (not as small as x) the right fragments are unique, independent of x. In fact for small x(and also for $-q^2$ finite, $\nu \to \infty$) the proton fragments in a unique way, the same way as it does in a hadron-proton collision at very high energy. Further, under these circumstances, the lepton also fragments in a characteristic way independent of what hadron was hit, proton or whatever. This is because we assume the wee and small x region is the same for all initial hadrons. (Further details on all these matters may be found in reference 4.)

In our exercise here we have assumed many things, some of which may be inconsistent (e.g. partons as quarks interacting only for low relative momentum yet unable to come apart into real quark states). Even the basis of partons may be quite incorrect. But what the example shows clearly, nevertheless, is that deep inelastic lepton scattering has already told us much that is fundamental about the strong interactions and shows every promise of yielding much more information of an equally basic kind.

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2. J.Kuti, this conference.

3. J.Bjorken, 1971 International Symposium on Electron and Photon Interactions, Cornell University.

4. R.P.Feyman, "Photon Hadron Interactions", W.A.Benjamin Company, (1972), (to be published).

5. For further inequalities see O. Nachtmann CERN Report LPTHE 71/29 (1971).

6. This can be applied to the total cross section of ν and $\bar{\nu}$ on nuclei, for which preliminary results from 2 to 7 GeV have been presented at this meeting.

Let us measure all cross sections in unites of $Gs/2\pi$ where G is the Fermi constant and s the square of the center of mass energy. For nucleons then, our unit is GME/π where E is the laboratory energy.

The total cross section of a neutrino with a spin 1/2 particle is 2. With an antiparticle it is 2/3. Hence on a proton the cross section is

$$\sigma^{\nu p} = 2 \int_0^1 x(d + \tfrac{1}{3}\bar{u})dx$$

the factor x coming because the cross section varies with s. For neutrons we replace d by u, etc, so the mean neutrino cross section of a nucleon is

$$\sigma = \frac{1}{2}(\sigma^{\nu p} + \sigma^{\nu n}) = \int_0^1 x(d + u + \tfrac{1}{3}(\bar{d} + \bar{u}))dx$$

The antineutrino cross section is

$$\bar{\sigma} = \frac{1}{2}(\sigma^{\bar{\nu}p} + \sigma^{\bar{\nu}n}) = \int_0^1 x(\bar{d} + \bar{u} + \frac{1}{3}(d+u))dx$$

Since $\bar{d}$, $\bar{u}$ are positive, but undoubtedly less than d,u, we see that $\bar{\sigma}/\sigma$ must be substantially less than 1, but greater than 1/3.

The sum is

$$\sigma + \bar{\sigma} = \frac{4}{3}\int_0^1 x(u + \bar{u} + d + \bar{d})dx$$

However integrating (7) we have

$$\int_0^1 x(f^{ep} + f^{en})dx = \frac{5}{9}\int_0^1 x(u + \bar{u} + d + \bar{d})dx + \frac{2}{9}\int_0^1 x(s + \bar{s})dx$$

Experimentally this integral is 0.31, so if we could forget the integral $\int x(s+\bar{s})dx$ we would have $\sigma+\bar{\sigma} = \frac{4}{3}\cdot\frac{9}{5}(.31) = 0.74$. But $s+\bar{s}$ must surely be less than $d+\bar{d}$ and $u+\bar{u}$ and, when weighted by x, surely much less. It would be hard to manage to make inclusion of the last term produce more than a 10% effect. Thus we have a very stringent test of our parton quark model: $\sigma+\bar{\sigma}$ cannot exceed 0.74 and yet almost surely cannot fall below 0.74 by more than 10%. One can also calculate upper limits for $\sigma^{\nu p} + \sigma^{\bar{\nu}p}$ and $\sigma^{\nu n} + \sigma^{\bar{\nu}n}$ separately (using other proportions of f^{ep} and f^{en}) they are .64 and .84 respectively.

These numerical estimates must be revised by a few percent for we have neglected $\sin^2\theta_c$. They are valid only at asymptotic energy, of course, but T.D.Lee has pointed out that electron data indicate that this should only require a few GeV. These results are those of Bjorken.

DISCUSSION

Moffat: Why have not quarks and partons been produced at SLAC and ISR?

Feynman: I am so used to the idea that quarks are not produced that I forgot to mention that there is a paradox: is it possible that quarks only have interactions for finite relative momenta and yet they do not get isolated, they cannot get separated? I do not understand that at all, and I am happy with that. I like paradoxes. So what I am trying to do is this: I deduce everything I can from a quark model, except that they should come apart. And in the struggle to be consistent, to have quarks inside which do not come apart, I have to figure all this out and I must say that we might be headed for a paradox. One of the two things can happen: either, one, we find that all this quark stuff, these quark quantum numbers, do not work and then that is very easy: there are no quarks, so they do not come apart; or, two,- mystery of mysteries!- All these predictions / 0.74 for average total cross-section of neutrinos and antineutrinos on nucleons, and so on/ all work, and yet the quarks do not come apart! That will be interesting. I was very interested to notice that if the quarks come apart into a kind of comet tail of hadrons, into a plateau in rapidity space, they can still disintegrate consistently that way, even tho they have non-integral quantum numbers. It is still possible that we do not have any inconsistency. So I am going to assume as long as I can, a paradoxical combination of things, that quarks cannot come apart as free and they are inside of the particles. I do not know how. But that is the fun.

Bell: Part of the trouble seems to come just from the idea that the interactions are restricted over a small interval in the x pace. Is that an essential part of the model, could you say a little where this particular idea comes from?

Feynman: It is essential to the totality of what I said. But it is not essential to everything. Various things that I said do not require that particular assumption, other things definitely do. I got that idea from the fact that transverse momenta were limited. That may be illegal and irrational. I do not know how well based this idea is. The ideas of limiting fragmentation due to Yang et al,/that the fragments going to one side and the other in hadron collisions depend only on the objects that were going in that direction/ if they continue to work, seem to me also to imply that interactions only occur over a finite range of y space /not x space/ which means relative momentum is finite. Well, I make this assumption I do not know why, and how its based.

Weisskopf: What you said I understand only as an assumption that there is just not much interaction between these particles of relatively high momentum, but not that the interaction is only between particles of almost equal y .

Feynman: You are absolutely right. The only thing I really need is that in the strong interaction the interaction of things with high relative momenta is small. And the only space left for me to put the interaction was at finite relative y.

Kögerler: Is there any theoretical reason that the final state interaction between the partons preserves scaling?

Feynman: If the final state interaction is over a limited range of relative momentum then we come to scaling.

Kögerler: This is an assumption?

Feynman: Yes, I use this all the time. I do not know exactly what I need to get to each place. The Babilonians did geometry by knowing a whole lot of theorems and when they forgot one theorem they proved it by those that they remembered, but did not organize in a way that they started at something called assumptions and everything else was deduced from that. And I got myself into a terrible tangle as I have all these facts. And when I lose one, I remember from an other one. But I cannot remember where I started any more. I start in the middle, I do not have a logical way of doing it. I think it scales I think the interaction is over a finite range; I think the transverse momenta are finite; I suspect there is a relation in the logic, that all these are not independent assumptions. But I do not know....

Marshak: Do you think that the reason we do not see quarks is they posses very large masses and interact strongly with each other in peculiar ways?

Feynman: This is a completely different direction, but not the direction I am going. If the quark masses are high, then you have strong interactions between them, strong interaction makes the scaling hopeless to understand, transverse momenta become large, the whole thing goes haywire, and the picture is very bad. My quarks have small masses, and they do not came apart because of something I'll tell you about 25 years from now. The masses that I want are so low that we would have absolutely definitely seen them. Thus perhaps the whole thing is nonsense, and that experiment will tell us very soon. Or if it is right, then this is very very exciting, because we are approaching a paradox, and the hope of physics is to find a paradox. This is the real way of making a revolution. We have to find a place where we are shocked. And I think we are getting near to one. I hope we are....

Weisskopf: If somebody detects a quark with a high mass - God forbid.....

Feynman: Right! God forbid!

Weisskopf: Still, we would not be lost, because it may be that the quark has an effective mass inside which is small /with a scalar interaction or something like that/ so it need not be in contradiction with all that you say.

Feynman: I would try, however to answer as you said, by "God forbid": a heavy quark, a real heavy quark is as embarrassing and difficult to understand as no quarks at all. It would not help me much except to confirm the reality of quark quantum numbers, a thing which I believe we can confirm without finding any quarks.

Achiman: In one of your guesses you said that neutral quarks probably exist also; Where did they come from, where can you put these neutral quarks in your theory?

Feynman: I did not say neutral quark, I said neutral parton, some particle other than a quark which is neutral. I have not found any way, by electron scattering or neutrino scattering, to tell us anything more about these neutral partons except their existence, induced by the fact that the conservation of momentum does not work with the charged quarks.

Marshak: I am suggesting a more specific model in analogy to the strong cubic W boson model where we introduce a new quantum number like cubic parity in the strong interaction among the triplet of W's. Let us translate this idea to a triplet of partons. Then you do not have a strong interaction between two partons, but one between the parton and the antiparton to give the pion and also one among the three partons to give the proton. Perhaps a new selection principle can reconcile scaling and large masses for the partons?

Feynman: Your are quite right, I have tried to do things vaguely like that. But at the present time I would rather like to have some kind of information showing quark quantum numbers are real before I have the energy to move on to such definite questions.

Budini: In view of the recent theory of Salam and Weinberg and so on to unify weak and electromagnetic interaction can you tell us what this would do?

Feynman: I believe, if I have "not-understood" Weinberg's theory correctly, this theory does not change the lepton-hadron interaction at all. All I have used here is the lepton-hadron interaction.

Somebody from the audience: What would be the effect of introducing partons with quantum numbers different from those of the normal quarks?

Feynman: Another system of partons with other quantum numbers /such as the triplet quark model and other models/ definitely have a big effect. It changes many of the numerical coefficients and in a paper by Nacthman /CERN Report LPTHE 71/29 /1971// for example, it is shown that already the crude total neutrino-nucleon cross section, if it remain near the present value, is almost ready to eliminate most of the alternatives. And new more accurate measurements of neutrino and antineutrino-nucleon cross section will clarify the situation concerning quark quantum numbers very soon.

DEEP-INELASTIC LEPTON-NUCLEON SCATTERING

J.Kuti, Massachusetts Institute of Technology, Cambridge
and Eötvös University, Budapest

I shall discuss the following topics:

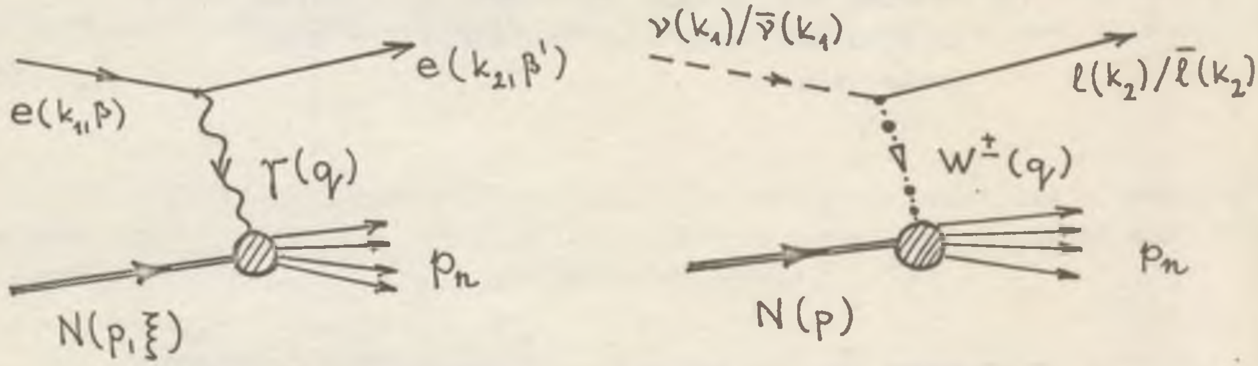
(1) Kinematics of deep-inelastic lepton-nucleon scattering
(2) Experimental results with comments on sum rules and inequalities
(3) Probing the partons and light-cone physics in spin-dependent deep-inelastic electroproduction

1. KINEMATICS OF DEEP-INELASTIC LEPTON-NUCLEON SCATTERING

I briefly summarize the kinematics mainly because not all of you are familiar with the spin-dependent scattering. Then in the parallel discussion of inelastic lepton-nucleon processes I can introduce the structure functions and cross sections which have been analyzed at this conference.

(i) The process of inelastic electron-nucleon scattering is shown in Fig.1 where an electron with four-momentum k_1, covariant spin β is incident on a nucleon of four-momentum p, covariant spin ξ and scatters with resulting final four-momentum k_2 by an angle θ due to the exchange of a single photon of four-momentum q. We do not observe the hadronic final state with four-momentum $p_n = q + p$, and sum over polarization β' of the scattered electron.

(i) inelastic electron scattering (ii) inelastic neutrino scattering



$e(k_1,\beta) + N(p,\xi) \rightarrow e(k_2,\beta') + \text{anything}$ $\nu(k_1)/\bar{\nu}(k_1) + N(p) \rightarrow \ell(k_2)/\bar{\ell}(k_2) + \text{anything}.$

Fig. 1

The differential cross section of process (i) is given by

$$d\sigma_{\xi\beta} = \frac{\alpha^2}{4\pi} q^{-4} \left[(pk_1)^2 - m^2 M^2\right] L^{\beta}_{\mu\nu} \cdot W_{\xi}^{\nu\mu}(p,q) \frac{d^3\vec{k}_2}{E_2} \tag{1.1}$$

where

$$L^{\beta}_{\mu\nu} = \sum_{\beta'} \bar{u}_{\beta'}(k_2) \gamma_\mu u_\beta(k_1) \bar{u}_\beta(k_1) \gamma_\nu u_{\beta'}(k_2).$$

is the leptonic piece of the cross section, and

$$W^{\xi}_{\mu\nu} = \int d^4x\, e^{iqx} \langle p,\xi | J_\mu(x) J_\nu(0) | p,\xi \rangle,$$

describes the hadronic matrix element.* $J_\mu(x)$ is the hadronic part of the electromagnetic current.

Next, I split $L^{\beta}_{\mu\nu}$ and $W^{\xi}_{\mu\nu}(p,q)$ into symmetric and antisymmetric parts in the μ, ν indices

$$L^{\beta}_{\mu\nu} = L^{S}_{\mu\nu} + i L^{A}_{\mu\nu} ,$$

$$W^{\xi}_{\mu\nu}(p,q) = W^{S}_{\mu\nu}(p,q) + i W^{A}_{\mu\nu}(p,q) ,$$

with

$$L^{S}_{\mu\nu} = 2\,(k_{1\mu} k_{2\nu} + k_{1\nu} k_{2\mu} - k_1 \cdot k_2\, g_{\mu\nu} + m^2 g_{\mu\nu}), \qquad (1.2)$$

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* normalization of states $\langle p', r | p, s \rangle = 2 p_0 (2\pi)^3 \delta(\vec{p} - \vec{p}\,') \delta_{rs}$

spinor normalization $\bar{u}(p) u(p) = 2M$

metric $p^2 = M^2, \quad k_1^2 = m^2, \quad \xi^2 = \beta^2 = -1$

$\epsilon_{0123} = 1$

$q^2 = (k_1 - k_2)^2 = -Q^2$ (in the lab system of the nucleon)

$Q^2 = 4 E_1 E_2 \sin^2 \frac{\theta}{2}, \quad k_{10} = E_1, \quad k_{20} = E_2,$

energy loss of the electron in the laboratory $\nu = \dfrac{pq}{M},$

invariant mass W of the hadron final state $W^2 = (p+q)^2,$

$\alpha = \dfrac{e^2}{4\pi} \doteq \dfrac{1}{137}.$

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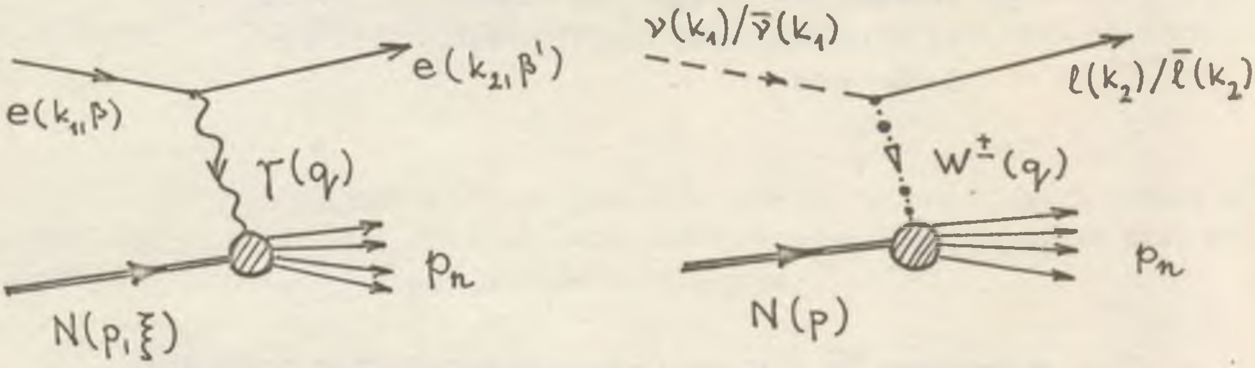
(i) inelastic electron scattering (ii) inelastic neutrino scattering



$e(k_1,\beta) + N(p,\xi) \rightarrow e(k_2,\beta') + \text{anything}$ $\quad$ $\nu(k_1)/\bar{\nu}(k_1) + N(p) \rightarrow \ell(k_2)/\bar{\ell}(k_2) + \text{anything}.$

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where

$$L^{\beta}_{\mu\nu} = \sum_{\beta'} \bar{u}_{\beta'}(k_2) \gamma_\mu u_\beta(k_1) \bar{u}_\beta(k_1) \gamma_\nu u_{\beta'}(k_2).$$

is the leptonic piece of the cross section, and

$$W_{\mu\nu}^{\xi} = \int d^4x\, e^{iqx} \langle p,\xi | J_\mu(x) J_\nu(0) | p,\xi \rangle,$$

describes the hadronic matrix element.* $J_\mu(x)$ is the hadronic part of the electromagnetic current.

Next, I split $L_{\mu\nu}^{\beta}$ and $W_{\mu\nu}^{\xi}(p,q)$ into symmetric and antisymmetric parts in the μ, ν indices

$$L_{\mu\nu}^{\beta} = L_{\mu\nu}^{S} + i L_{\mu\nu}^{A},$$

$$W_{\mu\nu}^{\xi}(p,q) = W_{\mu\nu}^{S}(p,q) + i W_{\mu\nu}^{A}(p,q),$$

with

$$L_{\mu\nu}^{S} = 2\,(k_{1\mu}k_{2\nu} + k_{1\nu}k_{2\mu} - k_1\cdot k_2\, g_{\mu\nu} + m^2 g_{\mu\nu}), \tag{1.2}$$

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* normalization of states $\langle p', r | p, s\rangle = 2p_0(2\pi)^3\delta(\vec{p}-\vec{p}\,')\delta_{rs}$

spinor normalization $\bar{u}(p)u(p) = 2M$

metric $p^2 = M^2,\quad k_1^2 = m^2,\quad \xi^2 = \beta^2 = -1$

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invariant mass W of the hadron final state $W^2 = (p+q)^2$,

$\alpha = \frac{e^2}{4\pi} \doteq \frac{1}{137}$.

and

$$L^{A}_{\mu\nu} = 2m\,\epsilon_{\mu\nu\varrho\sigma}\, q^{\varrho}\, \beta^{\sigma} \tag{1.3}$$

PT-symmetry allows us to write

$$W^{S}_{\mu\nu}(p,q) = \frac{1}{2}\sum_{\xi}{}' \int d^4x\, e^{iqx} \langle p,\xi | J_\mu(x) J_\nu(0) | p,\xi \rangle$$

and

$$W^{A}_{\mu\nu}(p,q) = \frac{1}{2}\int d^4x\, e^{iqx} \Big(\langle p,\xi | J_\mu(x) J_\nu(0) | p,\xi \rangle - \langle p,-\xi | J_\mu(x) J_\nu(0) | p,-\xi \rangle \Big)$$

$L^{S}_{\mu\nu}$ is independent of the electron's spin, and $W^{S}_{\mu\nu}(p,q)$ does not depend on the spin of the nucleon target, their combination yields the spin-averaged cross section. In what follows the lepton mass m will be neglected which is a good approximation for $-q^2 \gg m^2$

To work out explicit formulae for the cross sections, I define four structure functions in $W^{\xi}_{\mu\nu}(p,q)$:

$$W^{S}_{\mu\nu}(p,q) = \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right) 4M\pi\, W_1(q^2,\nu) +$$

$$+ \left(p_\mu - \frac{pq}{q^2} q_\mu\right)\left(p_\nu - \frac{pq}{q^2} q_\nu\right) \frac{4\pi}{M}\, W_2(q^2,\nu), \tag{1.4}$$

and

$$W^{A}_{\mu\nu}(p,q) = \epsilon_{\mu\nu\varrho\sigma}\, q^{\varrho} \xi^{\sigma} d(q^2,\nu) + (\xi q)\, \epsilon_{\mu\nu\varrho\sigma}\, q^{\varrho} p^{\sigma} g(q^2,\nu). \tag{1.5}$$

$W_1(q^2,\nu)$ and $W_2(q^2,\nu)$ have been measured in the SLAC-MIT experiment,[1] while $d(q^2,\nu)$ and $g(q^2,\nu)$ will appear in polarization effects only.[2]

The spin-averaged cross section measured in the laboratory is written as

$$\frac{d^2\sigma}{d\Omega dE_2} = \frac{d\sigma_M}{d\Omega}\left\{ W_2(q^2, \nu) + 2\tan^2\frac{\theta}{2}\cdot W_1(q^2,\nu)\right\}, \qquad (1.6)$$

where $d\sigma_M/d\Omega$ is the Mott cross section for a pointlike charge

$$\frac{d\sigma_M}{d\Omega} = \frac{\alpha^2\cos^2\frac{\theta}{2}}{4E_1^2\sin^4\frac{\theta}{2}} .$$

The derivation of (1.6) is straightforward algebra from the definition (1.1) using Eqs.(1.2) and (1.4).

The longitudinal and transverse structure functions are defined by

$$W_L = W_2\left(1+\frac{\nu^2}{Q^2}\right) - W_1 ,$$

$$W_T = W_1 .$$

With this definition, the ratio of the longitudinal photoabsorption cross section σ_L of virtual photons over the transverse cross section σ_T can be written as

$$R = \frac{\sigma_L}{\sigma_T} = \frac{W_L(q^2,\nu)}{W_T(q^2,\nu)} . \qquad (1.7)$$

The spin-dependent cross section, proportional to $L^A_{\mu\nu}\cdot W^{\nu\mu}_A$,is bilinear in ξ and β . Clearly, it is only the relative orientation of the spin vectors of the incident beam and the target which counts in the asymmetry

$$A = \frac{d\sigma^{\uparrow\downarrow} - d\sigma^{\uparrow\uparrow}}{d\sigma^{\uparrow\downarrow} + d\sigma^{\uparrow\uparrow}} . \qquad (1.8)$$

I write down an expression for the asymmetry which is valid if $d\sigma^{\uparrow\uparrow}$ is the cross section when the spins of the electron and the nucleon are parallel and along the direction of motion of incident electron (right-handed electron); $d\sigma^{\uparrow\downarrow}$ is the cross section for antiparallel spins

$$A = \frac{1}{\pi} \frac{M^{-1}(E_1 + E_2 \cos\theta)\cdot d + (E_1 + E_2)(E_1 - E_2\cos\theta)\cdot g}{4W_1 + 2\cot g^2\frac{\theta}{2}\cdot W_2} \qquad (1.9)$$

This expression is positive in pointlike scattering.

(ii) The kinematics of deep-inelastic neutrino (antineutrino)-nucleon scattering[3] can be calculated from an effective semileptonic Lagrangian $\mathcal{L}_{eff}$. Consider the process (Fig.1)

$$\nu(k_1)/\bar{\nu}(k_1) + N(p) \longrightarrow \ell(k_2)/\bar{\ell}(k_2) + \text{anything}$$

We assume

$$\mathcal{L}_{eff.} = \frac{G}{\sqrt{2}}\left(j^\lambda \cdot J_\lambda^{(+)} + \text{hermitian conjugate}\right), \qquad (1.10)$$

where $G = 10^{-5} M_p^{-2}$ is the weak coupling constant, $J_\lambda^{(+)}(x)$ is the weak current constructed from the hadronic fields, and j_λ denotes the leptonic current

$$j^\lambda(x) = \bar{\psi}_e(x)\gamma^\lambda(1-\gamma_5)\psi_{\nu_e}(x) + \bar{\psi}_\mu(x)\gamma^\lambda(1-\gamma_5)\psi_{\nu_\mu}(x)$$

Eq.(1.10) represents an effective Lagrangian for the semileptonic weak interactions; the existence of an intermediate vector boson $W^\pm$ would require the substitution

$$G^2 \longrightarrow \frac{G^2}{\left(1-\frac{q^2}{M_W^2}\right)^2},$$

in the cross section formulae. The differential cross section is given by

$$\frac{d^2\sigma^{\nu,\bar{\nu}}}{dQ^2\,d\nu} = \frac{G^2}{2\pi M^2}\,\frac{E_2}{E_1}\left(\cos^2\frac{\theta}{2}\,W_2^{\nu,\bar{\nu}} + 2\sin^2\frac{\theta}{2}\,W_1^{\nu,\bar{\nu}} \mp \right.$$
$$\left. \mp\,\frac{E_1+E_2}{M}\sin^2\frac{\theta}{2}\,W_3^{\nu,\bar{\nu}}\right) + O(m^2), \qquad (1.11)$$

where θ is the angle between the directions of the initial and final leptons in the laboratory with incoming energy E_1, and outgoing energy E_2; m denotes the mass of the lepton in the final state. The neutrino (antineutrino) structure functions are defined in the hadronic matrix element

$$W_{\mu\nu}^{\nu,\bar{\nu}} = \frac{1}{8\pi}\sum_{\xi}\int d^4x\, e^{iqx}\langle p,\xi|\, J_\mu^{(\mp)}(x)\, J_\nu^{(\pm)}(0)\,|p,\xi\rangle =$$
$$= -g_{\mu\nu}W_1^{\nu,\bar{\nu}} + \frac{p_\mu p_\nu}{M^2}W_2^{\nu,\bar{\nu}} - \frac{i\epsilon_{\mu\nu\alpha\beta}\,p^\alpha q^\beta}{2M^2}\cdot W_3^{\nu,\bar{\nu}} + \ldots, \qquad (1.12)$$

the contribution of the omitted structure functions to the cross sections (denoted by dots) is of the order of $O(m^2)$. $W_{\mu\nu}^{\nu,\bar{\nu}}$ in (1.12) is averaged over the spin states of the initial nucleon. You may have noticed that $W_1^{\nu,\bar{\nu}}$ and $W_2^{\nu,\bar{\nu}}$ are defined slightly differently from W_1 and W_2 in electroproduction.

The hadronic weak current can be separated into vector and axial vector parts

$$J_\lambda^{(+)} = V_\lambda^{(+)} - A_\lambda^{(+)}$$

In the language of the quark model

$$V_\lambda^{(+)}(x) = \bar{p}(x)\gamma_\mu \left(n(x)\cos\theta_c + \lambda(x)\sin\theta_c \right),$$

$$A_\lambda^{(+)}(x) = \bar{p}(x)\gamma_\mu\gamma_5 \left(n(x)\cos\theta_c + \lambda(x)\sin\theta_c \right), \qquad (1.13)$$

where the same Cabibbo angle θ_c was chosen both in the vector and axial vector currents; $J_\lambda^{(-)} = (J_\lambda^{(+)})^\dagger$. The quark fields are denoted in (1.13) by $p(x), n(x)$ and $\lambda(x)$

Scale invariance implies that in the deep inelastic limit the dimensionless structure functions $W_1^{\nu,\bar{\nu}}$; $\nu M^{-1} W_2$; $\nu M^{-1} W_3$ "scale" ; they are the functions of the scaling variable ω only:

$$W_1^{\nu,\bar{\nu}}(q^2,\nu) \xrightarrow[\substack{\nu\to\infty \\ \omega \text{ fixed}}]{} F_1^{\nu,\bar{\nu}}(\omega)$$

$$\nu M^{-1} W_i^{\nu,\bar{\nu}}(q^2,\nu) \longrightarrow F_i^{\nu,\bar{\nu}}(\omega), \quad i = 2, 3.$$

To discuss the consequences of this hypothesis I will define structure functions which describe the production of non-strange (f) and strange ($\tilde{f}$) final states

$$F_i^{\nu,\bar{\nu}}(\omega) = \cos^2\theta_c \cdot f_i^{\nu,\bar{\nu}}(\omega) + \sin^2\theta_c \, \tilde{f}_i^{\nu,\bar{\nu}}(\omega), \qquad i = 1, 2, 3.$$

Feynman used the notation $f_1^{\nu p}(x) = 2f_1^{\nu}(\omega = x^{-1})$ in his lecture .
Assuming scale invariance, Eq.(1.11) implies

$$\lim_{E\to\infty} \sigma^{\nu,\bar{\nu}}(E) = \frac{G^2 M}{\pi}\cdot E \int_1^{\infty} d\omega \left(\frac{F_2^{\nu,\bar{\nu}}(\omega)}{2\omega^2} + \frac{F_1^{\nu,\bar{\nu}}(\omega)}{3\omega^3} \mp \frac{F_3^{\nu,\bar{\nu}}(\omega)}{3\omega^3} \right). \tag{1.14}$$

Eq.(1.14) is the basic formula for comparison with the neutrino (antineutrino) data.

2. EXPERIMENTAL RESULTS WITH COMMENTS ON SUM RULES AND INEQUALITIES

In this Section I will review the most important results from the SLAC-MIT experiment. For an excellent report on the present status of the experiment I refer you to the review paper by Friedman and Kendall[5].

T.D Lee has discussed[6] Bjorken's conjecture that in the limit of $-q^2$ and ν much larger than the characteristic dimensional paramaters of the nucleon, with the ratio $\omega = 2M\nu/Q^2$ held fixed, the two dimensionless structure functions $2MW_1$ and νW_2 "scale", they become functions of the scaling variable ω only

$$2MW_1(q^2,\nu) \xrightarrow[\omega \text{ fixed}]{Q^2,\nu \to \infty} F_1(\omega)$$

$$\nu W_2(q^2,\nu) \longrightarrow F_2(\omega).$$

Feynman used the notation : $f^{ep}(x) = F_1(\omega = x^{-1})$ in his lecture.[4]

In order to test the scaling hypothesis, it is necessary to separate W_1 and W_2 from the measured differential cross sections.

This separation has another important application in the theoretical analysis: the values of $R = \sigma_L / \sigma_T$ yield information about the constituents' spin inside the nucleon.

The kinematic region where proton data was available and the subregion in which separation was carried out are shown in Fig.2. The heavy line bounds all data points for scattering of electrons measured at 6°, 10°, 18°, 26°, and 34° . The area marked "Separation Region" includes all points where data at three or more angles exist. In this region W_1 and W_2 can be separately calculated from the cross sections without assumptions about R. In Fig.3 the measured values of R are shown as a function of Q^2. Data from a range of W, (roughly from 2 to about 4 GeV) are averaged for each value of Q^2. If we assume that R is constant in the measured kinematic range, the average value of R for the proton is 0.18 ± 0.10 .

In a preliminary analysis of the new body of electron-proton data taken in conjunction with the deuterium measurements an average value of $R = 0.14 \pm 0.10$ was found.[5] Fig.4 shows the preliminary results for R, for hydrogen and for deuterium as functions of Q^2.

In the parton model with spin 1/2 constituents the longitudinal cross section vanishes,[7] therefore R=0. The same prediction has emerged of course, from the light-cone algebra of free quark fields.[8] This morning Feynman discussed the important role of $R = \sigma_L / \sigma_T$ in our understanding of the nucleon structure. If you look at the old data (Fig 3.) it is obvious that R is small, it may even vanish in the asymptotic scaling limit. The vague indication for a vanishing R at very large Q^2 is there from the preliminary analysis of the new body of data (Fig.4), however we should better wait for the final results of the experimentalists.

The validity of scaling for $(\nu W_2)_{proton}$ has been studied inside the "Separation Region" of Fig.2. Outside the separation region, only consistency with scaling can be studied. Region I in Fig.2 indicates the domain where the data are consistent with scaling in ω. Region II shows the extension of the scaling region if the data are analyzed in $\omega' = 1 + \frac{W^2}{Q^2}$.

I summarize here our present knowledge about the scaling behavior of the data:[5]

In Region A of Fig.2 $(\omega < 4)$ the experimental values of νW_2 scale for $W > 2.6$ GeV. This region covers data with W between 2.6 GeV and 4.9 GeV and with Q^2 between $2(GeV/c)^2$ and $20(GeV/c)^2$.

In Region B $(4 < \omega < 12)$ for $W > 2$ GeV and $Q^2 > 1(GeV/c)^2$, νW_2 is constant within the error bars and scales in ω; the data points are in the range 2 GeV $< W <$ 5 GeV, and $1(GeV/c)^2 < Q^2 < 7(GeV/c)^2$.

In Region C $(\omega > 12)$ there are only few points above $Q^2 = 1(GeV/c)^2$ and no points above $Q^2 = 2(GeV/c)^2$. There are no measurements of R in this region, and νW_2 is sensitive to the variation of R. Scaling cannot be tested critically in this region.

One of the most remarkable aspects of the above results is that already at $Q^2 \sim 1(GeV/c)^2$, $W \sim$ 2-3 GeV, the scaling limit is reached with considerable accuracy. This fact can hardly be explained by pure dimensional analysis. "Precocious scaling" is not well understood dynamically and the question is whether this feature of the data is a reasonable expectation or not.[15] No convincing answer has emerged yet.

A preliminary determination of R for the deuteron is consistent with the assumption $R^D = R^P$ which allows the extraction of νW_2^D from the available data. In Fig.5 (a) displays νW_2^D for $\theta = 6°$ and $10°$ and (b) for $18°$, $26°$, and $34°$, as functions of ω. Data points for νW_2^P measured in the same experiments are also shown there.[5] The deuteron results exhibit scale invariance within the experimental errors.

The neutron cross sections have been extracted from the deuteron cross sections in impulse approximation. However, a number of corrections have been considered to improve the impulse approximation:

(i) corrections for the internal motion of the nucleons within the deuteron which is called smearing;

(ii) Glauber correction (less than one percent)

(iii) mesonic exchange currents (they are assumed to be small from elastic electron-deuteron scattering)

(iv) final state interactions
(v) possible off-mass shell effects
} none of these effects can well be estimated at present

Fig.6 shows the values of $(D/H_s - 1)$ which represent within the limitations described in Ref. 5 the values of the smeared neutron cross sections divided by the smeared proton cross section. The points plotted against $x = \omega^{-1}$ represent data with $Q^2 \geq 1\,GeV^2$ and $W \geq 2$ GeV . The ratio of the neutron and proton cross sections is consistent with a single function of ω , therefore within the experimental errors the neutron cross sections exhibit scaling.

The effect of the calculated smearing correction is shown in Fig.7 The dashed line goes through the corrected points of Fig.6, the solid curve corresponds to the dashed one before smearing corrections.

The working hypothesis that quarks are partons has been used to derive an interesting inequality for the inelastic structure functions. Nachtman[9] has shown that the ratio of the deep-inelastic structure functions of the neutron and proton is limited by the values 4 and 1/4 if only isospin symmetry is assumed. To explain the low ratio W_{2n}/W_{2p} J I Friedman has pointed out last year at the Cornell conference[1] that if one supposes a strong anti-correlation in the I=1 state for two valence quarks with low relative momentum then there could be a considerable reduction in the scattering from neutron relative to the proton. Feynman has discussed this in the previous lecture[4]. He assumes that for x near 1 the difficulty of pushing all the quarks but one to low x may depend on the total quantum number of the state of these low quarks. If the I=0 (and quark number two) state dominates for $x \sim 1$, so W_2^n / W_2^p must approach 1/4 as $x \to 1$.

You have seen the data; the measured ratio W_2^n / W_2^p appears to fall to as low a value as 0.35 ± 0.07 at $x \simeq 0.8$. It will be extremely difficult to get closer to x=1 experimentally.

In Fig.8 $\nu(W_2^p - W_2^n)$ smeared is shown as a function of x. You may have noticed a maximum in the data at about $x = \frac{1}{3}$.

There are two integrals over the scaling function $F_2(\omega)$ which are important in testing sum rules of the parton model and the light-cone algebra of currents:

$$I_1^{p,n} = \int_1^\infty \frac{d\omega}{\omega^2} F_2^{p,n}(\omega) , \qquad I_2^{p,n} = \int_1^\infty \frac{d\omega}{\omega} F_2^{p,n}(\omega) . \qquad (2.1)$$

The experimental values of the integrals evaluated from interpolated data at fixed Q^2 are given in Table 1. The cut-off value (ω_m) of the integrals is indicated in the Table[5].

Table 1

	Measurement	ω_m	$Q^2 (GeV/c)^2$
I_1^P	0.159 ± .005	20	1.0
	0.165 ± .005	20	1.5
	0.172 ± .009	20	1.5
	0.154 ± .005	12	2.0
I_1^N	0.120 ± .008	20	1.0
	0.115 ± .008	20	1.5
	0.107 ± .009	12	2.0
I_2^P	0.739 ± .029	20	1.0
	0.761 ± .027	20	1.5
	0.780 ± .04	20	1.5
	0.607 ± .021	12	2.0
I_2^N	0.592 ± .051	20	1.0
	0.584 ± .050	20	1.5
	0.429 ± .036	12	2.0
$I_2^P - I_2^N$	0.147 ± .059	20	1.0
	0.177 ± .057	20	1.5
	0.178 ± .042	12	2.0

The values of the integrals $I_1^{p,n}$ are not sensitive to the upper limit of integration beyond $\omega_m = 20$. If we assume a constant value, of about 0.35, for νW_2 for $\omega > \omega_m$, the integrals in Feynman's notation have the following numerical values

$$I_1^p = \int_0^1 x f^{ep}(x)\, dx = 0.18 \pm 0.01,$$
$$I_1^n = \int_0^1 x \cdot f^{en}_{\omega}(x)\, dx = 0.14 \pm 0.01. \qquad (2.2)$$

Feynman has discussed what these numbers tell us in the quark-parton model.

The integrals I_2^{pn} represent the sum of the squared charge of the partons. Since the number of partons goes like $\log \frac{P}{M}$, these integrals diverge for $P \to \infty$. The divergence is removed from the difference $I_2^p - I_2^n$. A very simple and definite sum rule for the difference $I_2^p - I_2^n$ based upon the radical assumption that the core of $q\bar{q}$-pairs carries vacuum quantum numbers so that the isotopic spin of the nucleon is completely carried by the valence quarks[10] can be written as

$$I_2^p - I_2^n = e_u^2 - e_d^2 = \frac{1}{3}, \qquad (2.3)$$

where e_u^2 and e_d^2 are the squared charges of up quarks and down quarks, respectively. Unfortunately it is difficult to extract the correct value of the left-hand side in (2.3) due to ambiguities at large ω. If I extrapolate $\nu W_2^p - \nu W_2^n$ for $\omega > \omega_m$ on the basis of Regge theory, a rough estimate of $I_2^p - I_2^n = 0.22 \pm 0.07$ can be obtained.[5] We have to wait for more accurate data at large ω. The present data suggests that it may be a pretty rough simplification to picture the fast moving nucleon as three valence quarks decoupled from the infinite sea of $q\bar{q}$-pairs which carries vacuum quantum numbers.

3. SPIN-DEPENDENT DEEP INELASTIC ELECTROPRODUCTION

This experiment is feasible now. It is in the stage of preparation at the Stanford Linear Accelerator Center. The approved SLAC proposal of V.W.Hughes et al[10] describes a general scheme of the experiment which is similar to the experiments done at SLAC on deep inelastic electron scattering, and, in particular, would be very similar to the SLAC experiments which measured asymmetries in inelastic scattering of electrons from polarized protons in a search for T-invariance violation, and in elastic electron-proton scattering.

The scattered electron only is observed, and its energy and scattering angle is measured with either the 8 GeV/c or 20 GeV/c spectrometer. The principal differences of the proposed experiments are first that a polarized electron beam, obtained by accelerating polarized electrons from a low energy polarized electron source (injector) will be used. Secondly, the target will be a polarized proton target with proton polarization longitudinal to the direction of the incident electron beam. Reversal of either the electron polarization or the proton polarization can be done to obtain the antiparallel and parallel cases. The experiment is scheduled for the year 1973.

Spin dependent effects will be analyzed by measuring the asymmetry in counting rate

$$\epsilon = \frac{N^{\downarrow} - N^{\uparrow}}{N^{\downarrow} + N^{\uparrow}} ,$$

where $N^{\uparrow}$ ($N^{\downarrow}$) is the number of counts per unit incident beam for target polarization along (opposite to) the target direction. The target polarization has to be reversed periodically in order to avoid systematic errors. The asymmetry A defined in (1.8) is related to the experimentally measured asymmetry ϵ by

$$A = \frac{\epsilon}{P_T \cdot H_F} ,$$

where P_T is the target proton polarization and H_F is a function of the counts due to hydrogen in the target. The asymmetry A would be equal to ϵ for a hundred percent polarized target consisting of pure hydrogen. I wrote down the expression for A in terms of W_1, W_2 and the spin-dependent structure functions in (1.5)

A sizeable asymmetry would be expected for a proton with quark constituents.[10,12] It is important to note, however, that the asymmetry is bounded kinematically through the spin averaged structure functions W_1 and W_2. The most general restriction is given by the positivity condition[13]

$$0 \le \sum_n \delta(p+q-p_n) \left| \langle p,\alpha | J_\mu(0) | n \rangle a^\mu + \langle p,\beta | J_\mu(0) | n \rangle b^\mu \right|^2 =$$
$$= W^{\alpha\alpha}_{\mu\nu}(p,q) a^\mu a^{\nu *} + W^{\beta\beta}_{\mu\nu}(p,q) b^\mu b^{\nu *} + W^{\alpha\beta}_{\mu\nu}(p,q) a^\mu b^{\nu *} +$$
$$+ W^{\beta\alpha}_{\mu\nu}(p,q) b^\mu a^{\nu *}, \tag{3.1}$$

where

$$W^{\alpha\beta}_{\mu\nu}(p,q) = \int d^4x \, e^{iqx} \langle p,\alpha | J_\mu(x) J_\nu(0) | p,\beta \rangle .$$

α and β are arbitrary polarization vectors of the proton; a_μ and b_μ are arbitrary complex four-vectors. From (3.1) with straightforward matrix algebra we get two inequalities for the spin-dependent structure functions

$$d^2(q^2,\nu) \le \frac{R}{Q^2} 2M\pi W_1(q^2,\nu) \left\{ 4M\pi W_1(q^2,\nu) + \nu d(q^2,\nu) + M g(q^2,\nu)(\nu^2 + Q^2) \right\}$$

and

$$| \nu d(q^2,\nu) + M(\nu^2 - q^2) g(q^2,\nu) | \le 4M\pi W_T(q^2,\nu).$$

The inequalities imply rigorous upper and lower bounds on the asymmetry,

$$A^{+} = \frac{(E_1 - E_2\cos\theta)(E_1+E_2) + 2\sqrt{\frac{R}{Q^2}}\,E_1E_2^{2}\sin^2\theta}{Q^2\left(1+\frac{\nu^2}{Q^2} + \frac{R+1}{2}\cot g^2\frac{\theta}{2}\right)}$$

$$A^{-} = \begin{cases} \dfrac{(E_1 - E_2\cos\theta)(E_1+E_2) - 2\sqrt{\frac{R}{Q^2}}\,E_1\cdot E_2^{2}\sin^2\theta}{Q^2\left(1+\frac{\nu^2}{Q^2} + \frac{R+1}{2}\cot g^2\frac{\theta}{2}\right)} & \text{if } R > R_0 \\[2ex] \dfrac{(E_1 - E_2\cos\theta)(E_1+E_2) + \dfrac{R\cdot E_1E_2^{3}(1+\cos\theta)\sin^2\theta}{4(E_1+E_2)(E_1-E_2\cos\theta)}}{Q^2\left(1+\frac{\nu^2}{Q^2} + \frac{R+1}{2}\cot g^2\frac{\theta}{2}\right)} & \text{if } R \le R_0\,, \quad R_0 = \dfrac{8(E_1+E_2)^2(E_1-E_2\cos\theta)^2}{E_1E_2^3\sin^2\theta(1+\cos\theta)} \end{cases} \tag{3.2}$$

It is remarkable that the bounds in (3.2) depend on the dynamical details through the functional form of $R(q_1^2\nu)$ only. R is known to be small in the scaling region, and careful inspection shows that (3.2) is only slightly dependent on the actual numerical values of R . Upper bounds (A^+) and lower bounds (A^-) are plotted for $E_1 = 10\,GeV$, $\theta = 12^0, 18^0$ in Fig.9 as a function of $\omega' = 1 + W^2/Q^2$. An average value, $R = 0.18$ was chosen

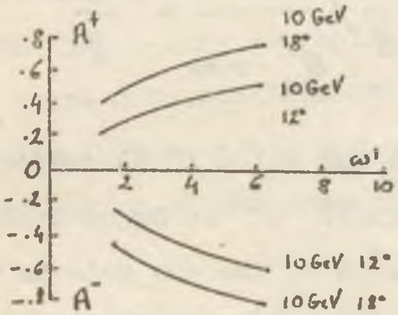

Fig. 9

The quark light-cone algebra of currents implies[2] that the following scaling laws hold:

$$\nu d(q_1^2\nu) \xrightarrow[x \text{ fixed}]{\nu\to\infty} \alpha(x),$$

and

$$\nu^2\cdot M g(q_1^2\nu) \xrightarrow[x \text{ fixed}]{\nu\to\infty} \beta(x),$$

where $\alpha(x)$ and $\beta(x)$ are dimensionless scaling functions. We introduce the sum of the two scaling functions

$$\gamma(x) = \alpha(x) + \beta(x)$$

and integrate over X

$$\frac{1}{2\pi}\int_0^1 \gamma(X)\, dX = Z, \tag{3.3}$$

where Z is given by $\langle p,\alpha | J_\mu^5(0) | p,\alpha\rangle = -2MZ\alpha_\mu$. The local operator $J_\mu^5(0)$ which has emerged is the $y \to 0$ limit of the bilocal operator :

$$\bar{\Psi}(y)\gamma_\mu\gamma_5 \lambda_Q^2 \cdot \psi(0) \xrightarrow[y\to 0]{} J_\mu^5(0), \tag{3.4}$$

$\lambda_Q^2 = \frac{2}{9}\lambda^0 + \frac{1}{3}\lambda^3 + \frac{1}{3\sqrt{3}}\lambda^8$ (Gell-Mann's λ matrices). The sum rule (3.3) was originally proposed by Bjorken[14] in the form

$$\frac{1}{4M\pi}\int_0^\infty \frac{d\nu}{\nu}\left(d + \nu M g\right) \xrightarrow[Q^2\to\infty]{} \frac{Z}{Q^2} \tag{3.5}$$

The nice thing about the sum rules (3.3) and (3.5) is that the integrals are convergent for fixed q^2 in Regge theory (Appendix)

The experimental evaluation of the sum rule in (3.3) would be very interesting. Unfortunately, the value of Z is model dependent:

$$Z = \begin{cases} \bar{Z} + \frac{1}{6}\left|\frac{G_A}{G_V}\right| & \text{proton target} \\ \bar{Z} - \frac{1}{6}\left|\frac{G_A}{G_V}\right| & \text{neutron target} \end{cases} \tag{3.6}$$

Here $|G_A/G_V| \simeq 1.2$ is the ratio of β -decay coupling constants and $\bar{Z}$ is an isoscalar contribution which depends upon the model of the nucleon. $|G_A/G_V|$ comes from the isovector part of $J_\mu^5(x)$ ($\frac{1}{3}\lambda^3$ in Eq. (3.4)). $SU(6)$ predicts $Z_p = \frac{5}{9}$ and $Z_n = 0$. The unknown isoscalar part is removed by writing the sum rule for the proton - - neutron difference:

$$\frac{1}{2\pi}\int_0^1 \left(\gamma_p(x)-\gamma_n(x)\right)dx = \frac{1}{3}\left|\frac{G_A}{G_V}\right|. \qquad (3.7)$$

It would be a fundamental test of the light-cone algebra of currents, as abstracted from the free quark field model, to check the validity of (3.7) ! Experimentally this is difficult but not impossible.

To get a qualitative idea about polarization effects in the parton picture, a simple relativistic quark-parton model was proposed[10] in which the core of $q\bar{q}$-pairs carries vacuum quantum numbers and does not contribute to spin-dependent effects. Even if the assumption turns out to be only a rough approximation to a more sophisticated picture, it is instructive to see the qualitative implications.

An immediate consequence of the model is that the spin-dependent diffractive component of the structure functions vanishes. The spin--dependence of the scattering is entirely given by the valence quark structure which represents nondiffractive scattering. The sum rule (3.3) is, of course valid in the model, and the precise value of $\bar{Z}$ depends on the spin distribution of the constituents quarks inside the fast moving nucleon.

Qualitatively we expect that the polarization asymmetry for the proton is large and positive over a considerable range of the energy loss ν The asymmetry is probably much smaller for neutron target. The sign of the asymmetry from the experiment will be informative. What a surprise for theoretists if next year they will find a sizable negative asymmetry. This would imply for example, that somehow more constituent spins point backward than forward when the fast moving proton is polarized in the forward direction.

There are many interesting points which I cannot discuss here. We will complete our long-delayed review paper[16] on this field soon, and I refer you to this paper or the original literature which will be found among the references.

APPENDIX

Gálfi and Patkós have extensively studied the J-plane expansion for spin-dependent virtual Compton scattering[12] in the forward direction. Their conclusion is that known Regge poles with intercepts $0 \leq \alpha(0) \leq 1$ do not contribute to the leading asymptotic behavior of $d(q^2, \nu)$ and $g(q^2, \nu)$ in the limit q^2 fixed and $\nu \to \infty$. The decoupling of the Pomeranchuk trajectory with $\alpha(0)=1$ intercept is a well-known consequence of theorems on the spin-dependence of high energy scattering amplitudes. The Pomeranchuk cut, however, does contribute to the asymptotic expansion in (A 1)

$$d(q^2, \nu) + \nu M g(q^2, \nu) \xrightarrow[\substack{\nu \to \infty \\ q^2 \text{ fixed}}]{} \beta_c(q^2) \frac{\nu^{\alpha_c(0)-2}}{\ln^2 \frac{\nu}{M}} + \beta_{A_1}(q^2) \nu^{\alpha_{A_1}(0)-1} + \ldots \tag{A 1}$$

where $\alpha_c(0) = 1$. The power behavior $\sim \nu^{\alpha_c(0)-2}$ corresponds to positive signature. A negative signature piece of the Pomeranchuk cut (not allowed if the product rule for the signature of cuts holds) would increase the power behavior by one unit. The A_1 trajectory with $\alpha_{A_1}(0) \sim -0.02$ is the simplest candidate for the leading term in the nondiffractive, I = 1 component. The asymptotic behavior in (A 1) guarantees the convergence of the integral in (3.5) for fixed q^2.

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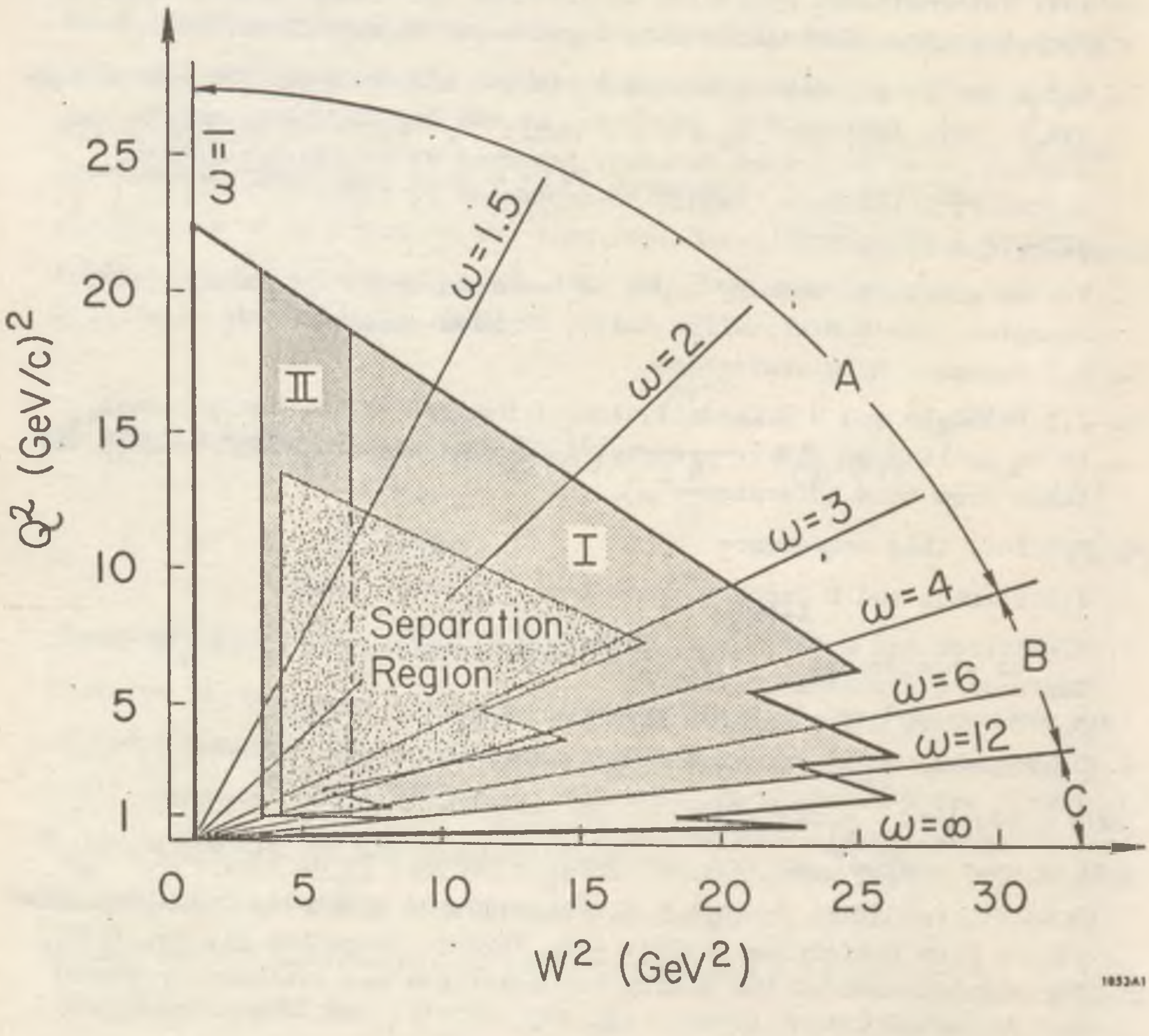
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A
B
C
I
II
Separation
Region
25
20
15
10
5
1
0
5
10
15
20
25
30
Q^2 (GeV/c)^2
W^2 (GeV^2)
1853A1

Fig. 2

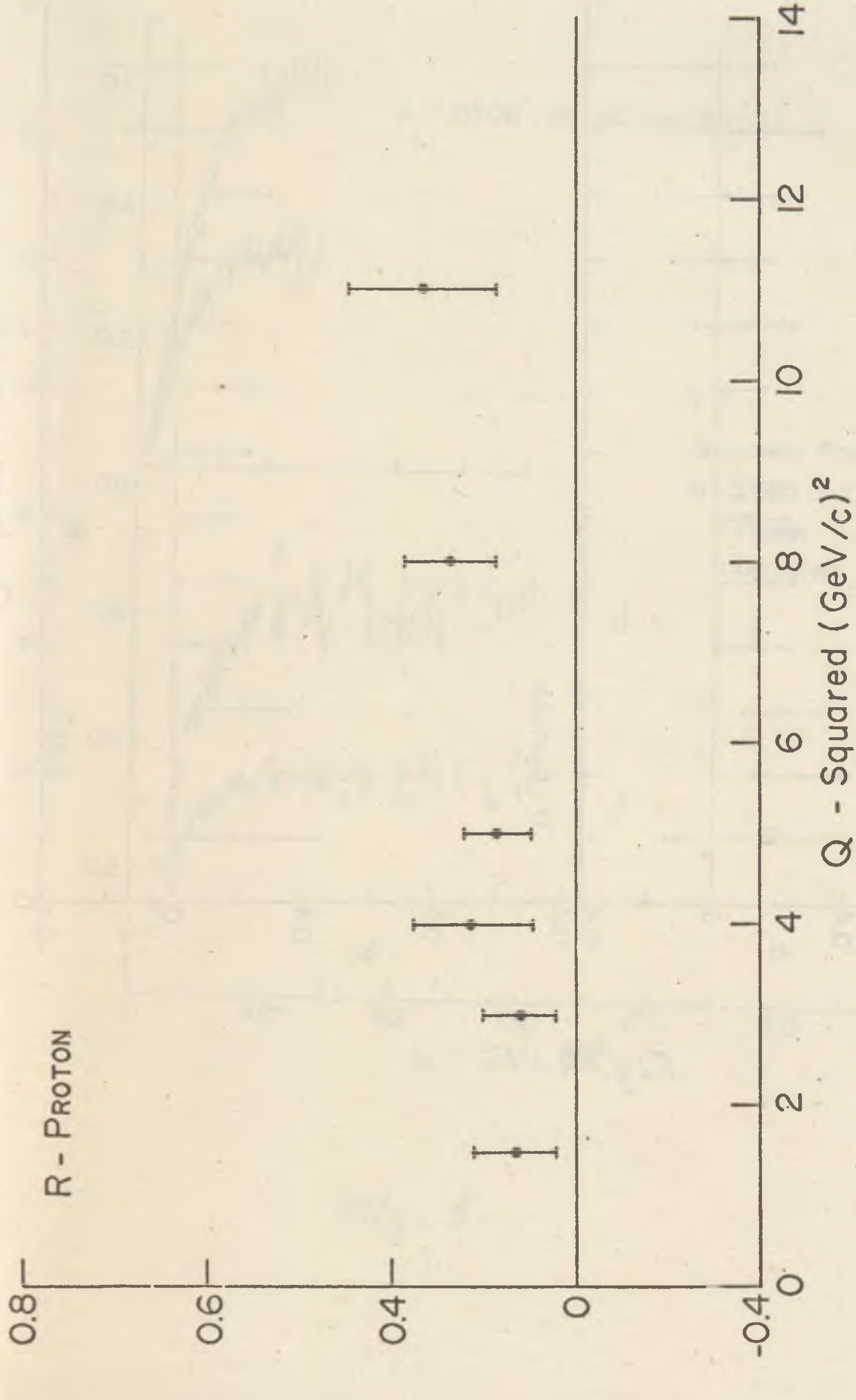


Fig. 3

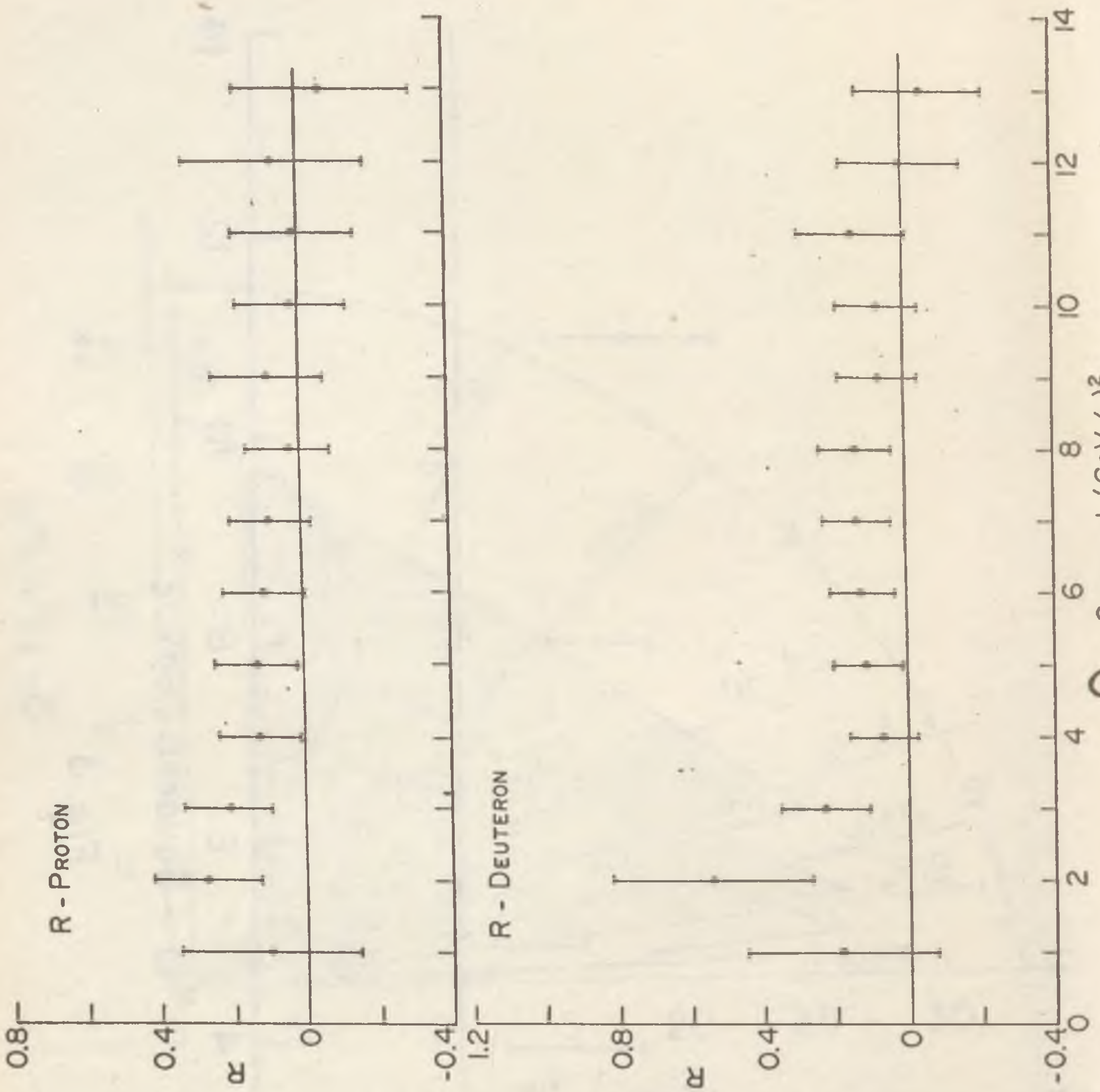

R - PROTON
R - DEUTERON
R
0.8
0.4
0
-0.4
1.2
Q - Squared (GeV/c)²
0
2
4
6
8
10
12
14

Fig. 4

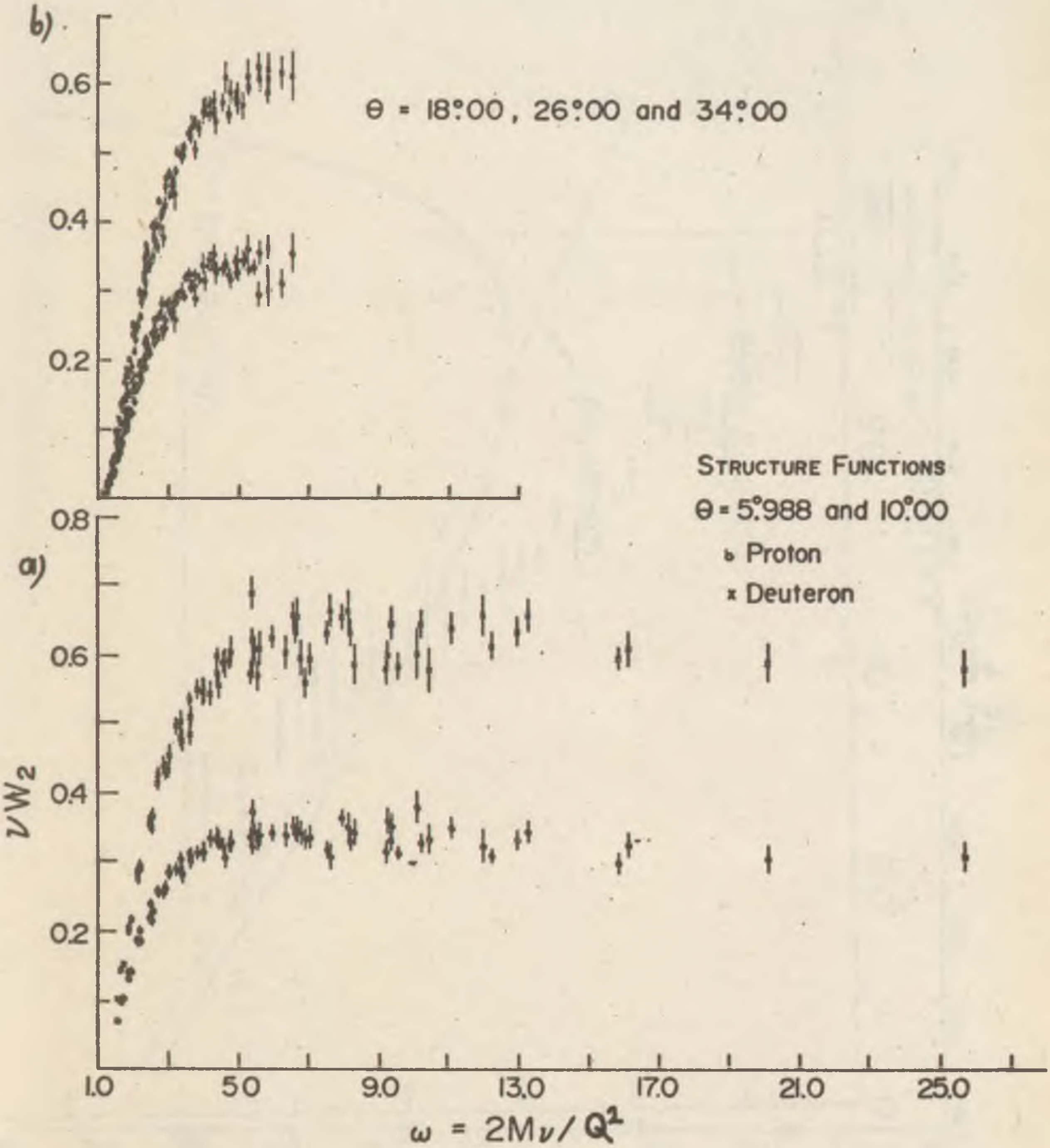

b)
θ = 18°.00, 26°.00 and 34°.00
0.6
0.4
0.2
0.8
a)
STRUCTURE FUNCTIONS
θ = 5°.988 and 10°.00
Proton
Deuteron
νW_2
1.0
5.0
9.0
13.0
17.0
21.0
25.0
$\omega = 2M\nu/Q^2$

Fig. 5

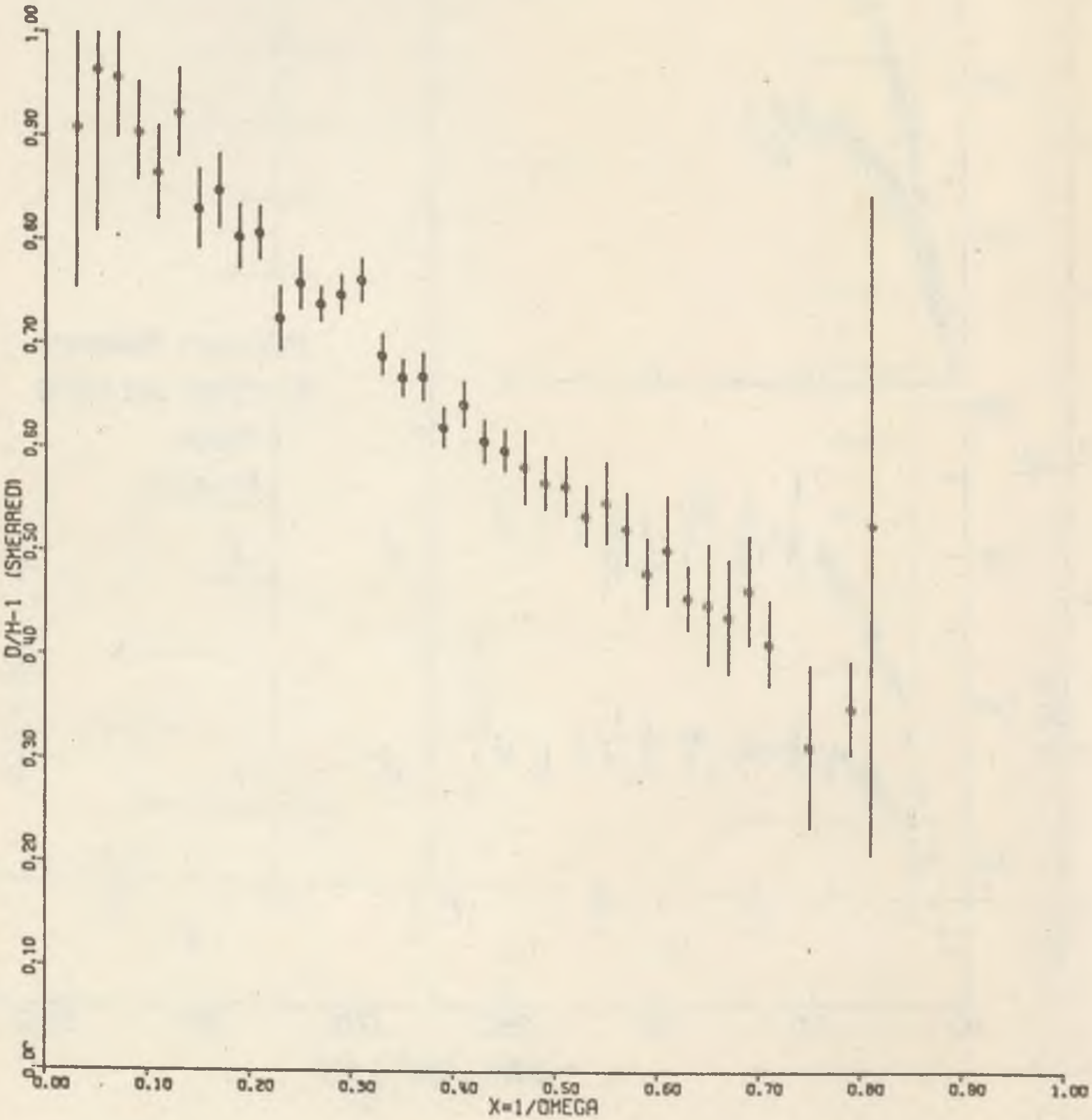
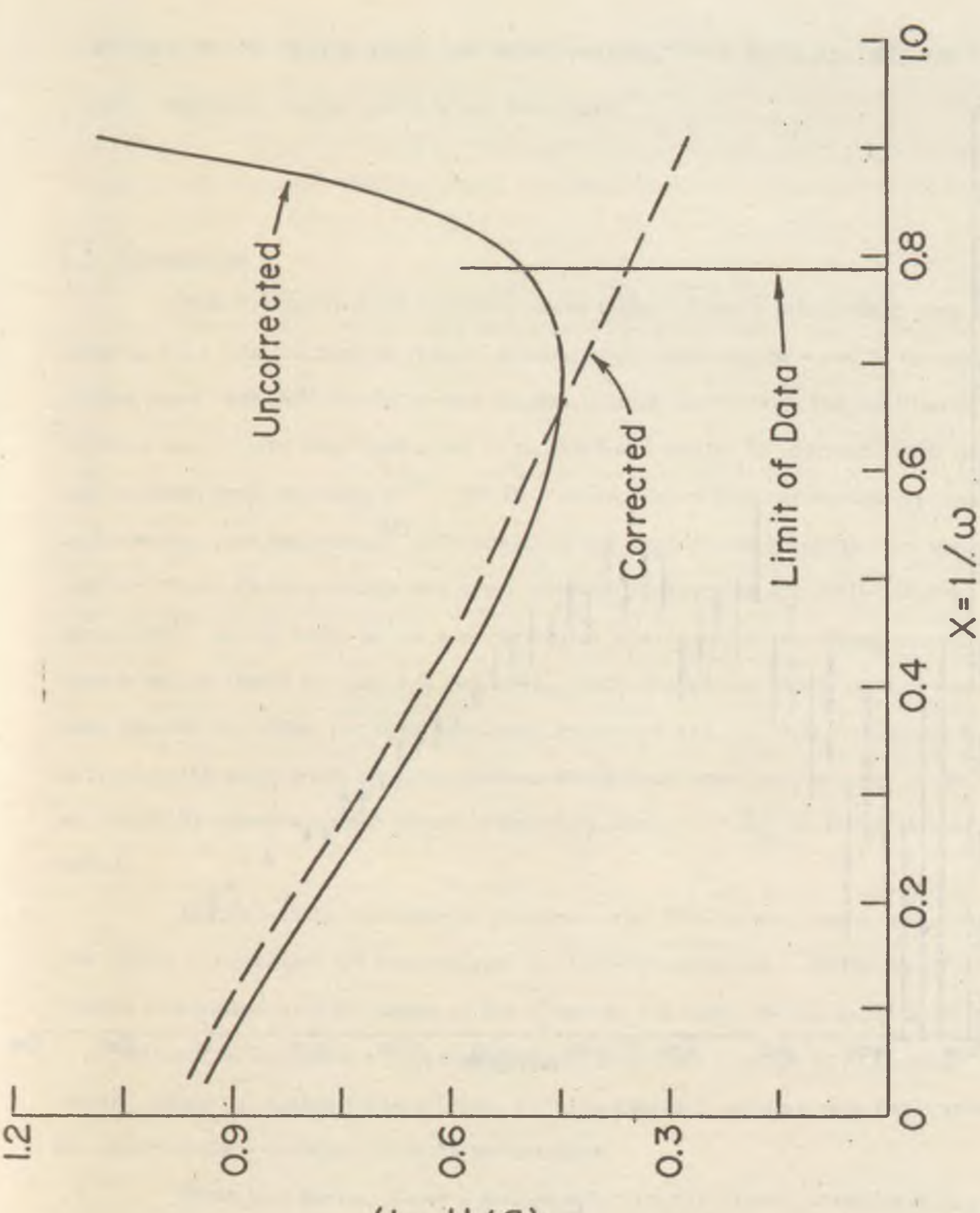

D/H-1 (SMEARED)
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00
X=1/OMEGA

Fig. 6


Uncorrected
Corrected
Limit of Data
(D/H* - 1)
X = 1/ω
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0.9
0.6
0.3
0
0.2
0.4
0.6
0.8
1.0

Fig. 7

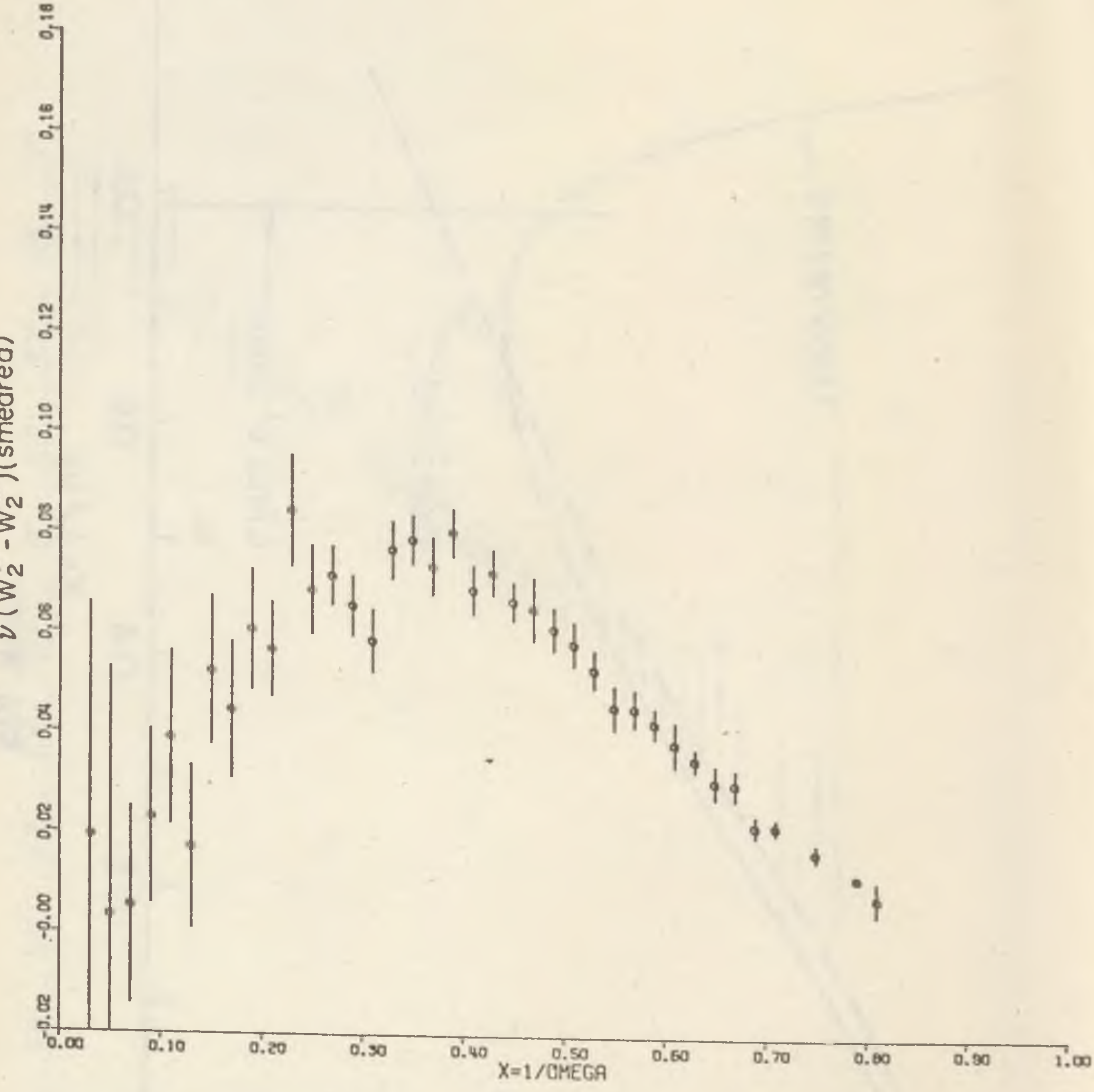
$\nu\,(W_2^P - W_2^N)$ (smeared)
X=1/OMEGA
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00
-0.02
-0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.16
0.18

Fig. 8

REPORT ON W BOSON MODEL OF WEAK INTERACTIONS WITH MAXIMAL CP VIOLATION

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§1 Introduction

The Universal (V-A) current-current theory of weak interactions was formulated in 1957 - in the face of several contradictory experiments - and in the ensuing fifteen years, the predictions of this current-current model (with the addition of the Cabibbo angle) have been confirmed in an enormous number of leptonic, semi-leptonic and hadronic weak processes[1]. The first derivation of this current-current theory by Sudarshan and the author[2] was based on the principle of chirality invariance for spin 1/2 Dirac fields and this was soon followed by Feynman and Gell-Mann's derivation[3] on the basis of the non-derivative interaction of two-component Klein-Gordon spinor fields for spin 1/2 particles. Both derivations of the correct theory were carried out within the current-current framework and can only artificially be applied to the semi-weak W boson-current interaction which can be used to generate the (V-A) current-current theory in the limit of $m_W \to \infty$ (m_W is the W boson mass).

Unfortunately, the remarkably successful (V-A) current-current theory has one defect - it predicts CP conservation in all weak processes. While the CP violation effects associated with the decay of the K_L^0 meson are small (of the order of 10^{-3}) and it is possible to introduce a phenomenological parameter into the (V-A) current-current theory to explain these effects, such an approach tells us very little about the origin of CP - violation in weak interactions.

There is a second major question which must be faced in connection with further refinements of the universal (V-A) current-current theory and that has to do with the very existence of the W boson. Experiment has already demonstrated[4] that if the W boson exists at all, its mass $m_W \geq 2$ Gev. This large mass explains why the experiments carried out until now (with $q^2 << m_W^2$ where q^2 is the four-momentum transfer squared) can not decide whether the current-current interaction is the basic interaction (whose field-theoretic content must still be delineated through a deeper study of higher order weak interactions[5] and a refined analysis of lowest order weak interactions for large q^2) or whether the current-current interaction is itself a second order effect resulting from the more fundamental semi-weak Yukawa-type interaction involving the W boson (or bosons). If we adopt the latter viewpoint and postulate the existence of a massive W boson (or bosons), we open up the very real possibility of developing a unified theory of CP-conserving and CP-violating weak processes on the basis of a single semi-weak W boson-current interaction. The strong cubic W boson model of weak interactions as developed by Okubo and the author[6], is the best example of such a theory and the one whose consequences have been most fully explored. The interest of this W boson model is further enhanced by the fact that it is the only extant theory which offers any hope of explaining the recent $K_L \rightarrow 2\mu$ puzzle[7] in a "natural" fashion.

In what follows we shall describe the essential features of the strong cubic W boson model (§ 2) and then show how this model can explain both the low rate for $K_L \rightarrow 2\mu$ decay and the high rate for $K_S \rightarrow 2\mu$ decay (§ 3). Finally, we shall summarize other experimental tests of the strong cubic W boson model (§ 4).

§ 2. Strong Cubic W Boson Model

The strong cubic W boson model of weak interactions arose out of the observation that the CP - violating parameter ϵ (which appears in the definition of $|K_L\rangle = |K_2\rangle + \epsilon\,|K_1\rangle$, with $|K_1\rangle$ and $|K_2\rangle$ the CP = +1 and CP = -1 combinations of $|K^o\rangle$ and $|\bar{K}^o\rangle$ respectively) is of the order[8] of the semi-weak coupling constant g. This line of argument leads to writing down a "pure" CP = -1 semi-weak W boson-current interaction which is capable of duplicating the results of the usual CP - conserving (V-A) current-current theory in order g^2 (in the limit $m_W \rightarrow \infty$) and the CP-violating effects (in K_L decay) in order g^3. This can be accomplished by postulating the existence of a triplet of W bosons[9] (with total charge 0) interacting strongly among themselves via a cubic interaction (hence the expression "strong cubic W boson model") and writing down a CP = -1 semi-weak interaction between this triplet of W bosons and suitable lepton and hadron currents.

More explicitly, we assume that the triplet of W bosons (W^o, W^-, W^+) are described by the following Lagrangian:

$$\mathcal{L}_o = -\tfrac{1}{2}\left[\partial_\nu \overline{W}_\mu^{(a)}(x) - \partial_\mu \overline{W}_\nu^{(a)}(x)\right]\left[\partial_\nu W_\mu^{(a)}(x) - \partial_\mu W_\nu^{(a)}(x)\right]$$

$$- m_o^2\, \overline{W}_\mu^{(a)}(x)\, W_\mu^{(a)}(x) - i f_o\, \epsilon_{abc}\left[W_\mu^{(a)}(x)\, W_\nu^{(b)}(x)\, W_\lambda^{(c)}(x) - \overline{W}_\mu^{(a)}(x)\, \overline{W}_\nu^{(b)}(x)\, \overline{W}_\lambda^{(c)}\right] \quad (1)$$

where a=1, 2, 3 (corresponding to charges 0, -1, +1), $W_\mu^{(a)}$ represents the vector W boson field, f_o is the strong coupling constant for the three W's and ϵ_{abc} is the usual antisymmetric tensor. Eq. (1) can be given a more convincing origin if we note that a quasi-Yang-Mills approach to the W Lagrangian suggests a new definition for the fields $F_{\mu\nu}^{(a)}(x)$ (note the bars over the W's in the second term):

$$F_{\mu\nu}^{(a)}(x) = \left[\partial_\mu W_\nu^{(a)}(x) - \partial_\nu W_\mu^{(a)}(x)\right] + i f_o\, \epsilon_{abc}\, \overline{W}_\mu^{(b)}(x)\, \overline{W}_\nu^{(c)}(x) \quad (2)$$

It is easy to show that if the $W_\mu^{(a)}$ fields transform according to the triplet representation of SU_3, the same will be true of the quasi-Yang-Mills fields $F_{\mu\nu}^{(a)}$. The W Lagrangian (1) can then be rewritten in terms of the $F_{\mu\nu}^{(a)}$ fields as[10]:

$$\mathcal{L}_o = -\tfrac{1}{2}\, \overline{F}_{\mu\nu}^{(a)}(x)\; F_{\mu\nu}^{(a)}(x) - m_o^2\; \overline{W}_\mu^{(a)}(x)\, W_\mu^{(a)}(x) \quad (3)$$

where $\mathcal{L}_o$ is manifestly invariant under SU_3.

Another important property of Eq. (1) [and Eq. (3)] is its invariance under the transformation:

$$W_\mu^{(a)}(x) \to \lambda_a\, W_\mu^{(a)}(x) \quad , \quad \overline{W}_\mu^{(a)}(x) \to \lambda_a^*\; \overline{W}_\mu^{(a)}(x) \quad (4)$$

where λ_a is a complex constant satisfying the cubic equation:

$$\lambda_a^3 = 1 \quad (4a)$$

The parameters λ_a, λ_b, λ_c are the cube roots of unity and effectively assign different "cubic parities" to the three W fields. The concept of "cubic parity" is basic to the strong cubic W boson model.

If we further define the charge conjugation operation for the W field by:

$$C : \quad W_\mu^{(a)}(x) \rightarrow -\overline{W}_\mu^{(a)}(x) \tag{4b}$$

the simplest CP= -1 total semi-weak interaction which can be written down is (θ is the Cabibbo angle):

$$\begin{aligned} H_{S.W.} = ig \Big\{ \Big[& W_\mu^{(o)} \left(\alpha J^2_{\mu 3} + \beta J^3_{\mu 3} \right) \\ & + W_\mu^{(-)} \left(\gamma J^1_{\mu 2} + \delta \ell_\mu \right) \\ & + W_\mu^{(+)} \left(\gamma' J^3_{\mu 1} + \delta' \bar{\ell}_\mu \right) \Big] - h.c. \Big\} \end{aligned} \tag{5}$$

where α, β, γ, δ, γ' and δ' are real coefficients, $\ell_\mu = i\bar{e}\gamma_\mu(1+\gamma_5)\nu_e + i\bar{\mu}\gamma_\mu(1+\gamma_5)\nu_\mu$ is the total (V-A) charged lepton current and $J^i_{\mu j}$ ($i, j = 1, 2, 3$) is the octet hadron current (in tensor notation). Note the coefficient i and the subtraction of the hermitian conjugate in Eq. (5); these features account for the CP= -1 property of the semi-weak interaction since C and P are defined by virtue of the strong cubic self-interaction of the W's. The interaction (5) also possesses the property that it is the most general semi-weak interaction which forbids $\Delta Y = 2$ (Y is the hyper-charge) transitions[11] to order g^2 and g^3.

It is now possible to show that the first-order effects[12] in g are forbidden by the invariance of Eq. (5) under the "cubic parity" transformation (4); this forbiddenness extends to any process which is first-order in g and of arbitrary order in e (electric charge) and therefore excludes the occurrence of an electric dipole moment of the neutron in this order[13]. The first non-vanishing effects in the strong cubic W boson model occur in order g^2 since a term like $\langle W_\mu(x) \overline{W}_\nu(x) \rangle_o$ is consistent with cubic parity conservation. In this way, one can derive an effective CP= +1 current-current interaction in order g^2 (in the limit $m_W \rightarrow \infty$) which can explain the whole range of CP-conserving leptonic, semi-leptonic and hadronic weak processes. Indeed, one can fix the coefficients in Eq. (5) by the requirement that they reproduce in order g^2 the results of the universal (V-A) current-current weak interaction:

$$H_W = \frac{G}{\sqrt{2}} \bar{\mathcal{J}}_\mu(x) \mathcal{J}_\mu(x) \tag{6}$$

where

$$\mathcal{J}_\mu(x) = \cos\theta\, J^1_{\mu 2}(x) + \sin\theta\, J^1_{\mu 3}(x) + \ell_\mu(x) \tag{6a}$$

is the Cabibbo current and $G/\sqrt{2} = g^2/m_W^2$. Eq. (6) yields a contribution of order g^2 to purely leptonic processes and $g^2 \cdot \cos^2\theta$ and $g^2 \sin^2\theta$ contributions to the $\Delta Y=0$ and $\Delta Y=1$ semi-leptonic weak processes respectively. These features are readily recaptured by the iteration of Eq. (5) through the unique choice:

$$\delta = \delta' = \tfrac{1}{2} \quad , \quad \gamma = \cos\theta \quad , \quad \gamma' = \sin\theta \tag{7}$$

The choice of (α and β) in Eq. (5) can not so easily be determined by comparing the iteration of Eq. (5) with the $g^2 \cos\theta \sin\theta$ contribution of Eq. (6) to the $\Delta Y=1$ weak hadron processes since we are asked to equate:

$$\alpha\beta \langle f | J^2_{\mu 3} J^3_{\mu 3} | i \rangle = \cos\theta \sin\theta \langle f | J^2_{\mu 1} J^1_{\mu 3} | i \rangle \tag{8}$$

Eq. (8) does not yield a simple determination of the coefficients α and β since its L.H.S. (originating from the W boson model) involves neutral hadron currents exclusively[14] and its R.H.S. (originating from the current-current theory) involves only charged hadron currents; consequently, at the present stage of the strong cubic W boson theory, the choice of α and β is dictated by experiment (we shall find below that $\alpha \sim \beta \sim 1$). The structure of the L.H.S. of Eq. (8) has the very desirable consequence that the $\Delta Y=1$ weak hadron processes automatically obey the $\Delta I=\frac{1}{2}$ rule (I is the isospin) in the W boson theory - in contrast to the artificial suppression of the $\Delta I=3/2$ contribution (through octet enhancement or some other mechanism) required in the current-current theory.

The CP = -1 <u>semiweak</u> W boson interaction which is consistent with the

CP = +1 weak current-current interaction thus becomes:

$$H_{S.W.} = \frac{ig}{\sqrt{2}} \left\{ \left[W_\mu^{(o)} \left(\alpha J_{\mu 3}^{2} + \beta J_{\mu 3}^{3} \right) \right.\right.$$
$$+ W_\mu^{(-)} \left(\sqrt{2} \cos\theta J_{\mu 2}^{1} + \ell_\mu / \sqrt{2} \right) \tag{9}$$
$$\left.+ W_\mu^{(+)} \left(\sqrt{2} \sin\theta J_{\mu 1}^{3} + \bar{\ell}_\mu / \sqrt{2} \right) - \text{h.c.} \right\}$$

Eq. (9) will be used to compute the matrix elements following from the W boson model. However, it is illuminating to recast Eq. (9) into the form:

$$H_{S.W.} = ig' \left\{ \left[W_\mu^{(o)} \left(\alpha' J_{\mu 3}^{2} + \beta' J_{\mu 3}^{3} \right) \right.\right.$$
$$+ \frac{\left(W_\mu^{(-)} - \overline{W}_\mu^{(+)} \right)}{\sqrt{2}} \left(\cos\theta J_{\mu 2}^{1} + \sin\theta J_{\mu 3}^{1} + \ell_\mu \right) \tag{10}$$
$$\left.+ \frac{\left(W_\mu^{(-)} + \overline{W}_\mu^{(+)} \right)}{\sqrt{2}} \left(\cos\theta J_{\mu 2}^{1} - \sin\theta J_{\mu 3}^{1} \right) \right] - \text{h.c.} \Big\}$$

where $g' = g/\sqrt{2}$, $\alpha' = \sqrt{2}\,\alpha$, $\beta' = \sqrt{2}\,\beta$. The second term now contains the interaction of the usual Cabibbo current (consisting of charged hadron and lepton currents) with the normalized combination $\frac{\left(W_\mu^{(-)} - \overline{W}_\mu^{(+)} \right)}{\sqrt{2}}$

In the third term, the orthogonal combination of the $W_\mu^{(-)}$ and $\overline{W}_\mu^{(+)}$ fields interacts with a purely charged hadron current while in the first term the neutral vector field $W_\mu^{(o)}$ interacts with a purely neutral hadron current. It is worth remarking that the combinations of $W_\mu^{(-)}$ and $\overline{W}_\mu^{(+)}$ which enter in Eq. (10) are precisely the ones that interact with the electromagnetic field when one adds this field to the quasi-Yang-Mills Lagrangian (3). This places the semi-weak interaction of the W boson model on an attractive theoretical foundation.

The basic difference between the current-current interaction model (6) and the strong cubic W boson model first arises in order g^3 (in the W boson model) where one encounters a term of the type $\langle W_\mu(x)\, W_\nu(x)\, W_\lambda(x) \rangle_o$ which conserves "cubic parity" and is large because of the strong cubic self-coupling of the W boson triplet. Thus, the strong cubic W boson model allows certain weak processes to occur in order g^3 which can only occur in the current-current theory in order $g^4 \sim \left(Gm_W^2\right)^2$ Moreover, the weak processes occurring in order g^3 receive CP = -1 contributions in this order and can exhibit CP-violation effects under suitable circumstances. When the CP = -1 g^3 amplitude interferes with the CP = +1g^2 amplitude, the CP-violating effect will be of the order $g \sim 10^{-3}$ whereas if it interferes with a CP = +1 ⟵ $g^2 e^n$ amplitude (a combined weak-electromagnetic amplitude), the CP-violating effect can be much larger (gross CP violation[15]). The CP-violating effect in $K_L \to 2\pi$ decay is an example of the former type of interference effect (and was the reason for proposing the strong cubic W boson model in the first place) while $K_L \to 2\mu$ decay would be an example of the latter type of interference effect. We conclude this section with a sketch of the calculation for $K_L \to 2\pi$ decay - to indicate the nature of the approximations invoked - and in the next section apply the strong cubic W boson model to the $K_L \to 2\mu$ problem.

The diagram contributing to $K_L \to 2\pi^0$ decay is given in Fig. 1 (a similar diagram can be drawn for $K_L \to \pi^+\pi^-$) and the matrix element following from Eq. (9) is:

$$M = \frac{g^3 \cos\theta \sin\theta\, \beta}{\sqrt{8 p_o k_{10} k_{20}}} \int d^4q\, \Delta^{W(+)}_{\mu\alpha'}(p-q)\, \Delta^{W(0)}_{\nu\beta'}(k_2)\, \Delta^{W(-)}_{\lambda\gamma'}(p-q-k_2) \tag{11}$$

$$\times\ \Gamma_{\alpha'\beta'\gamma'}(p,q,k_2)\,\Delta^{\pi}(q)\,\langle K_2(p)\,|V^3_{\mu 1}\,|\,\pi^-(q)\rangle\,\langle \pi^-(q)\,|V^1_{\lambda 2}\,|\,\pi^0(k_1)\rangle\,\langle \pi^0(k_2)\,|A^3_{\nu 3}\,|\,0\rangle$$

where $\Delta^W_{\mu\nu}$ is the W boson propagator, $\Gamma_{\alpha'\beta'\gamma'}$ is the triple W vertex and $V^i_{\mu j}$ and $A^i_{\mu j}$ are the vector and axial vector hadron currents with suitable tensor indices respectively. From symmetry considerations:

$$\Gamma_{\alpha'\beta'\gamma'} = f(q^2, q\cdot p, q\cdot k_2)\left[\delta_{\alpha'\beta'}(p-q+k_2)_{\gamma'} - \delta_{\alpha'\gamma'}(2p-2q-k_2)_{\beta'} + \delta_{\beta'\gamma'}(p-q)_{\alpha'}\right] \tag{12}$$

We assume that $f(q^2, q\cdot p, q\cdot k_2) \simeq f_o$ and, retaining the most divergent contributions, we get

$$M \simeq -\frac{3\Lambda^4}{256\pi^2}\,\frac{g^3}{m_W^6}\,\cos\theta \sin\theta\, \beta\, f_o\, f_\pi\, m_K^2 \tag{13}$$

where f_π is the pion decay amplitude. Hence:

$$| M (K_L \to 2\pi) / M (K_S \to 2\pi) | \simeq 10^{-3} f_o \left(\frac{\Lambda}{m_W} \right)^4 g\beta \tag{14}$$

Another relation between f_o and Λ is derived by calculating the self-mass of the W boson with cubic interaction; using the same approximations for the triple W vertex, one gets:

$$\delta m_W \simeq \frac{3}{4\sqrt{2}\,\pi} \quad \frac{f_o \Lambda^2}{m_W} \simeq m_W \tag{15}$$

or

$$f_o \Lambda^2 / m_W^2 \simeq 2\pi \tag{16}$$

These approximate calculations show that the correct order of magnitude of the CP-violating amplitude for $K_L \to 2\pi$ decay can be obtained from the strong cubic W boson model for a reasonable choice of parameters: $g \sim 3 \times 10^{-2}$ (corresponding to $m_W \simeq 10$ Gev), $\Lambda \simeq 2 m_W$ and $\beta \simeq 1$.

§ 3. Application of the Strong Cubic W Boson Model to $K_L \to 2\mu$ Puzzle

The strong cubic W boson model was not invented to explain the recent $K_L \to 2\mu$ puzzle[(7)]. It was put forward as the simplest W boson model capable of providing a unified description of both CP-conserving and CP-violating weak processes. However, it turns out that the same feature of the model which predicts the existence of the CP-violating $K_L \to 2\pi$ decay in order g^3 also predicts the existence of effective neutral lepton currents in the same order and this, when combined with the symmetry properties of the model, enables us to understand the low rate for $K_L \to 2\mu$ decay and a much higher rate for $K_S \to 2\mu$ decay. This interesting prediction of the strong cubic W boson model is now examined in some detail.

Let us write in the usual fashion:

$$|K_L\rangle = |K_2\rangle + \epsilon\, |K_1\rangle \tag{17a}$$

$$|K_S\rangle = |K_1\rangle + \epsilon\, |K_2\rangle \tag{17b}$$

where $\epsilon \simeq \bar{\epsilon}_o e^{i\pi/4}$ ($\bar{\epsilon}_o = 2 \times 10^{-3}$) and terms of higher order in ϵ have been dropped. It is easy to show that in the "local" approximation[16] to the g^3 matrix element, the CP = −1 $|K_2\rangle$ state does not decay into 2μ whereas the CP = +1 $|K_1\rangle$ state does. Indeed, the effective CP = −1 g^3 interaction in the local limit takes the CP = +1 $|K_1\rangle$ state into the CP = −1 (^1S_o) state of the 2μ system with the amplitude:

$$M(K_1 \to 2\mu) = i\bar{u}\, b\, \gamma_5\, v \tag{18}$$

where u and v are the Dirac spinors for the muons and b is computed from the diagram in Fig. 2. Using the same approximations made in computing $K_L \to 2\pi$ decay (see § 2), one obtains:

$$b = 2\sqrt{2}\, m_\mu\, G\, Z \sin\theta\, a\, f_K \tag{19}$$

where f_+ is the K^+ decay amplitude and

$$Z \cong -\frac{3}{16\pi^2} f_o \left(\frac{\Lambda}{m_W}\right)^2 g\cos\theta \simeq -\frac{3}{8\pi} g\cos\theta \tag{19a}$$

[using relation (16)]. The rate becomes:

$$\Gamma(K_1 \to 2\mu) = (G\sin\theta\, a\, |z|\, f_K\, m_\mu)^2\, (m_K^2 - 4m_\mu^2)^{1/2} \tag{20}$$

$$= 12\, |z|^2\, \Gamma_L \qquad (a \sim 1) \tag{21}$$

where we have expressed the decay rate in Eq. (21) in terms of $\Gamma_L = \Gamma(K_L \to \text{all})$. It is important to note that the amplitude (18) for $K_1 \to 2\mu$ is pure imaginary since the diagram in Fig. 2 gives a real contribution and the i comes from the fact that the CP = −1 part of the interaction is responsible for this contribution. It is this feature which enables (18) to partially cancel the CP = +1 absorptive contribution to the amplitude for $K_2 \to 2\mu$ given by the weak-electromagnetic diagram in Fig. 3.

Let us be more explicit: we may write the total amplitudes for $K_L \to 2\mu$ and $K_S \to 2\mu$ as follows:

$$M\left(K_{L,S} \to 2\mu_\pm\right) \equiv \langle 2\mu_\pm|T|K_{L,S}\rangle = \langle 2\mu_\pm|T|K_{2,1}\rangle + \epsilon\,\langle 2\mu_\pm|T|K_{1,2}\rangle \tag{22}$$

where the subscripts ± on μ denote the CP = ±1 final states of the $\mu\bar{\mu}$ system respectively. Separating the real and imaginary parts of the amplitudes, we write:

$$a_{\pm} = \mathcal{I}m \ \langle 2\mu_{\pm} | T_{\mp} | K_2 \rangle$$

$$b_{\pm} = \mathcal{I}m \ \langle 2\mu_{\pm} | T_{\pm} | K_1 \rangle$$

$$c_{\pm} = \mathrm{Re} \ \langle 2\mu_{\pm} | T_{\pm} | K_1 \rangle \qquad (23)$$

$$d_{\pm} = \mathrm{Re} \ \langle 2\mu_{\pm} | T_{\mp} | K_2 \rangle$$

where the subscripts $\pm$ on T denote the CP = ± 1 character of the effective interaction respectively. Using the definitions (23), we get:

$$\mathcal{I}m \ \langle 2\mu_{\pm} | T | K_L \rangle = a_{\pm} + \epsilon_0 b_{\pm}$$

$$\mathrm{Re} \ \langle 2\mu_{\pm} | T | K_L \rangle = d_{\pm} - \epsilon_1 b_{\pm} \qquad (24)$$

where $\epsilon_0 = \mathrm{Re}(\epsilon)$, $\epsilon_1 = \mathrm{Im}(\epsilon)$. The corresponding relations for the $K_S \to 2\mu$ amplitudes are:

$$\mathcal{I}m \ \langle 2\mu_{\pm} | T | K_S \rangle = b_{\pm} + \epsilon_0 a_{\pm}$$

$$\mathrm{Re} \ \langle 2\mu_{\pm} | T | K_S \rangle = c_{\pm} - \epsilon_1 a_{\pm} \qquad (25)$$

We repeat that $2\mu_+$ are in a $^3P_0$ state and $2\mu_-$ in a $^1S_0$ state.

Four of the quantities in Eq. (23), b_-, c_-, a_+, d_+, involve the CP = −1 part of the interaction and must be estimated on the basis of the g^3 diagram in Fig. 2: b_- is precisely the amplitude b defined by Eq. (18); $c_- \simeq 0$ since m_W is large; $a_+ \simeq 0$ in the local limit; $d_+ \simeq 0$ for the same reason as c_-. The other four quantities, a_-, d_-, b_+, c_+ involve the CP = +1 part of the interaction and must be estimated on the basis of the $g^2 e^4$ diagram in Fig. 3: a_- is the quantity computed by Sehgal[17]; d_- is the dispersive part of the $g^2 e^4$ contribution and is not expected to exceed the absorptive part a_- (although a good calculation has not yet been carried out as yet); b_+ is estimated to be the same order as a_- and the same can be said of c_+. Inserting our results into Eqs. (24) and (25), we get (all amplitudes are given in units of $\sqrt{\Gamma_L}$):

$$\begin{cases} \mathcal{I}m \ \langle 2\mu_+ |T| K_L \rangle = 7 \times 10^{-5} \ \epsilon_o \\ Re \ \langle 2\mu_+ |T| K_L \rangle = - D_L \ \epsilon_1 \end{cases} \tag{26}$$

$$\begin{cases} \mathcal{I}m \ \langle 2\mu_- |T| K_L \rangle = 7 \times 10^{-5} - 3.5 |z| \ \epsilon_o \\ Re \ \langle 2\mu_- |T| K_L \rangle = D_L + 3.5 |z| \ \epsilon_1 \end{cases} \tag{27}$$

$$\begin{cases} \mathcal{I}m \ \langle 2\mu_+ |T| K_S \rangle = 1 \times 10^{-4} \\ Re \ \langle 2\mu_+ |T| K_S \rangle = D_S \end{cases} \tag{28}$$

$$\begin{cases} \mathcal{I}m \ \langle 2\mu_- |T| K_S \rangle = - 3.5 |z| + 1 \times 10^{-4} \ \epsilon_o \\ Re \ \langle 2\mu_- |T| K_S \rangle = - D_S \ \epsilon_o \end{cases} \tag{29}$$

where D_L and D_S are the real parts of the weak-electromagnetic amplitudes for K_L and K_S decay into 2μ respectively[18] (via the two-photon mechanism of Fig. 3).

From Eqs. (26) - (29), we obtain the estimated partial transition rates for the 2μ decays of K_L and K_S into the CP = +1 (^3P_o) and CP = -1 (^1S_o) states:

$$\Gamma (K_L \rightarrow 2\mu_+) \simeq 5 \times 10^{-9} \cdot 2 \times 10^{-6} \cdot \Gamma_L \tag{30}$$

$$\Gamma (K_L \rightarrow 2\mu_-) \simeq \left(7 \times 10^{-5} - 3.5 \cdot 1.4 \times 10^{-3} \ |z|\right)^2 \Gamma_L \tag{31}$$

$$\Gamma (K_S \rightarrow 2\mu_+) \simeq 10^{-8} \cdot \Gamma_L \tag{32}$$

$$\Gamma (K_S \rightarrow 2\mu_-) \simeq 12 \ |z|^2 \ \Gamma_L \tag{33}$$

and hence for the total rates:

$$\Gamma (K_L \rightarrow 2\mu) \simeq \left(7 \times 10^{-5} - 3.5 \cdot 1.4 \times 10^{-3} \ |z|\right)^2 \Gamma_L \tag{34}$$

$$\Gamma (K_S \rightarrow 2\mu) \simeq 12 \ |z|^2 \ \Gamma_L \tag{35}$$

It is clear from Eqs. (34) and (35) that the $K_L \rightarrow 2\mu$ puzzle can readily be resolved by a suitable choice of the parameter $|z|$ (and therefore of m_W). Thus, we can match (34) to the upper limit 1.8×10^{-9} for the branching ratio

B.R. ($K_L \to 2\mu$) by choosing $|z| \simeq 5.5 \times 10^{-3}$ (and a fortiori $m_W = 15$ Gev). Inserting this value into (35) yields $\Gamma(K_S \to 2\mu)/\Gamma_S \simeq 0.6 \times 10^{-6}$ which is fairly close to Steinberger's upper limit[19] 0.8×10^{-6}. It should be emphasized that the predicted B.R. ($K_S \to 2\mu$) is very sensitive to the observed B.R. ($K_L \to 2\mu$); thus, if B.R. ($K_L \to 2\mu$) turned out to be 3×10^{-9} (a small change in such a difficult experiment), we would get $|z| \simeq 3 \times 10^{-3}$ ($m_W \simeq 10$ Gev) and the predicted $\Gamma(K_S \to 2\mu)/\Gamma_S$ would be 0.2×10^{-6}. It should also be pointed out that if only the $g^2 e^4$ diagram contributes to $K_S \to 2\mu$, the predicted $\Gamma(K_S \to 2\mu)/\Gamma_S$ would be $\sim 2 \times 10^{-10}$, an extremely small value; any evidence for a substantially larger branching ratio would require a modification of the (V-A) current-current theory.

§4. Further Predictions of the Strong Cubic W Boson Model

We have seen that one consequence of the strong cubic W boson model is the automatic prediction of an effective ($\mu \bar{\mu}$) current interaction with the K_1 meson in order g^3 which interferes destructively with the weak-electromagnetic current interaction in order $g^2 e^4$ for the K_L meson but not for the K_S meson. This is a negative sort of triumph and the true test of this model will lie with positive confirmation of a variety of predictions for weak processes involving the effective interaction of neutral lepton currents with hadrons in order g^3 and the competition, where appropriate, with weak-electromagnetic transitions to the same final states in order $g^2 e^2$ or $g^2 e^4$. Many of the relevant calculations in this regard have been given in previous papers[6] and only the more interesting results will be mentioned here.

A more perspicuous way to deduce the consequences of the strong cubic W boson model for neutral lepton pair effects in semi-leptonic processes is to derive the local limit (together with first-order corrections[20]) for the effective neutral lepton current-hadron current interaction. One derives the following effective interaction:

$$- \frac{i G \sin\theta}{\sqrt{2}} \; |z| \left\{ J^3_{\lambda 2} - J^2_{\lambda 3} \right\} \cdot \left\{ \left[i \bar{\nu}_e \gamma_\lambda (1+\gamma_5) \nu_e + i \bar{e} \gamma_\lambda (1+\gamma_5) e + i \bar{\nu}_\mu \gamma_\lambda (1+\gamma_5) \nu_\mu + i \bar{\mu} \gamma_\lambda (1+\gamma_5) \mu \right] - 2\rho' \left[m_e i \partial_\lambda (\bar{e} \gamma_5 e) + m_\mu i \partial_\lambda (\bar{\mu} \gamma_5 \mu) \right] \right\} \tag{36}$$

The first term in brackets in (36) is the strict local interaction whereas the second term is of order $\frac{q}{m_W}$ (q is the four-momentum transfer); terms of order q^2/m_W^2 (and m_ℓ^2/m_W^2) have been neglected. The current $\left[J^3_{\lambda 2}(x) - J^2_{\lambda 3}(x) \right] = 2i\, J^7_\lambda (x)$ (in the octet notation) is a $\Delta Y=1$, $CP=-1$ neutral hadronic current which can be written in the form:

$$J^7_\lambda (x) = A^7_\lambda + \delta V^7_\lambda \tag{37}$$

The parameters $|z|$, ρ (we set $\rho' m_K^2 = \rho - 1$) and δ can, in principle, be determined from three experiments and the results then used to make further predictions. However, in view of the unreliability of the experimental data (and the fact that only upper limits are actually known for the relevant transition rates), we shall make the reasonable assumption that $\delta \sim 1$ and see whether the estimated values of $|z|$ and ρ are consistent with the present data.

We can determine $|z|$ from B.R. ($K_S \to 2\mu$) if we choose that value, $|z| \sim 4 \times 10^{-3}$, which most plausibly reconciles B.R. ($K_L \to 2\mu$) and B.R. ($K_S \to 2\mu$) at the present time. A value of ρ can then be found from the predicted ratio for B.R. ($K_S \to 2\mu$) to B.R. ($K^+ \to \pi^+ \nu\bar{\nu}$), namely:

$$\frac{\text{B.R.}\ (K_S \to 2\mu)}{\text{B.R.}\ (K^+ \to \pi^+ \nu\bar{\nu})} = 0.23\,\rho^2 \tag{38}$$

The experimental upper limit[21] for B.R. ($K^+ \to \pi^+ \nu\bar{\nu}$) is 1.2×10^{-6} and if we insert this value into Eq. (38), we obtain $\rho^2 \sim 1.5$. The branching ratio $\Gamma(K^+ \to \pi^+ \nu\bar{\nu}) / \Gamma(K^+ \to \pi^0 e^+ \nu_e)$ itself gives an independent determination of $|z|$ through the relation:

$$\frac{\Gamma(K^+ \to \pi^+ \nu\bar{\nu})}{\Gamma(K^+ \to \pi^0 e^+ \nu_e)} = 2\,|z|^2 \tag{39}$$

whence $|z| \lesssim 3.5 \times 10^{-3}$ which is of the right magnitude.

We now list some predictions for several other interesting K meson decays involving the emission of a neutral lepton pair. Consider the decay $K_L \to \pi^o \ell \bar{\ell}$ which proceeds in order g^3 without violating any symmetries of the basic interaction. This implies that the CP = +1 $g^2 e^4$ weak-electromagnetic contribution to this decay may be neglected with respect to the CP = -1 g^3 contribution. The rate for $K_L \to \pi^o \mu\bar{\mu}$ decay involves ρ^2 whereas we can neglect this term for $K_L \to \pi^o e\bar{e}$ decay; we predict (using $|z| \sim 3 \times 10^{-3}$):

$$\text{B.R.}\,(K_L \to \pi^o e\bar{e}) = 0.41\,|z|^2 \simeq 3 \times 10^{-6} \tag{40}$$

$$\frac{\Gamma(K_L \to \pi^o \mu\bar{\mu})}{\Gamma(K_L \to \pi^o e\bar{e})} = \frac{1+\rho^2}{2}\left\{0.51 + \frac{\rho^2 - 1}{\rho^2 + 1}(0.17) + 1.5\lambda_+ - \frac{2\rho^2}{\rho^2+1}\left(0.16 - 0.13\,\xi - 0.03\,\xi^2\right)\right\} \tag{41}$$

which, for $\lambda_+ = 0.03$, $\xi = -0.6$ and $\rho^2 \approx 1.5$ yields the value[22] 0.40 so that the predicted B.R. $(K_L \to \pi^o \mu\bar{\mu}) \simeq 1.2 \times 10^{-6}$. It would be interesting to have measurements of these rare decay modes of K_L.

The decay process $K^+ \to \pi^+ \ell\bar{\ell}$ is more complicated because now the CP = +1 weak-electromagnetic amplitude involves one photon (and is therefore of order $g^2 e^2$) and is comparable with the CP = -1 g^3 amplitude. Estimates of the weak-electromagnetic contributions to the branching ratios have been made with the results:

$$\text{B.R.}\,(K^+ \to \pi^+ \mu\bar{\mu}) = 3.5 \times 10^{-7} \tag{42}$$

$(g^2 e^2)$:

$$\text{B.R.}\,(K^+ \to \pi^+ e\bar{e}) = 8.5 \times 10^{-7} \tag{43}$$

The g^3 contributions to $K^+ \to \pi^+ \ell\bar{\ell}$ are identical with the corresponding contributions to $K^+ \to \pi^+ \nu\bar{\nu}$ so that we have [cf. Eq. (39)]

$$\text{B.R.}\,(K^+ \to \pi^+ \mu\bar{\mu}) \simeq 5 \times 10^{-7} \tag{44}$$

(g^3):

$$\text{B.R.}\,(K^+ \to \pi^+ e\bar{e}) \simeq 12 \times 10^{-7} \tag{45}$$

If we recall that Eqs. (42) and (43) were obtained from absorptive amplitudes and that Eqs. (44) and (45) were derived from real amplitudes multiplied by i (to represent the CP= -1 nature of the W boson interaction), it follows that destructive interference may occur between the corresponding amplitudes and thereby reduce the actual branching rations. This may account for the measured lower upper bound for B.R. $(K^+ \to \pi^+ e\bar{e}) \lesssim 0.4 \times 10^{-6}$ compared to B.R. $(K^+ \to \pi^+ \nu\bar{\nu}) \lesssim 1.2 \times 10^{-6}$. If the above estimates are at all reasonable, the strong cubic W boson model would also predict gross CP violation effects for these rare decay modes of K^+. The magnitude of these effects and their modes of detection have already been discussed in considerable detail.[15] In the same paper will be found a discussion of gross CP violation effects in the related rare decay mode: $\Sigma^+ \to pe\bar{e}$ as well as in the W boson production reaction itself, namely, $\nu_\mu + N \to \mu^- + W + N$. In the latter case, the detection of an appreciable transverse polarization of the muon from the decaying W boson would provide evidence for gross CP violation.

We conclude our status report on the strong cubic W boson model by remarking that a search for neutral lepton pairs from decaying bosons or baryons at the g^3 level is of great interest independent of our model. CP violation is, so to speak, a large effect at this (g^3) level and it is difficult to believe that this departure from the "classical" (V, A) current-current interaction theory will not reflect itself in the occurrence of other unexpected phonomena at the same (g^3) level.

Acknowledgements:

I wish to thank Prof. K. Nishijima for emphasizing the possible importance of the approximate equality of the CP-violating parameter and the semi-weak coupling constant. Many valuable discussions with Prof. S. Okubo concerning the incorporation of this idea into the W boson model led to the triplet W boson model which is highlighted in this report. This report includes the results of more recent calculations (with special emphasis on the $K_L \to 2\mu$ puzzle) in which I was assisted by Prof. N.P. Chang and Dr. T. Ma, of The City College of New York. Conversations with Prof. B. Sakita are also gratefully acknowledged.

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7. A.R. Clark et al, Phys. Rev. Letters 26, 1667 (1971).

8. K. Nishijima and L.J. Swank Phys. Rev. 146, 1161 (1966) wrote down a CP = -1 semi-weak hadronic interaction in the form $f\partial_\lambda N_\lambda$ (N_λ is a CP = +1 neutral hadron current) where $f \sim g$. This term is in addition to the CP=+1 (V-A) current-current interaction.

9. One need not work with a triplet of W bosons - one can develop an equivalent strong cubic W boson model with an octet of W bosons as Okubo (ref. 6) first showed. I prefer the triplet formulation since it is simpler and requires the minimum number of new quantum numbers.

10. Eq. (3) contains an additional quartic interaction among the W's which prevents an infinite lower bound to the energy implied by a cubic interaction alone. Since the quartic interaction contributes to the W boson self-energy, it will

change the estimate of the coupling constant f_o; however, the qualitative conclusions will remain unchanged.

11. A $\Delta Y=2$ transition in order g^3 is forbidden by the small value of the $(K_L - K_S)$ mass difference (which is of order g^4).

12. The typical term which appears is $\langle W_\mu(x) \rangle_o$ (where the expectation value is understood with respect to the vacuum state of the W bosons defined by the W. Lagrangian) and this must vanish by "cubic parity" conservation.

13. Cf. P.D. Miller et al., Phys. Rev. Letters 19, 381 (1967); C.G. Shull and R. Nathans, Phys. Rev. Letters 19, 391 (1967).

14. This follows from the fact that $W^{(-)}$ and $\overline{W^{(+)}}$ are not antiparticles of each other - since they possess different "cubic parities" - and so there is no $J_1^2 J_3^1$ contribution to the hadronic weak processes.

15. See R.E. Marshak, Y.W. Yang and J.S. Rao, Phys. Rev. D3, 1640 (1971).

16. If SU_3 symmetry holds for the W bosons or even if $m(W^{(-)}) = m(W^{(+)})$ with $m(W^{(o)})$ being different, the g^3 amplitude for $K_2 \to 2\mu$ will vanish identically; neither condition is necessary in the "local" approximation to the matrix element.

17. Cf. L.M. Sehgal, Phys. Rev. 183, 1511 (1969).

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19. J. Steinberger (private communication).

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21. J.H. Klems, R.H. Hildebrand and R. Stiening, Phys. Rev. D4 66 (1971).

22. This is to be compared with the value of 0.32 obtained by Okubo and Bace (ref. 20) Independent computations of other neutral lepton processes are also carried out in this paper.

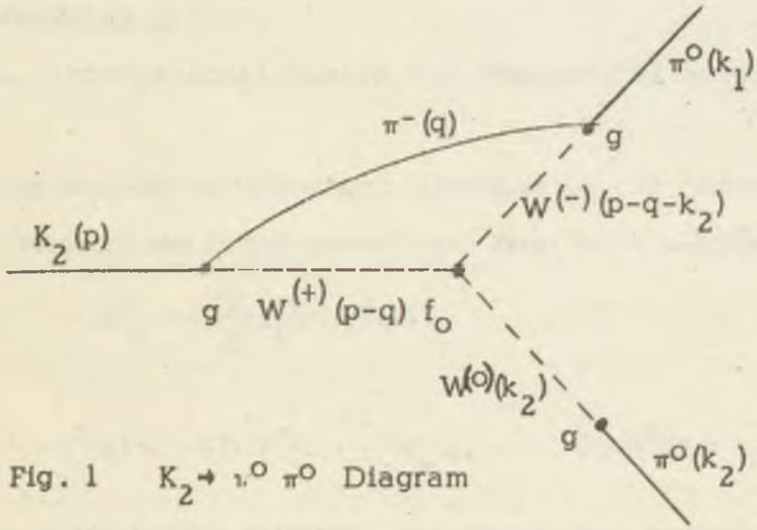

Fig. 1 $K_2 \rightarrow \pi^O \pi^O$ Diagram

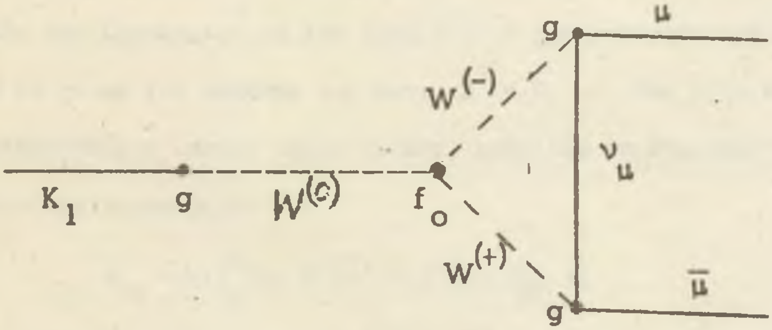

Fig. 2. $K_1 \rightarrow 2\mu$ Diagram (g^3)

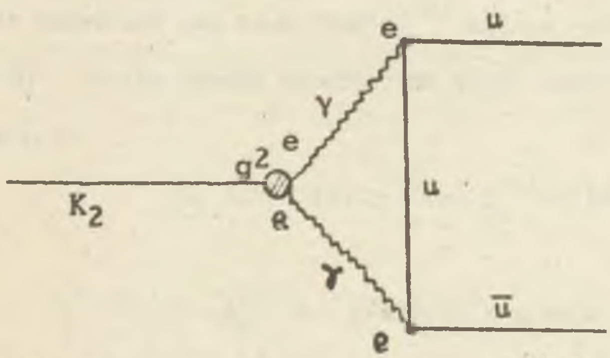

Fig. 3 $K_2 \rightarrow 2\mu$ Diagram (g^2e^4)

VIRTUAL NEUTRINO EFFECTS

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Let us consider the interaction Lagrangian density responsible for the lepton weak interactions in the conventional Fermi current-current form:

$$\mathcal{L}_W = -\frac{G_F}{\sqrt{2}} j_\rho^+(x)\, j^\rho(x) \tag{1}$$

with

$$j^\rho(x) = :\bar{e}(x)\gamma^\rho(1+\gamma_5)\nu_e(x): + :\bar{\mu}(x)\gamma^\rho(1+\gamma_5)\nu_\mu(x):$$

This Lagrangian implies the existence of forces between leptons due to the exchange of neutrinos (see Fig.1a) where ℓ stands for lepton).

Since the Lagrangian in the form (1) is non-renormalizable, it must be modified in an as yet unknown way near $x^2 = 0$ One such modification, as in the intermediate vector boson theory, consists in starting, instead of from (1), from the Lagrangian:

$$\mathcal{L}_W = g(j_\rho^+(x)\, W^\rho(x) + j^\rho(x)\, W_\rho^+(x)) , \tag{1'}$$

which in turn might be made renormalizable [1], and the corresponding neutrino exchange is represented in Fig.1b. These modifications near $x^2 = 0$, and hopefully renormalizations, will only modify the neutrino forces at short distance (high momenta) while their behaviour at large distance (low momenta) will be uniquely determined by the experimentally well-established behaviour of the Lagrangian (1) or (1') at large x^2 .

Their behaviour has been studied [2] and is represented by the behaviour (for $x^2 \neq 0$) of the vacuum expectation value (corresponding to the neutrino loop of Fig.1a):

$$\Pi_{\lambda\rho}(x) = \langle 0|T j_\lambda^{(\nu)}(x)\, j_\rho^{(\nu)}(0)|0\rangle \tag{2}$$

where

$$j_\lambda^{(\nu)} = :\bar{\nu}(x)\gamma_\lambda(1+\gamma_5)\nu(x):$$

At large distance, $\Pi_{\lambda\rho}(x)$ decreases with x as x^{-6} . The corresponding static potential in the three-dimensional space is:

$$V(r) = - \frac{G_F^2}{4\pi^3} r^{-5} , \qquad (3)$$

where $r = (x_1^2 + x_2^2 + x_3^2)^{\frac{1}{2}}$ and the corresponding r^{-6} - dependent force is repulsive between two leptons yet attractive between lepton and antilepton.

Due to the x^{-6} dependence, the neutrino forces may become strong at short distances. The problem is now up to which (short) distance does the well-established x^{-6} behaviour remain valid.

In the intermediate boson theory one would say up to the Compton wavelength of the W mass. In the current-current theory (1), since the only inbuilt length is $G_F^{\frac{1}{2}}$, it would be reasonable to take this as the lower limit for the validity of the large-distance behaviour (this corresponds to the unitary cut-off in momentum space).

Let us then examine which are the consequences of the following working hypothesis:

Hypothesis: The modification of the neutrino forces due to the modification of the Lagrangian (1) (and hopefully of its renormalization) happens, for leptons, at distances of the order of $G_F^{\frac{1}{2}}$:

$$\begin{array}{ll} \Pi_{\lambda\rho}(x) \text{ is given by (2)} & \text{for } x^2 \gtrsim G_F \\ \Pi_{\lambda\rho}(x) \text{ is not more singular than } x^{-2} & \text{for } x^2 \lesssim G_F \end{array} \qquad (4)$$

This hypothesis does not conflict with the knowledge we have at present of the lepton weak interactions and is particularly harmless for the neutrino forces.

We have in fact that if we admit the generally accepted hypothesis that neutrinos are both massless and chargeless, the function (2) obeys the equation:

$$\partial^\lambda \Pi_{\lambda\rho}(x) = 0$$

and, as a consequence, its Fourier transform can be put in the form

$$\Pi_{\lambda\rho}(k) = (k_\lambda k_\rho - k^2 g_{\lambda\rho})\Pi(k^2) \qquad (5)$$

and $\Pi(k^2)$ is represented by only a logarithmically divergent integral instead of quadratically, as one would deduce from power counting. (And this is true even at higher orders in G_F, that is, if we take the neutrino loop in higher orders, inserting into it an arbitrary number of lepton and neutrino loops. (see B.F.II)[2].)

The consequence of the logarithmic divergence of $\Pi(k^2)$ is that the unitary cut-off ($\Lambda \sim G_F^{-\frac{1}{2}} \approx 300$ GeV) or regularization of the neutrino loop is harmless for the higher-order weak lepton processes. It is plausible that (see for example A.T. Filippov's [3] report at the present meeting) the unitary cut-off is harmless for all lepton weak processes. Now precisely the unitary cut-off or regularization corresponds, in the x space, to the assumed hypothesis (4) that the neutrino forces have the behaviour (2) and (3) up to distances $x^2 \sim G_F$ and that the Lagrangian (1) has to be modified only for $x^2 \lesssim G_F$

Obviously, the most interesting consequence of these neutrino forces would be the formation of lepton-antilepton resonances or composite states.

The best instrument we have to examine this possibility is the Bethe-Salpeter equation, which, starting from the Lagrangian density (1), has the general form:

$$(i\hat{\partial}_1 - m_1)\chi_P(x_1x_2)(i\hat{\partial}_2 + m_2) = -\frac{G_F^2}{2}\gamma^\rho(1+\gamma_5)$$
$$\langle 0|T\, J_\rho^{(\nu)}(x_1)\ell_1(x_1)\bar{\ell}_2(x_2)J_\lambda^{(\nu)}(x_2)|P\rangle\ \gamma^\lambda(1+\gamma_5) \tag{6}$$

where the 4×4 function $\chi_P(x_1x_2) = \langle 0|T\ell(x_1)\bar{\ell}(x_2)|P\rangle$ represents the wave function of the possible bound state of total momentum P built up, due to neutrino forces, by the leptons $\ell_1(x_1)\ \ell_2(x_2)$. The left-hand side member of (6) is usually called the vertex function Γ :

$$\Gamma_P(x_1x_2) = (i\hat{\partial}_1 - m)\ \chi_P(x_1x_2)(i\hat{\partial}_1 + m_2)\ , \tag{7}$$

and, in the ladder approximation, is represented in Fig.2. Because of the projectors $(1+\gamma_5)$ in (6) the vertex (7) can have only the form

$$\Gamma(x_1 x_2) = \gamma^\rho(1+\gamma_5)\Gamma_\rho(x_1 x_2) \qquad (8)$$

and the Bethe-Salpeter equation in the ladder approximation assumes a particularly simple form both for the vertex Γ and for the bound state wave function χ_P and the sectors for their tensor components are decoupled (see B.F. II).[2] It is well known that the Bethe-Salpeter equation acquires a particularly simple and symmetric form for massless composite states $(P^2 = 0)$.

In our case the equation for the Γ_ρ appearing in (8) becomes, in momentum space and reference system $P_\mu = 0$, (see B.F.II Eq.(39')):[2]

$$\Gamma_\rho(p) = -4G_F^2 \int d^4q \, \frac{\Pi[(p-q)^2]}{(1-q^2)^2} \, (p-q)^2 \, [q^2 g_\lambda^\tau - 2q_\lambda q^\tau] \, \Gamma_\tau(q) \quad , \qquad (9)$$

where p is the relative momentum of the constituents.

In a general reference system the vertex appearing in (9) will be both a function of the total momentum P and of the relative momentum p. It is easy to show that the solution of the equation for $\Gamma_\rho(P,p)$ is invariant against the gauge transformation

$$\Gamma_\rho'(P,p) \to \Gamma_\rho(P,p) + P_\rho\Lambda(P,p) \qquad (10)$$

with $\Lambda(P,p)$ an arbitrary well-behaved function of P and p. This means that one can impose on the vertex $\Gamma_\rho(P,p)$ the condition

$$P^\rho\Gamma_\rho(P,p) = 0 \qquad (11)$$

and, consequently, further decompose it in the form

$$\Gamma_\rho^{(r)}(p) = \varepsilon_\lambda^{(r)}[g_\rho^\lambda S(p^2) + (4p^\lambda p_\rho - p^2 g_\rho^\lambda)D(p^2)] \qquad (12)$$

where $\varepsilon^{(r)}$ is a unit polarization vector and the equations for S and D can be reduced to one-dimensional integral equations (B.F.II).[2]

In order to solve Eq.(9) or the corresponding equations for S and D one has now to regularize the logarithmically divergent kernel $\Pi(k)$ representing the neutrino loop. This can be done in many, to some extent equivalent, ways. If one uses the non-polynomial technique in which the regularization depends on a regularizing coupling constant f (of dimensions M^{-2}) one finds that, for different approximations of the kernel and of the integral equations, the condition for Eq.(9) to admit a massless solution is *) (see B.F.I) [2]

$$f \simeq G_F \tag{13}$$

This condition in ordinary space implies that the behaviour of the neutrino forces will deviate from the long-distance behaviour (they will become less singular for $x^2 \to 0$) at distances of the order of

$$x^2 \approx f \approx G_F \tag{14}$$

that is Eq.(13) corresponds to the hypothesis (4).

From (11) and (12) we see further that the lepton-antilepton composite states may only have spin-1. Their coupling to the free leptons can also be computed, properly normalizing the wave function $\chi_P(x_1 x_2)$, and it turns out that in our approximations it is either strong or medium strong (see B.F.I).[1]

We have then that from the Lagrangian (1) the two following possible consequences can be drawn:

a) Hypothesis (4) is valid and as a consequence spin-one lepton composite states exist. Since they are presumably not weakly coupled to the free leptons it should be easy to verify their existence experimentally.

b) Neither the composite states nor resonances exist. But then

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*) The equations for S and D can easily be reduced to the Goldstein or Thirring [4] type of integral equations.

the hypothesis (4) must be discarded and the neutrino forces must deviate from their x^{-6} dependence at a distance much larger than $G_F^{\frac{1}{2}}$. The condition of non-existence of resonances or bound states could establish then a lower distance of validity of the Fermi current-current Lagrangian (1) which should be larger than $G_F^{\frac{1}{2}}$. In the intermediate vector boson theory it would establish an upper bound on the W mass.

Let us keep hypothesis (4) from which (a) follows, and let us draw some further consequences from it.

It is known [5] that a composite state can always be represented by a field, and in this frame the amplitude (8) could be considered to be derived (in the limit of long wavelength: $\lambda >> G_F^{\frac{1}{2}}$) from the effective Lagrangian:

$$\mathcal{L}^{eff} = e\,\bar{\psi}(x)\gamma^{\rho}(1 + \gamma_5)\psi(x)A_{\rho}(x) \tag{15}$$

where x stands for the c.m. co-ordinate of the composite system and $A_{\rho}(x)$ is defined, in momentum space, by

$$e\,A_{\rho}(P) = \Gamma_{\rho}(P, p_1^2 = m^2\ , p_2^2 = m^2) \quad . \tag{16}$$

Because of (10) and (11) we may impose on A_{ρ} the condition

$$\partial^{\rho}A_{\rho}(x) = 0 \tag{17}$$

The Lagrangian (15) implies the existence of a spin-1 field quasilocally (in the limit $\lambda >> G_F^{\frac{1}{2}}$) coupled to the electron and muon fields.

We shall suppose that the bare (with respect to the weak interactions) electron e_0 and muon μ_0 have the same mass. As a consequence, because of weak universality, Γ_{ρ} will obey the same equation, (9), both for electron and muon, and e in (16) will be the same for these two leptons. (15) will be explicitly:

$$\mathcal{L}^{eff} = e[\bar{e}_0(x)\gamma^\rho(1 + \gamma_5)e_0(x) + \bar{\mu}_0(x)\gamma^\rho(1 + \gamma_5)\mu_0(x)]A_\rho(x) \quad . \qquad (15')$$

Approximate solutions of Eq.(9) (see B.F.I)[2] give for e values between 0.1 and 1 . If these values are confirmed by further calculations, the composite states represented by A_ρ should be easily detectable.

It is known that the only spin-one boson medium-strongly coupled to the massive leptons is the photon.

One would then be naturally brought to identify the spin-one massless lepton composite state represented by the field A_ρ as the photon. But the difficulty in this interpretation is that from the weak Lagrangian the only permissible vertex of both electron and muon with the composite state is the parity non-conserving (15') while notoriously electromagnetic interactions conserve parity. One must enquire if there might be a mechanism by which the effective Lagrangian (15') deduced from the weak lepton Lagrangian might give rise to the space (and charge) reflection invariant form of the quantum electrodynamical Lagrangian.

The problem here is similar to that encountered in the recent successful attempts [1] to unify electrodynamics and weak interactions; there also, one has to find a mechanism to get rid of the γ_5 term in the neutral weak currents. One can then borrow from those models the mechanism for the restoration of parity (space and charge) conservation in going from weak to electromagnetic interactions.

One such possibility would be to take the Salam-Weinberg model [1] and choose the coupling constants g and g' in the intermediate boson Lagrangian in such a way as to make the diagonal weak Lagrangian parity-conserving for the massive leptons. In this way one would obviously obtain parity-conserving vertex for the free lepton-composite state amplitude. But the mathematical simplicity brought to our model by the projectors $(1 + \gamma_5)$

would be spoiled. Besides, one needs the introduction of more neutral vector bosons.

Another possibility is offered by the recent work of Georgi and Glashow [1)] in which no new neutral vector bosons are introduced besides the photon, which would be for us the lepton composite state. New unobserved massive leptons are introduced instead, building up two triplets of electronic states (and further two for the muon). In our case we would have that massless neutrino exchange generates composite states between these leptons; the charged and massive ones (we would bind a charged lepton and a neutral antilepton with different masses) would be the charged W meson and the only neutral massless one the photon. The O(3)-invariant coupling of these vector bosons to the fermions would give both the known weak and the parity-conserving electromagnetic coupling of the composite lepton-antilepton state with the leptons (the charged ones) plus weak and electromagnetic interactions of the unobserved leptons, as in the Glashow-Georgi model. Since the theory is renormalizable, we should obtain a well determined regularization of the kernel $\Pi(k^2)$. But still free parameters like the lepton masses would be left in the theory.

Yet we prefer to think that if hypothesis (4) and its consequence (a) are correct there should be a more economical way to restore parity (P and C) conservation. A possible way could be the following. Take the Konopinsky weak Lagrangian in the two equivalent forms:

$$\mathcal{L}_K^{(1)} = \frac{G_F}{\sqrt{2}} \bar{\mu}^c \gamma^\rho (1 - \gamma_5)\nu \; \bar{e}\gamma_\rho (1 + \gamma_5)\nu + \text{h.c.} \qquad (18)$$

$$\mathcal{L}_K^{(2)} = \frac{G_F}{\sqrt{2}} \; \bar{\nu}\gamma^\rho (1 + \gamma_5)\mu \; \bar{\nu}\gamma_\rho (1 - \gamma_5) e^c + \text{h.c.} \qquad (18')$$

Starting from these we could deduce for the vertex amplitude corresponding to (8):

$$\Gamma = \gamma^{\rho}(1 - \gamma_5)\Gamma_{\rho}^{(c)} \qquad (8')$$

both for the electron and for the muon, where the superscript (c) stands for charge conjugate (or, better, lepton conjugate since there is no charge yet).

As (15') was obtained from the amplitude (8) we may think that the amplitude (8') might be derived from the effective Lagrangian

$$\mathcal{L}_K^{eff} = e[\bar{e}_0^{(c)}(x)\gamma^{\rho}(1 - \gamma_5)e_0^{(c)}(x) + \bar{\mu}_0^{(c)}(x)\gamma^{\rho}(1 - \gamma_5)\mu^{(c)}(x)]A_{\rho}^{(c)}(x) \qquad (19)$$

Taking the charge conjugate of this we have

$$C\mathcal{L}_K^{eff}C^{-1} = e[\bar{e}_0(x)\gamma^{\rho}(1 - \gamma_5)e_0(x) + \bar{\mu}_0(x)\gamma^{\rho}(1 - \gamma_5)\mu(x)]A_{\rho}(x) \qquad (19')$$

where the hypothesis

$$C A_{\rho}^{(c)}C^{-1} = A_{\rho} \qquad (20)$$

was adopted.

To obtain the electromagnetic Lagrangian we need only "define" it as the half sum of (15') plus (19') to obtain:

$$\mathcal{L}_0 = e[\bar{e}_0(x)\gamma^{\rho}(e_0(x) + \bar{\mu}_0(x)\gamma^{\rho}\mu_0(x)]A_{\rho}(x) \qquad (21)$$

(This "definition" could be justified by the necessity of summing over opposite directions in closed lepton loops (see Fig.2).) The universality of the lepton electric properties follows from the universality of their weak interactions if we admit that the bare electron and muon have equal masses and that the (weak) renormalization will not alter the vector current densities and we obtain

$$\mathcal{L}_{em} = e[e(x)\gamma^{\rho}e(x) + \bar{\mu}(x)\gamma^{\rho}\mu(x)]A_{\rho}(x) \qquad (22)$$

Electric charge conservation follows from the gauge invariance (11) and it is different from lepton number conservation which follows from the gauge invariance of the weak Lagrangian (1).

Needless to say, in this model the lepton electrodynamics is finite, the unrenormalized electric charge is zero $(z_3 = 0)$ and the standard local electrodynamics should be valid up to distances of the order $G_F^{\frac{1}{2}}$ (if strong interactions are not taken into account).

In this model the electric charge should be computed as a function of G_F after solution of Eq.(9) or the corresponding ones for S and D in (12). It is clear that these solutions will depend on the way the kernel $\Pi(k^2)$ is regularized. In the way we have obtained the electrodynamical Lagrangian (22), this regularization is not defined because the current-current weak Lagrangians (1) and (18) are not renormalizable. If we started instead from a renormalizable Lagrangian like (1') then the renormalization of the weak interactions would fix uniquely (apart from parameters like the W masses which could be fixed by conditions in the frame of weak interactions) the behaviour of the kernel $\Pi(k^2)$ at high values of k^2 and the value of the electric charge should be computable uniquely.

Should the main line of this scheme for the origin of electromagnetic properties of leptons be at least partially true, many more questions remain open and await an answer. One of the first is the explanation of the electromagnetic properties of hadrons. There, because of baryon number conservation, one cannot exchange neutrinos between baryons (or, rather, baryon quarks); one is then unable to think of the photon as a composite of hadron quarks, as it was of lepton quarks (here we use the term lepton quark for bare leptons).

Nevertheless, if one accepts the idea that weak interactions are responsible for binding lepton quarks into the photon, they should also be able to bind hadron quarks into hadrons.*) (In the frame of recent models, one has only to bring the W mass one order of magnitude higher than the

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*) One such possibility was examined some time ago by Thirring and coworkers[4].

lower limits accepted at present.) The situation there is much more complicated by the fact that, roughly speaking, the baryon quarks differ much more from baryons than the lepton quarks do from leptons. Besides, one probably has not the $(1 + \gamma_5)$ projector but perhaps $(1 + \alpha\gamma_5)$ and then one will obtain $(\alpha^2 - 1)$ proportional pseudoscalar and tensor vertices besides the (8) vector and axial-vector one. One could nevertheless think that by some mechanism a massive vector composite system of baryon quarks is formed which has the same quantum numbers as the photon. Then, by weak interaction it will have the possibility of transition into the photon (the baryon constituents decay weakly in those of the lepton). This would then be a dynamical basis for the explanation of so-called vector dominance. Naturally, one could always suppose, at low momenta, that the lepton composite state, the photon is directly bound to the hadron and that the vertex obeys (10), from which electric charge conservation and universality follow. The intermediate of the massive vector boson would only be felt in the hadron electric form factor.

But many more problems regarding the renormalizability of this model and its implications for higher-order processes have yet to be investigated.

At this preliminary stage the model has the attractive features of allowing the derivation of electric from weak lepton universality and that for both electric and weak interactions the Fermi coupling constant represents the minimal distance at which the interactions are well represented by the local standard theory, while divergences disappear from electrodynamics and are left only in weak interactions.

But, apart from this and the possible self-consistency of the scheme, only nature, through experimental evidence, can tell us if all this is not only possible but also true.

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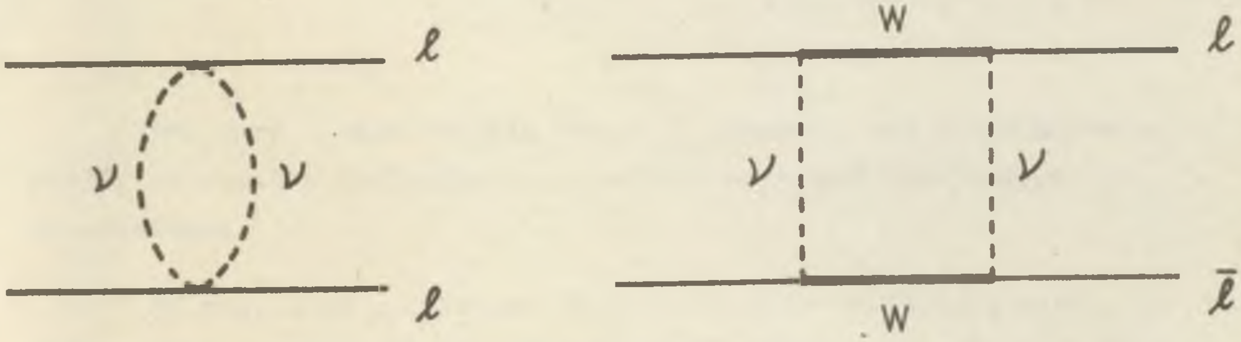

a) b)

Fig. 1

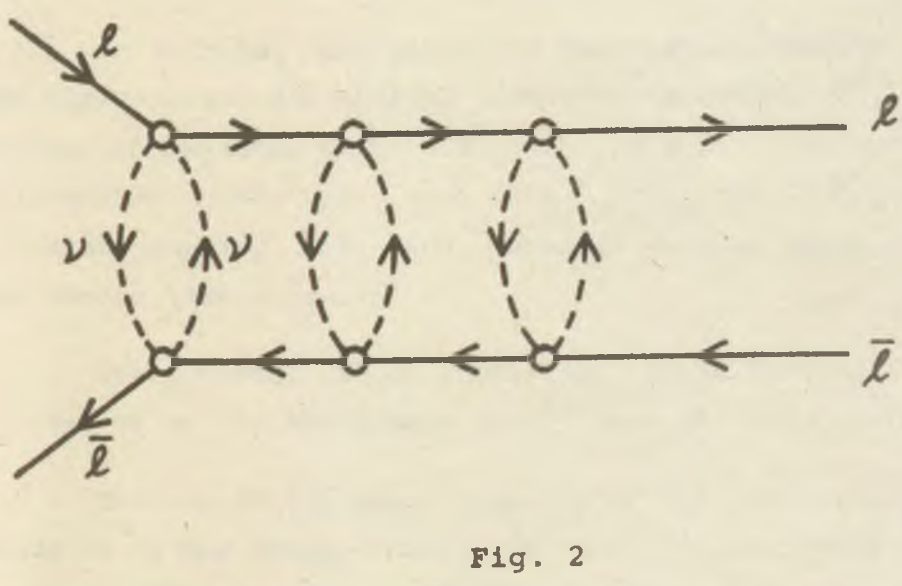

Fig. 2

CALCULATION OF STATIC QUANTITIES IN WEINBERG'S MODEL*

W.A. Bardeen, R. Gastmans+ and B. Lautrup, CERN - Geneva

The work I want to talk about [1] concerns the calculation of static quantities in Weinberg's model of weak and electromagnetic interactions [2].

By now, most people are acquainted with Weinberg's model, so let me just recapitulate briefly the main ideas. One starts with a gauge theory in which a triplet of gauge fields couples to electronic isospin and a singlet field to electronic hypercharge. One then introduces spontaneous breakdown of the symmetry - also called the Higgs-Kibble mechanism [3] - by the introduction of a scalar boson field which has a non-zero vacuum expectation value. The net result of all this is to produce masses for the electron and some of the bosons and various couplings among these particles.

In this way, one generates the electromagnetic interactions of the electron and the charged intermediate boson, $W^{\pm}$, the weak interactions of the usual type, $\bar{e}\gamma^{\mu}(1+i\gamma_5)\nu_e W_{\mu}^{-}$, plus neutral current interactions of the type $\bar{\nu}\gamma^{\mu}(1+i\gamma_5)\nu Z_{\mu}$ and $\bar{e}\gamma^{\mu}(1+ia\gamma_5)e Z_{\mu}$, a scalar coupling $\bar{e}e\phi$, and, finally, various other couplings among the bosons themselves.

The interest in the model lies in the fact that it belongs to a class of models which have a very good chance of being renormalizable.

Because of the gauge symmetry of the theory, one can in principle study it in any gauge. First, 't Hooft [4] examined a class of very similar models in the renormalizable gauge, i.e., the gauge in which the vector boson propagator behaves like $1/k^2$ for $k^2 \to \infty$. He can do this by introducing a set of ghost particles and additional vertices. He then could show that these models were unitary.

* presented by R. Gastmans

+ Aangesteld Navorser, N.F.W.O., Belgium

For practical purposes, it is, however, much more desirable to be able to calculate in the manifestly unitary gauge, with a vector boson propagator

$$-i \, \frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{M^2}}{k^2 - M^2 + i\varepsilon} ,$$

since one does not have to introduce unphysical particles or additional vertices.

Obviously, if the theory is renormalizable, then the calculation of physical quantities must lead to finite results in any gauge, also in the unitary gauge. On the one hand, the unitary gauge is the easiest one to calculate in, but on the other hand, the divergence difficulties look much more severe. In this work we wanted to verify the finiteness of physical quantities in the unitary gauge.

The first problem which then arises is how to regularize highly divergent integrals associated with Feynman diagrams involving closed loops, since it is very desirable to have a regularization scheme which respects the symmetry of the theory. Fortunately, a gauge invariant regularization procedure exists, and the main idea for it is due to 't Hooft and Veltman [5].

I shall now briefly describe this method. Suppose we live in a world with n space-time dimensions, then to evaluate Feynman diagrams, one would have to consider integrals of the type

$$I(n,m) = \int \frac{d^n k}{(2\pi)^n} \, \frac{1}{[k^2 - L + i\varepsilon]^m} .$$

For $n < 2m$, the integral exists and

$$I(n,m) = \frac{i^{1-2m}}{(2\sqrt{\pi})^n} \, L^{\frac{n}{2}-m} \, \frac{\Gamma(m-\frac{n}{2})}{\Gamma(m)}$$

This expression is a meromorphic function of n which can be analytically continued to complex values of n, and which then defines all integrals, even the ones which are divergent for $n \to 4$.

How do we now implement this in specific calculations ?

i) We calculate everything in n dimensions, but all physical four-vectors, like external momenta, polarization four-vectors, etc., have only their first four-components different from zero. (The other $n-4$ components vanish.) We do the same with the Dirac γ matrices. All other four-vectors are n dimensional. Our way of handling the γ matrices is different from 't Hooft and Veltman's way in which the matrices also are n dimensional. It is then impossible, however, to define a γ_5 without introducing many anomalies.

ii) Our prescription now regularizes in a gauge invariant way all loops involving meson lines.

iii) Only spinor loop subgraphs now remain divergent, and these have to be regularized separately, e.g., with a symmetric ε separation, as has been done by Bardeen [6). To make these spinor loops gauge invariant, we need only cancel anomalies in the lowest order loops, and this can be done by introducing supplementary quarks.

iv) At the end, one takes the limit $n \to 4$.

Since we now have a consistent regularization scheme, nothing should stop us from performing real calculations. Unfortunately, physical processes, like μ decay to fourth order, involve more than 20 diagrams, which means a considerable amount of work.

Therefore, we decided to look first into the calculation of the static quantities in the theory, since their calculation is much easier. It allows us to examine how our regularization scheme works, and to check the renormalizability of Weinberg's model.

The first quantity we looked at was the anomalous quadrupole moment of the charged intermediate boson. The five graphs of Fig. 1

contribute, and the superficial degree of divergence is logarithmic. It turns out that all graphs are separately finite, and we refer to Ref. 1) for their finite values.

A much more crucial test of the theory is provided by the calculation of the g-2 of the W. This time, there are ten graphs (Fig.2) of which only those involving the φ and the leptons are finite. The superficial degree of divergence is now quadratic. However, it turns out that the sum of all the other diagrams adds up to a finite contribution again [1].

Then, we looked at the electromagnetic properties of the neutrino. In lowest order, there are two diagrams which could give it a charge (Fig.3). For the theory to be consistent, the charge of the neutrino has to remain zero, and this turns out to be the case, using the n dimensional regularization procedure.

Let me now make a few comments about the charge radius of the neutrino, which is defined as

$$\langle r^2 \rangle = 6 \left. \frac{\partial F(q^2)}{\partial q^2} \right|_{q^2=0} ,$$

where

$$\mathcal{M}_\mu = ie\, F(q^2)\, \bar{u}\, \gamma_\mu (1 + i\gamma_5)\, u$$

is the matrix element of the electromagnetic current between nuetrino states. Four additional graphs now contribute also (Fig.4).

First, to measure a charge radius of the neutrino, one has "to get inside" the neutrino with virtual photons (say by doing $e\nu$ scattering). But then, one also has to consider the competing processes in Weinberg's model, like two Z or two W exchange, and radiative corrections to single Z exchange. Indeed, in Weinberg's model, all particles which couple to the photon must also couple to the Z. The consistency of the theory only requires the total scattering amplitude for $e\nu$ scattering to be finite, and indeed we find that $F'(0)$ is divergent in Weinberg's model. It is clear from this that the neutrino charge radius is not a physical quantity in the model.

Finally, we calculated the muon anomaly without any ambiguity since the gauge invariance of the theory is respected with our regularization schemes, in contradistinction to calculations using, e.g., the ξ - limiting procedure. The graphs from Fig. 5 now contribute, but their numerical value is very small. No limit on the mass of the φ can be deduced from the present or planned experiments of the muon g-2.

This completes our tour of the static quantities in Weinberg's model, and, in conclusion, we can say that our calculations support the claim that the model is indeed renormalizable.

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* * *

FIGURE CAPTIONS

Figure 1 : Feynman diagrams contributing to the anomalous quadrupole moment of the W.

Figure 2 : Feynman diagrams contributing to the dynamic anomalous magnetic moment of the W.

Figure 3 : Feynman diagrams contributing to the self-charge of the neutrino.

Figure 4 : Feynman diagrams which also contribute to the charge radius of the neutrino, but not to its self-charge, because of gauge invariance.

Figure 5 : Feynman diagrams contributing to the anomaly of the muon.

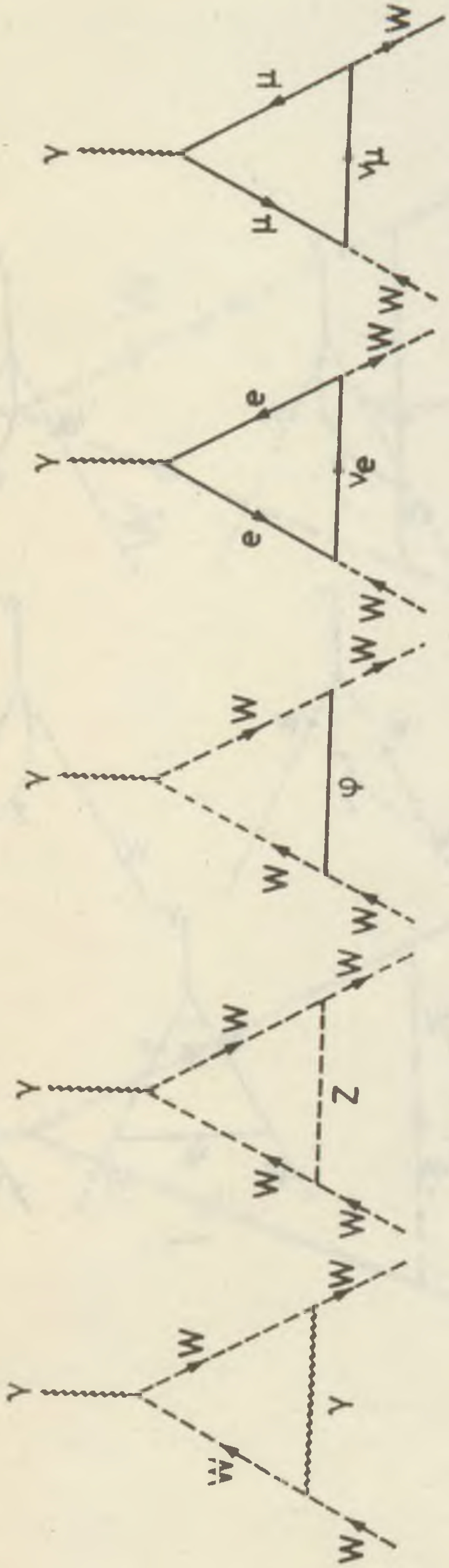

FIG.1

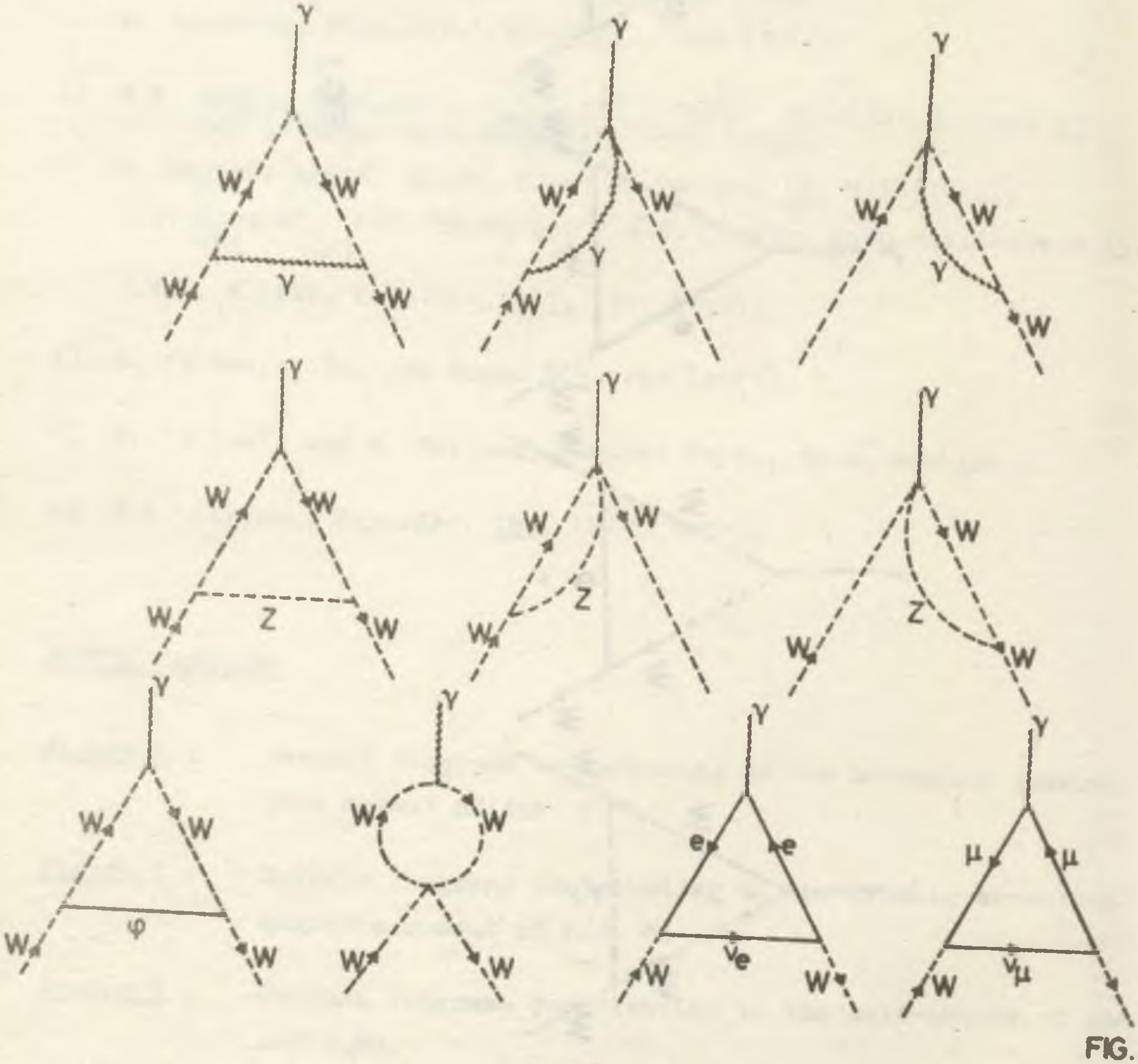

FIG. 2

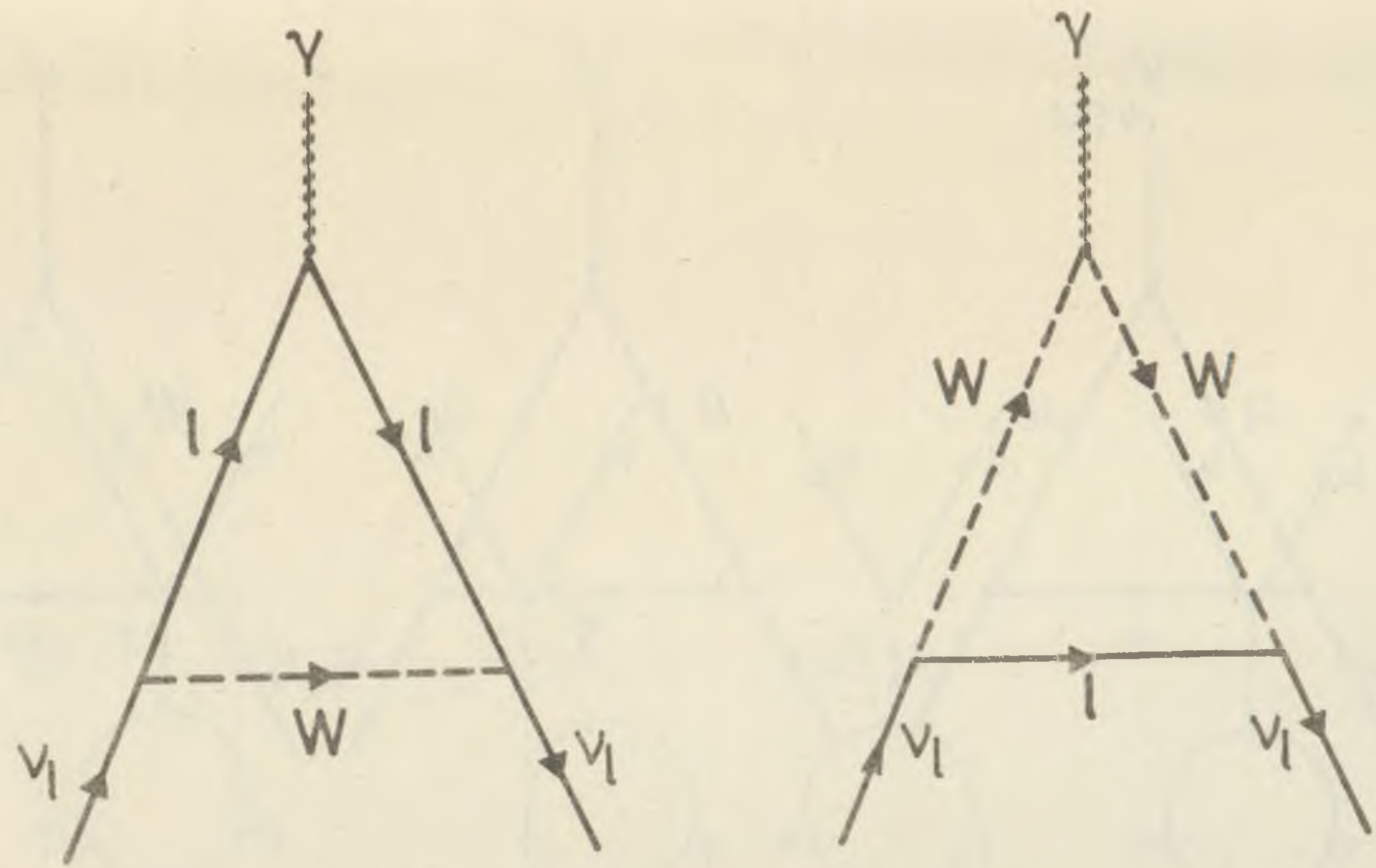

FIG.3

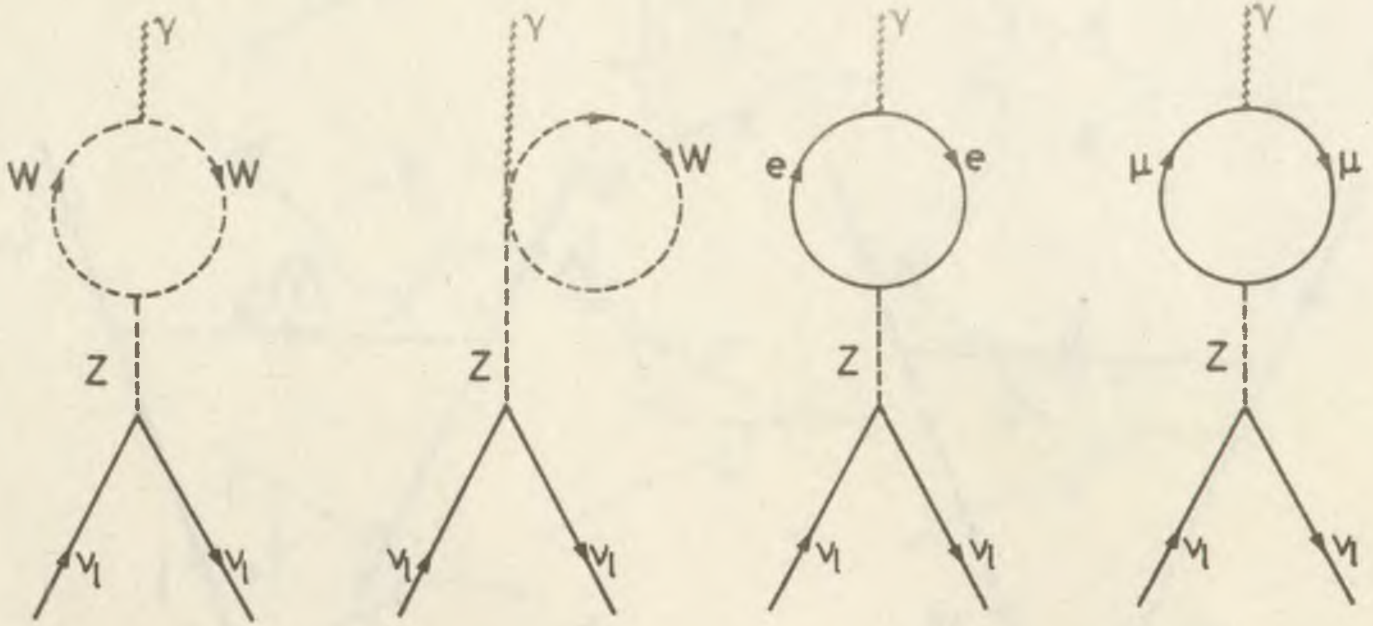

FIG.4

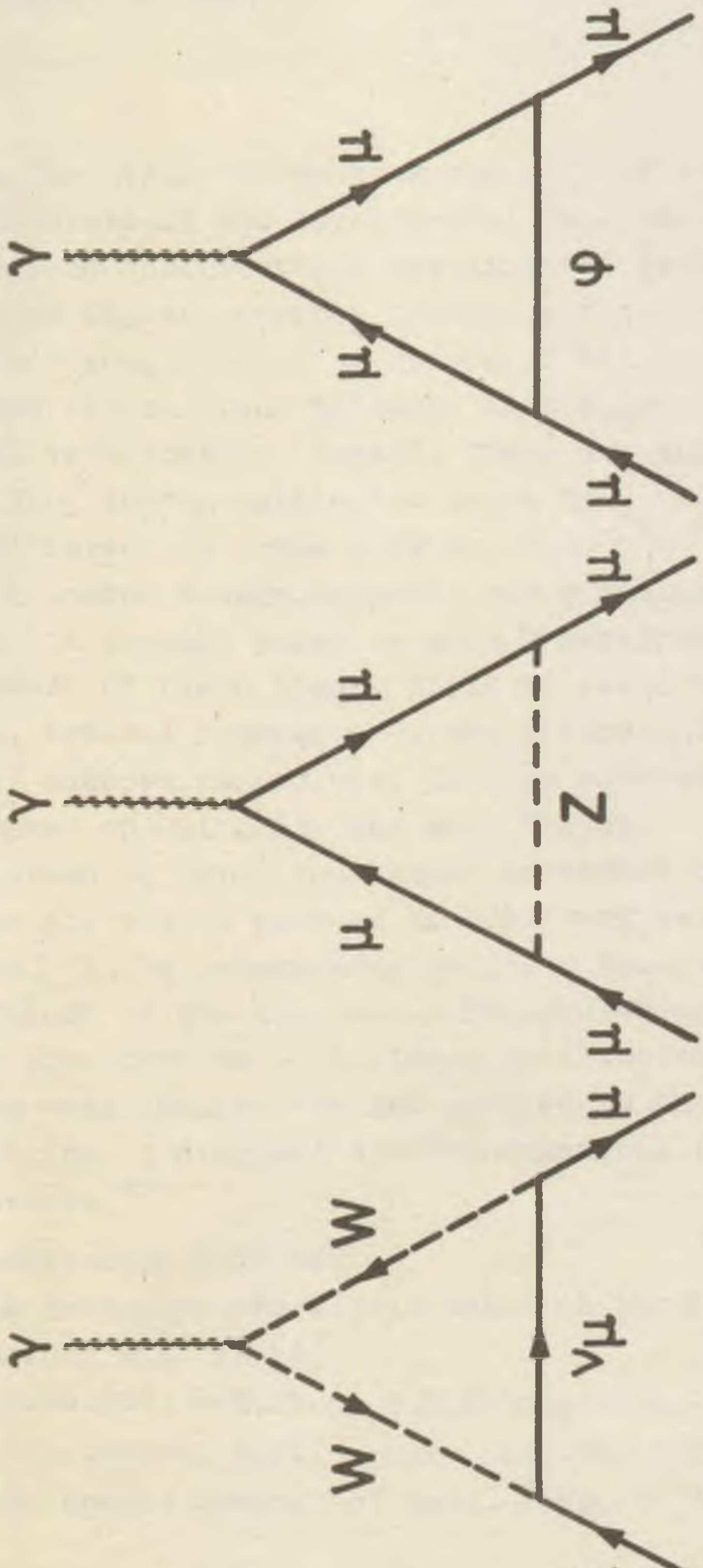

FIG. 5

ANALYTIC RENORMALIZATION AND ELECTRON - ANTINEUTRINO SCATTERING+

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1. The higher order weak corrections are interesting both for theoretical and experimental reasons. At low energies only negligible contributions are expected in higher orders of G, while at higher energies the corrections might become large due to the strongly singular nature of the conventional V-A four fermion interactions bringing in the possibility of the experimental verification. Indeed, there are arguments [1] that for colliding lepton-antilepton beams 2x250 GeV, $\Theta = 90^0$, the G^4-order differential cross section, $2.10^{-37} cm^2/sr.$, surpasses the corresponding electromagnetic contribution.

In the present paper we give a perturbation treatment in the framework of field theory which is valid below the unitarity limit, several hundred GeV. The method introduces a minimum number of unknown parameters. This is contrasted to the approach [2] based on unitarity and analyticity.

Instead of using the rather uncertain cutoff procedures, we define the finite part of an arbitrary Feynman graph by Speer's method [3]. In renormalizable field theories this method is equivalent to the Bogoliubov-Parasiuk-Hepp renormalization.

The procedure to be followed is illustrated in second order of the weak interaction and applied to the $e - \bar{\nu}_e$ elastic scattering. A diagonal electron-neutrino interaction is also introduced.++

+ Presented by G.Pócsik

++ The astrophysical significance of the $e - \bar{\nu}_e$ scattering is emphasized e.g. in [4]

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An experimental upper limit of the diagonal coupling constant has been reported by Reines [5] .At such energies, however, which we are interested in the $e-\bar{\nu}_e$ scattering is merely of theoretical interest, in any case it shows undisturbed weak interaction effects.

In the next Section we define the physical amplitude in order G^2 for $e-\bar{\nu}_e$ scattering, then the corrections are discussed to the cross section in Section 3.

2. Speer's method consists of two steps. At first, we continue each Feynman denominator in a formal graph into a meromorphic distribution, $(-p^2+m^2-i0)^{-1} \to (-p^2+m^2-i0)^{-1-\zeta}$ where $\mathrm{Im}\,\zeta \neq 0, \mathrm{Re}\,\zeta$ large enough /the only poles are located at $\zeta = 1, 2, \ldots$/ and carry out the momentum integrations. The result is a meromorphic function of many complex variables /called generalized Feynman amplitude/. In the second step a physical amplitude is defined by applying a generalized evaluator to the generalized Feynman amplitude [3]. Roughly speaking the generalized evaluator acts as if one would take out the constant term in the Laurent expansion of the generalized Feynman amplitude. The generalized evaluator can be conveniently identified by the operation

$$\frac{1}{k!}\frac{1}{(2\pi i)^k}\sum_{\sigma}\int_{C_{\sigma(1)}}\frac{d\zeta_1}{\zeta_1}\cdots\int_{C_{\sigma(k)}}\frac{d\zeta_k}{\zeta_k} \qquad /1/$$

where the summation runs over all the permutations of 1,...,k and the radius $|\zeta_i| = R_j$ of the circle C_j oriented counterclockwise is assumed to satisfy $0 < R_1 < \ldots < R_k, R_i > \sum_{j=1}^{i-1} R_j$

As we said, we are interested only in effects caused by a single $l\nu_l$ -loop which has the formal contribution

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$$\left(\frac{i m_j}{(2\pi)^4}\right)^2 \int \frac{\gamma_{\alpha\beta}\cdot P_1}{-P_1^2 - i0} \; \frac{\gamma_{\gamma\delta}\cdot P_2 + \delta_{\gamma\delta}}{-P_2^2 + 1 - i0} \; \delta^{(4)}(P_2 \pm P_1 - Q)\, d^4P_1\, d^4P_2 =$$

$$= -\frac{1}{2}\gamma_{\alpha\beta}\cdot\gamma_{\gamma\delta}\, d_1^{\pm}(Q^2)_j + (\gamma_{\alpha\beta}\cdot Q)(\gamma_{\gamma\delta}\cdot Q)\, d_2^{\pm}(Q^2)_j - \qquad /2/$$

$$- (\gamma_{\alpha\beta}\cdot Q)\,\delta_{\gamma\delta}\, d_3^{\pm}(Q^2)_j, \qquad Q = \frac{q}{m_j}, \quad P_k = \frac{p_k}{m_j}$$

Continuing the denominators we arrive at the following generalized version of the invariants $d_i^{\pm}$ for large ζ_i

$$d_1^{\pm}(Q^2, \zeta_1, \zeta_2)_j = \pm i\, \frac{m_j^2 \pi^2}{(2\pi)^8}\, \frac{\Gamma(\zeta_1+\zeta_2-1)\Gamma(2-\zeta_1)}{2\Gamma(1+\zeta_2)}\cdot$$

$$\cdot F(\zeta_1+\zeta_2-1, 1+\zeta_1; 3; Q^2+i0), \qquad /3/$$

$$d_2^{\pm}(Q^2, \zeta_1, \zeta_2)_j = \pm\left(\frac{m_j\pi}{(2\pi)^4}\right)^2 \frac{\Gamma(\zeta_1+\zeta_2)\Gamma(2-\zeta_1)\Gamma(2+\zeta_1)}{6\Gamma(1+\zeta_1)\Gamma(1+\zeta_2)}\cdot$$

$$\cdot F(\zeta_1+\zeta_2, 2+\zeta_1; 4; Q^2+i0),$$

$$d_3^{\pm}(Q^2; \zeta_1, \zeta_2)_j = \pm i\left(\frac{m_j\pi}{(2\pi)^4}\right)^2 \frac{\Gamma(\zeta_1+\zeta_2)\Gamma(2-\zeta_1)}{2\Gamma(1+\zeta_2)}\, F(\zeta_1+\zeta_2, 1+\zeta_1; 3; Q^2)$$

with F the hypergeometric function. The meromorphic structure is evident from the Γ -functions. Consider /3/ near ζ_i =0, here the poles of $d_i^{\pm}$ are of the form $(\zeta_1+\zeta_2)^{-1}$. Applying the generalized evaluator /1/ to /3/ we are led to the following physical amplitudes

$$d_1^{\pm}(q^2)_j^R = \mp\frac{i}{6}\left(\frac{m_j\pi}{(2\pi)^4}\right)^2 \left\{4 - \frac{11}{6}Q^2 + Q^{-4}\left((Q^2-1)^3 \ln(-Q^2+1-i0) - Q^2\right)\right\},$$

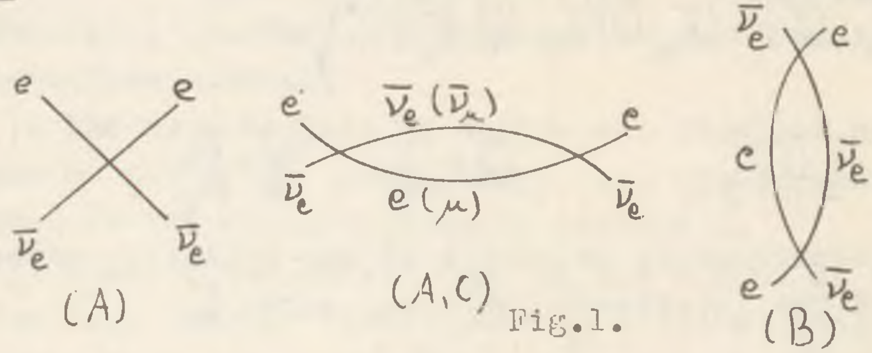
$$d_2^{\pm}(q^2)_j^R = \pm\frac{i}{6}\left(\frac{m_j\pi}{(2\pi)^4}\right)^2 \left\{\frac{11}{6} + Q^{-2}\left(-Q^{-2}\left(1+(1-Q^2)^2\right) - \right.\right. \qquad /4/$$

$$\left.\left. - Q^{-4}(1-Q^2)^2(2+Q^2)\ln(-Q^2+1-i0)\right)\right\},$$

$$d_3^{\pm}(q^2)_j^R = \pm\frac{i}{2}\left(\frac{m_j\pi}{(2\pi)^4}\right)^2 \left\{1 + Q^{-2} + Q^{-4}(1-Q^2)^2 \ln(-Q^2+1-i0)\right\}.$$

These functions are everywhere finite including $Q^2=0$ and $Q^2=1$.

3. The relevant graphs are drawn in Fig.1. Their contribut-



$$q = p_e + p_{\bar\nu} = p_e' + p_{\bar\nu}', \qquad p = p_e - p_{\bar\nu}'$$

Fig.1.

ions to the S-matrix can be found as

$$\langle e', \bar\nu' | T^{(1)} + T^{(2)} | e, \bar\nu \rangle = (2\pi)^{-6} G \sqrt{2 E_{\bar\nu} E_{\bar\nu}'} \cdot$$
$$\cdot \Big[(\tilde v_{\bar\nu} \gamma^{\omega} (1 - i\gamma_5) u_e)(\tilde u_e' \gamma_{\omega} (1 - i\gamma_5) v_{\bar\nu}') A(q^2) -$$
$$- (\tilde u_e' \gamma_{\omega} (1 - i\gamma_5) u_e)(\tilde v_{\bar\nu} \gamma^{\omega} (1 - i\gamma_5) v_{\bar\nu}') B(p^2) + \qquad /5/$$
$$+ (\tilde v_{\bar\nu} (1 + i\gamma_5) u_e)(\tilde u_e' (1 - i\gamma_5) v_{\bar\nu}') C(q^2) \Big]$$

and

$$A(q^2) = \lambda + \frac{(4\pi)^4 G}{2\sqrt{2}} \Big[\lambda^2 d_1^-(q^2)_e^R + d_1^-(q^2)_\mu^R - q^2 \big(\lambda^2 m_e^{-2} d_2^-(q^2)_e^R +$$
$$+ m_\mu^{-2} d_2^-(q^2)_\mu^R \Big],$$
$$B(p^2) = i \lambda^2 \frac{(4\pi)^4 G}{\sqrt{2}} \Big[-2 d_1^+(p^2)_e^R + m_e^{-2} p^2 d_2^+(p^2)_e^R \Big], \qquad /6/$$
$$C(q^2) = i \frac{(4\pi)^4 G}{\sqrt{2}} \Big[\lambda^2 d_2^-(q^2)_e^R + d_2^-(q^2)_\mu^R m_e^2 m_\mu^{-2} \Big]$$

λG denotes the strength of the diagonal $e-\nu_e$ Hamiltonian.

The cross section of the $e-\bar{\nu}_e$ scattering can be obtained by standard methods. In terms of the dimensionless variables [6]

$$\omega_\alpha = (\omega, \underline{\omega}), \; p_\alpha = (\varepsilon, \underline{p}), \; \underline{\omega} = m_e^{-1} \underline{p}_{\bar{\nu}}, \; \omega = m_e^{-1} E_{\bar{\nu}},$$
$$\underline{p} = m_e^{-1} \underline{p}_e, \; \varepsilon = m_e^{-1} E_e, \; \mu_{\bar{\nu}} = (E_{\bar{\nu}} E'_{\bar{\nu}})^{-1} \underline{p}_{\bar{\nu}} \underline{p}'_{\bar{\nu}} \qquad /7/$$

and for unpolarized electrons we get

$$\sigma = \frac{\sigma_0}{4\pi (p\cdot\omega)} \int \frac{d^3\omega' d^3p'}{\varepsilon' \omega'} \delta^{(4)}(p'_\alpha + \omega'_\alpha - p_\alpha - \omega_\alpha) \cdot$$
$$\cdot \Big[(p\cdot\omega')(\omega\cdot p') |A+B|^2 + \frac{(\omega\cdot\omega')}{4} \Big(C(A^* + B^*) + C^*(A+B) \Big) + \qquad /8/$$
$$+ (\omega\cdot p)(\omega'\cdot p') \frac{|C|^2}{4} \Big]$$

and

$$\sigma_0 = \frac{4 G^2 m_e^2}{\pi} = 1{,}7 \cdot 10^{-44}\, cm^2 \qquad /9/$$

From /8/ is easy to get various distributions.

[6] J.N.Bahcall: Phys.Rev. 136 /1964/ 1164

For instance, the angular distribution of the antineutrinos for an electron initially at rest is given by

$$\frac{d\sigma}{d\mu_{\bar\nu}} = \frac{\sigma_0}{2} \frac{\omega^2}{(1+\omega(1-\mu_{\bar\nu}))^2} \left[\frac{|A+B|^2}{(1+\omega(1-\mu_{\bar\nu}))^2} + \right.$$

$$\left. + \frac{1-\mu_{\bar\nu}}{4(1+\omega(1-\mu_{\bar\nu}))} \left(C(A^*+B^*) + C^*(A+B) \right) + \frac{|C|^2}{4} \right] \qquad /10/$$

From this result it follows that at low energies the higher order corrections are small as is expected. On the other hand, at about $s=/100\ \mathrm{GeV}/^2$ the corrections amount to 5-10 %. Numerical results are indicated in Table 1 for two extreme scattering angles and with λ =1.

	$\mu_{\bar\nu} = -1$		$\mu_{\bar\nu} = 1$	
	$\frac{G^3}{G^2}$ [%]	$\frac{G^4}{G^2}$ [%]	$\frac{G^3}{G^2}$ [%]	$\frac{G^4}{G^2}$ [%]
$\sqrt{S}$ =100 GeV	5,4	0,08	7	0,13
$\sqrt{S}$ =200 GeV	23,4	1,4	30	2,5

Table 1.

With growing energy the corrections become more and more significant even below the unitarity limit. On the basis of the preceding discussion a similar effect is expected in other lepton-lepton collisions too [1].

Nowadays, there is, however, a disadvantageous feature of the theories of higher order weak corrections and this is the appearance of arbitrary polynomials [1,2] making it difficult to draw clear conclusions. Clearly, in the present approach the non-uniqueness of the analytic continuations gives rise to polynomials, presumably with less coefficients than in analytic approaches considering the definite nature of field theoretic description. In this manner numerical informations concerning higher energies /e.g.Table 1/ are usable at best for first orientation.

Turning to the problem of polynomials, we first remark that by continuing the propagators in /2/ one may introduce a suitable function, furthermore, the general evaluator may contain another suitable function. In such a way $g(\zeta_1,\zeta_2)\,d_i^{\pm}(Q^2;\zeta_1,\zeta_2)_j$ is still a generalized Feynman amplitude where $g(\zeta_1,\zeta_2)$ is analytic around $\zeta_i = 0$ with real coefficients and $g(0,0)=1$. In renormalizable field theories $g(\zeta_1,\zeta_2)$ gives rise to finite renormalizations, in the present case it modifies the relevant amplitudes by the quantities

$$\Delta(A+B) = -\frac{G m_e^2}{6\pi^2\sqrt{2}}\left[C_1\lambda^2\left(\frac{15}{2}-\frac{p^2}{m_e^2}\right)+\frac{3}{2}\left(\frac{m_\mu^2}{m_e^2}\right)C_2\right],$$

$$\Delta C = \frac{G m_e^2}{6\pi^2\sqrt{2}}\left[\lambda^2 C_1 + C_2\right]. \qquad /11/$$

Two further constants were eliminated together with $d_3^{\pm}$ through calculations of traces. In a calculation working with massless leptons $C_1 = C_2$. In Table 2 we show leptonic processes of order G^2 where the constants C_1, C_2 appear and a comparison is made to Ref. [2].

In principle C_1, C_2 can be determined from not too low energy experiments, presumably they give contributions at most within the order of Table 1.

Processes	Parameters	Parameters in [2] /massless leptons/
$\nu_e e \to \nu_e e$	C_1, C_2 $(\lambda \neq 0)$ C_2 $(\lambda = 0)$	$\alpha_{20} - \gamma_{20} + \gamma_{11}$ $\alpha_{11} + \gamma_{11}$
$\nu_\mu \mu \to \nu_\mu \mu$	$C_1 C_2$ $(\lambda \neq 0)$ C_1 $(\lambda = 0)$	—"—
$\nu_e \mu \to \nu_e \mu$	C_1	γ_{20}, γ_{11}
$e \nu_\mu \to e \nu_\mu$	C_2	—"—
$\nu_e \mu \to e \nu_\mu$	C_1, C_2 $(\lambda \neq 0)$	α_{20}, α_{11}

Table 2

Let us assume C_1, C_2 give 7% at s=/100 GeV/^{2}, then we obtain c_1=25, c_2=15. These values provide small corrections at small and high energies.

Theoretically, analytic and ultra-high energy assumptions provide restrictions for the parameters [1,2] .

APPLICATION OF TWISTOR THEORY IN WEAK INTERACTIONS

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The purpose of this comment is to call attention to advantages of an interesting new particle theory in the study of weak interactions and, in particular, in neutrino-electron scattering. This, the twistor theory, is being developed by R. Penrose /Details will appear soon in a forthcoming paper by MacCallum and Penrose [1]/.

A twistor is, mathematically, the "spinor" for the SU/2,2/ group which is isomorphic to the 15 parameter group of conformal transformations of space-time. Geometrically, it is represented in a non-local way by a congruence of light-like lines. Some of the outstanding features of twistor theory are listed here:

1/ Fields can be described by means of analytic twistor functions. On introducing a scalar product, in an appropriate way, for these functions, they become elements of a Hilbert space. I suggest that we should call this the <u>Penrose representation</u>.

2/ Twistor theory offers a straightforward way of covariant quantization. This is ensured by the covariant canonical structure of the twistor equations of scattering obtained from the unquantized problem.

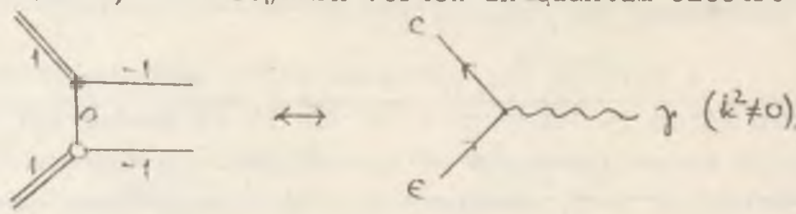
3/ No perturbation expansion is needed for the calculation of scattering amplitudes. From the point of view of the weak interactions, this is an important virtue of the theory. The calculation of matrix elements, involving contour integrations over several twistor variables, is usually strenuous, however.

4/ So far, divergences did not emerge in the theory.

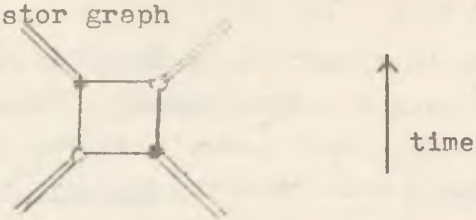
5/ The latest developments in twistor theory include the description of rest-mass.

The structure of scattering matrix elements can be represented by graphical methods analogous, to a certain extent, to the Feynman rules. It should be kept in mind however, that the twistor graphs represent the scattering process as a whole. Now some graphical rules follow.

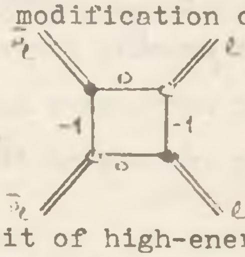
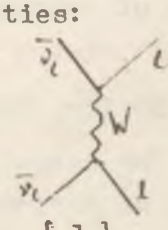
A dot • denotes a twistor variable /and a circle o means a conjugate twistor variable/ over which contour integration is to be performed. A line labeled by an integer n, joining a dot to a circle •—n—o shows that helicity n /in units of $\hbar/2$/ proceeds from the dot to the circle. A kind of <u>helicity conservation</u> can be formulated such that the sum of helicities at each vertex must be zero. We have the further rule that exactly four lines meet at each vertex. Particles are represented by pairs of twistor lines. "Virtual particles" of the conventional theory are replaced here by a pair of parallel lines joining <u>different</u> vertex pairs. For example, the twistor graph corresponding, in a sense, to a Feynman-vertex in quantum electrodynamics is



It turns out that a number of elementary processes can be described by the twistor graph



We might try to translate the V - A weak interaction Lagrangian into the twistor language. I am not quite sure, however, that this is the appropriate way to go. There are a number of uncertainties in the weak interaction theory such as the existence of the W boson or a self-current term[2]. This latter problem comes in when we consider the process $\bar{\nu}_\ell + \ell \to \bar{\nu}_\ell + \ell$. Let us start with the assumption, therefore, that the simple box diagram which already proved to be useful in the description of analogous well-understood processes, can be used here with the appropriate modification of the helicities:

Assumption  suggests 

In the limit of high-energy scattering [1], we can easily evaluate this diagram and obtain the cross-section $C.\mathrm{ctg}^4\frac{\vartheta}{2}$ with ϑ the CMS scattering angle. Though this result is in agreement with the first-order perturbation calculation based on V - A theory[2] , we cannot exclude the possibility that in the high energy limit, scattering can occur on the opposite helicity state of the electron giving an additional $C'\mathrm{ctg}^{-4}\frac{\vartheta}{2}$ term to the cross-section.

[1] R. Penrose and M.A.H.MacCallum, Physics Reports /to appear/
[2] R.E.Marshak, Riazuddin and C.P.Ryan: Theory of weak interactions in particle physics /Wiley, 1969/.

A THEORY OF UNIVERSAL WEAK INTERACTIONS OF LEPTONS

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The conventional theory of leptonic weak interactions[1] has the well known deficiency that, due to non-renormalizable nature of its divergences, it is in fact non-universal (see e.g.[2]). This deficiency is characteristic of the four-fermion theory with $(\nu \equiv \nu_e,\ \nu' = \nu_\mu)$

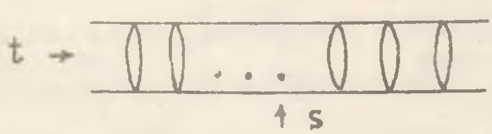
$$\mathcal{L}_{int} = \frac{G}{\sqrt{2}} j^+_\mu j^\mu, \quad j^\mu = \bar{e}\gamma^\mu(1+\gamma_5)\nu + \bar{\mu}\gamma^\mu(1+\gamma_5)\nu' \quad (1)$$

as well as the IVB-theory with

$$\mathcal{L}_{int} = g\,(j^+_\mu W^\mu + j^\mu W^+_\mu). \qquad (2)$$

Apparently, non-renormalizability is not such an unavoidable drawback of these theories as it seems at the first glance. Several methods for calculating higher order corrections in different non-renormalizable theories were recently proposed (see e.g.[3-6] and references cited there). Using these methods, based on a summation of infinity of diagrams, enables us to understand the characteristic difference between renormalizable (R) and non-renormalizable (N) theories. In both theories amplitudes of physical processes have a singularity in g at $g = 0$, but in R-theories it is an essential singularity while in N-theories it is a branch point. This assertion is not rigorously proved but seems to be quite plausible. In fact, it was found to be valid for some exactly solvable models with trivial S-matrix as well as for partial sums of perturbation theory diagrams (ladder diagrams[4,5], superpropagators and diagrams constructed by use of superpropaga - tors[3,7] etc.).

The easiest way to understand the principal features of the methods of calculating higher-order corrections in N-theories is to use the model of scattering on strongly singular potentials(for references see the review[8]). We note that the Schroedinger equation with a singular potential is not only a model of N-theories. Results obtained in this model theory may be directly applied to
calculating simplest corrections to lepton-lepton scattering amplitudes. For example, the Bethe-Salpeter equation for lepton-lepton scattering with vanishing total 4-momentum of colliding leptons can be reduced to Schroedinger equation with the potential $V(r) \sim \frac{G}{r^6}$ for $r \to 0$. In a more general case, one would obtain, instead of Schroedinger equation, some higher-order linear differential equation or a system of linear differential equations, the methods devoloped for solving Schroedinger equation with singular potential being applicable to this case as well. This enables us to estimate higher-order corrections to leptonic weak processes and to find the high energy behaviour of scattering amplitudes. It might be shown that the sum of the ladder diagrams



with $t > 0$, $s < 0$ gives us a reasonable quantitative approximation for $E \sim \sqrt{t} \ll G$, and with $t < 0$, $s > 0$ it gives us a reasonable qualitative approximation for $E \sim \sqrt{s} \sim G^{-1/2}$

The characteristic features of the approach to N-theories outlined above are as follows: 1) The renormalization of the X vertex is necessary; it corresponds to adding δ-type singularities to the potential. 2) Near $s \sim G^{-1}$ the behaviour

of the scattering amplitude significantly changes and its asymptotic behaviour for $s \to \infty$ is bounded by unitarity constraints (qualitatively, the asymptotics of the scattering amplitude is similar to one obtained for scattering on the hard core potential $V = \infty$ for $r \leq G^{1/2}$, $V = 0$ for $r \geq G^{1/2}$
3) The order of magnitude of the amplitude can be obtained by unitary cut-off after renormalizing the most divergent terms $\sim G^n \Lambda^{2n-2}$, i.e. these leading divergences are eliminated by use of the vertex renormalization constant Z_Λ and then the momentum cut-off parameter Λ is chosen to be of order $G^{-1/2}$. This observation is of special importance as the simplest recipe for the estimation of higher order corrections follows from it.

Unfortunately, using this recipe is not sufficient for obtaining a reliable quantitative result.

To this end, another methods not connected with a summation of infinity of diagrams were proposed - the differential interpolation method /9/ and the method based on using Padé approximations /10/. The idea of these methods has some resemblance to the renormalization group method /11/. One calculates a finite number of Frynman diagrams with a cut-off

$$F(G,\Lambda,\dots) = GF_1 + G^2F_2(\Lambda,\dots) + G^3F_3(\Lambda,\dots) \qquad (3)$$

and then transforms this expression into another one for which the limit $\Lambda \to \infty$ does exist and which coincides with (3) up to a given order of G. When Padé method is used, one simply rewrites this series in the form of a ratio of two polynomials such as the ratio has a finite limit for $\Lambda \to \infty$. When the differential interpolation method is used, one constructs a linear differential (with respect to Λ) equation satisfied by a given finite number of terms in eq.(3). The solution of the differential equation has the form

$$F_d(G,\Lambda,\dots) = \sum_{i=1}^{n_d} f_d^{(i)}(G\Lambda^2)\,\varphi_d^{(i)}(G,\dots)\,, \tag{4}$$

where $\varphi_d^{(i)}$ are determined by comparing eq.(3) and (4). After renormalizing the expression (4) we pass to the limit $\Lambda \to \infty$ and obtain a finite result (for the details of the procedure see refs.[9]).

Whichever method is used for treatment of leptonic weak interactions, the conclusion is: the universality of the original theory is strongly violated by higher orders. To illustrate this point consider perturbation theory diagrams for the processes $\nu\mu^- \to \nu' e^-$ and $\nu e^- \to \nu e^-$. The most divergent contributions to functions F_i are different for these two processes (the same is true for $f_d^{(i)}(G\Lambda^2)$). For example, up to the second order, the amplitudes have the form $G(1 + cG\Lambda^2 + \dots)$ with different values of c for different processes. If we try to eliminate the leading divergences by renormalizing the coupling constant, we obtain

$$\begin{aligned} F(\nu\mu \to \nu' e) &= Z\,G(1 + c\,G\Lambda^2 + \dots)(1 + G\cdot(d_1 m^2 \ln \Lambda^{-2} m^2 + d_2) + \dots)\,, \\ F(\nu e \to \nu e) &= Z' G\cdot(1 + c' G\Lambda^2 + \dots)(1 + G\cdot(d_1' m^2 \ln \Lambda^{-2} m^2 + d_2') + \dots) \end{aligned} \tag{5}$$

where $c' \neq c$ and therefore $Z \neq Z'$. So the effective coupling constants describing these processes need not be equal and the universality is completely destroyed. One arrives at the same conclusion if one uses summation methods (see e.g.[4,5]. Therefore, it is qute reasonable to consider this difficulty as the unavoidable intrinsic defect of the conventional theory of leptonic weak interactions.

Many attempts to construct a theory which is free from this difficulty were undertaken (see rev.[12,13]). In all these attempts, many new particles and new interactions are introduced to make a theory renormalizable. If we insist on using only presently detected particles we must consider modifications of four-fermion interactions. It is true that all four-fermion theories are non-renormalizable but we may say that some of them are "less non-renormalizable then others". In fact, there exists a four-fermion theory in which the universality holds for all leading divergences because of some internal leptonic symmetry and in which most dangerous leading divergences can be eliminated by use of one renormalization constant. Such a theory will be described below.

Note that the difficulty with universality does not seem to be so dangerous in the case of semileptonic and hadronic weak interactions. The well-known result concerning the quadratic divergence of second-order matrix elements of hadronic and semileptonic weak processes[15] is based on using Bjorken-Johnson-Low technique, which was strongly criticized because it did not lead to correct results in perturbation theory (see[16]). In addition to this, there is a possibility of compound nature of hadrons described by some universal dimensional parameter (say, the slope of the Regge trajectories). This parameter may play a role of a cut-off parameter Λ in weak interactions of hadrons. This hypothesis does not contradict to scaling in deep inelastic processes as was recently shown by S.Drell and T.D.Lee. Therefore, it is quite plausible to assume that in hadronic weak interactions there exists a cut-off for momenta larger than $\Lambda \sim 1$ GeV. This hypothesis is supported by the experimental data for $K_L \to \mu\mu$ process and

$K_L - K_S$ mass difference, which are consistent with $\Lambda \sim 3 \div 5$ GeV.

Now we describe a new theory of leptonic weak interac - tions which is free from the difficulty with universality due to higher leptonic symmetry. Higher leptonic symmetries were introduced previously by many authors. O(3) or SU_2 symmetry was proposed in refs./17/. Interactions invariant under O(5)-group were investigated in/18/. (B.Arbuzov/17/ and V.Kadyshevsky/18/ have shown that in O(5)-symmetric theory the decay $\mu \to e \bar{\nu} \nu'$ is forbidden and therefore this symmetry must be broken).

We suppose that the leptonic weak Lagrangian is invariant with respect to rotations and reflections (R(4)-group) in a four dimensional leptonic isospace (or leptospace). The leptons and antileptons are described by spinors of this space

$$\psi = (\nu, e^-, \nu', \mu^-)\,, \quad \bar{\psi} = (\bar{\nu}, \bar{e}^-, \bar{\nu}', \bar{\mu}^-)\,,$$

where $\bar{\ell} = \ell^+ \gamma_0$ and ℓ is the usual Dirac spinor. To construct the theory we introduce four 4 x 4 hermitian matrices α_i (i=1,2,3,4), satisfying the relations $\{\alpha_i, \alpha_j\} = 2\delta_{ij}$ and six matrices $\alpha_{ij} = (2i)^{-1} [\alpha_i, \alpha_j]$ representing the generators of the rotations in leptospace. Then the weak currents have the form $J^\mu_{ij} = \bar{\psi} \alpha_{ij} O^\mu \psi$ (where $O^\mu = \gamma^\mu (1+\gamma_5)$) and the Lagrangian describing leptonic weak interactions is*

$$\mathcal{L}_{int} = \frac{G}{8} J^\mu_{ij} J^{ij}_\mu = \frac{G}{8} (\bar{\psi} \alpha_{ij} O^\mu \psi)(\bar{\psi} \alpha^{ij} O_\mu \psi)\,, \qquad (6)$$

where $A_i \equiv A^i$ and the usual summation rule is used. We stress that it is the generators of the rotations represented by α_{ij} which are used for constructing the weak currents. This requi-

* The most general Lagrangian which is invariant under R(4)-group has the form $L_{int} = G/4\, [(\bar{\psi} \alpha_5 \psi)^2 + c(\bar{\psi}\psi)^2]$ where c is an arbitrary real parameter. To prove this it is sufficient to use identities, mentioned below.

rement may be considered as a generalization of the universality principle and it automatically follows from a geometric interpretation of weak interactions (see[17]). The Lagrangian (6) is invariant under the transformations of spinors $\psi \to \exp\left(\frac{i}{2}\alpha_{ij}\varphi^{ij}\right)\psi$ and $\psi \to \alpha_i \psi$ representing rotations and reflections of the leptospace. Furthermore, it is invariant under formal α_5-transformation $\psi \to \alpha_5 \psi$, $\bar\psi \to \bar\psi$ (where $\alpha_5 = \alpha_1\alpha_2\alpha_3\alpha_4$) which will be used later.

Consider now the subgroup of the three-dimensional rotations through the axes 1,2,3 of the leptospace and require the current J^μ_{12} to be coupled with the electromagnetic field and J^μ_{13} , J^μ_{23} -with weak hadronic (charged) currents. To this end it is convenient to choose the following representation for α -matrices: $\alpha_n = \tau_3 \otimes \tau_n$, $i\alpha_{mn} = \varepsilon_{mn\ell}\, 1 \otimes \tau_\ell$, where τ_n are the Pauli matrices. Then the charge matrix $Q_L=1/2(\alpha_{12}-1)$, connected with rotations in the plane (12), and the matrix $F_L = -i\alpha_1\alpha_2\alpha_3 = i\alpha_4\alpha_5$ representing the reflection of the fourth axis, are diagonal with the diagonal elements $Q_L=(0, -1, 0, -1)$ and $F_L=(+1, +1, -1, -1)$. Thus the multiplicatively conserved quantum number F_L may be identified with the multiplicative muonic charge (muonic parity)[19].

Fixing the representation tor the matrices α_4 and α_5, $\alpha_4 = \tau_1 \otimes 1$, $\alpha_5 = -\tau_2 \otimes 1$ *, and using the identity $\frac{1}{2} J_{ij} J^{ij} = (\bar\psi\psi)^2 + (\bar\psi\alpha_5\psi)^2$ and the Fierz identity for anticommuting spinors ℓ we get (omitting the matrices O^μ)

$$\mathcal{L}_L = \frac{G}{4}\left[(\bar\psi\alpha_5\psi)^2 + (\bar\psi\psi)^2\right] =$$

---

* $1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\alpha_5 = \begin{pmatrix} 0 & i1 \\ -i1 & 0 \end{pmatrix}$

$$= \frac{G}{4}\Big[2(\bar{e}\nu)(\bar{\nu}'\mu) + 2(\bar{\nu}e)(\bar{\mu}\nu') - 2(\bar{e}\nu')(\bar{\nu}\mu) - 2(\bar{\nu}'e)(\bar{\mu}\nu)$$

$$+ 4(\bar{e}e)(\bar{\mu}\mu) - 2(\bar{e}\mu)(\bar{e}\mu) - 2(\bar{\mu}e)(\bar{\mu}e) - \tag{7}$$

$$- (\bar{\nu}\nu')(\bar{\nu}\nu') - (\bar{\nu}'\nu)(\bar{\nu}'\nu) + 4(\bar{\nu}\nu)(\bar{\nu}'\nu') +$$

$$+ 2(\bar{\nu}\nu)(\bar{e}e) + 2(\bar{\nu}\nu)(\bar{\mu}\mu) + 2(\bar{\nu}'\nu')(\bar{\mu}\mu) +$$

$$+ 2(\bar{\nu}'\nu')(\bar{e}e) + (\bar{\nu}\nu)^2 + (\bar{\nu}'\nu')^2 + (\bar{e}e)^2 + (\bar{\mu}\mu)^2\Big].$$

The following essentially different processes predicted by this Lagrangian can be detected in near future/20/: $\mu \to e\bar{\nu}\nu'$, $\mu \to e\nu\bar{\nu}'$, $\nu e \to \nu e$, $\nu' e \to \nu' e$, $ee \to \mu\mu$.

The most important experiment is probably a search of $\bar{\nu}'e \to \bar{\nu}\mu$ at NAL, which was proposed by S.S.Gerstein. Our theory predicts the definite relations between these processes which are easily deduced from eq.(2).

Consider now the problem of higher order corrections. The most divergent terms do not depend on masses of leptons and on other interactions. So the highest divergences preserve the symmetries of the original Lagrangian (1), including α_5-symmetry. As the most general four-fermion interaction which is invariant under rotations and reflections in the leptospace and under α_5-transformation coincides with eq.(1), we conclude that these divergences may be factorized and eliminated by renormalizing the coupling constant G. Then the main corrections will be given by next to highest divergences $\sim G^n \Lambda^{2(n-2)}$ (up to a power of $\ln\Lambda$) which for $\Lambda \sim G^{-1}$ are of order G (up to a power of $\ln G$). These corrections may be different for different processes as the mass terms $m_\mu \bar{\mu}\mu$ and $m_e \bar{e}e$ break down the four-dimensional symmetry of the leptospace, the mass matrix having the form

$$M_L = \frac{1}{2} Q_L \left[\alpha_{54} \left(\frac{m_\mu}{m_e} - 1 \right) - \left(\frac{m_\mu}{m_e} + 1 \right) \right] \qquad (8)$$

It is worth noting that a T-violation may be easily incorporated in our theory by choosing the new representation for α_4 and α_5

$$\alpha_4' = \alpha_4 \cos\varphi + \alpha_5 \sin\varphi \,, \quad \alpha_5' = -\alpha_4 \sin\varphi + \alpha_5 \cos\varphi$$

which does not change α_1 , α_2 , α_3 and $\alpha_4 \alpha_5$. The Lagrangian (1) with these matrices violates T-invariance if

$$\varphi \neq 0, \pi/2 .$$

The theory also may be formulated in terms of the coupling of the currents J_{ij}^μ with six vector bosons W_{ij}^μ , the muonic parity being +1 for W_{12} , W_{23} , W_{31} and -1 for W_{14} , W_{24} , W_{34} . The semileptonic interaction can be introduced if we couple the hadronic currents only with the charge leptonic currents J_{23}^μ , J_{31}^μ or with W_{23}^μ , W_{31}^μ.

The weak hadronic currents may be constructed from baryonic fields by using the method of ref. [21]. With this aim we group all the baryons into two four-component spinors (cf. [26])

$$\Psi_+ = (p, n\cos\theta + Y\sin\theta, \Sigma^+, Z\cos\theta + \Xi^\circ \sin\theta) = (\Psi_{++}, \Psi_{+-})$$
$$(9)$$
$$\Psi_- = (-n\sin\theta + Y\cos\theta, \Sigma^-, -Z\sin\theta + \Xi^\circ\cos\theta, \Xi^-) = (\Psi_{-+}, \Psi_{--})$$

where $Y = 1/\sqrt{2}\,(\Lambda - \Sigma^\circ)$, $Z = 1/\sqrt{2}\,(\Lambda + \Sigma^\circ)$ and θ is a real parameter. These spinors correspond to antileptonic and leptonic spinors respectively. The charge matrix is $Q_H = \frac{\alpha_{12} + L_H}{2}$, $L_H \Psi_\pm = \pm \Psi_\pm$ and the analog of the muonic parity is represented by the matrix $F_H = i\alpha_4\alpha_5$, $F_H \Psi_{+(\pm)} = \pm \Psi_{+(\pm)}$, $F_H \Psi_{-(\pm)} = \pm \Psi_{-(\pm)}$. The matrices Q_H, L_H , F_H give us the complete set of quantum numbers defining the baryonic states. It is not hard to verify that from the conservation of L_H and F_H

(independently of L_L and F_L) the usual selection rules $|\Delta S| \leq 1$ and $\Delta S = \Delta Q_H$ follow. By defining the hadronic currents $J^{\mu}_{\pm} = \bar{\Psi}_{\pm} \alpha_{ij} O^{\mu} \Psi_{\pm}$, which transform under rotations and reflections just like the corresponding leptonic currents we write the semileptonic interaction in the form

$$\mathcal{L}_{HL} = \frac{G}{8}\left[J_{13}\left(J^{13}_{+} + J^{13}_{-}\right) + J_{23}\left(J^{23}_{+} + J^{23}_{-}\right)\right]. \qquad (10)$$

One may verify that the semileptonic $|\Delta \vec{T}| = 1/2$ rule for the processes with $|\Delta S| = 1$ is fulfilled for this Lagrangian. The most essential difference between this Lagrangian and that of Cabibbo is that eq. (5) predicts a nonvanishing vector coupling constant for the transition $\Sigma^{-} \to \Lambda e \bar{\nu}$. This prediction does not contradict available experimental data. Purely hadronic weak interactions will be considered in a subsequent paper together with a more detailed discussion of semileptonic interaction. Here we only mention that by using neutral currents the $|\Delta \vec{T}| = 1/2$ rule may be easily incorporated in the theory (cf.ref./22/). The conventional semileptonic weak interaction may be constructed also by use of the quark currents if we introduce the two component isospinor (p' , $n' \cos\theta + \lambda' \sin\theta$) (see e.g. a paper by S.Y.Tsai/17/).

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Appendix

It is worth noting that the original theory may be rewritten in the IVB-form by using only two neutral W-bosons. The Lagrangian is

$$\mathcal{L}_{int} = g\left(\bar{\Psi} O_\mu \Psi W_1^\mu + \bar{\Psi} O_\mu \alpha_5 \Psi W_2^\mu\right)$$

where W_1 and W_2 have the muonic parity +1 and -1 respectively. The intriguing feature of this Lagrangian is that S-matrix corresponding to it is renormalizable if masses of all the leptons are equal to zero. This is due to the conservation of the currents $\bar{\Psi} O_\mu \Psi$ and $\bar{\Psi} O_\mu \alpha_5 \Psi$ and can be proved by using Ward-type identities or by spinor field transformations /for the details see e.g. A.T. Filippov [6] /.

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COSMIC RAY NEUTRINOS*

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Introductory Remarks: A Partial Summary

The talk which I will give today is on an experiment of a completely different character than the report on the $\bar{\nu}_e, e^-$ scattering experiment which I gave on Tuesday. There, the goal is clear and the problem rather well defined. In contrast, the subject of cosmic ray neutrinos is still in an early stage and the experiment I will describe have goals which are of a more diffuse and exploratory character.

The Case Institute of Technology, University of Witwatersrand, University of California, Irvine (CWI) deep underground neutrino experiments have spanned a period of 8 years. Over 100 events have been identified as arising from the interaction of high energy neutrinos (> 100 MeV) with the nuclei of the rock target surrounding the detector.

Our motivation in undertaking the experiment was twofold: we wanted to learn about the neutrino component of the cosmic ray for its own sake and secondly we hoped to use the high energy neutrinos above the 10 GeV available at existing accelerators to learn more about the weak interaction.

These studies were unique in that they used a detector many times larger and located several thousand feet deeper than any previous work. Even with this large, $\sim 150\ m^2$, detector the count rate from muons penetrating the 2 miles of rock above the detectors was only a few per week. An effort of truly heroic proportions will be required to extend the muon depth-intensity curve to greater depths.

The neutrino induced muon data was scanned for features not expected for neutrinos produced in the atmosphere e.g. point or distributed sources of

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extraterrestrial origin, events of high multiplicity such as might arise from supernovae or gravitational collapse, events in time coincidence with signals from Weber's gravitational radiation detectors etc.

No such phenomenon of great interest was observed, but limits have been established on these various processes. For example, Steigman[1], have used our low energy ($\sim$100 MeV) ν data to set limits on the amount of anti-matter in the universe. Our data suggest that Seyfert galaxies seem not to derive their energy from matter-anti matter annihilation.

The limits on extraterrestrial neutrinos indicate how difficult any improved search for these extraterrestrial particles will be in the face of the relatively large flux of neutrinos of atmospheric origin.

Our experimental study of the weak interaction involves measurement of the rates and angular distributions of neutrino-induced muons. The specific goals were to examine the energy dependence of the inverse beta cross-section, and to search for the intermediate vector boson, spurred on by the knowledge of the high energy tail of the neutrino distribution. As we shall see, the experiment has now progressed to the point where our conclusions are no longer limited by our counting statistics. Further refinement of the results await more precise knowledge of its neutrino fluxes and cross-sections. An improved experiment would benefit from more explicit mass and charge measurements of the ν products involved.

With the advent of NAL and CERN prime with sizable fluxes of neutrinos up to $\sim$100 GeV it is evident that the study of neutrino interactions using cosmic rays becomes much less attractive. However the cosmic ray neutrino spectrum falls as $\frac{1}{E^{2.7}}$ so that the possibility of useful information from the cosmic rays still exists.

High energy neutrinos and antineutrinos from decay pions and kaons produced high in the earth's atmosphere by the primary cosmic radiation have long been considered a possible tool for the investigation of the weak interaction[2-4]. Interest in the investigation of the weak interaction was heightened by the discovery in 1962[5] of the neutrino induced muon although the considerations which led to the program I am here discussing dated back to ~ 1960. The extreme range of the product muon produced in such inverse beta reactions (as opposed to that of the electrons produced in ν_e reactions)

$$(\nu_\mu, \bar{\nu}_\mu) + N \rightarrow (\mu^-, \mu^+) + N' + \cdots \qquad (1)$$

provided the crucial advantage by allowing the identification of the neutrino interaction in target material far removed from the detector, in effect, adding the range of the muon to the physical size of the detector. The consequent enhancement of the counting rate from (1) makes it the most readily detectable of the cosmic ray neutrino induced reactions. The experiments here described were started in fall 1963 and the data collection was completed in October, 1971. The first clearly labelled neutrino event was observed on February 23, 1965.

In order to identify (1) through the detection of a single product muon, muons of atmospheric origin produced together with the neutrinos, sister muons, must be rejected. One possibility was to look for upward going muons with a detector situated near the surface of the earth. The background of cosmic ray muons was estimated to be $\sim 10^{10}$ times the neutrino induced signal. This truly formidable rejection ratio led us to seek a reduction in muon background by interposing a great earth shield. A survey of deep mine facilities was undertaken and a suitable location found ~ 3200 m below the surface of the earth at the bottom of the deepest mine existent, Hercules Shaft, East Rand Proprietary Mines, Boksburg Republic of South Africa. At such depth (8.74×10^5 gm/cm^2 standard rock) only cosmic ray sister muons with energy in excess of ~ 10 TeV are capable of reaching the detector. Furthermore, the angular distribution of cosmic ray muons, $dF_\mu/d\theta$, is sharply peaked in the vertical direction:

$$\frac{dF_\mu(\theta)}{d\theta} \sim \cos^{10}(\theta)\ ;\ \theta = \text{zenith angle}, \qquad (2)$$

in contrast to the neutrino induced muon flux which was expected to be approximately isotropic. Various estimates of the neutrino induced muon flux indicated a counting rate of a few counts/year in a detector of area 10^2 meters. A measurement of the neutrino induced muon flux was undertaken in anticipation that the higher energy neutrinos available in the secondary cosmic rays could aid the search for the hypothetical mediating vector boson of the weak interaction as well as reveal something of the character of the interaction at energies in excess of those available at accelerators.

To determine if the atmospheric neutrino induced signal could be observed in the deep mine experiment, a detector of limited angular resolution was operated at 76 level Hercules Shaft (8.74×10^5 gm/cm^2 std. rock). The results of this experiment clearly indicated a substantial and unambiguous neutrino signal. Subsequently, a detector with larger aperture and better angular resolution was operated at a slightly greater depth (8.98×10^5 gm/cm^2 std. rock). This detector determined the angular distribution of neutrino induced events with much greater accuracy ($\pm 1^\circ$ vs $\pm 15^\circ$).

II. Experiments

A. 76 Level

The initial detector installed on 76 level is shown in figure 3 and consisted of 54 liquid scintillation detector ranks arranged in two discontinuous rails, one on the east side of the tunnel and one on the west side, having a total area of 180 m^2. Each ultra violet transmitting tank ($5.5 \times 0.56 \times 0.13$ m) filled with liquid scintillator was viewed by four 5" photomultiplier tubes. We will refer to three tanks displaced vertically as a bay. A bay on the east side of the tunnel taken with the corresponding bay on the west side is referred to as a double bay.

The design of the detector was dictated by the need for:

a) A large and relatively inexpensive surface area viewed by a small number of photomultipliers.

b) A thickness such that energy deposited by a penetrating particle would be well above that due to radioactivity.

c) A height consistent with tunnel dimensions and the desired hodoscope resolution.

d) A response function such that pulse height variations over the length of the tank were not excessive. The event was located by the ratio of pulse heights seen at the two ends of the detector.

A modification of a technique originated at CERN[6] using plastic totally reflecting walls was found to satisfy our requirements. The detector contained some 20 metric tons of liquid scintillator, a chemically inert mineral oil based mixture.

Analog signals from 216 photomultiplier tubes on line continuously were partially encoded and passed to an environmentally controlled house for further encoding. These signals were displayed on two oscilloscopes and photographed when appropriate coincidence requirements were met. Thirty-five neutrino induced muons traversing the tunnel at large zenith angles triggering tanks on each side of the array were observed. The detailed arguments on which this statement is based may be found in Reines et al.[7] From these data the horizontal muon flux was found to be 4×10^{-13} μ/cm^2 - sec - sr, in good agreement with that obtained in the more sophisticated but smaller Kolar Gold Field (KGF)[8] detector. (The KGF detector recorded a total of 16 neutrino events). This agreement in spite of an order of magnitude larger cosmic ray background at KGF is by itself convincing evidence that we were observing cosmic ray neutrino. The success of the 76 level experiment stimulated efforts to determine the neutrino induced muon angular distribution with greater precision and accuracy, as well as to elucidate more clearly the events in which more than one particle penetrated the detector, and to this end the second phase experiment was performed at a slightly greater depth ($8.98 \times 10^{+5}$ gm/cm^2) on 77 level.

B. 77 Level

The 77 level scintillation detector physically embodied its predecessor in such manner as to nearly triple the geometric aperture for triggering, as shown in figure 4. The first four double bays of the 77 level detector repeat the geometry of the previous detector and serve as a common referent. In addition, an extensive Conversi Hodoscope array capable of defining muon trajectories to within $\pm 1^{\circ}$ and with a spatial resolution of a few cm was installed at the sides of the scintillation detector.

1. Scintillation Detector on 77

Similar to the previous detector on 76 level the amplitudes of the photomultiplier signals determine both the energy deposition in the scintillator and the position of traversal along the length of the tank. The discriminators were set for good efficiency for the detection of horizontal muons consistent with a low chance coincidence rate. However, the triggering scheme permits the system to be triggered by an acceptable fraction of the chance coincidences due to the natural radioactive background. These change coincidences are easily distinguished from real events and provide a valuable check on the operation of the system.

2. Conversi Hodoscope on 77

The Conversi Hodoscope was composed of commerically available neon flash tubes. The tubes, 200 cm. long and 1.8 cm. in diameter, were contained in elements consisting of 56 tubes arranged in a double layer. Six elements, 3 vertical and 3 horizontal, form a submodule. Three submodules designated alpha, beta and gamma form a module. Three modules covered one bay. The Conversi Hodoscope consisted of 48 modules containing a grand total of 48,144 flash tubes.

Each tube has an optical face at one end to which is attached a photocell. The photocell responds to the optical discharge of the tube and, in turn, latches on a thyristor serving as a memory device. For each trigger nine modules were interrogated sequentially and the state of the thyristor displayed on a lamp board which was photographed.

3. Experimental Results

The composite detector was operated for a period of two years (live-time), during which over 500 events of all types were recorded. Of these 105 were ascribed to neutrinos to which a zenith angle could be assigned. The experiment was terminated when the statistical uncertainty in the total neutrino rate approached that of systematic errors. Of the neutrino induced events two-thirds are essentially single particle and the remaining one-third are multiparticle.

Figure 5. Shows a simple event at zenith $< 70°$ it is ascribed to a ν interaction because of its great zenith $\angle$.

Figure 6. A forked event. The vertex of this event is too close to the side wall to infer that both particles are muons.

No events of this type display enough penetrability to be unambiguously labelled muon pairs such as might signify W production and decay. Not shown are examples of the occasional events in which a shower of particles penetrate the detector. In such cases it is not possible to unambiguously attribute to either a neutrino or cosmic ray interaction. This emphasizes the difficulty encountered in interpreting the results from detectors operating at lesser depths where cosmic rays are more copious.

In the absence of detailed particle identification in multiparticle events the interpretation of such events is open to dispute and in no such case can the specific type of interaction be unambiguously determined. However, the misinterpretation of multiparticle events is thought to produce an error of $< 5\%$ to the measured counting rate for neutrino events.

The experimentally recorded muon angular distribution is shown in figure 7. A clear separation between the cosmic ray muon and the neutrino induced muons is seen for zenith angles $> 40°$.

The number of counts in zenith angle bin $85° - 90°$ is a direct measure of the horizontal neutrino induced muon flux, $F_{\mu\nu}(\theta = 90°)$. We find

$$F_{\mu\nu}(\theta = 90°) = (4.0 \pm 0.9) \times 10^{-13}\ \mu/\mathrm{cm}^2\text{-sec-sr}.$$

The solid curve of figure 7 is a fit to the experimental data. We find in general that the calculated angular distributions are quite insensitive to variations of model parameters. The fit to the data gives:

$$F_{\mu\nu}(\theta = 90°) = (4.1 \pm 0.4) \times 10^{-13}\ \mu/\mathrm{cm}^2\text{-sec-sr}.$$

In good agreement with that inferred from the statistically poorer, less sophisticated 76 level experiment:

$$F_{\mu\nu}(\theta = 90°) = (4.2 \pm 0.7) \times 10^{-13}\ \mu/\mathrm{cm}^2\text{-sec-sr}.$$

and that measured at KGF:

$$F_{\mu\nu}(\theta = 90^{\circ}) = (3.5 \pm 0.9) \times 10^{-13}\ \mu/cm^{2} - sec - sr.$$

III. Analysis

A. High Energy Interactions

In the following, we give results of calculations of the expected fluxes for the "76" and "77" level experiments. The difference between the calculated and the observed rates can then be used to place limits on the behaviour of neutrino cross-sections with respect to: (1) possible deviation from linear growth of the inverse beta cross-section. You will recall that this linear growth is expected from scale invariance, and (2) mass limits on (or existence of) intermediate bosons of weak interactions.

In the calculation, we consider several possible sources for the observed muons. These are (1) the muons directly produced in inverse beta reactions, (2) the direct muon appearing in the production of the (hypothetical) intermediate vector boson, $W_{1}^{\pm}$.

Cross-sections for the inverse beta reaction have been measured at accelerators for neutrino lab energies less than 10 GeV.[9] The behaviour of the cross-section at high energies is as yet completely unknown. Theoretical arguments suggest that the cross-section continues to rise linearly with energy until cutoff by some mechanism such as higher order weak interaction or the production of the W boson. We assumed in the calculations that neutrino cross-sections per nucleon measured at CERN can be directly applied to standard rock. Furthermore we assume that antineutrino cross-sections per nucleon are equal to neutrino cross-sections except where explicitly calculated. As an alternative to the latter assumption, an extreme case envisaged by Drell[10], for which the antineutrino cross-section is one third the neutrino cross-section at high energy is also considered. In the detailed approach explicit use is made of the structure functions for the various interactions. The inverse beta reaction is divided into three categories: (a) "elastic", (b) "quasielastic", and (c) deep-inelastic.

With scale invariance, the calculated 'deep-inelastic' inverse beta cross-section grows linearly with incident neutrino lab energy. In order to investigate a possible deviation from this linear growth, we insert a cutoff at large momentum transfer in the amplitude of the form

$$(1 + q^2/M^2)^{-1} . \qquad (3)$$

The parameter M sets a scale in an otherwise scale invariant process. The resulting cross-section no longer increases linearly but increased logarithmically at large incident neutrino energies. The magnitude of M determines the neutrino lab energy at which the 'deep-inelastic' cross-section departs noticeably from linearity. At

$$E_\nu = 3M^2/m_p , \qquad (4)$$

Physically:
- If M is ∞ then point interact until unitary cutoff
- If M_w is ∞ then point interaction and $\sigma \sim E$ (until unitary cutoff)
- If M_w small then σ also rises rapidly ergo can in principle obtain upper and lower bounds on M_w.

Mention that a similar {upper / lower} limit analysis has been made earlier and independently by Dr. Z. Kunszt. (m_p is the proton mass) the resulting 'deep-inelastic' cross-section is roughly one-half the linearly increasing cross-section. We expressly point out that should the boson $W_1^\pm$ exist, the parameter M is the mass of this boson and that in any event the factor (1) plays a natural role in all inverse beta processes.

In addition there should be contributions to the cross-section from the production of other resonances. As described in our 1971 Phys. Rev. paper (Chen[11]) these contributions are not large ($\sim 10\%$) and are reasonably model independent.

In order to calculate the contribution of the direct muon from $W_1^\pm$ production, we have taken results of recent calculations on all $W_1^\pm$ production reactions. The energy transferred to the direct muon is relatively small in the important energy regions for all $W_1^\pm$ production reactions. The process resulting in a large muon flux is the coherent reaction

$$\nu_\mu + Z \rightarrow \mu + Z + W_1$$

since the high energy neutrinos, where this reaction dominates, are available in the cosmic rays. The result of these calculations is that only a small contribution is expected to the counting rate for boson mass above about 4 GeV.

Because the $W_1^\pm$ carries off a large fraction of the incident neutrino energy in production, the decay muon can be important. However, the decay muon preferentially comes out backwards relative to $W_1^\pm$ motion and so carries on the average, only 1/4 of the $W_1^\pm$ energy. Nevertheless, the average decay muon energy is larger than the average direct muon energy. In our prediction of this contribution to the rate we chose a branching ratio of 1/4 for

$$W_1 \rightarrow \mu + \nu_\mu .$$

The neutrino induced muon counting rate is given by the expression

$$R_i = \iint I\,(E_\nu)\, j(E_\nu,\theta)\, \frac{dA}{d\theta}\, N(E_\nu)\, \sigma_i(E_\nu)\, d\theta\, d E_\nu \qquad (5)$$

where

a) $I(E_\nu)\, j(E_\nu,\theta)$ is the incident neutrino or antineutrino flux taken from the literature assuming a K/π ratio of $0.2^{+0.2}_{-0.1}$. The total error in this flux is calculated to be $^{+13\%}_{-\ 8\%}$;

b) $dA/d\theta$ is the differential aperture for the detect, n of charged particles. In view of the large momenta involved, the direction of the resultant muon is taken to be that of the incident neutrino. The total error in differential aperture is $\sim 5^\circ$;

c) $N(E_\nu)$ is the number of target nucleons per unit volume multiplied by the range of the resultant muon or electron in rock.

d) $\sigma_\nu(E_\nu)$ is the cross-section per nucleon per incident neutrino or antineutrino for the particular product under consideration.

The calculated, minus observed rates due to (1) muons produced in inverse beta decay reactions for the assumptions

(i) $\sigma(\nu p) = \sigma(\bar{\nu} n) = \sigma(\nu n) = \sigma(\bar{\nu} p)$ (6),(7)

(ii) $\sigma(\nu p) = 3\sigma(\bar{\nu} n) = \sigma(\nu, n) = 3\sigma(\bar{\nu} p)$

introduced by uncertainties in the neutrino, antineutrino flux $\begin{pmatrix} +13\% \\ -8\% \end{pmatrix}$, inverse beta cross-sections measured at CERN (± 25%), measured horizontal muon flux (~ ± 15%).

For (i) the experimental result favors saturation by 1 standard deviation. The intersection between the calculated and observed rates occurs at $M \approx 4.0$ GeV. Evaluating (E_s) with $M = 4.0$ GeV gives a 50% reduction in the linear rise of the inverse beta cross-section with energy at $E_s \cong 50$ GeV. For (ii), the experimental result if $M \approx 30$ GeV, $E_s = 300$ GeV and in view of the uncertainty, no upper limit can be placed on M. The linear growth of the cross-section would therefore be unrestricted.

Calculated rates including, (1) muons from inverse beta reactions, (2) direct muon production of $W_i^{\pm}$, (3) $W_i^{\pm}$ decay into lepton pairs, are compared with experiment as a function of M_w^{-1} in figure 8.

For (i) the experimental result favors no W production. At one standard deviation, we find

$$40 \gtrsim M_w > 3.0 \text{ (GeV/c}^2\text{)}. \qquad (8)$$

For (ii) a value of $M_w \gtrsim 2.0$ is indicated at one standard deviation.

B. Cosmic Ray Muons

The background of cosmic ray muons recorded by this deepest underground detector is of considerable interest in itself. So as an aside we give the result -- it is interesting that the neutrino signal contributes a non-negligible background in this connection.

Preliminary analysis of the cosmic ray vertical intensity, I_μ, indicate

$$I_\mu = (1.7 \pm 0.2) \times 10^{-11} \ \mu/\text{cm}^2\text{-sec-sr} \qquad (9)$$

with an angular distribution not inconsistent with that obtained in the less deep KGF experiment[12].

C. Astrophysical Implications

One of the most intriguing recently discovered phenomena is the signal detected by Weber and attributed by him to gravitational radiation. Using the results of the underground neutrino detector, we have set limits on $(\nu_\mu + \bar{\nu}_\mu)$ flux possibly associated with gravitational radiation[13]

To a reasonable approximation, the neutrino rate in the detector is giv[en] by

$$R = \beta \int_E E\, f(E)\, \sigma(E)\, dE \qquad (10)$$

where $f(E)$ is the differential neutrino spectrum, $\sigma(E)$ is the reaction cross section, the range of the product muon is proportional to E and β is a para[-] meter characterizing the detector geometry. Since there is no information a[s] to the energy spectrum of neutrinos which might conceivably be associated wi[th] Weber pulses limits are deduced for monoenergetic neutrinos.

To evaluate the integral assume $\sigma(E) = \alpha E$, $f_a(E) = 0.6\, E^{-3} \nu/cm^2$ - sec - GeV and $E > 1$ GeV. The result is insensitive to the precise value of the saturation energy taken here as 1000 GeV, and the form for $f_a(E)$ is accura[te] to within twenty percent of the actual value. A search for coincidences betwe[en] Weber pulses and charged particles penetrating our detector during 227 days [of] common run time from 22 December 1969 to 17 September 1970 gave two even[ts] penetrating the detector within $\pm$ 2 min. of a Weber pulse. The expected number of random coincidences during this time was 0.7.

In view of the poor statistics, the observed 2 counts are consistent with accidental, and we take an upper limit of 2 counts per year. The energy flux ratio reduces to:

$$\mathcal{E}/\mathcal{E}_g < 5 \times 10^{-9}/E_o\, \eta \qquad (11)$$

where η is the efficiency with which Weber observes his events. Weber estimates this to be $\eta \approx 0.1$. The upper limit on $(\nu_\mu + \bar{\nu}_\mu)$ fractional energy at $E_o = 1$ GeV is $< 5 \times 10^{-8}$. Incidentally Davis & Bahcall have set upper limits at lower energies using results from the solar neutrino detector.

Extraterrestrial Sources

In view of our detector's good angular resolution there exists the possibility of detecting $\nu_\mu + \bar{\nu}_\mu$ sources should they be sufficiently strong.

I remark at the outset that the observed neutrino induced muons can be entirely accounted for through atmospheric processes.

The small contribution due to the interactions of primary cosmic rays with the average 4 gm/cm^2 of interstellar matter traversed in their tortuous path from source to earth is completely negligible relative to the atmospheric ν signal.

Incidentally, the data are not inconsistent but lend no support to a uniform distribution of high energy ν's in space.

We find from a search of the sky as a limit on point sources a flux

$$\frac{J\ \text{pt. source}}{J\ \text{atmosph.}} < 1/5 .$$

In addition, an extraterrestrial source could manifest itself in several ways.

a) Spatial and temporal coincidences with time dependent sources whose positions are known.

b) Single or multiple events from a region of special interest in the sky.

c) Repetitive events from an arbitrary point in the sky.

As an example we have examined the neutrino signals for correlations with observed novae and supernovae. If we require a spatial, i.e., angular coincidence $\sim 5^\circ \times 5^\circ$ and a time coincidence within one month of observed neutrino events, we eliminate all but one candidate. A supernova, 1970F, was observed 31 May 1970 and in the same direction (180° ambiguity) on a neutrino event was observed 2 June 1970. The probability that the coincidence is accidental is quite large ($\sim$ 10%) mainly due to the high rate at which extragalactic supernova are observed ($\sim$ 0.5/week). Furthermore, the relative timing of the events would imply that they are uncorrelated if 1970F conforms to general notions of supernovae formation which suggests prompt short neutrino burst. Incidentally there are no time coincidences between the neutrino event of 2 June 1970 and Weber pulses. We conclude, in the absence of additional information, that the coincidence is most likely accidental.

We have also compared the observed distribution of neutrino events with the coordinates of celestial objects and find no correlation between neutrino events and pulsars nor with supernovae remnants.

<u>Conclusion</u>: And so has ended this first foray into the study of "naturally produced" neutrinos (> solar ν energies). It is evident that we have found no surprises, an observation which is in some sense in itself a surprise since it is almost an article of faith, that given a more sensitive probe, new phenomena will reveal themselves. But it might simply be that we are using a hand magnifying glass where we should be using a microscope.

Acknowledgment

I wish to extend my deep appreciation to our research engineer, Mr. A. A. Hruschka, for his indefatigible and brilliant support in the design and construction of the experimental equipment.

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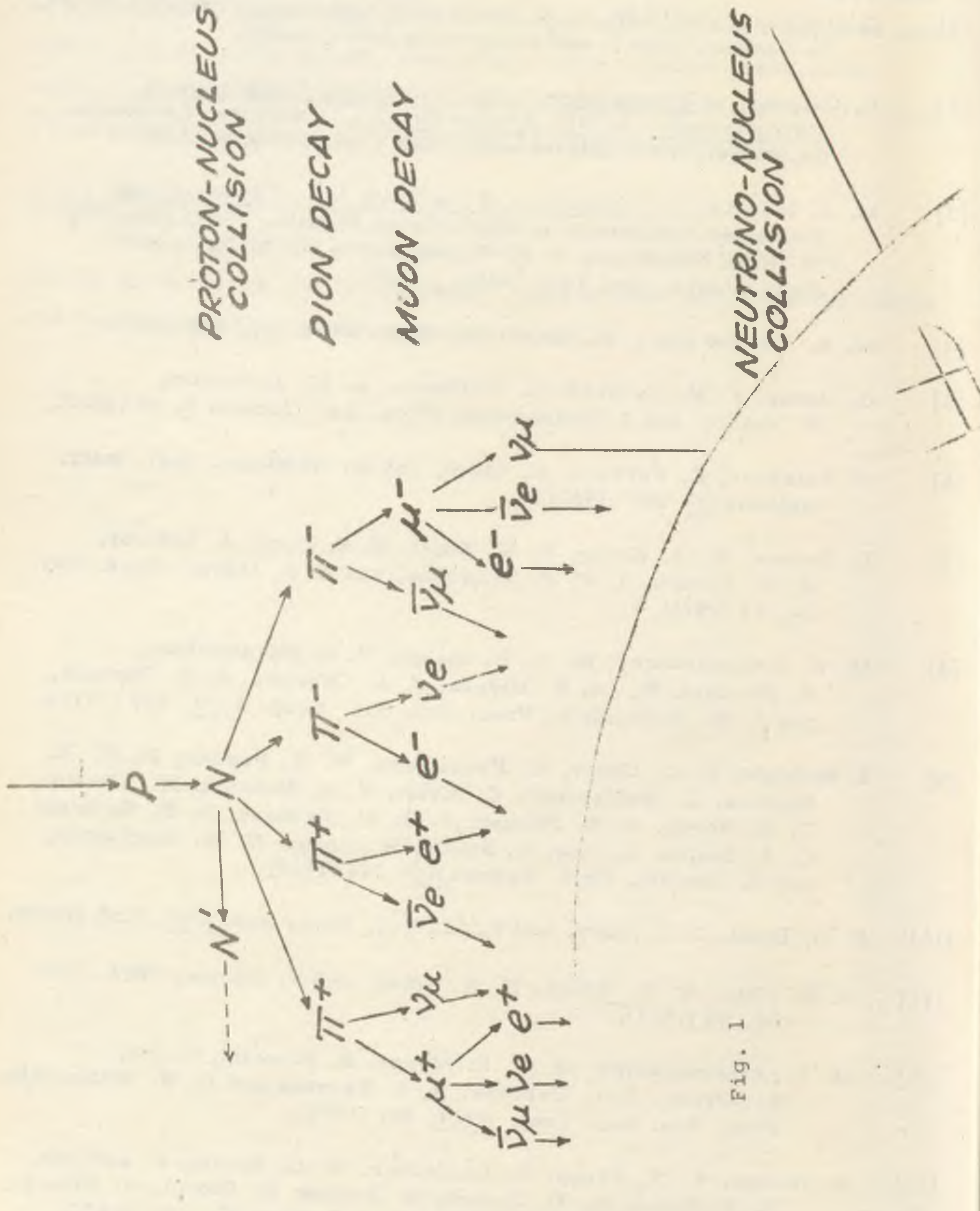
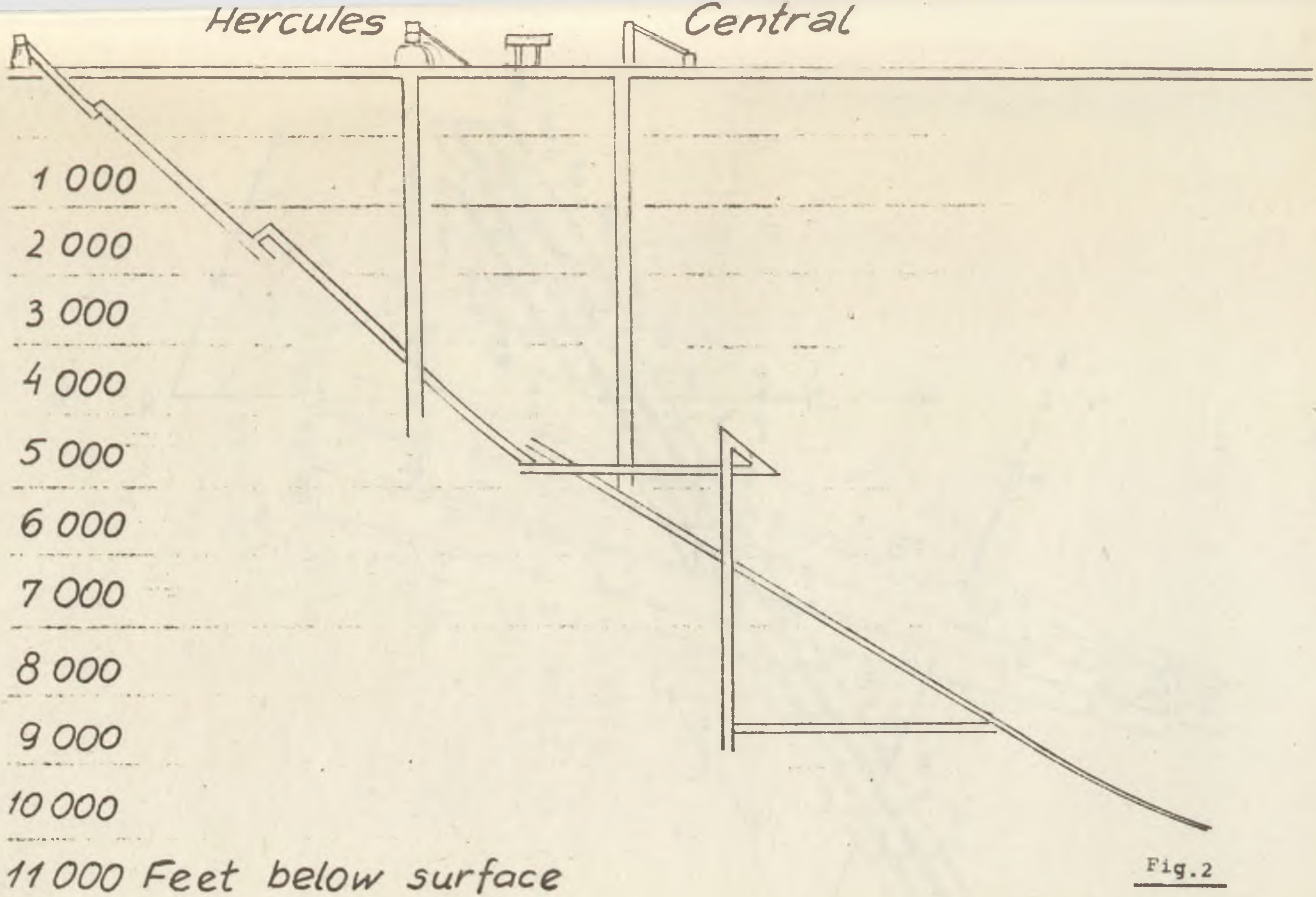

PROTON-NUCLEUS COLLISION
PION DECAY
MUON DECAY
NEUTRINO-NUCLEUS COLLISION

Fig. 1


Hercules
Central
1 000
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10 000
11 000 Feet below surface

Fig.2

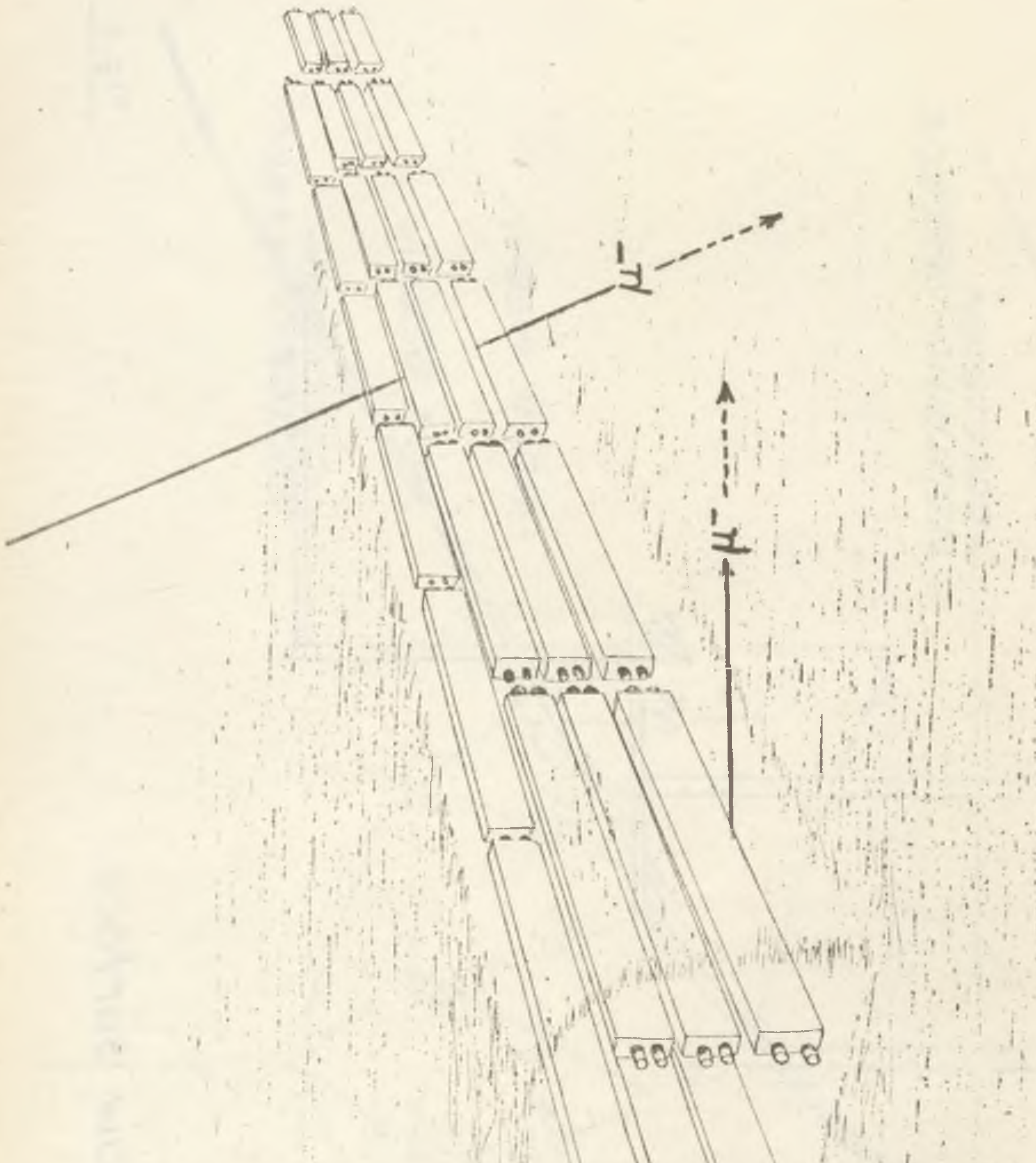
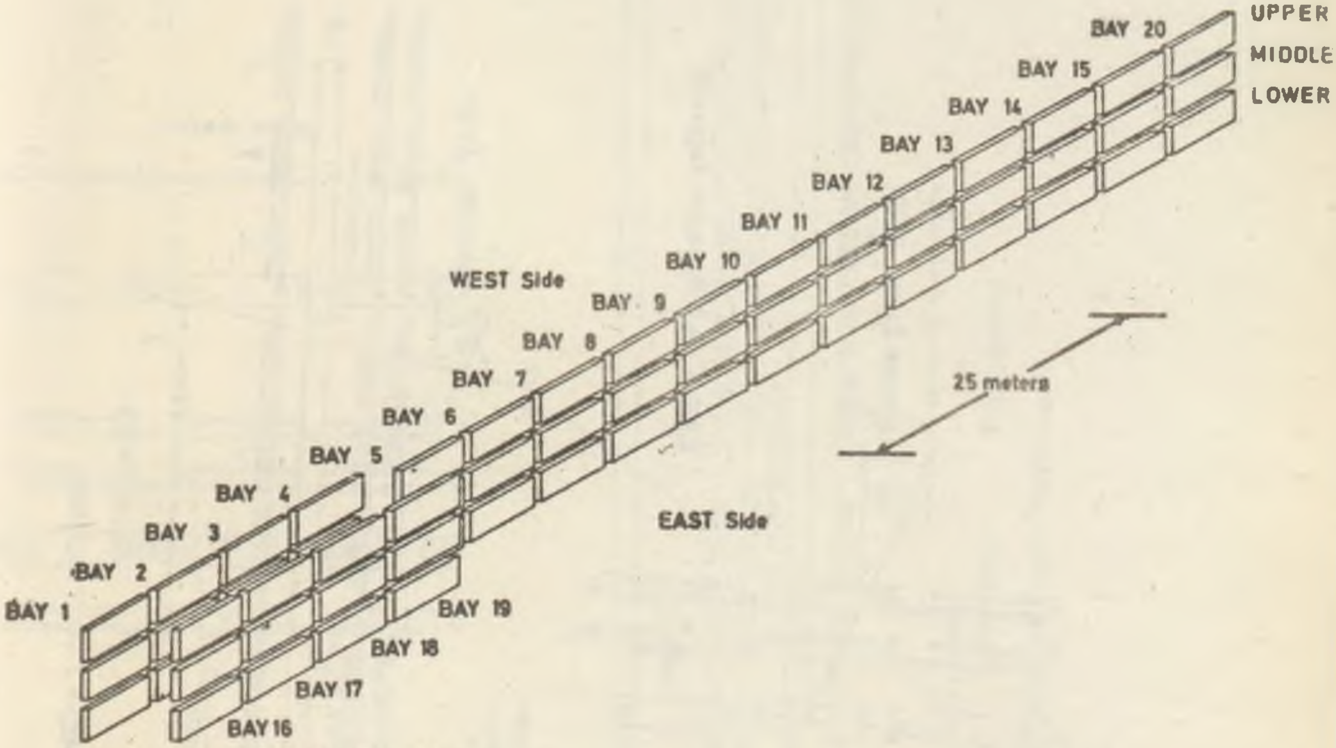
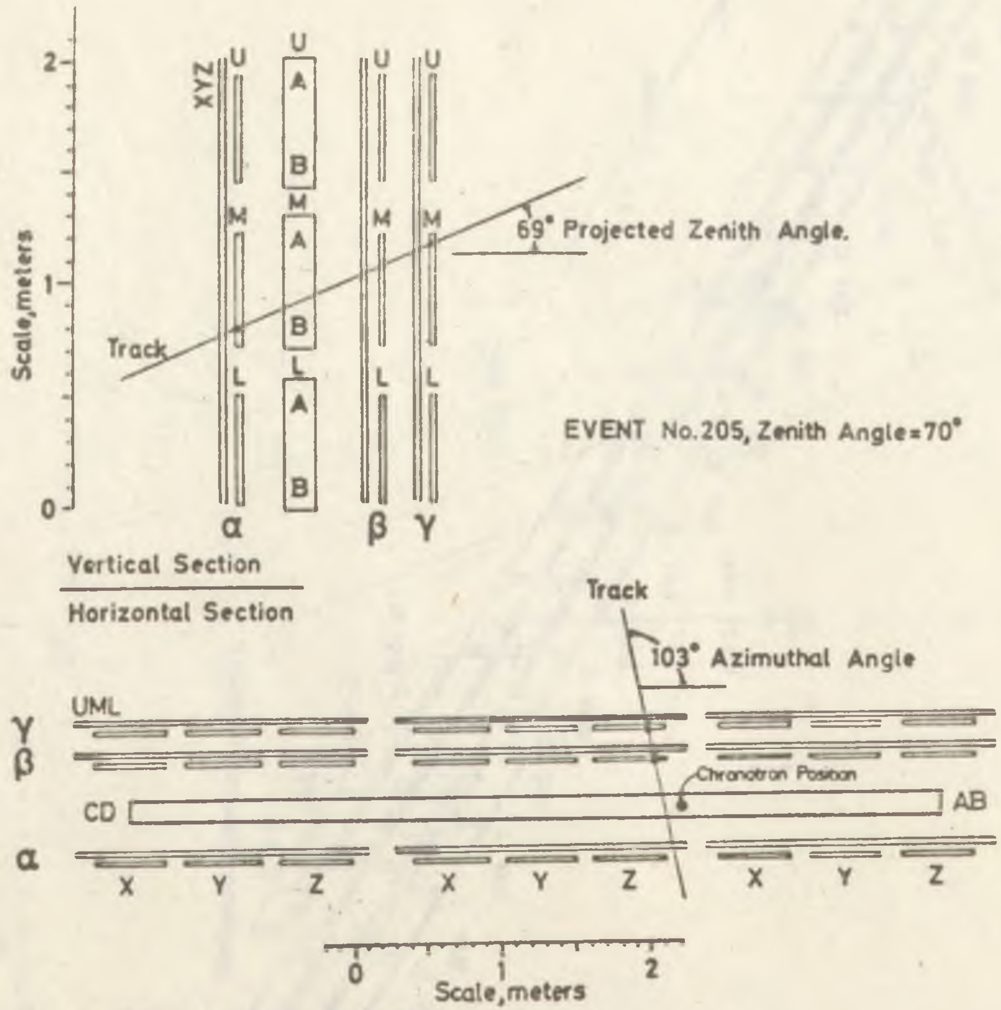


Fig.3



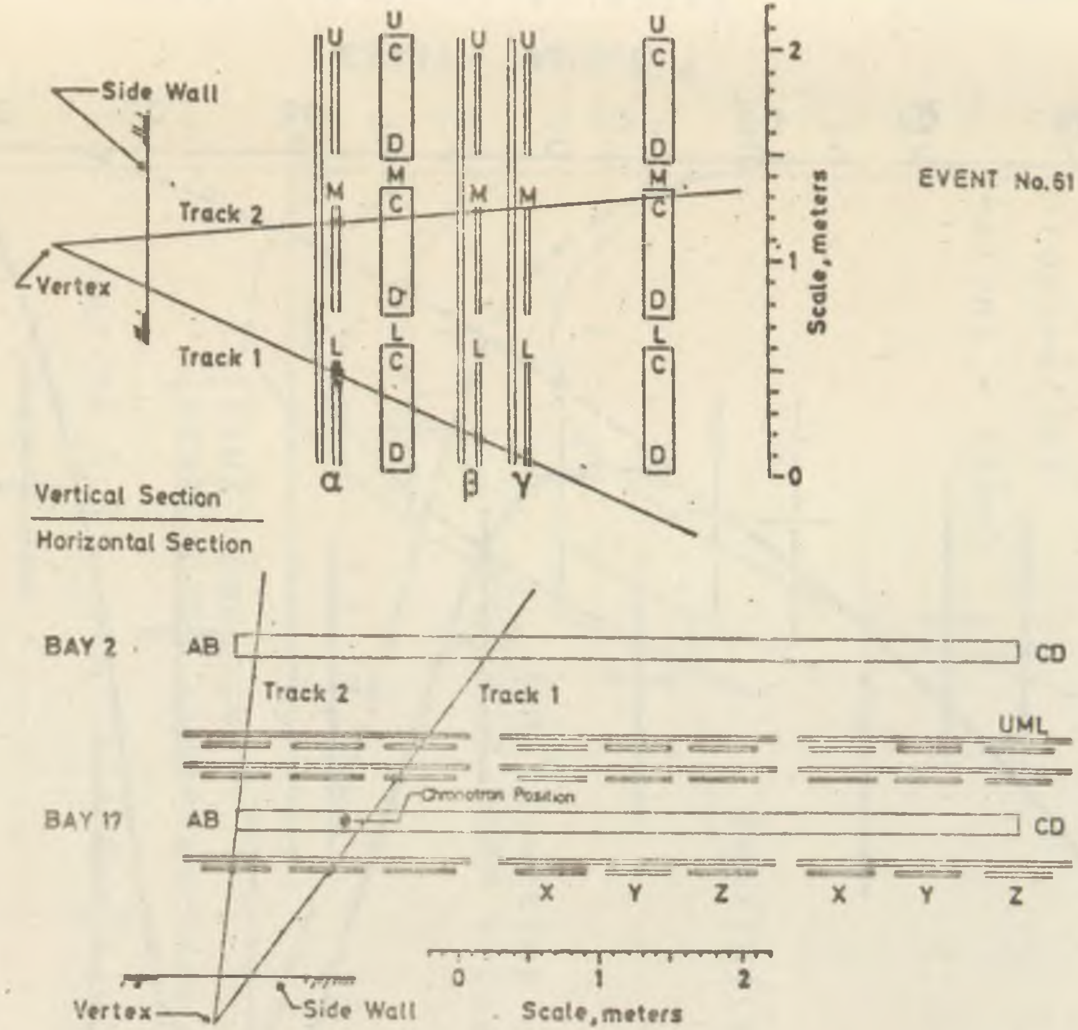
Scintillation Detector Geometry

Fig. 4



Conversi Hodoscope Track Projections

Fig. 5



Conversi Hodoscope Track Projections

Fig. 6

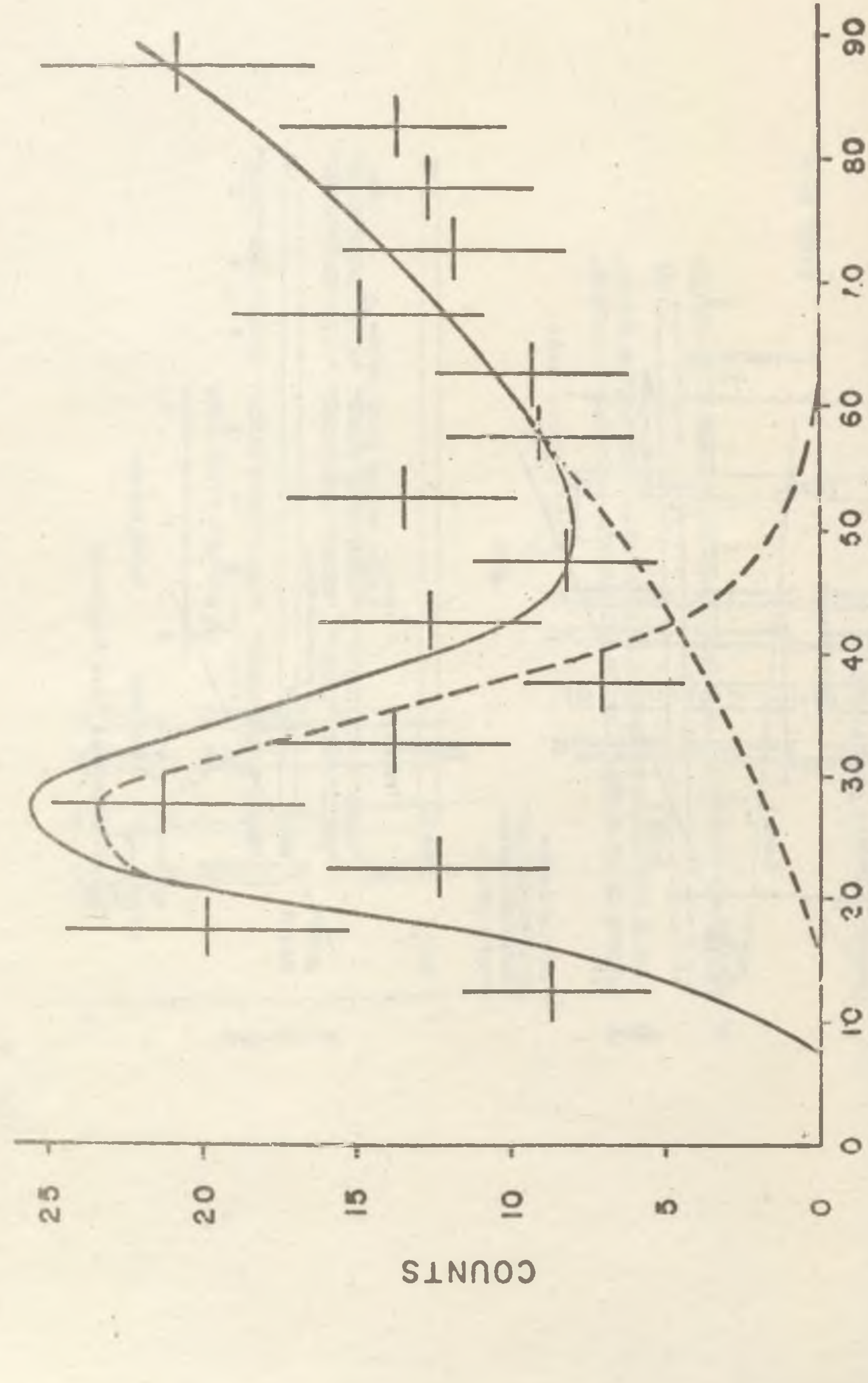


Fig. 7

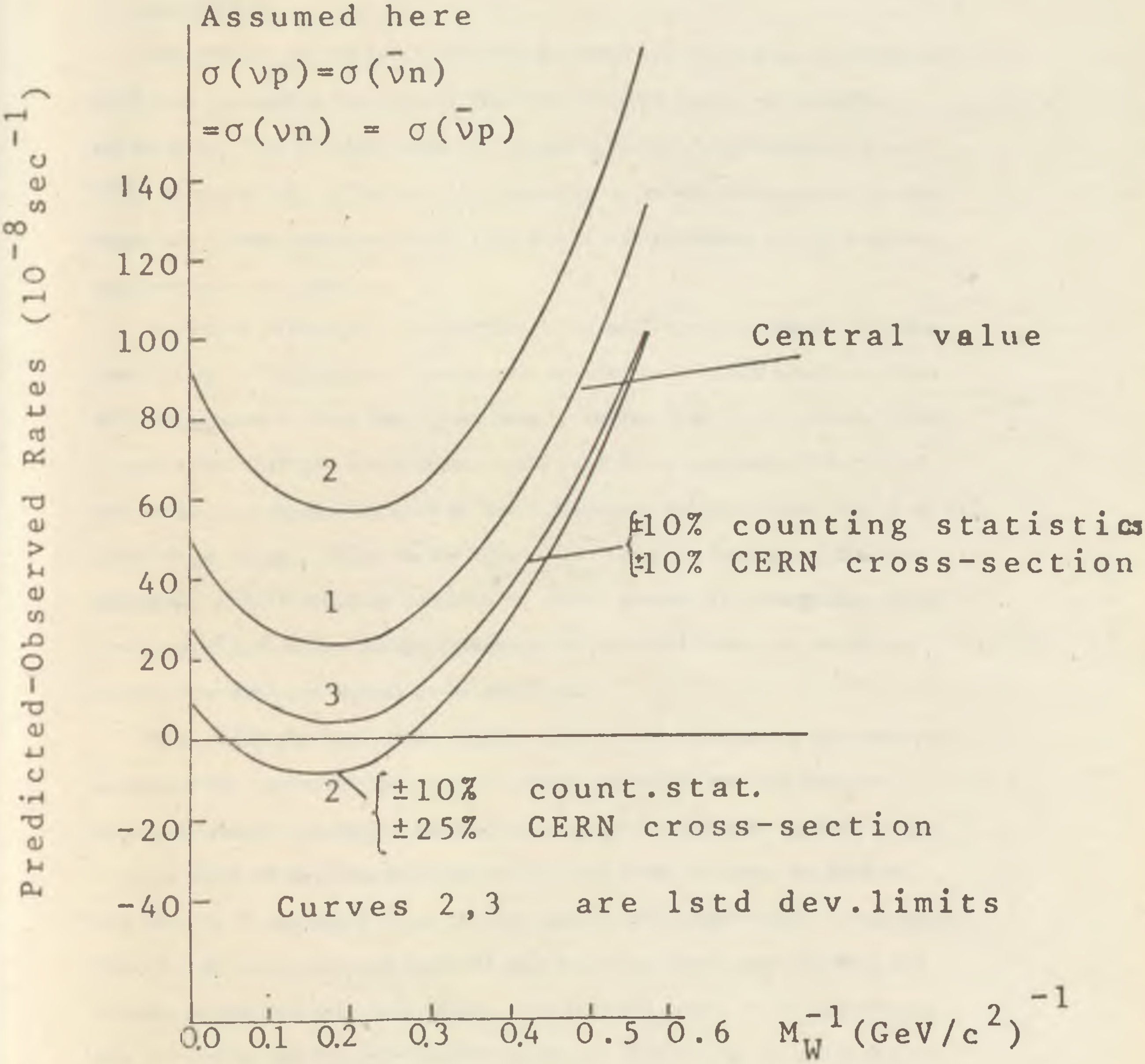

Assumed here
$\sigma(\nu p)=\sigma(\bar{\nu}n)$
$=\sigma(\nu n) = \sigma(\bar{\nu}p)$
Predicted-Observed Rates ($10^{-8}sec^{-1}$)
140
120
100
80
60
40
20
0
-20
-40
2
1
3
2
Central value
±10% counting statistics
±10% CERN cross-section
±10% count.stat.
±25% CERN cross-section
Curves 2,3 are 1std dev.limits
0.0 0.1 0.2 0.3 0.4 0.5 0.6
$M_W^{-1}(GeV/c^2)^{-1}$

Fig. 8

AN ANALYSIS OF COSMIC RAY MUON NEUTRINO EXPERIMENTS

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/Presented by E.C.M.Young/

1. Introduction

The present review has a two-fold purpose: (i) to briefly summarise the situation concerning the results from the two main cosmic ray neutrino experiments, both of which have now ceased operation (Krishnaswamy et al. 1971; Reines et al. 1971a) and (ii) to refer to recent developments in the magnitude of the neutrino intensities and in our knowledge of the neutrino interaction cross section.

By way of historical introduction it is sufficient to remark that the feasibility of carrying out cosmic ray experiments to study neutrino interactions appears to have been first made by Markov (1960) and Greisen (1960). It was clear that the measurements would need to be made deep underground and it was the pioneering work of the India-Japan collaboration (Miyake et al., 1964; Menon et al., 1963) in the Kolar Gold Fields in Southern India that indicated that it would be possible to choose accessible underground sites which would give a low enough background of unwanted events to enable an unambiguous neutrino signal to be recorded.

The philosophy behind the cosmic ray neutrino experiments has been to calculate the expected muon neutrino energy spectrum from the measured muon spectrum as a function of zenith angle and to use this together with various forms of neutrino-nucleon interaction cross sections to predict the numbers of secondary muons for the appropriate experimental arrangements. The form of cross sections favoured was then that which gave the best fit between prediction and observation. An important aspect of the experiments was the search for the intermediate boson, W, both by way of its effect on the measured rate of muon secondaries and, more important in the form of identifiable signatures (i.e. the detection of muon pairs).

2. The Kolar Gold Field Experiment

In this experiment five telescopes and two magnet spectrographs were used as detectors which were located in the Kolar Gold Mines in South India at a depth equivalent to 7500 hg cm^{-2} of 'standard rock'. The disposition of these detectors in the tunnel is shown in Figure 1. At this depth the intensity of atmospheric muons in the vertical direction is attenuated by a factor of 10^8 compared with that at ground level.

The three types of detectors used are shown diagrammatically in Figure 2. In each of these detectors there were two walls of plastic scintillators, separated horizontally, between which were located arrays of neon flash tubes as visual detectors. In telescopes 1 and 2 and in both spectrographs only the projected angle of the muon could be measured, whereas in telescopes 3, 4 and 5 the flash-tube arrays were placed in a crossed geometry and measurements of the spatial angle with an error of only $\pm 1^{\circ}$ could be achieved. Vertical walls of absorber were used in all the detectors and it was possible to distinguish between electrons and muons or pions which traversed the detectors. By studying the secondary particles produced in the absorber it was possible in some cases to determine the sense of direction of the incident particle.

The telescopes 1 and 2 started operation in early 1965, followed by telescopes 3, 4 and 5 in 1966 and the two magnet spectrographs in 1967. The neutrino project was terminated in June, 1969. During the period of operation a total of sixteen events were recorded which could be attributed to neutrinos.

The distinction between atmospheric muons and neutrino-induced muons could be achieved by studying the arrival direction of the incident particle at the detectors. At the depth of the experiment the vertical intensity of atmospheric muons is about 10^{-10} $cm^{-2}s^{-1}sr^{-1}$ and this intensity falls off very rapidly with increasing zenith angle. The neutrino intensity, however, is fairly isotropic, with some excess towards the horizontal direction. A spatial angle of 50° was adopted as the boundary beyond which it could be safely assumed that the events were due to neutrinos. The aperture x running time appropriate to the neutrino-induced events ($\phi \geq 50^{\circ}$) for the whole experiment was 4.71×10^{13} $cm^2 s\ sr$ and the horizontal intensity of neutrino-induced muons is found to be

$(3.5 \pm 0.9) \times 10^{-13}$ $cm^{-2}s^{-1}sr^{-1}$. (Krishnaswamy et al. 1971).

3. The Case-Wits-Irvine Experiment

This experiment was performed by groups from the Case Western Reserve University, University of Witwatersrand and the University of California (Irvine). The apparatus was located in the E.R.P. Mines near Johannesburg at a depth of 8.74×10^3 hg cm^{-2} (standard rock). The detectors comprised 54 liquid-scintillation-detector elements on two sides of the tunnel. Visual detectors in the form of large arrays of neon flash tubes were introduced at the later stages of the experiment. The coincidence requirements for a neutrino-induced muon were a pulse from a scintillator detector element on each side of the tunnel.

The experiment spanned a period of eight years and more than 100 events attributable to neutrinos were recorded (Reines et al., 1971b). Based on part of the events the horizontal muon flux due to neutrinos is $(4.2 \pm 0.7) \times 10^{-13}$ $cm^{-2}s^{-1}sr^{-1}$ (Chen et al. 1971).

4. Recent Developments

4.1 Expected neutrino intensities

In order to calculate the expected frequencies of neutrino-induced muons the energy spectrum of muon neutrinos at the experimental locations must be known. The neutrino spectra adopted in the analysis of the K.G.F. data were those calculated by Osborne et al. (1965). The calculations were based on the sea level vertical muon spectrum of Osborne et al.(1964) as datum. A necessary feature of the high energy nucleon-nucleus collisions, the ratio of kaons to pions produced (K/π ratio) was taken as 20%; this value was shown later by Ashton et al. (1966) to be not inconsistent with the measurements of the muon spectrum as a function of zenith angle.

The vertical muon spectrum of Osborne et al. (1964) was based, in part, on a direct measurement of the vertical muon spectrum by Hayman and Wolfendale (1962) and although this measurement has stood the test of time insofar as its spectral shape is concerned, it seems very likely that the absolute values are a little too low through being normalised to a datum intensity at 1 GeV/c which has since undergone upward revision. To be specific, the datum was the value of 2.45×10^{-3} $cm^{-2} sec^{-1} . sr^{-1}$ GeV/c^{-1} given by Rossi (1948) - an intensity that had been used by many authors - but recent measurements strongly suggest that this is some 10% too low. This conclusion comes from the work of Allkofer et al. (1971), Crookes et al. (1971) and Ashton et al. (1972) and draws support from measurements of Bateman et al. (1971).

There are still some problems concerning the muon intensities (small inconsistencies in spectral shape between different sets of measurements) but it appears at present that in the important muon energy region 20 - 100 GeV the intensity of near vertical muons should be raised by $\sim$ 10%.

The situation at large zenith angles is less clear. The measurements referred to (Ashton et al., 1966) were absolute values and some measure of confirmation has come from the work of Flint and Nash (1971) and Asbury et al. (1970). If these intensities are correct and the vertical intensities are raised then some increase in the K/π ratio is indicated. In fact, however, the angular measurements are not sufficiently precise to enable the K/π ratio to be determined with any precision.

As a first approximation, allowance can be made for any (small) changes in the predicted neutrino spectrum as a result of changes in the muon intensities, by relaxing the predicted frequencies of events by the ratio of the new and old intensities at the median neutrino energy for the detected events. Menon et al.(1967) have estimated median energies for various neutrino interaction cross sections and the value appropriate to the cross section giving a best fit to the data is in the region of 15 GeV. This can, in turn, be related to a median muon energy at sea level such that these muons are associated with 15 GeV neutrinos and a change in muon intensity would perforce cause the same change in the neutrino intensity. Calculation using data given by Osborne (1966) indicates a median muon energy of $\simeq$ 30 GeV in the vertical direction, which was used as a datum in the calculations.

As has been remarked already the K/π ratio cannot be determined with precision from angular measurements, and certainly not at interaction energies corresponding to 30 GeV muons. However, in view of what has been said it would appear safer to assume a K/π ratio of 40% in the calculations (a more detailed examination of this problem will be given in a later publication). Increasing the K/π ratio to 40% would by itself increase the ν_μ intensity at 15 GeV by ~ 8% in the horizontal direction and ~ 18% in the vertical direction (Osborne et al., 1965) to which would be added the 10% increase in the muon intensity. When allowance is made for the fact that the large angle neutrino intensity is higher than that in the vertical direction and the increased aperture of the detectors for large angle events it appears that a reasonable procedure is to increase the relevant muon neutrino intensities by 15%.

4.2 Neutrino cross sections

The neutrino-induced muons come from the following neutrino interactions:

(i) Elastic $\nu_\mu + N \rightarrow \mu + N'$

(ii) Inelastic $\nu_\mu + N \rightarrow \mu + N' + \pi$'s etc.,

and if the intermediate boson, W, exists:

$\nu_\mu + N \rightarrow \mu + N + W$ (incoherent production)

$\nu_\mu + Z \rightarrow \mu + Z + W$ (coherent production)

The leptonic decay of the W will further contribute to the muon flux.

Neutrino cross sections for processes (i) and (ii) have been measured in accelerator experiments up to neutrino energies of about 10 GeV (Budagov et al. 1969, a, b and c). The cross sections for the elastic and N^* production reactions approach constancy at high energies because of the presence of the structure functions. The total neutrino cross section has been found to increase linearly with neutrino energy up to 10 GeV (Budagov et al., 1969c). Beyond that the behaviour of the cross section is not known. Theoretical considerations suggest that the total neutrino cross section is expected to rise linearly with neutrino energy until some mechanism, e.g. the existence of the intermediate boson or higher-order weak interaction, cuts off this increase (Bjorken, 1969).

It should be emphasized that the difference between the interaction cross sections for neutrinos and antineutrinos needs examination. In the case of the elastic process it is expected that the neutrino cross section rises faster than that for the antineutrino at low energies and they both reach the same asymptotic limit a little over 1 GeV. This prediction has been confirmed by accelorator experiments (Budagov et al, 1969a). In the case of the inelastic interaction the situation is not so clear. The neutrino cross section measured up to 10 GeV has rather large errors: $\sigma_{tot} = (0.8 \pm 0.2)\, E_\nu\, (10^{-38}\ cm^2\ GeV^{-1}\ nucleon^{-1})$ (Budagov et al., 1969c) and measurements of the antineutrino cross section are in progress in CERN. Preliminary results seem to suggest that the antineutrino cross section is considerably lower than that for the neutrino.

In most of the analyses of the cosmic ray neutrino data in the past it has been conveniently assumed that the inelastic cross sections for the neutrino and the antineutrino are equal at high energies. Recent theoretical considerations (Bjorken and Paschos, 1970; Drell et al., 1969; Gross and Llewellyn-Smith, 1969) have indicated the possibility that the antineutrino cross section is lower and, as has just been remarked, this seems to be confirmed by the available data from the CERN experiment. Bjorken and Paschos (1970) have shown that a difference in the neutrino and antineutrino cross sections is characteristic of the parton model and for these and other models Drell et al. (1969) and Gross and Llewellyn-Smith (1969) calculate that the cross section for $\bar{\nu}$-N will be one-third of that for ν-N at high energies. A result of this is a considerable reduction in the expected ν-induced muon flux underground as will be shown shortly.

The main aims of the cosmic ray neutrino experiments are to determine whether the total neutrino cross section saturates at high energies and, if so at what energy, and whether the intermediate boson, W, exists, and, if it exists, what limits can be placed on its mass. Conclusions drawn on these questions will depend on the accuracy of our knowledge about the form of the neutrino and antineutrino cross sections. In the present analysis both cases are examined, i.e. the ratio of the inelastic cross sections for the neutrino and the antineutrino is assumed to be one and three respectively. If there is no boson, it is simply assumed that the inelastic cross section continues to rise linearly with neutrino energy up to a particular cutoff energy E_0. Above this energy the cross section is assumed to remain constant. For neutrino energy below 10 GeV the cross sections used are those from the CERN experiment (Budagov et al. 1969a, b,c) and the mean fraction of energy taken by the muon are the same as fiven by Krishnaswamy et al. (1971).

If the boson exists the cross sections must be modified by the propagator factor and the inelastic cross section will rise more slowly. Total cross sections for the coherent and incoherent production of the boson have been calculated by a number of authors (Burns et al., 1965; Von Grehlen, 1963; Chen, 1970; Brown et al., 1971). The cross sections and energy transfers to the muon adopted are those used in the analysis of Krishnaswamy et al. (1971). It should be pointed out that the existence of the boson contributes to the muon rate both by the directly produced muon and the decay muon.

Calculations have been made of the expected muon rate for the various assumptions. The results are shown in Figures 3 and 4 for the cases of the ratio of the inelastic ν to $\bar{\nu}$ cross section equal to one and three respectively. The predicted numbers of neutrino-induced muons are those pertaining to the running time and geometrical acceptance of the K.G.F. experiment.

5. Comparison of Prediction with Observations

The measurements of the K.G.F. experiment are shown in Figures 3 and 4, together with the CWI (Case-Wits-Irvine) results which have been converted to comparable K.G.F. numbers. It can be seen that the two experimental results agree well within their uncertainties. The uncertainties on the expected numbers are at the one standard deviation level. A comparison with the measurements brings out the striking fact that the uncertainties on the prediction are considerably bigger. The weighted mean of the experimental measurements from the two experiments has been used in the present analysis to compare with expectation.

In both Figures 3 and 4, (a) shows the variation of the expected muon flux for various boson masses assuming that the linearly rising cross section is modified only by the existence of the boson. The shallow minimum (at a boson mass of 4.5 GeV/c^2) arises because as the boson mass increases the contribution from muons associated with the boson falls but this is, eventually, more than compensated by muons from the 'normal' inelastic process. In case (b) it is assumed that the total neutrino cross section rises linearly to a neutrino energy E_o where it reaches saturation due to some process which does not lead to further muon generation.

We now examine the possible behaviour of the total neutrino cross section. If the boson does not exist and $\sigma_\nu = \sigma_{\bar{\nu}}$, then a comparison favours the saturation of the total cross section at energies not much greater than 10 GeV. However, it is not possible to rule out the case of no saturation at better than 95% confidence level. Assuming $\sigma_\nu = 3\sigma_{\bar{\nu}}$ the predicted muon flux is lower and the predicted number for no saturation is only 1.5 standard deviations above the observed number.

However, if the boson exists and the linear rise of the inelastic cross section is modified only by the boson propagator, then a lower limit on the boson mass M_W can be set. In the case of $\sigma_\nu = \sigma_{\bar{\nu}}$, with 95% confidence M_W is greater than 3 GeV/C^2.

In the case of $\sigma_\nu = 3\sigma_{\bar{\nu}}$, at one standard deviation its mass is greater than 3.4 GeV/c^2 and with 95% confidence $M_W > 2.6$ GeV/c^2.

6. Conclusions

In view of the likely possibility of a difference between the total cross sections for neutrinos and antineutrinos, it has become more difficult to establish firmly whether and at what energies the total cross section approaches the limiting value. If there is a difference, the lower limit on the mass of the boson, if it exists, will also fall and the possibility of a total inelastic cross section rising continuously with energy is somewhat enhanced. It is clear that firm conclusions must await more precise data from the accelerators. The largest uncertainty at present is the total cross section which has been measured in the CERN experiment up to energies of 10 GeV with some 25% uncertainty and there is also significant uncertainty in the cosmic ray neutrino intensities.

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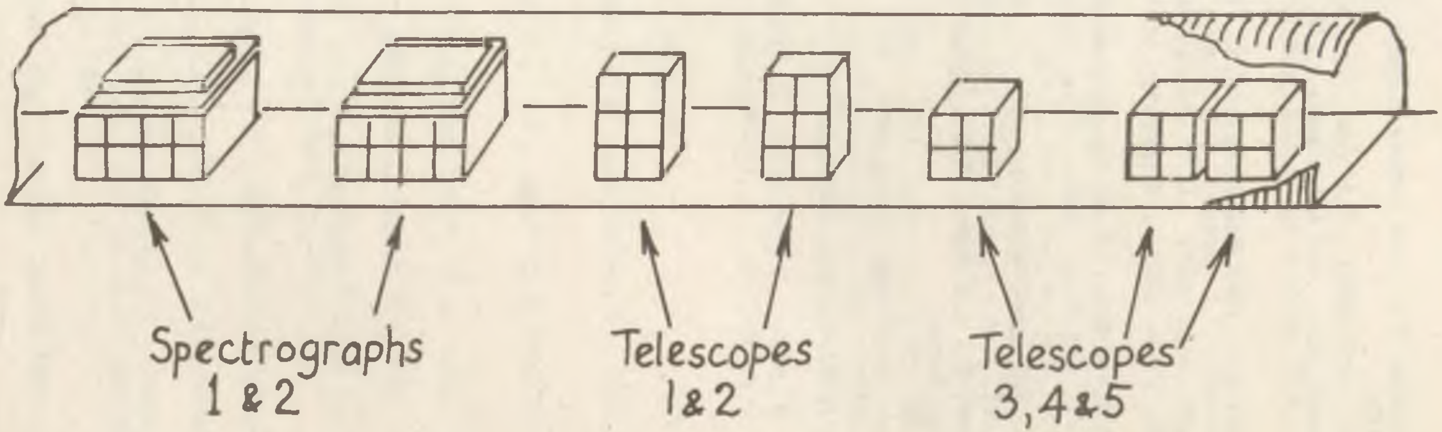

Fig. 1 Disposition of apparatus at K.G.F. (7500 m.w.e.)

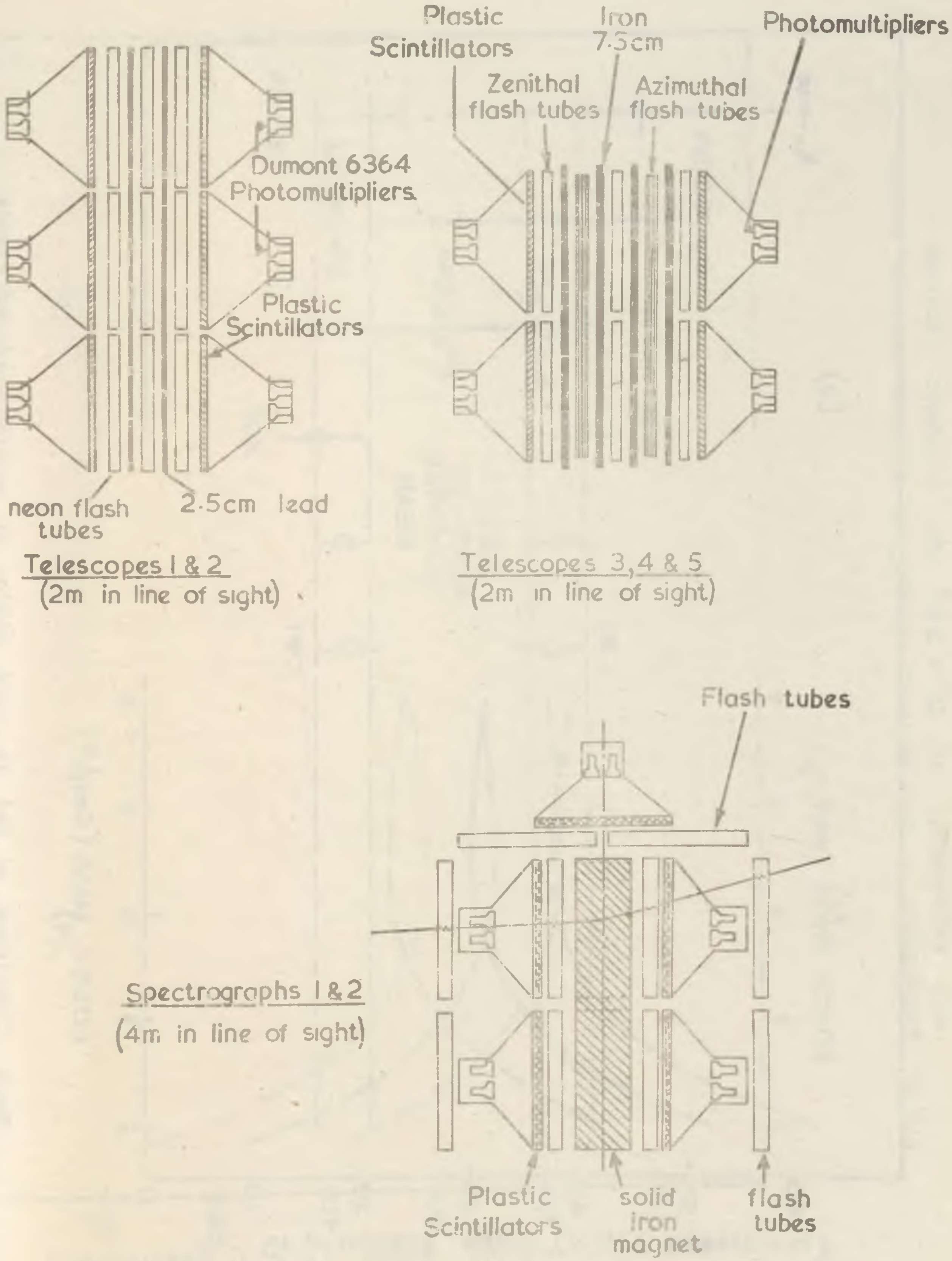

Fig. 2 K.G.F. detectors.

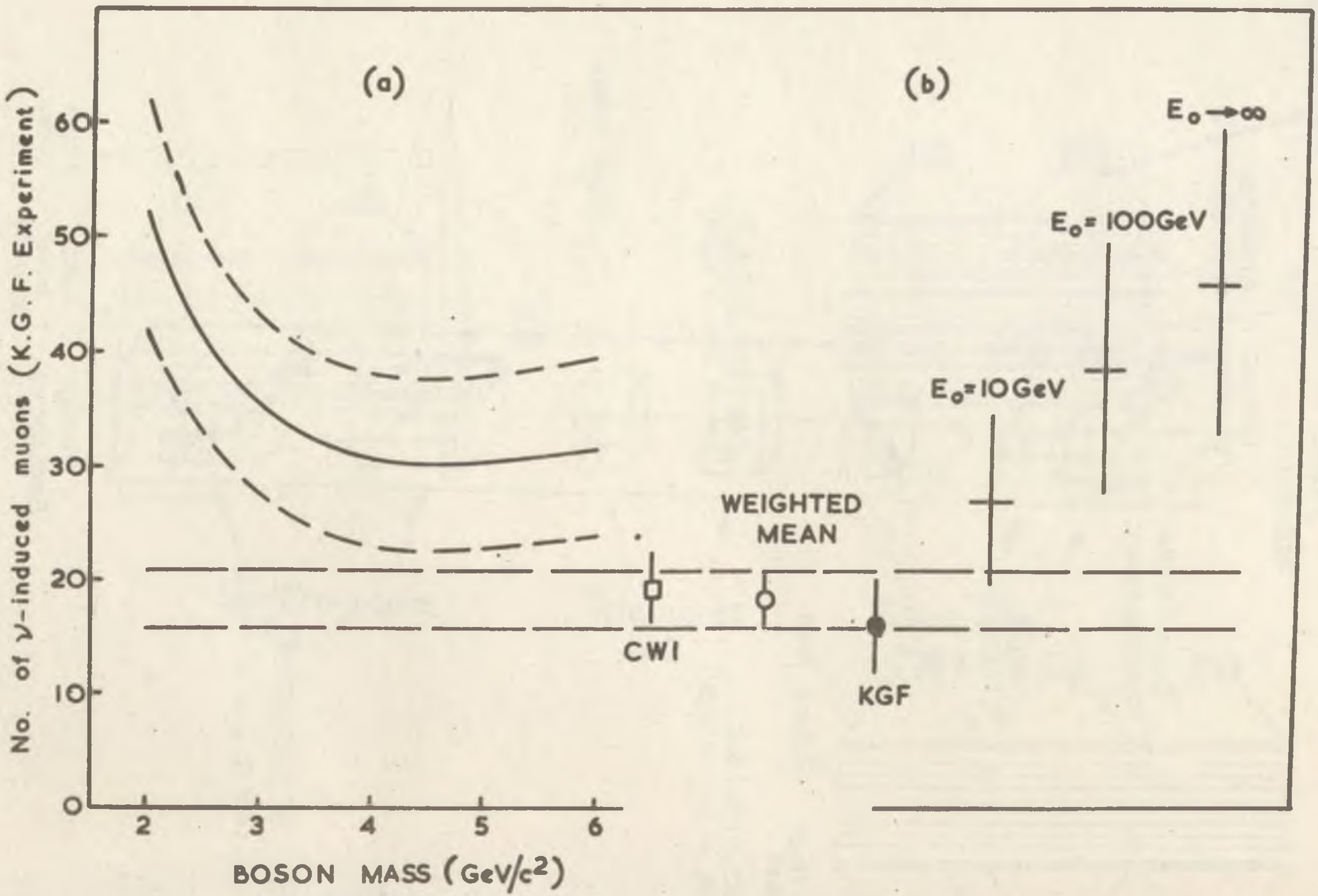

Fig.3. Comparison of the observed number of neutrino-induced events with expectation for $\sigma_{\nu} = \sigma_{\bar{\nu}}$ for inelastic reaction.

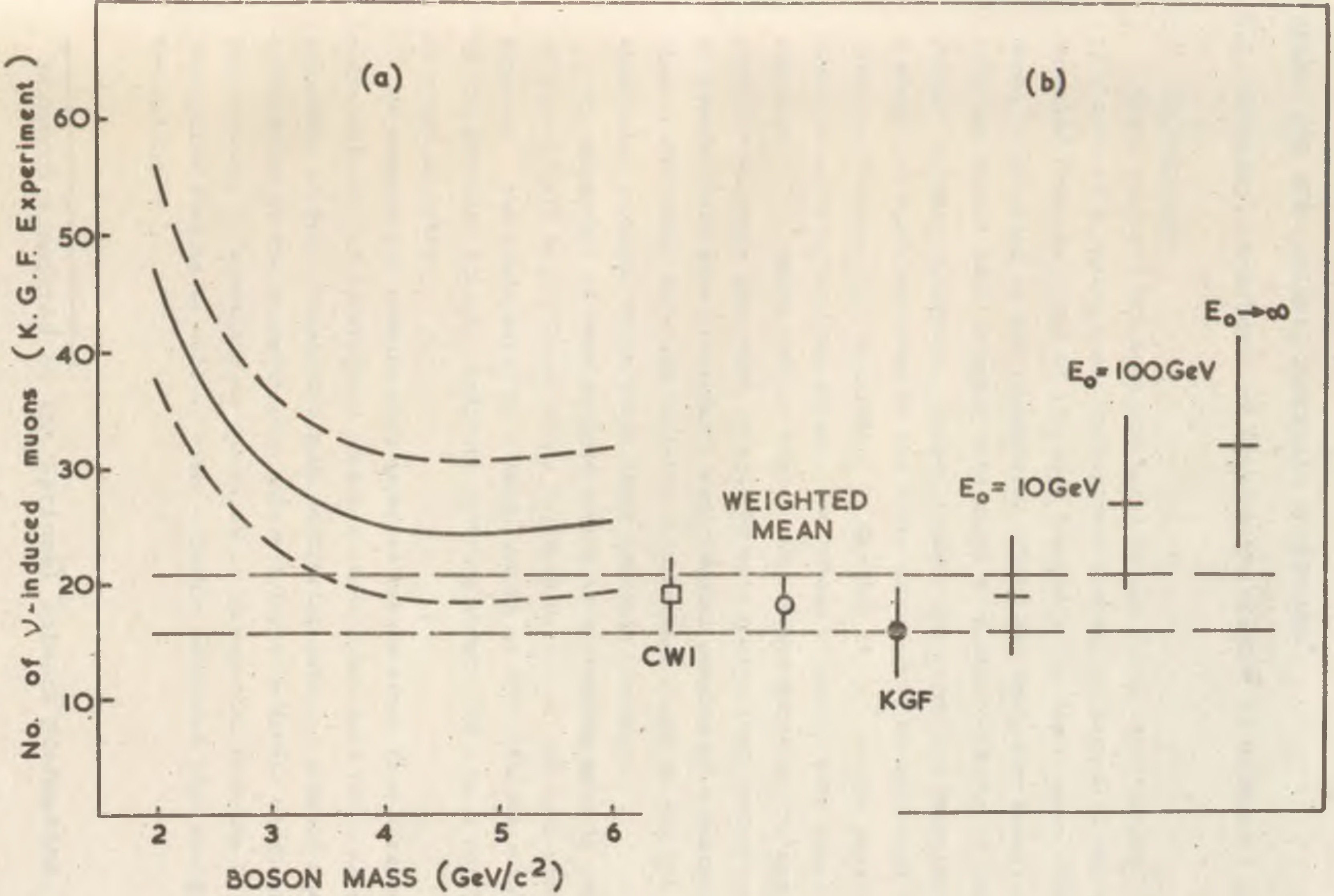

Fig. 4. Comparison of the observed number of neutrino-induced events with expectation for $\sigma_{\nu} = 3\sigma_{\bar{\nu}}$ for inelastic reaction

SEARCH FOR NEW LEPTONIC PROCESSES UNDEGROUND*

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I. Introduction

Great interest has been generated by the Turin observations[1,2,3] of a rate of stopping muons underground greatly in excess of the rate expected from the slope of the muon intensity vs. depth curve (WSDI) of muons originating in the atmosphere. There has been some speculation that the excess muon stopping rate might be a manifestation of new high energy leptonic processes. In particular, the anomolous behaviour of underground muons observed by the Turin group could be accounted for with the W-boson model considered by Keuffel et al[4] as one possible model to explain the Utah effect. A number of papers have since appeared[6,7,8,9] which seek to explain the excess muons as the decay products of pions generated locally by more conventional mechanisms. We present here some preliminary experimental results of a search for those underground muon and neutrino interactions (such as via the W-absorption process) which yield large hadronic cascades.

The signature of such an interaction is a stopping muon in one of four liquid scintillator drums deployed on top of the main Utah detector[10] and accompanied by a large amount of identifiable activity in the detector itself. Ordinary stopping events are rarely accompanied by great activity.

We compare our results with expectations obtained from detailed calculations of underground particle production rates based on extrapolations of SLAC inelastic muon scattering data and a Monte Carlo calculation of the stopping pion multiplicity in hadronic cascades. Furthermore, we speculate on the rates to be expected from the W-absorption process as well as other possible anomolous high energy interactions.

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*Research supported by the National Science Foundation, U.S.A.

II. Expected Rates

The sources of stopped muons considered here are: (1) stopping atmospheric muons; (2) muons from the decay of slow pions. (These pions are produced in hadronic cascades generated locally by interactions of the through-going muons.) Muons induced by neutrinos are negligible for depths up to 6000 hg cm^{-2}, as demonstrated by Grupen et al.[6] We present here the essential results of detailed calculations by Cassiday, Keuffel and Thompson (CKT).[11] All results are quoted for an isotropically active detector whose effective thickness is 100 g cm^{-2} of standard rock ($z = 11$, $A = 22$), hereafter referred to as a standard detector.

The stopping atmospheric muon rate is obtained by differentiating the WSDI, folding in the standard muon angular distribution $G(h,\theta)$[12,13] and integrating over the detector aperture. The expectations for our experiment are presented in Table 1 and the results for a standard detector as a function of depth are shown in figure 2. The results in figure 2 are all normalized to the through-going muon rate.

The calculation of the rate of stopping muons generated by through-going ones is greatly facilitated by considering an underground detector to be a target as well. This is permissible only if as many muons stop per gram of rock as are generated per gram, a situation which holds if the attenuation length of the through-going muons is much greater than the range of the generated secondaries, which is indeed the case.

In order to calculate the stopping rate due to local muoproduction, an estimate is needed of: (i) the multiplicity $m(\nu)$ of stopping charged pions generated in a hadronic cascade of energy , (ii) the differential cross section $\frac{d\sigma}{d\nu}$, (iii) and the local muon spectrum J(E,h,0). Here we extract the essential results of CKT for $m(\nu)$ in which several extreme models of hadronic shower development were considered. The best estimate is $m(\nu) = 0.8\nu^{0.75}$. The differential cross-section for muoproduction $\frac{d\sigma}{d\nu}$ has been obtained from fits to the existing accelerator data. Estimates of the local differential muon spectrum were obtained by differentiating the WSDI and using the range-energy relation for muons. The rate of stopping muons generated by muoproduction is then obtained by folding the local spectrum into the cross section and stopping pion multiplicity and integrating. The results are then multiplied by a factor η which takes into consideration the cavern surrounding the detector. Since slow negative pions are readily absorbed by matter only those slow negative pions which emerge from the cavern roof and decay in flight will register a stopping muon. Since all such slow positive pions yield a stopping muon we take η to be 0.7. The results for the stopping rate induced by muoproduction are shown in figure 2.

In addition we have considered the effects of electromagnetic showers initiated by muon knock-on, bremsstrahlung, or pair production. The energy transfers involved are quite small (< 1 GeV) and the resultant contribution to the stopping rate is of the order of 10% of that due to direct muoproduction as demonstrated by CKT. These results are also shown in figure 2.

III. Summary of Expectations and Comparison With Other Experiments

Shown in Table 1 are the results for all contributions of stopping muons to be expected at our depth of 1400 hg cm^{-2} Conventional production of muons by neutrinos is negligible compared to the above contributions except at very great depths underground (depths greater than 6000 hg cm^{-2}. For example, Wolfendale[6] estimates approximately a 2-3% contribution to the Turin observation at 4270 hg cm^{-2}).

Shown in figure 1 are the results of our estimates for the rate of observed stopping positive pions as a function of depth underground. The experimental results are seen to agree with our calculations. At shallow depths locally generated pions arise primarily from low energy muons ($E\mu$ ~ tens of GeV) where the physics is well known except perhaps, for the cascade slow pion multiplicity factor m(ν). This agreement provides a strong overall confirmation of our estimate for m(ν).

Shown in figure 2 are the normalized stopping muon ratios. Most experimental results are in agreement with our calculations except for the Turin observations. It should be pointed out that the Turin observations at 110, 175 and 300 hg cm^{-2} were performed at a vertical depth of 60 hg cm^{-2} with a two counter telescope inclined at 60^{o} and 78^{o} employed to look through the advertised slant depths. Consequently, these results are quite sensitive to the locally generated muon flux which principally results from vertical going muons. For example, a 2% response of the Turin 78^{o} trigger would easily explain their observed stopping ratio. From calculations on the distribution of events as a function of energy transfer, we find that 2% of the stopping rate at that depth is initiated by events involving over 100 GeV energy transfer which yield about 25 slow muons and 100 electrons, so such a response is not unreasonable.

However, the 4270 hg cm^{-2} data point obtained with no external triggering criteria imposed are a factor of 4 above our estimates. The probability of agreement is .003. Even if we take an extreme limit for our estimates we underestimate the Turin result by a factor of 2, the probability of agreement then becoming .03. Possibly, an anomoly is indicated.

IV. Present Experiment

The experiment employs four 150 kg liquid scintillators (drums) designed to look for stopping muons. These drums are mounted on top of the main Utah detector (10m x 12m x 6m, located at an effective depth of 1400 hg cm^{2}) permitting for the first time visual classification of the stopping particles. Furthermore, two large arrays of cylindrical spark counters (CSC) have been mounted on top of the Utah detector on each side of the drums. These counters provide a further check on particle activity accompanying a muon stopping in one of the drums. In addition, in favorable cases, these counters serve to identify neutral-induced, hadronic cascades occurring in the main detector (see fig 3.)

Five phototubes view each drum. The trigger requirement is a simple coincidence between two pairs of phototubes at a bias level of about 10 MeV. The fifth tube is operated in a space-charge limited mode at the anode, which permits a decay electron to be seen in the presence of large prompt energy releases. The lower energy cut on the decay electrons is presently at about 20 MeV at which level a decay can be identified following a prompt pulse of several GeV. At this level approximately 10% of the electrons are lost.

The prompt anode pulses of the space-charge limited tube in each of the four drums are coded and displayed along with the decay electron pulses on a 20 μsec scope trace. The last 8 μsec of the trace is gated open also for the display of the time-coded dynode pulses from each of these tubes. The main Utah detector is triggered in conjunction with a drum trigger. The Utah detector is then interrogated for accompanying activity.

V. Results

From a plot of the time distribution of delays observed underground, of all classifications, and including some events where the main detector was inoperative, a 2.2 μsec signature was seen which insures we are seeing stopping muons. The greatest source of background stems from random scintillator pulses in conjunction with a through-going muon. Two such events have been observed in which the delayed pulse lies beyond 12 μsec. Since they occur mostly with a through-going muon, they can be rejected from the main detector information if desired.

From table I, it is evident that there is good agreement (within the as yet rather crude statistics) between the observed and predicted rates. The existence of locally-produced muons including the hadronic cascade multiplication process is thus directly confirmed. On the other hand, at this depth (1400 hg/cm^2) there is no evidence for the Turin effect, which at a depth of 4270 hg/cm^2 shows up as a fourfold increase over our calculated locally-produced muon rate. Since the depth dependence of our calculations is much more accurate than the absolute values, we can say that this discrepancy is well established.

However, our result by no means stands in contradiction to the Turin effect for the following reason.

If the Turin effect is real, it is very likely due to some new high-energy process. In this case, it may have quite a different depth dependence than the conventional local muon interactions. For example, if muons of 2 TeV are responsible, the depth dependence would (apart from fluctuations) be about the same as the limiting exponential behaviour of the depth-intensity curve. In that case, (an attenuation length of 800 hg/cm^2) our result is still just compatible with Turin. (Interestingly enough, such a slope would also still be compatible, within errors, with the Case-Wits-Irvine[25] stopping muons, as analysed by Grupen et al.)

A similar depth-dependence would be expected for the W-boson model for the Utah effect, and the absolute rate would be expected to be (at our depth) 0.2 day^{-1} .

Shown in figure 3 is a typical event obtained. This visual identification allows us to classify stopping events into 3 categories (a) atmospheric (b) locally induced within in the roof of the surrounding cavern (c) locally induced within a drum.

The interesting events are those locally induced with lots of accompanying activity. We note that at our depth about 10% of the local events should involve energy transfers greater than 100 GeV. Neutrino or muon induced W-boson events should involve energy transfers greater than 500-1000 GeV with about the same frequency as the 100 GeV "normal" events. Thus far we have three candidates for high-energy events in which the detector memory was saturated (> 130 sparks in all counters.) This rate is consistent with conventional predictions if the energy transfers are of the order of 100 GeV. An expanded memory plus operation of the CSC's in the proportional mode should provide us with a method of determining the energy release to a degree of accuracy necessary to distinguish between ultra high energy transfer events and those induced by ordinary muon inelastic scattering.

Furthermore, we are searching for neutral induced events inside the main detector. We have two candidates, but again since the memory was saturated, we cannot with certainty ascertain the energy of the event or the neutrality of its origin.

Consequently, in view of the previous questions raised and in view of our current lack of ability to tag the energy of an event or conclusively establish the neutrality of its origin we consider the matter of anomalous underground production still open to question.

Table I

Scintillator Characteristics

Number of Scintillators	4
Effective thickness, each	17.0 g/cm^2
Effective area, each	8.4 x 10^3cm^2
Volume, each	1.65 x 10^5cm^3
Density	0.865 g/cm^3
Stopping power (rel. to st'd. rock)	1.14
Total mass, all 4 (equiv. st'd rock)	654 kg.

Rates Per Day	Predicted	Observed
Stopping atm.	0.84	0.78$\pm$ .2 (12 events)
Locally produced	1.15	0.87$\pm$ .3 (11 events)
Total stopping	1.99	1.55$\pm$ .4
Through-going	583	730$\pm$ 85

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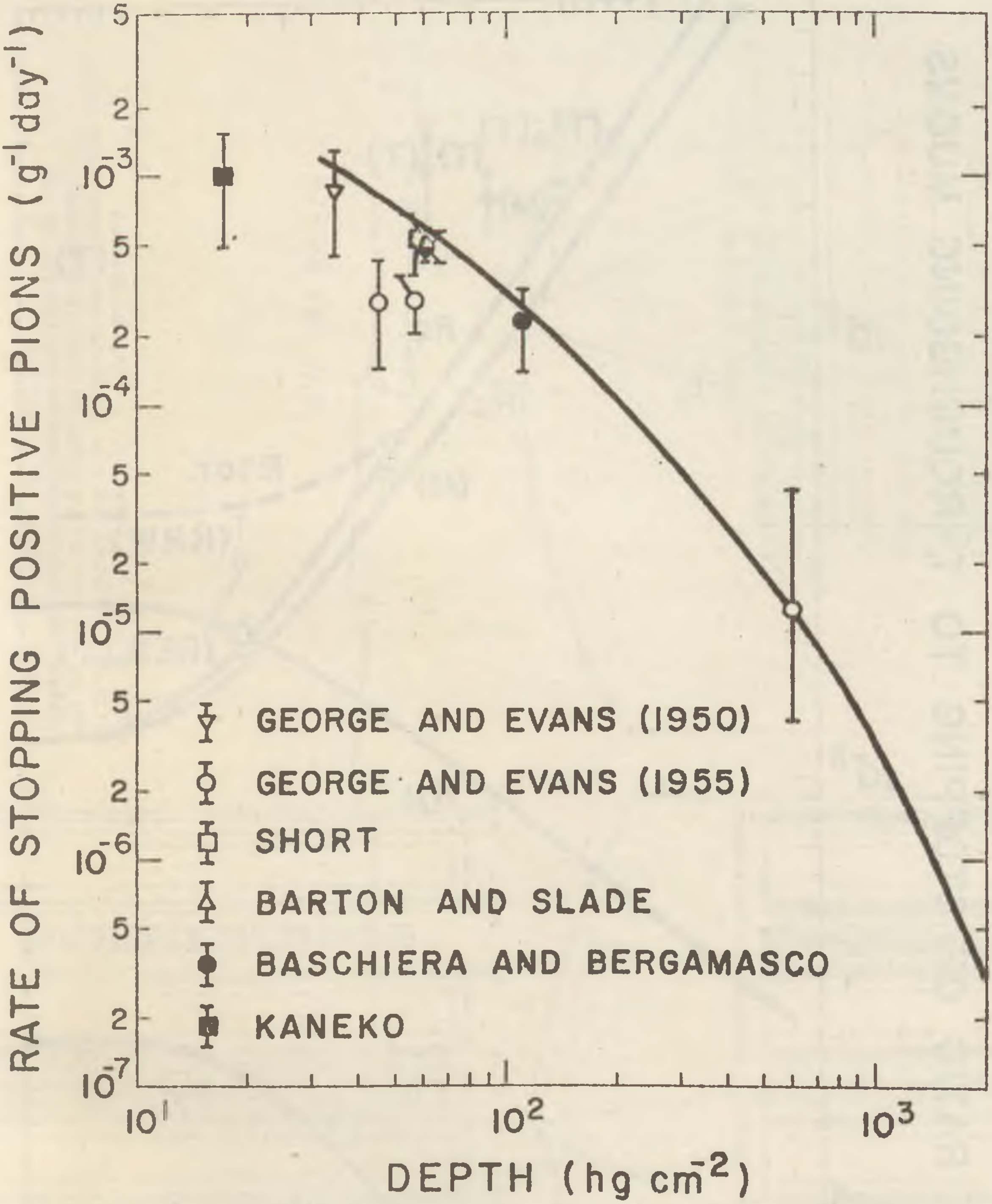


Figure 1.

Calculated rate of stopping positive pions vs. depth compared with those experiments where the positive pion is identified as such.

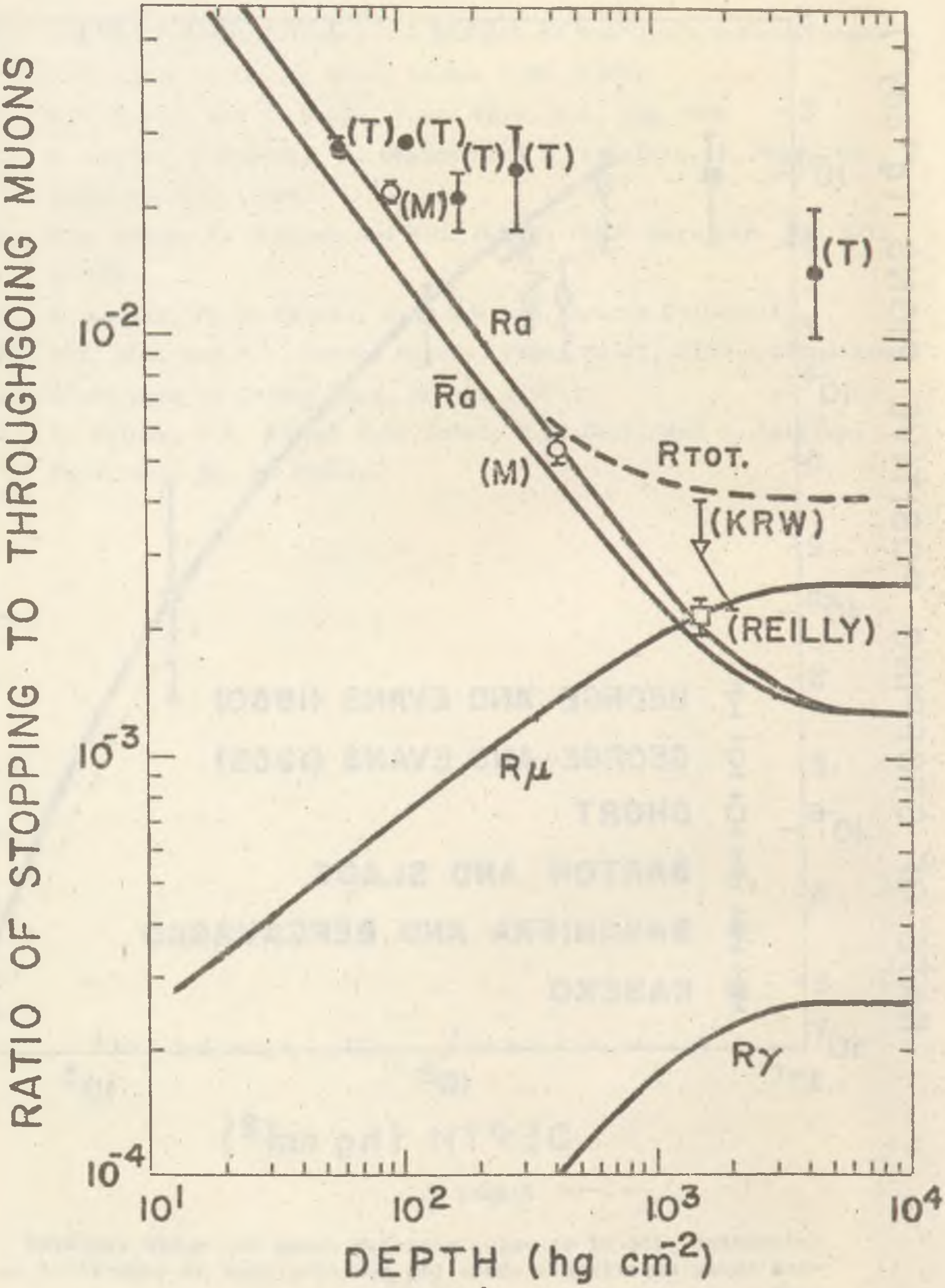


Figure 2.

Ratio of stopping to through-going muons. R_a refers to vertical atmospheric muons, $\overline{R}_a$ to all atmospheric muons; R_μ, R_γ to locally produced muons by virtual and real photons.

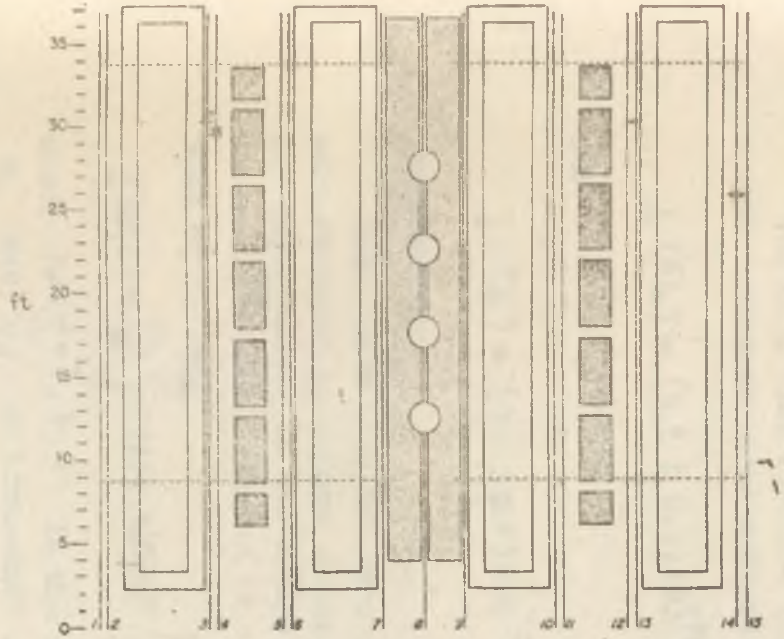
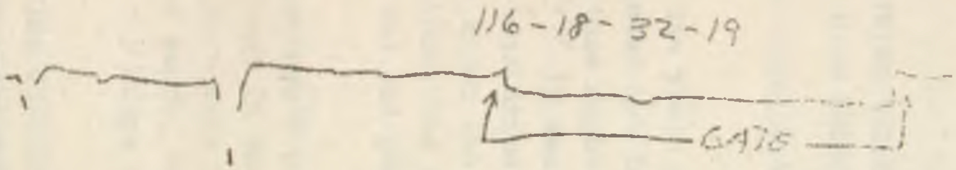
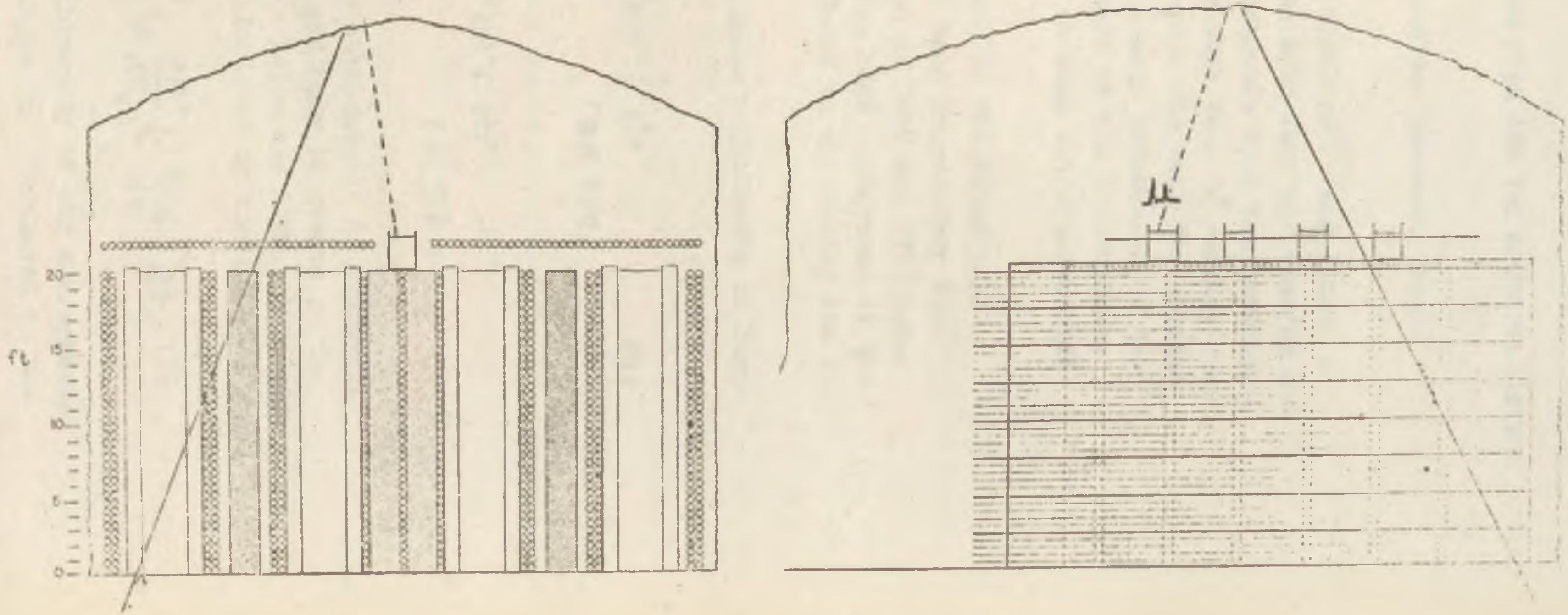


Figure 3.
Stopping muon resulting from an interaction in the roof of the cavern. A decay is seen in the drum while the accompanying through-going muon passes elsewhere through the main Utah detector.





ATMOSPHERIC NEUTRINO INDUCED MUON FLUX AND THE NEUTRINO NUCLEON INTERACTIONS AT HIGH ENERGIES

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More than ten years ago M.A.Markov[1] and K.Greisen[2] have pointed out that the muon flux produced by atmospheric neutrinos might be observed. From the quantitative estimate[3,4] it was clear that this ν-induced muon flux may be considered as a possible tool for obtaining information about the high energy properties of the νN interactions, the existence of the intermediate vector boson (IVB) and perhaps on some extraterrestrial sources[2].

In the experiments performed in the Kolar Gold Fields[5] (KGF) and in the East Rand Propriatery Mine (ERPM)[6] detectors were located at 7000 and 8710 meter water equivalent depth, where the huge background of the primary cosmic ray muons is reduced to acceptally low values.

The muon flux obtained in these experiments in horisontal direction is as follows

$$I(\pi/2) = (4.2 \pm 0.7)\times 10^{-13}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}\mathrm{str}^{-1} \quad \text{(ERPM)} \qquad (1)$$

and

$$I(\pi/2) = (3.5 \pm 0.9)\times 10^{-13}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}\mathrm{str}^{-1} \quad \text{(KGF)}$$

respectively.

Since more than 100 events attributable to neutrinos were recorded by Reines' group and only 16 events in the KGF experiments, in the analysis below the ERPM value will be used. The muon flux can be calculated by the integral formula as follows

$$I(\theta,\varepsilon) = \frac{N_A}{A}\int_{\varepsilon}^{\infty} N(\nu+\bar{\nu},E,\theta)\,dE \int_{\varepsilon}^{E} E_\mu \frac{d\sigma^{a}}{dE_\mu}\left(-\frac{dE_\mu}{dx}\right)^{-1} dE_\mu \qquad (2)$$

where $N(\nu+\bar{\nu},E,\theta)$ is the intensity of the atmospheric ν_μ and $\bar{\nu}_\mu$ at horizontal angle θ (measured to the

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vertical), $d\sigma^A/dE_\mu$ is the differential cross section per nucleus for the reaction $\nu_\mu + Z \rightarrow \mu +$ anything, $-dE_\mu/dx$ is the average energy loss of muons in the rock, ε is the threshold energy for the muon detection, N_A is the Avogadro's number, A is the nucleon number of the nucleus. If we assume that $dE_\mu/dx \approx$ const, then we get a simpler formula

$$I(\theta, \varepsilon) \approx \frac{N_A}{A} \int_{\varepsilon/k}^{\infty} N(\nu+\bar{\nu}, E, \theta)\, \sigma_T^A(E) R(kE-\varepsilon) \quad (3)$$

where σ_T^A is the total cross section per nucleus for the same reaction, R is the effective range-energy function of muons in the corresponding rock and k is the average energy ratio transferred to the muon from the neutrino.

In order to estimate the muon flux produced by the atmospheric neutrino we have to know i) the intensity of the atmospheric neutrinos $N(E, \pi/2)$; ii) the average energy loss of the muons or their effective range-energy function; iii) the differential cross section $d\sigma/dE_\mu$ and the average energy ratio transferred to the muon from the neutrino k. We adopted the values of the intensity as calculated by Osborne et al.[7] for a ratio of 20% and for energies between $1-10^4$ GeV. Beyond this region it is sufficiently accurate to take a straight line extrapolation on a log-log plot.

For the average rate of the energy loss of the muon in standard rock we used the expression proposed by Hayman et al.[8]

$$-\frac{dE_\mu}{dx} = 10^{-3}(2.053 + 0.154 \ln E - 0.077 \ln(0.93+E) + 10^{-3} bE) \times \text{GeV cm}^2 \text{gr}^{-1} \quad (4)$$

where for standard rock on the average $b = 4.0$. In the first estimations of the atmospheric neutrino induced muon flux[9], for energies less than 10 GeV the linearly rising total cross section obtained in the CERN heavy liquid bubble chamber experiment[10] was used. At higher energies it was assumed to increase linearly up to critical energy value where the total cross section became constant. From the measured muon rate the authors were able to derive restrictions on the value of the energy where the saturation began. There was a big uncertainty since the value of k was not known. The theoretical developments concerning the deep-inelastic lepton-nucleon scattering stimulated by the SLAC-MIT experiments[11], however, called for the revision of these analyses. The first estimation which assumed the scale invariance of the deep-inelastic scattering was done more than two years ago[12] (see also[13]).

We shall review and revise the results of this analy sis from the point of view of the quark-parton model predictions. We shall see that the region of the slope values of the neutrino-nucleon and antineutrino-nucleon scattering (ξ and $\bar{\xi}$) allowed by the Reines' experiment involves neither the whole region obtained in the CERN experiment not the whole region allowed by positivity conditions in the quark parton model[14]. But the interception of the parton and the CERN regions is completely inside the region allowed by this cosmic ray experiment. (See Fig. 2). Since the muon flux produced by atmospheric neutrinos obtains the main contributions from very energetic($\sim$ 100 GeV) neutrinos this result may be regarded as an independent and non-trivial support (of course not strong and not very conclusive) for the predictions of the parton model.

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*In accord with recent measurements of the direct vertical muon spectrum it would appear safer to assume a K/π ratio of 40% in the calculations of the neutrino intensities which would increase the value of the relevant muon neutrino intensities by 15% /6/.

Muon Flux Produced by Neutrinos with Energy Less than 10 GeV

In the neutrino energy region 0.12-12 GeV information about the neutrino nucleon interactions has been obtained in the bubble chamber and spark chamber experiment at CERN/10/ and in a spark chamber experiment at ANL/16/*.

The most important points of the results relevant to the estimation of the muon flux have been summarized by Pattinson/10/. In accord with these experimental results we propose to use the cross sections as given on Fig. 1 which should give good approximation with the present uncertainties. At low energies ($E_\nu \lesssim 1$ GeV) it is assumed that the quasi-elastic and Δ -production reactions dominate and that $\sigma^\nu = 3\sigma^{\bar\nu}$. In the interval E_ν =1-10GeV the measured value of the total cross section is used

$$\sigma^\nu = (0.52 \pm 0.13)\, \frac{G^2 M E_\nu}{\pi} \Big/ _{\text{per nucleon}} \qquad /(5)$$

For the $\bar\nu_\mu$ reaction in the range 3-10 GeV we assumed linearly rising cross section with a slope of one-half of the slope given in /5/. The used k values are given in Table I.

Using the ν_μ and $\bar\nu_\mu$ intensities calculated by Coswik/9/ we have obtained that the neutrinos with energy less than 10 GeV give about one-half of the measured horizontal muon intensity

$$I_L(\pi/2) = (2.10 \pm 0.80) \times 10^{-13}\ cm^{-2} s^{-1} str^{-1} \qquad (6)$$

The error bars come from the uncertainties of the slopes and the neutrino fluxes.

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*See also the new antineutrino data presented at this conference (Degrange' talk).

Reaction	Quasi-Elastic		Δ -Production		Inclusive	
Neutrino Energy(GeV)	ν_μ	$\bar{\nu}_\mu$	ν_μ	$\bar{\nu}_\mu$	ν_μ	$\bar{\nu}_\mu$
1	0.8	0.8	0.2	0.2	-	-
1- 3	-	0.8	-	0.5	0.5	-
3-10	-	-	-	-	0.5	0.65

T a b l e I.

Average energy transfer ratio k for various processes in various energy range.

Contributions of High Energy Neutrinos ($E_\nu > 10$ GeV)

In accelerator experiments the neutrino energy range extends only up to 10 GeV (Although it will soon be vastly extended at NAL*), therefore if we want to estimate the contributions of the high energy neutrinos to $I(\theta)$ we are forced to abstract those features of the low energy data and some other measured processes (deep-inelastic electroproduction e.q.) which might hopefully be valid for the high energy neutrino nucleon interactions as well.

For the sake of clarity we should here dwell on defining the kinematics. In notation we follow Llewelyn Smith with slight modifications[16].

Let us consider the process

$$\nu(k)/\bar{\nu}(k) + N(p) \longrightarrow \mu^-(k')/\mu^+(k') + \text{hadrons} \qquad (7)$$

where k, k' and p denote the four-momenta of the initial neutrino, the final muon and the target nucleon. The cross section for this process is determined by a second rank Lorentz tensor as follows

* See Barish's and Baltay's talk, this conference.

$$W_{\mu\nu}^{\nu/\bar{\nu}}(q,p)=\frac{1}{4\pi}\bar{\sum}\int d^4x\, e^{iqx}\langle p\,|\,[\,J_\mu^{\mp}(x),J_\nu^{\pm}(0)]\,|\,p\rangle \quad (8)$$

We use the Cabibbo current*

$$J_\mu^{\pm}(x)=\left(V_\mu^{\pm}(x)+A_\mu^{\pm}(x)\right)^{\Delta S=0}\cos\theta_c+\left(V_\mu^{\pm}(x)+A_\mu^{\pm}(x)\right)^{\Delta S=1}\sin\theta_c \quad (9)$$

Assuming P , T , PT invariance, $W_{\mu\nu}^{\nu/\bar{\nu}}$ can be expanded in terms of five invariant functions

$$W_{\mu\nu}^{\nu/\bar{\nu}}(p,q)=-g_{\mu\nu}W_1^{\nu/\bar{\nu}}(q^2,\nu)+\frac{1}{M^2}p_\mu p_\nu W_2^{\nu/\bar{\nu}}(q^2,\nu)-$$
$$-\frac{i\epsilon_{\mu\nu\lambda\rho}q^\lambda p^\rho}{2M^2}W_3^{\nu/\bar{\nu}}(q^2,\nu)+\frac{q_\mu q_\nu}{M^2}W_4^{\nu/\bar{\nu}}(q^2,\nu)+\frac{p_\mu q_\nu+p_\nu q_\mu}{2M^2}W_5^{\nu/\bar{\nu}}(q^2,\nu) \quad (10)$$

where M is the proton mass, $M\nu=pq$. To order m^2/EM (m - is the muon mass) the cross section is determined by W_1 , W_2 and W_3

$$\frac{d\sigma^{\nu/\bar{\nu}}}{dQ^2 d\nu}=\frac{G^2E'}{2\pi ME}\Big[\cos^2\tfrac{\theta}{2}W_2^{\nu/\bar{\nu}}(q^2,\nu)+2\sin^2\tfrac{\theta}{2}W_1^{\nu/\bar{\nu}}(q^2,\nu)$$
$$\mp\frac{E+E'}{2M}\sin^2\theta/2\;W_3^{\nu/\bar{\nu}}(q^2,\nu)\Big]+O(m^2/EM) \quad (11)$$

where $Q^2=4EE'\sin^2\theta/2$ and θ is the angle between the direction of the initial and final leptons in the laboratory frame. From positivity of the tensor $W_{\mu\nu}^{\nu/\bar{\nu}}$ it follows that

$$\frac{1}{M}\sqrt{\nu^2+Q^2}\,|W_3^{\nu/\bar{\nu}}|\le W_1^{\nu/\bar{\nu}}\le\left(1+\frac{\nu^2}{Q^2}\right)W_2^{\nu/\bar{\nu}} \quad (12)$$

The charge symmetry gives

$$W_i^{\nu p}=W_i^{\bar{\nu}n},\quad W_i^{\bar{\nu}p}=W_i^{\nu n}\quad (i=1,2,..5,\ \Delta S=0) \quad (13)$$

---

*In the following the strangeness changing part will be ignored i.e. we take $\theta_c=0$.

Introducing dimensionless structure functions

$$2W_1(\nu, q^2) = G_1(\omega, Q^2)$$
$$\nu/M \, W_i(\nu, q^2) = G_i(\omega, Q^2) \quad , \quad i = 2,3,4,5 \tag{14}$$

where $\omega = x^{-1} = 2\nu M/Q^2$. The differential cross section may be written in the form

$$\frac{d^2\sigma^{\nu/\bar\nu}}{d\omega\, dy} = \frac{G^2ME}{\pi}\Big[\big(1-y-\frac{My}{2\omega E}\big)\frac{G_2^{\nu/\bar\nu}}{\omega^2} + y^2\frac{G_1^{\nu/\bar\nu}}{2\omega^2} \mp \mp y(1-1/2\, y)\frac{G_3^{\nu/\bar\nu}}{\omega^3}\Big] \tag{15}$$

where $y = \nu/E$. The scale invariance or automodelity /17/ found in the SLAC-MIT experiments for the structure functions of the deep-inelastic electroproduction at high Q^2 (20 $GeV^2 > Q^2 > 1 GeV^2$) and ν (ν < 12 GeV) can be extended for this structure functions. Thus as $Q^2 \to \infty$ at fix ω the G_i functions approach a finite nonzero value at all

$$\lim_{Q^2\to\infty,\ \omega\ fix} G_i(\omega, Q^2) = f_i(\omega) \tag{16}$$

This scaling law gives simple asymptotic forms both for the total cross section /17/ and the average energy transfer ratio k /12/.

$$\lim_{E_\nu\to\infty} \sigma_T^{\nu/\bar\nu}(E) = \frac{G^2ME}{\pi} Z^{\nu/\bar\nu} = \frac{G^2ME}{\pi}\Big[\frac{1}{2}K_2^{\nu/\bar\nu} + \frac{1}{6}K_1^{\nu/\bar\nu} \mp \mp \frac{1}{3}K_3^{\nu/\bar\nu}\Big] \tag{17}$$

where

$$K_1^{\nu/\bar\nu} = \int_1^\infty \frac{d\omega}{2\omega^3} f_1^{\nu/\bar\nu}(\omega)\ ;\ K_2^{\nu/\bar\nu} = \int_1^\infty \frac{d\omega}{2\omega^2} f_2^{\nu/\bar\nu}(\omega)\ ;\ K_3^{\nu/\bar\nu} = \int_1^\infty \frac{d\omega}{\omega^3} f_3^{\nu/\bar\nu}(\omega) \qquad (18)$$

and

$$k^{\nu/\bar\nu} = \frac{8 + \frac{K_1^{\nu/\bar\nu}}{K_2^{\nu/\bar\nu}} \mp 3\frac{K_3^{\nu/\bar\nu}}{K_2^{\nu/\bar\nu}}}{12 + 4\frac{K_1^{\nu/\bar\nu}}{K_2^{\nu/\bar\nu}} \mp 8\frac{K_3^{\nu/\bar\nu}}{K_2^{\nu/\bar\nu}}} \qquad (19)$$

Due to the inequalities (12) we can write

$$|K_3| \leq K_1 \leq K_2 \qquad (20)$$

$$1/3 \leq z^{\nu/\bar\nu} \leq 1 \qquad (21)$$

and

$$1/2 \leq k^{\nu/\bar\nu} \leq 3/4 \qquad (22)$$

We can now discuss the high energy aspects of the results of the CERN heavy liquid bubble chamber experiment. In the energy region (E = 1 - 12 GeV) of this experiment* we do not expect scale in variant behaviour unless the scaling occurs in an average sense even in the non-asymptotic region. Bloom and Gilman /18/ and more extensively Rubinstein et al. /19/, however, have pointed out that in an average sense all the electroproduction and photoproduction data can be fitted with an universal scaling curve in the variable $\omega' = 2M\nu + M_c^2 / Q^2 + \alpha^2$ ($M_c^2 \approx 1.5$ GeV^2, $\alpha^2 \approx 0.4$ GeV^2). By analogy, Myatt and Perkins /20/ assumed in their analysis of the combined propane and freon data that scaling in $\omega' = \frac{2\nu M + M^2}{Q^2}$ occurs in the non-asymptotic region and so they used all the events with $E_\nu > 1$ GeV. For the present discussion the important points of their results are as follows

i) The scaling hypothesis in the energy region 1-12GeV is supported by the linear rise of the cross section and

* $Q^2_{max} = 2.5$ GeV^2 and $\bar{Q}^2 \leq 0.5$ GeV^2 for 40% of the events.

the constancy of the energy transfer ratio

$$k_{exp} = 0.52 \pm 0.06 \tag{23}$$

ii) The data are compatible with the Callan-Gross relation which is also suggested by the electroproduction data $(f_1 = \omega . f_2)$.

iii) The data suggest that K_3 is negative, which implies that* $\sigma^{\nu N} > \sigma^{\bar{\nu} N}$.

iv) The scaling function $f_2(x')$, $x' f_1(x')$, $x' f_3(x')$ can be fitted by the formula $(1- x'^2)^3$.

If in the description of the high energy (anti)neutrino-nucleon interaction we accept scale invariance and the Callan-Gross relation, in the calculation of the neutrino induced muon flux only two free parameters remain, It is convenient to use as free parameters the first moment integrals (18) $K_3 = \frac{1}{2}(K_3^{\nu p} + K_3^{\nu n})$ and $K_1 = K_2 = \frac{1}{2}(K_1^{\nu p} + K_1^{\nu n})$ or the slopes (21) $Z = 2/3 K_1 - 1/3 K_3$ and $\bar{Z} = 2/3 K_1 + 1/3 K_3$. The measured values of Z and $\bar{Z}$ give the following limits on and

$$(0.54 \pm 0.13) \leq K_1 \leq 0.81 \pm 0.20 \; ; \quad -0.175 \leq -\frac{K_3}{K_1} \leq 1 \tag{24}$$

The lower and upper limits on K_1 correspond to $K_3 = -K_1$ and $K_3 = +0.175 K_1$, respectively. The allowed region in the plane$(Z'\bar{Z})$ is given on Fig. 2.

In the quark parton model from positivity conditions and the electroproduction data the following limits can be obtained on K_1 /14/

$$(0.18 \pm 0.13) \leq K_1 \leq 0.50 \pm 0.08 \tag{25}$$

---

* The antineutrino data from the CERN measurement give $R = \sigma^{\bar{\nu} N} / \sigma^{\nu N} = 0.42 \pm 0.08$ (see Degrauge's talk).

In the $(z, \bar{z})$ plane, the region allowed by the positivity condition in the quark parton model is inside the contour "a" of Fig. 2. In the quark parton model we have the expression for the slopes as follows/21/

$$z = \int_0^1 x\left[u(x) + d(x) + \frac{1}{3}\bar{u}(x) + \frac{1}{3}\bar{d}(x)\right]dx$$
$$\bar{z} = \int_0^1 x\left[\frac{1}{3}u(x) + \frac{1}{3}d(x) + \bar{u}(x) + d(x)\right]dx \qquad (26)$$

where $u(x)$, $d(x)$ is the number of up, down quarks with momentum fraction x per dx in the proton.

Since u , d , $\bar{u}$, $\bar{d}$ are positive functions and the main contribution comes from the region $x \approx 1$ we obtain

$$1/3 \leq \bar{z}/z \leq 1 \qquad (27)$$

The allowed region in this case is one-half of the region given by the positivity conditions. Furthermore as we have learned from Feynman's talk the sum rule

$$\int_0^1 x\left(f^{ep}(x) + f^{en}(x)\right)dx = \frac{5}{9}\int_0^1 x\left[u(x) + \bar{d}(x) + \bar{u}(x) + \right. \qquad (28)$$
$$\left. + \bar{d}(x)\right]dx + \frac{2}{9}\int_0^1 x\left[s(x) + \bar{s}(x)\right]dx \approx 0.30$$

suggests additional restrictions on $z + \bar{z}$: if we accept that the strange-quark contribution to this sum rule is less or about 10% (which is quite plausible) then we can obtain that

$$0.65 \leq z + \bar{z} \leq 0.76 \qquad (29)$$

which is a small area in the $(z, \bar{z})$ plane (the dark spot of Fig. 2) having comparatively large interception with the new CERN data. Treating z and $\bar{z}$ as free parameters in the estimation of the contribution of the high energy neutrinos to the atmospheric neutrinos induced horizontal muon flux we have obtained that only the region inside the contour "b" of Fig. 2 is allowed by the measured value of the muon flux. Since it involves neither the whole region allowed by the CERN ex it involves neither the e region allowed by the positivity condition in the parton model, but their interception is involved in it, this result may be regarded as a nontrivial (although not conclusive) support given by hgih energy ($\gtrsim$ 100 GeV) neutrinos for parton models.

Contributions and Corrections by Assuming that IVB Exists

If IVB exists we have to take into account the damping effect of its propagator in the amplitudes of reactions (7) at high energies and we have to calculate the contpibutions induced by IVB production reactions to the muon flux. The modification introduced by the IVB propagator is very simple: we should use the factor $G^2/(1-q^2/M_W^2)^2$ instead of G^2 in the expression of the cross section (11), (15). Estimating the underground muon-flux we need only the differential cross section $d\sigma/dy$. Since the propagator $\left(\omega+\frac{2ME}{M_W^2}y\right)^{-2}$ cannot be factorized in ω and y, the integration over ω can be performed only if we know explicitely the scaling functions. Since they are, as indicated by the experiment[20], smooth function of x $(\sim(1-x^2)^3)$ it is easy to see that for the present purposes (regarding other uncertainties of the calculation) it is sufficiently accurate to approximate the scaling functions with constants. We have calculated the contribution of the process (17) as a function of the IVB's mass M_W for neutrinos with energy higher than 10 GeV, using the cross section formula as follows

$$\frac{d\sigma^{\nu/\bar\nu}}{dy}=\frac{G^2ME}{\pi}\cdot\frac{(1-y+\frac{1}{2}y^2)\mp y(1-\frac{1}{2}y)r}{1+2MEy/M_W^2}K_1^{\nu/\bar\nu} \qquad (31)$$

where
$$r = K_3/K_1$$

IVB can be produced in the Cb field of the nucleus by the reactions

$$\nu_\mu + Z \longrightarrow \mu + W + Z' \qquad (32)$$
/ coherent and incoherent /

and

$$\nu_\mu + N \longrightarrow \mu + W + \text{hadrons} \qquad (33)$$

The total cross sections and k values for these processes have recently been calculated for a wide range of W -boson masses and neutrino energies [22]. Chen et al. [13], using these results have made a very thorough analysis calculating the contributions of these reactions to the neutrino-induced muon flux. Muon flux values induced by neutrinos via reaction (32) was also calculated by Coswik et al. [23], he, however, used unrealistically high value of k. In paper [13] Coswik's estimation corrected by a factor 1/5 is used for the contribution of reaction (32) and the contribution of reaction (33) is neglected. Since the results of Chen et al.[22] are roughly in agreement with such an approximation, for the present analysis we adopt again these values.

Results, Discussions

If it is assumed that IVB exists, the neutrino induced muon flux is the sum of the contributions from the low energy neutrinos (6), the high energy neutrinos and the IVB production reactions (31), (32)

$$I(\pi/2, M_W) = I_L(\pi/2) + I_H(\pi/2, M_W) + I_W(\pi/2, M_W) \qquad (34)$$

The horizontal muon flux is calculated for different K_2 and r values / see eqs. (31) /. Assuming that

$z = \bar{z} = 0.53$ and using $I_L = 2.1 \times 10^{-13}\ cm^{-2} s^{-1} str^{-1}$ and $I_W = \frac{1}{5} I_W^c$ where I_W^c is the muon intensity via the reaction (32) given by Coswik et al./23/, we obtain the curve A of Fig. 3. The curve B is obtained by using $z = \bar{z} = 0.38$*. The curve C corresponds to $z = 0.46$, $\bar{z} = 0.23$, the values suggested by Feynman/21/, with $I_L = 1.5 \times 10^{-13}\ cm^{-2} s^{-1} str^{-1}$ and $I_W = \frac{1}{5} I_W^c$. The curve D represents a lower limit on $I(\pi/2, M_W)$ at 2 standard deviation.

Since according to the deep-mine experiment

$$I_{exp}(\pi/2) < 5.0 \times 10^{-13}\ cm^{-2} s^{-1} str^{-1} \qquad (35)$$

We see that the assumptions used for the curve A are in contradiction with experiment. Even if we take into account the uncertainties in the neutrino intensities the contradiction remains. Only the region inside the contour b of Fig. 2 gives values of z and $\bar{z}$ which are compatible with the measured muon flux. With the assumption used for the curve B we obtain upper and lower limits on M_W

$$3.5\ GeV < M_W < 12\ GeV \qquad (36)$$

With the slope values preferred by the quark parton model /curve C on Fig. 3/ we can obtain only a lower limit $M_W \geq 2.8$ GeV. Finally at confidence level more than 96% we can say that $M_W > 2$ GeV.

If IVB does not exist, but in order to simulate the saturation of the cross section at high energies, the IVB propagator is preserved, the characteristic energy where

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*These slope values are obtained in the Kuti-Weisskopf parton model, as well /24/.

the saturation begins can be defined by $E_c = M_W^2/M_N$
The horizontal muon flux will be the sum $I = I_L + I_H$
and we obtain instead of the curves A, B, D of Fig. 3 the curves A', B', D' of Fig. 4. The restrictions on E_c given by A', B' and D' are as follows /in GeV/

$$7.6 \leq E_c^A \leq 17 \; ; \quad 7.6 \leq E_c^B \leq 100 \; ; \quad E_c^D > 10^4 \qquad (37)$$

Since there is no sign of saturation at energies up to 12 GeV, the contradiction with the curve A' remains , therefore the allowed region in the $(\xi , \bar{\xi})$ plane / Fig. 2. contour b / will only slightly be modified.

Acknowledgement:

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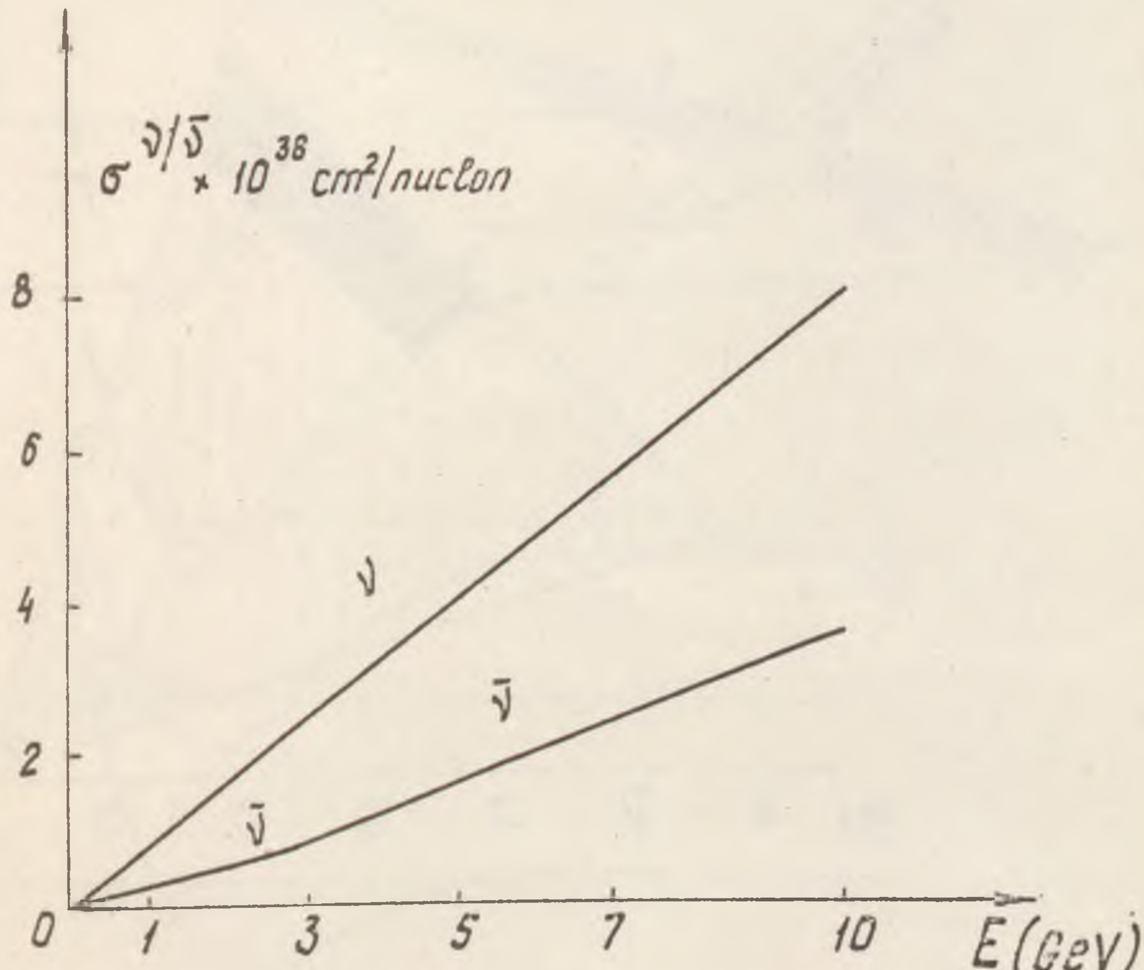

Figure 1.

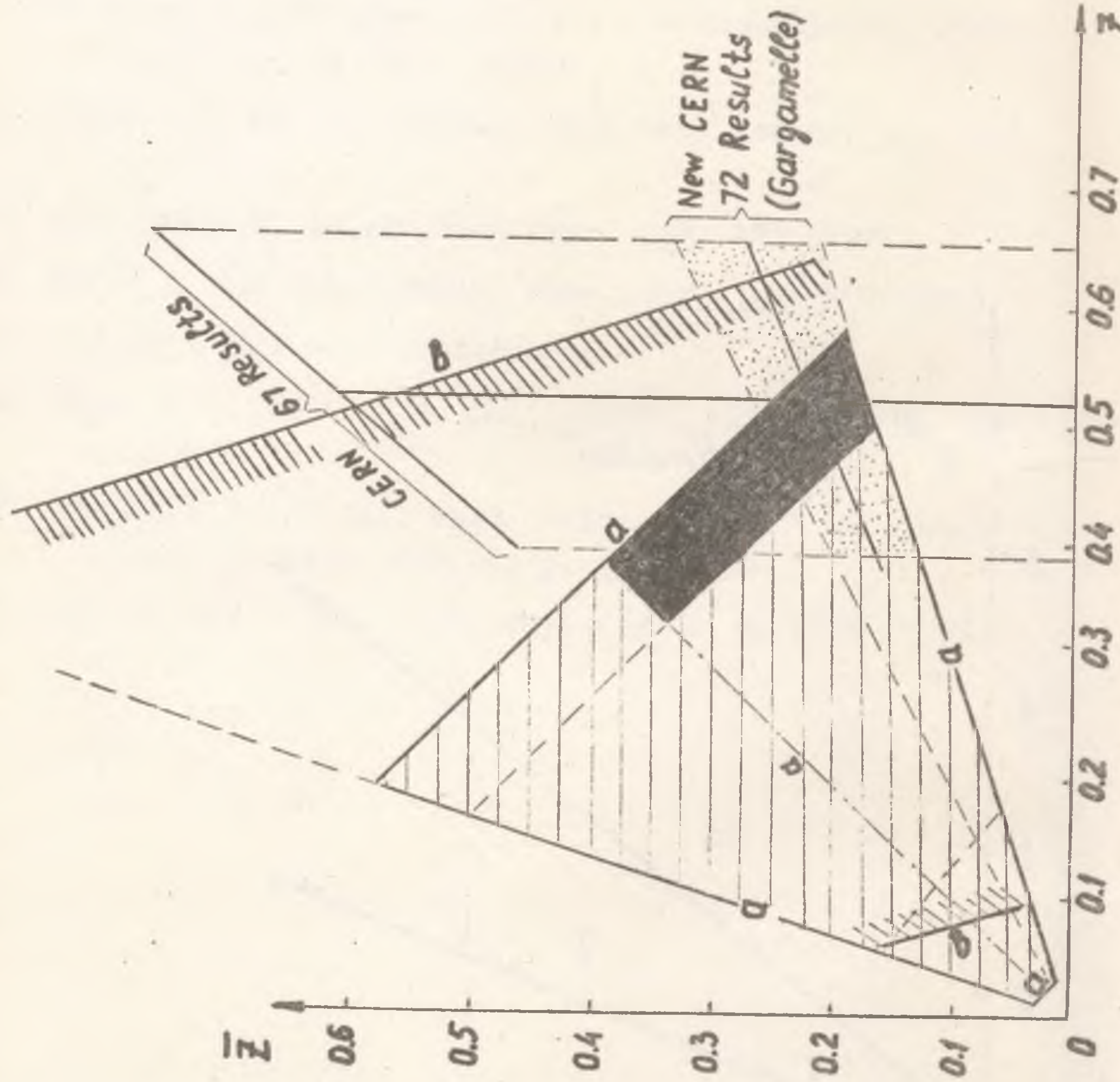
New CERN
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Figure 2.

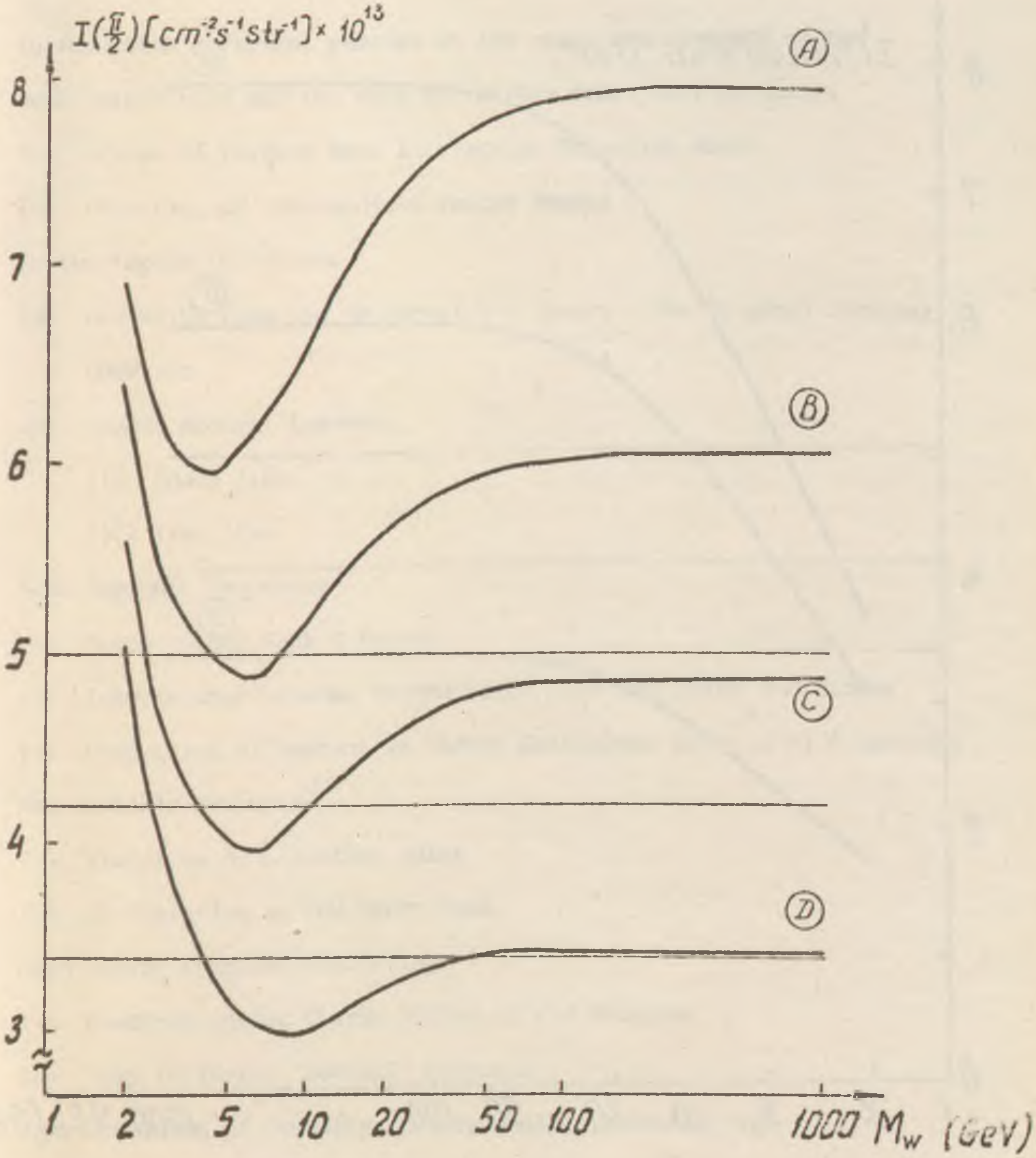

Figure 3.

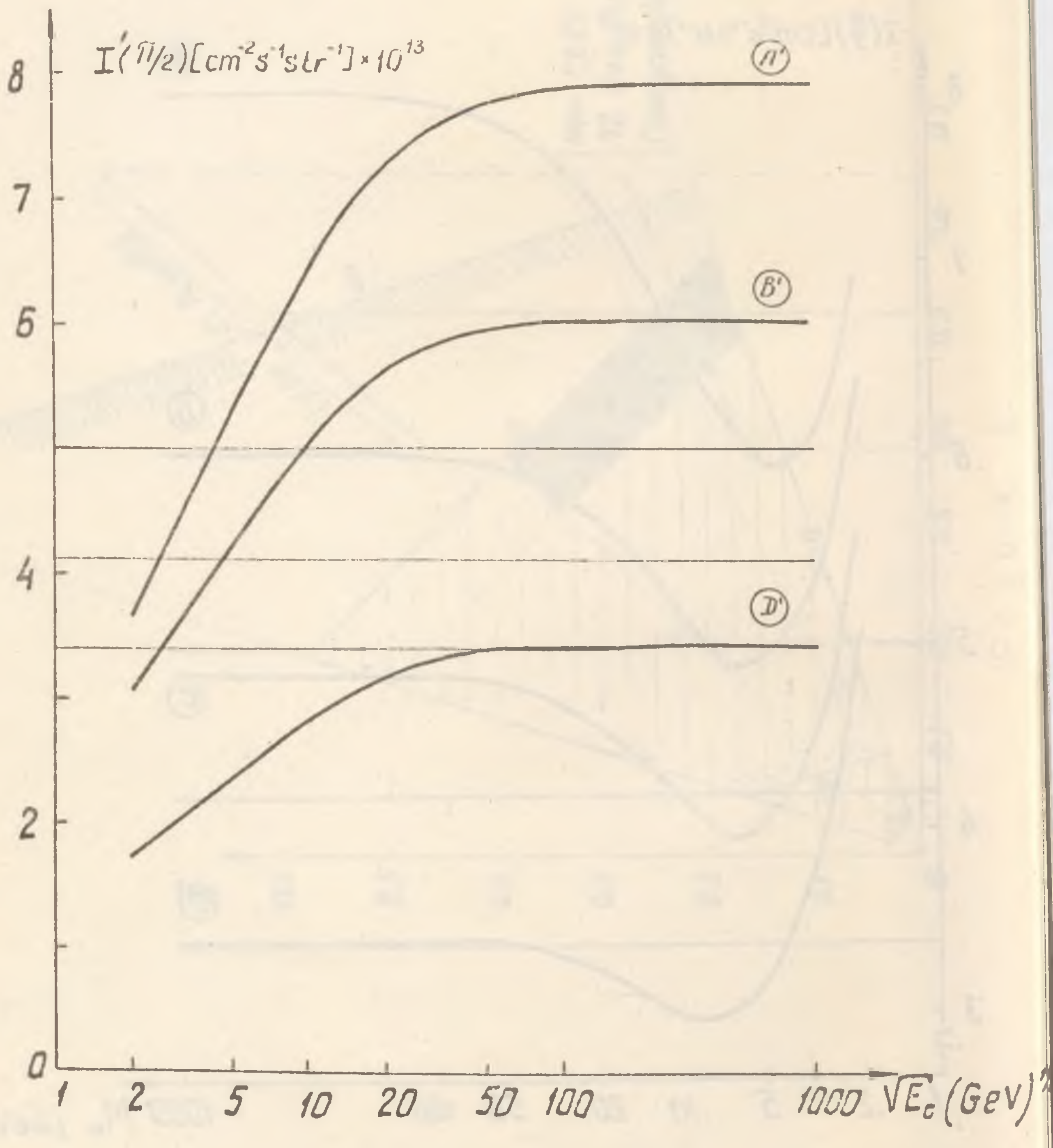

Figure 4.

PROSPECTS FOR THE DETECTION OF HIGHER ORDER WEAK PROCESSES AND THE STUDY OF WEAK INTERACTIONS AT HIGH ENERGY*

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Contents

1. Introduction

The first observation of weak interactions is now over 75 years old.[1] An impressive array of understanding of a vast number of phenomena has been achieved for low energy processes, and yet some of the simplest questions that can be asked about the basic nature of the weak interaction can not presently be answered. In many ways we know less about this interaction than we do about the strong interaction. Apparently Heisenberg was the first to recognize the significance of the dimensionality of the coupling constant of the lowest order current-current interaction.[2] The lowest order interaction being

$$H_{eff} = \frac{G}{\sqrt{2}} j_\lambda j_\lambda^+ \qquad (1)$$

where j_λ, j_λ^+ are appropriate currents and G is the coupling constant. G has the dimensions of $(\text{length})^2$ or $(1/m)^2$ with a numerical value

$$G = (1.01 \times 10^{-5})/(m_p)^2$$

In order to form a dimensionless parameter for the weak interaction it is frequently suggested to use s and to form the parameter[3]

$$\lambda = Gs,$$

s being the only parameter of the scattering process that sets a length (or m^{-2}) scale (s is the center of mass energy squared).

There is at present no experimental information that sets the length scale of the weak interactions. However there are two theoretical suggestions as to what the length scale might be.

1. The 'length' at the unitarity limit. If the weak interaction was pointlike all two body cross sections would rise like

$$\frac{G^2 s}{\pi} \qquad (2)$$

and being pointlike only the S wave interaction is allowed. However, the unitarity limit for the cross section for S wave scattering goes as π/s, thus at a large value of s the weak interaction cross section must be modified to avoid a unitarity violation (at the energy $\sqrt{s_u} = \frac{1}{G}$). The length associated with this value of s (which was called the 'fundamental length' by Heisenberg) is

$$\ell_f = \frac{1}{\Lambda_u} = \frac{1}{\sqrt{s_u}} = \sqrt{G} \sim 10^{-17}\text{cm} \quad (3)$$

Note that, by definition, the dimensionless coupling constant

$$\lambda_u = Gs_u \simeq 1, \quad (4)$$

thus indicating that the weak interactions actually become 'strong' at these very high energies. It appears that the intrinsic strength and the range of force of the weak interactions are therefore intimately tied together. The interaction is strong in the sense that the S wave cross section is as large as any S wave cross section can be. (In strong interactions the low partial waves are strongly absorbed and thus the S wave cross section probably does not stay at the unitarity limit; thus at the unitarity limit the weak interaction cross section would likely exceed the strong interaction S wave cross section, however, the actual cross section would only be $\sigma \sim \frac{G}{\pi} \sim 10^{-33}\text{cm}^2$ compared to $\sim 3 \times 10^{-26}\text{cm}^2$ for hadron scattering cross sections, because of the large number of angular momentum states excited in the hadron scattering.)

2. A second way to set the 'length' scale for weak interactions is to imagine that the exchange of a massive boson is responsible for the weak force between two particles.[4] The mass (M_W) of this hypothetical boson then sets the scale

$$\ell_W \sim \frac{1}{M_W} \quad (5)$$

and the coupling constant for the W coupling to say two leptons is semiweak and given by

$$g^2 \simeq GM_W^2 \simeq \lambda_W.$$

Thus the larger the mass M_W, the stronger the semiweak interaction becomes. This illustrates again that the fundamental nature of the weak interaction is presently indeterminate, there being a tradeoff between the strength and the range of the interaction. Experimentally it is, therefore, necessary to determine either the fundamental dimensionless coupling constant or directly measure the range of the interaction. Clearly measurement of a distance of 10^{-17}cm is a very ambitious undertaking since momentum transfers of $\sim(300)^2$ GeV/c^2 would be required. Nevertheless as discussed later we might contemplate observation of momentum transfers of $(30)^2$ within the decade, in forthcoming neutrino experiments allowing a probe of distance down to $\sim 10^{-16}$cm.

There have been other suggestions as to a fundamental length of weak interactions in terms of the exchange of scalar bosons and a variety of other postulated particles.[5] These particles were invented to provide a renormalizable theory of weak interactions.[5]

Recently a dispersion theoretic approach has been applied to the question of the high energy behavior of weak interactions starting with the posthumous paper by Pomeranchuk.[6,7] Other calculations have followed this lead.[8] There are no firm conclusions to be drawn from such analyses but some very interesting speculation about the processes that may dominate the weak interactions at high energy are made. Also, as shown by Pomeranchuk,[6] if the weak interaction becomes long ranged at high energy with a cross section that approaches that of strong interactions, such a behavior cannot set in before an energy of the unitarity energy $\sqrt{s_u}$. Dolgov, Okun and Zakharov have attempted a dispersion theoretic estimate of the lower limit of the contribution from higher order weak diagrams for lepton-lepton collisions.[8]

Other theoretic attempts at handling the higher order weak interactions have focused on a summation of the contributions from all higher order diagrams.[9,10] The first such attempt known to us was made by Feinberg and Pais and more recently by Arbuzov.[9]

An interesting proposal for modifying the weak interaction was made by Gell-Mann, Goldberger, Kroll and Low.[11] Their proposal would lead to a modification of the universality of first order weak interactions such that the diagonal and nondiagonal lepton-lepton processes would proceed with different rates.

Many other suggestions have been made for calculating the higher order diagrams or for formulating a renormalizable theory of weak interactions. (See Refs. 12, 13, 14, 15, 16 for an incomplete list).

A promising way to separate (or estimate) the range and 'intrinsic' strength of the weak interaction is through the observation of a certain class of higher order weak interaction processes. While the validity of such calculations is certainly not proved, as order of magnitude estimates these calculations make some sense, especially when applied to pure leptonic systems.[15,16,17,18,19] If higher order weak processes are suppressed in all systems relative to first order processes then the observation of higher order weak processes will likely be carried out with low energy weak interaction processes such as a rare decay mode of K mesons because of the possible large abundance of such decay particles.

At the same time study of high energy weak interactions bring us closer to the unitarity limit where we expect surprises. These studies will likely be carried out with high energy neutrino beams or colliding lepton beams. In fig. 1 we attempt to summarize the present and projected range of energies available for weak interaction studies as well as the present range of transition rates that have been studied for K decays, in particular, in this figure we attempt to show the regions in these variables where new surprises in the weak interaction might be expected. The moral to be gained from this graph is that already experiments have covered a large range of energy and transition rates and we are close to the regions where surprises might be expected.

A short summary of the experimental measurements needed to 'unravel' the range and 'intrinsic' strength of the weak interaction is in order. The 'intrinsic' range and 'intrinsic' strength are assumed to be tied together in such a way that

$$G \sim g^2 \cdot 1/(m_\ell)^2 \tag{6}$$

where g is the intrinsic coupling strength and m_ℓ is a mass that characterizes the range of forces.

There are basically three ways to detect or measure the value of m_ℓ

1. Study high momentum transfer processes observing the effects of m_ℓ in the form factor

$$\frac{d\sigma}{dq^2} \propto \frac{1}{\left[1 + \left(\frac{m_\ell}{q}\right)^2\right]^2} \tag{7}$$

2. Study very high energy scattering; in the vicinity $\sqrt{s} \sim m_\ell$ where higher partial waves will enter the weak interactions and a 'break down of locality' will occur.

3. Observe processes that can only proceed by 2nd or higher order weak interactions and assume (on the basis of the perturbation theory

allogrim) that the rate for such processes related to that for first order processes is, order of magnitude,

$$\frac{\Gamma(\text{2nd order})}{\Gamma(\text{1st order})} \sim G^2 m_\ell^4. \tag{8}$$

In a more careful perturbation calculation the ratio of second to first order rates becomes[19]

$$\xi^2 = \frac{G^2 \Lambda^4}{32\pi^4} \tag{9}$$

where Λ is a cut off mass that is used to remove the divergence of the integrals associated with second order contributions. For nonleptonic or semileptonic processes these calculations assume that the range or size of the strong interactions does not provide a cutoff to the integral.[15,20] Such an assumption can be justified on the grounds of current algebra or the quark model or any model where the weak current couples to pointlike objects inside the hadron (like the parton model).[21,22] However, this assumption does seem to violate simple minded intuition that the hadrons can not generally support high momentum transfers. Recent observations of inclusive processes where hadrons appear to be capable of supporting high momentum transfers,[24] can be explained by parton or quark pointlike structures.[22,23] However, it is not clear that pointlike structure is <u>necessary</u> to explain this phenomena (nor in fact that it is really sufficient) and more mundane explanations of the deep inelastic scattering have been proposed.[25] Therefore, it is not presently clear that the higher order processes are not cut off by the strong interaction in semileptonic or nonleptonic processes. For this reason it is very important that leptonic processes be studied.

Experimentally techniques 1 and 2 require high energy particles and the possibilities for such studies are only now becoming available with the advent of high energy machines such as NAL and the CERN 300 GeV machine. In practice such studies will likely be carried out using high energy neutrino beams.

The direct observation of higher order weak processes will likely depend on the intervention of a selection rule in first order weak interactions that are violated by the higher order processes. However, in some cases it may be necessary to separate higher order weak processes from first order contributions by observing the nonlocality generated by the higher order process.[26,27] Generally, therefore, the detection of higher processes will only be as sensitive as the validity of the selection rule. So far the best obeyed selection rules appear to be the absence of neutral currents in semileptonic processes and the $|\Delta S| < 2$ rule for nonleptonic processes.[28] In the next section we review the present status of the selection rules obeyed by the weak interaction.

It is interesting to note the different dependence on m_ℓ in techniques 1 - 3. For 1 and 2 the larger m_ℓ the more difficult it becomes to 'measure' m_ℓ (or to detect a deviation from $m_\ell \to \infty$). However, for the higher order corrections, especially for lepton-lepton collisions, the larger m_ℓ the easier it is to 'measure' m_ℓ. Of course perturbation intuition may fail here but if it does not then these techniques are complementary and should all be pursued. For example, it is difficult to foresee in the near future experiments that attain momentum transfers of $(300)^2$ GeV/c^2 and therefore $m_\ell \sim 300$ GeV would be hard to observe by techniques 1 or 2. However, for $m_\ell \sim 300$ GeV the higher order corrections become maximal and might be detected eventually in e^+e^- collisions as discussed below.

In table 1 we have attempted to summarize the present guesses for the limit on Λ from various viewpoints, the low values of Λ all come from semileptonic processes or nonleptonic processes. This table might be viewed in the following way; there are hints that the weak interaction cutoff is low and therefore something interesting is expected to occur in weak interaction processes for $\sqrt{s} \lesssim 10$ GeV. Also if the weak force is transmitted by an intermediate vector boson the mass is expected to be relatively low compared to the unitarity limit. However, these speculations are based on calculations that in all cases involve hadrons in the weak process. It may still be that the low values of Λ in table 1 are (i) determined by the strong interaction range or (ii) that perturbation theory is not relevant. To answer the first question will require the study of leptonic processes at large s. Probably the answer to question (ii) will require study of weak interaction processes very near $s \sim 1/G$.

The plan of this paper is essentially spelled out in the index. We first review the status of various weak interaction selection rules and discuss briefly the prospects for detecting intermediate vector bosons in the near future. The rest of the paper is broken up into sections that are classified by the kinds of particles that participate in the weak process. Each section deals with the processes suitable for detecting higher order weak processes or the high energy behavior of the weak interaction for that particular system.

a. Status of Various Selection Rules

The selection rules in weak interactions are not presently required by any basic theory; the rules being almost completely empirical. For this reason it is not known how exact such rules should be, and in fact some selection rules are known to be broken at the 5% level in the amplitude. However, some selection rules are suspected to be exact in first order weak interactions, but perhaps broken in higher orders. If this is true then the observation of a violation of the rule would be a signature for higher order processes; but, it need not be since the rule might simply be broken by the first order weak interaction. Since the observation of the violation of CP invariance, we know that sometimes very small violations in weak amplitudes (or super weak) can occur, and perhaps small violations of other selection rules might equally be observed. However, in the case of the absence of neutral semileptonic currents ($\Delta Q = 0$, $\Delta S \neq 0$ processes), the upper limit on the violation has now been shown to be three orders of magnitude lower than the CP violation rate,[29] perhaps indicating that the absence of neutral currents is a better selection rule than CP invariance.

In table 2 the current upper limits on the amount of violation for weak amplitudes for the selection rules is presented for:

$\Delta Q \neq 0$	leptonic processes
$\Delta Q \neq 0$	semileptonic processes
$\Delta S = \Delta Q$	semileptonic processes
$\Delta S < 2$	semileptonic processes
$\Delta S < 2$	nonleptonic processes

A notable point in this table is the absence of any useful limit on the $\Delta Q \neq 0$ selection rules for purely leptonic systems. Remarkably, the only well tested selection rule is the $\Delta Q \neq 0$, semileptonic rule, and only for the $\Delta S \neq 0$ subclass.

The $\Delta T = 1/2$ selection rule is now known to be broken by about 5% in the amplitude for several processes suggesting that the rule is only approximate in <u>all</u> cases. We, therefore, neglect this rule in table 2. Similarly, second class current in semileptonic amplitudes may come in at the same level.

One moral that might be drawn from table 3 is that when searching for higher order weak processes, violations of the ($\Delta Q \neq 0$, semileptonic) rule would be more likely to pay off because the other selection rules have yet to be tested to a sensitive level. For example, if the higher order processes come in at the relative amplitude level of 10^{-6}, this is 4-5 orders of magnitude in the amplitude lower than these selection rules have been tested, but only one or two orders below the ($\Delta Q \neq 0$, $\Delta S \neq 0$ semileptonic) rule. Even if the second order process comes in (1-2) orders of magnitudes below a primitive neutral current, it might still be possible to separate the higher order process as discussed below.

b. Detection of Intermediate Vector Bosons

The discovery of one or more bosons that couple semiweakly to leptons and hadrons and thus are candidates for the 'mediators' of weak interactions would go a long ways towards answering the basic questions about weak interactions posed in the introduction. Thus the search for such hypothetical but crucial states is of great importance and experimenters are well aware of this as can be proved by looking at the current proposals for experiments at the NAL.[32]

With the advent of high intensity neutrino beams at NAL or CERN it should be possible to produce, in a massive detector, adequate numbers of W vector bosons to discover such a particle if the mass is below $\sim$ 12-15 GeV.[33] It also appears that the boson can be detected independent of the relative branching fraction into leptonic and hadronic final states and, therefore, a conclusive search can be made in this mass range.[34]

Higher mass bosons might be detected in hadronic or photonic interactions at NAL or CERN up to the mass of 30-40 GeV, provided the cross sections for the production are comparable to the estimates of Lederman and Pope and provided the boson decays via the leptonic decay mode.[35] We emphasize that in the range of 15-40 GeV it will likely be impossible to conclusively exclude the existence of the intermediate vector boson because of the uncertainty of production cross sections and decay rates. Thus, up to ∿ 15 GeV an exhaustive search can be made and if conditions are favorable a W of mass 15-40 GeV could be detected.

The observation of a scalar charged meson is virtually impossible due to the expected small production cross section and the suppression of the leptonic decay mode.[36] If neutral vector bosons exist (perhaps producing so far undetected neutral leptonic current processes) and have any mass above the kaon mass, they likely would not have been detected up to the present. A neutral W° could be produced in e^+e^- collisions, but sensitive experimental searches have yet to be carried out in these processes.[37] It has been proposed to search for the existence of W° bosons using the process $e^+e^- \rightarrow \mu^+\mu^-$.[38] This search should be sensitive to the existence of any W° boson with mass below 8 GeV using colliding beam facilities such as SPEAR.[38]

2. Lepton-Lepton Collisions

Without the obscuring effects of the strong interactions, lepton-lepton scattering provides a 'clean' study of weak interactions. Experimentally, the detection of weak lepton-lepton processes is just coming into the range of experimental feasibility. There are basically three kinds of processes that may yield practical and interesting results:

$$\nu_\ell + z \rightarrow \ell + \overline{\ell} + \nu_\ell + z \quad (10)$$

$$\nu_\ell + \ell \rightarrow \nu_\ell' + \ell' \quad (11)$$

$$e^+ e^- \xrightarrow{\text{weak}} \mu^+ \mu^- \quad (12)$$

Study of the first two processes is becoming feasible because of the advent of high energy-high intensity neutrino beams at NAL and CERN. The s available to such processes, however, is likely to be limited to the range

$$s \sim 2m_e E_\nu \lesssim 5 \times 10^{-1} \text{ GeV}^2$$

For processes like 10 the requirements of coherence limits the mass of the three leptons to equally small values. Process 12 is the only one where values of s can be obtained where surprises and perhaps departures from the standard lowest order weak interaction theory may occur. In this case, s values in the vicinity of

$$s \sim 10 - 64 \text{ GeV}^2$$

might be attained with storage ring machines that are presently being constructed.

Unfortunately, since weak interactions are in general overwhelmed by electromagnetic interactions in process 12 , a special dispensation is required to observe weak interactions. It has been recently speculated that such a dispensation may occur under special circumstances at colliding beam facilities such as SPEAR.[38]

a. Deviations from the Universal V-A Theory in Lowest Order--the Diagonal Coupling

Gell-Mann, Goldberger, Kroll and Low[11] have suggested a theory of weak interactions in which the leading divergences occur only in the diagonal interactions (i.e. $(\nu_e e)(\nu_e e)$ terms), which are thus speculated to be quite unconnected with the off diagonal interactions (i.e. $(\nu_e e)(\nu_\mu \mu)$ terms). Thus, higher order weak corrections may be manifested in a resulting difference between the diagonal and off diagonal coupling constants, which in turn would be observable in $s \to 0$ processes. In order to test this idea it will be necessary to compare processes like

$$\nu_\mu + \mu^- \to \nu_\mu + \mu^- \tag{13}$$

$$\nu_e + e^- \to \nu_e + e^- \tag{14}$$

with processes like

$$\nu_\mu + e^- \to \mu^- + \nu_e \tag{15}$$

Fortunately, these processes will likely be measured in the near future and the issue can be resolved.

Observation of process (14) may be accomplished in neutrino experiments currently underway at CERN using the Gargamelle bubble chamber or in early experiments at NAL using the 15' bubble chamber filled with neon.[39]

Reaction (13) is the most problematic since free muon targets do not exist in nature. A convenient substitute for this process is the process[40]

$$\nu_\mu + z \to \mu^+ \mu^- \nu_\mu z \qquad (16)$$

This process can likely be detected also at NAL and the Harvard-Penn-Wisconsin Collaboration experiment (E1A) has been designed with this process in mind. I will not go into detail concerning the projected experimental difficulties in studying this process since Professor Mann has described this in his talk. If this process can be separated from background at NAL, it should be possible to make a 10% measurement of the cross section. Incidentally, the calculations of the rate for process (16) are presently only good to $\sim$ 10%.[40]

We must emphasize, however, that the bulk of the events detected at NAL, even though the neutrinos are high energy, will likely have a low $\mu^+\nu_\mu$ invarient mass and thus the study of process (13) via (16) is at small s.[33,40] Nevertheless, it should soon be possible to experimentally compare the diagonal and off-diagonal coupling constants at <u>low s</u> and thus decide on the GGKL conjecture.

b. <u>Psuedo Neutral Leptonic Currents</u>

(i) Spacelike

At present there is no evidence to support the absence of first order neutral currents coupled only to leptons (see table 2). Recently it has been conjectured by Weinberg and others that such currents could exist in a renormalizable theory of weak and electromagnetic interactions.[13]

The most convenient processes to use to search for neutral leptonic currents in first order are

$$\nu_\mu + e^- \to \nu_\mu + e^- \tag{17}$$

$$\nu_\mu + Z \to e^+ e^- \nu_\mu Z \tag{18}$$

Again process (17) is on the verge of detectability in present or near future experiments. For example, process (17) can perhaps be detected in the present CERN studies with Gargamelle if the cross section is no less than ∿ 5 times smaller than the present limit on this process.[41] The present limit on the cross section for (17) relative to the cross section expected for process (14) (on the basis of the universal V-A theory) is[42]

$$\frac{\sigma(\nu_\mu + e^- \to \nu_\mu + e^-)}{\sigma(\nu_e + e^- \to \nu_e + e^-)} < 0.4 \tag{19}$$

The lower limit of this ratio predicted by the theory of Weinberg is[13]

$$\frac{\sigma(\nu_\mu + e^- \to \nu_\mu + e^-)}{\sigma(\nu_e + e^- \to \nu_e + e^-)} \geq 0.125 \tag{20}$$

The search for process (17) in the neon bubble chamber at NAL is likely to be even more definitive. The study of process (18) is problematic because of the large background of Dalitz pairs in neutrino collisions.

If process (17) is not detected at the level of first order weak in bubble chambers it becomes interesting to see at what level the higher order corrections may come in and if the resulting cross section can be measured by massive target-counter techniques. An estimate of the cross section for process (17) proceeding through second order weak processes and assuming that the weak interaction cutoff is at the unitarity limit ($\Lambda \sim \sqrt{s_u}$) gives[19]

$$\sigma(\nu_\mu + e^- \to \nu_\mu + e^-) \simeq 1.5 \times 10^{-44}(E_\nu)\text{cm}^2/\text{GeV}$$

where E_ν is the ν_μ energy in GeV. Using full design intensity of the NAL machine and a 500 ton Pb detector approximately 2 events of type (17) would be produced per day. Thus, in principle, a purely leptonic higher order weak process could be detected at NAL, provided the unitarity limit provides the weak interaction cutoff. We do not mean, to imply, however, that it is presently known how to separate these two events/day from the large background, but only that the process seems in principle detectable under favorable circumstances. Note, however, that even at this level the ratio of cross sections is

$$\frac{\sigma(\nu_\mu + e^- \to \nu_\mu + e^-)}{\sigma(\nu_e + e^- \to \nu_e + e^-)} \sim 10^{-3}$$

and thus the resulting limit on first order weak neutral currents would only be at best $\sim 3 \times 10^{-2}$ in the amplitude. Thus, it appears difficult to put limits on the absence of first order neutral leptonic currents to the level that $\Delta S \neq 0$ semileptonic neutral currents have reached.

(ii) Timelike

Process (12) can proceed via weak interactions in several speculative ways: (1) direct channel production of a W° on the mass shell; (2) a first order weak neutral current coupling of the form $(ee)(\mu\mu)$; (3) an induced neutral current coming from higher order weak interactions.

Experimentally, the detection of any of these weak processes requires a suppression of the dominant electromagnetic amplitudes and a unique signature for the weak process. It appears that a sizable suppression of the first order electrodynamic contribution can be obtained if the initial leptons in process (12) are highly polarized in opposite transverse directions. A 'hole' appears in the angular distribution of the outgoing muons at favored values of θ and ϕ ($\cos\theta = \hat{p}_\mu \cdot \hat{p}_e$, $\cos\phi\sin\theta = \hat{p}_\mu \cdot \hat{a}$, where $\hat{a}$ is a unit vector along the e^- polarization vector).[38,43] This 'hole' is illustrated in fig. 2 as the ratio of the differential cross section for reaction (12) for completely polarized initial leptons to the cross section for unpolarized initial leptons, and in fig. 3 in a projection drawing of the differential cross section for the two cases. At the bottom of the 'hole' should be a sensitive place to search for any anomolies in process (12) including a weak interaction process.[38] In particular the μ longitudinal polarization will likely be sensitive to interference between first order EM and perhaps weak amplitudes. The polarization will be enhanced in the 'hole'.[38,44] It is too early to conclusively conclude that amplitudes can be uniquely extracted in this way, but there seems to be an intriguing possibility here that should be pursued. It seems very likely that the existence of a W° boson with mass below $\sim$ 8 GeV could be directly observed in this way.[38] Careful theoretical calculations of this polarization and the background from higher order EM processes would be very useful in planning experiments.

3. Semi-leptonic Processes

a. Second Order Weak K Decays

The studies of K meson decays over the past two decades have provided a rich field for the study of nature and the weak interaction. Nearly every symmetry principle of particle physics has been successfully tested or found to be violated using K meson decays. The primary reason for this richness of the K meson system is due to the large mass of the Kaon relative to the leptons and π mesons. It is fortunate indeed that K mesons exist. Higher order corrections could, in principle, show up in any K decay including the nonleptonic decays. If the intrinsic coupling constant were large then the higher order corrections might be of comparable magnitude to the first order processes. For this reason exhaustive searches for rare decay modes of K mesons is of considerable importance. Any rare decay that is observed with an anomalous rate relative to the best theoretical guesses for the rate based on first order theory, is a candidate for evidence concerning higher order weak processes. In fig. 4 is shown the branching fraction levels to which exhaustive searches for rare decays have been made. In this figure are examples of processes with the lowest branching ratios that have been presently studied. As a rough rule of thumb exhaustive searches for rare K^+ decay modes have been extended down to a branching ratio of $\sim 10^{-5}$ to 10^{-6}.[45] For K_L^o decays the corresponding branching ratio is $\sim(10^{-3}$ to $10^{-4})$ and for K_S^o mesons the branching ratio is only $\sim(10^{-2}$ to $10^{-3})$. For K^- mesons the branching ratio is $\sim 10^{-2}$, however, CP invariance requires the K^+ and K^- decay ratios to be the same and the results from K^+ decays can then be inferred for K^- decays. In some cases it is possible to relate K_L^o and K^+ decays of K_S^o and K^+ decays and therefore the results for K^+ decays can be applied to the K_L^o, K_S^o decays.

Recently searches for special individual rare decay modes have been extended down to the branching ratio of $\sim (10^{-8}$ to $10^{-9})$.[29] Although only a few experiments of this kind have been attempted we may hope that the branching ratios region of 10^{-6} to 10^{-10} will be searched considerably more in the future. The advent of high intensity $K^{\pm}$ and K^o beams at the AGS and the Bevatron will be the key factor in these studies.

The study of rare decay modes of K mesons therefore naturally divides into two parts. Studies of the branching ratio region of 10^{-2} to 10^{-6} where nearly exhaustive searches for all rare decay modes have been made and the branching ratio of 10^{-6} to 10^{-10} where studies are just beginning.

It appears that no important surprises are found in the K decay processes observed down to the level of $\sim 10^{-6}$. It seems likely that the higher order processes are not important in this region.

At lower levels the search for HOW processes has been associated with the $\Delta Q \neq 0$ selection rule and this seems to be the logical place to push for definitive evidence of HOW processes. The most important decay processes in this respect are

$$K_L^o \to \mu^+\mu^- \quad (13)$$

$$K_S^o \to \mu^+\mu^- \quad (14)$$

$$K^+ \to \pi^+e^+e^- \quad (15)$$

$$\to \pi^+\mu^+\mu^- \quad (16)$$

$$\to \pi^+\nu\bar{\nu} \quad (17)$$

$$K_L^o \to \pi^o e^+e^- \quad (18)$$

In the first four cases the decay can also proceed through a first order weak and first or second order electromagnetic transition. Unless interference is invoked between the HOW and the electromagnetic processes, these processes can only be used to search for HOW amplitudes down to the level of the electromagnetic amplitudes. In both processes 13 and 15 the present experiments have approximately reached the level where the E.M. processes should be seen. These processes will probably not be useful to pursue the search to lower levels unless something is amiss in our present understanding of the electromagnetic corrections.

Processes 17 and 18 are likely to provide the most sensitive way to unambiguously search for HOW processes and push lower the limit $\Delta Q = 0$, $\Delta S \neq 0$ currents. The first order weak-electromagnetic amplitude for process 17 is expected to be highly suppressed due to the zero charge of the neutrino. However since the neutrino is likely to have distribution of charge the amplitude does not vanish. A crude guess is that the rate for this process should be at least down by $q^4 \cdot \langle r^2 \rangle^2$, where r is the electromagnetic radius of the neutrino. The best guess for $\langle r^2 \rangle$ is $\sim 10^{-32} cm^2$ and for $q^2 \sim m_\pi^2$ we obtain a suppression factor of 10^{-12} in the rate.[46] Thus process 17 should be safe as a signature for HOW or neutral currents down to a branching ratio of $\sim 10^{-18}$.

The electromagnetic contribution to process 18 is likely to be strongly suppressed because CP invariance forbids the single photon intermediate state contribution to this process.[26] The lowest order E.M. process will then be due to diagrams with two photon intermediate states. We can crudely estimate the lower limit due to such contributions using a recent experimental limit on $K^+ \to \pi^+\gamma\gamma$[45]

$$\frac{\Gamma(K_L^\circ \to \pi^\circ e^+e^-)}{\Gamma(K_L^\circ \to \text{all})} \lesssim \alpha \frac{\Gamma(K^+ \to \pi^+\gamma\gamma)}{\Gamma(K^+ \to \text{all})} \sim 10^{-5} \cdot 2 \times 10^{-5} \sim 10^{-10}.$$

Using current theoretical estimates for the rate of $K^+ \to \pi^+\gamma\gamma$ we find a branching factor of $\sim 10^{-12}$ or less.[45] The contribution coming from CP violation in the first order weak process is expected to be much smaller.

Experimentally, process 17 has been searched for in two experiments each covering a different region of the available phase space.[45,52] The best limit for the process that is independent of the behavior of the matrix element is $\sim 4 \times 10^{-5}$ at the 90% confidence level.[45] If a phase space or V-A matrix element is assumed the limit is reduced by an order of magnitude.[52] It seems feasible to search for this process, in the near future down to the level of $\sim 10^{-10}$.

Process 18 has yet to be searched for in any definite way. Considering all factors this process is likely the best candidate for a realistic search for HOW process if the branching ratios are below 10^{-9}.

It is possible to estimate the rate for processes 17 and 18 due to HOW in perturbation theory as discussed in the introduction. Primakoff has estimated that[19]

$$\frac{\Gamma(K \to \pi\nu\bar{\nu}, \pi^\circ \ell\ell)}{\Gamma(K \to \pi\ell\nu)} \sim 8\xi^2 \cos^2\theta_c$$

where θ_c is the Cabbibo angle. If these processes are not detected before 10^{-12} in this ratio, Λ the resulting cutoff would be reduced to $\sim$ 1 GeV.

b. Interference Between Second Order Weak Amplitudes and Others

A possibly more sensitive technique to search for HOW is to observe a large sample of events of the kind

$$K^+ \to \pi^+e^+e^- \qquad (15)$$

that likely proceeds dominately through first order weak-first order E.M. processes. An asymmetry in the momentum spectrum of the e^+ and e^- could come about because of the HOW amplitude interfering with the lowest order process. Estimates of this effect have been presented in reference 27. Until process 15 is experimentally observed, it is impossible to estimate the experimental feasibility of this approach.[47]

c. Production of Leptons in Hadron Collisions ($NN \to (\ell,\nu)$ + hadrons)

If (ℓ,ν) lepton pairs were observed in hadron collision direct evidence for weak transitions in these processes would be obtained. Lederman has suggested that at a high energy pp colliding beam facility it might be possible to observe such processes.[48] He has used an analogy with the process $pp \to (\ell,\ell)$ + hadrons and attempted to extrapolate available data at low energies to these very high C.M. energies. Provided this all works, we might expect that high mass (ℓ,ν) pairs would be produced. In fact it might be possible to obtain events where

$$m_{\ell\nu}^2 \sim s_u.$$

Since the lepton system is at the same s as the unitarity limit we might expect appreciable (perhaps observable) HOW amplitudes.

4. Non-leptonic Processes

(a) Violation of Selection Rules

As can be seen from table 3, the only important selection rule for nonleptonic processes seems to be the $\Delta S < 2$ rule. The only obvious way to search for HOW non-leptonic amplitudes is to search for $\Delta S \geq 2$ transitions. The only experimentally detected non-leptonic processes with $\Delta S > 0$ are kaon and hyperon decays. The only $\Delta S \geq 2$ kaonic process is the interaction responsible for the K^o_S - K^o_L mass difference. It is presently thought that the mass difference is due to HOW which break the $\Delta S < 2$ rule. Unfortunately, the mass difference is only one very small number and it has not yet been calculated reliably. The search for other HOW amplitudes is likely to be best accomplished by looking for the decays of $|S| > 1$ hyperons into $S = 0$ final states. For example:

$$\Xi^- \rightarrow n\pi^- \quad (\Delta S = 2) \tag{19}$$

$$\Xi^o \rightarrow p\pi^- \quad (\Delta S = 2) \tag{20}$$

$$\Omega^- \rightarrow \pi^- n \quad (\Delta S = 3) \tag{21}$$

$$\rightarrow \pi^- \Lambda \quad (\Delta S = 2) \tag{22}$$

With the advent of high energy proton beams it becomes feasible to produce copious high energy hyperon beams. Process (20) is the easiest to detect because of the two charged particles in the final state and the characteristic Q value of the process relative to $\Lambda \rightarrow \pi^- p$ decay. There is an approved experiment at NAL which will likely be sensitive to this process.[49] It has been estimated that a branching ratio limit of $\sim 10^{-8}$ can be reached within a modest running time if the NAL machine runs at design intensity.[50] March estimates that a limit of $\sim 10^{-10}$ might eventually be achieved.[50]

Theoretical estimates of the possible HOW contribution to these processes seem to be nonexistent and would be appreciated.

b. CP Violation as 2nd Order Weak

In the Wolfenstein superweak theory of CP Violation, the violation occurs in the mass matrix with $\Delta S = 2$. It seems to us quite possible (but we know of no theoretical suggestions along this line) that the CP violation is a direct manisfestation of HOW processes.

5. High Energy Neutrino Scattering

Clearly the most likely place to observe departures from the expectations of conventional, lowest order weak theory is at large s, in neutrino scattering. It is fortunate indeed that under certain circumstances the hadronic systems in such collisions will likely behave as though they were massive, pointlike scattering centers. Thus we expect that very high momentum transfers can be achieved in early experiments at NAL and the CERN SPS.

As before we expect HOW process to lead to violations of certain selection rules in neutrino processes. In addition it may be possible to directly observe the nonlocality that HOW process may produce.

a. Electromagnetic Charge Radius of the Neutrino

The small distance behavior of weak interactions will be sensitively probed by observing the charge radius of the neutrino. The best guess for this radius leads to a cross section ratio of[46]

$$\frac{\sigma(\nu_\mu + N \to \nu_\mu + N)}{\sigma(\nu_\mu + N \to \mu^- + N)} \sim 10^{-5}$$

We would also expect by analogy that the contribution to deep inelastic ν_μ scattering would also behave the same way with

$$\frac{\sigma(\nu_\mu + N \to \nu_\mu + \text{all})}{\sigma(\nu_\mu + N \to \mu^- + \text{all})} \sim 10^{-5}$$

The process

$$\nu_\mu + N \to \nu_\mu + (\text{all}) \qquad (19)$$

could also arise from $\Delta S = 0$, $\Delta Q = 0$ first order semileptonic currents and from HOW induced neutral currents. Thus, we expect that the search for such induced currents will not be confused by EM processes (i.e. the ν_μ charge radius) unless the resulting cross section is only $\sim 10^{-5}$ of the charged current cross sections.

The measurement of the charge radius is in itself an interesting experiment. In order to separate the charge radius from the neutral currents the Z^2 behavior of the electromagnetic process would need to be observed.

b. Deep Inelastic 'Neutral' Currents

The SLAC experiments have given evidence that hadrons can 'act' point like if appropriate processes are studied (inclusive processes).[24] Using high energy neutrinos, and hitting these 'pseudo point like hadrons' allows very high momentum transfers in the lepton-lepton system. To the extent that the hadrons act point-like, the HOW divergent intergals may truly be cutoff by the weak interactions and not the hadronic size. It is thus possible that if the weak interactions cutoff is near $\sqrt{s_u}$ the HOW amplitudes may be relatively much larger than in the case of semileptonic decay processes. Thus, these processes may be almost 'lepton-lepton like'

Experimentally it would be necessary to study the processes

$$\nu_\mu + N \rightarrow \nu_\mu + (\text{all}) \tag{19}$$

and separate this from the large background of events

$$\nu_\mu + N \rightarrow \mu^- + (\text{all}) \tag{20}$$

In particular it would be necessary to prove that there is no μ^- in the final state. It is likely that this can be easily done in a Ne bubble chamber or the detector for E1A at NAL if the ratio of cross sections for these reactions is $10^{-2} - 10^{-3}$.[39] Going to smaller ratios would likely require a major change of the experimental setup for E1A or the use of the Ne bubble chamber with an External Muon Identifier to reject a larger fraction of events of type (20).

Primakoff has estimated the ratio of these cross section to be[19]

$$\frac{\sigma(\nu_\mu + N \rightarrow \nu_\mu + \text{all})}{\sigma(\nu_\mu + N \rightarrow \mu^- + \text{all})} = 3\xi^2$$

for the integrated cross section. This ratio would likely be larger if only large q^2 $(= (p_\nu - p_\mu)^2)$ events were used. For $\Lambda \sim \sqrt{s_u}$ we obtain a theoretical ratio of $\sim 10^{-3}$. Thus, if the weak interaction cutoff is at $\sqrt{s_u}$ and if the hadronic system in reaction (19) does not provide a cutoff of the divergent integral and if the cutoff procedure is valid, then the HOW induced process (19) will likely be observed at NAL.

c. Breakdown of Locality in Deep Inelastic Scattering

We now turn to a brief discussion of the possibility of direct locality tests in deep inelastic processes of the type[53]

$$\nu_\mu + N \rightarrow \mu^- + (\text{all}) \tag{20}$$

and thus the direct observation of the 'range' of weak interactions. We use the ordinary definitions of the variables for process 20

$$q^2 = 4E_\nu E_\mu \sin^2\theta_\mu/2$$

$$\nu = E_\nu - E_\mu$$

$$x = q^2/2\nu m_p;\ y = \nu/E_\nu$$

If scale invariance holds the differential cross section can be expressed entirely as a function of x and y. We assume that scale invarience holds and proceed to discuss locality tests (which test the locality at the lepton-lepton vertex if these assumptions are valid). We must distinguish two kinds of nonlocality in this regard.

(a) Type 1. In the (ν-μ) system an orbital angular momentum of > 0 is observed. Tests for this kind of nonlocality were pointed out long ago by Lee and Yang.[51] These tests take on a particular significance when high momentum transfer collisions are studied. The most general expression for the differential cross section for inelastic neutrino scattering, if locality holds, is of the form

$$\frac{d^2\sigma}{dxdy} = G(q^2,x)\ f(y;x,q^2)$$

with $f = \sum_{n=0} a_n y^n$ and $a_n = 0$ for $n > 2$.

(b) Type 2. This is the type of nonlocality that comes from a meson propagating from the leptonic vertex to the hadronic vertex. The mesonic propagator is then expected to modify the differential cross section for deep inelastic scattering. If scale invariance holds it would then be possible to write the differential cross section as a product of three functions (taking the diffraction model)

$$\frac{d^2\sigma}{dxdy} = \frac{G^2}{\pi} M E\ [\nu\beta]\ [1 - y + y^2]\ [f(q^2)]$$

where, in particular we take the meson mass to be the W mass,

$$f(q^2) = \frac{1}{(1 + q^2/m_W^2)^2}\ .$$

This might allow us to search well above the mass range covered by the direct production of W's by neutrinos. If scale invariance is badly broken it would be difficult to use deep inelastic scattering to probe this form of nonlocality.

In fig. 5 is shown graphically the type of measurements that would be used to test for a breaking of the two types of locality. We have assumed that the NAL machine only runs at 200 GeV for this graph. In one case (q^2,x) would be fixed and the behavior of the resulting cross section with y would be studied. If y^3 or higher powers of y are needed to explain the data, evidence for nonlocality of type 1 would be obtained. In the second case (x,y) would be fixed and the resulting q^2 behavior of the cross section will be studied.

In fig. 5 is also shown the possible sensitivity of this probe of locality. Present tests of type 1 locality have reached the level of $\sim 10^{-13}$cm (in K-decay) whereas the experiment proposed here offers the possibility of studying distances of the order of 10^{-15}cm. An increase of two orders of magnitude in the locality check would clearly be of great interest.

We now briefly turn to the question of event rates for the deep inelastic process. We use as an example the predicted rates for E1A.[33] This detector which is schematically illustrated in fig. 6 will have a target mass of $\sim$ 400-500 tons. This is to be compared with the large H_2 bubble chamber at NAL with a target mass of $\sim$ 1 ton and the Ne filled chamber with a mass of $\sim$ 20 tons.

In table 3 we present the expected rates/day for events where $q^2 > 200$ GeV/c^2, under a variety of assumptions concerning the incident neutrino beam for 500 GeV/c protons in the machine. Even in the most pessimistic case an adequate number of events can be obtained to carry out the the locality test described above. Thus it seems likely that a definitive statement can be made concerning the range of weak interactions down to $\sim 10^{-15}$cm. With good luck and a 1000 GeV NAL proton beam perhaps 10^{-16}cm could be reached.

6. Summary and Conclusions

The short ranged behavior of the weak interaction is not presently known. Within the framework of conventional theory a pointlike interaction leads to divergent integrals which must be cutoff. It is probably necessary to consider different cutoffs depending on the type of process being investigated. For example, the cutoffs might be arranged as Λ_{NL}, Λ_{SL}, Λ_L denoting the nonleptonic, semileptonic and leptonic processes, respectively. We suggest that a further subdivision of the semileptonic taking into account the quasi-point-like behavior of the hadrons in deep inelastic processes. We denote this cutoff as Λ_{SLDI} for semileptonic-deep inelastic. Possibly this cutoff is more directly related to the Λ_L whereas the Λ_{SL} is more directly related to Λ_{NL}. However, arguments based on the Bjorken technique would likely not differentiate these cutoffs.

Within this framework we can summarize the conclusions of this paper

1. The search for $\Delta Q = 0$ semileptonic decay processes limits $\Lambda_{SL} \lesssim 15$ GeV. Reducing this limit further will require the search for $\Delta Q = 0$ processes that have strongly suppressed electromagnetic corrections. Two processes were suggested where the electromagnetic correction is likely sufficiently small to allow a limit on Λ_{SL} of ~ 1 GeV. The search for these processes requires new high intensity K beams.
2. The search for $\Delta Q = 0$ leptonic processes, in principle, allow an upper limit to be set on Λ_L of $\sim$(100-300) GeV. The experimental detection of such processes will be very difficult.
3. The search for $\Delta Q = 0$ semileptonic-deep inelastic processes will

probably allow an upper limit of $\sim$ 100 GeV to be set on Λ_{SLDI}. The experiment looks feasible at NAL either using the Ne bubble chamber or the massive calorimeter-target detector.

4. A lower limit on Λ_{SLDI} can likely be set by observing the resulting nonlocality (type 2). We guess that $\Lambda_{SLDI} > 30$ GeV can be obtained at NAL with the large calorimeter-target dectectors.
5. The existence of a W° with mass less than 8 GeV and a $W^{\pm}$ with mass less than (11-15) GeV can be determined using $e^+e^- \rightarrow \mu^+\mu^-$ and neutrino production, respectively. First order neutral leptonic currents at high Q^2 might also be detected in $e^+e^- \rightarrow \mu^+\mu^-$.
6. A breakdown of locality of type 2 in the weak interaction might be detected at high Q^2 using deep inelastic neutrino scattering.
7. A crude limit can be set on Λ_{NL} by searching for $\Delta S \geq 2$ decays.

Thus within this conventional picture it would be possible to bracket Λ_{SLDI} by $\Lambda_{SLDI} < 100$ GeV and $\Lambda_{SLDI} > 30$ GeV. This is about the best we can hope for. If $\Lambda_{SLDI} \sim \Lambda_{SL}$ then the present limits on Λ_{SL} would lead to interesting-observable nonlocal effects in the neutrino experiments.

The most exciting possibility is of course that totally new phenomena dominate weak interactions at large s and Q^2. In this regard neutrino microscopy also offers the exciting possibility of probing nature in the new region of small distances.

We wish to thank Profs. J. D. Bjorken, A. K. Mann, C. Rubbia, and S. Treiman for helpful discussions. This is not to imply that these people share the same optimistic viewpoint as expressed in this paper.

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Table 1.

Present Information On The Weak Interaction Cut off

Λ	PROCESS	COMMENTS	AUTHORS
~ 2600 GeV	$\nu+\bar{\nu} \rightarrow W+\overline{W}$	Intermediate Boson theory	Gell Mann et al.[11]
~ 320 GeV	$\nu+e \rightarrow \nu+\mu$	Simple unitary limit	
< 100 GeV	$K^+ \rightarrow \pi^+ \ell\ell$	Cut off of divergent integral	Ioffe and Shabalin[15]
~ 30 GeV	$\nu+\ell \rightarrow \nu+\ell$	Crossing symmetry included in calculation	Applequist and Bjorken[7]
≤ 14 GeV	$K_L \rightarrow \mu\mu$	Cut off of divergent intergrals using Bjorken technique	Ioffe and others[17] (LRL Experiment)[29]
Λ ~ 8 GeV	$K_S \rightarrow \pi\pi$	Soft π and K techniques	Glashow, Schnitzer and Weinberg[31]
Λ ~ (4-8) GeV	$K_L - K_S$ Mass difference	Bjorken technique and cut off of divergent intergrals	Ioffe et al.[15,17] Mohapatra et al.[16]
$\Lambda \simeq$ Small	Rare Electro-magnetic decays of K mesons	Electromagnetic processes with virtual photon diverge quadratically	Geshkenbein and Ioffe[30]
$\Lambda \simeq$ Small	Nonleptonic decays	$f \sim G\Lambda^2 \sim 10^{-5}\text{-}10^{-6}$	
Λ > 2 GeV	W production	Assume $\Lambda \sim M_W$	CERN bubble chamber and counter experiments

Table 2

Selection Rule	Approximate limit on the Ratio of $\left(\frac{\text{Violating Amplitude}}{\text{Nonviolating Amplitude}}\right)$	Processes that Violate the Rule
$\Delta Q \neq 0$, leptonic	~ 1	$\nu_\mu + e^- \to \nu_\mu + e^-$
		$e^+e^- \to \nu_\mu \bar{\nu}_\mu$
		$e^+e^- \to \mu^+\mu^-$
$\Delta Q \neq 0$, semileptonic	$\sim 4 \times 10^{-5}$	$K^\circ_L \to \mu^+\mu^-$
$\Delta S \neq 0$	$\sim 10^{-3}$	$K^+ \to \pi^+ e^+ e^-$
	$\sim 3 \times 10^{-3}$	$K^+ \to \pi^+ \nu\bar{\nu}$
$\Delta S = 0$	$\sim 5 \times 10^{-1}$	$\nu_\mu + n \to \pi^- p\, \nu_\mu$
$\Delta S = \Delta Q$, semileptonic	$\sim 10^{-1} - 4 \times 10^{-2}$	$K^\circ \to \pi^+ e^- \nu$
	$\sim 10^{-1}$	$K^+ \to \pi^+\pi^+ e^- \nu$
	$\sim 10^{-1}$	$\Sigma^+ \to n e^+ \nu$
$\Delta S < 2$, semileptonic	~ 1	$\Xi^- \to n e^- \nu$
	~ 1	$\Omega^- \to n e^- \nu$
$\Delta S < 2$, nonleptonic	3×10^{-2}	$\Xi^- \to \pi^- n$
		$\Xi^\circ \to \pi^- p$
		$\Omega^- \to \pi^- n$
		$\Omega^- \to \pi^- \Lambda$

Table 3

Rate for Selected Deep Inelastic Scattering Events with $q^2 > 200 (GeV/2)^2$

(Based on the Parton Model)*

E_ν	Quad Focus H-R	No Focus H-R	Quad Focus CKP	No Focus CKP	Quad Focus H-R H_2 Target ($2\frac{1}{2}$ Tons)
135-145	12	5	2	1	.2
145-155	67	28	10	4	.9
155-165	125	53	16	7	1.6
165-175	172	77	17	8	2.2
175-185	238	103	19	8	3.1
185-195	280	118	18	8	3.7
195-205	308	132	16	7	4.0
205-215	300	128	13	6	3.9
215-225	280	120	12	5	3.6
225-235	280	120	11	5	3.6
235-245	235	104	10	4	3.1
245-255	200	81	8	3	2.6
Total Events/Day					32.5 H_2 Target Rate
(192 Ton Detector)	2497	1070	152	66	
(20 Ton Detector)	260	107	16	6.6	

*Folding in the correct detection efficiency may drop all of these rates by factors of at least 2.

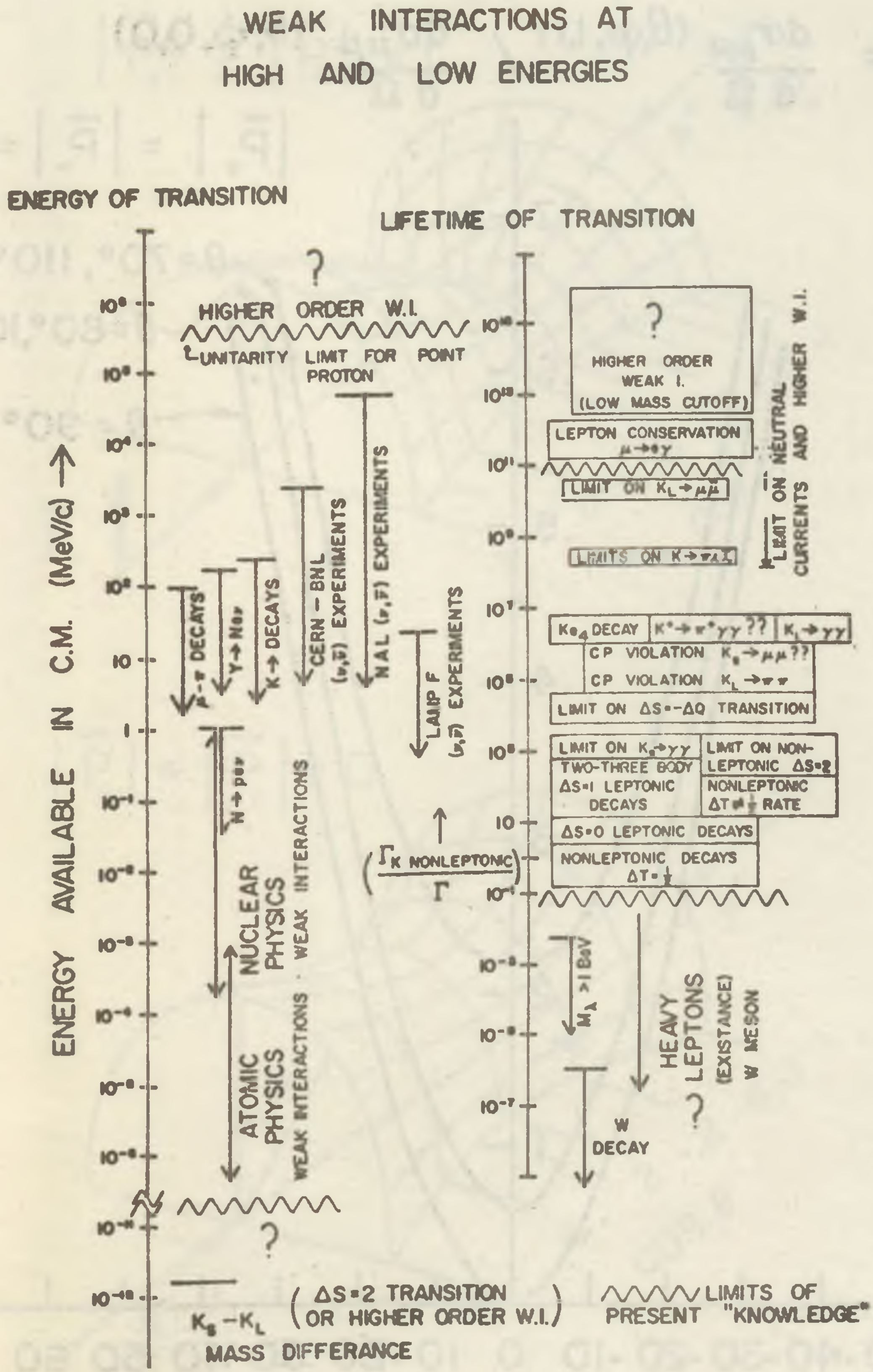
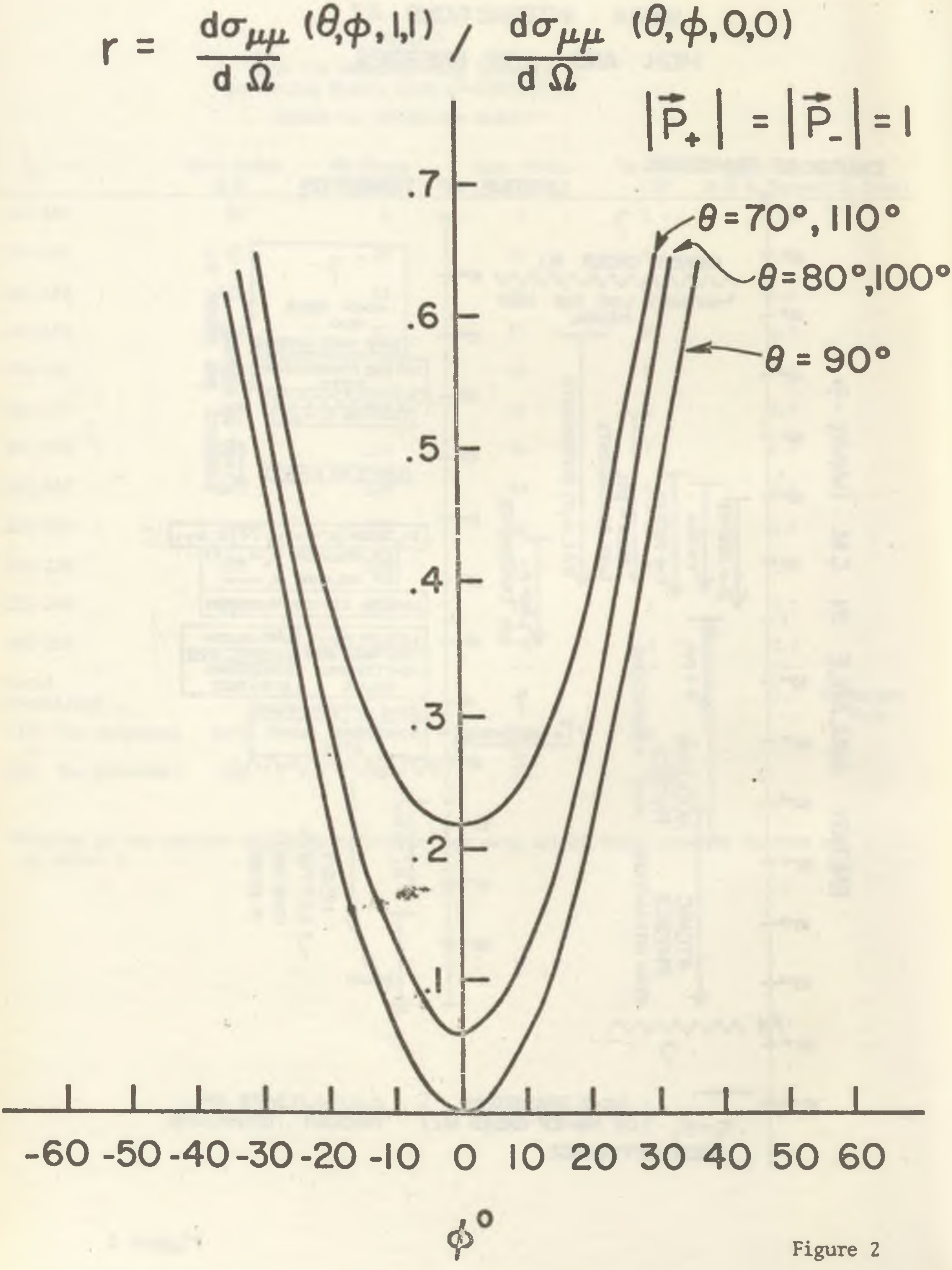
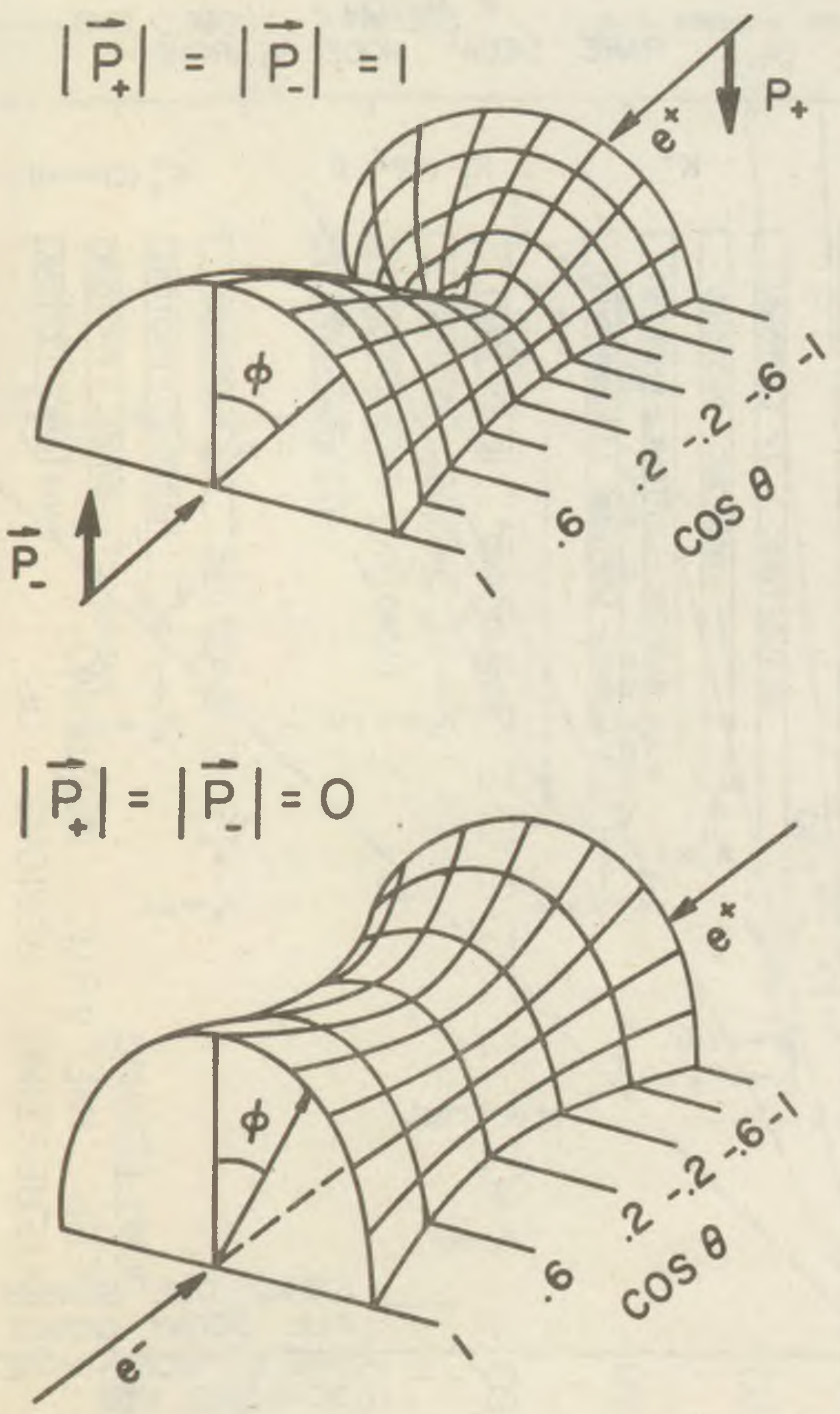

WEAK INTERACTIONS AT HIGH AND LOW ENERGIES
ENERGY OF TRANSITION
LIFETIME OF TRANSITION
ENERGY AVAILABLE IN C.M. (MeV/c)
HIGHER ORDER W.I.
UNITARITY LIMIT FOR POINT PROTON
CERN - BNL ($\nu,\bar{\nu}$) EXPERIMENTS
NAL ($\nu,\bar{\nu}$) EXPERIMENTS
LAMPF ($\nu,\bar{\nu}$) EXPERIMENTS
K → DECAYS
NUCLEAR PHYSICS · WEAK INTERACTIONS
ATOMIC PHYSICS WEAK INTERACTIONS
HIGHER ORDER WEAK I. (LOW MASS CUTOFF)
LEPTON CONSERVATION $\mu \to e\gamma$
LIMIT ON $K_L \to \mu\bar{\mu}$
LIMIT ON NEUTRAL CURRENTS AND HIGHER W.I.
K_{e4} DECAY
$K^+ \to \pi^+\gamma\gamma$??
$K_L \to \gamma\gamma$
CP VIOLATION $K_S \to \mu\mu$??
CP VIOLATION $K_L \to \pi\pi$
LIMIT ON ΔS = −ΔQ TRANSITION
LIMIT ON $K_S \to \gamma\gamma$
LIMIT ON NON-LEPTONIC ΔS = 2
TWO-THREE BODY ΔS = 1 LEPTONIC DECAYS
ΔS = 0 LEPTONIC DECAYS
NONLEPTONIC DECAYS ΔT = ½
$\left(\frac{\Gamma_{K\ NONLEPTONIC}}{\Gamma}\right)$
$M_\Lambda > 1$ BeV
HEAVY LEPTONS (EXISTANCE) W MESON
W DECAY
$K_S - K_L$ MASS DIFFERANCE
(ΔS = 2 TRANSITION OR HIGHER ORDER W.I.)
LIMITS OF PRESENT "KNOWLEDGE"

Figure 1



r = dσμμ/dΩ (θ,φ,1,1) / dσμμ/dΩ (θ,φ,0,0)
|P+| = |P-| = 1
θ=70°, 110°
θ=80°,100°
θ = 90°
.7
.6
.5
.4
.3
.2
.1
-60 -50 -40 -30 -20 -10 0 10 20 30 40 50 60
φ°

Figure 2



$|\vec{P}_+| = |\vec{P}_-| = 1$
e^+
P_+
ϕ
$\vec{P}_-$
1
.6
.2
-.2
-.6
-1
COS θ
$|\vec{P}_+| = |\vec{P}_-| = 0$
e^+
ϕ
e^-
1
.6
.2
-.2
-.6
-1
COS θ

Figure 3

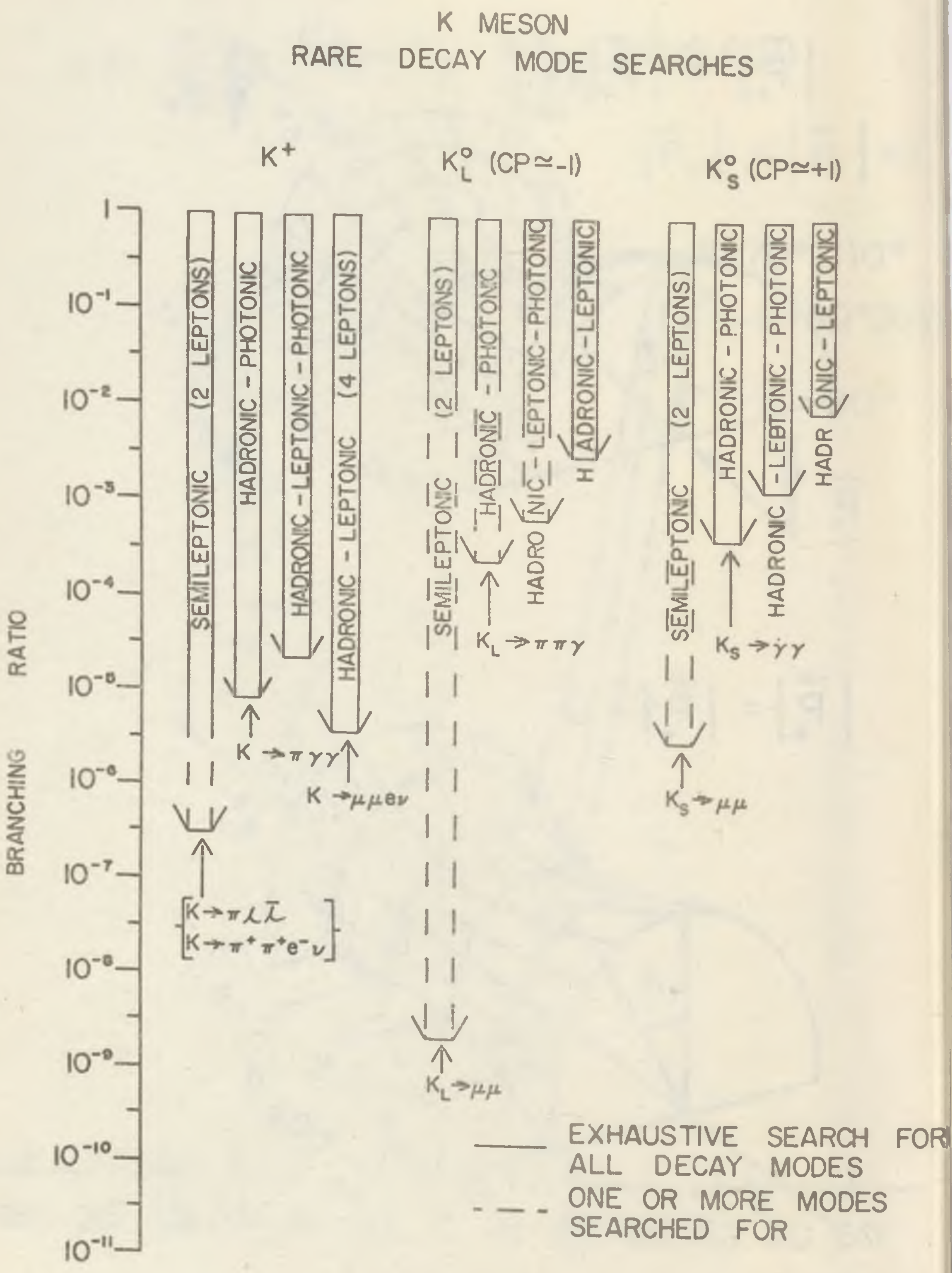

K MESON
RARE DECAY MODE SEARCHES
K+
K°L (CP≃-1)
K°S (CP≃+1)
BRANCHING RATIO
1
10⁻¹
10⁻²
10⁻³
10⁻⁴
10⁻⁵
10⁻⁶
10⁻⁷
10⁻⁸
10⁻⁹
10⁻¹⁰
10⁻¹¹
SEMILEPTONIC (2 LEPTONS)
HADRONIC - PHOTONIC
HADRONIC - LEPTONIC - PHOTONIC
HADRONIC - LEPTONIC (4 LEPTONS)
HADRONIC - LEPTONIC
K → π γ γ
K → μ μ e ν
K → π l l̄
K → π⁺ π⁺ e⁻ ν
K_L → π π γ
K_L → μ μ
K_S → γ γ
K_S → μ μ
HADRONIC - LEBTONIC - PHOTONIC
EXHAUSTIVE SEARCH FOR ALL DECAY MODES
ONE OR MORE MODES SEARCHED FOR

Figure 4

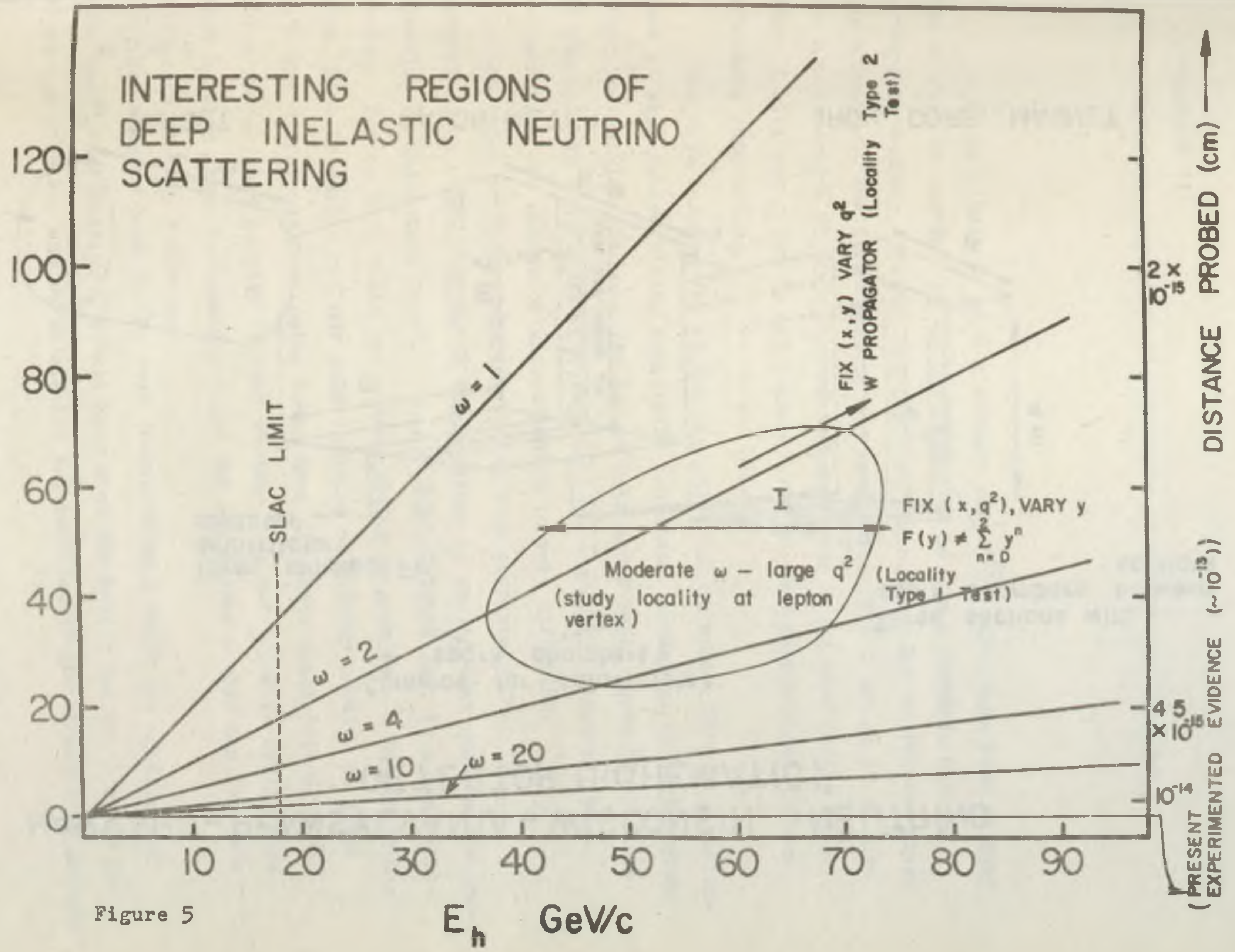

INTERESTING REGIONS OF DEEP INELASTIC NEUTRINO SCATTERING
q^2 (GeV/c)^2
120
100
80
60
40
20
0
10
20
30
40
50
60
70
80
90
E_h GeV/c
SLAC LIMIT
ω = 1
ω = 2
ω = 4
ω = 10
ω = 20
FIX (x,y) VARY q^2
W PROPAGATOR (Locality Type 2 Test)
I
FIX (x,q^2), VARY y
F(y) ≠ Σ_{n=0}^{2} y^n
Moderate ω – large q^2
(study locality at lepton vertex)
(Locality Type I Test)
DISTANCE PROBED (cm)
2 × 10^-15
4.5 × 10^-15
10^-14
(PRESENT EXPERIMENTED EVIDENCE (~10^-13))

Figure 5

HARVARD-PENNSYLVANIA-WISCONSIN NEUTRINO DETECTOR (SCHEMATIC)

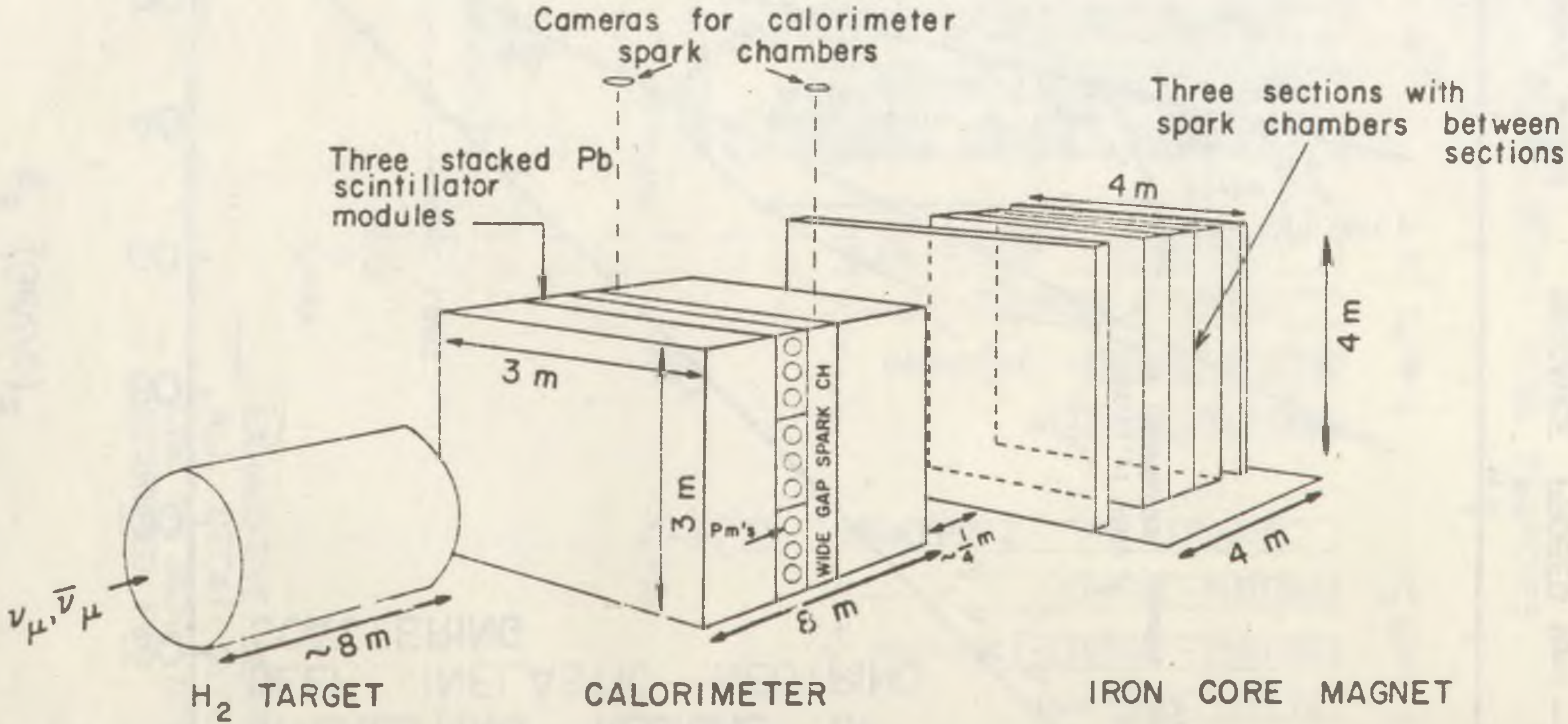


Figure 6

CONCLUSION II.

V.F.Weisskopf, Massachusetts Institute of Technology, Cambridge

It is difficult to give a summary since the talks have been so good and so full of content. So do not expect of me now a summary, even an incomplete summary, of what has happened since Bruno Pontecorvo has given his summary of the first part of the conference. All I can do is to give you a perspective of what kind of thoughts came to my mind during these days, I will not react to all papers, even to some papers which I found extremely interesting.

Now with this excuse at the beginning let me start. This is a conference on weak interactions, especially neutrinos. Actually, we talked about other interactions because you cannot talk about weak interactions alone. If you look at the famous four interactions - gravity, electric, weak, and strong - our degree of understanding is very different.
I do not think we understand any interaction completely, but let me normalize our understanding of interactions by saying that quantum electrodynamics we understand "one". This, I should rather say, is a renormalization. Now on this basis our understanding of gravity - I am not saying mine, mine is zero - but our understanding of gravity physics is about 0.7. The difference from unity comes from all these black-hole-problems which appear when gravity becomes strong. I do not think we really know what happens then. Fortunately - for the summarizer - this has not been discussed at this conference.

Now when we come to the weak interactions I think the coefficient goes down considerably. I do not know what the metric is, but I would put it at 0.2 at best, and we should put the coefficient for strong interactions at zero, should we not? So we do understand something about weak interactions. I gave it a finite fraction, at least.

Before showing the first transparecy I would like to make a general remark which is a request to physicists who give talks. Since about a year people always cover the transparent sheet by a piece of blank paper and proceed from the top, line by line. I find this procedure extremely awkward for the listeners or viewers or whatever you call them. The reason is this: Nobody is able to grasp everything when the speaker says it. You have to integrate over a time interval both in the past and also into the future. A transparency shows something that will be said later and, therefore, helps your understanding. The reason which people give for covering the sheets is that they would like to keep their surprises. If you really need a surprise then put it on the next sheet. I am not sure whether other people feel the same way but I would like to make this request very strongly.

Our understanding of the weak interactions is rather old, in fact it is forty years since Fermi published his famous paper. Something has happened since then but in a way not much.

What has happened since Fermi? There was the Sudarshan-Marshak idea of V-A the Lee-Yang P-violation, and then the idea and verification of the two neutrinos by Pontecorvo, Schwartz, Steinberger and Lederman. And there was Landau, of course, with the formulation $(1-\gamma_5)$ of the maximum P-violation. But in principle it is the same thing since forty years, namely it is a weak interaction, so we are permitted to make use only of the first approximation, and we consider the leptons as pointlike. This is also true if one introduces a W-meson. The leptons still act as pointlike particles at the vertices and higher approximations are supposed to be neglected. This is the story since forty years. Now we come to the troubles. There are four types of themes:

1. Unitarity $<$ 300 GeV
2. Second order approximation $\rightarrow$ cut-off necessary $\sim$10 GeV
"bad processes" and "nonrenormalizable" quantities:

$K \rightarrow \mu^+ + \mu^-$

$K_L - K_S$ mass difference

3. $K_L \rightarrow 2\mu$ trouble
4. CP-violation

The first trouble comes from unitarity: at 300 GeV and higher the theory breaks down, by giving cross sections which are larger than the unitarity limit.

The second trouble - a very grave trouble, I think - is usually called the nonrenormalizable structure of the theory. This is a very bad word, Nature must be renormalizable; if not it would not exist . But what one means by the word "nonrenormalizable" is that if one calculates certain effects in second order one gets divergences and one has to cut off. That again is not necessarily something very bad if the cut-off is at very high energy, or at some energy where you have definite reason to believe that something new and different happens. But if the cut-off is already at 10 GeV or less, then one is in trouble. It means that there is something there which one does not understand. Here are two processes, (there are more) where this cut-off is important. One is the mass-difference between the two uncharged K's , for which one must cut off the theory at about 4 GeV in order to keep it as small as observed. The other is the $K \rightarrow \mu^+ + \mu^-$ decay, which appears in second approximation. In order to keep it as a small as the observed upper limit, there must be a cut off at less than 10 GeV. (This trouble has nothing to do with trouble N° 3; the second approximation has a real amplitude and cannot interfere with the predicted imaginary one. See Oakes talk.)

The third trouble is the famous $K_L \rightarrow 2\mu$ process where we should have found an effect on the basis of the observed $K_L \rightarrow 2\gamma$ process, and did not find an effect. We have heard a talk by Oakes about it; If the experiment is correct, we may be in trouble and have to look for direct interactions. Marshak has proposed a theory which does that job.

I do not know whether you call the fourth a trouble or not. I find it disappointing that all this beautiful symmetry that Landau, Pontecorvo, and Lee-Yang have introduced is actually not true; there is a difference between world and antiworld - very disagreeable and unsymmetric. We do not know at this point whether the CP-violation is really a disease of our theory of weak interactions or simply a new superweak interaction of some form.

I believe that one aspect has been neglected in the dicussion of weak interactions at this conference. It is the developments of Weinberg and Salam and other people who have looked at the problem in a somewhat different way than most of the discussions during this conference. An important exception was the work which Dr.Filippov has presented here; it is a most interesting method to avoid the troubles mentioned under 2; these are the troubles which are the object of the Weinberg-Salam attempts.

We have discussed Marshak's "extension", of the weak interaction theory, which is based on problem four; He takes the CP-violation very seriously, but ignores the troubles of N° 2. Weinberg's procedure forgets CP-violation - he keeps it in the dark - and bases his ideas on attacking problem two; So therefore I thought it may be a good idea if I say a few words about Weinberg's theory. It will be a very vague "al fresco" description of the way I see it, which may or may not be identical with Weinberg.

Weinberg wants to unify quantum electrodynamics and weak interactions. The fact that they have something to do with each other was already indicated since Feynman-Gell-Mann's CVC - both fields have the same sources, partially - so they may be related and the idea is to exploit this possibility. But there is a difference; it is the difficulty with "problem two", the nonrenormalizable structure in weak interactions, in contrast to the renormalizable electromagnetic interactions. I repeat, the word "renormalizable" is used to indicate that you are able to calculate higher approximations and do not get into trouble in higher orders. If you want to formulate the weak interactions in analogy with quantum electrodynamics you must introduce an intermediate boson in analogy to the light quantum. The intermediate boson, however must be a massive and charged vector field. We know that such a thing gets into trouble in higher approximation, it is not renormalizable. It is only renormalizable if you put the masses of these vector fields zero. So that is the idea of Weinberg.

Let us start with a theory, with Lagrangian, in which the masses of all particles are zero; then everything is renormalizable. We know that charged vector fields with zero mass are gauge invariant and renormalizable (Yang-Mills). One then removes the fact that the masses are zero by introducing something new, namely a symmetry breaking field. It is a scalar isospinor field φ, one component of which has a non-zero vacuum expectation value. The field φ is supposed to break gauge symmetry and therefore introduces masses for the particles. But let us leave that aside for a moment. Now how many fields do I need to describe quantum electrodynamics and weak interactions? We leave out hadrons from the discussion. Weinberg has an isovector vectorfield A which is coupled to the leptons with a coupling constant g and an isoscalar vector field B which is coupled with g':

isovector	A_1	A_2	A_3	isoscalar B
coupling constant		g		g'

Later on, when the symmetry is broken, these fields are broken into two groups: A_1 and A_2 the charged ones are going to become the two charged W, A_3 and B will undergo linear combinations to produce another uncharged vector boson Z, and the photon field A :

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(A_\mu^1 \pm i A_\mu^2\right),$$

$$Z_\mu = \frac{1}{\sqrt{g^2+g'^2}}\left(g A_\mu^3 + g' B_\mu\right),$$

$$A_\mu^{EM} = \frac{1}{\sqrt{g^2+g'^2}}\left(-g' A_\mu^3 + g B_\mu\right).$$

The symmetry breaking thing, as I said, is an isospinor scalar field which has the property that one of its components - at least that is one way of doing it - has a vacuum expectation value different from zero

$$\langle \varphi^0 \rangle_{vac} = \lambda,$$

that makes the masses nonvanishing

$$M_{W^\pm} = \frac{1}{2}\lambda g, \qquad M_Z = \frac{1}{2}\lambda (g^2+g'^2)^{\frac{1}{2}}$$

but leaves the mass of the A^{EM}-field zero: $M_{A^{EM}} = 0$. Things are arranged that way that one of the fields, A^{EM}, remains mass-less.

In order to do all this in a systematic way you will also have to ascribe to the leptons - electron, neutrinos, and muon - a certain isospin, and that can be done in different ways. Weinberg introduced an isospinor of the left-handed neutrino and left-handed electron, and an isoscalar, the right-handed electron

$$\begin{pmatrix} \nu_e^L \\ e^L \end{pmatrix} \qquad\qquad e^R$$

isospinor — isoscalar

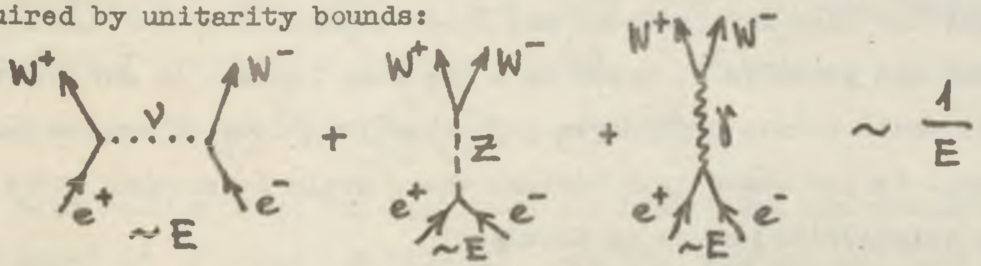
The same is done for the muon and muon neutrino. We finally get relations between the g and g' and the charge e and the Fermi constant G :

$$e = \frac{gg'}{\sqrt{g^2+g'^2}} , \qquad \frac{G}{\sqrt{2}} = \frac{g^2}{8M_W^2} .$$

It is seen that there is a smallest possible value for M which turns out to be 37 GeV. The natural choice $g = g'$ gives a $\sqrt{2}$ times larger value.

In addition this theory contains neutral lepton currents, a feature which, in principle, can be tested.

This procedure is a little arbitrary and perhaps not very elegant. It can be done in different ways, about which I do not want to say too much here. There is, however, a particularly pleasing feature of the theory which I would like to demonstrate on the process $e^+ + e^- \to W + \bar{W}$. Because the W has an "anomalous" magnetic moment, with gyromagnetic ratio $g_W = 2$ the electromagnetic pair-production amplitude grows like E as $E \to \infty$. Further, the neutrino t-channel and Z s-channel diagrams do not cancel here, so that the weak pair-production amplitude also grows like E . However, the weak and electromagnetic amplitudes cancel each other as $E \to \infty$, leaving a scattering amplitude which vanishes like 1/E as $E \to \infty$, as required by unitarity bounds:



This cooperation between the weak and electromagnetic interactions in solving each other's problems is one of the most satisfying features of this theory. Of course the whole approach disregards the CP-violation completely. It assumes that there is something besides, a small unknown superweak interaction. That is one of the weakness of the approach.

I have spent some time discussing Weinberg's theory because it has not been systematically presented here. Marshak's theory has been presented systematically and ably by its author and therefore I do not want to say much. Marshak's approach is complementary in a way to Weinberg's, namely he disregards - leaves in the dark - the cut-off problem just as Weinberg does the CP-problem. Marshak emphasizes the CP-violation. He considers it as something fundamental and basic to the weak interaction. In order to avoid strong CP-violating processes he assumes that there is a law of Nature that allows his W's only to appear in triplets - the triality quantum number. This will probably displease some people. On the other hand Nature seems to love the number three as we know from quarks, so maybe there is something true in it. His argument against just disregarding CP as the Weinberg approach does, is that after all, the CP-violation is a strong violation in weak terms; namely it is of the order of g_W^3 (the small g_W is the semiweak coupling constant), whereas the cut-off difficulties appear in the order of g_W^4 which is further away. Well, that is a possible philosophy.

Both approaches are very interesting but have certainly not yet raised the coefficient of understanding of the weak interaction by much, although I am personally quite impressed by the Weinberg kind of approach, namely to get the weak interactions and electrodynamics in one unified form. If it is true and possible it would be a big step forward in our under - standing: It would reduce the forces from four to three. Of course Marshak would say that is not quite true because there would be another force (superweak interaction) so it is four again.

The rest of what I want to say is not directly connected with weak and electromagnetic interactions, but with the fact that weak and electromagnetic interactions are useful tools to investigate hadrons. There were talks about strong interactions in spite of the title of the conference, for example the talks by Lee and Feynman.

Now my impression (this is what I want to give here only), when listening to the present situation in strong interactions is that the strong interactions are strong but not "superstrong." I mean that the strong interactions in a hadron produce momentum exchanges of the order of 1 GeV/c or less but not more. The only direct proof of this - I am not even sure whether it is a proof, but the only direct indication of this - - is the famous result that the perpendicular momenta in very high energy collisions so far observed are always smaller then one GeV. From this "smallness" we can conclude, that"scaling" exists. The word "conclude" is an exaggerated expression but from the small perpendicular momenta we may hope to understand why things scale. It means if q^2 and the energy s are all larger than one GeV, then the effects of strong interactions are not important and can be neglected along with the masses. Cross-sections should depend on dimensionless quantities which do not contain masses or coupling constants. Therefore you get the famous scaling laws which Lee has so ably demonstrated, in particular in electron and neutrino induced processes. We get neutrino cross sections which are essentially functions only of the scaling variable $x=q^2/2P.q$

$$d\sigma^{\nu} \sim G^2 f(q^2/2p.q)$$

The elctroproduction cross section apart from a trivial factor contains essentially two scaling functions $f_1(x)$ and $f_2(x)$

$$d\sigma^{e} \sim \frac{\alpha^2}{q^4}(f_1(x), f_2(x)).$$

All that follows from the fact that the masses and the interactions can be neglected if q^2 and $s=(p+q)^2$ are big. Now can we? Here we must look at two approaches, namely the Lee talk and the Feynman talk.

They look at the same problem by putting one aspect in the shadow and the flashlight on another aspect. For example, Lee puts the flashlight on field-theoretic consideration and Feynman puts it on other things (partons) . Now let me talk a little about Lee.

If one puts the flashlight on field-theoretical consideration one would say, scaling is suspicious because we know after all from electrodynamics that, even if there are not very strong interactions, the scaling law is really not true because of the famous logarithmic term $\sim \alpha \log \frac{s}{m^2}$ in higher approximations. When such a logarithmic term appears scaling is broken because you cannot put m=0. Now, surely logarithms may not be important, in particular in those processes which are induced by electrons and neutrinos, when they are multiplied by a small number, the coupling constant α. But if there is a strong interaction field theory of the ordinary type then such terms will not be so small because even the one GeV-interaction is a relatively strong interaction. So be aware of those terms, says Lee, scaling cannot really be there forever, for infinite q^2. He says that scaling can only be valid in the interval $1 GeV^2 \leq s \leq \log \frac{1}{g} GeV^2$ when the logarithms are not important whatever the coupling constants g is. Well, maybe. Now, Feynman says don't bother with field theory, we know so little about it. So, let us apply simple concepts. We have no superstrong interactions as concluded from the famous perpendicular momentum distribution and therefore at very high energies we can consider the hadron essentially as an assembly of free partons. We can use the impulse approximation which is just that. Of course, we have heard of quarks and the analysis of the spectra of hadrons seems to indicate that quarks are there. Who can resist the temptation to say that the hadrons consist of quarks?

Let me say here a word about this kind of looking at a hadron that moves fast. One must be very careful. If I have a hadron at rest and I look at its spectroscopy then we find the usual quark SU(3) or SU(6) spectrum for baryons with three quark constituents. Now let us look at this thing moving very fast. Then one should not expect that this fast moving proton should consist only of those three quarks, in the impulse approximation.
The three quarks of the spectroscopic considerations are "dressed" quarks. After all the quarks have an interaction, even 1 GeV is not so weak, therefore they are surrounded by $q\bar{q}$-pairs or maybe by gluons. When we speak

about a system of three particles, exhibiting certain spectra, we speak just like we do in the case of a nucleus of dressed nucleons. If things move fast and if q^2 is large compared to $1GeV^2$ and the strength of the clothing is only one GeV, then you would expect that you really look at undressed quarks. That means we look not only at the three quarks but also at the material (quarks and gluons) which made up the dressing. This is why one should not take the three-quark-picture too literally if one looks at fast moving hadrons.

We speak about quarks and try to understand Nature in terms of quarks. Let us not forget that there are grave difficulties with quarks, not only the difficulties which were much discussed here. If the quarks are ordinary particles bound by a 1 GeV interaction, than they should have been observed; furthermore they appear to have the wrong statistics. We believe they have spin 1/2. That is one of the nicest results of the SLAC-MIT experiment. On the other hand, from the spectroscopic observations they seem to have Bose statistics. That is terrible. Nobody knows what it means, so let us keep that in mind. I do not know what is worse of two things: that quarks do not exist or that they have Bose statistics. I am inclined to feel that the second is worse.

Feynman expressed very nicely in his talk the fact that they do not exist as free particles, when he said that an isolated quark is really a state in the continuum of the spectrum of hadrons. Well, maybe Nature is made in such a way that, if quarks are close together, then the quark state is well defined, has a well defined spin, fractional charge, and so on; but if they are far apart then they are just not eigenstates of this strange Hamiltonian that Nature has given us. The wrong statistics worries me more, maybe without reason, but it does.

Feynman remarked that the quantum numbers of the quarks are actually observable, in some sense. Consider a quark, which is ejected by a neutrino or an electron. It then exists in space - he says - as a state of the continuum, a "trail of hadrons". That state of the continuum has an average charge, has an average spin, has an average isospin and it is possible to measure this in principle, at least. It is difficult, of course, because in a high-energy process an ejected quark, say, with a charge 2/3 makes a lot of trail. It has a comet trail of hadrons behind it and you have to observe all of them to find out that the charge on the average is 2/3 and not unity. It may be difficult to measure this. The main point here is conceptual: that you can really speak of a quark and its quantum numbers even if the quark cannot exist in free space.

We do not know whether the quarks exist, but we have new handles to find out about the quarks. I do not want to go into all details because Feynman's talk was clear enough. But still, I would like to point out a few of these handles which can be used to check the fractional charges of the quarks and other qualities. Some are inequalities, and some are equalities.

The first one is the ratio of the famous structure functions of the neutron and the proton near x = 1

$$\frac{1}{4} \leq \frac{W_2^N}{W_2^P} \leq 4 .$$

If that ratio turns out to be less than 1/4 or more than 4, then the quark picture is wrong. We have seen the experiment; it seems that it is just about 1/4, so we do not need to worry so far, but one never can tell what the experiments will give at the end. So this is one of the critical results for the idea of a fractionally charged quark.

Another most important relation is the Llewellyn Smith sum rule; it has an equality sign:

$$\int_0^1 \left(f_3^{\nu p}(x) + f_3^{\bar{\nu} p}(x) \right) dx = -6$$

$f_3(x)$ is the third neutrino structure function in its scaling form.

As Feynman has shown us, these structure functions are combinations of probabilities of spin or isospin up or down, of the three different type of quarks. Therefore whenever you have a relation between those distribution functions then you can usually catch a critical statement about charge or spin of the quarks. You can get relations between distribution functions which are connected with the properties of these partons (quarks) and the Llewellyn Smith sum rule is one of those. If the quarks do not have fractional charges of 1/3 and 2/3, the right hand side would not be -6. This sum rule is very interesting and maybe we will be able to check it.

The other relation contains the neutrino cross sections. Until we get into the energy region where the W-boson plays a role, the neutrino cross sections are linear in E, and they are usually expressed in this way:

$$\sigma^{\nu} = G' \cdot Z \cdot E$$

G' is some kind of constant that contains G, and Z is a number which is of the order of unity. Well, Feynman gave us the precise definiton. Now Z and $\bar{Z}$ ($\bar{Z}$ is the corresponding parameter for the antineutrino cross section) can be limited at least in two ways. The quark theory requires that

$$Z + \bar{Z} \leq \frac{3}{4}$$

and as Feynman pointed out the sum $Z + \bar{Z}$ should not deviate from 3/4 by more than ten percent. Now that is a pretty strong requirement and so far it is all right. The other, less stringent condition, is valid for the ratio $\bar{Z} / Z$ as Feynman has told us

$$\frac{1}{3} \leq \frac{\bar{Z}}{Z} \leq 1$$

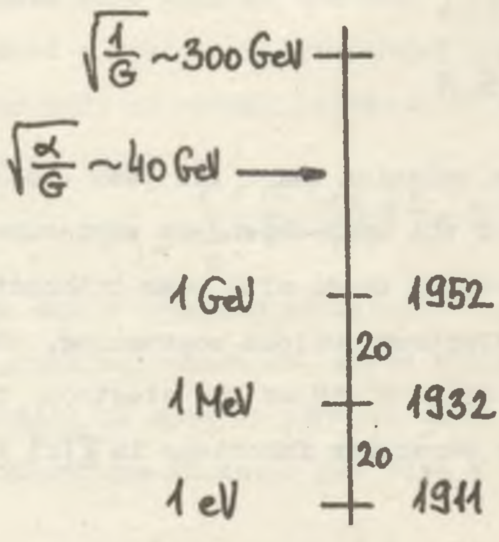
There is another very fine relation which has been discussed by Julius Kuti. It is a relation for the spin-dependent structure functions $\gamma^{p}(x)$ and $\gamma^{n}(x)$. They have nothing to do with weak interactions; they have to do with deep-inelastic electron-nucleon scattering, when you look at the polarization both of the nucleon and of the electron. Then you have more functions, and one of those structure functions is $\gamma(x)$ in its scaling form which was precisely defined by Kuti. The integral over the difference between proton and neutron has the wonderful property of being equal to $\frac{1}{3}|G_A/G_V|$

$$\int_0^1 \left(\gamma^p(x) - \gamma^n(x)\right) dx = \frac{1}{3}\left|G_A/G_V\right|$$

This relation is quark-dependent. Of course, one has ascribed to a single quark the property that G_A is equal to one. The actual G_A comes from the specific quark structure of the proton. Whatever that structure is, this relation must hold. This sum rule is going to be a very good test with an equality sign.

I am not sure whether my list is complete; there are probably more relations, but I mentioned the most important ones. So the quark picture still lives, but let me assure you that even if this all comes out right we do not really know anything about strong interactions. The whole theory in some way is crazy. What are the forces? What are those gluons? Is there a new interaction? Nevertheless, here is a new way of looking at Nature, and this is already some progress.

Now let me at the end refer to the so-called quantum ladder. T.D.Lee has made such a diagram:



The 1 eV-region here is the region of atomic and molecular physics, the physics in which we live. Some people say it is the most interesting physics and I cannot quite disprove them. Biology is there after all, and everything else in Nature around us. Nuclear physics, nuclear phenomena are associated with the MeV-region; the next step is the hadron spectroscopy in the GeV-region, the resonance states which have been discovered in recent decades.

And then what do we expect? There is the famous energy $(1/G)^{\frac{1}{2}}$, which is above 300 GeV where the weak interaction theory will be completely different. Only after having experimented in that region, will we know whether Weinberg or somebody else is right. But there may be something in between. In fact the Weinberg theory itself does point directly towards the energy $\sqrt{\frac{\alpha}{G}}$ which is of the order of 40 GeV. In Weinberg's theory this would be the mass of the intermediate boson. Nobody knows whether it exists, but obviously it must be found in that region whether it is Weinberg's theory or not. So here we may get something new from experiments in an energy region which will be reached in this decade.

Let me, at the end, tell you an interesting historic observation. You know when all this started. It began in 1911 when Rutherford (I always look at experiments) made his famous experiment. The next step, nuclear physics, started in the year 1932 which was a wonderful year. Chadwick discovered the neutron, Anderson and Neddermeyer discovered the antielectron, and Fermi wrote his weak interaction paper. It was the year when strong interactions and weak interactions were recognized, and also quantum electrodynamics got a big lift because of the antielectron. This was in 1932, 20 years after Rutherford. In the subsequent years nuclear physics has developed and in 1952 again something new happened; the third step began: Fermi discovered the first excited hadron level, the Δ-resonance. So it is again 20 years, right? Now I do not need to say more. The year 1972 is the beginning of a new period in physics.

Thank you.

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