CONDITIONALS, QUANTIFICATION AND BIPARTITE MEANINGS

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0. Introduction

In this paper I will present a non-conventional approach to conditional and quantification sentences. The conventional dynamic interpretation of conditionals can be paraphrased as follows:

(1) a. If \( \varphi \), then \( \psi \).
   b. \( \forall w \in W(\varphi(w) \rightarrow \psi(w)) \)

The formula in (1b) is a so-called test: it does not have any dynamic effect in the sense that it does not foreground any entity that can be referred back to by anaphoric expressions in subsequent discourse. This, however, is incorrect:

(2) If a client turns up, offer him a cup of coffee. Show him around the premises.

The second sentence in (2) is to be interpreted in the same possible worlds in which the consequent in the first (conditional) sentence (namely, the possible worlds in which the antecedent is true); the anaphoric pronoun him in the second sentence in (2) refers back to the eventual client in the first sentence. (This phenomenon has been called modal subordination since Roberts (1987).) Under the traditional view formulated in (1b), this should be excluded: neither the possible worlds in which the antecedent is true nor a client may be foregrounded by the first sentence if it is indeed translated as a test.

According to the solution to be proposed in this paper, conditional sentences are interpreted as the dynamic conjunction of a formula of assignment and the consequent. Informally:

(1') a. If \( \varphi \), then \( \psi \).
   b. \( (W := \{ w \in W : \varphi(w) \}) \land \forall w \in W(\psi(w)) \).

The formula in (1'b) is not a test: it has the dynamic (foregrounding) effects of both of its conjuncts. In particular, the set \( W \) of possible worlds in which \( \varphi \) holds is foregrounded as a consequence of the first conjunct (the so-called assignment). This can be taken up by subsequent anaphoric expressions. (In fact, it is picked up by the consequent itself, which mentions \( W \); note that then in the consequent is itself an anaphoric pronoun.) This predicts that the interpretation of subsequent sentences may be relativized to the possible worlds in which \( \varphi \) is true, as is the case in (2). The first conjunct stores all the \( \varphi \)-worlds to \( W \), and the second asserts \( \psi \) about each of those worlds. Moreover, it seems that the formula embedded in the assignment (i.e., \( \varphi \)) may also have dynamic effects (namely, foregrounding the client in (2)).

Note that the dynamic effects of a formulae embedded in a conditional must be restricted to the possible worlds singled out by the antecedent. For example,
in order for the client in (2) to be an available antecedent for a subsequent anaphor, the sentence in which the anaphor occurs has to take up the possible worlds foregrounded by the antecedent of the conditional (as is the case in (2)). The latter phenomenon will not be explained in this paper, because the logic that I will introduce is far too simple to deal with modalities. What would be required for its treatment is a concept of information states which store information about several possible worlds at the same time (see section 3.1).

In conventional dynamic theories, quantificational sentences are effectively treated as conditionals:

\[(3) \quad \text{a. } \text{Every } P \text{ Q.} \]
\[\text{b. } \forall x (P(x) \rightarrow Q(x))\]

So quantificational sentences are also translated as tests. This predicts that a quantificational sentence cannot foreground any entity for further reference. This is incorrect:

\[(4) \quad \text{Every client left. They didn't buy a single piece of furniture.}\]

The anaphoric pronoun they in the second sentence of (4) refers back to the clients mentioned in the first sentence, which should be impossible if we were to translate the first sentence as a test. On the other hand, the translation that I will propose in this paper can be informally characterized as follows:

\[(3') \quad \text{a. Every } P \text{ Q.} \]
\[\text{b. } (X := \{x \in U : P(x)\}) \land \forall x \in X(Q(x)).\]

This schematic translation is analogous to the translation schema for conditional sentences in (1'), so the parallelism is preserved under this approach. However, the intended dynamic interpretation of the formula in (3'b) has the effect of foregrounding the set \(X\) of \(P\)-entities, which yields the prediction that that set will be foregrounded, and subsequent anaphors may refer back to it, as is the case in (4).

As can be seen in the translations proposed in (1'b) and (3'b), we will need a device that allows us to model set-type entities' (discourse referents') ability to be foregrounded in discourse. To that effect, in section 1, I will develop a pluralized version of Dynamic Predicate Logic (DPL, Groenendijk and Stokhof (1991)), different from (and simpler than) van den Berg's (1990). Pluralized DPL allows variables to be assigned sets of individuals as values, which makes it possible to formulate translations similar to those in (1'b) and (3'b) with their intended interpretation.

After introducing the necessary formal tools and formulating the proposed translations, I will further develop the treatment to deal with presuppositions in section 2. This means that I have to develop a partialized version of the logic proposed so that the translations of sentences may be assigned undefined semantic
values if their presuppositions fail to hold. The reason for this is that the most problematic properties of conditional and quantificational sentences are their presuppositional properties. Accordingly, I have to examine the ‘projection’ (inheritance) properties of semantic value gaps and see how they correspond to existing views on the ‘projection problem’ for presupposition in general, and the projection of presuppositions in conditional and quantificational sentences, in particular. I will show that the predicted inheritance phenomena are essentially correct: they harmonize with Karttunen and Peters’ (1979) and Heim’s (1983) observations, but they fare better than Heim’s (1983) treatment for quantificational sentences.

In section 3 I will briefly summarize the residual problems touched upon in earlier sections which I believe are independent of the main topic of this paper.

Finally, section 4 is only tangentially related to the topic of conditional and quantificational sentences. It is about the methodological assumptions that underly the proposed treatment. Kálmán and Szabó (1990) argued on independent grounds that utterances have a bipartite semantic structure, separating the pieces of meaning which determine how certain referents are to be grounded in the previous context from what the sentence claims. I will look at some consequences of that view, including the treatment of quantificational structures.

I will conclude that the semantic structure of utterances is partly independent of both the underlying logic and the syntactic structure, which means that not all semantic properties of natural-language sentences need to be predictable from the logical or syntactic properties of the building blocks from which their semantic representations are constructed. This implies a sort of autonomy of semantics. For example, using the formulae of a pluralized and partialized dynamic predicate logic as the basic elements of our bipartite meaning representations does not mean that that logic must account for every relevant semantic phenomenon. This challenges some syntactically oriented definitions of the principle of compositionality.

1. Formal Tools: Pluralizing DPL

As I have said in the Introduction, we need to extend the language of Dynamic Predicate Logic (DPL, Groenendijk and Stokhof (1991)) to cope with plural anaphors. My pluralization of DPL is similar to van den Berg’s (1990), with technical differences that I will not go into. Instead of collecting assignment functions, I will rely on assignment functions that assign sets of individuals rather than individuals to variables. In section 1.1, I introduce DPL, then in section 1.2 I develop the pluralized version. Section 3 summarizes the most basic consequences of the translations proposed.
1. Formal Tools: Pluralizing DPL

1.1. DPL

The syntax of DPL is that of first-order predicate logic with identity. (Here and in the following, the syntactic clauses are implicit in the definition of the semantics.) I define the semantic-value function \([-\cdot]\)^\mathcal{M}. The model \(\mathcal{M} = (\mathcal{U}, \mathcal{I})\) consists of the non-empty universe of individuals \(\mathcal{U}\) and the interpretation function \(\mathcal{I}\). The interpretation function assigns each n-ary predicate constant \(P\) a set of n-tuples over the universe \(\mathcal{U}: \mathcal{I}(P) \in \mathcal{P}(\mathcal{U}^n)\). The value of \(\mathcal{I}(P)\) is also called \(P\)'s extension in the model \(\mathcal{M}\), and will be written as \(P^+\). The semantic-value function will be written as \([-\cdot]\) (without the superscript) for the sake of simplicity. In this logic, a context or information state is characterized with a set of assignment functions, i.e., it expresses potential values of variables. This means that contexts are represented as partial knowledge about certain individuals (identified by variables or discourse markers) that have been foregrounded ('introduced') in earlier discourse. The semantic value of a formula is a function from contexts to contexts: it expresses how the interpretation of a sentence may update an information state. The original formulation of DPL uses a slightly different terminology, but is effectively equivalent to what follows here.

\[ ([\,]) \, [P(x_1,\ldots,x_n)](G) = \{ g \in G : (g(x_1),\ldots,g(x_n)) \in P^+ \}. \]

Applying the n-ary predicate \(P\) to \(n\) arguments \((x_1,\ldots,x_n)\) are variables) is a so-called test. This means that its value is a subset of its argument: it simply selects some assignments from it. The assignments that it selects from \(G\) are the ones which assign appropriate values to those variables (the n-tuple that they constitute must be in \(P\)'s extension).

\[ ([\,]) \, [x = y](G) = \{ g \in G : g(x) = g(y) \}. \]

Another test. It selects those assignments in \(G\) that yield identical values for \(x\) and \(y\).

\[ ([\,]) \, [-\varphi](G) = \{ g \in G : \varphi(G) = \emptyset \}. \]

Negation creates tests as well.

\[ ([\,]) \, [\varphi \land \psi](G) = \varphi(G)(\psi(G)). \]

Dynamic conjunction, defined as function composition. The dynamic effects of the formulae \(\varphi\) and \(\psi\), if any, are preserved, because we use the value of \(\varphi\) at \(G\) as the argument of \(\psi\).
1. Formal Tools: Pluralizing DPL

\[(v) \quad [\varepsilon_x](G) = \{ h \in \text{Var} : \bigvee_{g \in G} g[x]h \} .\]

Here \(g[x]h\) means that \(h\) differs from \(g\) at most in the value that it assigns to \(x\). This type of formulae, called random assignment, is the only source of dynamism, i.e., the only one whose semantic value is not a subset of the argument set (it is not eliminative). It assigns ('stores') an arbitrary value to the variable \(x\). Ordinary (dynamic) existential quantification can be expressed with \(\varepsilon_x\):

\[\exists x(\varphi) =_{\text{def}} \varepsilon_x \land \varphi,\]

whereas universal quantification can be expressed in terms of existential quantification, as usual:

\[\forall x(\varphi) =_{\text{def}} \neg \exists x(\neg \varphi) \equiv \neg (\varepsilon_x \land \neg \varphi).\]

So universal quantification is by definition a test.

The usual connectives '\(\lor\)' and '\(\rightarrow\)' can be expressed in terms of conjunction and negation as usual:

\[\varphi \lor \psi =_{\text{def}} \neg (\neg \varphi \land \neg \psi);\]
\[\varphi \rightarrow \psi =_{\text{def}} \neg (\varphi \land \neg \psi).\]

Accordingly, every disjunction is a test, and both of its members act as tests as well (they may have no dynamic effect on each other), so disjunction is said to be internally static. Material implication is a test as well, but it is internally dynamic in the sense that the eventual dynamic effects of the antecedent affect the interpretation of the consequent (because of the dynamic conjunction in its definition). So it is important that we do not use the alternative definition of material implication, in terms of disjunction:

\[\varphi \rightarrow \psi \neq \neg \varphi \lor \psi \equiv \neg (\neg \varphi \land \neg \psi).\]

Defining material implication in terms of disjunction would give rise to an interpretation that is effectively internally static: although it would contain a dynamic conjunction, the double negation of the antecedent would make it static. In general,

\[\neg \neg \varphi \neq \varphi\]

in DPL, because \(\neg \neg \varphi\) (sometimes also written \(! \varphi\)) is a test, even if \(\varphi\) happens not to be.
1.2. Pluralizing DPL

As I have anticipated, the pluralized version of DPL to be used here allows us to assign sets of individuals as values to discourse markers (variables). The set Var of variables will contain only one sort, but their values will all be sets. Individual variables will be assigned singleton sets as values. The empty set as a value of a variable will be treated as a degenerate value, so the assignment functions are effectively partial. This raises some additional complication in the definitions. I introduce two more implicit syntactic clauses: $A_x$ (for all variables $x$) assigns $x$ non-empty sets of any cardinality ('absolutely random assignment'). We will also need formulae of the form $M_X, x(\varphi)$, which select assignments that assign the largest possible sets to $X$ satisfying $\varphi$, they assign some element of $X$ to $x$, and update the resulting assignments with $\varphi$ ('maximization'). A formula of the form

$$A_X \land \epsilon_x \land M_X, x(\varphi)$$

will store these largest sets to the value of $X$, assign an arbitrary element of $X$ to $x$, and do the updates that $\varphi$ requires.

The domain and the range of the semantic-value function are the same as before, but I will use the notation $[]^M$ (or, rather, $[]$, for the sake of simplicity) to distinguish it from the previous one.

\[(\mathbb{I})\]

$$[P(x_1, \ldots, x_n)](G) = \{g \in G : \bigwedge_{1 \leq i \leq n} g(x_i) \neq \emptyset \land$$

$$\land \bigwedge_{u_1 \in g(x_1)} \ldots \bigwedge_{u_n \in g(x_n)} \langle u_1, \ldots, u_n \rangle \in P^+\}. $$

That is, we select those assignments in $G$ that are defined for each argument. Every $n$-tuple with components that are chosen from the values of the respective arguments must be in the extension of $P$.

\[(\mathbb{II})\]

$$[x = y](G) = \{g \in G : g(x) \neq \emptyset \land g(y) \neq \emptyset \land g(x) = g(y)\}. $$

The same as the singular definition, except that we have to check whether the values of $x$ and $y$ are defined at all.

\[(\mathbb{III})\]

$$[-\varphi](G) = \{g \in G : [\varphi](\{g\}) = \emptyset\}. $$

Same as for $[\cdot]$. 


(iiv) \[[\varphi \land \psi](G) = [\psi](\varphi(G))\].

Same as for [].

(iv) \[[\epsilon_x](G) = \{h \in \mathcal{V}_x \mathcal{U} : \bigvee_{g \in G} g[x]h \land |h(x)| = 1\}\].

As for [], but we want the new value of \(x\) to be a singleton set.

(vi) \[[A_x](G) = \{h \in \mathcal{V}_x \mathcal{U} : \bigvee_{g \in G} g[x]h \land h(x) \neq \emptyset\}\].

The same as \(\epsilon_x\), but we only want the new value of \(x\) to be non-empty.

(vi) \[[Mx, x(\varphi)](G) = \{h \in \mathcal{V}_x \mathcal{U} : \bigvee_{g \in G} (g(X) \neq \emptyset \land |g(x)| = 1 \land g(x) \subseteq g(X)
\land \bigwedge_{u \in U} ([\varphi][\{g[X : \{u\}]][x : \{u\}])] \neq \emptyset \Rightarrow u \in g(X) \land
\land h \in [\varphi]\{g\})\}\].

The notation \(g[x : \{u\}]\) refers to a function that assigns the same values to the same variables as \(g\), except for \(x\), to which it assigns \(\{u\}\). Interpreting a formula of the form \(MX, x(\varphi)\) means to select those assignments from the information state which assign to \(X\) the largest set of individuals satisfying \(\varphi\) (so that \(x\) is evaluated to a member of that set), and update them with \(\varphi\). For example, if we calculate

\[[A_x \land \epsilon_x \land MX, x(\text{farmer}(x) \land \epsilon_y \land \text{donkey}(y) \land \text{owns}(x, y))](G) = G'\]

then, if \(g' \in G'\), then \(g'(X)\) is the set of all farmers who own a donkey, \(g'(x)\) is a singleton set of one of those farmers, and \(g'(y)\) a singleton set of a donkey that that farmer owns.
1.3. Conditionals and Quantification

As I said in the Introduction, conditionals and quantified sentences will not be translated with material implication but with dynamic conjunction. I will call the first conjunct an assignment, because its only effect is to store the possible values of a variable. The second conjunct will be called the assertion, because it expresses what we assert about those possible values.

The relevance of this move should be clear from the presentation of DPL in section 1.1: as opposed to material implication, dynamic conjunction is not only internally, but also externally dynamic: it may foreground entities (or, in the pluralized version, sets of entities) for subsequent discourse.

1.3.1. Conditionals

The translation schema for conditionals presented in the Introduction corresponds to the following type of formulae in our pluralized language:

\[(1') \quad \text{a. If } \varphi, \text{ then } \psi.\]
\[\text{b. } (A_W \land M_W \land w(\varphi(w))) \land \text{every } W, w(\psi(w))\]

(I have enclosed the assignment in the above formula in parentheses for better readability. As I have mentioned earlier, I will not dwell on how variables referring to possible worlds enter the language; we can assume for the moment that \(W\) and \(w\) are just ordinary variables.) Interpreting this formula stores all the \(\varphi\)-worlds to \(W\) for good, and asserts that \(\psi\) holds for every element \(w\) of \(W\), as desired. How the operator every in it must be interpreted will be clarified shortly. The translations of similar sentences with (potentially implicit) quantifiers like usually, mostly etc. are analogous: the translations of such quantifiers replace every in the above formula.

Most importantly, if \(\varphi\) or \(\psi\) have any dynamic effect, those will be preserved. Consider:

\[(5) \quad \text{If Joe is smart, he bought a bicycle.}\]

The output assignments that the interpretation of this sentence gives rise to when applied to a set of input assignments will assign to some variable \(W\) the possible worlds in which ‘Joe is smart’ is true; moreover, each assignment will also assign to some variable \(c\) one bicycle that Joe bought in some of the possible worlds \(w\) in the set that it assigns to \(W\). Our semantics is not rich enough to express that the existence of such a bicycle is not guaranteed for the actual world (unless it is also guaranteed that ‘Joe is smart’ is true in the actual world), so we cannot explain within this language why ‘the bicycle that Joe bought’ is not available if the subsequent sentences are about the actual world. (As I have said in the Introduction,
the availability of a discourse referent foregrounded within a conditional sentence should be relativized to the possible worlds that the conditional foregrounds: see section 3.1.) If, however, the subsequent discourse is about possible worlds in which ‘Joe is smart’ is true, a bicycle becomes available for anaphoric reference:

(5') *If Joe is smart, he bought a bicycle. Then he keeps it locked into his garage.*

(As it often happens in consequents of conditionals as well, the anaphoric pronoun *then* refers to the possible worlds that the first sentence introduces, namely, the ones in which ‘Joe is smart’ is true.)

Note that the so-called ‘donkey-equivalence’

$$\exists x(\varphi) \rightarrow \psi \equiv \forall x(\varphi \rightarrow \psi)$$

does not hold for the translations proposed here. That is,

$$A_w \land MW, w(\exists x(\varphi(w)) \land \text{every} W, w(\psi(w)) \neq \forall x(\forall w(\varphi(w) \rightarrow \psi(w))).$$

As a consequence, a ‘donkey-sentence’ such as

(6) *Mostly, if a farmer owns a donkey, he beats it*

will not be assigned the ‘strong’, double-universal reading ‘for most farmer/donkey pairs, if the farmer owns the donkey, he beats it’. As a matter of fact, nothing more is really certain about what reading we will assign to (6) until we know exactly how possible worlds will enter the picture. At any rate, as is often emphasised in the literature, speakers have very vague and varying intuitions about the interpretation of this sentence: whether to count farmer/donkey pairs, donkey-owning farmers or donkeys owned by farmers, and how farmers who own more than one donkey behave. (It is seldom asked how donkeys owned by several farmers are treated, but that is just because it is unusual for one donkey to be owned by more than one person.) Given these judgments, it would not be wise to assign such sentences a double-universal interpretation automatically. The possible explanation of the vagueness of the judgments awaits further investigation in any theory currently on the market (see Heim (1990)).

1.3.2. Quantificational sentences

As I have said in the Introduction, the translations of quantified claims are very similar to those of conditionals:

(3'') a. *Every* P Q

b. *(A_X \land \epsilon_x \land MX, x(P(x))) \land \text{every} X, x(Q(x)).*
Note that, in terms of the definition in ([vii]) above, the formula in (3"b) leads to an empty (contradictory) information state if it is known already that there are no Ps at all. On the other hand, since variables which have an empty value are exceptional (‘degenerated’), we will be able to say that a sentence of the form in (3"a) is infelicitous rather than false in a context in which there cannot be any Ps. We just need the necessary tools required for the treatment of presuppositions to do that (see section 2). That is, we will say that a formula like (3"b) has an existential presupposition rather than existential import in that section.

As with the translation of conditionals in (1"), the set of all P-individuals becomes an available antecedent for subsequent anaphors. Moreover, this set is already available in the (local) context in which the main assertion is interpreted, although in English it is not possible to refer back to it with a plural pronoun for grammatical reasons: *Every farmer believes that they must beat their donkeys. I believe that this is due to the simple fact that every-phrases are grammatically singular in English unlike in, say, French:

(7) Tous les fermiers croient
   every-MPL the-PL farmers believe-3PL
   ‘Every farmer believes
   qu’ ils doivent battre leurs ânes.
   that they-M must-3PL beat-INF their donkeys
   that he must beat his donkeys.’

Since ‘every farmer’ is expressed analogously to all farmers in French, the anaphoric pronouns in the assertion are plural. One could speculate that, since our translations introduce both a plural and a singular variable, it is a grammatical issue which one is considered the grammatical subject. On the other hand, this does not apply to anaphors referring to discourse referents introduced in the assignment: those will agree with their antecedents as usual:

(7) Tous les fermiers qui ont un âne croient
   every-MPL the-PL farmers who have-PL a donkey believe-3PL
   ‘Every farmer who has a donkey believes
   qu’ ils doivent le battre.
   that they-M must-3PL him beat-INF
   that he must beat it.’

The way in which we might want to interpret quantifiers in our language is a type of generalized quantification, namely, a relation between sets:

\[ ([\text{viii}]) \quad [\text{OP}X, x(\varphi)](G) = \{ g \in G : g(X) \neq \emptyset \land g(x) \neq \emptyset \land \\
& \quad \text{OP}(\{u \in g(X) : [\varphi](\{g[x : \{u\}][X : \{u\}]) \neq \emptyset\}, \\
& \quad \{u \in g(X) : [\varphi](\{g[x : \{u\}][X : \{u\}]) = \emptyset\})) \}, \]
where $\text{OP} \in \{\text{every, most, few, ...}\}$, and $\text{OP}'$ is the corresponding relation over $\mathcal{P}(\mathcal{U})$.

The 'donkey-equivalence'\
\[\forall x(\exists y(\varphi) \rightarrow \psi) \equiv \forall x(\forall y(\varphi \rightarrow \psi))\]
does not hold for our translations of quantified sentences, either. That is,
\[\begin{align*}
& \left(\forall X \land \epsilon_x \land M X, x(\exists y(P(x, y)))\right) \land \text{every} X, x(Q(x, y)) \neq \\
& \neq \forall x(\forall y(P(x, y) \rightarrow Q(x, y))).
\end{align*}\]

As a consequence, a donkey-sentence like

\[(8) \text{Every farmer who owns a donkey beats it}\]

will not be assigned a double-universal reading ('every farmer beats every donkey he owns'). Again, speakers' judgments vary as to what this sentence says about farmers who own more than one donkey, so this result is desirable.

The other problem often raised in connection with quantifiers since Evans' (1980) paper is what sets should a quantificational sentence foreground as available antecedents:

\[(9) \text{Few congressmen admire Kennedy. They are very junior.}\]

Under the treatment that I have just proposed, the set of 'all congressmen' rather than that of 'the few congressmen who admire Kennedy' becomes available for they to refer back to. Indeed, this is the most natural reading of (9), though 'all congressmen are very junior' is usually untrue. As Anna Szabolcsi (p.c.) points out to me, the other reading can be produced with co-ordination (and an eventual demonstrative pronoun):

\[(9') \text{Few congressmen admire Kennedy, \{and \quad \text{they} \quad \text{but} \quad \text{those}\} are very junior.}\]

The phenomenon in (9') seems mysterious to me. What I find most likely is that the heavily stressed personal or demonstrative pronoun, together with the co-ordination, serve as a clue to the presence of an ellipsis: in that case, they or those should be interpreted as 'those who do' in (9'). Note that, in Hungarian, where the presence of a subject personal pronoun is rather unusual unless it bears contrastive stress, the type of reading illustrated in (9') can only be produced with a heavily stressed overt pronoun and the word is 'even':
1. Formal Tools: Pluralizing DPL

(10) a. *Kevés diák* szól hozzá, és/de *nagyon fiatalok.*
   few student speak-up and/but very young-PL
   ‘Few students speak up, and/but they (the students) are very young’

b. *Kevés diák szól hozzá, és azok is nagyon fiatalok.*
   and those even
   ‘Few students speak up, and even those (who do) are very young.’

Since the treatment proposed here predicts a uniform behaviour of quantifiers in terms of anaphoric reference (modulo agreement facts), we expect that even negative quantifiers may give rise to antecedents:

(11) *No salesman is walking in the park. They are at home asleep.*

As desired, the prediction is that *they* can refer back to ‘all salesmen’, but not to the empty set of ‘salesmen walking in the park’. Accordingly, the ‘even those’-version of such a sentence is infelicitous in Hungarian:

   a salesman not even walks they-all sleepy-PL
   ‘No salesman is walking. They are all sleepy.’

b. *# Egy kereskedő sem sétál, és azok is álmosak.*
   and those even
   # ‘No salesman is walking, and those who do are sleepy.’

2. Dealing With Presuppositions

Two very important questions related to conditional and quantificational sentences are treated in this paper: their foregrounding features and their presuppositional behaviour, which I am turning to now. I start from the very common assumption that presuppositions are to be captured in terms of the definedness of semantic values. That is, in both static and dynamic theories, the most usual way of accounting for the oddity that arises when the presupposition of a sentence fails to hold (in a model or an information state) is to show that the semantic value of the corresponding formula is undefined (in that model or information state).

In order to talk about the definedness conditions of formulae, we first have to develop an interpretation where this makes sense at all, that is, a *partial* interpretation, in which the semantic value of a formula is not always defined. I do this in section 2.1. Then, in section 2.2, I examine what that interpretation predicts with regard to the *projection* properties of various types of complex expression, i.e., how definedness properties are *inherited* from simpler expressions to more complex ones that contain them as constituents. I will argue that the predictions about the behaviour of conditional and quantificational sentences are plausible.
2. Dealing With Presuppositions

2.1. Partializing the Language

The partial version of the interpretation function defined in the previous section will be written as \([\cdot]\). It is not surprisingly different from other partial dynamic interpretation functions, such as Dekker’s (1992). This function will assign a set of assignments or the value-gap ‘*’ to every set of assignments. This will complicate the definitions, of course. Partiality will arise from two sources. First, the interpretation of predicate constants accounts for presuppositions stemming from the lexical content of a predicate, which specifies that the predicate may be true or false only for a certain type of objects. For example, one can assume that ‘mammal’ is not interpreted for individuals that are not animals, or that ‘left’ is not interpreted for individuals for which ‘was here/there’ is not true. Second, the effective partiality of assignments, which I have ignored so far, may also give rise to undefined semantic values. We will see one example of this in this section, namely, the existential presupposition of quantified sentences is due to this fact (generalized quantification over an empty domain will lead to a semantic-value gap, see section 2.2). Whether other types of presupposition require this mechanism will be discussed in section 4.

According to the above, we will assume that the interpretation function \(I\) associates each \(n\)-ary predicate constant \(P\) with two domains: the extension \(P^+\) and the anti-extension \(P^-\):

\[
P \in \text{Con}^{(n)}_{\text{pred}} \Rightarrow I(P) = (P^+, P^-) \in \mathcal{P}(\mathcal{U}^n) \times \mathcal{P}(\mathcal{U}^n).
\]

The first component of \(I(P)\) is \(P^+\), i.e., \(P\)'s extension, while the second component is \(P^-\) or \(P\)'s anti-extension. We stipulate that \(P^+ \cap P^- = \emptyset\) for every \(P\). The set \(\mathcal{U}^n \setminus (P^+ \cup P^-)\), i.e., the \(n\)-tuples that are in neither the extension nor the anti-extension, will be referred to as \(P^*\).

In order to keep the remaining definitions simple, I will define two functions rather than one, with simultaneous recursion: the partial interpretation function \([\cdot]\), and the function \([\cdot]^+\), which assigns every formula the set of information states in which \([\cdot]\) is defined at all.

\[
([i]^+) = \text{Val}^i U : \bigvee_{g \in G} \left( \bigwedge_{1 \leq i \leq n} g(x_i) \neq \emptyset \land 
\right. \\
\left. \land \bigwedge_{u_1 \in g(x_1)} \ldots \bigwedge_{u_n \in g(x_n)} \langle u_1, \ldots, u_n \rangle \notin P^* \right).
\]

Predicate application is defined for those information states in which some assignments assign a value to each argument and, furthermore, every \(n\)-tuple that we
can form from the values that they assign to the respective arguments is either in
the extension or the anti-extension of \( P \).

\[ ([iii]^+) \quad [x = y]^+ = \{ G \subseteq \text{Var} \mathcal{U} : \bigvee_{g \in G} (g(x) \neq \emptyset \& g(y) \neq \emptyset) \}. \]

The identity of \( x \) and \( y \) is defined for those information states in which some
assignments assign a value to both \( x \) and \( y \).

\[ ([iii]^+) \quad [\neg \varphi]^+ = [\varphi]^+. \]

The negation of a formula is defined for the same information states as the formula
itself.

\[ ([iv]^+) \quad [\varphi \land \psi]^+ = \{ G \in [\varphi]^+ : [\varphi](G) \in [\psi]^+ \}. \]

I use \([\cdot]\) in this definition, hence the simultaneously recursive character of the
definitions of \([\cdot]^+ \) and \([\cdot]\). For dynamic conjunction as function composition to be
declared, we must make it sure that the first conjunct is defined for the argument,
and that the second conjunct is defined for the value of the first conjunct.

\[ ([v-vi]^+) \quad [\varepsilon_x]^+ = [A_x]^+ = \{ G \subseteq \text{Var} \mathcal{U} : \bigwedge_{g \in G} g(x) = \emptyset \}. \]

For the sake of simplicity, we only define the values of these ‘assignment formulae’
for information states in which the variable that they introduced has an undefined
value. This decision, however, will play no role in the following. We could as well
define them for all information states.

\[ ([vii-viii]^+) \quad [MX, x(\varphi)]^+ = [OPX, x(\varphi)]^+ = \{ G \in [\varphi]^+ : \bigvee_{g \in G} (g(X) \neq \emptyset \& g(x) \neq \emptyset) \}. \]

Looking at the values of \( X \) and \( x \) in terms of \( \varphi \) requires that \( X \) and \( x \) be defined
variables and that the value of \( \varphi \) be defined.

Finally, the only clause that we need for the definition of the partial semantic-
value function \([\cdot]\) is this: For every formula \( \varphi \),

\[ ([ii]) \quad [\varphi](G) = \begin{cases} * & \text{if } G \notin [\varphi]^+; \\ [\varphi](G) & \text{otherwise}, \end{cases} \]

where \([\cdot]\) is the total semantic value function that we have defined earlier.
2.2. Projection Properties

The next question to be examined after this simple extension of DPL is how presuppositions are ‘projected’ (inherited) from the sub-formulae of our representations to the entire representation. Since the way in which I have partialized the language is sort of trivial, we do not expect surprising results for the types of formulae in ([i–vi]). In fact, the results mostly harmonize with Karttunen and Peters’ (1979) observations, and partly with Heim’s (1983) proposal. The latter is more liberal in certain respects, which I will mention only briefly. The crucial difference is between the predicted presuppositional behaviour of quantified sentences and what Heim (1983) predicts about them, which are related to the definitions in ([vii–viii]⁺), i.e., the definedness properties of M and operators like every.

Negation. First, negation is a ‘hole’ for presuppositions, since [¬φ] is undefined whenever [φ] is. Accordingly, the following pieces of discourse are correctly predicted to be odd:

(13) a. I don’t regret that Joe left. # Joe is still here.
    b. I don’t regret that Joe left. # He’s never been here.

Conjunction. The value of formulae of the form φ ∧ ψ will be undefined at G if either [φ] is undefined at G or [ψ] is undefined at [φ](G), which is desirable:

(14) a. Joe left. He was here.
    b. Joe was here. He left.

The first sentence in (14a) is odd if ‘Joe was here’ cannot be taken for granted, even though the second sentence provides the missing presupposition. So the presupposition of the first sentence of the conjunction must be satisfied in the initial (‘global’) context. On the other hand, if the second sentence of a conjunction contains a presupposition, as in (14b), then its presupposition may be fulfilled by the immediately preceding (‘local’) context, created by the interpretation of the first sentence, as we see in (14b): irrespective of the initial (‘global’) context of the utterance of the conjunction, the second sentence is not odd if the first provides the required presupposition ‘Joe was here’.

Disjunction. Since φ ∨ ψ can be defined as ¬(¬φ ∧ ¬ψ), the predicted behaviour of presuppositions in disjunctions is as follows. ¬φ and ¬ψ inherit the presuppositions of φ and ψ, and ¬φ ∧ ¬ψ inherits all those presuppositions. Since ¬φ is a test, the conjunction here is effectively commutative:


\[ ¬φ \land ¬ψ \equiv ¬ψ \land ¬φ. \]

Finally, the outermost negation ‘lets through’ all these presuppositions. For example, consider:

(15) Joe left Paris or he quitted smoking.
Our semantics correctly predicts that (15) presupposes both ‘Joe was in Paris’ and ‘Joe smoked’.

On the other hand, consider the following disjunction:

(16) The king of France or the president of France called.

This sentence does not sound as odd as it should if it presupposed both ‘France has a king’ and ‘France has a president’. This is a serious problem, which is to be remedied by assigning different types of translations to the sentences in (15) and (16). Obviously, (16) does not presuppose the existence of either the king of France or the president of France (let alone both). What we presuppose in it is that France has either a king or a president. So, effectively, we should say that (15) and (16) are to be translated with two different types of disjunction.

Before proceeding, let me point out that the contextual restrictions on using (15) and (16) are quite different. Uttering (15) naturally is only possible if the two clauses express alternative, competing explanations of the same fact:

(17) — Joe is in wonderful shape these days. What could have happened?
- Either he left Paris or he quitted smoking.

The two alternatives in (16), on the other hand, are elaborations or instances of one and the same (implicit) statement, namely, ‘the ruler of France called’, and the alternative lies in who rules France (a king or a president). So the competition is between two propositions in (15), whereas it is between two predicates (‘king’ vs. ‘president’) in (16). Accordingly, the translations of (15) and (16) should differ in that the former contains disjunction as a sentential connective, whereas the latter contains predicate disjunction. (How the two translations can be produced is irrelevant here.) We could introduce predicate disjunction into our language so that it has definedness properties different from those of propositional disjunction:

\[
([ix]) \quad [P^\lor Q(x_1, \ldots, x_n)](G) = \{g \in G : \bigwedge_{1 \leq i \leq n} g(x_i) \neq \emptyset \& \bigwedge_{u_1 \in g(x_1)} \ldots \bigwedge_{u_n \in g(x_n)} (u_1, \ldots, u_n) \in P^+ \cup Q^+ \};
\]

\[
([ix]^+) \quad [P^\lor Q(x_1, \ldots, x_n)]^+ = \{G \subseteq \text{Var} U : \bigvee_{g \in G} \bigwedge_{1 \leq i \leq n} g(x_i) \neq \emptyset \& \bigwedge_{u_1 \in g(x_1)} \ldots \bigwedge_{u_n \in g(x_n)} (u_1, \ldots, u_n) \notin P^* \cap Q^* \};
\]

Under this definition,

\[ P(x_1, \ldots, x_n) \lor Q(x_1, \ldots, x_n) \neq P^\lor Q(x_1, \ldots, x_n) \]
because the former is undefined iff either $P(x_1,\ldots,x_n)$ or $Q(x_1,\ldots,x_n)$ is undefined, whereas the latter is undefined if both are. Under this approach, we could have the following translations:

(15') *Joe left Paris or he quitted smoking.*
\[
\text{left-Paris}(j) \lor \text{quitted-smoking}(j)
\]

(16') *The king of France or the president of France called.*
\[
\epsilon_x \land \text{king-of-France}(x) \land \text{president-of-France}(x) \land \text{called}(x)
\]

There is a different type of problematic disjunctive sentences as well, in which the first disjunct explicitly denies the presupposition of the second (see Karttunen and Peters (1979)):

(18) *Either Joe wasn't in Paris or he left Paris already.*

I believe that this type of sentences should be analysed in terms of *ellipsis*: the second disjunct in (18) is to be translated as if it was *or, if he was there, he left Paris already*. The motivation for this analysis comes from the celebrated *bathroom-sentence*:

(19) *Either there is no bathroom here, or it is in a funny place.*

I submit that the only reasonable way to account for the possibility of using an anaphoric pronoun (*it*) in the second disjunct to refer to a bathroom (while the first disjunct denies its existence) is to assume ellipsis and translate the second disjunct as if it was *or, if there is one, it is in a funny place*. Under this analysis, the anaphoric pronoun refers back to the referent introduced by *one* in the ellipted antecedent of the elliptical conditional sentence. If the ellipsis mechanism is there for the explanation of (19), then we can use it in the analysis of (18) as well.

If the ellipsis-based explanation of (18) and (19) is correct, then exchanging the two disjuncts should matter: while it seems reasonable that the positive version of the first disjunct is an implicit antecedent of the second in (18) and (19), we do not expect an implicit antecedent of this sort in the first member of a disjunction. As a matter of fact, exchanging the two disjuncts in these sentences yields odd sentences unless we add some indication that the presupposition of the first disjunct is to be withdrawn:

(18') *Either Joe left Paris already, or *
\[
\begin{cases}
\text{he wasn't there.} & \text{OK he wasn't there in the first place.} \\
\text{he wasn't there in the first place.} & \text{OK he wasn't there in the first place.}
\end{cases}
\]

(19') *Either the bathroom is in a funny place, or *
\[
\begin{cases}
\text{there isn't one.} & \text{OK there isn't one at all.} \\
\text{there isn't one at all.} & \text{OK there isn't one at all.}
\end{cases}
\]

**Conditionals.** Turning now to conditionals, $\varphi \rightarrow \psi$ is defined as $\neg(\varphi \land \neg \psi)$ rather than $\neg \varphi \lor \psi$ to capture the internal dynamism of conditional sentences:

(20) *If Joe has a cat, he likes it.*
2. Dealing With Presuppositions

According to this definition,

\[[\varphi \rightarrow \psi]^+ = [\varphi \land \psi]^+\],

that is, the value of \(\varphi \rightarrow \psi\) is undefined at \(G\) if either \([\varphi]\) is undefined at \(G\) or \([\psi]\) is undefined at \([\varphi](G)\). Thus, we correctly predict that the presupposition of the consequent may be satisfied by the antecedent (in the 'local' information state that results from interpreting the antecedent) even if it is not at \(G\): \([\psi]\) may be defined at \([\varphi](G)\) even if it is not in the 'global' context \(G\):

(21) If Joe was in Paris, he left Paris by now.

On the other hand, if the antecedent does not guarantee that the presupposition of the consequent is satisfied, then the initial context must guarantee it:

(22) If Joe is in a good shape, then he left Paris.

Since interpreting the translation \(\varphi\) of the first clause of (22) does not provide the presupposition of the second clause, the translation of the second clause will be defined at \([\varphi](G)\) only if it is already in \(G\). Thus (22) as a whole presupposes that Joe is or was in Paris.

As a matter of fact, we will not translate conditional sentences using material implication, but, since the projection properties of conjunction are the same as those of implication, the inheritance properties of the translations that I have proposed in the previous section are the same. This is the type of presuppositional behaviour adopted by Karttunen and Peters (1979) and Heim (1983) as well, except that Heim (1983) potentially allows for 'local accommodation', i.e., a mechanism that ensures that presuppositions which should be satisfied by the 'global' context are somehow introduced only locally. Such a move would be motivated by a sort of apparent counterexamples that sometimes appear in the literature (e.g., van der Sandt (1989)), but which simply exhibit the effect of focus on the presuppositional behaviour of utterances:

(23) If Joe has a son, his kid must be happy.

The apparent problem with (23) is that the presupposition of the consequent ('Joe has a kid') is satisfied by the context created by the antecedent (because 'Joe has a son' entails 'Joe has a child'), which would mean that the context of uttering the conditional as a whole need not satisfy it. However, the entire sentence in (23) may presuppose that Joe has a child. The correct explanation of this phenomenon is that \(son\) in the antecedent may be understood as focussed and, in that case, the fact that Joe has a child is taken for granted by the antecedent already. This is pretty obvious if we translate (23) into Hungarian, where focus is marked syntactically (with word order):
2. Dealing With Presuppositions

(24) a. Ha Jóskának van egy fia,…
   'If Joe has a son (*rather than a girl)…'

b. Ha Jóskának fia
   'If Joe has a SON (rather than a girl)…'

A Hungarian sentence that starts with (24a) will not presuppose that Joe has a child, whereas one that starts with (24b) will. In sum, I believe that 'local accommodation' is not needed to explain the projection behaviour of conditionals.

Quantification. Finally, the definedness conditions that we would predict for universally quantified sentences if we agreed to translate them with the universal quantifier definable in our representation language would be as follows. If we translated a sentence of the form Every x that \( \varphi \), \( \psi \) as

\[
\forall x (\varphi \rightarrow \psi) \equiv \neg (\varepsilon_x \land \varphi \land \neg \psi),
\]

then its semantic value would be defined at \( G \) iff some \( g \in [\varepsilon_x](G) \) assigns a value to \( x \) for which \( [\varphi] \) is defined, and \( [\psi] \) is defined at the value of \( [\varphi] \) applied the set of those assignments. That is, the value of the translation of Every dog barks would be defined whenever 'dog' is defined for at least some individuals, and 'barks' is defined for some of them as well. (The same presupposition is predicted for quantified sentences in general.)

This predicted projection behaviour is acceptable in general, but it differs sharply from Heim's (1983) result, according to which all quantified sentences should have universal presuppositions. With her mechanism, 'barks' should be defined for all dogs in order for Every dog barks to make sense. Similarly, 'every nation has a king' should hold in an information state in which either Every nation cherishes its king or No nation cherishes its king can be uttered felicitously. As a matter of fact, it is hard to tell what these sentences presuppose, i.e., how many nations must have a king to utter them felicitously. It seems that the same kind of vagueness is involved here as with generic sentences in general: these sentences presuppose the generic statement 'nations have kings' rather than the universal sentence 'every nation has a king' (or the existential sentence 'some nation has a king', which the translation with \( \forall \) in the current framework would yield).

Maximization and quantifiers. Let me now look at the projection properties of the interpretation of quantificational sentences proposed in this paper. M\( X, x(\varphi) \) inherits the presuppositions of \( \varphi \), so Everyone who knows that Joe left presupposes 'Joe left'. On the other hand, Everyone who beats his donkey will yield an undefined value only if nobody has a donkey at all. Similarly, since O\( P X, x(\varphi) \) also inherits the presuppositions of \( \varphi \), Everyone knows that Joe left
presupposes 'Joe left', but *Everyone beats his donkey* only presupposes 'somebody has a donkey'.

In terms of presuppositional behaviour, these results are the same as if we were to translate quantifiers in the conventional way, with the improvement that we do not predict universal presuppositions à la Heim (1983). The big difference lies in the 'existential import' of quantification: since *Everybody who has a donkey* will assign an empty set to the variable that I have called $X$ above if nobody has a donkey, the semantic value of *Everybody who has a donkey beats it* will be undefined rather than false in that case. This clearly corresponds to the intuitive interpretation of this sentence. Sometimes negative quantifiers must be prevented from having this sort of existential import, so *No one who has a donkey beats it* must be translated as 'it is not the case that anyone who has a donkey beats it'. Accordingly, this type of sentences will not have any foregrounding effect, either: they do not allow plural anaphors to refer back to the people who have a donkey.

The obvious shortcoming of this treatment is the same as what we have seen with $\forall$: an existential rather than generic presupposition is predicted for quantificational sentences. It would take a full-fledged view of genericity to formulate the requirement that $[MX, x(\varphi)]$ and $[OPX, x(\varphi)]$ are defined if $[\varphi]$ is defined 'generically' for the individuals in $X$'s value. As with the problem of *donkey*-sentences discussed in sections 1.3.1 and 1.3.2, I believe that the solution lies in independent factors complicating the picture drawn here.

3. Residual Problems

The following problems, which are clearly crucial to the issues discussed in this paper, have been raised in the previous sections but have not been solved.

3.1. Modalities

Information states or contexts of DPL (and its extension in the previous sections) express partial knowledge about individuals in the actual world, given a complete knowledge of the world (the model). In actual fact, an information state should also capture a partial knowledge about the actual world itself. This could be implemented by conceiving of information states as sets of possibilities, i.e., pairs of the form $(g, w)$ where $g$ is an assignment and $w$ is a possible world, which also acts as a model with its own universe and its own interpretation function. Moreover, an information state does not contain just information about the actual world, but also about various alternative worlds, such as hypothetical, counterfactual, past and future worlds. It should be possible to update partial knowledge about various alternative worlds separately in an information state to translate modal,
past-tense etc. sentences. Under this approach, the variables referring to possible worlds in formulae range over those partial possible worlds in the information state rather than complete possible worlds. Various partial possible worlds may be foregrounded in an information state pretty much as discourse referents (partial individuals) can. This is the kind of apparatus that should be developed to successfully treat the modal character of conditional sentences and the phenomena related to modal subordination.

3.2. Genericity

Whenever collections of individuals (or possible worlds) are involved in a natural-language utterance, the linguist faces the problem of genericity. I use this term in a rather broad sense, referring to all sorts of cases when a predication about a collection of individuals is vague as to the extent in which individual members of the collection can be made responsible for the truth of the predication. The conditions determining whether genericity arises in a sentence involving collections are not well-understood. The examples below illustrate some cases which, for some reason, involve generic meanings in the above sense. In (25a), a property is predicated about a collection; for lack of an explicit distributive quantifier (such as every), such sentences allow exceptional members in the collection, which do not have the given property (this is generic predication in the narrow sense). In (25b), the collection is seen as the agent in a particular event; under one reading, it is left vague what role each member plays, if any, in achieving the result in question (collective predication). In (25c), a certain type of event is said to occur regularly, i.e., there is a collection of time intervals somehow evenly distributed in each of which the event occurs at least once (habitual predication). This is similar to (25a) in that exceptional time intervals are allowed.

(25) a. Ravens are black.
    b. The boys carried the piano upstairs.
    c. Joe goes to the library.

I argued in earlier papers (e.g., Kálmán (1990)) that, in spite of the diversity of the cases in which genericity arises, the generic aspect of meanings should be given a uniform treatment. True, generic meanings cannot be captured in terms of inference properties: it is not possible to specify what the above sentences entail about the proportion of black ravens, boys who actually helped carrying the piano upstairs or time intervals (say, weeks) in which Joe has been to the library. On the other hand, if we look at these meanings from a different perspective than their inference properties, we can capture rather precisely what information they carry.

The information carried by generic expressions can be best captured in terms of the possibilities of accommodating presuppositions in an information state.
Informally speaking, a presupposition $\varphi$ can be accommodated in a context if either $\varphi$ is entailed by the context or assuming $\varphi$ is plausible in the context. For example, for every raven $x$, it is possible to accommodate $\text{black}(x)$ in a context created by (25a). Similarly, for every boy $x$ in ‘the boys’ in (25b), the presupposition that $x$ performed a part of carrying the piano upstairs can be accommodated in the context that the sentence yields. Finally, the context created by (25c) makes it possible to accommodate, for any member of a set of implicit time intervals (e.g., weeks), that Joe has been to the library within that interval.

If the above approach to genericity is correct, then the genericity involved in the presuppositional behaviour of quantified sentences must be captured in terms of which presuppositions can be accommodated in a context. This would imply a formal definition of plausibility, which probably involves a relation over information states: if $\sigma_1$ and $\sigma_2$ are information states, then $R(\sigma_1, \sigma_2)$ holds just in case we can get from $\sigma_1$ to $\sigma_2$ by updating it with a piece of information that is plausible in it. As a matter of course, the characterization of the formal properties of the relation $R$ lies outside the scope of this paper.

4. Bipartite Meanings

In this section, I will briefly put the framework outlined above into a broader perspective on meaning representations for natural language. I start from the question how anaphoric expressions and the presuppositions that they are associated with are to be characterized in the framework proposed in this paper (section 4.1). I argue that the bound-variable approach to anaphors should be given up in favour of bipartite semantic representations, in which the sub-formulae that determine the grounding of referents are explicitly separated from the sub-formulae that express what a sentence claims. As a consequence of giving up the bound-variable approach, the individuals about which information is stored in a context are anonymous. Then I explain some consequences of this approach for the semantics of natural language (section 4.2). To conclude, in section 4.3, I draw attention to some theoretical consequences of this build-up, in particular, to the fact that the semantic properties of natural languages are not uniquely determined by the underlying logic or syntax, i.e., it is autonomous to a certain extent.

4.1. Variables and Discourse Markers

The dynamic semantic theories akin to the one I have developed in the previous sections (in particular, the DPL of Groenendijk and Stokhof (1991)) adopt the bound-variable view of anaphors: the discourse markers corresponding to individuals foregrounded in a context are represented with variables, the possible values of which are a fundamental characteristics of a context. Under this view,
an anaphoric expression can be translated as an occurrence of one of the variables which is assigned possible values in the context. These approaches share the very unattractive feature that we have to ensure that the right variable is used in the translation of an anaphoric expression. This implies looking at the information state in which the translation will be interpreted, since that is where we store information on the possible values of variables. So these theories cannot capture the intuition that sentences can be translated independently of the information state in which they are uttered even though, of course, they cannot be interpreted without reference to an information state.

The other alternative is to say that no variable binding is possible across formulae, but the fact that a discourse referent must be familiar from the discourse context is encoded in the translation of sentences in which an anaphoric expression occurs. (This is similar to the approach taken by Heim (1982).) The advantage of this way of proceeding is that sentences can be assigned translations and even context-changing potentials without taking the information state of their utterance into account. The following paragraphs discuss the main features of the formal machinery required to implement such an approach.

First, the assignment functions in information states do not assign values to named variables, but to anonymous discourse markers (similar to the ‘file cards’ in Heim (1982) or the ‘pegs’ of Landman (1986)), which can be represented by integers from 0 to some n ≥ 0. I will call such a function a peg function in the following. The domain of a peg function (and, for that matter, the domain of an entire information state, because all peg functions in it must have the same domain) is such a set of pegs or natural numbers: for other natural numbers, the peg functions yield the empty set as a value. So, if Π is an information state, then it is a set Π ⊆ N\mathcal{P}(U) (where N is the set of natural numbers). We can write Dom(Π) = n for the domain of Π if for every π ∈ Π, the domain of π is n.

Second, the variables that must correspond to discourse markers (pegs) already present in the discourse context (i.e., that have anaphoric properties) are to be marked somehow in the translations. The less restrictive approach is to introduce a special type of anaphoric variables, which could occur anywhere in a translation. In what follows, I will take a more restrictive approach, originating from Kálmán and Szabó (1990). According to this, we have to use bipartite semantic representations, i.e., separate those sub-formulae which are responsible for the grounding of those referents that must be anchored in the input information state (we have called these ‘anchors’ in Kálmán and Szabó (1990)) from those parts of the translation which express what the sentence claims (we called these the ‘predicate’ in our earlier paper). Variable binding across sub-formulae can only go in one direction: variables introduced in the grounding part may occur free in the claim, but not the other way round, hence the more restrictive character of this alternative.
Third, variables and assignment functions play a role similar to their role in classical (static) interpretation. (As a matter of course, the assignment functions map variables to integers, because they must establish associations between variables and pegs.) The sets of assignment functions that constitute information states are only relevant formula-externally. So a semantic-value function \([\cdot]'\), very similar to the function \([\cdot]\) defined earlier, can be used for sub-formulae in bipartite formulae. It yields functions from sets of ordered pairs consisting of an assignment function \(g \in \text{Var}\mathbb{N}\) (where \(\mathbb{N}\) is the set of natural numbers) and a peg function \(\pi\). The functions \([\cdot]'\) and \([\cdot]'\) can be defined in terms of a function \([\cdot]\), just the same as \([\cdot]\) was defined together with \([\cdot]\) in terms of \([\cdot]\). The modifications are in fact very straightforward, so I will not comment the following definitions.

\(([i]'')\)
\[
[P(x_1, \ldots, x_n)]'(\Gamma) = \{ (g, \pi) \in \Gamma : \bigwedge_{1 \leq i \leq n} \pi(g(x_i)) \neq \emptyset \land \bigwedge_{u_1 \in \pi(g(x_1))} \ldots \bigwedge_{u_n \in \pi(g(x_n))} (u_1, \ldots, u_n) \in P^+ \}.
\]

\(([i]'')\)
\[
[P(x_1, \ldots, x_n)]'' = \{ (g, \pi) \in \text{Var}\mathbb{N} \times \mathbb{N}^\mathbb{P}(U) : \bigvee_{(g, \pi) \in \Gamma} \bigwedge_{1 \leq i \leq n} \pi(g(x_i)) \neq \emptyset \land \bigwedge_{u_1 \in \pi(g(x_1))} \ldots \bigwedge_{u_n \in \pi(g(x_n))} (u_1, \ldots, u_n) \notin P^+ \}.
\]

\(([iii]'')\)
\[
[x = y]'(\Gamma) = \{ (g, \pi) \in \Gamma : \pi(g(x)) \neq \emptyset \land \pi(g(y)) \neq \emptyset \land \pi(g(y)) = \pi(g(y)) \}.
\]

\(([iii]'')\)
\[
[x = y]'' = \{ (g, \pi) \in \text{Var}\mathbb{N} \times \mathbb{N}^\mathbb{P}(U) : \bigvee_{(g, \pi) \in \Gamma} (\pi(g(x)) \neq \emptyset \land \pi(g(y)) \neq \emptyset) \}.
\]

\(([iii]'')\)
\[
[-\varphi]'(\Gamma) = \{ (g, \pi) \in \Gamma : [-\varphi]'(\{ (g, \pi) \}) = \emptyset \}.
\]

\(([iii]'')\)
\[
[-\varphi]' = [\varphi]''.
\]
4. Bipartite Meanings

([iv]') \quad [\varphi \land \psi]'(\Gamma) = [\psi]'([\varphi]'(\Gamma)).

([iv]+') \quad [\varphi \land \psi]^+ = \{ \Gamma \in [\varphi]^+ : [\varphi]'(\Gamma) \in [\psi]^+ \}.

([v]') \quad [\varepsilon_x]'(\Gamma) = \{(h, \pi^+) \in \text{Var} N \times \text{Dom}(\Gamma) + 1 \mathcal{P}(\mathcal{U}) : \\
\forall (g[x]h \& h(x) = \text{Dom}(\Gamma) \& \pi(h(x))\pi^+ \& |\pi^+(h(x))| = 1) \}.

That is, random assignment extends the domain of the peg functions in \Gamma, and ensures that the value of x will be the new peg.

([vi]') \quad [\mathbf{A}_x]'(\Gamma) = \{(h, \pi^+) \in \text{Var} N \times \text{Dom}(\Gamma) + 1 \mathcal{P}(\mathcal{U}) : \\
\forall (g[x]h \& h(x) = \text{Dom}(\Gamma) \& \pi(h(x))\pi^+ \& \pi(h(x)) \neq \emptyset) \}.

([v-vi]+') \quad [\varepsilon_x]^+ = [\mathbf{A}_x]^+ = \{ \Gamma \subseteq \text{Var} N \times N \mathcal{P}(\mathcal{U}) : \\
\bigwedge_{(g, \pi) \in \Gamma} : \pi(g(x)) = \emptyset \}.

([vii]') \quad [\text{MX}, x(\varphi)]'(\Gamma) = \{(h, \pi^+) \in \text{Var} N \times N \mathcal{P}(\mathcal{U}) : \\
\forall (g(x)) \neq \emptyset \& |\pi(g(x))| = 1 \& \pi(g(x)) \subseteq \pi(g(X)) \& \\
\& \bigwedge_{u \in \mathcal{U}} (\varphi)'(\{(g, \pi[g(X)] : \{u]\}[g(x) : \{u]\}))) = \emptyset \Rightarrow \\
\Rightarrow u \in \pi(g(X)) \& \\
\& (h, \pi^+) \in \varphi]'(\{(g, \pi)\})).

([viii]') \quad [\text{OPX}, x(\varphi)]'(\Gamma) = \{(g, \pi) \in \Gamma : \pi(g(X)) \neq \emptyset \& \pi(g(x)) \neq \emptyset \& \\
\& \text{OP}'( \\
\{u \in \pi(g(X)) : [\varphi]'(\{(g, \pi[g(x)] : \{u]\}[g(X) : \{u]\}))) = \emptyset\} , \\
\{u \in \pi(g(X)) : [\varphi]'(\{(g, \pi[g(x)] : \{u]\}[g(X) : \{u]\})) = 0\}).
Fourth, if we take the approach proposed here, we have to state how the semantic value of a bipartite representation is to be calculated. This is simple: all the grounding sub-formulae are presuppositional, because a semantic-value gap must arise if a referent cannot be grounded (for example, if the antecedent of an anaphor cannot be retrieved). The top-level semantic-value function, written $\llbracket \cdot \rrbracket_G$, assigns every bipartite formula a partial function from information states (sets of peg functions) to information states. The set $G$ of assignment functions is just a parameter that plays a role formula-internally: the semantic value of a closed bipartite formula does not depend on $G$. So, if $(g, c)$ is a bipartite representation with grounding $g$ and claim $c$, and $\Pi$ is an information state (a set of peg functions), then

$$[(g, c)]_G(\Pi) = \begin{cases} * & \text{if } [g]'(G \times \Pi) = * \text{ or } [g]'(G \times \Pi) = \emptyset; \\ \{\pi : \bigvee_{h \in \text{Var} \cap \text{N}} \langle h, \pi \rangle \in [g \land c]'(G \times \Pi)\} & \text{otherwise.} \end{cases}$$
4. Bipartite Meanings

4.2. Grounding and Claim in Natural Language

As is clear from the previous section, the bipartition of semantic representations corresponds to a distinction between two types of presupposition, namely, the ones arising from the definedness conditions of \([\cdot]\)', on the one hand, and the ones that stem from the definition of \([\cdot]_G\), on the other. These two types of presupposition correspond to what we have called external and internal presuppositions in Goldberg et al. (1990), where we have argued that these are different in various respects.

An external presupposition is essentially propositional: \textit{Joe left Paris} externally presupposes ‘Joe was in Paris’ because the proposition ‘Joe was in Paris’ must be true in order for \textit{Joe left Paris} to make sense. On the other hand, \textit{Joe called the director} presupposes that the hearer is able to identify a unique individual as the possible referent of \textit{the director}, and this piece of information is not propositional in character. Its non-propositional nature is clear from its instability. Under normal circumstances, if an information state entails a proposition, then all its extensions (i.e., more specific, more informative information states) will also entail it. This is true even for downward-entailing formulae: after uttering \textit{At most two people are in the park}, it will lead to a contradiction to suggest that there are three or more people in the park. So propositional information in an information state is stable. On the other hand, if the hearer is able to identify a unique individual as ‘the man walking in the park’ in an information state, and then \textit{Another man is watching him from the bushes} is uttered, that does not lead to a contradictory information state. So external presuppositions behave differently from internal ones in terms of stability.

In general, internal presuppositions require that the speaker identify some entity, which involves the retrieval of some information provided earlier, whereas external presuppositions must be verified by the speaker. In this sense, external presuppositions are ‘speaker-oriented’: if \(S\) presupposes \(\varphi\), then a speaker uttering \(S\) is bound to commit himself/herself to the truth of \(\varphi\), which the hearer can either accept or reject. Internal presuppositions, on the other hand, are of an ‘interpersonal’ character: if \(S\) has the anaphoric presupposition that a certain entity \(e\) can be successfully identified, then not only the speaker, but also the hearer must be able to identify \(e\) in order for the utterance of \(S\) to be successful.

The linguistic manifestations of these differences are numerous. Among the examples of presuppositions that I have used in earlier examples, ‘lexically-induced’ presuppositions, such as the presuppositions of factive or inchoative verbs are prototypical external presuppositions, whereas anaphoric expressions, especially anaphoric pronouns, are prototypical elements carrying internal presuppositions.
The unidirectionality of variable binding between the grounding part and the claim in a translation embodies a substantive prediction about the semantics of possible natural-language sentences. No internal presupposition may refer to entities that are introduced in the claim, whereas the claim may mention entities grounded in the previous context. As we will see below, entities introduced by claims are the referents of predicate-internal, ‘non-specific’ indefinite noun phrases, new reference times introduced by accomplishment and achievement verbs, etc. The prediction is, thus, that nothing can be internally presupposed about such entities. For example, no translation can be assigned to the following sentence (under the reading in which the subscripted noun phrases are obligatorily coreferent):

(26) *The captain of [this ship] sank [a ship].

Note that this applies to internal presuppositions only: although *Joe left someone presupposes ‘Joe was with that person previously’, which mentions the referent corresponding to the ‘non-specific’ someone, this presupposition is typically external: it is of the ‘lexically induced’ kind.

One interesting feature of the approach based on bipartite representations is that various types of grounding can be distinguished. The type of grounding particular for anaphoric pronouns, for example, consists in retrieving a unique discourse referent that must have a value identical to the one in the translation. This can be achieved by translating such expressions with a quantifier expressing ‘exactly one’:

(27) The king of France is bald.
   ([∃!x(king-of-France(x))], [bald(x)])

(I have enclosed the grounding and the claim in square brackets in the above translation for better readability.) True, the king of France usually is not ‘anaphoric’ in the same way an anaphoric pronoun is, but that difference is probably an independent issue. In terms of its grounding properties, it patterns together with anaphoric pronouns in that it requires an antecedent to be considered identical with its referent.

The existence of another main type of grounding, that of discourse-linked indefinite referents, provides an excellent motivation for the assumption of bipartite meanings. Consider:

(28) a. I met a nun yesterday.
    b. A nun has not arrived yet.

The (predicate-internal) indefinite noun phrase a nun in (28a) corresponds to a non-discourse-linked (‘non-specific’) indefinite: it gives rise to a novel referent which the hearer may safely introduce as ‘the nun that the speaker met yesterday’
without checking any contextual condition. In (28b), the same indefinite noun phrase occurs predicate-externally, and acts as a so-called discourse-linked ('specific') indefinite. It also gives rise to a novel referent, but one that must be linked somehow to some entity available in the context (namely, it must be an element of a set familiar from the context). (The concept of discourse-linked indefinites was introduced by Eng (1991).)

Obviously, there is no difference between (28a) and (28b) in terms of how the indefinite noun phrase in them contributes to either the truth conditions or the context-changing potentials of the respective sentences. There need not even be a difference in terms of 'speaker’s reference' ('whether the speaker has a particular individual in mind'). So it is not possible to account for their difference in a semantic theory that lacks a concept of grounding. The only difference in the interpretation of the noun phrase in question between (28a) and (28b) lies in the way in which the hearer is supposed to introduce the corresponding referent. The existence of a nun is claimed in (28a), whereas it is internally presupposed in (28b). The translations must differ in whether the existential sub-formula figures in the first or the second component of the bipartite representation:

\[
(28')
\begin{align*}
(a) & \quad I \text{ met a nun yesterday.} \\
& \quad ([T], \exists x(\text{nun}(x)) \land \text{met-yesterday}(i, x)) \\
(b) & \quad A \text{ nun has not arrived yet.} \\
& \quad ([\exists x(\text{nun}(x))], \neg \text{arrived}(x))
\end{align*}
\]

The semantic value of the special formula \(T\) in (28'a) is the identity function: its effect is that no grounding is needed to interpret the claim. The translation in (28'b), on the other hand, presupposes the existence of at least one nun: a set of individuals some of which are nuns must be retrieved by the hearer. If the discourse context contains exactly one nun, then (28'b) is not felicitous for Gricean reasons (the speaker is supposed to use the nun in that case). So the above translations yield appropriate interpretations under a plausible assumption on the pragmatics of grounding.

Another important phenomenon that involves grounding and internal presuppositions is free focus. As we have argued in Kálmán and van Leusen (1993), a sentence containing a focussed constituent presupposes the contextual availability of an open sentence or ‘predicate’ which must apply exhaustively to the free variable that it contains; the sentence claims that the unique entity in question is identical to what the focussed constituent refers to:

\[
(29) \quad \text{Joe met MARY.} \\
& \quad ([A X \land \exists x(MX, x(\text{met}(j, x)))], [X = m])
\]

According to this translation, the sentence in (29) internally presupposes the existence of a maximal entity \(X\) such that \(X\) is the set of people that Joe met, and it
claims that the only member of that set is Mary. The presupposition of such sentences is typically internal: as we have argued in Kálmán and van Leusen (1993), the piece of information that such an exhaustive set exists must be present in the immediate discourse context, just like the antecedent of a pronominal anaphor. Bipartite meaning representations offer a natural way of producing this type of interpretation.

Note that the grounding part of the translation in (29) is exactly like the assignment part of a quantificational structure. As a matter of fact, the layout of a quantificational structure may be similar to the formula in (29):

\((30)\) *Every dog barks.*

a. \([\text{Ax} \land \exists x(MX, x(dog(x)))), \text{everyX, } x(barks(x))])\]

b. \([\{T\}, \text{Ax} \land \exists x(MX, x(dog(x))) \land \text{everyX, } x(barks(x))]])\]

The translation in (30a) is more similar to the translation of (29) than the one in (30b). It corresponds to ‘all the(se) dogs bark’, i.e., to the reading in which a set of dogs is to be grounded in the previous context; the formula in (30b), on the other hand, corresponds to a reading with an unlimited domain: there is no need for context-based grounding in that case.

4.3. Conclusion: Compositionality Issues

The similarity between the translations of conditional and universal sentences, characteristic for other dynamic theories (such as Kamp (1981)) as well, is certainly attractive. (In other dynamic theories, such as Heim (1982) or Groenendijk and Stokhof (1991), they are only semantically equivalent.) However, as Kamp (1981) points out, it leads to a mismatch between the syntactic similarity between quantificational noun phrases and other noun phrases, on the one hand, and their semantic dissimilarity, on the other. The same type of mismatch characterizes the translations that I have proposed for definite descriptions (the translation of which belongs to the grounding part irrespective of their syntactic position), and for sentences containing free focus (where the translation of the focussed constituent figures in the main assertion irrespective of its syntactic role). These are *prima facie* violations of the principle of *compositionality* which, under certain interpretations (e.g., Groenendijk and Stokhof (1991)) would require an exact match between the syntactic constituents of a sentence and the sub-formulae of its translation. For Kamp (1981), compositionality is an empirical issue rather than a methodological principle; in my view, it is a methodological principle, but it should not be interpreted as a requirement of ‘sub-formula preservation’.

I wish to make two empirical remarks and a more general methodological remark in this connection.
The first empirical remark is that quantified noun phrases are not *that* similar to other noun phrases after all. It is true that *every dog* patterns together with *the dog* in many respects, but there are important differences between them. First, in many languages, the quantifier in the noun phrase, unlike a regular determiner, has an obviously intimate relationship with the predicate and can be ‘floated’ there, as the following French examples show:

(31) a. *Tous les bergers ont vu un messager.*
    all the shepherds have seen a messenger
    ‘Every shepherd saw a messenger’

b. *Les bergers ont tous vu un messager.*
    the shepherds have all seen a messenger
    ‘The shepherds all saw a messenger’

Second, quantificational noun phrases, unlike regular noun phrases, cannot be topicalized (*As for every dog, ⋯*). In Hungarian, they cannot act as ‘neutral topics’ (‘what the sentence is about’):

(32) a. *Józsi tegnap felhívott.*
    Joe yesterday up-called
    ‘Joe called me yesterday’

b. *Mindenki tegnap felhívott.*
    everyone

c. *OK Tegnap mindenki felhívott.*
    ‘Everyone called me yesterday’

As can be seen, in Hungarian, a regular noun phrase such as *Józsi ‘Joe’* in (32a) can be separated from the predicate by an adverbial, whereas a quantified one cannot (see (32b)). This suggests that quantificational noun phrases must be predicate-internal, which would harmonize with the assumption that quantifiers always belong to the main assertion. (The fact that the translation of the quantifier is the main operator in the semantic representation of quantified sentences in theories of generalized quantifiers such as Barwise and Cooper (1981) also points in this direction.)

The second empirical remark is that, while quantified noun phrases and definite descriptions can be given ‘compositional’ treatments (in the sense of ‘sub-formula preservation’) in various theories, no such option is available for free focus. Every formal semantic theory of free focus has assumed that the translation of a sentence containing free focus contains the translation of the focussed constituent and that of the ‘remnant’ as sub-formulae, even though the syntactic relationship between these varies from one sentence to the other. The focussed constituent may be a full noun phrase, a modifier in a noun phrase, an adverbial, the verb itself, and so on. So mismatches seem inevitable at least in these cases.
I submit that the concept of 'compositionality' in the sense of 'sub-formula preservation' is both too strong and too weak. Too strong, because it excludes what seem the only reasonable approaches to the analysis of free focus (and, probably, quantificational structures). Too weak, because it hinges on how firm our beliefs are about syntactic structure. For example, in recent 'government and binding' theory, quantifiers and focus as 'functional categories' can easily count as separate, maybe even immediate, constituents of sentences.

An alternative view of compositionality would reduce it to a sort of context-independence: when calculating the translation of a sentence from its syntactic and lexical build-up, all and only the linguistic information coming from the syntax and the lexicon must be taken into account. No information may be lost, and no information may come from external sources. This is a perfectly legitimate reading of the slogan 'The meaning of a complex expression is a function of the meanings of its parts an the way in which they are put together', yet it does not imply anything about 'sub-formula preservation'.

It is in this sense that the present paper has been building on the assumption of an autonomous semantics: the structural properties of the proposed meaning representations, especially their bipartition, is only superficially related to either the underlying logic or the underlying syntactic structure. The logic would allow us to posit entirely different structures, as would the various different syntactic structures observed across languages. The structures proposed are purely motivated by (hopefully adequate) generalizations about the semantic structure of possible natural-language utterances.

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References


Heim, I. 1983. ‘On the projection problem for presuppositions’. In M. Barlow et al., eds., *Proceedings of the 2nd WCCFL*. Stanford University, Stanford CA.


