

TIK 156.436



KFKI
17 / 1966

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INTERACTION INDUCED BY PARAMAGNETIC IMPURITIES
AND ITS EFFECT IN SUPERCONDUCTING ALLOYS

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BUDAPEST

Published
by the
Publishing Group of the Central Research
Institute for Physics
Budapest
on Romayor mimeograph

19, Dez. 1966.

Registration No: 2857

No. of copies: 175

Responsible for duplication: I. Gyenes

New type of anomaly in the electron - electron
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its effect in superconducting alloys

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A new type of anomaly has been found in the electron - electron interaction induced by paramagnetic impurity spin excitations. In the case of antiferromagnetic conduction - electron - impurity - spin interaction this anomalous term is an attractive one and may give rise to the increase of the superconducting transition temperature.

In the recent years Matthias and his co-workers¹ performed numerous experiments to investigate the transition temperature of superconducting alloys. They have found that in the majority of cases the transition metal impurities depress T_c . In e.g. Ti based alloys², however, an extra increase of the transition temperature has been found. To account for this, Matthias et al.³, have postulated an enhanced attraction caused by virtual levels interacting by magnetic exchange.

The effect of paramagnetic impurities upon the critical temperature has been theoretically investigated by Abrikosov and Gorkov⁴, using a ladder approximation. The effective electron - electron interaction has been determined in the Born approximation, but this calculation can account only for the decrease of T_c . After the discovery of the Kondo anomaly⁵ and the Abrikosov - Suhl resonance^{6,7} in the scattering of conduction electrons on paramagnetic impurities, Griffin⁸ discussed the effect of this resonance on T_c . Only electron - electron scattering without energy exchange was considered, and the effect of the paramagnetic impurities was shown to be the lowering of T_c .

Calculating the electron - electron scattering with energy exchange, we have found a new type of anomaly in the effective electron - electron interaction mediated by impurity spin excitations. The first non-vanishing anomalous term appears in the third order of the perturbation theory and gives an attractive or repulsive interaction depending on the sign of the s-d exchange interaction. It will be shown that in some peculiar cases this

attractive interaction may overcome the repulsive elastic one mentioned above. This then can cause the increase of the transition temperature in dilute superconducting alloys with increasing concentration of the impurity.

Abrikosov's method /and notations/⁶ are used with the fictitious fermion particles in place of impurity spin S /dotted lines in the diagrams/. The general effective electron - electron interaction appearing in Abrikosov and Gor'kov's ladder approximation may be represented by the diagram in Fig. 1, where Γ stands for the general vertex function.

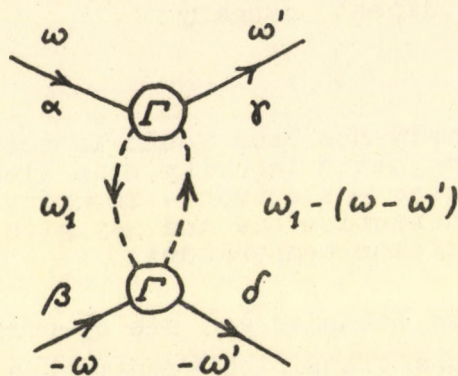


Fig.1.

General electron - electron interaction appearing in the ladder approximation.

Abrikosov and Gor'kov used the unrenormalized vertex /Fig.2.a/ which gives contribution only for $\omega = \omega'$. Griffin adopted the vertex part calculated by Abrikosov and Suhl to take into account the resonance scattering but neglected the energy exchange.

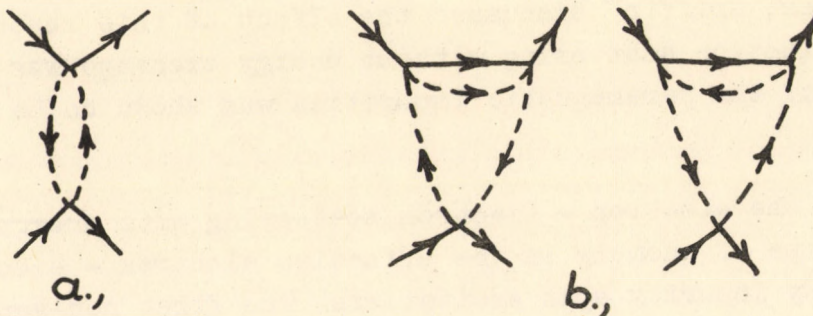


Fig.2.

Second - /a/ and third-order /b/ diagrams in the electron - electron scattering.

The calculation of the energy exchange seems to be very important because it corresponds to the virtual excitations of the impurity-spin. In order to perform this calculation, the vertex function with off - energy - shell electron - and spin - energy variables should be evaluated but this is not feasible. The fact that the Kondo effect determined in the third order of the perturbation theory is in good agreement with the experimental results, suggests to carry out the calculation only up to the first non - vanishing term. This is likely to be a good approximation for not too low temperatures. In the first non - vanishing contribution /third order/ one of the vertex functions is the unrenormalized one and the other is corrected in the second order /Fig.2.b/.

A straightforward calculation gives for the contribution of the diagrams in the second and third order using the Kondo Hamiltonian

$$V^{(2)}(\omega, \omega') = \frac{1}{T} \frac{1}{3} S(S+1) \left(\frac{J}{N}\right)^2 (\vec{\sigma}_{\alpha\gamma} \vec{\sigma}_{\beta\delta}) d_{\omega, \omega'} \quad /1/$$

$$V^{(3)}(\omega, \omega') = V_{Kondo}^{(3)} + V_{anomalous}^{(3)} \quad /2/$$

where

$$V_{Kondo}^{(3)} = -\frac{1}{T} \frac{1}{3} S(S+1) \left(\frac{J}{N}\right)^2 \rho^{(0)} \int_{-\epsilon_F}^{\epsilon_F} d\epsilon \frac{\epsilon}{\epsilon^2 + \omega^2} \text{th} \frac{\epsilon}{2T} (\vec{\sigma}_{\alpha\gamma} \vec{\sigma}_{\beta\delta}) d_{\omega, \omega'} \quad /3/$$

$$V_{anomalous}^{(3)} = \frac{2}{3} S(S+1) \left(\frac{J}{N}\right)^2 \rho^{(0)} \frac{\pi}{\omega - \omega'} (sg \omega - sg \omega') (\vec{\sigma}_{\alpha\gamma} \vec{\sigma}_{\beta\delta}) \quad /4/$$

where $\rho^{(0)}$ is the density of states per atom, and $\vec{\sigma}_{\alpha\beta}$ is the Pauli matrix. The first term in $V^{(3)}$ is the usual Kondo anomaly without energy exchange and this may be neglected here⁹. The second term shows a new type of anomaly as it contains the factor $(|\omega| + |\omega'|)^{-1}$ for processes in which the energy variable changes its sign. The origin of this strong new divergency is the virtual polarization of the impurity - spin system and it is expected that higher-order terms would contain similar divergencies. This interaction is attractive or repulsive depending on the sign of J.

To determine the transition temperature, Abrikosov and Gor'kov⁴ investigated a $K_{\omega \alpha, \beta}(p_1, p_2)$ quantity which is derived from the scattering amplitude of an electron pair. Including the new anomalous term the equation for $K_{\omega \alpha, \beta}(p_1, p_2)$ is modified as follows:

$$K_{\omega \alpha, \beta}(\rho, -\rho) = \tilde{G}_{\omega}(\rho) \tilde{G}_{-\omega}(-\rho).$$

$$\left\{ g_{\alpha\beta} + nT \sum_{\omega} \frac{1}{(2\pi)^3} \int d^3\rho' [u_1^2 + \right. \\ \left. + \left[\frac{1}{T} \frac{1}{3} S(S+1) \left(\frac{\tau}{N}\right)^2 d_{\omega, \omega'} + \frac{2}{3} S(S+1) \left(\frac{\tau}{N}\right)^2 \rho(0) \frac{\pi}{\omega - \omega'} (sg\omega - sg\omega') (\vec{\sigma}_{\lambda\alpha} \vec{\sigma}_{\rho\beta}) \right] \right. \\ \left. \cdot K_{\omega \lambda, \rho}(\rho_i - \rho') \right\}$$

where $\tilde{G}_{\omega}(\rho)$ is the renormalized electron Green's function, and

$$g_{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}_{\alpha\beta}$$

and n is the number of impurities, and u_1 the amplitude of the potential scattering.

Introducing the notation

$$\frac{1}{2\pi} \int_{-\epsilon_F}^{\epsilon_F} K_{\omega \alpha, \beta}(\rho, -\rho) d\epsilon = K_{\omega} g_{\alpha\beta} \quad /6/$$

where $\epsilon = \frac{p^2}{2m} - \epsilon_F$, we get

$$K_{\omega} = \frac{1}{2|\omega|\eta_s} \left[1 - \frac{n}{N} 4\pi^2 S(S+1) \tau^3 \rho(0) T \sum_{\omega' > 0} \frac{1}{|\omega| + \omega'} K_{\omega'} \right] \quad /7/$$

where η_s is the same as in the classic paper of Abrikosov and Gor'kov⁴.

The transition temperature can be determined from an equation which is similar to that of Abrikosov and Gor'kov's:

$$\log \frac{T_{c0}}{T_c} = 2 \sum_{n=0}^{\infty} \frac{1}{2n+1} (1 - 2|\omega_n| K_{\omega_n}) \quad /8/$$

Solving /7/ by an iteration procedure, the transition temperature can be obtained as a power series expansion in the concentration, $c = \frac{n}{N}$. For the sake of simplicity we retain only the term, proportional to the concentration, i.e. we calculate $\frac{dT_c}{dc}$ for $c=0$.

The final result is:

$$\frac{dT_c}{dc} \Big|_{c=0} = 2 \tau^2 S(S+1) \rho(0) \left\{ -\frac{2}{3} \frac{\pi^2}{8} - 2 \tau \rho(0) A \right\} \quad /9/$$

where $A = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{1}{2n+1} \frac{1}{2m+1} \frac{1}{(2n+1)+(2m+1)} \approx 1.05$

Thus in the case of antiferromagnetic coupling the transition temperature may be raised by the anomalous electron - electron interaction via paramagnetic impurities if the effective coupling $f\rho(0)$ is strong enough:

$$(-f)\rho(0) > 0,39$$

/10/

It is worth mentioning that this condition is independent of the spin value and is likely to be fulfilled in metals with high density of states. Supposing $|f| \sim 0,33 \text{ eV}$ the requirement is $\rho > 1,17 (\text{e.V. atom})^{-1}$ which is the case e.g. for Ti, V, Os, Ru, considering only non - magnetic metals. The experimental data obtained by Matthias et. al.² for Ti based alloys may be fitted with $\rho = 1,48 / \text{eV atom}^{-1}$ ¹⁰ and $J = 0,26 - 0,3 \text{ eV}$.

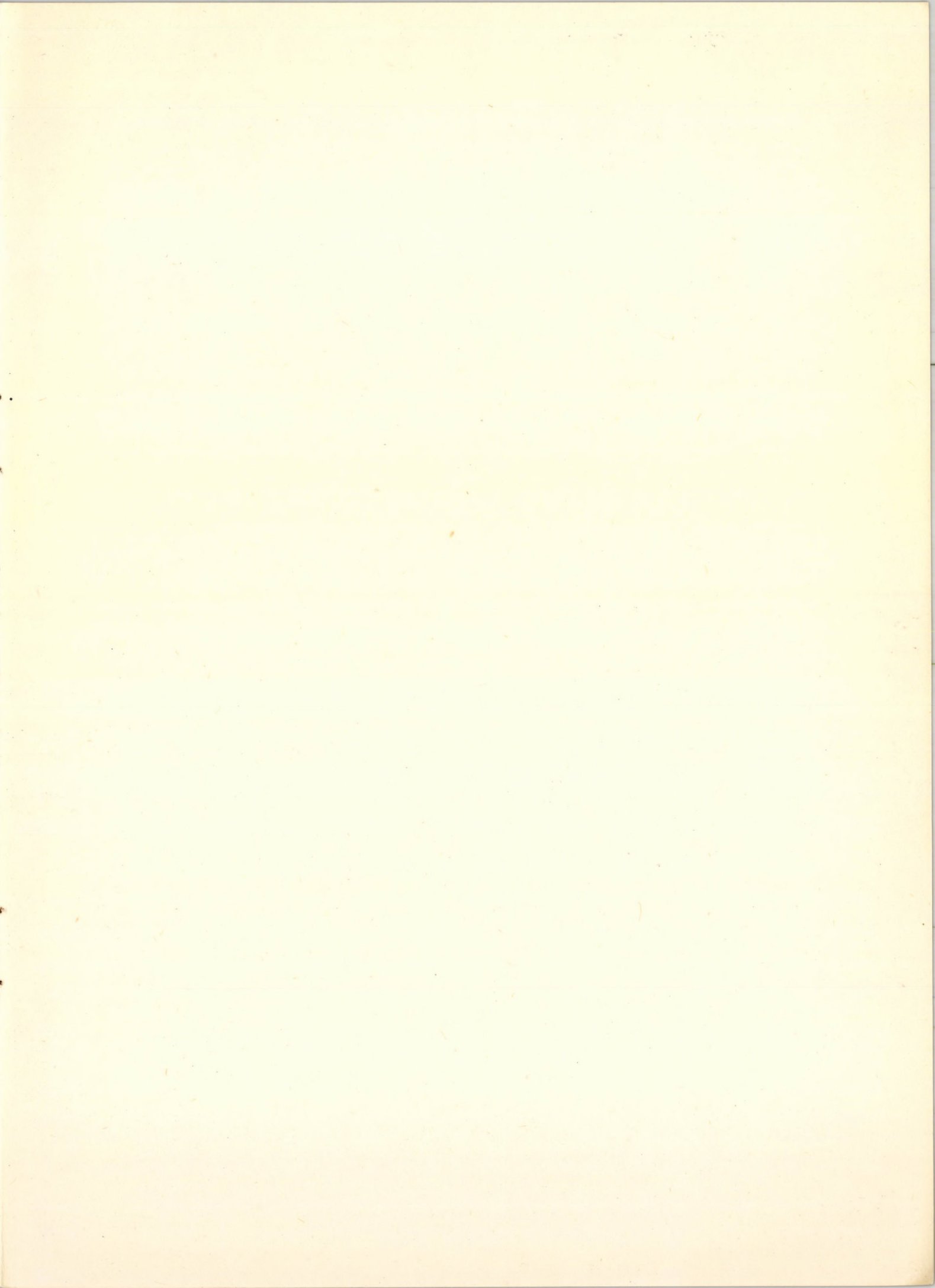
It should be stressed that in the above considerations the occurrence of localized moments on the impurities is quite irrelevant¹¹.

In view of some contradictory experimental results¹² and another theoretical explanation¹³ it is hard to say if the proposed mechanism is of great importance in Ti based alloys. We think, however, that in those cases for which Matthias and his co - workers supposed the existence of some exchange type interaction this anomalous term may be at least in part responsible for the unusual concentration dependence of T_c .

We are grateful to Prof. L.Pál for his continuous interest and Dr. Cs. Hargitai for critical discussions.

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65231