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GENERATION OF FREQUENCY TUNED PICOSECOND PULSES BY FOUR-WAVE MIXING

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ABSTRACT

The parametric nonstationary four wave phase conjugation process is investigated in nonlinear media without absorption and inertia. The reflected phase conjugated pulse is shortened by shortening the duration of one of the reference pulse propagating in the same direction. In noncollinear scheme of four wave phase conjugation the frequency of the reflected short pulse can be tuned by tuning the frequency of the signal wave of much longer duration than the one of the reflected and the reference short pulse beam.

АННОТАЦИЯ

Исследован нестационарный режим четырехволнового параметрического обращения волнового фронта /ОВФ/ в безинерционной непоглощающей нелинейной среде. Показана возможность укорочения импульса обращенной волны за счет укорочения импульса попутной опорной волны. При этом в неколлинеарной схеме невырожденного режима ОВФ предложен способ перестройки частоты генерируемого короткого импульса обращенной волны за счет перестройки частоты сигнальной волны, имеющей существенно бо́льшую длительность импульса.

KIVONAT

Abszorpció és tehetetlenség nélküli nemlineáris közegben megvizsgáljuk a parametrikus nemstacionarius négyhullámu keverés folyamatát. A reflektált fáziskonjugált hullám követi a legrövidebb impulzus alakját, a frekvenciája pedig a beeső "jel" hullám frekvenciáját. Igy hangolható pikoszekundumos impulzusok állithatók elő.

1. INTRODUCTION

The scheme of phase conjugation by four wave mixing is well known [1]. The signal wave is reflected with phase conjugation if it is mixed with two reference waves propagating in opposite to direction to each other in the nonlinear material. This process can be theoretically easily described if the interacting waves are stationary sinusoidal waves.

The process of four-wave mixing is investigated for the case of the interacting beams are pulses of different duration and different frequencies (see also [2,3]). It can be found, that the reflected pulse can be shortened by shortening one pulse of the reference beam and that the frequency of the reflected beam can be tuned by tuning the frequency of the signal beam.

2. COLLINEAR BEAMS

The scheme of the interacting beam can be seen in *Fig. 1.* It is supposed that the medium is transparent at frequencies of the beams and the relaxation time of the medium is shorter than the duration of the shortest pulse in the interaction. The electrical field strength of the four interacting wave E_j takes the following form

$$\underline{E}_j = \frac{1}{2}\underline{e}_j A_j(\underline{r},t)e^{i(\underline{k}_j\underline{r}-\omega t)} + c.c$$

where

$$j = 1, \dots 4$$

The amplitude of the reference waves, the signal and the reflected waves are $A_1(\omega_1)$, $A_2(\omega_2)$, $A_3(\omega_3)$ and $A_4(\omega_4)$, respectively. $\underline{e}_j$, $\underline{k}_j$, ω_i are the polarisation vector, the wave vector and the frequencies. The polarisation of the waves are the same and A_1, A_2, A_3 are given at the beginning. Then the reflected wave amplitude fullfils the equation

$$\left(\frac{\partial}{\partial z} + \frac{1}{v_4} \cdot \frac{\partial}{\partial t}\right)A_4 = i\gamma e^{i\Delta k_z \cdot z} \cdot A_1\left(t-\frac{z}{v_1}\right) \cdot A_2\left(t+\frac{z}{v_2}\right) \cdot A_3^*\left(t-\frac{z}{v_3}\right) \tag{1}$$

in the approximation of the slowly varying amplitude. v_j $(j=1,2,3,4)$ are the group velocities of the waves where $v_1=v_2=v$. γ is the factor containing the susceptibility of third order of the nonlinear material. Δk_z is the projection of the phase mismatch $\Delta\underline{k} = \underline{k}_1+\underline{k}_2-(\underline{k}_3+\underline{k}_4)$ on the z axis, which is the direction of propagation of the beams. If $\omega_1=\omega_2=\omega$, $\omega_3=\omega+\delta$ and $\omega_4=\omega-\delta$, $\Delta\underline{k}= -(\underline{k}_3+\underline{k}_4)$. The mismatch is minimum if the direction of propagation of the signal wave is opposite to the propagation direction of the reflected wave i.e.

$$\Delta k_z = \Delta k = k_4-k_3 = \frac{\omega-\delta}{c}n(\omega-\delta) - \frac{\omega+\delta}{c}n(\omega+\delta)$$

where $n(\omega)$ is the index of refraction of the material. The solution of the equation (1) at z=0 (see *Fig. 1*) has the form

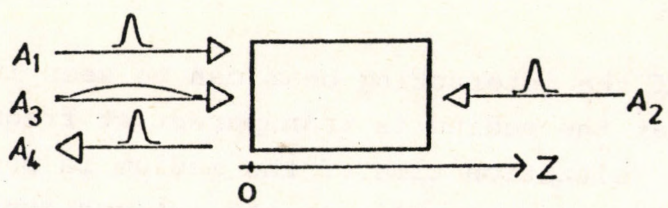

Fig. 1.

Collinear scheme of nondegenerated parametric four wave phase conjugation

$$A_4(z=0,t) = i\gamma A_2(t)\int_0^L dz'\; r^{i\Delta k_z z'}\cdot A_1\left(t-z'\left(\frac{1}{v}+\frac{1}{v_4}\right)\right)\cdot A_3^*\left(t-z'\left(\frac{1}{v_4}+\frac{1}{v_3}\right)\right) \tag{2}$$

where

$$\left(\frac{1}{v} - \frac{1}{v_4}\right)L \ll \tau$$

was supposed. L is the length of the nonlinear material and τ is the width of the pulse A_2. In the following, take the amplitude of the beams $A_1 = A_3$. If τ is much less than the duration of the signal ($\tau \ll \tau_3$) and the propagation time back and forth through the medium, then the duration of the reflected pulse τ_4 is roughly equal to τ ($\tau_4 \simeq \tau$). Let us investigate this effect in more detail. Suppose - as an example - that the interacting waves are of Gaussian type

$$A_{1,2} = A^o_{1,2}\exp\left(-\frac{2t^2}{\tau^2}\right); \qquad A_3(t) = \exp\left(-\frac{2(t-\Delta t)^2}{\tau_3^2}\right) \tag{3}$$

Δt is the time lag between the signal (A_3) and the reference ($A_{1,2}$) beams. The intensity of the reflected wave $I_4(o,t) = |A_4(o,t)|^2$ is given according to the expression (2) as

$$I_4(o,t) = \frac{\pi|\gamma|^2}{32} I_{10}\cdot I_{20}\cdot I_{30}\cdot v^2\cdot\tau^2\cdot(1-\xi)\cdot \exp\left\{-\frac{4t^2}{\tau^2} - \frac{4\Delta t^2}{\tau_3^2} - \frac{\tau^2\cdot\Delta k^2\cdot v^2}{16}(1-\xi)\right\}\cdot \left|\mathrm{erf}\left\{\frac{\sqrt{2}}{\tau}\left[\frac{2L}{v} - t - \frac{iv\cdot\Delta k\cdot\tau^2}{8\sqrt{2}} + \frac{\xi}{2}\left(\frac{2L}{v} - t + 2\cdot\Delta t + i\frac{v\Delta k\cdot\tau^2}{4\sqrt{2}}\right)\right]\right\} + \mathrm{erf}\left\{\frac{\sqrt{2}}{\tau}\left[t + i\frac{v\cdot\Delta k\cdot\tau^2}{8\sqrt{2}} + \frac{\xi}{2}\left(t - 2\Delta t - i\frac{v\Delta k\cdot\tau^2}{4\sqrt{2}}\right)\right]\right\}\right|^2 \tag{4}$$

where

$$\mathrm{erf}(X) = \frac{2}{\sqrt{\pi}} \int_0^X e^{-t^2}\, dt; \qquad \xi = \frac{\tau^2}{\tau_3^2} << 1$$

It can be seen from expression (4) that the intensity of the reflected wave decreases with increasing mismatch Δk in the case of the four wave mixing discussed. The bandwidth of the possible frequency tuning can be given as

$$\Delta\omega < \frac{c}{n(\omega+\delta)+n(\omega-\delta)} \cdot \frac{1}{\tau \cdot v} , \qquad (5)$$

which means that the bandwidth of frequency tuning is about the same as the bandwidth of the Gaussian reference beam of width τ.

In the case of "short" nonlinear medium i.e. if $\tau > \frac{2L}{v}$ the expression (4) is the same as that well known expression given for the quasitationary scattering.

In the opposite case of "long" nonlinear medium i.e. if $\tau << \frac{2L}{v}$ we can get from expression (4)

$$I_4(o,t) = \frac{\pi}{32}|\gamma|^2 \cdot I_{10} \cdot I_{20} \cdot I_{30} \tau^2 \cdot v^2 (1-\xi) \cdot$$

$$\exp\left\{- \frac{4t^2}{\tau^2} - \frac{4\Delta t^2}{\tau_3^2} - \frac{\tau^2 \cdot \Delta k^2 \cdot v^2}{16}(1-\xi)\right\} \cdot \qquad (6)$$

$$\left| \mathrm{erf}\left\{\frac{\sqrt{2}}{\tau}\left[\frac{2L}{v} - i\frac{v\Delta k \cdot \tau^2}{8\sqrt{2}} + \frac{\xi}{2}\left(\frac{2L}{v} + 2 \cdot \Delta t + i\frac{v \cdot \Delta k \cdot \tau^2}{4\sqrt{2}}\right)\right]\right\} \right|^2$$

It can be seen from expression (6) that the pulse width of the reflected wave is the same as that of the reference wave τ. The bandwidth of frequency tuning $\Delta\omega \sim 10^{12}$ Hz if $\tau = 10^{-12}$ sec. Consequently - in case of the collinear four wave mixing discussed - it is possible to generate reflected pulse of picosecond duration and with conjugated wave front. But the range of frequency tuning is very restricted. To widen the tuning range let us investigate the case of noncollinear four wave mixing.

3. NONCOLLINEAR FOUR WAVE MIXING

The geometry of noncollinear four-wave mixing is given in *Fig. 2*. Here we have supposed that wave vectors of the interacting beams are in the XZ plane. The solution of the equation (1) is given in that case as

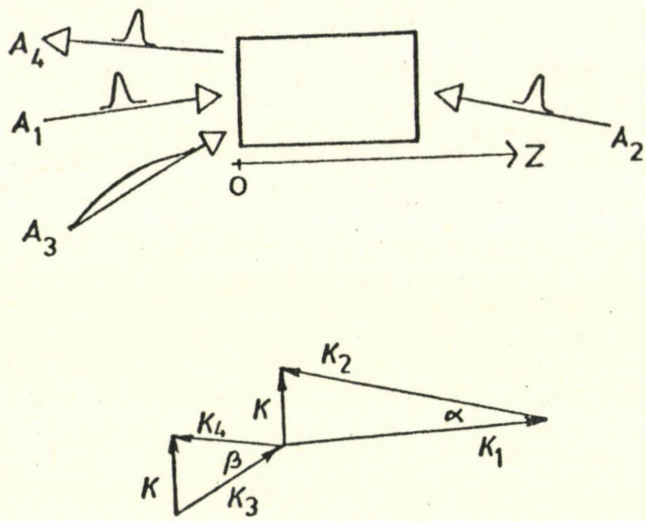

Fig. 2.

Noncollinear scheme of nondegenerated parametric four wave phase conjugation process. $\underline{k}=\underline{k}_1+\underline{k}_2=\underline{k}_3+\underline{k}_4$

$$A_4(z=o,x,t)=i\gamma\int_0^L A_1\left\{t-\frac{x\sin(\beta+\alpha/2)}{v}-\xi\left[\frac{1}{v_4}+\frac{\cos(\beta+\alpha/2)}{v}\right]\right\}\cdot$$

$$\cdot A_2\left\{t+\frac{x\sin(\beta-\alpha/2)}{v}-\xi\left[\frac{1}{v_4}-\frac{\cos(\beta-\alpha/2)}{v}\right]\right\}\cdot A_3^*\left\{t-\frac{x\sin\beta}{v_3}-\xi\left[\frac{1}{v_4}+\frac{\cos\beta}{v_3}\right]\right\}d\xi .$$

In case of interacting beams of Gaussian type given by expression (3) and in the most interesting case of "long" nonlinear medium ($\tau<<\frac{2L}{v}$) the intensity of the reflected wave is given as

$$I_4(o,x,t)=|\gamma|^2\cdot I_{10}\cdot I_{20}\cdot I_{30}\cdot\frac{\pi}{32}\cdot\tau^2\cdot v^2\cdot$$

$$\exp\left\{-\left[\frac{8(t-x/v_4\cdot(\beta+\alpha/2\cdot v_4/v))^2}{2}\right]\right\}\cdot \tag{7}$$

$$\exp\left\{-\left[\frac{f(t,x)}{\tau_3^2}\right]\right\}\cdot\left|\mathrm{erf}\left(-\frac{2\sqrt{2}\cdot L}{\tau\cdot v}\right)\right|^2 .$$

α and β are the angles between the vectors $\underline{k}_1$, $\underline{k}_2$ and $\underline{k}_3$, $\underline{k}_4$ respectively (see *Fig. 2*) and

$$f(t,x) = \left\{-\left[\frac{(v-v_3)}{2v_3}\cdot t - \frac{x}{2vv_3}\left[v\left(\beta-\frac{\alpha}{2}\right)-v_3\left(\beta+\frac{\alpha}{2}\right)\right]+\Delta t\right]\right\}^2$$

Furthermore $v_4=v$ was also supposed. The relation between the angle α and β is given by the phase synchronism $\underline{\Delta k}=0$ from where

$$\sin\frac{\alpha}{2} = \pm\sqrt{\sin^2\frac{\beta}{2}+\frac{\delta^2}{\omega^2}\cdot\cos^2\left(\frac{\beta}{2}\right)} \tag{8}$$

For simplicity the dispersion of the medium is neglected in the range $\omega\pm\delta$.

If the frequency offset δ is changed i.e. the frequency of the signal beam us tuned $(\omega+\delta)$ and the angle between the signal and the reflected beam is not changed (β=const.) the angle between the reference beams has to be changed according to the expression (8). Consequently the frequency of the picosecond reflected beam can be tuned in the whole range $(\frac{\Delta\omega}{\omega}<1)$ of the frequency tuning of the signal beam.

If the spectrum of the signal beam is wide the frequency of the picosecond reflected beam can be tuned by changing the angle $\beta(\delta)$ according to the expression

$$\sin\frac{\beta}{2} = \pm\frac{1}{\sqrt{1-\delta^2/\omega^2}}\sqrt{\sin^2\frac{\alpha}{2}-\frac{\delta^2}{\omega^2}} \tag{9}$$

gotten from expression (8) supposing α=const. Here

$$-\omega\sin\frac{\alpha}{2}<\delta<\omega\sin\frac{\alpha}{2}$$

is also supposed.

4. CONCLUSION

The study of the nonstationary nondegenerate four wave mixing process shows that it is possible to get picosecond pulses with tunable frequency by tuning the frequency of comparatively long (nanosecond) pulse of the signal beam. Moreover the wavefront of this picosecond "reflected" pulses in the four wave mixing process is conjugated in respect to the signal wave of the "long" pulse.

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