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COSMOLOGY AND THE LARGE MASS PROBLEM OF THE KALUZA-KLEIN THEORY

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ABSTRACT

It is shown that in five dimensional Kaluza-Klein theories the large mass problem leads to circulus vitiosus: the huge present e^2/G produces the large mass problem, which restricts the ratio e^2/Gm^2 to the order of unity, in contradiction with the present 10^{40} value for elementary particles.

АННОТАЦИЯ

Показывается, что в пятимерных теориях Калуца-Клейна проблема большой массы приводит к ошибочному кругу: нынешней огромной величиной частного e^2/G дается проблема большой массы, которая ограничивает e^2/Gm^2 на порядок 1, напротив нынешней величины $\sim 10^{40}$ для элементарных частиц.

KIVONAT

Megmutatjuk, hogy 5 dimenziós Kaluza-Klein elméletben a nagy tömeg probléma ördögi körre vezet: e^2/G jelenlegi nagy értéke okozza a nagy tömeg problémát, ami viszont e^2/Gm^2-et 1 körüli értékre korlátozza, szemben a mai kb. 10^{40}-nel elemi részecskékre.

Looking for the common origin of fundamental interactions one may arrive at Kaluza-Klein type theories realizing the unification in a geometrical framework assuming more than 3 spatial dimensions [1]. Fundamental interactions seem to be described by gauge theories; under specific assumptions the gauge groups appear in the geometrical approach in a -more or less- natural way (cf. e.g. Ref. 2 and citations therein).

Nevertheless, the three dimensional character of the space is an elementary experience; if the extra dimensions possess geometric meaning, the usual explanation is that they are compact with microscopic radii (e.g. in the order of Planck length). While such an anisotropic Universe may seem to be unnatural, there are some arguments in the literature that this state is a result of a cosmologic evolution [3], [4] and that this particular evolution has led to the enormous ratio of the electromagnetic and gravitational forces as well [3] (in which case this is a manifestation of the original idea of Dirac [5]). However, there may be some problems when realizing this very promising idea; within the framework of the five dimensional theory the so called large mass problem seems to exist, i.e. particles with usual elementary charge must be very massive (in the order of Planck mass). This result can be derived from the analysis of a matter field coupled to the five dimensional metric (cf. e.g. Refs. 2 and 3) as well as from pure classical investigation of the geodesic motion [6].

In this paper we shall show that the original idea of Chodos and Detweiler [3] about the origin of the huge ratio of the electromagnetic and gravitational forces between elementary particles cannot be realized, at least in a simplified model; the value of this ratio in a five dimensional Universe is obtained of order 1, while the present value is $\sim 10^{40}$, and this fact is in close connection with the large mass problem.

Consider a five dimensional Universe. Assuming the usual symmetries in four dimensions one gets

$$d\hat{s}^2 = -dt^2 + R^2(t)[dx^2 + f^2(x)(d\vartheta^2 + \sin^2\vartheta d\varphi^2)] +$$

$$+ S^2(t)(dx^5)^2 \tag{1}$$

$f(x) =$	$\sin x$	x	$\mathrm{sh}\, x$
$k =$	$+1$	0	-1

where k is the curvature constant of the three-space. The fifth dimension, as usual, is assumed to be compact, $0 \leq x^5 < 2\Pi s$, where s is a length scale.

The metric obeys the usual Einstein equation (in five dimensions); since electromagnetism has been geometrized by means of the fifth dimension, if other interactions are negligible, then the energy-momentum tensor is that of a mixture of incoherent fluids of different particles. The simplest model containing charges is a two fluid one:

$$\hat{T}^{ik} = \sum_{\Gamma=1,2} \hat{m}_\Gamma \hat{n}_\Gamma \hat{u}^i_\Gamma \hat{u}^k_\Gamma \tag{2}$$

(the Latin indices take the values 0,1,2,3 and 5). Here $\hat{m}_\Gamma$ stands for the invariant mass, $\hat{n}_\Gamma$ is the comoving five dimensional number density, and $\hat{u}^i_\Gamma$ is the five-velocity; because of the symmetries

$$\hat{u}^i_\Gamma = (\hat{a}_\Gamma, 0, 0, 0, \hat{b}_\Gamma) \tag{3}$$

with the usual normalization $\hat{g}_{rs}\hat{u}^r_\Gamma \hat{u}^s_\Gamma = -1$, whence

$$\hat{b}_\Gamma = \pm\sqrt{\hat{a}^2_\Gamma - 1}\,/S \tag{4}$$

Since the fifth component of $\hat{u}^i$ is connected with the charge [6], the $\pm$ signs in eq. (4) express the existence of opposite charges, when one cannot use a common comoving system as seen in eq. (3). The (1) form of the line element reflects the symmetries observed by macroscopic neutral observers.

With the assumed metric all the offdiagonal components of the Ricci tensor vanish [7]; hence

$$\sum_\Gamma \hat{T}_\Gamma{}^{o5} = 0 \tag{5}$$

expressing the fact that the Universe is neutral in average. The remaining components yield

$$\begin{aligned}
&\left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} + \frac{\ddot{RS}}{RS} = \frac{8\Pi\hat{G}}{3}\sum_\Gamma \hat{m}_\Gamma \hat{n}_\Gamma \hat{a}_\Gamma^2 \\
&2\frac{\ddot{R}}{R} + \left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} + \frac{\ddot{S}}{S} + 2\frac{\dot{R}\dot{S}}{RS} = 0 \\
&\frac{\ddot{R}}{R} + \left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} = -\frac{8\Pi\hat{G}}{3}\sum_\Gamma \hat{m}_\Gamma \hat{n}_\Gamma (\hat{a}_\Gamma^2 - 1)
\end{aligned} \tag{6}$$

Here $\hat{G}$ is the five dimensional gravitational coupling constant, related to the four dimensional coupling as $\hat{G} = 2\Pi sSG$ [6].

Eqs. (5) and (6) yield four relations for the six unknown quantities R, S, $\hat{n}_\Gamma$ and $\hat{a}_\Gamma$. However, all interactions are either geometrized or neglected, therefore the two fluids obey separate balance laws

$$\hat{T}_\Gamma{}^{ir}{}_{;r} = 0 \tag{7}$$

while the balance equation for the sum is a consequence of the Einstein equation; this yields extra relations for one constituent , with two nontrivial components. Thus we have arrived at a complete system of evolution equations.

In the generic case one cannot expect analytic solutions.

Nevertheless, eq. (7) can be integrated as

$$\tilde{n}_\Gamma = \hat{n}_\Gamma \hat{a}_\Gamma = \frac{K_\Gamma}{R^3 S}$$
$$\hat{a}_\Gamma^2 = 1 + \frac{C_\Gamma{}^2}{S^2} \tag{8}$$

Here $\tilde{n}_\Gamma$ can be interpreted as the particle number density measured by a neutral observer, $\bar{u}^i = \delta^i_o$; it is inversely proportional to the four-volume. Then the neutrality equation (5) becomes a constraint for the constants of integration, while the contracted Bianchi identity makes one of eqs. (6) superfluous. The remaining two govern the evolution of geometry.

Now, the picture proposed in Ref. 3 requires a particular solution with increasing R and decreasing S; here we do not discuss the conditions for the existence of such solutions, similar evolution paths are well known for a single neutral component [4]. Nevertheless, a realistic Universe model has to produce the high value of the specific charge which existed at the end of the plasma era (note that the present simplified model is a plasma). Then, in a mixture of protons and electrons, the ratio of charge and energy densities was cca. $e/m_p \sim \sqrt{G} \cdot 10^{20}$. However, in our model Universe this ratio is limited. The charge of a particle measured by a neutral observer is [6]

$$q_\Gamma = \sqrt{16\pi G}\, S^{-1} \hat{m}_\Gamma \hat{u}_{\Gamma 5} \tag{9}$$

Then the corresponding charge and energy densities are

$$\hat{\rho}_\Gamma = \sqrt{16\pi G}\, S^{-1} \hat{T}_{\Gamma o5}$$
$$\hat{\mu}_\Gamma = \bar{u}_r \bar{u}_s \hat{T}_\Gamma^{rs} = \hat{T}_{\Gamma oo} \tag{10}$$

whence

$$\frac{\hat{\rho}_\Gamma}{\hat{\mu}_\Gamma} = \sqrt{16\pi G}\, \sqrt{1 - \hat{a}_\Gamma^{-2}} \le \sqrt{16\pi G} \tag{11}$$

While these quantities are five-dimensional densities, nothing depends on x^5, therefore a simple $2\Pi sS$ factor connects them with the four-dimensional ones, which cancels in the ratio. Thus clearly a 10^{20} factor is missed.

When the matter is already very diluted, the ratio (11) gives q/m, whence the ratio of electromagnetic and gravitational forces can be obtained in the order of unity. So, the ratio of the observed forces cannot be a consequence of the age of the Universe, as was conjectured in Ref. 3.

Observe that here we are confronted with the large mass problem. For a particle moving on timelike geodesic a neutral observer measures a charge according eq. (9) and a mass [6]

$$m_{eff} = \hat{m}\sqrt{1 + S^{-2}\hat{u}_5^2} \qquad (12)$$

Then

$$\frac{q^2}{Gm_{eff}^2} = 16\Pi \frac{x}{x+1} \qquad (13)$$

with $x = S^{-2}\hat{u}_5^2$. So the ratio must be between 0 and 16Π. This result is clearly independent of the particle composition of the model Universe, which confirms our model calculation about the impossibility of obtaining a correct ratio in an evolution process. Note that -apart from the value- the behaviour of the ratio is decent; it grows as S is shrinking (i.e. as the Universe is ageing).

Of course, this result valid only for timelike geodesic motion. Ref. 6 proposed spacelike motion in five dimensions. This is an unnatural hypothesis from the viewpoint of General Relativity as emphasized in Ref. 6; moreover, it will not solve our particular problem. Namely then

$$\frac{q^2}{Gm_{eff}^2} = 16\Pi \frac{x}{x-1} \qquad (14)$$

which decreases when the extra dimension is shrinking.
The other possibility is that the motion is not geodesic; nevertheless, as we mentioned, the electromagnetic interaction

is geometrized, so an incoherent energy-momentum tensor is physically expected,when the geodesic motion is a consequence.Of course, an internal structure may lead to deviations from geodesic motion, nevertheless such effects are not expected on macroscopic level.

This analysis indicates that the large mass problem has an unpleasant consequence: the Dirac hypothesis cannot be built into the Kaluza-Klein framework in a natural way.

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