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CRITICAL BEHAVIOUR OF THE 2D HEISENBERG MODEL

II. CRITICAL EXPONENTS FROM SERIES EXPANSIONS

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E. PRAVECZKI

Central Research Institute for Physics
H-1525 Budapest 114, P.O.B. 49, Hungary

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ABSTRACT

High temperature series have led to the conclusion that phase transition occurs in the 2D Heisenberg model at a non-zero temperature. High temperature susceptibility diverges at the critical point and the low temperature susceptibility diverges at all non-zero temperatures.

Critical exponents are determined. It is found that high temperature susceptibility diverges logarithmically; the critical isotherm is similar to the classical one (δ=3); and the specific heat has no singularity if the magnetic field is zero but is singular as a function of the magnetic field.

It is also concluded that critical phenomena occouring in the 2D Heisenberg model can correctly be described and understood only when both long- and short-range effects are taken into account (duality hypothesis).

АННОТАЦИЯ

Высокотемпературное разложение приводит к возникновению фазового перехода в двумерной модели Гейзенберга. Высокотемпературная восприимчивость расходится в критической точке, а низкотемпературная - везде ниже нее. Определены критические экспоненты. Сделан вывод, что высокотемпературная восприимчивость расходится логарифмически, критическая изотерма похожа на классическую (δ=3), и теплоемкость не имеет сингулярностей. Наконец, сделан вывод, что критическое явление, возникающее в двумерной модели Гейзенберга, становится понятным только при одновременном учете близко- и дальнодействующих эффектов.

KIVONAT

Magashőmérsékleti sorfejtés arra az eredményre vezetett, hogy a 2D Heisenberg modellben fázisátalakulás lép fel. A magashőmérsékleti szuszceptibilitás divergál a kritikus ponton, és az alacsonyhőmérsékleti szuszceptibilitás divergál mindenütt ez alatt.

Meghatározzuk a kritikus exponenseket. Azt kapjuk, hogy a magashőmérsékleti szuszceptibilitás logaritmikusan divergál-, a kritikus izoterma a klasszikushoz hasonló (δ=3); és a fajhőnek nincs szingularitása.

Végül arra következtettünk, hogy a 2D Heisenberg modellben fellépő kritikus jelenség csak akkor érthető meg, ha mind a rövid-, mind a hosszutávu effektusokat figyelembe vesszük (dualitási hipotézis).

1. INTRODUCTION

We have concluded in one of our earlier works (Praveczki, 1980) that phase transition occurs in the 2D Heisenberg model at a non-zero temperature, $T_c>0$, although in the low temperature phase there is no spontaneous magnetization (Mermin and Wagner, 1966). The critical temperature is defined by the divergence of the susceptibility. It should be mentioned that a similar conclusion is drawn from the investigations by Stanley and Kaplan (1966). Note that an opposite conclusion is drawn for the model Brezin and Zinn-Justin (1976). Although the conclusion of these authors is strongly critized in the first part of the present work (Praveczki, 1985a).

The question now arises as to which are the values of the critical exponents characterizing the non-analytic behaviour of physical quantities at the critical point.

We have made an attempt to determine the critical exponents in a mannar similar to that applied to investigation of the 3D Heisenberg- and Ising model (Praveczki, 1985b). The results are as follows:

$$\begin{array}{ll} \alpha=\alpha'=-1, & \phi=2/3 \\ \beta=1/2, & \rho=2, \\ \gamma=0\ (\log), & \rho'=4/3, \\ \gamma'=\infty & \lambda=2, \\ \delta=3 & \lambda'=0, \end{array} \qquad (1.1)$$

where ϕ characterizes the susceptibility below the critical temperature and ρ, ρ' λ and λ' are defined by the specific heat

$$\chi(T,H)=\chi_o(T)H^{-\phi} \qquad (T<T_c),$$

$$C(T,H)=\begin{cases} C_o(T-T_c)^{-\alpha}+C_1(T-T_c)^{-\lambda}H^{\rho} & (T>T_c) \\ C'_o(T_c-T)^{-\alpha'}+C'_1(T_c-T)^{-\lambda'}H^{\rho'} & (T<T_c). \end{cases} \qquad (1.2)$$

Note that β is interpreted as the limiting ($d\to 2$) value of $\beta(d)$ where d is the dimensionality.

We also looked into the question regarding the equation-of-state corresponding to the critical exponent estimates given above (Eq.1.1) and have arrived at the corrected critical equation-of-state

$$\frac{1}{M} = \int \frac{k^2\, dk}{\Theta[H+M\rho(T,M)\omega_k]} \qquad (1.3)$$

where $\Theta = I/k_B T$; $\omega_k \sim k^2$ at small values of k; and $\rho(T,H)$ has the non-analytic properties

$$\rho(T,O) \sim \ell n\, \ell n \frac{T_c}{T - T_c} \qquad (T>T_c);$$

$$\rho(T_c,M) \sim \ell n \frac{1}{M} \quad . \qquad (1.4)$$

In order to compare our results for critical exponents with other theoretical results known from the literature, we would mention that the estimate obtained by Stanley and Kaplan (1966) for γ is remarkably higher than that suggested above but the difference can be understood from the difference in the values of the critical temperature obtained in the two works.

Unfortunately, there are very few experimental results with which we could compare our results. Karimov (1972)discovered that the susceptibility of the quasi-two dimensional Heisenberg ferromagnet, obtained by the diffusion of $FeCl_2$ molecules into crystalline graphite, diverges under a non-zero temperature in agreement with our results.

2. SUSCEPTIBILITY

Because the critical temperature is defined by the divergence of the high temperature susceptibility, the first question we have to answer concerns the value of the critical exponent characterizing that divergence.

To answer the question, we used the high temperature expansion result of Stanley and Kaplan (1966) for the susceptibility and our result (Praveczki, 1980) for the critical temperature. The investigations are accomplished in three different ways due to the essential role of the exponent. The first way is the simple ratio method; the second is based on the formulae derived in the first part of our work (Praveczki, 1985b) ; the third way is connected with the fact that if the susceptiblity, χ, has a divergence at $\Theta=\Theta_c$ ($\Theta=I/k_B T$) characterized by the exponent γ, then Θ as a function of $K=1-\chi^{-1}$ has a singularity at K=1 which is characterized by the exponent $1/\gamma$.

The results of the simple calculations are shown in Fig.1. It can be seen that extrapolation of approximate estimates leads uniquely to the conclusion $\gamma=0$ in all three cases. Regarding the susceptibility to be divergent at $\Theta=\Theta_c$ by the definition of Θ_c, the conclusion means that susceptibility diverges logarithmically at Θ_c.

More precisely, the above result is consistent with the general expression

$$\chi=\chi_o \ \ell n^{\mu} \frac{T_c}{T-T_c} \tag{2.1}$$

where μ is positive. In order to decide whether χ diverges as a simple logarithmic function ($\mu=1$) or whether it exhibits more complicated behaviour, we examined the function

$$\psi= \exp\chi$$

which behaves asymptotically (Eq. 2.1),

$$\psi \sim (T-T_c)^{-\lambda(\mu)},$$

where

$$\lambda(\mu) = \begin{cases} \infty & \text{if} \quad \mu>1, \\ \chi_o & \text{if} \quad \mu=1 \quad \text{and} \\ o & \text{if} \quad \mu<1 \ . \end{cases} \tag{2.2}$$

The results of the calculations are shown in Fig.2. Extrapolation leads us to the result that λ is finite. This means that $\mu=1$ and the susceptibility diverges as a simple logarithmic function, Eq.2.1.

Taking into account that $\lambda(\mu)=\chi_o$ if $\mu=1$ (Eq.2.2), we can establish, Fig.2, that

$$\chi = 7.39 \ell n \frac{T_c}{T-T_c} \qquad (T>T_c) \tag{2.3}$$

3. CRITICAL ISOTHERM

Next, we undertake the task of determining the critical isotherm and the exponent δ. With this purpose, we investigate the equation-of-state on the basis of the results obtained by Baker et al. (1970) with the use of the high temperature series expansion method.

If $\Theta<\Theta_c$, $K=\text{th}\Theta H/M$ can be expanded into a series in powers of Θ as

$$K = K_o(\Theta) + K_1(\Theta)M^2 + K_2(\Theta)M^4+\dots \tag{3.1}$$

where $K_n(\Theta)$ is given in the form

$$K_n(\Theta) = \sum_{m=n}^{\infty} K_{nm}\Theta^m \quad , \tag{3.2}$$

and here, K_{nm} are determined up to m=8 (Baker et al.,1970)

Because $K_o(\Theta)$ is the inverse susceptibility, we have to begin the investigations with $K_1(\Theta)$. If we define

$$L_1 = \frac{1}{7\Theta}(2\Theta - K_1) \ ,$$

$$L_2 = \frac{1}{26\Theta^2}(5\Theta^2 - K_2), \tag{3.3}$$

$$L_3 = \frac{1}{92\Theta^3}(13\Theta^3 - K_3),$$

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and invert the series (3.2) with the use of the Legendre inversion formulae, we get the equations

$$\Theta = \sum_{m=1}^{\infty} \Theta_{nm} L_n^m \tag{3.4}$$

where the coefficients Θ_{nm} have the values listed in Table 1.

As can be seen from Table 1, the coefficients are all positive and increase as functions of n as well as m. If we generalize these properties for all values of n and m, we arrive at the conclusion that $K_n(\Theta_c)$ are finite, Eq.3.3, satisfying the inequalities

$$2\Theta_c \geq K_1(\Theta_c) \geq 2\Theta_c(1 - \frac{7}{2} R_1),$$

$$5\Theta_c^2 \geq K_2(\Theta_c) \geq 5\Theta_c^2(1 - \frac{26}{5} R_2),$$

$$13\Theta_c^3 \geq K_3(\Theta_c) \geq 13\Theta_c^3(1 - \frac{92}{13} R_3),$$

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where R_n is the convergence radius of the series in powers of L_n.

A more detailed investigation of the series shows that all $K_n(\Theta_c)$ have positive values. What is more, those values can be determined approximately with the use of the simple successive approximation method. It turns out that the approximants of $K_n(\Theta_c)$ converge well.

As a final result, we write down the equation determining the critical isoterm,

$$\Theta_c H = 0.53M^3 + 0.35M^5 + 0.22M^7 + \ldots \tag{3.5}$$

As can be seen from the equation, we have reason to conclude that H is an analytic function of M even at the critical temperature. Furthermore, if the analyticity postulate holds, then Eq. 3.5 gives us that δ has the classical value of 3 as given by Eqs. 1.1.

4. SPONTANEOUS MAGNETIZATION

It is well-known that the 2D Heisenberg model cannot have spontaneous megnetization (Mermin and Wagner, 1966). It is, therefore, an intriguing question whether that exact result can be confirmed by the approximation methods applied in the present work.

Here, we briefly present the results of investigations accomplished by us on the basis of the inversion method. Let us introduce the notation

$$L = 1-K \tag{4.1}$$

and invert the series 3.1 as functions of Θ. We obtain

$$\Theta = \sum_{n=0}^{\infty} \varphi_n(L) M^{2n} \tag{4.2}$$

where

$$\varphi_o(L) = 0.25L+0.125L^2+0.08333L^3+0.06380L^4+0.05977L^5+\ldots \tag{4.3}$$

$$\varphi_1(L) = 0.25L-0.0625L^2+0.03125L^3+0.03385L^4+0.06901L^5+\ldots$$

We have to investigate series 4.2 at the critical temperature which corresponds to L=1, Eq. 4.1. It is easy to see that the series of $\varphi_o(L)$, Eq. 4.3, gives the critical temperature at L=1, $\varphi_o(1)=\Theta_c$ (see one of our earlier works, 1980). As far as series of $\varphi_1(L)$ is concerned, the second term is negative but all the others are positive and decrease slowly with increasing index. If all higher order coefficients are positive and decrease so slowly as is confirmed at lowest orders, then the series diverges. Therefore, if we express M from Eq. 4,2, we get

$$M = B[\Theta - \Theta_c]^{1/2} + \ldots$$

where $B=[\varphi_1(L)]^{1/2}$ tends to zero when $H \to 0$ (L=1).
This confirms the exact result that there is no spontaneous magnetization at non-zero temperatures.

5. SPECIFIC HEAT

In this section, we investigate the specific heat near the critical temperature and at small values of the megnetic field. Firstly, we examine the zero-field specific-heat.

Richi and Fisher (1973) have determined some low-order terms of the high temperature series for the zero-field specific heat. In the special case of $S = 1/2$, it reads as

$$C_o(\Theta) = 1-0.75\Theta-0.9844\Theta^2+1.7576\Theta^3+0.4288\Theta^4-1.2832\Theta^5+... \qquad (5.1)$$

As a means of invetigating the series near the critical temperature, we apply the inversion method. Let us define

$$L = \frac{1}{4} C_o(\Theta) .$$

Then, from Eq. 5.1, we get

$$\frac{1}{2}\Theta = L+1.5L^2+8.4375L^3+32.3478L^4+170.0205L^5+782.0053L^6+... \qquad (5.2)$$

It can be seen that in the series obtained from the above, all the coefficients are positive. What is more, the coefficients increase rapidly with increasing value of the index which means that the value of Θ runs from zero to infinity when the value of L runs from zero to a finite estimate $L_r<1$ (convergence radius of the series). More precisely,

$$0 \leq L(\Theta) \leq L_r < 1$$

for all values of Θ. With regard to this result and definition 5.1, we can conclude that specific heat has a finite value at the critical temperature,

$$0 \leq C_o(\Theta_c) < 1/\Theta_c .$$

What is more, we can establish, using the successive approximation method, that $C_o(\Theta_c) = 0.11$.

Once the specific heat has a finite value at the critical temperature, it is expected that it is an analytic function at the same value of the temperature

$$C_o(\Theta) = C_{oo}+C_{o1}(1-\frac{T_c}{T})+C_{o2}(1-\frac{T_c}{T})^2+...$$

With the aim of conforming to the expectation, we differentiate series 5.1 with Θ, once then twice. We arrive at the series

$$\frac{1}{2}\,\Theta = L_1+5.357L_1^2+60.875L_1^3+ 835.785L_1^4+12834.626L_1^5+\ldots$$

$$\frac{1}{2}\,\Theta = L_2+10.714L_2^2+240.020L_2^3+6604.207L_2^4+203758.770L_2^5+\ldots \qquad (5.3)$$

where

$$L_1 = \frac{1}{7.875}\left[1.5+\frac{dC_o(\Theta)}{d\Theta}\right],$$

$$L_2 = -\frac{\Theta}{15.75}\,\frac{d^2C_o(\Theta)}{d\Theta^2} .$$

The results (5.3) show that L_1 as well as L_2 has a finite value at the critical temperature as long as series 5.3 are continued with terms having the same properties as can be established at low orders. Furthermore, we can determine approximate values of C_{o1} and C_{o2} with high accuracy. As a summary of the above investigations of the zero-field specific heat, we may write.

$$C_o(\Theta)= 0.11 - 2.57(1-T_c/T)+0.321(1-T_c/T)^2+\ldots \qquad (5.4)$$

From the above, we assume that $C_o(\Theta)$ is an analytic function of the temperature at the critical point. If this is so then $\alpha=\alpha'=-1$ as given by Eqs.1.1. (Note that we can write $\alpha=\alpha'=0$ instead of $\alpha=\alpha'=-1$ as in the classical theory of phase transitions).

Let us now turn to the problem of determining the field-dependence of the specific heat at the critical point. On the basis of the results obtained in Section 3 for the critical isotherm, it seems reasonable to state that free energy is an analytic function of M^2 at all temperatures, that is

$$F(T,M) = F_o(T)+F_1(T)M^2+F_2(T)M^4+\ldots \qquad (5.5)$$

where the coefficients $F_o(T)$, $F_1(T)$, ..., are finite and are given by the coefficients $K_o(T)$, $K_1(T)$,..., Eqs. 3.1, as

$$F_1(T) = K_o(T)/2,$$

$$F_2(T) = K_1(T)/4, \qquad (5.6)$$

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at small values of H. Therefore, the specific heat is determined as

$$C(T,H) = C_o(T)+C_1(T,H) ,$$

where $C_o(T)$ is determined above, Eq. 5.4, and

$$C_1(T,H) = \frac{\partial^2}{\partial T^2}\left[\frac{K_o(T)}{2} M^2(T,H) + \frac{K_1(T)}{4} M^4(T,H) + \ldots\right] \quad (5.7)$$

When investigating $C_1(T,H)$, we have to distinguish between the two cases corresponding to $T>T_c$ and $T<T_c$.

If $T>T_c$,

$$K_o(T) = 0.14/\ell n \frac{T_c}{T-T_c}$$

which yields, neglecting the second term in Eq. 5.7,

$$C_1(T,H) = 2.43(1-\frac{T_c}{T})^{-2} H^2 . \quad (5.8)$$

If $T<T_c$, the first term is identically equal to zero in Eq. 5.7 which is a consequence of the zero-field susceptibility diverging at all temperatures below the critical temperature, see Stanley and Kaplan, 1966 and Section 6. So we obtain from Eq. 5.7 and Eqs. 3.1 and 3.5, that

$$C_1(T,H) = C_{11}(T)H^{4/3} \quad (5.9)$$

where

$$C_{11}(T) = \frac{\partial^2}{\partial T^2}\, \Theta^{-5/3} K_1^{-1/3}/4 .$$

As we have seen in Section 3, $K_1(\Theta)$ has a finite value at Θ_c. Furthermore, the first derivative of $K_1(\Theta)$ with respect to Θ is also finite which can be seen from the series obtained from the series of $K_1(\Theta)$, Eq.3.2, as

$$\Theta = R+3.0314R^2+12.7004R^3+ 59.2234R^4+294.7472R^5+\ldots$$

where

$$R = \frac{1}{14}\,(2 - \frac{d}{d\Theta} K_1(\Theta))\ldots$$

In fact, we get

$$\left[\frac{d}{d\Theta} K_1(\Theta)\right]_{\Theta_c} = -0.8$$

Unfortunately, it is impossible to determine the value of the second derivative of $K_1(\Theta)$ at Θ_c with satisfactory accuracy. However, it can be supposed that it has a finite value. A consequence, however, is that $C_{11}(T)$ has a finite value at the critical temperature, Eqs. 1.1 and 1.2.

As a summary of the present section, we can establish that zero-field specific heat has no singularity at the critical temperature. This result can be well undestood on the basis of the fact that critical exponents of other physical quantities cannot be related to the zero field specific heat due to the absence of the spontaneous magnetization. Furthermore, field-dependent specific heat is singular at the critical point as given by Eqs. 1.1 and 1.2.

6. EQUATION-OF-STATE ON THE BASIS OF THE RENORMALISED SPIN-WAVE THEORY

With the aim of writing down the critical equation-of-state and casting more light on the nature of the phase transition in the 2D Heisenberg model, we refer to one of our previous works (1977) where it is ahown that the equation-of-state for the Heisenberg model (at any value of the dimensionality) can be given in the form

$$\frac{1}{M} = \int \frac{d^D k}{\mathrm{th}\Theta[H+\bar{\Lambda}(k,\Theta,\bar{H})} \qquad (6.1)$$

where $\bar{\Lambda}$ is the exchange part of the renormalised spin-wave energy and it can be given as a series in powers of H.

$$\bar{\Lambda}(k,\Theta,\bar{H}) = \sum_{n=o}^{\infty} \bar{\Lambda}(k,\Theta)\bar{H}^{\,2n+1} \qquad (6.2)$$

where $\bar{H} = \Theta H$ and $\bar{\Lambda}_n(k,\Theta)$ can be expanded into a high temperature series, for instance, as

$$\bar{\Lambda}_n(k,\Theta) = \sum_{m=o}^{\infty} \bar{\Lambda}_{nm}(k)\,\Theta^m \quad . \qquad (6.3)$$

Equations 6.1 - 6.3 are based on the canonical ensemble of the states of the model and do not converge at $T<T_c$.To get rid of we suggested previously that H in the expression of $\bar{\Lambda}(k,\Theta,\bar{H})$, Eqs. 6.2 and 6.3, must be eliminated by the magnetization and the temperature using the equation-of-state obtained by the usual type high temperature expansion,

$$\bar{M} = \mathrm{th}\Theta[H+\omega(\Theta,\bar{M})] \quad , \qquad (6.4)$$

where

$$\omega(\Theta,\bar{M}) = \sum_{n=o}^{\infty} \omega_n(\Theta)\bar{M}^{\,2n+1}$$

in which $\omega_n(\Theta)$ is given by a series of the form

$$\omega_n(\Theta) = \sum_{m=o}^{\infty} \omega_{nm}\Theta^m \quad . \qquad (6.5)$$

If we insert the expression of $\bar{H}$, Eq. 6.4, into the expression of $\bar{\Lambda}(k,\Theta,\bar{H})$, Eq. 6.2, we arrive at the equation-of-state

$$\frac{1}{\bar{M}} = \int \frac{d^D k}{th\Theta[H+\Lambda(k,\Theta,\bar{M})]} \tag{6.6}$$

instead of Eq. 6.1 where Λ is given as

$$\Lambda(k,\Theta,\bar{M}) = \sum_{n=o}^{\infty} \Lambda_n(k,\Theta)\bar{M}^{2n+1} \tag{6.7}$$

and here, the coefficients are determined, Eqs. 6.2 and 6.3, in the form of the high temperature series

$$\Lambda_n(k,\Theta) = \sum_{m=o}^{\infty} \Lambda_{nm}(k)\Theta^m \ .$$

In the above equations, $\bar{M}$ denotes the magnetization as well as M being based, both of Eqs. 6.1 and 6.4, on exact definitions. Therefore, we may write $\bar{M}$=M which results from Eq. 6.6 being a self-consistent equation for M as a function of T and H. However, there is another possibility, viz. that we regard $\bar{M}$ in Eq. 6.6 as a parameter determined by Eq. 6.4.

The second approach is reasoned by the situation that $\omega_n(\Theta)$ and $\Lambda_n(k,\Theta)$ as functions of Θ are generally determined in different orders. However, there is a second and more deeper reasoning which prefers the second approach to the first one.

Equation 6.7 as well as Eq. 6.1 reflects first of all the long-range effects (wave properties of the elementary excitations) whereas Eq. 6.4 reflects mainly the short-range effects (particle properties of the spins).This means that if we conserve the notation in Eq. 6.7, we have to conserve it in Eq. 6.4, too. Thereby, we arrive at a pair of equations which reflects the long-range effects as well as the short-range effects (dual properties of excitations).

Here, as a means of illustrating the above statements we carry out a brief investigation based on the lowest order approximations for $\omega(\Theta,\bar{M})$ and $\Lambda(k,\Theta,\bar{M})$ which are

$$\omega(\Theta,\bar{M}) = \bar{M} \ ,$$

$$\Lambda(k,\Theta,\bar{M}) = \bar{M}\omega_k \ .$$

With the use of these expressions, we get from Eqs. 6.4 and 6.6 the equations-of-state

$$\bar{M} = th\Theta[H+\bar{M}] \ , \tag{6.8}$$

and

$$\frac{1}{\bar{M}} = \int \frac{d^D k}{\text{th}[H+\bar{M}\omega_k]\Theta} \quad . \tag{6.9}$$

Equation 6.9 can be seen to be the result of the mean field approximation which reflects the local properties only (particle approach). The spontaneous magnetization and the zero-field susceptibility are determined as

$$\bar{M}_o = [3(1-\Theta_c(\Theta)]^{1/2} ,$$

$$\bar{\chi}_o = \frac{\Theta}{\Theta_c - \Theta} \qquad (\Theta_c = 1) \quad . \tag{6.10}$$

Next, insert $\bar{M} = M$ into Eq. 6.9. We then get the equation-of-state which is known as that obtained with the random phase approximation. We can easily see that $\Theta_c(D)$ (D is the dimensionality) is given as

$$\Theta_c(D) = \int \frac{d^D k}{\omega_k}$$

which diverges when $D \to 2$. Consequently, there is no phase transformation according to this approach (Wave-approach). Note that the same dimension-dependence is obtained for the critical temperature by applying Brezin and Zinn--Justin's (1972) field-theoretical method as well as by the renormalization group method of Migdal (1975) and of Forgács and Zawadowski (1970).

Finally, let us investigate Eqs. 6.8 and 6.9 in accordance with the duality hypothesis which tells us that critical phenomena (especially those occurring in the 2D Heisenberg model, etc.) can correctly be described and understood by taking into account both the long-range and short range effects.

From Eq. 6,8, at small values of H and $\bar{M}$, we get

$$\frac{1}{\bar{M}} = \int \frac{d^D k}{\Theta[H+\bar{M}\omega_k]}$$

which yields for the zero-field susceptibility, Eqs. 6.10,

$$\chi_o = \begin{cases} \dfrac{1}{\Theta_c - \Theta} \Big/ \ln \dfrac{1}{\Theta_c - \Theta} & \text{if } T>T_c \text{ and} \\ \infty & \text{if } T<T_c \quad . \end{cases} \tag{6.11}$$

At the same time, spontaneous magnetization is given as

$$M_o \sim (D-2)(\Theta-\Theta_c)^{1/2} \underset{\to}{} 0 \tag{6.12}$$

if $D \to 2$.

As we see, the results obtained from the system-of-equations 6.8 - 6.9 are in full qualitative agreement with the results obtained in the former sections.

It can be shown that increasingly better results can be obtained for the critical behaviour of the magnetization, susceptibility, etc. by using better approximations for $\omega_n(\Theta)$ and $\Lambda_n(k,\Theta)$ as high temperature series. Some of the critical exponents (β, δ, etc.) take their correct values at lowest order approximation and do not change their values when we use longer and longer high temperature series approximants. Similary, we get that zero-field susceptibility diverges at all temperatures below the critical one in all orders of approximations. Some other critical exponents (γ, for instance) have approximate critical exponents that are different in different approximations for high temperature series. However, it can be ensured that they converge to the right value.

7. CORRECTED CRITICAL EQUATION-OF-STATE

Based on the results achieved in the previons section, it is possible to construct the corrected critical equation-of-state. We accomplish the task by accounting for the duality hypothesis.

First, we would mention that some of the critical exponent estimates are given correctly by the system-of-equations 6.8 and 6.9, as was mentioned in Section 6. In view of this we need only renormalize in that system-of-equations and then combine them into one equation.

The result is given by Eq. 1.3 where $\rho(T,M)$ is an analytic function of T and M if $M\neq 0$ and $T\neq T_c$, respectively, and it has the singular properties given by the expressions 1.4, otherwise.

8. REMARKS

To complete this (present) work, we would mention that the scalling law (as given by the critical equation-of-state, etc.) is invalid for the 2D Heisenberg model judging by our investigations. A similar statement is true for some of the critical exponent relations ($\alpha+2\beta+\gamma=2$, for instance).

As further support for our conviction that the critical phenomena (especially those occurring in the 2D Heisenberg model, etc.) cannot be described and understood correctly without accounting for both the long- and the short--range forces simultaneously (duality hypothesis), we briefly show the main steps accomplished in one of our former works(1976) in which Heisenberg films were investigated.

If we define the Green function

$$G_{ff'}(t,t') = \langle\langle S_f^-(t);\ S_{f'}^+(t')\rangle\rangle$$

we get the equation-of-state

$$\frac{d}{dt} G_{ff'}(t,t') = M\delta_{ff'}\delta(t-t')-i\sum_g I_{f-g}[\rho(g,f,f';t,t')G_{ff'}(t,t')-$$
$$-\rho(f,g,f';t,t')G_{gf'}(t,t')]$$

where

$$\rho(g,f,f';t,t') = \frac{<[S_g^o S_f^-(t);\ S_{f'}^+(t')]>}{<[S_f^-(t);\ S_{f'}^+(t')]>} \ .$$

The long-range effects can be taken into account by an approximation for ρ satisfying the asymptotic equality

$$\lim_{g-f \to \infty} \rho(g,f,f';t,t') = M \qquad (8.1)$$

It can be satisfied using the approximation $\rho(g,f,f';t,t')\simeq M$ which gives the equation-of-state obtained in the RPA. However, if we also wish to take into account the short range effects, it is necessary to find an approximation which satisfies the equality

$$\lim_{f-g \to 0} \rho(g,f,f';t,t') = 1 \qquad (8.2)$$

at least.

Equation 8.1 and 8.2 are both satisfied by the approximation

$$\rho(g,f,f';t,t') = \frac{M}{1-2<S_f^+S_g^->} \ .$$

This approximation leads to the result that there is a phase transition at a non-zero temperature (where the high temrepature susceptibility is divergent (but the spontaneous magnetization is absent below the critical temperature as well as above.

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Table 1. Values of the coefficients Θ_{nm} determining Θ as a function of L_n, Eqs. 3.3 and 3.4.

	1	2	3	4	5	6
1	1	2.0476	5.0193	14.978	44.156	133.31
2	1	3.0224	11.501	48.022	210.88	955.70
3	1	4.0290	20.533	115.22	681.51	-

Table 1.

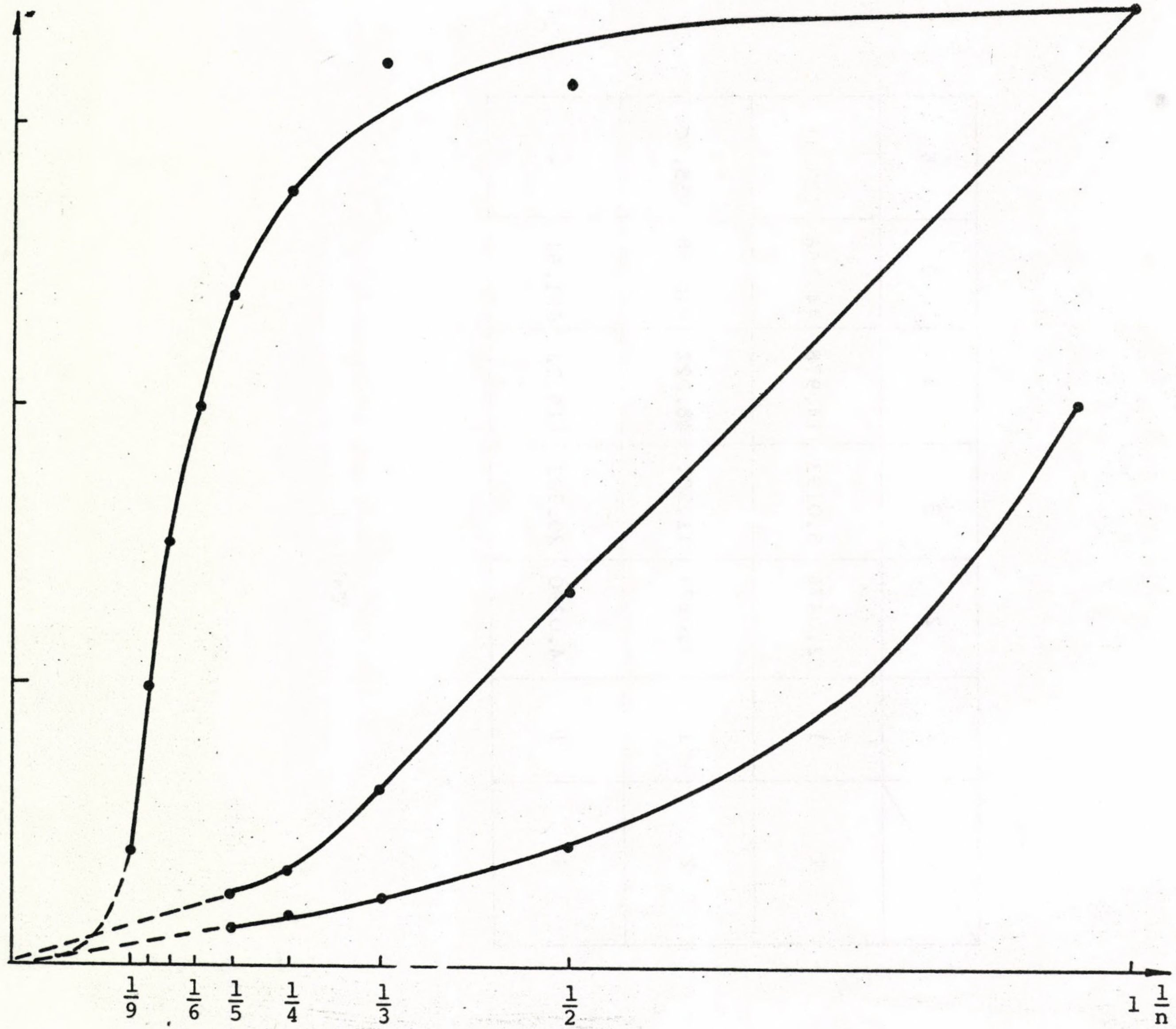

Fig.1

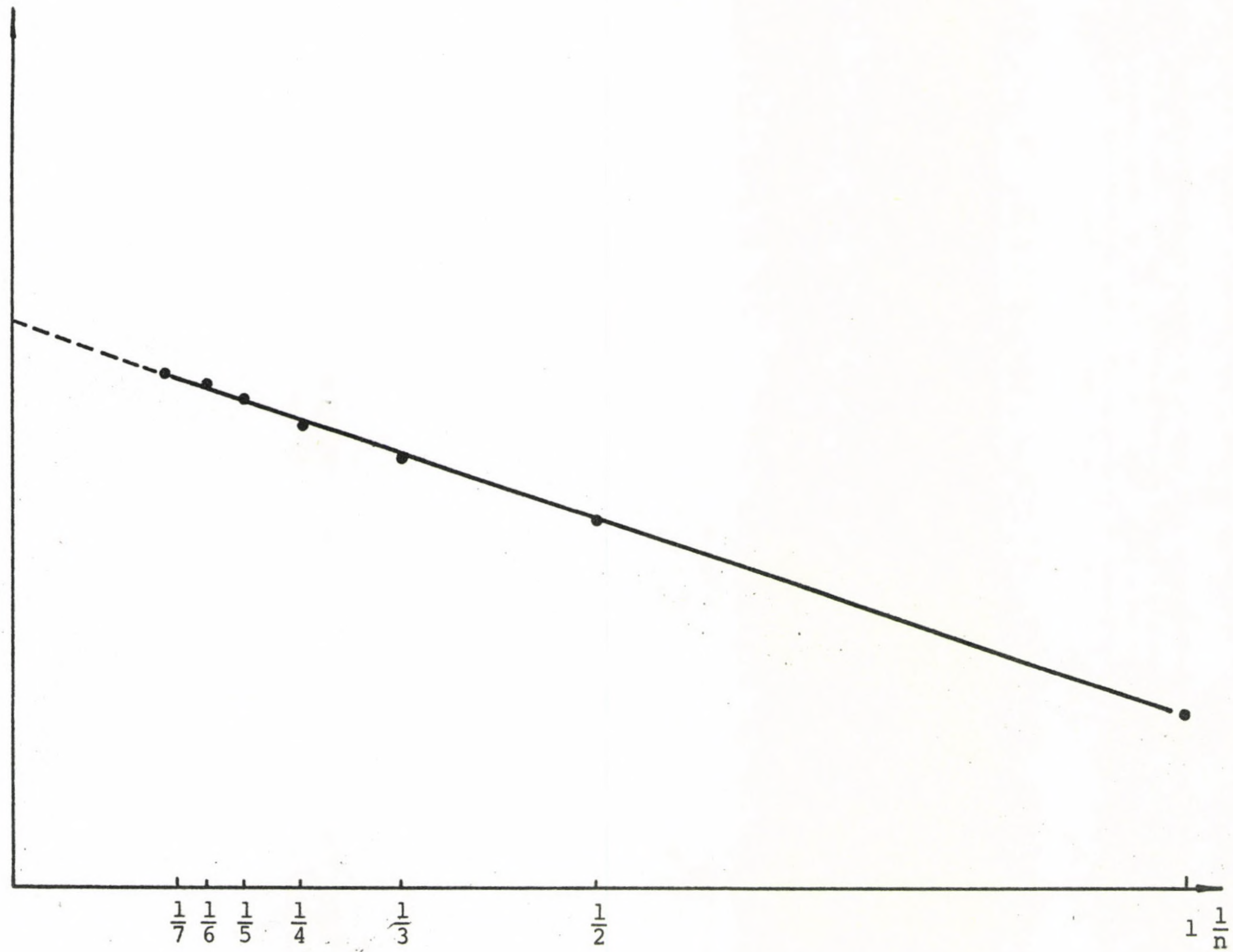

Fig. 2

63.718

Kiadja a Központi Fizikai Kutató Intézet
Felelős kiadó: Kroó Norbert
Szakmai lektor: Dr. Siklós Tivadar
Nyelvi lektor: Harvey Shenker
Gépelte: Kovács Gyuláné
Példányszám: 280 Törzsszám: 85-294
Készült a KFKI sokszorosító üzemében
Felelős vezető: Töreki Béláné
Budapest, 1985.junius hó