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ABSTRACT

It is a general belief that in some early stages of the evolution of the Universe unequilibrium situations played important role. In order to incorporate some deviations from thermal equilibrium into the description of the evolution, here an extension of the thermodynamic formalism is given, where, using the notion of ekaentropy, new terms are introduced into the Gibbs-Duhem relation for representing the deviation. Two situations are investigated in simplified models: the primordial thermalization in the symmetric phase of GUT, and the development of a nonthermal distribution for massive neutrinos.

АННОТАЦИЯ

Общепринятым является предположение, что на ранних стадиях эволюции Вселенной равновесные условия играли важную роль. В целях включения в описание эволюции различных отклонений от теплового равновесия в данной работе приводится расширение термодинамического формализма, в котором, используя понятие экаэнтропии, для описания отклонений в соотношение Гиббса-Дюгема вводятся новые члены. В упрощенных моделях рассматриваются две ситуации: первичная термализация в симметричной фазе теории большого объединения и установление неравновесного распределения массивных нейтрино.

KIVONAT

Általánosan elfogadott vélemény szerint az Univerzum fejlődésének korai szakaszaiban a nemegyensulyi folyamatok fontos szerepet játszottak. Hogy a hőmérsékleti egyensulytól való eltéréseket beépitsük az evolució leirásába, kiterjesztjük a termodinamikai formalizmust: az ekaentrópia fogalmának segitségével uj tagokat vezetünk be a Gibbs-Duhem relációba, amelyek az eltérést jellemzik. Két esetet vizsgálunk egyszerüsitett modellek segitségével: a primordiális termalizációt a GUT szimmetrikus fázisában, és a tömeges neutrinók nemegyensulyi eloszlásának kialakulását.

1. INTRODUCTION

In the last decade the horizon of cosmology has remarkably marched backward into the very early past, and now it almost reaches the Planck time, which is the absolute limit until the unified theory of gravitation, relativity and quantum effects is not known. This extension of the scope of cosmology is a consequence of the birth of new particle physical theories (as e.g. the Weinberg-Salam unification of the electromagnetic and weak interactions [1] and Grand Unified Theories [2]), which can yield predictions until at least 10^{15} GeV energy. Of course, these new theories describe exotic enough situations, thus it is not so easy to collect sufficient amount of evidences for them. Concerning the unified electroweak theory, two of the three new predicted particles, the W and Z coupling bosons have been experimentally found [3], with the predicted mass. Nevertheless, the Higgs boson is still unobserved, therefore one parameter of the Higgs self-interaction is unknown. The situation is even more obscure at higher energies, where some version of the Grand Unified Theory (GUT) is expected to describe the physics; no new particles or effects predicted by GUT have been observed until now. (For a review see Refs. 4 and 5.) Nevertheless, here we are not going to discuss the validity of these new theories; they are the simplest ones for the corresponding energy ranges, so, if one needs physical theories for constructing cosmological models for the very early Universe, then they are the most obvious candidates until clear counterevidences are not shown.

Nevertheless, even fully accepting these theories, some additional informations are necessary for a cosmological model. Generally speaking, the description of a physical process needs a theory for the interactions, and the specification of the initial condition. The new theories may correctly describe the interactions, but cannot specify the initial conditions, and one may have some doubts, how to choose the latter ones for our Universe, which should have to be unique.

This problem is not very explicite in the usual models. Namely, generally the degrees of freedom of the matter of the Universe are represented by several macroscopic data (which is, of course, necessary from technical reasons), as e.g. particle numbers, energy and volume. Then the necessary initial conditions are either directly connected to conserved observables (as e.g. the total electric charge), or they can be calculated from observables. Then, of course, there remains the problem, what is the fundamental reason of these specific initial data, nevertheless the practical problems are solved. In the same time, it is very convenient to use such a restricted set of characteristic data of the matter, and this is conform with our ideas about the simplicity of the Universe. Such a restricted set is yielded by thermodynamics [6-9].

Nevertheless, the axioms of thermodynamics do not guarantee that a generic physical system can be described by a finite set of data, even approximately; it is stated only for equilibrium states [10]. Therefore, the use of a thermodynamic formalism implicitely implies that one believes in the existence of (at least local) thermal equilibrium. Now, this is not a serious problem in a static system; either the initial conditions were lucky, or, if not, the interactions will equilibrate the system in a finite characteristic time anyway. Nevertheless, the Universe is par excellence nonstatic. Therefore, the realization of the second possibility is a question of the ratio of different characteristic times, while it would be unfair to a priori assume fulfilment of the first one (and it is well known that in dynamically evolving systems even the prearranged equilibrium may break down if the changes are too rapid [11]). In addition, there are some signals for the importance of deviations from equilibrium in the very early Universe: e.g. it seems that equilibrium processes cannot have led to generation of a nonvanishing total baryon number from a symmetric initial state (which generation is generally permitted in GUT) [5].

Therefore in this paper we want to discuss the possibility and the consequences of unequilibrium states in the early Universe. Two specific situations have been chosen, when unequilibrium states seem to be probable: the initial very rapid evolution of the Universe, when the characteristic time of the expansion was too short, and the decoupling of the neutrinos. In the second case it is obvious that after the decoupling there are no interactions maintaining the equilibrium between the neutrinos and other particles, but if the neutrinos are massless, their momentum distribution remains thermal with a false temperature; if $m_\nu \neq 0$, the distribution cannot remain thermal. Of course, when discussing such effects, one must not use the thermodynamic description. Nevertheless, that formalism can be extended to states without full thermalization [12], at the expense of introducing new characteristic quantities; in principle an infinite set is needed, but, if partial information is available, a finite set may be practically sufficient.

It is necessary to emphasize that nonequilibrium effects caused by comparable characteristic times have been investigated in the cosmologic literature. An excellent discussion can be found in Ref. 13. There it was shown that the finite collision time leads to viscous irreversibilities. Nevertheless, as the classical limit investigated in Ref. 12 clearly shows, there exist genuinely unequilibrium irreversibilities which cannot be described by transport terms as e.g. viscosity, although in such symmetric space-times as a Universe model different irreversible processes can simulate each other.

Since even a partial information seems to be absent about so early states of the Universe when the particle theories do not work, this paper is to be regarded as only an attempt to get some insight into the primordial irreversibilities. However, it is interesting to see which predictions of the standard model are direct consequences of the equilibrium hypothesis, definitely not too physical for some stages of the evolution.

2. PSEUDOTHERMODYNAMICS

Generally in realistic nonequilibrium situations there are two different types of deviations from equilibrium (although they may be connected with each other in some extent): either there are inhomogeneities in the spatial distributions of the characteristic quantities, or the momentum distributions deviate from the equilibrium ones. Obviously, spatial inhomogeneities generate transport fluxes, which then maintain certain deviations in the momentum distribution, too [14]. Nevertheless, this type of nonequilibrium situations is widely discussed, and there exist some standard formalisms to handle the problem. Furthermore, there are genuine unequilibrium situations, when the sources of the unequilibrium behaviour are not gradients and currents but the incomplete thermalization of some initial configurations (as e.g. in relativistic heavy-ion collisions [15]). Finally, in cosmology the inhomogeneities automatically vanish on large scale. Therefore, it seems that a formalism would be useful which can handle unequilibrium situations without direct connection with inhomogeneities. For simplicity's sake for this paper, which deals with cosmology, we completely ignore spatial inhomogeneities and currents in the local description of the matter, since such terms must actually vanish.

Obviously, if the system is out of equilibrium, as in the presently discussed situation, we cannot hold the usual assumption that the equilibrium extensives sufficiently describe the system; in the best case new parameters are necessary. A way to introduce "unequilibrium" thermodynamic parameters describing the unequilibrium features was given in Ref. 12. There, using the notion of ekaentropy P [16], and by the help of the generalised Callen Postulates [17], it was shown that deviations from the thermal distribution result in a corrected entropy function, which, in first approximation, reads as

$$S = S_o(X^{i\prime}) - \frac{1}{2}\sum_{i,k} g_{ik}(X^i - X^i_o)(X^k - X^k_o)\,\frac{1}{V} \tag{2.1}$$

where the $X^{i\prime}$s are the usual extensives, X^i stands for some extra, higher momenta with equilibrium values X^i_o, while

$$g_{ik} = g_{ik}(\rho^m) \sim \frac{\partial^2 (S/V)}{\partial\rho^i \partial\rho^k} \tag{2.2}$$

with $\rho^i = X^i/V$ (the densities). The physical meaning of this formula is transparent enough: the unequilibrium state is characterized by the higher momenta too, and the entropy takes its maximal value in equilibrium.

Ref. 12 gave some formulae how to handle unequilibrium situations by means of these extra parameters, at least for dilute classical Boltzmann gases. These formulae should be generalized for (dilute) relativistic quantum gases in order to discuss the early Universe; this generalization can be done. Nevertheless, a slight detour seems to be useful.

It is necessary to clearly state the physical assumptions before going into technical details, and the discussed situation is exotic enough to have to be careful. Now, a postulate system is the standard way to collect all the assumptions; and an axiomatic formulation guarantees the self-consistency of the formalism, although, naturally, not its applicability to real physical problems. Thus, before manufacturing our formulae for the actual situation, here we give a postulate system following Callen's thermodynamical postulates as closely as possible.

Definition: Extensive parameters X^i are parameters proportional with the extension of the system, obeying balance equations and characterizing the equilibrium states of the system investigated.

Definition: Pseudoextensive parameters $\bar{X}^i$ are parameters proportional with the extension of the system, obeying balance equations, whose equilibrium values are 0.

Postulate 1: There exist particular states of macroscopic systems (called pseudothermodynamic systems or PSTS) which, macroscopically, can be completely characterized by a finite set $\{X^i, \bar{X}^i\}$.

Postulate 2: There exists a function P, called ekaentropy, for any PSTS, which is a homogeneous function of first order of the parameters $\{X^i, \bar{X}^k\}$.

Postulate 3: There exists the limit

$$\lim_{\bar{X}\to 0} P(X,\bar{X}) = P_o(X) \tag{2.3}$$

Postulate 4: The function $P_o(X)$ is differentiable, and monotonously increases with the energy; and

$$\lim_{\frac{\partial P_o}{\partial E}\to\infty} P_o = 0 \tag{2.4}$$

Postulate 5:

$$\lim_{\bar{X}\to 0} \frac{\partial P}{\partial \bar{X}} = 0 \tag{2.5}$$

Postulate 6: The matrix of the second derivatives of P with respect to $\bar{X}$'s is negative definitie.

We note that in these Postulates "finite" stands for "at most countably infinite".

If there exists a physical process $\bar{X}\to 0$, then at the end the system reaches a thermodynamic state. Namely that state will be characterized purely by extensives X^i; P reduces to a P_o fulfilling the equivalents of the Callen Postulates for ekaentropy, and for such functions there exists a transformation $P_o\to S$ [17]. Note that we have not postulated that for all the real processes $\bar{X}^i\to 0$; a counterexample will be shown in Sect. 9. Postulate 6 means that the unequilibrium correction for the entropy (if one can define it) starts as a negative quadratic term, being entropy one of the ekaentropies [17].

3. THE EINSTEIN EQUATION

According to fundamental assumptions of General Relativity, the geometric properties of the space-time is determined by the matter content. In the simplest realization of this idea it happens via the Einstein equation. First, for weak gravitation and slow motion $\frac{1}{2}(g_{oo}-1)$ plays the role of the gravitational potential [18], therefore the gravitational equation has to contain second derivatives in order to get the Newtonian limit. Then the simplest possible equation is linear in the second derivatives of g_{ik}. The only such tensor of two indices (up to trivial algebraic manipulations) is the Ricci tensor R_{ik}, and then the gravitational equation has the form

$$R_{ik} = \kappa Q_{ik} \tag{3.1}$$

where κ is an appropriate constant, and Q_{ik} is a still undefined tensor characterizing the distribution of the matter.

Now, the combination $R_{ik}-(R/2)g_{ik}$ is divergence-free by construction, thus

$$(Q^{ik} - \frac{1}{2} g^{ik}Q^r{}_r)_{;k} = 0 \tag{3.2}$$

That is, this combination is an object of four conservation laws. However, we know that in a closed system the energy and 3-momentum are conserved, therefore (except for trivially conserved terms) we has to identify this combination with the energy-momentum tensor. Hence

$$Q_{ik} - \frac{1}{2} Q^r{}_r g_{ik} = T_{ik}-(\lambda/\kappa)g_{ik} \tag{3.3}$$

where λ is a new constant. Then eq. (3.1) obtains the form

$$R_{ik} - \frac{1}{2} R^r{}_r g_{ik} + \lambda g_{ik} = \kappa T_{ik} \tag{3.4}$$

which is the usual form of the Einstein equation. By means of a proper definition of the zero point of the energy (i.e. that of the vacuum) the cosmological constant can be made 0 [20]. This form of the gravitational equation will be accepted here. From the Newtonian limit one obtains [19]

$$\kappa = 8\pi G \tag{3.5}$$

where G is the gravitational constant.

4. THE HYDRODYNAMICAL APPROXIMATION

The energy-momentum tensor characterizes the distribution and motion of the matter; its form may be quite complicated. By a formal way it can be decomposed with respect to any timelike unit vector field u^i as

$$\begin{aligned} T^{ik} &= \rho u^i u^k + q^i u^k + u^i q^k + p^{ik} \\ u^r u_r &= -1,\ u^r q_r = p^{ir} u_r = 0 \end{aligned} \tag{4.1}$$

Because of the structure of the energy-momentum tensor ρ is the energy density, q^i is the energy flux 3-vector and p^{ik} is the spatial stress tensor, measured by an observer whose velocity is u^i. If such a velocity field is preferred by certain physical reasons, then the other quantities on the right hand side of eq. (4.1) possess some physical meaning, otherwise the decomposition is a purely mathematical operation.

If there is a unique and physically important flow velocity field in the matter, then it is natural to choose this vector field as u^i. In this case the quantities ρ, q^i and p^{ik} are measured in a system comoving with the matter, one may expect no substantial local velocities, and therefore the relations among ρ, q^i and p^{ik} may be similar to the classical relations [14, 21]. (For an example, when this program cannot be realized, see the two fluid model [22].)

Nevertheless, it is definitely not obvious how to choose the velocity field. The so called Landau gauge can always be constructed. Then u^i is the flow velocity of the energy [23]:

$$T^{ir} u_r = -t u^i \tag{4.2}$$

Then $q^i = 0$.

If there are identifiable particles in the system, one can investigate their current densities. The currents can be decomposed as

$$n_A^{\ i} = n_A u^i + \nu_A^{\ i}$$
$$\nu_A^{\ r} u_r = 0 \qquad (4.3)$$

where $_A$ labels the different particle degrees of freedom. If these degrees of freedom are physically important, then it would be necessary to choose the flow velocity of the particles as u^i, since the agents of interactions are the particles. Unfortunately, generally there is no guarancy that all the current vectors are proportional to each other. If, however, they are from some physical reason, or if there is only one such degree of freedom, then one can choose

$$u^i = n^i/n$$
$$n = (-n^r n_r)^{1/2} \qquad (4.4)$$

where n is the particle density. This velocity vector is generally not an eigenvector of T^{ik}, therefore $q^i \neq 0$.

In our special case, as we shall see, the two gauges coincide, since the high symmetry of the space-time guarantees the uniqueness of the velocity field. Therefore here it is not necessary to discuss further the physical differences between these gauges.

We have seen that the energy-momentum tensor is divergence-free:

$$T^{ir}_{\ \ ;r} = 0 \qquad (4.5)$$

These four equations can be decomposed in such a way that three equations of motion are obtained for the three independent components of u^i, while the fourth one is the differential form of the First Law of Thermodynamics [24] (i.e. a balance equation for the energy). In the simplest case when the matter possesses only a single thermal degree of freedom, these four equations completely determine the evolution of the matter (an obvious example is the blackbody radiation), and eq. (3.4) yields the geometry. When additional degrees of freedom exist, extra evolution equations are needed. E.g., for the particle densities the continuity equations

$$n_A^{\ r}{}_{;r} = \psi_A \qquad (4.6)$$

are valid. However, there is no a priori information about the source terms ψ_A; (except for some unequalities coming from the Second Law of Thermodynamics); they can represent e.g. chemical transmutations, or even spontaneous annihilation or creation of particles [25]. Conservation laws give algebraic constraints for the source terms.

The explicite forms of the balance laws and the Second Law will be discussed after imposing the symmetry principles on the system.

5. SYMMETRIES

It is a more or less general belief that the Universe (on a large scale) possesses a symmetry which is maximal in some sense. This belief is often referred as cosmological principle [26]. Here, using Occam's razor, we accept that the symmetry is the maximal which is compatible with the observations, and that it is valid for the individual components building up the energy-momentum tensor, too. (As a counterexample see Ref. 27.) This can be formulated in such a way that for some set of Killing vectors K^i_α the Killing equation

$$K_{\alpha i;k} + K_{\alpha k;i} = 0 \tag{5.1}$$

holds, and the Lie-derivatives of the material fields, ρ, q^i, u^i and p^{ik} vanish along the K^i_α fields [28]:

$$\underset{K}{\mathcal{L}}\omega^{ik\dots} = 0 \tag{5.2}$$

where $\omega^{ik\dots}$ represents the relevant quantities connected with the matter.

Now, the maximal number of Killing vectors in four dimensions is 10 [29], nevertheless, this case is excluded by the observations [18]. Furthermore, imposing Conds. (5.2) on u^i one gets that no such timelike vector field can exist. Therefore we can conclude that such a high symmetry is impossible from some reason; the next natural symmetry principle is full spatial symmetry, i.e. the existence of six space-like Killing vectors. Then there are only three possibilities for the symmetry group:

$k =$	$+1$	0	-1
Group:	SO(4)	E(3)	SO(3,1)

[29]. In the lack of serious counterevidence, we use this symmetry group. Then eq. (5.1), in adapted coordinates, leads to the line element

$$ds^2 = -dt^2 + R^2(t)\{dr^2+\chi(r)^2(d\vartheta^2+\sin\vartheta d\varphi^2)\}$$

$$\begin{array}{llll} k = & +1 & 0 & -1 \\ \chi(r) = & \sin r & r & \mathrm{sh}\, r \end{array} \tag{5.3}$$

By evaluating Conds. (5.2) in these coordinates, one gets

$$\begin{aligned} \rho &= \rho(t) \\ q^i &= 0 \\ u^i &= \delta^i_o \\ p^{ik} &= P(t)(g^{ik}+u^iu^k) \end{aligned} \tag{5.4}$$

all the currents in the matter are parallel to u^i, and the scalars can have t-dependence only. One can see that for such a symmetry the stress tensor is isotropic, the stresses are represented by a single scalar P.

Now, with such an isotropic p^{ik} and with $q^i = 0$ eqs. (4.5-6) yield the following balance equations for ρ and n^A:

$$\dot{\rho} + (\rho+P)u^r{}_{;r} = 0$$
$$\dot{n}^A + n^A u^r{}_{;r} = \psi^A \tag{5.5}$$

where the dot derivative is $u^r\nabla_r$. Because of the spatial symmetry the entropy current s^i has the form

$$s^i = su^i \tag{5.6}$$

s denotes the entropy density. Here, for reasons to be fully understood in Sect. 7, we assume that

$$s = s(\rho,\ n^A,\ z^\Gamma) \tag{5.7}$$

where z^Γ are some scalars representing extra degrees of freedom, with the balance laws

$$\dot{z}^\Gamma + \alpha^\Gamma z^\Gamma u^r{}_{;r} = \zeta^\Gamma$$
$$\alpha^\Gamma = \alpha^\Gamma(\rho, n^A, z) \tag{5.8}$$
$$\zeta^\Gamma = \zeta^\Gamma(\rho, n^A, z)$$

Then the Second Law of Thermodynamics can be formulated as [14]

$$s^r{}_{;r} = \dot{s} + su^r{}_{;r} \geq 0 \tag{5.9}$$

Using the balance equations, and separating the terms of different $u^r{}_{;r}$ dependence, uneq. (5.9) leads to

$$s - s_\rho(\rho+P) - S_R n^R - \alpha s_\Gamma z^\Gamma = 0 \tag{5.10}$$

$$s_R\psi^R + s_\Gamma\zeta^\Gamma \geq 0 \tag{5.11}$$

where, as a shorthand notation, a lower index of s means derivation.

While uneq. (5.11) is a constraint for the source terms, eq. (5.10) expresses P as a function of ρ, n^A and z^Γ, if the form of the entropy function is given. For the chosen symmetry the only two nontrivial components of the Einstein equation (3.4) with $\lambda = 0$ are as follow:

$$\dot{R}^2 = \frac{8\pi G}{3} \rho R^2 - k$$
$$\ddot{R} = -\frac{4\pi G}{3} (\rho + P) R \qquad (5.12)$$

while the balance equations for n^A and z^Γ read as

$$\dot{n}^A + 3 \frac{\dot{R}}{R} n^A = \psi^A$$
$$\dot{z}^\Gamma + 3\alpha^\Gamma \frac{\dot{R}}{R} z^\Gamma = \zeta^\Gamma \qquad (5.13)$$

where the dot derivative is the t derivative. Having the function $s(\rho, n^A, z^\Gamma)$, and the functions ψ^A, α^Γ, ζ^Γ fixed, the system of eqs. (5.10), (5.12-13) completely determines the evolution of the Universe.

In this paper, for obtaining simple formulae for the particle physical relations, we use such units that

$$\hbar = c = 1 \qquad (5.14)$$

In these units

$$G = \frac{1}{M^2} \qquad (5.15)$$

where M is the Planck mass, $1.22 \cdot 10^{19}$ GeV.

6. CONDITIONS FOR EQUILIBRIUM

The description of the continuum is the simplest if it is in thermal equilibrium. Then, for simple systems [14] the local state is determined by the parameters ρ and n^A. For simplicity's sake, consider a dilute gas of one type of particles. If it is sufficiently dilute, then the two-particle correlations are negligible, the local state is described by a momentum distribution function $f(\underline{p})$

$$dN = f(\underline{p}) dV \frac{d^3 p}{p^o} \qquad (6.1)$$

and the right hand side of the Einstein equation is built up from some momenta of f. Then the coupled Einstein-Boltzmann equations are to be solved [14]. In thermal equilibrium f is known up to two continuous and one discrete parameters as

$$f_o(\underline{p}) = (e^{(-u_r p^r - \mu)/T} + q)^{-1} \qquad (6.2)$$

where u^i is the flow velocity, μ is the chemical potential, T is the temperature, and q depends on the quantum properties of the particles: q=+1 for fermions, -1 for bosons, and 0 if quantum statistics can be ignored. Obviously, ρ and n are functions of μ and T; these functions can be inverted, so the local state is determined by ρ and n, indeed, in equilibrium.

Nevertheless, generally the equilibrium is incompatible with the Einstein-Boltzmann equations. Some exact negative statements are known for exact equilibrium [14]: it cannot hold, if the particles possess nonvanishing rest mass, and there is no timelike Killing vector; for massless particles the necessary condition for equilibrium is the existence of a timelike conformal Killing vector. The line element (5.3) possesses timelike conformal Killing vector, so a gas of massless particles may remain in thermal equilibrium during the expansion, however, it is not necessary that it reach equilibrium starting from a generic initial state.

Nevertheless, the above mentioned theorems tell us very little about approximate equilibrium. Without specifying the system, consider a matter of some particles, near to thermal equilibrium and with an equilibrating process of some characteristic time τ. If T is changing, the distribution of particles has to be being continuously rearranged, and this cannot be effectively done if

$$\left|\frac{\dot{T}}{T}\right| \geq \frac{1}{\tau} \tag{6.3}$$

[11,30]. On the other hand, if the temperature change is sufficiently moderate, then the equilibrating process dominates, and so the system probably remains near to equilibrium.

There exist some indications that unequilibrium situations were important in some stages of the evolution of the Universe; e.g. Ref. 31 lists four such situations, namely

a) supercooling in the GUT symmetry breaking phase transition, resulting in inflation;
b) subsequent unequilibrium decay of the X bosons leading to baryon excess;
c) supercooling in the quark-hadron phase transition; and
d) inequilibrium in the nucleosynthesis, leading to the present chemical composition of the Universe.

In cases a) and c) there were two phases, but possibly both phases were in thermal equilibrium with themselves; in the other two cases the chemical equilibrium did not hold, but the thermal one might be valid. Now, obviously, there is a third possibility that the momentum distribution is not thermal.

For simplicity's sake, assume that the Universe is radiation-dominated. In equilibrium both ρ and p are determined by T [4]

$$\rho = 3p = \frac{\pi^2}{30} NT^4 \tag{6.4}$$

where N is the number of helicity states, $N=N_b+7N_f/8$. The characteristic time τ is in the order of the time between two subsequent collisions of the same particle, therefore it can be estimated as

$$\tau \simeq \frac{1}{\tilde{n}\sigma} \tag{6.5}$$

where $\tilde{n}$ is a characteristic density sum for all kinds of particles with which the collision is possible, and σ is an average cross section. Both quantities are expected to depend on T, for $\tilde{n}$

$$\tilde{n} \simeq \frac{\zeta(3)}{\pi^2} N^* T^3 \tag{6.6}$$

where $\zeta(3)$ is the Riemann function, $N^*=N_b+3N_f/4$ for the particles considered. In a gas of point particles, up to a number constant depending on the details of the differential cross section,

$$\sigma \sim \frac{\alpha^2(T)}{T^2} \tag{6.7}$$

[4], where $\alpha(T)$ is an (effective) coupling constant. In a radiation-dominated Universe $RT \simeq$ const., therefore, using eqs. (5.12) and (6.3-7), one obtains that the approximate equilibrium cannot hold if

$$\frac{T}{M} > \sqrt{\frac{45N}{4\pi}} \, \frac{N^*}{N} \, \frac{\zeta(3)}{\pi^3} \, \alpha(T)^2 \tag{6.8}$$

We are going to evaluate this unequality for two cases. The first is the decoupling of (massive) neutrinos (if $m_\nu = 0$, the decoupling does not alter the form (6.2) because there exists timelike conformal Killing vector; the upper limit for any neutrino mass is several dozen eV [32]). Then, for weak interaction at low energies [33]

$$\alpha_w \sim \alpha_{e.m.} \frac{T^2}{E^2} \tag{6.9}$$

where $\alpha_{e.m.} = 1/137$, and $E \sim 100$ GeV is in the order of the rest mass of the W or Z bosons, and the scale of the symmetry breaking in the Weinberg-Salam theory. Then uneq. (6.8) holds for temperatures lower than a certain limit. Using the numerical values $N_b=2$ (photons), $N_f = 4(e^+ + e^-)+3x2$ (neutrinos), one gets that this limiting temperature is cca. 9.4 MeV; below this value the distribution of massive neutrinos starts to deviate from a thermal one.

The second case is the thermalisation of the primordial distribution of the GUT particles. At temperatures definitely higher than 10^{15} GeV it is believed that all the particle masses are negligible; N_b=82 and N_f=90 in minimal SU(5) GUT, and $\alpha \simeq 1/45$ [4]. Using these numbers, one gets that uneq. (6.8) holds above $T \simeq 6.2.10^{15}$ GeV. Thus at such temperatures thermal equilibrium is not expected, unless there were some evidence that the primordial distribution had been thermal, which is not probable. Notice the coincidence between this temperature value and the critical value of the GUT scale parameter at which viscosity is strong enough at the phase transition to produce sufficient entropy and to stop the cooling [34].

7. RELATIVISTIC UNEQUILIBRIUM STATES

Here we give the relativistic version of a recently developed unequilibrium formalism [12] which keeps thermodynamic language for unequilibrium states, so will be referred as pseudothermodynamics. The essence of this formalism was to introduce higher momenta as new extensive variables; then an entropy function can be defined which depends on both the equilibrium and the unequilibrium parameters, and starts quadratically with the unequilibrium ones near equilibrium; the evolution of the system is determined by the form of this function and by the specific evolution equations for the unequilibrium parameters. Now, mutatis mutandis, we are going to repeat the steps of Ref. 12.

For simplicity's sake, let us start from the assumptions of Sect. 6. So the system is sufficiently dilute to be described by one particle distribution functions (there are some arguments that even the GUT continuum before the symmetry breaking phase transition at 10^{-37} s can be regarded as a dilute gas [4]); in addition we assume full spatial homogeneity with vanishing conductive currents. (This second assumption may be strong, nevertheless it is the proper assumption for the Universe; without it one could expect the familiar transport terms too.) For such a gas an entropy flux can be defined [14]:

$$s^i = -\int [f\ln f + q(1-qf)\ln(1-qf) - (1-q^2)f]\, p^i \frac{d^3p}{p^o} \tag{7.1}$$

(the parameter q was defined in the previous Section). The distribution function f is governed by the relativistic Boltzmann equation:

$$L(f) = C(f) \tag{7.2}$$

where L is the Liouville operator,

$$L(f) = [p^r\partial_r - \Gamma_{rs}{}^t p^r p^s \frac{\partial}{\partial p^t}] f(p^i, x^k) \tag{7.3}$$

and C(f) is the collision integral; its specific form depends on the particular statistics and on the interactions [13]. For ideal gases the equation

$$C(f_o) = 0 \tag{7.4}$$

possesses the solutions given in (6.2) without assuming thermal equilibrium. From hence f_o stands for the solutions of eq. (7.4); its parameters u^i, T and μ may depend on the coordinates (here, due to the spatial homogeneity only on time). Therefore, if the actual distribution function f is sufficiently near to one of the set f_o, then C(f) is linear in $f-f_o$; here we restrict ourselves to the relaxation time approximation

$$C(f) \simeq \frac{1}{\tau}(u_r p^r)(f-f_o) \tag{7.5}$$

where u^i is the flow velocity of the matter defined by eq. (4.4); for this specific form see the Appendix. Of course, because of the high symmetry in the discussed case, f is not depending on the spatial coordinates.

Now, consider an arbitrary given distribution function f; one has to establish a connection between the actual f and the corresponding member of the set f_o. This can be done by requiring as many constraint equations for f_o as the number of its free parameters. If that number is five, then one may use the Eckart form of matching conditions [14]:

$$\begin{aligned} n^i &= \int f p^i \frac{d^3p}{p^o} = n^i_o \\ \rho &= \int f(p_r u^r)^2 \frac{d^3p}{p^o} = \rho_o \end{aligned} \tag{7.6}$$

if the particle number density is not an independent characteristic (e.g. for charge-symmetric situations, cf. Ref. 35) a different system may be necessary.

Via eq. (4.4) the matching conditions (7.6) guarantee that $n=n_o$ and $u^i=u^i_o$. These conditions single out an f_o for any given f. Now, take any convenient basis of functions $\{\varphi_\alpha(\underline{p})\}$. (On mass shell, the p^o dependence in f is superfluous.) Then $f-f_o$ can be expanded on this basis as

$$f(\underline{p},x^i) = f_o(\underline{p},x^i) + \sum_\rho a^\rho(x^i)\varphi_\rho(\underline{p}) \tag{7.7}$$

The basis $\{\varphi_\alpha(\underline{p})\}$ is completely arbitrary, except for the condition that its functions keep the matching conditions, i.e.

$$\int \varphi_\alpha \, p^i \frac{d^3p}{p^o} = 0$$
$$\int \varphi_\alpha \, (p^r u_r)^2 \frac{d^3p}{p^o} = 0 \qquad (7.8)$$

Then the local state is completely determined by the five parameters of f_o and by the new parameters a^α. The evolution of a^α's can be deduced from the Boltzmann equation and in an equilibrating process $a^\alpha \to 0$.

Because of the assumed symmetries

$$s^i = su^i \qquad (7.9)$$

the extra terms would be proportional with the heat conduction flux q^i [14], and we ignore here such fluxes. Then s is the entropy density. By construction s depends on T,μ and a^α. Instead of a^α one can introduce a (generally infinite) set of extra tensorial momenta $b^{ik...}$ as

$$b^{ik...} = \int f(\underline{p}) p^i p^k \ldots \frac{d^3p}{p^o} \qquad (7.10)$$

n^i and ρ are among the members of this set. Then, by means of eqs. (4.4), (6.2), (7.5) and (7.10), one can introduce n, ρ and the extra $b^{ik...}$'s instead of T,μ and a^α.

Consider a fixed volume V. Then, from the density-like $b^{ik...}$'s one can form parameters proportional to the extension of the system as

$$C^{ik...} = F^{ik...}(V, N=nV, E=\rho V)\, b^{ik...}$$
$$F^{ik...}(\lambda V, \lambda N, \lambda E) = \lambda F^{ik...}(V,N,E) \qquad (7.11)$$

otherwise the functions $F^{ik...}$ are arbitrary convenient functions. Thus

$$S = sV = S(V,N,E,C^\Gamma) \qquad (7.12)$$

where the capital Greek index is a shorthand notation for combinations ik... other than in N and E. Because of the assumed homogeneity S is additive for subsystems

$$S(1+2) = S(1) + S(2) \qquad (7.13)$$

whence S is a homogeneous function of first order of its variables. Consider equilibrium states of the matter. The C^Γ parameters take some values there

$$C^\Gamma_{eq} = C^\Gamma_{eq}(V,N,E) \qquad (7.14)$$

and one can form new parameters vanishing in equilibrium:

$$z^{\Gamma} = C^{\Gamma} - C^{\Gamma}_{eq} \tag{7.15}$$

introducing z^{Γ} instead of C^{Γ} in S. Because of its homogeneous linearity, S fulfils the Euler identity

$$V \frac{\partial S}{\partial V} + N \frac{\partial S}{\partial N} + E \frac{\partial S}{\partial E} + z^{\Gamma} \frac{\partial S}{\partial z^{\Gamma}} = S \tag{7.16}$$

which can be read as

$$S = \frac{p}{T} V - \frac{\mu}{T} N + \frac{1}{T} E + \frac{Q_{\Gamma}}{T} z^{\Gamma} \tag{7.17}$$

This equation defines some homogeneous quantities of zero order.

If there is equilibration in the system, then $z^{\Gamma} \to 0$. The time evolution of z^{Γ} can be obtained via the Boltzmann equation, using the defining eqs. (7.10-11), (7.14-15) too; e.g. in the relaxation time approximation,

$$b^{ik...r}{}_{;r} = \frac{1}{\tau} u_r (b^{ik...r} - b_o^{ik...r}) \tag{7.18}$$

by using Gauss' theorem [14]. Then, for $z^{\Gamma} = Z^{\Gamma}/V$ one can get balance equations, generally with right hand sides, so the quantities Z^{Γ} will not be conserved. The proper choice of the functions $F^{ik...}$ can be used to get convenient form for these balance equations; if the equilibrium can be preserved in the system, then [13]

$$\begin{aligned} f_{eq} &= f_o \\ L(f_o) &= 0 \end{aligned} \tag{7.19}$$

and then the form (5.8) can be achieved. (Nevertheless, the existence of the equilibrium is not trivial [14]; this question will be discussed in Sect.9.)

Now, let us stop for a moment to look over the results of this section. We have a system, which, at least in stationary space-times, can be in equilibrium at f_o. Then one can recognize the z^{Γ} quantities as pseudoextensives (obeying balance equations, with vanishing equilibrium values), while V, N and E are extensives. Evaluating S via eqs. (7.1), (7.6), (7.9) and (7.12), one gets

$$S = S_o(V,N,E) - \frac{1}{2} \frac{V}{8\pi^3} \int \sum_{\rho} (a^{\rho} \varphi_{\rho})^2 \frac{f_o}{-qf_o+1} d^3p + \vartheta(a^3) \tag{7.20}$$

where S_o is the entropy in the equilibrium, so it starts quadratically with a^α, so with Z^Γ too:

$$S = S_o(V,N,E) - \frac{1}{2} S_{\Gamma\Delta}(V,N,E) Z^\Gamma Z^\Delta + \vartheta(Z^3) \qquad (7.21)$$

where $S_{\Gamma\Delta}$ is a homogeneous function of order -1. Then S satisfies Postulates 2, 3, 5 and 6; S_o obviously satisfies Postulate 4, and, since f is determined by f_o and a^α, the actual state is completely characterized by V, N, E and Z^Γ, the only question is whether the necessary set of Z^Γ is countable or not. It would be difficult to decide this in the general case, but this question is immaterial for practical purposes, when the set is truncated somewhere anyway. Therefore one can conclude that our present formalism is conform with the postulate system given in Sect. 2, so it will be called pseudo-thermodynamics.

In the formulation of the postulate system we assumed the notion of the equilibrium states but did not assume that the system can in fact remain in these states. The meaning of this distinction will be explicitely shown in Sect. 9.

Obviously, for practical use, a moderate set of pseudoextensives would be needed. This is equivalent with the problem how to choose the most proper set of functions $\{\varphi_\alpha(\underline{p})\}$, whose truncation causes the smallest possible error. This question was discussed in some extent in Ref. 12; the answer would need the knowledge of the initial conditions and the exact form of the collision integral. For the primordial equilibration of the Universe one obviously cannot know the initial conditions; for the neutrino decoupling the problem could be solved in principle, but this would involve serious technical difficulties. So here we give only a model calculation with maximal simplifications: the pseudothermodynamical states are characterized by a single pseudoextensive, which is chosen according to maximally analytic expressions. For the primordial equilibration it is the use of Occam's razor; for the neutrino decoupling the model can be improved if necessary.

8. EQUILIBRATION IN THE VERY EARLY UNIVERSE

Grand Unification Theories enable us to look back into the very early stages of the evolution of the Universe, almost until Planck time (cca. 10^{-43} s). The picture suggested by GUTs may or may not be correct, considering that clear evidences for these theories (e.g. proton decay) are still absent, the extrapolations are very strong (for example, the desert between 10^2 and 10^{15} GeV), and some predictions (as the monopole dominance) are disturbing [4,5]. Nevertheless, there are no competitors for so early times, therefore here we do not discuss the validity of any specific GUT.

GUTs contain a spontaneous symmetry breaking at some energy determined by the energy scale parameter, which is believed to be roughly 10^{15} GeV [36, 37]. Above this energy the fermions and Gauge bosons are massless; at asymptotic temperatures radiation-dominated behaviour is generally assumed [4,5]. Since the situation is charge-symmetric, in these stages the only independent extensive density is the energy density ρ. Assuming complete thermal equilibrium one arrives at the standard model [4] discussed in Sect. 6. Since

$$s = \frac{2\pi^2}{45} NT^3 \tag{8.1}$$

one can directly express s by ρ, through eq. (6.4).

Now, observe that for k=0, which is a good approximation for so early stages [4], eqs. (5.12) and (6.4) yield

$$\frac{\dot{R}}{R} + \frac{\dot{T}}{T} = 0 \tag{8.2}$$

But then, with a velocity field (5.4), the vector field $v^i=u^i/T$ fulfils the conformal Killing equation

$$v_{i;k} + v_{k;i} = Vg_{ik} \tag{8.3}$$

in Robertson-Walker metrics. Therefore, evaluating the Liouville equation for f_o with such u^i and T, and with $\mu=0$, one obtains

$$L(f_o) = -\frac{m^2}{T}\frac{\dot{R}}{R} f_o \tag{8.4}$$

Therefore for massless particles $L(f_o) = 0$, the set of distributions f_o is a stable endpoint of the evolution of f [14].

Now, consider a state when $f \neq f_o$, but f is still isotropic according to the assumed symmetries. Then, being $p^{o2} = \underline{p}^2$,

$$\rho = 3p \tag{8.5}$$

In the absence of particle density as extensive density, one can define u^i in the Landau gauge (4.2), and then the only remaining matching condition is

$$\rho = \rho_o \tag{8.6}$$

Then the parameter $\tilde{T}$ of f_o belonging to the actual f fulfils eq. (6.4), so the statements of the standard model remain valid for R(t) and $\tilde{T}(t)$:

$$R = R_o \sqrt{t}$$
$$\tilde{T} = \sqrt{\frac{3M}{4\pi}} \left(\frac{5}{N\pi}\right)^{1/4} t^{-1/2} \tag{8.7}$$

The evolution of f can be calculated on this geometry.

Let us use the relaxation time approximation (7.5), with a relaxation time according to eqs. (6.5-6), (6.9). Then, as it was shown in Sect. 6, there is a characteristic temperature $T_o \simeq 6.2.10^{15}$ GeV, above which the collisions were ineffective to produce equilibrium. For earlier stages we do not have any information, and it would not be fair to assume equilibrium as initial condition. Therefore we expect that at T_o f essentially differs from f_o, however, of course, do not know how. From this reason there is no way to find the optimal basis $\{\varphi_\alpha(\underline{p})\}$, i.e. only a guess can be done. This will happen according to the principle of maximal simplicity (Occam's razor).

Consider one particle component, either fermion or boson, and assume that the most important mode of deviation is

$$f = f_o\{1 + a[e^{-2x}(e^x+q)+A]\}$$
$$x = E/\tilde{T} \tag{8.8}$$

where A is determined by the matching conditon (8.6) as

$$A = -\frac{45}{\pi^4}\frac{2}{15-q} \tag{8.9}$$

The chosen deviation function has a decent behaviour; it is simple enough with a maximum at $E \sim \tilde{T}$ and with an exponential tail, it would be difficult to tell more for its favour. The most convenient extra momentum is the particle number excess:

$$z = \frac{1}{2\pi^2}\int (f-f_o)E^2 dE = \frac{a}{8\pi^2}[1 - \frac{45}{\pi^4}\zeta(3)]\tilde{T}^3 \tag{8.10}$$

Because of the relaxation time approximation its evolution equation is

$$\dot{z} + 3\frac{\dot{R}}{R}z = -\frac{1}{\tau}z \tag{8.11}$$

where, using the approximations of Sect. 6,

$$\frac{1}{\tau} = \alpha_{GUT}^2 \frac{N^*\zeta(3)}{\pi^2}\tilde{T} \tag{8.12}$$

and this particular component gives a contribution to s (up to quadratic terms) as follows:

$$s = s_o - \frac{32\pi^2}{[1 - \frac{45}{\pi^4}\zeta(3)]^2}\left[\frac{1}{125} + \frac{q}{32} + \frac{1}{27}\left(1 - \frac{180q}{(15-q)\pi^4}\right) - \frac{45}{2(15-q)\pi^4} + \left(\frac{90}{(15-q)\pi^4}\right)^2\right]\frac{1}{\tilde{T}^3} z^2 \quad (8.13)$$

where $\tilde{T}$ can be expressed by ρ via eq. (6.4).

Now, the calculations should start from T_o given by eq. (6.8) with $\alpha(T) = \alpha_{GUT} \simeq 1/45$. Using eqs. (8.7), (8.11-12), the result is

$$z^{\Gamma} R^3 = (z_o^{\Gamma} R_o^3) \exp\{-(\frac{T_o}{\tilde{T}} - 1)\} \quad (8.14)$$

where the initial conditions z_o^{Γ} represent the complete lack of knowledge about the situation above T_o. In any case, one can guess that for so early stages the deviations were substantial. Therefore the assumption of equilibrium is groundless for such temperatures when the exponential factor is not sufficiently small.

Of course, we do not have any objective measure for the smallness of this factor, not knowing the initial conditions decaying. Nevertheless, it seems to be decent to require that the factor should be at most some percents to get approximate equilibrium. It is cca. 5% at $T=T_o/4$, which seems to be $1.5.10^{15}$ GeV. Above this value the completely unknown initial conditions can be felt, therefore, although the expansion of the Universe is not affected, the predictions of GUT calculations cannot be regarded as unique. This gives a technical limit for the energy scale parameter lower by a factor 4 than in Ref. 34; for higher values the symmetry breaking phase transition would be affected by the unknown initial conditions.

9. NEUTRINO DECOUPLING

In Sect. 6 we gave an estimation for the temperature where the neutrinos became decoupled; the result was cca. 10 MeV. The history of the Universe was quite complicated in that temperature range; possibly with individual temperatures for the different types of neutrinos and for the electromagnetic sector (e^+, e^- and γ); there is an annihilation process for the e^+-e^- pairs at 0.5 MeV [30]. Nevertheless, the decoupling of the neutrinos does not automatically lead to the development of unequilibrium distributions; if some process disturbs the equilibrium distribution then the collisions may be ineffective to reestablish it, nevertheless it seems that if the neutrinos are massless, there is no obvious candidate for such a process. However, they are not necessarily massless; cosmological observations are compatible with a several dozen eV mass [38], and some measurements seem to indicate such a mass [32]. But then f_o is not a solution of the Liouville equation [14], therefore the particles cannot remain in equilibrium during the expansion of the Universe. Of course, for high temperatures, when m^2/T^2 is small enough and the cross sections of weak interaction are substantial, f_o is a good approximation for f, the deviations will continuously develop. Here we want to get some insight into this process.

Of course, as we mentioned above, the real situation during this process was complicated, so some computer simulation would be necessary; this will be done in a subsequent paper; here we are going to manufacture a simple caricature to emphasize the important features of the process. Therefore

a) we ignore the electromagnetic sector, and consider three kinds of neutrinos only, with a common mass m and temperature T;

b) the pair annihilation will be neglected below some temperature T_o, which may be e.g. the decoupling temperature, there the neutrinos will be approximated by a Boltzmann distribution; and

c) everything will be calculated only up to m^2 terms.

The first simplification underestimates slightly the expansion rate, the first of b) is not very rough, because without mass terms the pair number $\sim T^3$ would be conserved in equilibrium, the second is a technical trick; none is explicitely causing or forbidding thermal unequilibrium. As a support for c), we shall see that they are indeed the leading terms of unequilibrium contributions.

Now we can proceed as follows. We have two balance equations for the extensive densities:

$$\dot{n} + 3\frac{\dot{R}}{R}n = 0$$
$$\dot{\rho} + 3\frac{\dot{R}}{R}(\rho+P) = 0 \tag{9.1}$$

where P is not necessarily the pseudothermodynamic pressure given by eq. (7.17). Since the quantities n and ρ can be calculated from f_o too, via the matching conditions (7.6), eq. (9.1) determines the evolution of the parameters of f_o. The right hand side of the Boltzmann equation can again be expanded around f_o, and we take the form (7.5); nevertheless now $L(f_o) \neq 0$, thus f_o is not a solution. Using the operator (7.3) the term $L(f_o)$ can be evaluated; it contains $\dot{\tilde{\mu}}$, $\dot{\tilde{T}}$ and $\dot{R}/R$. If $\mu/\tilde{T}$ were constant and $u^i/\tilde{T}$ were a conformal Killing vector, $L(f_o)$ would be proportional to m^2 (cf. eq. (8.4)); these conditions do not hold, but this is caused by mass terms too (compare the present situations with that of discussed in Sect. 8), so one expects $L(f_o)$ to remain proportional to m^2; we shall see that this is, indeed, the situation. Neglecting first any unequilibrium contribution in P, for a Boltzmann distribution f_o eqs. (7.3) and (9.1) give

$$L(f_o) = -\frac{m^2}{\tilde{T}}\frac{\dot{R}}{R}\left(\frac{1}{6}\frac{E^2}{\tilde{T}^2} - \frac{E}{\tilde{T}} + 1\right)f_o + \vartheta(m^4) \qquad (9.2)$$

So, calculating $L(f-f_o)$ one gets two competiting terms: the collision integral attracts the state towards f_o, while $L(f_o)$ repels it thence. The actual state is the result of a momentary balance between these forces.

The optimal basis could be determined from the details of the differential cross sections. Here we are manufacturing only a caricature, therefore we simply assume that the most important mode of deviation is

$$(f-f_o)/f_o \simeq a \frac{1}{m\sqrt{E^2-m^2}}(E^2+AmE+Bm^2) \qquad (9.3)$$

This is a function leading to less emphasized thermal peak. The corresponding extra density is chosen as

$$\begin{aligned} z &= c-c_o \\ c &= n^{-2/3}(d-nm^2) \qquad (9.4) \\ d &= \int fE^3 \frac{d^3p}{p^o} \end{aligned}$$

Then the evolution equation is

$$\dot{z} + 3\frac{\dot{R}}{R}z = -\frac{1}{\tau}z + \frac{\dot{R}}{R}m^2n^{1/3} \qquad (9.5)$$

Observe that there is a source term, which does not depend solely on pseudoextensives, as it was assumed in eq. (5.8), therefore P in eq. (5.10) cannot be the pseudothermodynamic pressure. Repeating that calculation for the entropy production with eq. (9.5) one gets that the Second Law requires

$$P = p + \frac{1}{3}m^2n^{1/3}\frac{S_z}{S_\rho} \qquad (9.6)$$

Eq. (9.5) suggests that after a previous thermalization, until the momentary balance between C(f) and $L(f_o)$ does not break down, z is expected in the order of $m^2(\dot{R}/R)$. Therefore either the pseudothermodynamic corrections in p, or the corrections (9.6) in P are proportional to m^4, that is, our neglections have been justified. The entropy production is

$$\dot{s} + 3\frac{\dot{R}}{R}s = -\frac{1}{\tau}s_z z \qquad (9.7)$$

starting as $z^2/\tau \sim m^4(\dot{R}/R)^2/\tau$.

Now one should calculate $s(n,\rho,z)$ for a Boltzmann gas with a deviation function (9.3); then P could be taken from eqs. (7.17) and (9.6), and the system of equations to be solved would consist of eqs. (5.12), (9.1) and (9.5). Nevertheless, ours is a simplified model anyway, so here we perform an analytic approximation yielding some insight into the global behaviour of the system.

Introduce a fictitious particle density conserved even above the decoupling temperature, $\bar{n}$, and write

$$z = \bar{n}y \tag{9.8}$$

(Above the decoupling the difference between n and $\bar{n}$ is proportional to m^2.) Then substitute R(t) and T(t) from the standard model; in this approximation y satisfies the equation

$$\dot{y} + At^{-5/2}y = B$$

$$A = \frac{\zeta(3)}{\pi^2} N^* \alpha_{em}^2 \frac{1}{E^4} \left(\frac{3M}{4\pi}\right)^{5/2} \left(\frac{5}{N\pi}\right)^{5/4} \tag{9.9}$$

$$B = \left(\frac{\pi^2}{N^*\zeta(3)}\right)^{2/3} \left(\frac{4\pi^3 N}{45}\right)^{1/2} \frac{m^2}{M} + \vartheta(m^4)$$

$$E \sim 100 \text{ GeV}$$

A and B can be scaled out by writing

$$y = A^{2/3}B\eta$$
$$t = A^{2/3}x \tag{9.10}$$

and then

$$\eta_{,x} + x^{-5/2}\eta = 1 \tag{9.11}$$

The solution of this equation, starting from equilibrium, can be written as

$$\eta = e^{2x^{-3/2}/3} \int_0^x e^{-2x'^{-3/2}/3} dx' \tag{9.12}$$

There are clearly three different regimes in the evolution. For $x \ll 1$ the solution of eq. (9.11) is

$$\eta = x^{5/2} \tag{9.13}$$

The other asymptotic solution for $x \gg 1$ is

$$\eta = x \tag{9.14}$$

while $x \sim 1$ is the transition period. Now one can directly see the characteristic features of the asymptotic stages from eq. (9.11).

For $x \ll 1$ the second term of the left hand side dominates the first one. There is some balance between the equilibrating tendency of collisions and the effects of the time-dependent geometry; the situation is a near-equilibrium one. In fact, restoring the dimensions and using eq. (9.7) one sees that z is proportional to $m^2\ \tau(\dot{R}/R)$, the pressure correction is proportional to $m^4\ \tau(\dot{R}/R)$, while the entropy production is proportional to $m^4\ \tau(\dot{R}/R)^2$. This is just the result which could be obtained by using a viscous model with Stewart's bulk viscosity at $m/T \ll 1$ [39].

On the other hand, when $x \gg 1$, the first term of the left hand side of eq. (9.11) dominates the second one; this is a drift driven by the disturbing force of the changing geometry. Here no cross section occurs in the entropy production and in z; the state is monotonously evolving away from the equilibrium.

The transition period is, as we have seen, at $x \sim 1$, i.e. at $t \sim A^{2/3}$; the corresponding temperature is in the order of magnitude of the decoupling temperature estimated in Sect. 6. Here the evaluation of eq. (9.12) yields the evolution of the pseudoextensive z, therefore our model is a demonstration for calculating continuous decoupling (or, in the langauge of heavy ion physics, break up).

Thus we get the following picture. The deviations from equilibrium are small far above $T_{dec} \simeq 10$ MeV; they are increasing with decreasing temperature as $\sim T^{-5}$, this increase is governed by an energy scale $(m^2E^4/M\alpha^2)^{1/5}$ ~ 0.1 MeV. The deviations would become substantial at this temperature, nevertheless, somewhere not far above T_{dec} the near-equilibrium formalism breaks down. Then there is a continuous transition into a collisionless Knudsen gas, which ends somewhere not far below T_{dec}, and there the deviations begin to increase linearly with t, i.e. with the inverse square of T. This indicates that after some time the system effectively forget the history of the transition period; the energy scale of the increase is $\sqrt{BM} \sim m$, therefore the extreme unequilibrium features are developed at $T \sim m$.

Since this history is in agreement with our knowledge collected from different approximations, the presented unequilibrium formalism can indeed be used for describing a continuous decoupling, if informations are needed about the transition period.

10. CONCLUSIONS

In this paper we have demonstrated that deviations from the thermal equilibrium can be incorporated into a mathematical treatment analogous with thermodynamics, which is consistent with general relativity and relativistic continuum mechanics. By means of this formalism two steps of evolution of the early Universe have been investigated in simplified models: the primordial equilibration in the symmetric phase of the GUT continuum, and the breakdown of the thermal distribution of neutrinos. In the first case the initial conditions are completely unknown; nevertheless the model calculation indicates that the continuum cannot effectively forget the initial conditions until cca. $1.5.10^{15}$ GeV temperature, while below this value the thermalization is very rapid. Therefore GUT calculations may be questionable above $1.5.10^{15}$ GeV, which is thus a technical constraint for the energy scale parameter, four times lower than given in Ref. 34.

For massive neutrinos our calculation reproduces both the neutrino viscosity at high temperatures, and the collisionless dethermalization well below the decoupling temperature, together with an intermediate stage of evolution where none of these approximations can be use. In this formalism the direct source of dethermalization is the time dependence of the geometry, inevitable in cosmology. The rate of this unequilibration is proportional to m^2.

There is some intimate connection between the unequilibrium processes discussed here and the relativistic bulk viscosity effects, and, in fact, the results are very similar in both formalisms for the high temperature stage of the evolution of the neutrino distribution. Nevertheless, these mechanisms are not identical, as it is directly shown by the fact that the bulk viscosity vanishes for the massless particles in the symmetric phase of the GUT continuum, while deviations from equilibrium in the momentum space still lead to entropy production.

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APPENDIX

Here we give some arguments for the form of the relaxation time approximation (7.5). Near the equilibrium - i.e. near the f_o distribution defined by (7.4) - one expects that the collision integral can be "expanded" in a power series:

$$C(f) = C(f_o) + \frac{\delta C}{\delta f}\bigg|_{f_o} (f-f_o) + \vartheta((f-f_o)^3) \tag{A.1}$$

where the symbol $\frac{\delta C}{\delta f}\big|_{f_o}$ denotes some derivative of C(f) taking it at f_o. The actual form of this expression depends on the details of the interactions. It is a reasonable assumption that the matter four-velocity appears in it while the invariance of the Boltzmann equation requires some scalar function: the simplest one which is dimensionally correct:

$$\frac{\delta C}{\delta f}\bigg|_{f_o} = u_r p^r \frac{1}{\tau(f_o)} \tag{A.2}$$

where τ is some scalar functional with time dimension.

In the nonrelativistic limit - i.e. in flat spacetime and at slow motion - in the comoving coordinate system the (7.3) Liouville operator takes the form

$$m \frac{\partial f}{\partial t} + m(\underline{v}\ \mathrm{grad}) f = L(f) \tag{A.3}$$

while (A.2) becomes

$$\frac{\delta C}{\delta f}\bigg|_{f_o} = -m \frac{1}{\tau(f_o)} \tag{A.4}$$

One sees that in this case (7.2) reduces the usual nonrelativistic Boltzmann equation in the relaxation time approximation and τ is the nonrelativistic relaxation time.

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