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ON THE TREATMENT OF EXCHANGE EFFECTS  
IN DIRECT REACTIONS

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ON THE TREATMENT OF EXCHANGE EFFECTS IN DIRECT REACTIONS

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#### ABSTRACT

Exchange effects in direct reactions are investigated in the frame of the general algebraic theory of identical particle scattering. It is shown that effects due to the permutational symmetry of the system can be separated from the treatment of reaction dynamics.

#### АННОТАЦИЯ

На основе общей алгебраической теории рассеяния тождественных частиц изучались обменные эффекты, возникающие в прямых ядерных реакциях. Показано, что эффекты, связанные с перестановочной симметрией, могут быть интерпретированы независимо от динамики реакций.

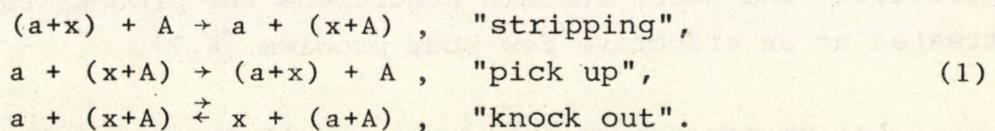
#### KIVONAT

Az azonos részecske szórás általános algebrai elmélete alapján tanulmányozzuk a direkt magreakciókban fellépő kicserélődési effektusokat. Megmutatjuk, hogy a rendszer permutációs szimmetriájából adódó hatások a dinamikától elkülönítve tárgyalhatók.

The treatment of exchange effects in direct reactions is not trivial, especially if it is combined with some kind of dynamical approximation. The underlying theoretical difficulties have not always been fully appreciated. Textbooks published in the seventies state that the general treatment of exchange effects is straightforward [1] and the antisymmetrisation itself does not pose any difficulty [2]. On the other hand a most recent work [3] refers to it as a "long standing problem" and claims to have provided a solution to it.

In the present work the problem is reconsidered on the basis of the general algebraic theory of identical particle scattering developed earlier [4,5]. As it will be shown, exchange effects can in fact be exactly separated from the treatment of dynamics. As a result physical inconsistencies can be avoided in the various approximation schemes.

Direct reactions are intuitively described in terms of only a small number of degrees of freedom /e.g. clusters/ of the multiparticle collision process. Consequently they can be associated with simple mechanisms. In particular, rearrangement reactions can be described by the following simple three-body mechanisms:



Conventional theory and DWBA models further classify mechanisms according to the type of interaction assumed to dominate the transition /e.g. "heavy particle" mechanisms/ [1,2]. Such a classification, however, depends strongly on dynamical properties and approximation schemes hence is of no general validity. The mechanisms listed in (1) share the common feature that the system is dominated by the three-cluster partition  $(a)(x)(A)$ .

If the multiparticle system contains identical and indistinguishable particles, a physical reaction may proceed by various mechanisms. In an exact multiparticle scattering theory the notion of mechanism is purely formal since the permutation symmetry is carried by the relevant physical operators [4,5]. On the other hand in conventional reaction theory "direct" and "exchange" mechanisms have well defined physical meaning and describe various aspects of the reaction under consideration. In the following we try to bridge the gap between the two approaches so that the underlying physics becomes transparent.

Let us consider a system of  $N$  distinguishable particles and a rearrangement reaction  $\alpha \rightarrow \beta$ , connecting two-cluster channels  $\alpha$  and  $\beta$ . Aside from the necessary quantum numbers, the relevant physical quantity that describes the process is the transition operator  $T^{\beta\alpha}$ . It is convenient to define the mechanism of the reaction  $\alpha \rightarrow \beta$  as the lattice theoretical meet of the partitions [4]:

$$\gamma = \alpha \cap \beta \quad (2)$$

One can immediately see that definition (2), which determines a partition, is in fact the abstract generalisation of the intuitive picture in (1). Thus the mechanism selects a "dominant partition" and under certain conditions the process can be treated as an effective few-body problem [6,7].

Let us now assume that some or all the particles of the system are identical. In this case a physical reaction involves various mechanisms. The dominant, or "direct" mechanism of conventional theory is usually chosen by physical intuition. The basic problem is then to assess the importance of the coherent "exchange" mechanisms which in principle should also be taken into account.

The on-shell T-matrix element of a physical rearrangement reaction  $a \rightarrow b$ , between physical channels  $a$  and  $b$  can be written on the basis of the general theory [4,5] as follows:

$$T_{ba} = \sqrt{\frac{N_a}{N_b}} \sum_{\beta \in b} \langle \phi_{\beta} | T^{\beta \alpha_0} | \phi_{\alpha_0} \rangle \quad (3)$$

The physical channels  $a$  and  $b$  are in fact equivalence classes of partitions which can be transformed into each other by permuting identical particles. The respective number of elements of these equivalence classes are  $N_a$  and  $N_b$ . Partition  $\alpha_0$  is arbitrary but fixed, and the summation runs over the equivalence class  $b$ . The channel wave functions  $\phi_{\alpha_0}$  and  $\phi_{\beta}$  are products of the internal wave functions of the bound clusters and a plane wave describing their relative motion. The internal wave functions are assumed to be properly symmetrised [4,5].

Expression (3) has been derived by symmetrising the final asymptotic state. If symmetrisation is performed in the initial state, the following exactly equivalent expression can be obtained [4,5]

$$T_{ba} = \sqrt{\frac{N_b}{N_a}} \sum_{\alpha \in a} \langle \phi_{\beta_0} | T^{\beta_0 \alpha} | \phi_{\alpha} \rangle \quad (4)$$

It is a paradox that even though strictly equal, expressions (3) and (4) involve a different number of terms in the sum. This paradox is resolved by introducing equivalence classes for reaction mechanisms along the lines indicated in refs [4,5].

Formula (3), by taking into account (2) defines a whole set of reaction mechanisms

$$\{\gamma\} \equiv \{\beta \cap \alpha_0; \beta \in b\} \quad , \quad (5)$$

whose number is clearly  $N_b$ . Another set of  $N_a$  mechanisms is defined by eq. (4),

$$\{\gamma'\} \equiv \{\beta_0 \cap \alpha; \alpha \in q\} \quad (6)$$

While the two sets (5) and (6) are not identical, it will be shown that the relevant quantity is the properly introduced equivalence class, whose number is the same in both cases [8].

Indeed, if  $\gamma'$  and  $\gamma$  are mechanisms, which can be transformed into each other by a permutation  $P \in S$  of the systems identical particles, where  $S$  denotes the permutation group of the system, they are physically equivalent. Thus it is straightforward to introduce the equivalence relation

$$\gamma' R \gamma \text{ iff } P \in S, \quad \gamma' = P\gamma \quad (7)$$

This equivalence relation splits up both sets (5) and (6) into disjoint equivalence classes, which will then play the role of physical reaction mechanism. This equivalence relation can also be formulated for both  $\{\beta \in b\}$  and  $\{\alpha \in a\}$  as follows

$$\beta' R \beta \text{ iff } P \in S_{\alpha_0} \subset S, \quad \beta' = P\beta, \quad (8)$$

$$\alpha' R \alpha \text{ iff } P \in S_{\beta_0} \subset S, \quad \alpha' = P\alpha, \quad (9)$$

where  $S_{\alpha_0}$  and  $S_{\beta_0}$  denote subgroups of permutations that leave  $\alpha_0$  and  $\beta_0$  invariant, respectively [4,5].

It is a simple combinatorial exercise to prove that sets (5) and (6) have identical number of equivalence classes under the relation (7) [4,8].

By taking into account equivalence relation (7) and the

label transforming property of the transition operators, (3) can be rewritten in the form

$$T_{ba} = \sqrt{\frac{N_a}{N_b}} \sum_{i=1}^m n_i \langle \Phi_\beta | T^{\beta_i \alpha_0} | \Phi_{\alpha_0} \rangle, \quad (10)$$

where  $m$  denotes the number of equivalence classes /mechanisms/ with respective number of elements  $n_i$  and representatives  $\beta_i$ . Clearly,

$$\sum_{i=1}^m n_i = N_b \quad (11)$$

Expression (10) is our final result as far as permutation symmetry is concerned. The relative importance of the various mechanisms is determined by  $N_a$ ,  $N_b$  and  $n_i$ , which can be calculated by simple combinatorics. It is important to emphasise that (10) is exact, if no approximation has been introduced into the matrix elements. Depending on the approximation scheme for  $T^{\beta_i \alpha_0}$ , eq. (10) yields at once the properly "symmetrised" version, taking into account exchange effects in a consistent way.

In DWBA models for direct reactions, appropriate distorting potentials are introduced in the two-cluster channels. For systems with identical particles, these distorting potentials  $U_a$ ,  $U_b$  should depend only on the physical channels /"antisymmetrised optical potentials"/. Distorted waves can be introduced in terms of the wave operators

$$|\chi_\alpha^{(\pm)}\rangle = \Omega_a^{(\pm)} |\Phi_\alpha\rangle \quad (12)$$

where  $\alpha \in a$ , and

$$\Omega_a^{(\pm)} = 1 + (z - H_\alpha - U_a)^{-1} U_a; \quad \alpha \in a. \quad (13)$$

The distorted transition operators  $T^{\beta_i \alpha_o}$  can be related to  $T^{\beta_i \alpha_o}$  as follows [9]

$$T^{\beta_i \alpha_o} = \Omega_b^{(-)+} T^{\beta_i \alpha_o} \Omega_a^{(+)} \quad (14)$$

Correspondingly, for a "distorted" transition operator formalism (10) can be rewritten as

$$T_{ba} = \sqrt{\frac{N_a}{N_b}} \sum_{i=1}^m n_i \langle \chi_{\beta_i}^{(-)} | T^{\beta_i \alpha_o} | \chi_{\alpha_o}^{(+)} \rangle \quad (15)$$

Expression (15) is then the exact starting point for introducing DWBA in a consistently symmetrised theory. However, there are two facts which should be kept in mind. Various exact scattering theories yield different driving terms for the distorted transition operators. Also it is well known that DWBA violates post-prior symmetry [1,2]. Hence, no unique DWBA approximation to (15) exists, contrary to the conclusion of ref. [3].

In order to illustrate how symmetrisation combinatorics works, let us consider (p,d) and (p,<sup>3</sup>He) reactions [3]. Let the system consist of Z protons and N neutrons and take

$$\begin{aligned} \alpha_o &= (p_1)(p_2, p_3, \dots, p_Z; n_1, n_2, \dots, n_N) \\ \beta_o &= (p_1 n_1)(p_2, p_3, \dots, p_Z; n_2, n_3, \dots, n_N) \end{aligned} \quad (17)$$

Simple enumeration yields the number of elements  $N_a$  and  $N_b$  of the respective equivalence classes,

$$N_a = Z \quad ; \quad N_b = ZN \quad (18)$$

As for reaction mechanisms, it is easy to see that the following physical three-cluster channels are possible

$$(p)(n)(Z-1; N-1) \quad ; \quad (p)(pn)(Z-2; N-1) \quad (19)$$

Their number of elements can again be enumerated by simple methods [8]. What should be noticed, however, that the first mechanism corresponds to a pick-up process while the second is clearly of knock out type. Let us denote a typical T-matrix element for these mechanism  $T^{pu}$  and  $T^{ko}$ , respectively. The corresponding numbers are  $n_1=N$ ,  $n_2=(Z-1)\cdot N$ .

By taking into account all the known combinatorial factors, the exact, symmetrised T-matrix element for a (p,d) reaction can be written as

$$T^{(p,d)} = \sqrt{N} \{T^{pu} + (Z-1)T^{ko}\} \quad (20)$$

The T-matrix elements, corresponding to the two simple basic mechanisms can be calculated by any approximate method of conventional theory [1,2].

The (p,  $^3\text{He}$ ) reaction can be treated by the same simple combinatorics to yield

$$T^{(p, ^3\text{He})} = \sqrt{2N(Z-1)} \{T^{pu} + (Z-2) T^{ko}\} \quad (21)$$

Again, the dynamical features are contained in the two T-matrix elements corresponding to pick-up and knock out mechanism, respectively. Of course all the spectroscopic information on the clustering of the system and the details of dynamics is contained in them.

The most important exact result that emerges from (20) and (21) is that, aside from the trivial and unphysical case of  $Z=1$  or  $Z=2$ , a knock out term must always be present, contrary to a claim in ref. [3], that its effect can be included in the distorted waves. As a matter of fact, it would be surprising if any consequence of a nondynamical symmetry would show up in a dynamical quantity, e.g. potential.

The main conclusion can be summarised as follows. Effects due to permutation symmetry can be taken into account exactly, independently of the dynamical treatment. Dynamical approximations can then be introduced after exchange symmetry has been taken care of. However, due to the post-prior symmetry violation of the DWBA, no unique DWBA amplitude can be derived from the exact theory. Another important result is that reaction mechanisms appear in the exact formalism as equivalence classes of dominant partitions. Hence reaction mechanism is not a dynamical concept, so that possible mechanisms cannot be transformed out by any dynamical scheme.

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