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# A STOCHASTIC APPROACH TO STATIONARY TWO-PHASE FLOW

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ABSTRACT

It is suggested that stationary two-phase flow can be described by the local one-time correlation function of its density fluctuations. It is surmized that for most flow regimes, the correlation function is locally isotropic and decays rapidly in space, but the actual attenuation law can vary slowly in space, depending on the local structure of the two-phase substance. Conditions for the existence of such a correlation function are determined for random bubbly flow, and the correlation function is explicitly calculated.



АННОТАЦИЯ

Для описания двухфазного потока предлагается использовать локальную одновременную пространственную корреляционную функцию флуктуаций плотности среды. Предполагается, что в большинстве режимов потока корреляционная функция локально изотропная и быстро затухает в пространстве, но форма актуального затухания может медленно изменяться в пространстве, в зависимости от локальной структуры двухфазной среды. Для случайного пузырькового потока определяются условия существования такой корреляционной функции и дается ее расчет.



KIVONAT

Kétfázisu áramlás leirására a közeg sürüségfluktuációinak lokális egyidejü térbeli korrelációs függvényét javasoljuk használni. Azt sejtjük, hogy a legtöbb áramlási rezsim esetére, a korrelációs függvény lokálisan izotróp és térben gyorsan lecseng, de az aktuális attenuáció alakja lassan változhat a térben, a kétfázisu közeg lokális szerkezetétől függően. Véletlen buborékos áramlásra egy ilyen korrelációs függvény létezésének feltételeit meghatározzuk, és a korrelációs függvényt explicite kiszámoljuk.

CONTENTS

## 1. INTRODUCTION

Two-phase flow, that is concurrent transport of a random mixture of gas and liquid, is a rather complex phenomenon. An important notion in the qualitative description of two-phase flows is that of flow regimes or two-phase regimes. This means that the experimentally observed diverse flow patterns are classified into categories of different qualitative appearance, the categories constituting the different flow regimes (Hewitt & Hall-Taylor, 1970). In vertical flow the four most important regimes are termed as bubbly, slug, churn-turbulent and annular flows, respectively (Fig. 1.) (Vince & Lahey, 1982; Rouhani & Sohal, 1983).

In a number of scientific and engineering applications, the particular flow regime appearing in a problem plays an important role. Thus, methods of determining flow regimes (flow identification) are of paramount importance. The problem is nevertheless far from being completely settled and a great deal of research is devoted to this particular item. One type of possible approach, that is predicting flow regimes from initial and boundary thermohydraulic conditions is indeed an invidious task and it does not appear to be a practical alternative at present. Even direct measurements, aiming at flow regime identification in cases where the flow is not directly visually accessible, cope with serious problems. The latter suffer from the fact that the notion of flow regimes is a geometrically motivated one, and it is usually difficult both to define and to measure a suitable set of parameters whose disjoint regions would characterise different flow regimes. Such parameter combinations with subdivisions corresponding to different flow regimes are called flow regime maps (Govier & Aziz, 1972). The difficulties noted above are well reflected by the fact that different flow regime maps are usually not compatible to each other (Rouhani & Sohal, 1983).

In this paper we try to illuminate a somewhat lesser exposed aspect of two phase flow description, which may eventually make it possible to formulate identification methods alternative to the already existing ones. We propose to describe the propagating two-phase substance by the spatial correlation function of its density fluctuations. It is assumed that two different distance scales can be separated in the correlations, a fast isotropic decay of correlations that describes the local structure of the flow, which however can change its shape on a much larger scale which is comparable with system dimensions. By system dimensions we refer to the dimensions of the confinement of the flow, e.g. tube diameter etc. Particular flow regimes can be characterised by the aggregate of local correlation functions over the cross-section of the flow. Actually, for such a correlation function, it is possible to define a (space-dependent) correlation distance, and we surmize that the different flow regimes can be characterized by the differing spatial dependence of their correlation distances.

Admittedly, the above description of two-phase flow is geometrical rather than dynamical since it makes no explicit use of fluid dynamic and thermohydraulic equations. We believe that at the same time this can constitute its merit, as it may fit in well with the rather geometrically motivated concept of flow-regimes. Besides, a conceptually rather simple non-intrusive method, based on the cross-correlation of time signals arising from radiation attenuation measurements can be derived by which the local correlation functions can be determined at any point of the flow (Pázsit, 1984). The latter circumstance constituted much of the motivations for the present work.

The paper consists of two parts. In Sect. 2 the general assumptions on the correlation function of density fluctuations are expounded. In Sect. 3 a stochastic model of bubbly two-phase regime is formulated, conditions for the existence of the postulated correlation function are examined, and the correlation function is explicitly calculated.

## 2. GENERAL PRINCIPLES

The two-phase substance will be characterised by its space and time dependent density $\varrho(\underline{r},t)$. At any point $\underline{r}$, $\varrho(\underline{r},t)$ is regarded a stationary and ergodic process, but one whose statistical properties are space-dependent, that is the process is not stationary in space. Due to temporal stationarity we have

$$\langle \varrho(\underline{r},t) \rangle = \varrho_0(t) \tag{1}$$

and we introduce density fluctuations by

$$\delta\varrho(\underline{r},t) = \varrho(\underline{r},t) - \varrho_0(\underline{r}) \tag{2}$$

The main assumptions are then as follows

a.) The two-phase substance is characterised by its one-time spatial correlation function $R(r,r')$:

$$R(\underline{r},\underline{r}') = \langle \delta\varrho(\underline{r},t)\, \delta\varrho(\underline{r}',t) \rangle \tag{3}$$

b.) Correlation effects are characterised by two different distance scales. One is the fast spatial relaxation of the correlation function over a range that is small compared to system dimensions. A volume $V_{\underline{r}}$ at a point $\underline{r}$ having the local correlation distance $d_{\underline{r}}$ as its radius will be called a control volume. The other scale is large, comparable to system dimensions and characterises distances over which the shape or amplitude of the short range local relaxation effects can change significantly

c.) At a given point $\underline{r}$, the correlation function is locally isotropic and homogeneous, and is given as

$$R(\underline{r},\underline{r}') = R_{\underline{r}}(\xi) \quad ; \qquad \xi = |\underline{r}-\underline{r}'| \tag{4}$$

In the above, dependence on $\xi$ describes the short range, on $\underline{r}$ the long range effects. No factorisation between the $\underline{r}$ and $\xi$ dependence is assumed.

The most crucial of the above assumptions is the existence of a short correlation distance $d_{\underline{r}}$, or with other words, a small control volume. The rest of the assumptions is justified if statistical properties of the flow can be taken constant over the control volume. If the latter is indeed small compared to system dimensions, this condition is usually met.

The philosophy behind the above approach is much the same as that of time dependent frequency spectra: we assume that fast changes characterise the current dynamics (here: spatial structure) of the system, but the dynamics can change slowly in time. Similar assumptions are apparently in use in many related fields, such as in turbulence studies etc. (Hinze, 1969, Goldstein, 1983).

It is an important feature of the recent model that the description is based completely on the spatial structure of density fluctuations, and fluid flow does not explicitly enter the description. However, the presence of fluid propagation yields a possibility of determining $R_{\underline{r}}(\xi)$ by time signal measurements, in which the autocorrelation function of density fluctuations at a given $\underline{r}$ is determined. The latter is defined by

$$R_{\underline{r}}(\tau) \equiv \langle \delta\varrho(\underline{r}, t+\tau)\, \delta\varrho(\underline{r}, t) \rangle \qquad (5)$$

Assuming that the density fluctuations propagate in the flow with a speed $v_{\underline{r}} = |\underline{v}_{\underline{r}}|$ which can be taken constant over the control volume, we may write

$$\langle \delta\varrho(\underline{r}, t+\tau)\, \delta\varrho(\underline{r}, t) \rangle = \langle \delta\varrho(\underline{r}-\underline{v}_{\underline{r}}\tau, t)\, \delta\varrho(\underline{r}, t) \rangle \qquad (6)$$

that is we have

$$R_{\underline{r}}(\tau) = R_{\underline{r}}(\xi) \Big|_{\xi = v_{\underline{r}}\tau} \qquad (7)$$

which is a well-known formula. The above assumption also neglects the fact that Eq. (6) is violated by the vapour generation and condensation between $\underline{r}$ and $\underline{r} - \underline{v}_{\underline{r}}\tau$. Again, if the control volume is sufficiently small, this effect is negligible.

There are of course situations, when conditions for the above approximations are not met, e.g. when there is a spatially quick (step-wise) change in the statistical properties of the system, and/or the correlation distance is not small compared to system dimensions. Examples are certain cases of annular and slug flow. However, the idea may be useful for flow identification purposes even in such cases, for $R_{\underline{r}}(\xi)$ still may exist in substantial part of the flow, probably except the very vicinity of large void-fluid interfaces.

Feasibility of the model for flow identification depends on two conditions. The first is that to each separate flow-regime there need to belong a characteristic $R_{\underline{r}}(\xi)$ which differs from that of other flow regimes. It can be surmized that this is wery well the case since $R_{\underline{r}}(0)$ and the cut-off point $d_{\underline{r}}$ of $R_{\underline{r}}(\xi)$, $R_{\underline{r}}(d_{\underline{r}}) = 0$, determining local void fraction and control volume respectively, are indeed dependent on the flow regime; but the whole structure of $R_{\underline{r}}(\xi)$ may carry even more information. The second condition is the validity of the model itself which requires the existence of a short correlation distance.

The above questions can be answered by further experimental and theoretical investigations. To give an example for the latter, in the next Section a simple stochastic model of bubbly two phase flow is constructed and $R_{\underline{r}}(\xi)$ is explicitly calculated. The results yield a positive answer in that for a bubbly regime, the correlation distance is found to be small and therefore the conditions for the existence of $R_{\underline{r}}(\xi)$ are mild.

## 3. A STOCHASTIC MODEL OF BUBBLY FLOW

In order to calculate $R_{\underline{r}}(\xi)$, we need to make assumptions about the process $\delta\rho(\underline{r},t)$ . We assume that $\rho(\underline{r},t)$ is a binary variable:

$$\rho(\underline{r},t) = \rho \qquad \text{if there is fluid}$$
$$\rho(\underline{r},t) = 0 \qquad \text{if there is void}$$

at point $\underline{r}$ at time t. In what follows, for simplicity we take $\rho = 1$. Temperature- and pressure - dependent fluctuations within the fluid, as well as density of the gas, will be neglected. The average density and void fraction are defined respectively as

$$\begin{aligned} \rho_0(\underline{r}) &= \langle \rho(\underline{r},t) \rangle \\ \alpha(\underline{r}) &= 1 - \rho_0(\underline{r}) \end{aligned} \tag{8}$$

From (1) and (8) we have

$$\delta\rho = \begin{cases} \alpha(\underline{r}) \equiv \rho_+ & \text{in the fluid} \\ -\rho_0(\underline{r}) \equiv \rho_- & \text{in the void} \end{cases} \tag{9}$$

Let $P_0(r)$ denote the (time - independent) probability of having fluid at $\underline{r}$, then, since

$$\langle \delta\rho(\underline{r},t) \rangle = P_0\rho_+ + (1-P_0)\rho_- = 0 \tag{10}$$

we have the trivial result

$$\begin{aligned} P_0 &= \rho_0(\underline{r}) \\ 1 - P_0 &= \alpha(\underline{r}) \end{aligned} \tag{11}$$

Here and in the following, notation of dependence of $P_o$ on $\underline{r}$ will be dropped. Likewise, we obtain

$$\langle [\delta\varrho(\underline{r},t)]^2 \rangle = P_o(1-P_o) = \alpha(\underline{r})(1-\alpha(\underline{r})) \qquad (12)$$

The particular model of bubbly regime we select is specified as follows. We select a line through an arbitrary point $\underline{r}$ of the flow in an arbitrary direction $\underline{\omega}$, and parametrise points along the line by the variable s. We assume, that momentary value of the density fluctuations along the line constitute a binary random process $\delta\varrho(s)$ which is defined by

$$\delta\varrho(s) = \begin{cases} \varrho_- & \text{if } s \in [s_i - d_i/2,\ s_i + d_i/2] \\ \varrho_+ & \text{otherwise} \end{cases} \qquad (13)$$

Here the random points $s_i$ will be called bubble centres, and we assume that they obey a Poisson statistics with a space-dependent parameter $\lambda_{\underline{r}}(s)$. The $d_i$ are called bubble diameters. Their distribution is described by a function $p_{\underline{r}(s)}(w)$, which gives the probability that a bubble, whose centre falls between s and s + ds, will have a diameter between w and w + dw. At any point r(s) we shall assume a finite maximum bubble diameter $d_{\underline{r}}^{max}$ such that

$$p_{\underline{r}}(w) = 0 \qquad \text{if } w > d_{\underline{r}}^{max} \qquad (14)$$

The processes $s_i$ and $d_i$ are assumed independent, hence this construction allows for bubble overlapping. We have furthermore

$$\varrho_+ - \varrho_- = 1$$

and $\qquad (14)$

$$\langle \delta\varrho(s) \rangle = 0$$

which makes the model complete.

The correspondence between the three-dimensional bubble distribution and $\delta\varrho(s)$ is illustrated in Fig. 2 for the case $\underline{\omega} = \underline{i}$. The parameters $\lambda_{\underline{r}(s)}$ and $p_{\underline{r}(s)}(W)$ can be derived from the distributions $\Lambda_{\underline{r}}$ and $F_{\underline{r}}(w)$ that the bubble centres and diametres follow in three-dimensional space. For instance, $p_{\underline{r}(s)}(W)$ will be the distribution of chords falling inside the bubbles (Sandervåg, 1971) whose diameters follow of course a different distribution $F_{\underline{r}}(w)$.

It is easy to see that the conditions for the existence of $R_{\underline{r}}(\xi)$ are satisfied if $\lambda_{\underline{r}(s)}$ and $P_{\underline{r}(s)}(w)$ can be regarded constant over $V_{\underline{r}}$. First, as is easy to confirm, in this case the one-dimensional distribution $\lambda_{\underline{r}(s)}$ will indeed be independent of $\underline{\omega}$ at $\underline{r}$ and will only depend on s. Second, the one-dimensional bubble centre distributions and diameters remain independent processes. We shall also see, that the correlation distance will be equal to the local maximum (or average) bubble diameter, which provides for the desired fast decay of correlations.

That is, with the above assumptions, the locally isotropic correlation function $R_{\underline{r}}(\xi)$ exists and it can be easily calculated from the properties of $\delta\varrho(s)$. Details of the calculations are found in the Appendix, here we only quote the results:

$$P_o = e^{-\lambda_{\underline{r}}\langle d_{\underline{r}}\rangle} \tag{16}$$

$$R_{\underline{r}}(\xi) = e^{-\lambda_{\underline{r}}\langle d_{\underline{r}}\rangle}\left[e^{-\lambda_{\underline{r}}\int_0^{\xi} g_{\underline{r}}(w)\,dw} - e^{-\lambda_{\underline{r}}\langle d_{\underline{r}}\rangle}\right] \tag{17}$$

where

$$g_{\underline{r}}(w) = \int_{w}^{\infty} p_{\underline{r}}(w')\,dw' \tag{18}$$

is the probability of bubble diameter being larger than w. Since

$$\int_0^{\infty} g_{\underline{r}}(w)\,dw = \int_0^{d_{\underline{r}}^{max}} g_{\underline{r}}(w)\,dw = \langle d_{\underline{r}} \rangle \tag{19}$$

we have

$$R_{\underline{r}}(\xi) = 0 \quad \text{if} \quad \xi \geq d_{\underline{r}}^{max} \tag{20}$$

That is, the correlation distance equals $d_{\underline{r}}^{max}$, as is expected. The actual decay of $R_{\underline{r}}(\xi)$ can of course be faster, depending on bubble diameter distribution. For a deterministic bubble diameter distribution $p_{\underline{r}}(w) = \delta(w - d_{\underline{r}})$, we have

$$R_{\underline{r}}(\xi) = e^{-\lambda_{\underline{r}} d_{\underline{r}}}\left[e^{-\lambda_{\underline{r}}\xi} - e^{-\lambda_{\underline{r}} d_{\underline{r}}}\right]\Theta(d_{\underline{r}} - \xi),$$

$\Theta(x)$ being the unit step function. For a uniform bubble diameter distribution with $d_{\underline{r}}^{max} = 2d$,

$$P_{\underline{r}}(w) = \frac{1}{2d}\,\Theta(2d - w),$$

we have $\langle d_{\underline{r}} \rangle = d$, and

$$R_{\underline{r}}(\xi) = e^{-\lambda_{\underline{r}} d}\left[e^{-\lambda_{\underline{r}} d\left[\frac{\xi}{d} - \frac{1}{4}\left(\frac{\xi}{d}\right)^2\right]} - e^{-\lambda_{\underline{r}} d}\right] \tag{21}$$

The $R_{\underline{r}}(\xi)$ of Eq. (21) is depicted in Fig. 3 with $\lambda_{\underline{r}} d = 0.3$. Fig. 3 suggests that although the cut-off of $R_{\underline{r}}(\xi)$ is given by $d_{\underline{r}}^{max}$, the decay is determined by $\langle d_{\underline{r}} \rangle = d$. Accordingly, the average bubble diameter will be called the correlation distance, whereas $d_{\underline{r}}^{max}$ will be termed as maximum correlation distance or correlation length. This distinction may be important for instance for slug flow where $d_{\underline{r}}^{max}$ is comparable with system dimensions, nevertheless $\langle d_{\underline{r}} \rangle$ may be substantially smaller in some cases so that the concept of $R_{\underline{r}}(\xi)$ still remains applicable.

Formally, with $\delta\rho(s)$ constructed as above, $R_{\underline{r}}(\xi)$ is valid for any $\lambda_{\underline{r}}$ and $p_{\underline{r}}(w)$ values. For $\lambda_{\underline{r}} \langle d_{\underline{r}} \rangle \ll 1$ (sparse bubbles, small void fraction) one has

$$R_{\underline{r}}(\xi) \cong \lambda_{\underline{r}} \langle d_{\underline{r}} \rangle \left[ 1 - \frac{1}{\langle d_{\underline{r}} \rangle} \int_0^{\xi} g_{\underline{r}}(w)\,dw \right]$$

whereas in the opposite limit $\langle d_{\underline{r}} \rangle \gg 1$ (dense bubbles) one obtains

$$R_{\underline{r}}(\xi) \cong e^{-\lambda_{\underline{r}} \langle d_{\underline{r}} \rangle} \, e^{-\lambda_{\underline{r}} \int_0^{\xi} g_{\underline{r}}(w)\,dw}$$

However, regarding the boiling process that we are to model, the latter case is not realistic. In case of dense bubbles there is heavy overlapping in the present model, and the previous independence of separate bubble birth and also that of bubble centres and positions for close bubbles, cannot be expected. For large void fraction ($\alpha \cong 1$) it is more realistic to assume that the droplets of liquid, travelling in the gas, possess the independence properties formerly attributed to bubbles. The derivation given above remains valid if we reverse the roles of gas and liquid, that is make the substitutions

$$\begin{aligned} \rho_0(\underline{r}) &\longleftrightarrow \alpha(\underline{r}) \\ P_0 &\longleftrightarrow 1 - P_0 \end{aligned} \tag{23}$$

and let $\lambda_{\underline{r}}$ and $p_{\underline{r}}$ denote distributions of the droplets.

For the case $\alpha \sim 0.5$, the situation cannot be described either by random bubbles in the liquid or by random droplets in the gas. In such a case it is the simplest to assume that along any fixed line in the medium, the void-fluid interfaces follow a random distribution with parameter $\lambda_r$. Then, for $\alpha = 0.5$, the process $\delta\rho(s)$ becomes what is known as the random telegraph signal with a correlation function (Papoulis, 1965)

$$R_r(\xi) = \frac{1}{4} e^{-2\lambda_r \xi}$$

Then the mean lengths of the sections of the line falling into the fluid or the liquid both equal

$$\langle d_r \rangle = 1/\lambda_r$$

which plays the role of the mean bubble and droplet diameters in this case. Imposing an upper limit on these diameters as in the preceding example would have naturally led to a finite correlation length.

These simple examples show that if the randomness of the two-phase medium is provided in the above sense, the decay of the spatial correlations is determined by the diameter of the void or fluid packages, whichever is smaller. If these diameters are small compared to system dimensions, the suggested description of two-phase flow should work reasonably well. Then the model has also some diagnostical importance since by measuring $R_r(\xi)$, parameters of the process can be determined. For instance, in case of bubbly (droplet) regimes, the distribution of bubble or droplet diameters and the frequency parameter $\lambda_r$ can be determined from Eq. (17) as follows:

$$g_{\underline{r}}(w) = \frac{\lambda_{\underline{r}} \left. \frac{dR_{\underline{r}}(\xi)}{d\xi} \right|_{\xi = w}}{R_{\underline{r}}(w) + P_0^2} \tag{24}$$

where $g_{\underline{r}}(w)$ is the integral of the droplet or bubble diameter distribution, depending on the situation, $P_0$ is the smaller root of

$$P_0^2 - P_0 + R_{\underline{r}}(0) = 0 \tag{25}$$,

and

$$\lambda_{\underline{r}} = \frac{1}{P_0} \frac{dR_{\underline{r}}(\xi)}{d\xi} \tag{26}$$

Of course, as noted before, $p_{\underline{r}}(w)$ , as determined from (24), will give the distribution of random chord lengths lying within the bubbles or droplets, but true bubble or droplet diameter distribution can be determined from it by straightforward methods (Sandervåg, 1971).

## 4. CONCLUSIONS

The main suggestion of the paper is that the two-phase flow can be described by the structure of the spatial correlation function of density fluctuations in which a fast and a slow tendency can be separated. The model was given some credibility through calculating a case modelling a simple flow-regime, although the range of spatial correlations in that model is somewhat underestimated. Nevertheless, the concept might prove useful for general two-phase flow identification purposes. Methods of measuring $R_{\underline{r}}(\xi)$ under engineering circumstances can be easily devised and will be reported in a technical journal (Pázsit, 1984).

## 5. APPENDIX

Assume that $\underline{r}$ and $\underline{r}'$ lie at s=0 and $s=\xi$, respectively. We introduce the function F(a,s) which is the probability that no bubble, whose centre falls between $(-\infty, a)$ will extend to s, that is will lead to the occurrence of void at s. Then F(a,s) must obey the master equation

$$\frac{\partial F(a,s)}{\partial s} = -\lambda_{\underline{r}}\, g_{\underline{r}}(2|s-a|)\, F(a,s) \tag{A1}$$

with the boundary condition

$$F(-\infty, s) = 1 \tag{A2}$$

The solution is

$$F(a,s) = e^{-\lambda_{\underline{r}} \int_{-\infty}^{\infty} g_{\underline{r}(s)}(2|s-s'|)\, ds'} \tag{A3}$$

In the above, as previously stated, we assume that $\lambda_{\underline{r}}$ and the w dependence of $g_{\underline{r}}(w)$ can be taken constants in $(s-d_{\underline{r}}^{max}, s+d_{\underline{r}}^{max})$. Having said this, $P_o(\underline{r})$, that is the probability of having no void at $\underline{r}$ for any bubbles, is given by symmetry reasons as

$$P_o = F^2(0,0) = e^{-\lambda_{\underline{r}} \int_{-\infty}^{\infty} g_{\underline{r}}(2|w|)\, dw} = e^{-\lambda_{\underline{r}} \langle d_{\underline{r}} \rangle} \tag{A4}$$

which confirms Eq. (16).

To calculate the correlation function, we write

$$R_{\underline{r}}(\xi) = \Sigma P_{\varrho_i \varrho_j}\, \varrho_i \varrho_j \tag{A5}$$

where both $\varrho_i$ and $\varrho_j$ stand for either $\varrho_+$ or $\varrho_-$, introduced in (9), and related to $P_o$ in (9) and (11). $P_{\varrho_i \varrho_j}$ is the joint probability of having $\rho_i$ at s = 0 and $\rho_j$ at s = $\xi$. The three different terms in (A5) will now be calculated.

1. $P_{\rho_i\rho_j}$ gives the probability of having no void at both 0 and $\xi$. This is achieved if no bubbles with centre s $\leq \xi/2$ reach the point s = 0, and no bubbles with centre s $\geq \xi/2$ extend to $\xi$ (Fig. 4). Due to symmetry, this is given by

$$P_{\rho_+\rho_-} = F^2(\xi/2,a) = e^{-\lambda_{\underline{r}}\langle d_{\underline{r}}\rangle}\, e^{-\lambda_{\underline{r}}\int_0^{\xi} g_{\underline{r}}(w)\,dw} \qquad \text{(A6)}$$

Introducing the notation

$$P_{\underline{r}}(\xi) \equiv e^{-\lambda_{\underline{r}}\int_0^{\xi} g_{\underline{r}}(w)\,dw} \qquad \text{(A7)}$$

we have

$$P_{\rho_+\rho_-} = P_0\, P_{\underline{r}}(\xi) \qquad \text{(A8)}$$

Since $P_0$ is the probability of no void at s = 0, $P_{\underline{r}}(\xi)$ is the conditional probability of having no void at s = $\xi$, given that there is no void at s = 0.

2. $P_{\rho_+\rho_-} = P_{\rho_-\rho_+}$ is the probability of having fluid at s = 0 and void at $s=\xi$, and vica versa, respectively. With the interpretation of $P_{\underline{r}}(\xi)$ given above, this can be written as

$$P_{\rho_+\rho_-} = P_{\rho_-\rho_+} = P_0\left[1 - P_{\underline{r}}(\xi)\right] \qquad \text{(A9)}$$

3. $P_{\rho_-\rho_-}$ is the probability of having void both at s = 0 and s = $\xi$. This situation can be the result of two mutually exclusive events, that is it can be brought about by one common bubble

covering both points, or it may be the result of two separate bubbles. In both cases, the number of bubbles that cover only one of the points, can be arbitrary.

3.a $P^{C}_{\varrho-\varrho-}$ is the probability that there is one bubble covering the points $s = 0$ and $s = \xi$. The probability of these points sharing no joint bubble is given, due to symmetry, as

$$F^2(\xi/2, \xi) = P_0 \cdot P_{\underline{r}}^{-1}(\xi)$$

from where we have

$$P^{C}_{\varrho-\varrho-} = 1 - P_0 \cdot P_{\underline{r}}^{-1}(\xi) \qquad \text{(A10)}$$

3.b $P^{S}_{\varrho-\varrho-}$ is the probability of having void at $s = 0$ and $s = \xi$, with no joint bubbles. Here from

$$F(\xi/2, 0) = F(\xi/2, \xi) \cdot P_{\underline{r}}(\xi)$$

and the interpretation of $F(a,s)$ we obtain that $P_{\underline{r}}(\xi)$ is also the conditional probability that for all bubble centres lying between $(-\infty, \xi/2)$, there will be no void at both $s = 0$ and $s = \xi$, given that there will be no void at $s = \xi$. Thus,

$$P^{S}_{\varrho-\varrho+}\Big|_{s \leq \xi/2} = F(\xi/2, \xi)\left[1 - P_{\underline{r}}(\xi)\right]$$

and, again from symmetry, we obtain that

$$P^{S}_{\varrho-\varrho-} = F^2(\xi/2, \xi)\left[1 - P_{\underline{r}}(\xi)\right]^2$$

or

$$P^{S}_{\varrho-\varrho-} = P_0 \cdot P_{\underline{r}}^{-1}(\xi)\left[1 - P_{\underline{r}}(\xi)\right]^2$$

(A11)

Putting (A8)-(A11) into (A5), and making use of Eq. (11), we have

$$R_{\underline{r}}(\xi) = P_o \left[ P_{\underline{r}}(\xi) - P_o \right] \tag{A12}$$

Substituting (A4) and (A7) into (A12), Eq. (17) immediately follows.

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## 7. FIGURE CAPTIONS

Fig. 1 Flow regime types in vertical two-phase flow

Fig. 2 Density fluctuation $\delta\varrho(s)$ along a line in three-dimensional bubbly flow

Fig. 3 $R_{\underline{r}}(\xi)$ for a uniform bubble diameter distribution $p_{\underline{r}}(w) = \frac{1}{2d}\Theta(2d - w)$; $\langle d_{\underline{r}} \rangle = d$

Fig. 4 To the calculation of $R_{\underline{r}}(\xi)$

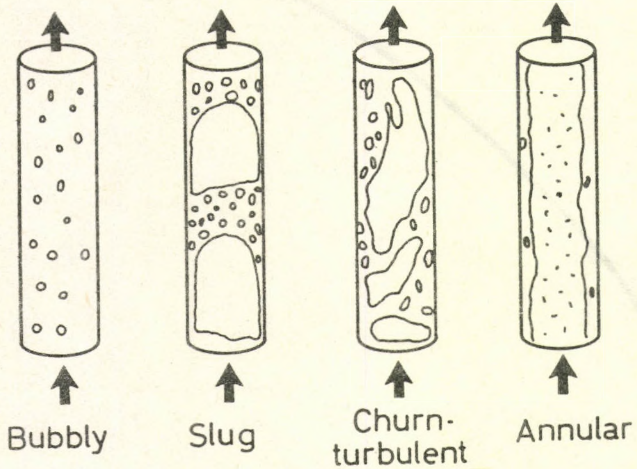
Bubbly
Slug
Churn-turbulent
Annular

Fig. 1

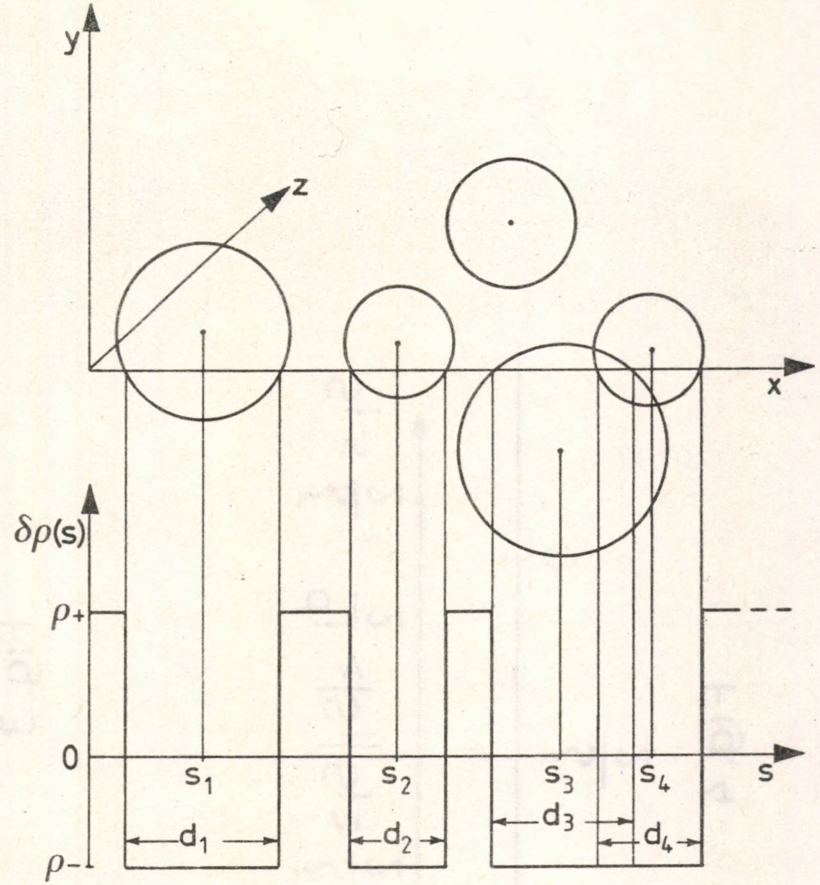
y
z
x
δρ(s)
ρ+
0
ρ-
s
s1
s2
s3
s4
d1
d2
d3
d4

Fig. 2

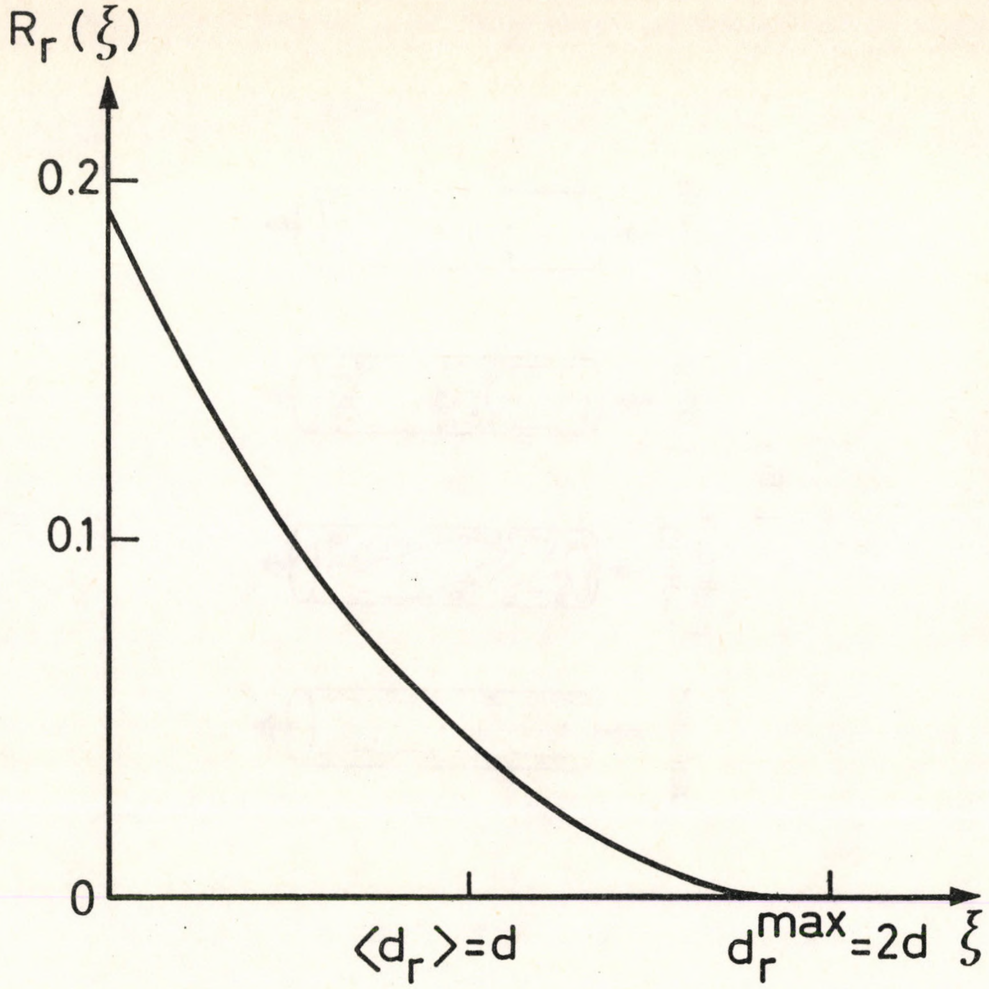
$R_r(\xi)$
0.2
0.1
0
$\langle d_r \rangle = d$
$d_r^{max} = 2d$
$\xi$

Fig. 3

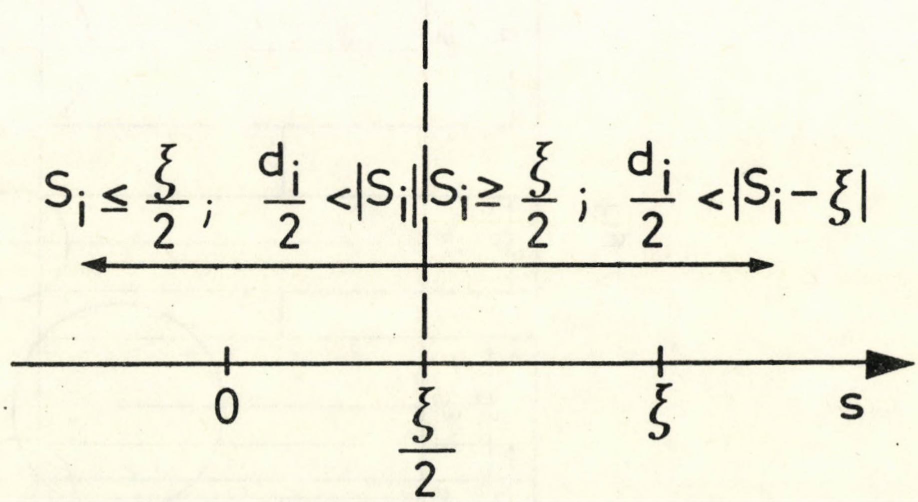
$S_i \le \frac{\xi}{2}$ ; $\frac{d_i}{2} < |S_i|$
$S_i \ge \frac{\xi}{2}$ ; $\frac{d_i}{2} < |S_i - \xi|$
0
$\frac{\xi}{2}$
$\xi$
s

Fig. 4

Kiadja a Központi Fizikai Kutató Intézet
Felelős kiadó: Gyimesi Zoltán
Szakmai lektor: Glöckler Oszvald
Nyelvi lektor: Pázsit Mária
Példányszám: 170 Törzsszám: 84-649
Készült a KFKI sokszorosító üzemében
Felelős vezető: Töreki Béláné
Budapest, 1984. november hó