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L. DIÓSI
BETTINA KESZTHELYI
B. LUKÁCS
G. PAÁL

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Hungarian Academy of Sciences

CENTRAL RESEARCH INSTITUTE FOR PHYSICS

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L. DIÓSI, BETTINA KESZTHELYI*, B. LUKÁCS, G. PAÁL**

Central Research Institute for Physics
H-1525 Budapest 114, P.O.B. 49, Hungary

*Roland Eötvös University, H-1088 Puskin u. 5-7,
Budapest, Hungary

**The Konkoly Observatory of the Hungarian Academy of
Sciences, H-1525 Budapest 114, Pf. 67, Hungary

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ABSTRACT

It is shown that GUT monopoles with energy about 10^{16} GeV might themselves solve the so-called monopole problem of GUT cosmologies by their unavoidable viscosity, which may lead to a quasi-inflationary scenario even without appeal to a "false vacuum".

АННОТАЦИЯ

Указано, что вторая вязкость газа, состоящего из ГУТ-монополей с массой 10^{16} ГэВ, может привести к показательному расширению Вселенной без предположения ложного вакуума.

KIVONAT

Megmutatjuk, hogy a 10^{16}GeV körüli tömegü GUT monopólusok gázában szükségképpen fellépő viszkozitás olyan nagyságrendü, amely elegendő az Univerzum exponenciális tágulásának létrehozására, azaz egy felfuvódó szcenárió generálására, mindenféle hamis vákuum nélkül.

1. INTRODUCTION

Although GUT-type theories are very promising from several viewpoints as e.g. unification of interactions or explanations for the baryon-antibaryon asymmetry of the present Universe, they do have a prediction which definitely cannot be correct: the calculated monopole density in the present Universe tends to be too high, so that the mass density of the other kinds of particles are completely negligible. While astronomical observations seem to indicate the existence of some non-luminous matter with density up to 30 times of the observed matter (Faber S.M. and Gallagher J.S., 1979), this mass ratio cannot be as high as 10^{15}, predicted by decent and conservative monopole calculations (Kibble, 1982). In spite of the fact that there are some ways to diminish the predicted mass ratio by several orders of magnitude, generally there remains some serious discrepancy.

Guth has mentioned a mechanism to decrease the present monopole density (and to solve both the flatness and the horizon problems in the same time) (Guth, 1981). He argues that if the expansion of the Universe were non-adiabatic, then the present monopole/entropy ratio would be lower than in the standard models, because some part of the entropy would emerge after the monopole creation. Since in this case the expansion rate would be higher, the flatness and horizon problems might also be simultaneously solved.

To carry out this idea, he has constructed the so-called inflationary scenario. There first the initial phase transition ends in a "false vacuum" (a local but not global minimum of the Higgs potential), or the Universe remains in the high-temperature phase. Then the Universe cools down adiabatically for a long time, and finally there is a non-equilibrium phase transition into the true vacuum, producing a great amount of entropy. He estimated that the temperature should have decreased by a factor 10^{28} (i.e. until some 0.1 K^o) in the false vacuum to explain both the flatness and the horizon problems, but, obviously, such a tremendous supercooling is not necessary to solve the monopole problem only; probably during the second violent non-equilibrium phase transition the domain (and thus the monopole) structure is being disarranged, and a new one is emerging. Thus if the temperature after reheating is sufficiently low (there are some estimations that $T_r<10^{11}$ GeV would be sufficient), then the new monopole density will be low enough. On the other hand, there are some arguments suggesting that supercooling probably cannot continue below 10^{11} GeV (Kibble, 1982). Since reheating may produce a temperature comparable to $T_{tr} \simeq 10^{14}$ GeV, supercooling may not help in solving the monopole problem (Kibble, 1982).

Independently of the mechanism producing the extra entropy, Guth's original idea was that after the monopole creation the expansion should have been nonadiabatic. Thus various irreversible processes might lead to extra expansion and dilution of the monopoles.

On the other hand, the emergence of the monopoles may have caused serious changes in the evolution of the Universe because of their uniquely high rest mass (which is not too far even from the Planck mass). If the initial monopole/photon ratio was around the estimated 10^{-8} or less (Kibble, 1982), then the blackbody radiation remained dominant in the energy density and pressure. Nevertheless, if a phenomenological gas description can be used for this stage of the evolution of the Universe (which is at least silently assumed generally), the presence of very massive objects in the early hot Universe results in essential viscosity. Since the viscosity increases with increasing mass and temperature, it seems to be worthwhile to discuss the influence of viscosity on the evolution of the Universe just after the monopole creation.

Since GUT monopoles are not elementary objects but do have some internal structure, their elementary processes behind the dissipation cannot be calculated at the present state of understanding the theory. Thus here we want to make only decent estimations about the influence of the viscosity. If the effects turned out to be negligible, the more detailed calculations would be unnecessary.

2. VISCOSITY IN HOMOGENEOUS ISOTROPIC UNIVERSES

There are some calculated viscous model Universes in the literature (see e.g. Murphy, 1973; Heller et al., 1973; Heller and Suszycki, 1974; Lukács, 1976; Weinberg, 1972). The original motivation was mainly to avoid the initial singularity, or to produce the observed entropy/baryon ratio. In fact, for k=0 it was possible to get solutions without any singularity either for ζ = const., or for $\zeta \sim n^{1/3}$, or for $\zeta \sim \rho$, where ζ stands for the bulk viscosity coefficient. For ζ = const. Heller et al. (1973) note that the evolution equations of the Universe are identical with those of Hoyle's steady-state cosmology, and it was possible to obtain one true steady-state Universe (with constant local data and exponential expansion, see Sol. BIII in their paper). This fact shows the close relationship of such viscous models with the inflationary scenario.

Viscosity is a phenomenological description of different irreversible processes, thus the viscosity coefficients should be calculated from these processes. If the deformation velocities are not too great, one can stop at the linear terms in the stresses, and then there are two viscosity coefficients (Maugin, 1973; Heller et al., 1973):

$$T_{ik} = \rho u_i u_k - (p - \zeta u^r{}_{;r}) h_{ik} + \eta (u_{r;s} + u_{s;r} - \frac{2}{3} g_{rs} u^t{}_{;t}) h_i{}^r h_k{}^s$$
$$h_{ik} = g_{ik} - u_i u_k; \quad u^r u_r = 1 \tag{2.1}$$

Here the energy density ρ and pressure p fulfil some thermodynamical relations, while η and ζ stand for the coefficients of the (pure) shear and bulk viscosities, respectively. These coefficients depend on the local thermodynamical data. It is difficult to decide if this linear approximation for the viscosity is sufficient or not in the early Universe. Nevertheless, if viscosity turns out to be important already in this approximation, one can be sure that it is not negligible in the more general case either.

Now consider homogeneous isotropic, i.e. Robertson-Walker Universe models (clearly this is the case when the influence of the viscosity is minimal). Using the energy-momentum tensor given by eq. (2.1), these Universes are governed by the equations (Heller et al., 1973):

$$\ddot{R} = -\frac{1}{6}\kappa[\rho + 3p - 9\zeta \frac{\dot{R}}{R}]R$$
$$\dot{R}^2 = (-\frac{k}{R^2} + \frac{\kappa}{3}\rho)R^2 \tag{2.2}$$
$$\kappa = 8\pi\gamma$$

where R is the scale factor of the Universe. Thus only the bulk viscosity works in such Universes, because of the high symmetry. Eq. (2.2) shows that for $\dot{R}>0$ the bulk viscosity imitates a negative pressure, and so it can accelerate the expansion. The multiplicative factor of ζ can be quite large for early Universes.

It is possible to eliminate the second order equation for R, obtaining a balance equation for the entropy. In order to see this, assume first that there is no conserved particle number in the system (as it is in a photon-Universe or in a radiation-dominated model). Then the only independent intensive is the temperature, and the characteristic quantities fulfil the following relations:

$$p = p(T)$$
$$\rho = Tp_T - p \tag{2.3}$$
$$s = p_T$$

Combining eqs. (2.2) and (2.3) one then gets:

$$(sR^3)^{\cdot} = 9\frac{\zeta}{T}\dot{R}^2 R \tag{2.4}$$

There we have assumed that there is only one phase in the system. During a phase transition the situation is more complicated, and the result depends on the equilibrium nature of the transition (Lukács, 1983; Csernai and Lukács, 1983).

If there are some particles in the system obeying balance equations, then the thermodynamical relations are:

$$p = \sum_r n_r f_{n_r}(n_i, T) - f$$
$$\rho = f - Tf'_T \qquad (2.5)$$
$$s = -f_T$$

where f is the free energy density. If there are no source terms for the particles, then

$$(n_i R^3)^{\cdot} = 0 \qquad (2.6)$$

and then eq. (2.4) again holds. With source terms there may be some extra entropy production.

Having fixed the form of the proper thermodynamical potential p(T) or $f(n_i, T)$, and the viscosity coefficients, eqs. (2.2), (2.4) and (2.6) completely determine the expansion and thermal history of the Universe.

3. THE INVESTIGATED SCENARIO

In GUT-type theories there are some scalar Higgs fields, whose self-interactions can lead to spontaneous symmetry breaking. Generally the symmetry is not broken at high temperatures, while at some T_{tr} the system undergoes a phase transition, and the Higgs fields obtain some nonvanishing expectation values. Since these theories contain only one characteristic energy scale, thus the expectation values, the masses of the Higghs bosons, and the transition temperature T_{tr} are all similar to each other, and the numerical factors are model-dependent. So here we represent all these quantities by T_{tr}, neglecting the details. This characteristic energy is a free parameter, nevertheless it is expected to be of the order of 10^{15} GeV (Preskill, 1979). Definitely lower values would lead to observable proton decay, while higher values close to the Planck mass $m_P = 1.2.10^{19}$ GeV generally are regarded improbable (Preskill, 1979).

In the new phase the absolute value of the Higgs field is unique, but the orientation is not. Thus some bubble or domain structure can be expected with topological knots, which are the monopoles (Guth, 1981). The mass of such a monopole is cca. $\langle\Phi\rangle/\alpha$, where $\alpha \approx 1/45$ is the grand unified coupling constant. Now, if the phase transition is of second order, then (Kibble, 1982)

$$\langle\Phi\rangle \approx \sqrt{T_{tr}^2 - T^2} \qquad (3.1)$$

so first the monopoles are relatively light, but with decreasing temperature m_m rapidly approaches the final mass $\approx T_{tr}/\alpha \approx 10^{16}$ GeV. If the transition is of strongly first order, then the monopoles are massive from the beginning.

By estimating the initial monopole density (Kibble, 1982) one gets that the energy density of the radiation dominates, when the situation is similar to that considered by Heller and Suszycki (1974) and by Lukács (1981). Then (Guth, 1981)

$$\rho = 3p = \frac{\pi^2}{30} NT^4$$

$$s = \frac{2\pi^2}{45} NT^3 \tag{3.2}$$

$$N \simeq 160$$

In this approximation eqs. (2.2), (2.4) give:

$$\dot{R}^2 = \frac{4\pi^3}{45} N \frac{T^4}{m_P^2} R^2$$

$$\dot{T} = 6\pi \frac{T}{m_P^2} \left[\zeta(T) - \sqrt{\frac{N\pi}{405}} T^2 m_P \right] \tag{3.3}$$

If the bracketed term in the second of eq. (3.3) is small for some reasonable temperature, then there the expansion is almost isothermal, i.e. the viscosity strongly affects the evolution of the Universe. Then the first of eq.(3.3) requires exponential expansion as in the inflationary scenario. Let us see if this can indeed happen to a Universe containing photons and monopole gas.

4. THE BULK VISCOSITY OF THE MONOPOLE GAS

For simple systems there are some model calculations for the viscosity coefficients. In thin classical gases the shear viscosity tends to be independent of the density, and for classical gases of rigid spheres it has the form

$$\eta = \frac{5\sqrt{\pi}}{16} \frac{\sqrt{mT}}{\sigma} \tag{4.1}$$

where σ is the cross section. (Reed and Gubbins, 1973.) Model calculations (as e.g. Kohler, 1948), and measurements for real gases indicate that this value does not essentially depend on the molecular structure.

On the other hand, the bulk viscosity strongly depends both on the density and the molecular structure. It vanishes for classical monatomic ideal gas (Lifshic and Pitaevskii, 1979), and starts linearly with the number density n when using the rigid sphere model (Reed and Gubbins, 1973). This is a consequence of the fact that in these cases the total energy of the internal motion is carried by the translation of the particles (Waldmann, 1958). If the particles do have some internal structure, other modes are also excited, carrying comparable part of the internal energy because of equipartition, and then ζ and η are in the same order of magnitude. A specific model calculation was performed by Kohler (1948) verifying the above expectation, and the two viscosity coefficients are similar to each other in many real gases (Waldmann, 1958; Landau and Lifshic, 1953).

Since GUT monopoles are not true elementary particles but complex entities with rich internal structure (Barrow and Turner, 1982), it is not quite obvious, which approximation would be correct for calculating the bulk viscosity of a monopole gas. Obviously ζ is limited by two extremal values, namely $\zeta \sim \eta$ (case a/) and the value yielded by the rigid sphere model (case b/). Of course, the exact calculation of the momentum transfer processes in the continuum just below the phase transition temperature T_{tr} would require some complicated quantum field theoretical treatment. Nevertheless, at the present state of the study of this problem we use cases a/ and b/ as upper and lower estimates for the bulk viscosity of the continuum.

Eq. (4.1) gives the classical value of the shear viscosity. In the ultrarelativistic case, when $T \gg m(T)$, $\eta \sim T/\sigma$ (Stewart, 1973). Since $m_m > T$, the shear viscosity is higher when the full monopole mass has been built up, nevertheless the difference is not great. If $\zeta \sim \eta$, then eqs. (3.3) and (4.1) give the following condition for isothermal expansion:

$$T_o \simeq \frac{192\ N}{(15)^3} \alpha^5 \frac{m_P^2}{T_{tr}} \tag{4.2}$$

This means that in case a/ there can be an approximately isothermal expansion beginning just after the creation of the monopoles if $T_o \simeq T_{tr}$, which holds if T_{tr} is somewhere at 10^{15} GeV. (If one accepts all the numerical factors in eq. (4.2), then $T_o = T_{tr}$ if $T_{tr} = 2.7 \cdot 10^{15}$ GeV.) Such a value seems to be slightly but not absurdly high.

Nevertheless, it is possible that $\zeta \sim \eta$ overestimates the bulk viscosity of the monopole gas. So we turn to case b/ in order to underestimate it. As it has been mentioned, the bulk viscosity vanishes for a classical thin monatomic gas, and the same is true in the ultrarelativistic limit (Lifshic and Pitaevskii, 1979). However, between these limiting cases even such a gas has a bulk viscosity, because a relativistic equilibrium distribution is incompatible with nonrigid motion (Ehlers, 1971; Stewart, 1973). This bulk viscosity has been calculated only for special models, but the phenomenon being connected with such an elementary irreversible process, one may accept the calculated values for estimation. One can find (Stewart, 1973) that, neglecting some modeldependent numerical factors,

$$\zeta = \begin{cases} T^{5/2}m^{-3/2}\sigma^{-1} & \text{if } T \ll m \\ T^{-3}m^{4}\sigma^{-1} & \text{if } T \gg m \end{cases} \tag{4.3}$$

where σ is some cross section. Since $\sigma \sim \alpha^2/T^2$ (Preskill, 1979), eq. (4.3) can be rewritten as

$$\zeta = \begin{cases} T^{9/2}m^{-3/2}\alpha^{-2} & \text{if } T \ll m \\ T^{-1}m^{4}\alpha^{-2} & \text{if } T \gg m \end{cases} \tag{4.4}$$

The two approximations yield the same value if $T = m$, so one can expect the maximal bulk viscosity there:

$$\zeta \simeq T^3\alpha^{-2} \quad \text{when } m(T) \simeq T \tag{4.5}$$

Since α^{-2} comes from σ, and T^3 is expected from dimensional considerations, (4.5) can indeed be accepted as lowest limit for ζ.

$T=m$ cannot hold for strongly first order transitions, for them $m_m > T_{tr}$. Nevertheless, such a relation can be valid for second order (and weakly first order) transitions, when m_m is increasing with decreasing temperature, according to eq. (3.1). Then the viscosity reaches the maximal value near to T_{tr}, and if there the bracketed term in eq. (3.3) is small, i.e. if

$$\zeta(T_{tr}) \simeq T_{tr}{}^3/\alpha^2 \simeq \sqrt{\frac{N\pi}{405}}\, T_{tr}{}^2 m_P \tag{4.6}$$

then an almost isothermal expansion begins. Hence the critical value of T_{tr} is cca. 4.10^{15} GeV, which is only slightly higher than in case a/.

5. VISCOSITY AND THE INFLATIONARY SCENARIO

If eq. (4.2) or (4.6) holds, then eq. (3.3) yields $\dot{T} = 0$. But then $\dot{R} = \text{const.}\, R$, so there is an exponential expansion. Such an exponential behaviour for the early Universe is not unusual nowadays (see e.g. Starobinskii, 1980; Zel'dovich, 1981; Guth, 1981), but here the driving mechanism is the irreversibility caused by viscosity. The local thermodynamical quantities of our solution remain constant, the entropy production compensates the cooling by expansion. So we arrived at an inflationary scenario, without appeal to false vacuum or vacuum fluctuations. The viscosity is certainly an existing mechanism, but it cannot be very efficient if there is no tuning between T_{tr} and m_P, which quantities are unrelated in GUT theories (but may be related in supergravity). The tuning, however, need not be too fine. In order to decrease the present monopole density to a tolerable level, some 10^5 times isothermal expansion is necessary. This is possible if

ζ does not differ more than 5% from the ideal value. This leads to cca. the same 5% tuning in T_{tr}.

Guth (1981) estimated the necessary exponential expansion for solving both the cosmological horizon and the flatness problems. The same can be ensured by our mechanism supposing a tuning of accuracy of cca. 1%.

In the absence of such a 5% tuning two different cases are possible. If T_{tr} is too high, then the entropy production is too great, the Universe is heated up, and the symmetric phase is restored again. This definitely did not happen to the Universe. If T_{tr} is too low, the irreversibility is not sufficient to keep T constant, so then the expansion is not exponential. Nevertheless, even in this case there happens an extra expansion diluting the monopoles, although not sufficient to eliminate the monopole problem.

In the later history of the Universe the bulk viscosity is only a small perturbation, because it rapidly decreases with the temperature. When the temperature passes through a usual elementary particle mass, a new bulk viscosity is generated. However, then $\zeta \sim m^3/\alpha^2$, and the second of eq. (3.3) indicates that this cannot produce an exponential expansion if $m \ll m_P$. (Heavy bosons of GUT, however, may also significantly contribute to the bulk viscosity.)

6. CONCLUSIONS

Here we investigated the influence of the bulk viscosity of the monopole gas on the expansion of the Universe just after the spontaneous symmetry breaking. The result is that this influence is not negligible. Although the determination of the correct value of the bulk viscosity would require some detailed quantum field theoretical calculations for a continuum containing monopole-antimonopole plasma and all kinds of GUT particles at high temperature, decent estimations can be done for the effect. It can be seen that if the GUT phase transition temperature T_{tr} is tuned to a critical value of about a few times 10^{15} GeV (with some 5% accuracy), then after second order transitions the entropy production of the viscosity can drive an exponential expansion, producing an inflationary scenario. This remains true for first order transitions too, provided that the bulk viscosity is in the same order as the shear one. Then the bulk viscosity in itself dilutes the monopole gas below the present experimental limit. With a 1% tuning even the cosmological horizon and flatness problems can also be simultaneously eliminated. In GUT type theories T_{tr} and m_P are unrelated quantities, thus such a tuning is possible but only accidental.

If the phase transition producing the monopoles is of second order (or weakly first order), then even the lowest estimation for ζ is high enough to prevent the cooling of the Universe forever, provided that T_{tr} is higher than cca. 4.10^{15} GeV. Since the Universe obviously has cooled down, this case is ruled out by observational evidence, and this yields a cosmological upper limit for the GUT mass parameter. Unfortunately, the picture is not so clear for strongly first order phase transitions, when a similar limit could only be obtained if $\zeta \sim \eta$, which is quite possible, however not confirmed by GUT calculations so far.

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