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BUBBLE GROWTH IN VARIABLE PRESSURE FIELDS

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GY. EGELY

Central Research Institute for Physics
H-1525 Budapest 114, P.O.B.49, Hungary

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ABSTRACT

The paper presents a study for the bubble behaviour in variable pressure fields. The presentation is started with the deduction of mass momentum and energy conservation equation in spherical form, and continued until the solution is presented in a closed integral form. The paper outlines the numerical solution, too. The main object of the paper is to present a useful numerical tool for the study of growing or collapsing bubbles, in order to study the interface mass and energy transferm terms, and the interface area.

АННОТАЦИЯ

В работе исследуется поведение пузырька в бесконечной среде под влиянием изменения давления. Представленная физическая модель содержит уравнения сохранения массы, импульса и энергии, сформулированные в сферических координатах. В работе сначала дается вывод этих уравнений, а затем дается такая замкнутая форма решений, которая уже непосредственно пригодна для численных расчетов. Исследование роста пузырька можно осуществить при наличии самых разнообразных внешних воздействий, но время, необходимое для расчета, сильно зависит от длительности транзиента. Главная цель представленного метода - это возможность исследования временных поведений фазовых переходов, сопровождающихся обменом массой и энергией.

KIVONAT

A kutatási jelentés a végtelen közegben a nyomásváltozás hatására növekvő vagy összeomló buborék vizsgálatára készült. A bemutatott fizikai modell gömbi koordinátarendszerben, a buborék középpontjára felirt tömeg, impulzus és energiamegmaradási egyenletet oldja meg. A megoldás az egyenletek alakjának levezetésével kezdődik, majd megadja azt a zárt formát, amelyet már csak numerikusan lehet megoldani. A buboréknövekedést tetszőleges környezeti nyomás-idő függvény esetén lehet vizsgálni, de a számitási időszükségletet a tranziens időtartama erősen befolyásolja. A bemutatott módszer fő célja az, hogy segitségével vizsgálni lehessen kétfázisu áramlás esetén a fázisok közötti energia és tömegcsere tagokat, valamint a határfelület időbeni változását.

Contents

page

Introduction

There are three major fields in the reactor safety studies, where the behaviour of vapour bubbles is important.

a./ Interface mass and energy transfer terms for blowdown calculations;
b./ noise analysis for boiling detection;
c./ bubble collapse, or cavitation for coolant pump studies.

Vapour bubbles may grow, or collapse for two reasons:

1./ The pressure inside the bubble is different from that of the outside. The growth or collapse is controlled then by inertial forces.
2./ The temperature of vapour bubble is different from that of the bulk liquid, there is evaporation or condensation on the liquid/vapour interface. This is the thermal effect.

Usually these effects are unseparable, they act together and there is a definite interaction between them.

In practice, the pressure and temperature difference between the phases is not constant, but changing during transient processes. But the widely used analytical relations are derived for steady superheat case with negligible inertial effects.

The aim of the paper is to solve the governing equations of bubble growth with no such restrictions, for the following reason:

During flow transients, like the Loss of Coolant Accident, the bulk liquid pressure is changing rapidly, there are even pressure oscillations sometimes, and the inertial effects could be important for the subcooled part of boiling.

The model discussed below is based upon the following physical limitations:

1./ The liquid is incompressible. This restriction may cause error only near to the critical point.

2./ As a consequence of surface tension, the shape of the bubble is spherical.

3./ The pressure and the temperature in the liquid at a given r radius from the bubble center is uniform. /See Fig.1,2/

4./ The spatial distribution of pressure and temperature is uniform within the bubble.

5./ The vapour and liquid temperature is the same at the interface.

6./ The system has only one component, there is no dissolved gas in the liquid or in the vapour.

7./ The kinetic energy of vapour is negligible compared to the liquid.

8./ The liquid density will be taken at the bulk liquid temperature. /It is different only in the thin thermal boundary layer./

Only the restrictions 2,3 and 4 could be serious limitations, but for small bubbles, where the spatial change of pressure is still not significant, one may use the spherical approximation.

For reactor coolants, when the liquid is chemically pure, the limitations 5 and 6 are not significant.

Derivation of Equations

In order to be able to determine the behaviour of a bubble one must solve a set of conservation equations, namely the mass, momentum and energy.

From the conservation of mass /see Appendix I./ one obtains:

$$u/_R = \frac{dR}{dt} \qquad /1/$$

The momentum equation for sphere, with angular independence:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 u\right) = 0 \qquad /2/$$

therefore, excluding r = 0: $\frac{\partial}{\partial r}\left(r^2 u\right) = 0$

Integrating the left-hand side with respect to r yields:

$$\int_R^r \frac{\partial}{\partial r}(r^2 u)\,dr = \left[r^2 u\right]_R^r = r^2\frac{dr}{dt} - R^2\frac{dR}{dt} = 0$$

as the right-hand side = 0, the velocity of liquid, u will be:

$$u = \frac{dr}{dt} = \frac{dR}{dt}\left(\frac{R}{r}\right)^2 \qquad /3/$$

For incompressible liquids the conservation of momentum is the same as the conservation of kinetic energy.

The kinetic energy of the liquid /that is the system, as the vapour kinetic energy is neglected/ is given by the

$$E_k = \int_{r=R}^{r=\infty} \frac{\rho_l}{2} u^2 \, dV \qquad /4/$$

integral, i.e. substituting /3/ into /4/ yields:

$$E_k = \frac{\rho_l}{2} \int_R^{\infty} \left[\frac{dR}{dt} \left(\frac{R}{r} \right)^2 \right]^2 4 r^2 \pi \, dr \equiv$$

$$\equiv \left(\frac{dR}{dt} \right)^2 \frac{\rho_l}{2} 4 \pi R^4 \left[- \frac{1}{r} \right]_R^{\infty} \equiv 2 \rho_l \pi R^3 \left(\frac{dR}{dt} \right)^2 \qquad /5/$$

The work done against the surface tension and liquid pressure is:

$$E_l = \int_{R_0}^{R} P_w(t) \cdot 4 R^2(t) \cdot \pi \, dR - \int_{R_0}^{R} P_\infty(t) \cdot 4 R^2(t) \pi \, dR \qquad /6/$$

where the first integral stands for the work done against the surface tension forces, while the second one stands for the work done against the ambient P_∞ /t/ pressure.

The relation between the vapour pressure and the P_W pressure at the outer wall:

$$P_w = P_{sat}(T) - \frac{2\sigma(T)}{R(t)} \qquad /7/$$

Therefore substituting /7/ into the first term of /6/, and adding the integrals yields:

$$E_\ell = \int_{R_0}^{R} \left[P_{sat}(T) - \frac{2\sigma(T)}{R(t)} - P_\infty(t) \right] 4R^2(t)\,\pi\, dR =$$

/8/

$$= \int_{0}^{R} \left[P_{sat}(T) - \frac{2\sigma(T)}{R(t)} - P_\infty(t) \right] 4R^2 \pi\, dR - \int_{0}^{R_0} \left[P_{sat}(T) - \frac{2\sigma(T)}{R(t)} - P_\infty(t) \right] 4R^2 \pi\, dR$$

When the initial radius $\mathbf{R_0}$ is small enough, the second integral is negligible in /8/.

As the work done on the liquid minus the work of surface tension forces and ambient pressure is equal to the kinetic energy of the liquid: $E_\ell = E_K$, that is:

$$\int_{0}^{R} 4R^2(t)\,\pi \left[P_{sat}(T) - \frac{2\sigma(T)}{R(t)} - P_\infty(t) \right] dR = 2\pi \rho_\ell \left(\frac{dR}{dt} \right)^2 R^3(t) \qquad /9/$$

substituting $dR = \frac{dR}{dt} dt$ into the integrational variable, then differentiating by time both sides, one obtains:

$$4R^2 \pi \left[P_{sat}(T) - \frac{2\sigma(T)}{R} - P_\infty(t) \right] \dot{R} = 2\pi \rho_\ell \left(2R^3 \dot{R} \ddot{R} + 3\dot{R}^2 R^2 \dot{R} \right)$$

or in simplified and rearranged form:

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{1}{\rho_\ell} \left[P_{sat}(T) - \frac{2\sigma(T)}{R} - P_\infty(t) \right]$$

When the effect of viscosity is taken into account, a new term is added to the right-hand side:

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{1}{\rho_\ell} \left[P_{sat}(T) - \frac{2\sigma(T)}{R} - P_\infty(t) \right] - \frac{4\nu(T)\,\dot{R}}{R} \qquad /10/$$

This is a second order nonlinear ordinary differential equation, but it is easily transformed into a set of linear differential equations.

Substituting $\dot{R} = z$,

$$R\dot{z} + \frac{3}{2}z^2 = \frac{1}{\rho_l}\left[P_{sat}(T) - \frac{2\sigma(T)}{R} - P_\infty(t)\right] - \frac{4\nu z}{R} \qquad /11/$$

This system of equations gives the motion of spherical vapour bubbles. It is obvious, that P_∞ /t/ must be given and T_{sat}/t/ must be calculated in order to close the system of equations, where T_{sat}/t/ is the temperature of vapour being in saturated condition.

The conservation of the thermal energy

The thermal energy equation for spheres, in the absence of heat source, when the temperature distribution is spherically symmetric is as follows:

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial r} = \frac{\lambda}{\rho_l \cdot c} \cdot \frac{1}{r^2} \cdot \frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right)$$

combining with /3/ yields:

$$\frac{\partial T}{\partial t} + \frac{R^2}{r^2}\frac{dR}{dt}\frac{\partial T}{\partial r} = \frac{\lambda}{\rho_l \cdot c} \cdot \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) \qquad /12/$$

This linear, second order partial differential equation will be transformed into a practically usable integral form.

A new variable

$$h = \frac{1}{3}\left(r^3 - R^3\right) \qquad /13/$$

is introduced to replace r, so h = h/r,t/ for $R \leqq r$

and

$$\frac{\partial h}{\partial r} = r^2 \qquad /14/$$

that is

$$\partial h = r^2 \partial r$$

Instead of the temperature a new variable will be introduced, which is proportional to the energy conducted to the bubble:

$$U(r,t) = \int_R^r r^2 \left(T_1 - T(r',t)\right) dr'$$

instead of r, according to /14/, the new variable **h** is introduced as $\partial r = \frac{1}{r^2} \partial h$

so $$U(h,t) = \int_0^h \left(T_1 - T(h',t)\right) dh' \qquad /15/$$

For this new variable U/h,0/ = 0
U/0,t/ = 0

(See Fig.2.)

As the U/h,t/ function has been derived from T/r,t/, one may write for the first term of the left hand side in /12/:

$$\frac{\partial T(r,t)}{\partial t} = \frac{\partial U}{\partial h} \cdot \frac{\partial h}{\partial t} + \frac{\partial U}{\partial t} \cdot \frac{dt}{dt}$$

but $\frac{\partial h}{\partial t} = -R^2 \frac{dR}{dt}$ from /13/,

so $$\frac{\partial T}{\partial t} = -\frac{\partial U}{\partial h} R^2 \frac{dR}{dt} + \frac{\partial U}{\partial t} \qquad /16/$$

In the second term of the left hand side of /12/

$$\frac{\partial T}{\partial r} = \frac{\partial U}{\partial h} \cdot \frac{\partial h}{\partial r} + \frac{\partial U}{\partial t} \cdot \frac{dt}{dr} \qquad /17/$$

but $\frac{dt}{dr} = 0$, and substituting /14/ into /17/ one obtains:

$$\frac{\partial T}{\partial r} = r^{-2} \frac{\partial u}{\partial h} \qquad /18/$$

/16/ and /18/ is substituted into /12/ in order to eliminate T/r,t/, the result is:

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial h} \cdot R^2 \frac{dR}{dt} + \frac{R^2}{r^2} \frac{dR}{dt} \cdot r^2 \frac{\partial u}{\partial h} \equiv \frac{\partial u}{\partial t} \qquad /19/$$

That is, the left hand side of /12/ is a single term now.

Transformation of the right hand side of /12/:

As $\frac{\partial}{\partial r} = r^2 \frac{\partial}{\partial h}$ from /13/ or /14/,

$$\frac{\lambda}{\rho_\ell \cdot c} \cdot \frac{1}{r^2} \cdot \frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) \equiv \frac{\lambda}{\rho_\ell \cdot c} \cdot \frac{\partial}{\partial h}\left(r^2 \frac{\partial T}{\partial r}\right) \equiv$$

but with /18/

$$\equiv \frac{\lambda}{\rho_\ell \cdot c} \cdot \frac{\partial}{\partial h}\left(r^4 \frac{\partial u}{\partial h}\right) \qquad /20/$$

If r is expressed from /13/, r^4 will have the following form:

$$r^4 = R^4(t)\left[\frac{3h}{R^3(t)} + 1\right]^{\frac{4}{3}} \qquad /21/$$

Therefore the combination of /20/ and /21/ yields:

$$\frac{\lambda}{\rho_\ell \cdot c} \cdot \frac{\partial}{\partial h}\left\{R^4(t)\left[\frac{3h}{R^3(t)} + 1\right]^{\frac{4}{3}} \frac{\partial u}{\partial h}\right\} \qquad /22/$$

that is

$$\frac{\lambda}{\rho_l \cdot c}\left\{\frac{\partial u}{\partial h}\cdot\frac{\partial}{\partial h}\left[R^4\left(\frac{3h}{R^3}+1\right)^{\frac{4}{3}}\right]+R^4\left(\frac{3h}{R^3}+1\right)^{\frac{4}{3}}\cdot\frac{\partial^2 u}{\partial h^2}\right\}$$

or

$$\frac{\lambda}{\rho_l \cdot c}\left[\frac{\partial u}{\partial h}\cdot 4r+R^4\left(\frac{3h}{R^3}+1\right)^{\frac{4}{3}}\cdot\frac{\partial^2 u}{\partial h^2}\right] \qquad /23/$$

As $\frac{\partial u}{\partial h} = 0$ outside the thermal boundary layer, and the magnitude of r is always comparable with R, for small bubbles the first term is negligible, therefore:

$$\frac{\partial u}{\partial t}=\frac{\lambda}{\rho_l \cdot c}\,R^4\left(\frac{3h}{R^3}+1\right)^{\frac{4}{3}}\cdot\frac{\partial^2 u}{\partial h^2}$$

or

$$\frac{1}{R^4}\,\frac{\partial u}{\partial t}=\frac{\lambda}{\rho_l \cdot c}\left(\frac{3h}{R^3}+1\right)^{\frac{4}{3}}\cdot\frac{\partial^2 u}{\partial h^2}$$

Introducing a new time variable, $\upsilon = \int_0^t R^4(t')\,dt'$

therefore $$\frac{1}{R^4}\,\frac{\partial}{\partial t}=\frac{\partial}{\partial \upsilon} \qquad /24/$$

finally $$\frac{\partial u}{\partial \upsilon}=\frac{\lambda}{\rho_l \cdot c}\left(\frac{3h}{R^3}+1\right)^{\frac{4}{3}}\frac{\partial^2 u}{\partial h^2} \qquad /25/$$

As $r \cong R$ in the thermal boundary layer, so $r^2 \cong R^2$ and $r \cdot R \cong R^2$, so the following approximation holds true:

$$\frac{h}{R^3}=\frac{r^3-R^3}{3\cdot R^3}\cong\frac{r-R}{R}=\frac{\delta}{R}$$

where δ is the thickness of the thermal boundary layer. This expression for example in case of water seldom exceeds 10^{-2}, but usually is around 10^{-3}.

In this case Eq./25/ is simplified to

$$\frac{\lambda}{\rho_l \cdot c} \frac{\partial^2 u}{\partial h^2} = \frac{\partial u}{\partial \upsilon} \qquad /26/$$

/26/ is a second order, linear transient heat conduction equation, named after Fourier. /See Appendix II. for the solution./

The solution transformed to the original variables is as follows:

$$T_1 - T(r,t) = \left(\frac{\lambda}{\rho_l \pi c}\right)^{\frac{1}{2}} \int_0^t \frac{r^2(\xi)}{\left[\int_\xi^t R^4(\xi)\, d\xi\right]^{-\frac{1}{2}}} \cdot \frac{\partial T(r(\xi),\xi)}{\partial r}\, d\xi \qquad /27/$$

One should specify $\left.\frac{\partial T}{\partial r}\right|_r$ in order to get the integrand, with the help of the boundary condition for the bubble.

The temperature of the vapour will be:

$$T_{sat}(R,t) = T_1 - \left(\frac{\lambda}{\rho_l \pi c}\right)^{\frac{1}{2}} \int_0^t \frac{R^2(\xi)}{\left[\int_\xi^t R^4(\xi)\, d\xi\right]^{-\frac{1}{2}}} \cdot \frac{\partial T(R(\xi),\xi)}{\partial r}\, d\xi \qquad /28/$$

$\left.\frac{\partial T}{\partial r}\right|_R$ is obtained from the conservation of energy at the bubble interface:

$$4R^2 \pi \lambda \left.\frac{\partial T}{\partial r}\right|_R = \frac{d}{dt}\left[\frac{4}{3} R^3 \pi \rho_v(T) G(T) + 4R^2 \pi \sigma(T) + \frac{4}{3} R^3 \pi \left(P_\infty(t) - P_{sat}(T)\right)\right] \qquad /29/$$

The left hand side is the energy conducted to the bubble, the first term on the right hand side is the evaporation heat, the second is the energy requirement of work against surface tension forces, the last is the work against the ambient pressure.

Substituting /7/ into the last term, and neglecting $P_w - P_\infty$ the sum of the second and last term will be $\frac{4}{3} R^2 \sigma(T) \pi$, thus after simplification:

$$\lambda R^2 \frac{\partial T}{\partial r}\bigg|_R = \frac{d}{dt}\left[\frac{1}{3} R^3 \rho_v(T) G(T) + \frac{1}{3} R^2 \sigma(T)\right]$$

or after differentiation and rearrangement:

$$\frac{\partial T}{\partial r}\bigg|_R = \frac{1}{3\lambda R^2}\left[R^3 \frac{d(G(T)\rho_v(T))}{dt} + G(T)\rho_v(T) 3R^2 \frac{dR}{dt} + 2\sigma R \frac{dR}{dt}\right] \quad /30/$$

The second term is dominant in this expression, while the last is usually considerably smaller, practically negligible.

The final form is obtained by substituting /30/ into /28/, that is:

$$T^* = T_{sat}(R,t) =$$

$$= T_1 - \frac{1}{3}\left(\frac{1}{\lambda \rho_l c \pi}\right)^{\frac{1}{2}} \int_0^t \frac{3G\rho_v R^2 \frac{dR}{dt} + R^3 \frac{d(G\rho_v)}{dt} + 2\sigma R \frac{dR}{dt}}{\left[\int_\xi^t R^4(\xi)\, d\xi\right]^{-\frac{1}{2}}} d\xi \quad /31/$$

That is, /12/ has been transformed into a second order Volterra-type integral equation, which has always a solution, and from numerical point of wiew is easier to solve.

Now a closed system of equations has been obtained, that is: **F** = **F**/T/ , the state variables as a function of temperature /or pressure/

$$1./\quad \frac{dR}{dt} = z$$

$$2./\quad R\dot{z} + \frac{3}{2}z^2 = \frac{1}{\rho_l}\left[P_{sat}(T) - \frac{2\sigma(T)}{R(t)} - P_\infty(t)\right] - \frac{4\nu(T)z}{R}$$

/32/

$$3./\quad T_{sat} = T_1 - \frac{1}{3}\left(\frac{1}{\lambda\rho_l c\pi}\right)^{\frac{1}{2}}\int_0^t \frac{3G\rho_v R^2\dot{R} + R^3\frac{d(G\rho_v)}{dt} + 2\sigma R\dot{R}}{\left[\int_\xi^t R^4(\xi)\,d\xi\right]^{-\frac{1}{2}}}\,d\xi$$

$P_\infty = P_\infty/t/$ is specified as boundary condition, and the initial conditions are:

$$R/_{t=o} = R_o$$

$$\left.\frac{dR}{dt}\right|_{t=o} = 0$$

$$T/r,0/ = T_1$$

The solution of the system

Obviously, this system of equations has no analytical solution, only numerical iterative methods may yield results. Unfortunately, the ordinary differential equations have stiff behaviour, $\dot{R}/t/$, $R/t/$, t may change several orders of magnitudes. The step-length control is significant in this case, therefore a fifth order Sarafian-Butcher method has been applied.

In the denominator of /31/, the functional has been integrated by second order approximation.

The method of iteration is a "learning" algorithm devised for this system.

Appendix I.

Conservation of mass

The continuity for vapour:

$$\dot{m} = \frac{1}{4R^2\pi}\frac{d}{dt}\left(\frac{4}{3}R^3\pi\rho_v\right) \equiv \frac{1}{4R^2\pi}\left(\frac{4}{3}R^3\pi\frac{d\rho_v}{dt} + \frac{4}{3}\pi\rho_v 3R^2\frac{dR}{dt}\right)$$

after simplification:

$$\dot{m} = \frac{R}{3}\frac{d\rho_v}{dt} + \rho_v\frac{dR}{dt}$$

for small bubbles the first term is negligible, so

$$\dot{m} = \rho_v\frac{dR}{dt} \qquad /I1/$$

Continuity for liquid:

$$\rho_l\left(\dot{R} - u(R,t)\right) = \dot{m} \qquad /I2/$$

where u is the interface velocity, where the radius increment caused by evaporation is not neglected, while in $\dot{R}$ only the density variation is considered.

Combining /I1/ and /I2/ yields:

$$\rho_l\dot{R} - u\rho_l = \rho_v\dot{R}$$

or

$$\dot{R}\,\frac{\rho_l - \rho_v}{\rho_l} = u \qquad /I3/$$

If $\rho_v \ll \rho_l$ /I3/ is as follows:

$$\dot{R} = u, \quad \text{or} \quad \frac{dR}{dt} = u$$

the interface velocity.

Near to the critical point this relation yields considerable error.

Appendix II.

The solution of the Fourier equation

The equation is: $$\frac{\partial^2 U}{\partial h^2} = \frac{\varrho_\ell \cdot c}{\lambda} \cdot \frac{\partial U}{\partial v} \qquad /II.1/$$

For U the initial and boundary conditions are respectively:

$$U(h,0) = 0 \qquad /II.2/$$

$$U(0,v) = 0 \qquad /II.3/$$

$$\lim_{h \to \infty} U(h,v) = 0 \qquad /II.4/$$

Introducing $\frac{\lambda}{\varrho_\ell \cdot c} = D$, eq./II.1/ will be solved by Laplace transformation.

The formal transformation yields: $$\frac{d u}{d h^2} = s \frac{u}{D} \qquad /II.5/$$

where $\mathcal{L}\left\{U(h,v)\right\} = u(h,s) = \int_0^\infty e^{-sv}\, U(h,v)\, dv$

This is a second order linear ordinary differential equation. The general form of the solution will be:

$$u = A e^{h\sqrt{\frac{s}{D}}} + B e^{-h\sqrt{\frac{s}{D}}}$$

and from /II.4/ $$\left.\frac{du}{dh}\right|_{h=\infty} = 0$$

therefore $$\left.\frac{du}{dh}\right|_{h=\infty} = A e^{\infty} + B e^{-\infty} = 0$$

yielding $A = 0$, that is, $$u = B e^{-h\sqrt{\frac{s}{D}}} \qquad /II.6/$$

Starting from the definition of $U(h,t) = \int_0^h \left(T_1 - T(h',t)\right) dh'$

we obtain $\left.\frac{\partial^2 U}{\partial h^2}\right|_{h=0} = -\frac{\partial T(h,t)}{\partial h} = -\frac{1}{R^2}\frac{\partial T(0,t)}{\partial r} = F(t)$

Therefore $\mathcal{L}\left\{F(\vartheta)\right\} = f(s)$

So $$\left.\frac{d^2 u}{dh^2}\right|_{h=0} = f(s) \qquad /II.7/$$

After the derivation of eq. /II.6/ with respect to h twice, and combining with eq. /II.7/ the result is

$$\left.\frac{d^2 u}{dh^2}\right|_{h=0} = f(s) = B \cdot \frac{s}{D}$$

therefore $B = f(s)\,\frac{D}{s}$

Now u will have the following form:

$$u(h,s) = f(s) \cdot \frac{D}{s} \cdot e^{-h\sqrt{\frac{s}{D}}} \qquad /II.8/$$

The inverse Laplace transformation will be carried out not for u , but for $\frac{du}{dh}$, as this is more favourable;

as $$\mathcal{L}^{-1}\left\{\frac{du}{dh}\right\} = \frac{\partial U}{\partial h} = T_1 - T(h,\vartheta)$$

that is $$T(h,\vartheta) = T_1 - \mathcal{L}^{-1}\left\{-\sqrt{\frac{D}{s}} \cdot f(s) \cdot e^{-h\sqrt{\frac{s}{D}}}\right\} \qquad /II.9/$$

The inverse transformation has been carried out by /1/ table, yielding

$$T(h,\nu) = T_1 + \sqrt{\frac{D}{\pi}} \int_0^{\nu} \frac{F(\xi)}{(\nu-\xi)^{\frac{1}{2}}} \cdot e^{-\frac{h^2}{4D(\nu-\xi)}} \, d\xi \qquad /II.10/$$

If $F(\xi) = -\frac{1}{r^2} \cdot \frac{\partial T(h,\xi)}{\partial r}$ is substituted into the nominator, one obtains

$$T(h,\nu) = T_1 + \sqrt{\frac{D}{\pi}} \int_0^{\nu} \frac{\frac{\partial T(h,\xi)}{\partial r}}{r^2\sqrt{\nu-\xi}} \cdot e^{-\frac{h^2}{4D(\nu-\xi)}} \, d\xi$$

Transformation to original variables:

As the transformed time variable is $\xi = \int_0^x R^4(x')dx'$

one may replace $d\xi$ with dx , because of $\frac{d\xi}{dx} = R^4(x)$

one obtains

$$T(h,t) = T_1 - \sqrt{\frac{D}{\pi}} \int_0^t \frac{r^2 \frac{\partial T}{\partial r} \cdot e^{-\frac{h^2}{4D\int_x^t R^4(y)dy}}}{\left[\int_x^t R^4(y)dy\right]^{\frac{1}{2}}} \, dx$$

and

$$T(r,t) = T_1 - \sqrt{\frac{D}{\pi}} \int_0^t \frac{r^2 \frac{\partial T}{\partial r} \cdot e^{-\frac{r^3-R^3}{12D\int_x^t R^4(y)dy}}}{\left[\int_x^t R^4(y)dy\right]^{\frac{1}{2}}} \, dx$$

or at the bubble wall

$$T(R,t) = T_1 - \sqrt{\frac{D}{\pi}} \int_0^t \frac{R^2(x) \left.\frac{\partial T}{\partial r}\right|_{r=R(t)}}{\left[\int_x^t R^4(y)\,dy\right]^{\frac{1}{2}}}\,dx \qquad /II.11/$$

Finally, when $R \rightarrow \infty$, that is, for a plane, in eq. /II.11/ R is regarded as a constant, therefore

$$T(t) = T_1 - \sqrt{\frac{D}{\pi}} \int_0^t \frac{\left.\frac{\partial T}{\partial r}\right|_R}{\left[\int_x^t dy\right]^{\frac{1}{2}}}\,dx$$

yielding

$$T(t) = T_1 - \sqrt{\frac{D}{\pi}} \int_0^t \frac{\left.\frac{\partial T}{\partial r}\right|_R}{\sqrt{t-x}}\,dx$$

Inasmuch as $\frac{\delta}{R} \ll 1$ inequality is not fulfilled, a correction is to be applied. Applying the method of successive approximation . T is approximated as
$T = T^{*} + T^{**} + T^{***} + \ldots\ldots$

T^{*}, as the first approximation is given by /II.11/

By similar methods, as T^{*} was obtained, an inequality is derived for T^{**} :

$$\left.T^{**}(t)\right|_R \leqq -\frac{4}{3}\sqrt{\frac{\lambda}{\rho_l \cdot c \cdot \pi}} \int_0^t \frac{\Theta(R,\xi)}{R^3(\xi)\sqrt{t-\xi}}\,d\xi$$

and $$-\sqrt{\frac{\lambda}{\rho_l \cdot c \cdot \pi}} \cdot \frac{1}{R^3(t)} \int_0^t \frac{\Theta(R,\xi)}{\sqrt{t-\xi}}\,d\xi \leqq \left.T^{**}(t)\right|_R$$

where $\Theta(R,t) = T_1 - T^*(R,t)$

According to the calculations, the correction was very small, usually negligible.

[1] A. Erdélyi et al.: Tables of Integral Transforms. Vol. 1., McGrow Hill

Comparison of results with test data

The previously presented numerical model has been compared with three test data. The data has been published by M. Nüno in the "Study of Single Bubble Generation and Growth by Laser beam", Ph. D. Thesis Univ. of Tohoku, Japan, 1975, and by Hewitt and Parker in the "Bubble Growth and Collapse in Liquid Nitrogen", ASME Journal of Heat Transfer, Vol. 90, 1968, pp. 22-26.

The data is plotted in a paper by O. Jones, Jr and N. Zuber: "Bubble Growth in Variable Pressure Fields" ASME Journal of Heat Transfer, Vol. 100, Aug. 1978, pp. 453-459.

On Fig. 1. and Fig. 2. water has been used in the experiment, while in case of Fig. 3., liquid nitrogen. The experimental results of bubble radius measurements are plotted on the radius curves as black dots. The agreement is reasonable between the calculated and measured values. It has to be emphasized that the method is general, the pressure history around the bubble could be arbitrary. Smooth pressure functions are, however, more convenient to calculate, jumps or sudden changes require more computational time.

In case of Fig. 1. and Fig. 2. at the beginning of the calculations the time has been plotted logarithmically. This period, although starts in real time, requires a significant protion of all computational time.

The method is appropriate for bubble collapse calculation, until the bubble radius becomes smaller then the critical radius.

Nomenclature

r	radius, independent variable
t	time, independent variable
$R(t)$	bubble radius
R_O	initial bubble radius at $t = 0$
u	velocity
P	pressure
$P_\infty(t)$	ambient pressure
T	temperature
T_1	initial temperature
V	bubble volume
P_w	pressure at the outer wall of bubble

State variables

$\rho_\ell(T)$	liquid density
$\rho_v(T)$	vapour density
$\sigma(T)$	surface tension
$P_{sat}(T)$	saturation pressure
$\lambda(T)$	liquid heat conductivity coefficient
$c(T)$	liquid specific heat
$G(T)$	evaporation heat
$\nu(T)$	dynamic viscosity

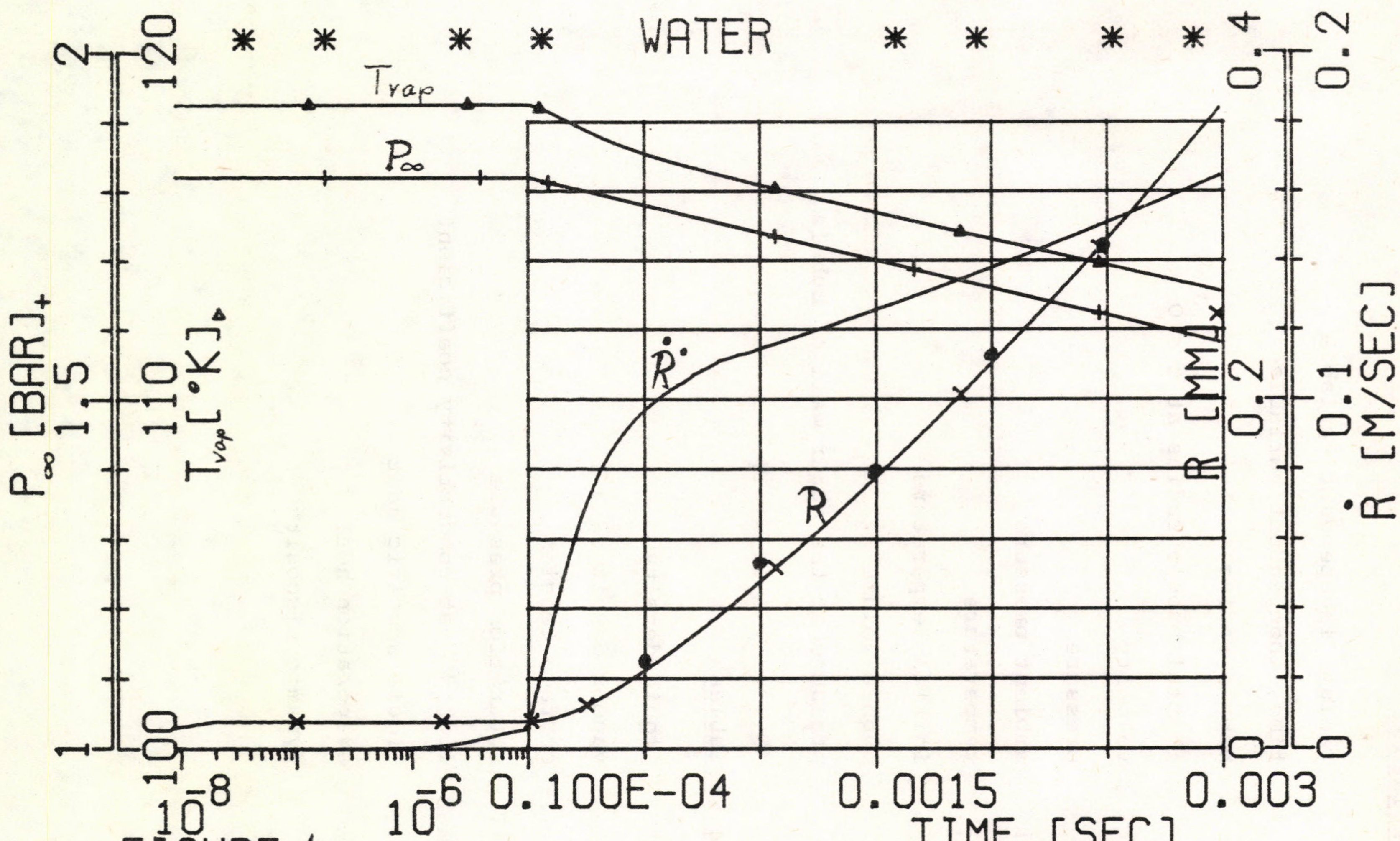
* * * * WATER * * * *
P∞ [BAR]+
T_vap [°K]▲
2
1.5
1
120
110
100
T_vap
P∞
Ṙ
R
R [MM]×
Ṙ [M/SEC]
0.4
0.2
0
0.2
0.1
0
10^-8
10^-6
0.100E-04
0.0015
0.003
TIME [SEC]

FIGURE 1

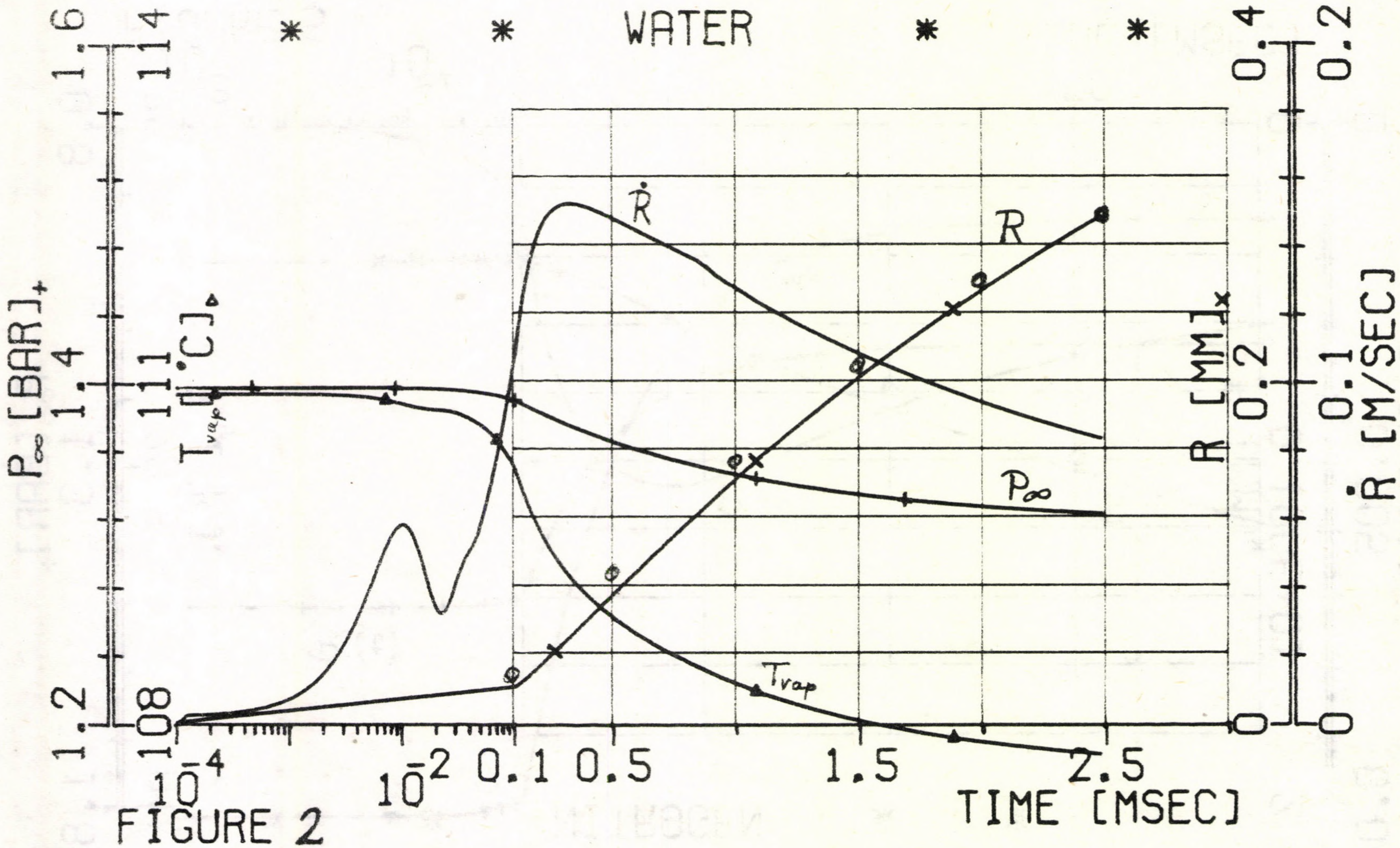
WATER
P∞ [BAR]+
1.2
1.4
1.6
T_vap [°C]▵
108
111
114
R [MM]×
0
0.2
0.4
Ṙ [M/SEC]
0
0.1
0.2
10⁻⁴
10⁻²
0.1
0.5
1.5
2.5
TIME [MSEC]
Ṙ
R
P∞
T_vap

FIGURE 2

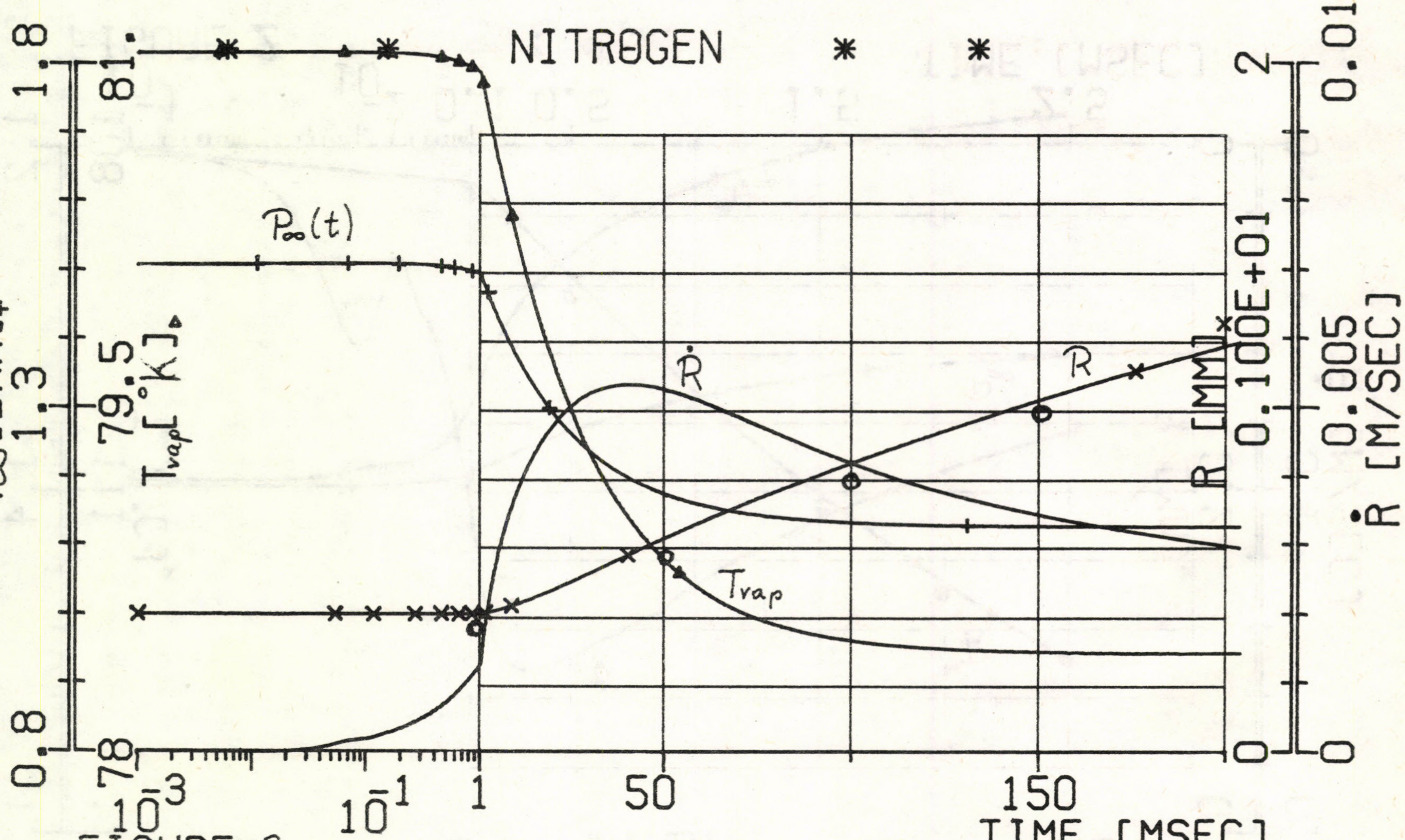
NITROGEN
P∞ [BAR]
T_vap [°K]
1.8
1.3
0.8
81.
79.5
78
$\mathcal{P}_\infty(t)$
Ṙ
R
T_vap
10^{-3}
10^{-1}
1
50
150
TIME [MSEC]
R [MM]
0.100E+01
2
0
0.01
0.005
Ṙ [M/SEC]

FIGURE 3

Kiadja a Központi Fizikai Kutató Intézet
Felelős kiadó: Gyimesi Zoltán
Szakmai lektor: Katona Tamás
Nyelvi lektor: Dus Magdolna
Példányszám: 225 Törzsszám: 83-452
Készült a KFKI sokszorositó üzemében
Felelős vezető: Nagy Károly
Budapest, 1983. junius hó