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ABSTRACT

A scheme for linearizing the source free Yang-Mills field equations is given. An infinite parameter invariance group of the gauge field equations is discovered.

АННОТАЦИЯ

Дана схема для линеаризации уравнений поля Янга-Миллса без источника. Найдена группа симметрий уравнений калибровочного поля, обладающая бесконечным числом параметров.

KIVONAT

Egy módszert adunk a forrásmentes Yang-Mills egyenletek linearizálására. A téregyenletek egy végtelen sok paraméteres invariancia csoportját is felfedeztük.

Recent developments have convincingly shown that soliton theoretic ideas borrowed from two dimensional models can successfully be applied to obtain some special solutions / self-dual ones, e.g. instantons and monopoles / of four dimensional gauge theories [1,2,3] . Alternatively, these results can be derived [4] by techniques originating from an interpretation of self-duality / SD / equations in twistor space [5] . Motivated by this success Witten [6] and Isenberg et al [7] gave a similar twistorial interpretation of the sourceless gauge field equations. However, this construction was not carried far enough to be practically implementable. The aim of the present paper is to extend the ideas of [1,8] for non self-dual solutions of gauge theories.

In his approach Witten [6] proposed a system of equations in eight dimensional space that on the physical subspace yields the wanted gauge solutions. We show that this system can be linearized just in the same way as the SD equations. Similarly to the procedure of Ref. [8] we connect the linear equations to matrix Riemann-Hilbert problems / RHP / for two and one variable that, in principle, are capable of generating all local solutions of the system. Furthermore, we reformulate the RHP in terms of linear singular / matrix / integral equations and solving them in the infinitesimal case we point out the existence of an infinite parameter invariance group of the system. Finally, we make some comments on how these properties will be inherited by the gauge field solutions on the physical subspace. The next step would be to construct explicit non self-dual solutions which appears to be feasible using our construction. However, an even more important possibility may be to uncover the nonperturbative structure of gauge theories using the linearization property. In fact a detailed analysis of the classical sector may provide the key for understanding the quantum theory in analogy with two dimensional completely integrable models.

In the eight dimensional space with coordinates a^i, b^i $i = 1,\dots 4$ we consider the following system of equations [6]:

$$F_{a_i a_j} = {}^{*}F_{a_i a_j} \; ; \quad F_{b_i b_j} = -{}^{*}F_{b_i b_j} \; ; \quad F_{a_i b_j} = 0 \,, \qquad (1)$$

where $F_{ij} = \partial_i B_j - \partial_j B_i + [B_i, B_j]$ and ${}^{*}F_{a_i a_j}$ denotes the dual of $F_{a_i a_j}$. An important property of system (1) is that it's solutions satisfy on the $x^i = \frac{1}{2}(a^i + b^i)$ diagonal subspace the four dimensional source free Yang-Mills equations: $D^\mu F_{\mu\nu} = 0$. This paper is devoted to a thorough study of system (1).

Our first main claim is that system (1) can be linearized. To show this we introduce complex coordinates defined as

$$y_1 = b_1 + ib_2,\; y_2 = b_3 + ib_4,\; z_1 = a_1 + ia_2,\; z_2 = a_3 - ia_4,$$
$$\bar{y}_1 = b_1 - ib_2 \;\; \text{etc.} \qquad (2)$$

Using these coordinates the system (1) can be written as

$$F_{y_1 y_2} = F_{z_1 z_2} = F_{\bar{y}_1 \bar{y}_2} = F_{\bar{z}_1 \bar{z}_2} = 0 \; ; \qquad (3a)$$
$$F_{y_i z_j} = F_{y_i \bar{z}_j} = F_{\bar{y}_i z_j} = F_{\bar{y}_i \bar{z}_j} = 0 \,, \; (i,j = 1,2) \; ; \qquad (3b)$$
$$F_{y_1 \bar{y}_1} + F_{y_2 \bar{y}_2} = 0 \; ; \; F_{z_1 \bar{z}_1} + F_{z_2 \bar{z}_2} = 0 \,. \qquad (3c)$$

Following Yang's ideas [9] equations (3a) and a part of (3b) can be solved explicitly in complexified space if the gauge potentials, B_i, are expressed in terms of a $D = D(y_i, z_i) \in SL(N,C)$ element as

$$B_{u_i} = -D_{,u_i} D^{-1},\ u = y,z\ ;\ B_{\bar{u}_i} = D^{+-1} D^{+}_{,\bar{u}_i}\ ,\ \bar{u} = \bar{y}, \bar{z},\ i = 1,2 \qquad (4)$$

The remaining equations in (3) take the form in terms of a hermitian $g = D^+D$ as

$$\left(g_{,u_1} g^{-1}\right)_{,\bar{u}_1} + \left(g_{,u_2} g^{-1}\right)_{,\bar{u}_2} = 0,\quad u = y,z \qquad (5a)$$

$$\left(g_{,y_i} g^{-1}\right)_{,\bar{z}_j} = 0\ ;\ \left(g_{,z_i} g^{-1}\right)_{,\bar{y}_j} = 0,\quad i,j = 1,2 \qquad (5b)$$

In this formalism an – eight dimensional – gauge transformation is defined as $D \rightarrow GD$, $D^+ \rightarrow D^+G^+$, $G \in SU(N)$, therefore, g is gauge invariant. However, system (5) admits a group of invariance transformations: if g is a solution of (5) then $\Omega(y_i, z_i) g\, \Omega^+$ with $\Omega \in SL(N,C)$ also solves (5). The easiest way to read off the components of the gauge potentials from g is to go into Yang's R-gauge [9] where D takes a lower triangular form.

We linearize system (5) similarly to what we applied successfully in the case of selfduality equations [1] i.e. using the notion of prolongation structures. Indeed, a straightforward generalization of the procedure in [1] yields the following system of linear equations:

$$D_1 \psi \equiv \left(-\lambda_1 \partial_{\bar{y}_2} + \partial_{y_1}\right)\psi = g_{,y_1} g^{-1} \psi\ ,$$

$$D_2 \psi \equiv \left(\lambda_1 \partial_{\bar{y}_1} + \partial_{y_2}\right)\psi = g_{,y_2} g^{-1} \psi\ , \qquad (6)$$

$$D_3 \psi = \left(-\lambda_2 \partial_{\bar{z}_2} + \partial_{z_1}\right)\psi = g_{,z_1} g^{-1} \psi\ ,$$

$$D_4 \psi \equiv \left(\lambda_2 \partial_{\bar{z}_1} + \partial_{z_2} \right) \psi = g_{,z_2} g^{-1} \psi \tag{6}$$

In equations (6) λ_1, λ_2 are two arbitrary complex parameters and $\psi = \psi(\lambda_i, u_i, \bar{u}_i)$ is a $N \times N$ matrix for which we also impose the boundary condition $\psi(0, u_i, \bar{u}_i) = g$. We guarantee this by considering only solutions, ψ , analytic in λ_1, λ_2 at $\lambda_1 = \lambda_2 = 0$.

It is easy to verify that the compatibility conditions of system (6) are just equations (5) : the commutators of D_1, D_2 and D_3, D_4 yield eqs (5a) and the others give (5b) . An essential, new feature of this system is that it consists of four commuting operators with two spectral parameters. To make contact with the geometrical interpretation of Refs [6,7] it is worth pointing out that these equations, after a suitable reinterpretation of the λ_i parameters, can be conceived to live on a coordinate patch of $CP^3 \times CP^3$. To make this correspondence more transparent it is more appropriate to consider a slightly different system expressing the pure gauge nature of the vector potentials on $CP^3 \times CP^3$:

$$\left(- \lambda_1 \left[\partial_{\bar{y}_2} + B_{\bar{y}_2}\right] + \partial_{y_1} + B_{y_1}\right)\phi = 0 \, ; \left(- \lambda_2 \left[\partial_{\bar{z}_2} + B_{\bar{z}_2}\right] + \partial_{z_1} + B_{z_1}\right)\phi = 0;$$
$$\left(\lambda_1 \left[\partial_{\bar{y}_1} + B_{\bar{y}_1}\right] + \partial_{y_2} + B_{y_2}\right)\phi = 0 \, ; \left(\lambda_2 \left[\partial_{\bar{z}_1} + B_{\bar{z}_1}\right] + \partial_{z_2} + B_{z_2}\right)\phi = 0. \tag{7}$$

This system is the generalization for the present case of the Ward-Belavin-Zakharov [10] linear system for the selfduality equations.

Our next step is to connect both system (6) and (7) to a two dimensional matrix Riemann-Hilbert problem. The simplest way to achieve this is to note that for any $\psi(\lambda_i)$ solution of (6) $g\, \psi^{+-1}(-\bar{\lambda}_i^{-1})$ solves the same equation exploiting the hermiticity of g thus

$$g\,\psi^{+-1}(-\bar\lambda_1^{-1}, -\bar\lambda_2^{-1}) = \\ = \psi(\lambda_1,\lambda_2)\, G(\lambda_1,\lambda_2,\lambda_1 y_1+\bar y_2, \lambda_1 y_2-\bar y_1, \lambda_2 z_1+\bar z_2, \lambda_2 z_2-\bar z_1), \tag{8}$$

where G is a $N\times N$ matrix satisfying $D_iG = 0$, $i= 1,\dots 4$. If ψ is analytic around $\lambda_i=0$ then $g\,\psi^{+-1}(-\bar\lambda_i^{-1})$ is analytic near $\lambda_i = \infty$ / in fact $g\,\psi^{+-1}(-\bar\lambda_i^{-1})\to I$ as $\lambda_i\to\infty$ /, therefore, G is analytic in the overlapping region of the two domains. This suggests the following way to construct solutions of (6) : given a G nonsingular / i.e. $\det G = 1$ / satisfying $D_iG = 0$, $i= 1,\dots 4$; $G^+(-\bar\lambda_i^{-1}) = G(\lambda_i)$ and analytic in $\Gamma_\epsilon = \{\lambda_i;\ 1-\epsilon<|\lambda_i|<1+\epsilon\ ;\ \epsilon>0,\ i= 1,2\}$, for a certain domain in our eight dimensional space, split it according to (8) into the product of two matrices with nonvanishing determinants analytic in λ_i for $|\lambda_i|<1$ / Γ_{++}/ and $|\lambda_i|>1$ / Γ_{--} / respectively, with the additional property that the matrix in Γ_{--} is normalized to I at infinity. This is the regular matrix Riemann-Hilbert problem / RHP / for two variables. Of course, we still have to show that a solution of this RHP satisfies system (6) as well. To prove this we apply D_i / $i = 1,\dots 4$ / to eq(8) on $\Gamma_o = \{\lambda_i;\ |\lambda_i|^2 = 1\}$:

$$D_i(g\,\psi^{+-1})\,\psi^+ g^{-1} = D_i\psi\,\psi^{-1} \qquad \text{on } \Gamma_o, \tag{9}$$

where we denoted the matrix analytic in Γ_{++} and Γ_{--} by ψ and $g\,\psi^{+-1}$ respectively. As ψ /$g\,\psi^{+-1}$/ are analytic in their domain eq (9) defines the analytic continuation of both sides from Γ_o into Γ_{++} and Γ_{--}. Now using the generalization of Liouville's theorem [11] for many complex variables and the fact that our matrix at infinity is normalized to I we conclude that both

$D_i \psi \psi^{-1}$ and $D_i(g \psi^{+-1}) \psi^+ g^{-1}$ are independent of λ_i in Γ_{++} and Γ_{--} respectively, thus they indeed solve (6) . The ψ constructed this way yields at $\lambda_i = 0$ a g that solves (5) . It is also a solution of our initial problem as detg = 1 is guaranteed by the assumption detG = 1 on Γ_o.

This argument applies in a similar way to eq (7) as well. In this case we find that eq (8) is replaced by

$$\phi^{+-1}(-\bar{\lambda}_1^{-1}, -\bar{\lambda}_2^{-1}) =$$
$$= \phi(\lambda_1, \lambda_2)\, F(\lambda_1, \lambda_2, \lambda_1 y_1 + \bar{y}_2, \lambda_1 y_2 - \bar{y}_1, \lambda_2 z_1 + \bar{z}_2, \lambda_2 z_2 - \bar{z}_1), \qquad (8')$$

where the analytic properties of ϕ^{+-1} and ϕ are the same as for $g \psi^{+-1}$ and ψ but ϕ^{+-1} does not tend to the identity matrix at infinity. Therefore, having split F into the product of two matrices ϕ and ϕ^{+-1} analytic in Γ_{++} and Γ_{--} respectively, from the analogue of eq (9) we can conclude – using the generalized Liouville's theorem – that the general form of $D_i \phi \phi^{-1}$ in Γ_{++} / $D_i \phi^{+-1} \phi^+$ in Γ_{--} / contains both a term linear in λ_i and a λ_i independent one, i.e. eq (7) are satisfied.

However, the above construction is not as simple as it may seem as an arbitrary G / F / cannot be split according to eq (8) , (8') in general. The conditions on G / F / guaranteeing the possibility of this splitting are not known in general. Nevertheless, in the special cases when G = I + v where v is infinitesimal or G is upper triangular " Ansätze like" these conditions are easily obtained.

The / RHP / can be reformulated in terms of linear singular / matrix / integral equations. Indeed, adopting the integral representation for ψ in Γ_{++}, $\Gamma_{+-} = \{\lambda_i, |\lambda_1| < 1\}$ and

$$\Gamma_{-+} = \left\{ \lambda_i,\ |\lambda_2| < 1 \right\}$$

$$\psi = I + \frac{1}{(2\pi i)^2} \int_{\Gamma_0} \frac{\sigma(t_1,t_2)}{(t_1-\lambda_1)(t_2-\lambda_2)}\, dt_1 \wedge dt_2 + $$
$$+ \frac{1}{2\pi i} \int_{\partial D_1} \frac{\hat{\sigma}_1(t_1)}{t_1-\lambda_1}\, dt_1 + \frac{1}{2\pi i} \int_{\partial D_2} \frac{\hat{\sigma}_2(t_2)}{t_2-\lambda_2}\, dt_2 , \qquad (10)$$

and in Γ_{--} with the signs of the last three terms reversed from eq (9) one obtains

$$I - \frac{1}{4}\left(I_1 - I_2 - I_3 + \sigma(\lambda_1,\lambda_2)\right) - \frac{1}{2}\left(\hat{\sigma}_1(\lambda_1) + \hat{\sigma}_2(\lambda_2)\right) =$$
$$= \left\{ I + \frac{1}{4}\left(I_1 + I_2 + I_3 + \sigma(\lambda_1,\lambda_2)\right) + \frac{1}{2}\left(J_1 + J_2\right) + \frac{1}{2}\left(\hat{\sigma}_1(\lambda_1) + \right.\right.$$
$$\left.\left. + \hat{\sigma}_2(\lambda_2)\right)\right\} G + \frac{1}{2}\left(J_1 + J_2\right) \quad , \quad \text{on } \Gamma_0, \qquad (11)$$
$$\frac{1}{2}\left(J_1 + J_2\right) = -\frac{1}{4}\left(I_1 - \sigma(\lambda_1,\lambda_2)\right) \ ; \ \hat{\sigma}_2(\lambda_2) - \hat{\sigma}_1(\lambda_1) = \frac{1}{2}(I_3 - I_2),$$
on Γ_0,

where I_i, / $i = 1,2,3$ / and J_i / $i = 1,2$ / are defined on Γ_0 by

$$I_1 = -\frac{1}{\pi^2} P \int_{\Gamma_0} \frac{\sigma(t_1,t_2)}{(t_1-\lambda_1)(t_2-\lambda_2)}\, dt_1 \wedge dt_2, \quad I_2 = \frac{1}{\pi i} P \int_{\partial D_1} \frac{\sigma(t_1,\lambda_2)}{t_1-\lambda_1}\, dt_1,$$

$$I_3 = \frac{1}{\pi i} P \int_{\partial D_2} \frac{\sigma(\lambda_1,t_2)}{t_2-\lambda_2}\, dt_2, \quad J_1 = \frac{1}{\pi i} P \int_{\partial D_1} \frac{\hat{\sigma}_1(t_1)}{t_1-\lambda_1}\, dt_1 ,$$
$$J_2 = \frac{1}{\pi i} P \int_{\partial D_2} \frac{\hat{\sigma}_2(t_2)}{t_2-\lambda_2}\, dt_2. \qquad (12)$$

Here ∂D_i denotes the circles $|\lambda_i| = 1$ and P stands for the principal value prescription. This reformulation of RHP in terms of linear singular integral equations makes the problem more amenable to simple minded analysis such as finding solutions in the infinitesimal case. Furthermore, it may be easier to find the class of G's for which solutions of (11) exist. In the case G = I + v / v infinitesimal / we found that (11) is solvable if $v(\lambda_1, \lambda_2)$ has the form

$$v(\lambda_1, \lambda_2) = \sum_{n,m>0} a_{nm} \lambda_1^n \lambda_2^m + \sum_{n,m\geq 0} b_{n,m} \lambda_1^{-n} \lambda_2^{-m},$$

and in this case the infinitesimal solution of (11) is given by

$$\sigma(\lambda_1, \lambda_2) = -v(\lambda_1, \lambda_2), \quad \hat{\sigma}_1(\lambda_1) = \sum_{n>0} b_{no} \lambda_1^{-n},$$

$$\hat{\sigma}_2(\lambda_2) = \sum_{m>0} b_{om} \lambda_2^{-m} \quad .$$

Another nice property of eqs (11) is that they reduce in a simple way to a form describing selfdual /SD / / antiselfdual / solutions. Indeed, a SD /ASD/ g does not depend on $z_i, \bar{z}_i$/ $y_i, \bar{y}_i$/ and a possible way to achieve this is to consider ψ's / or ϕ's/ independent of $\lambda_2, z_i, \bar{z}_i$/ $\lambda_1, y_i, \bar{y}_i$/ which in turn implies that G / F / shares the same property. In this case we deduce from (11) by making the ansatz $\sigma(t_1, t_2) = 2\tilde{\sigma}(t_1)$:

$$\left(I + \frac{1}{\pi i} P \int_{\partial D_1} \frac{\tilde{\sigma}(t_1)}{t_1 - \lambda_1} dt_1\right)(I - G) - \tilde{\sigma}(\lambda_1)(I + G) = 0 \quad \text{on } \Gamma_o$$

$$\hat{\sigma}(\lambda_1) = \frac{1}{\pi i} P \int_{\partial D_1} \frac{\tilde{\sigma}(t_1)}{t_1 - \lambda_1} dt_1 - \tilde{\sigma}(\lambda_1), \quad \hat{\sigma}_2(\lambda_2) = 0 \text{ on } \Gamma_o.$$

As it should have become apparent the use of the RHP with two variables presents considerable mathematical difficulties therefore, any simplification would be welcome. Obviously, $\psi(\lambda_i)$ contains a lot more information we needed / it is enough to know it in a neighbourhood of $\lambda_i = 0$ /. Putting it another way we observe that in eqs (6) / or (7) / the use of just one spectral parameter is sufficient: we take $\lambda_2 = \lambda_1^3$ / this is of course not unique, e.g. $\lambda_2 = \lambda_1^{2n+1}$; $n > 0$ and integer would also do /. This indicates that in fact it is sufficient to work just on a suitable algebraic submanifold of $CP^3 \times CP^3$. This submanifold is clearly different from the 5 complex dimensional hypersurface / quadric / used in [6,7].

Our choice of λ_2 guarantees that the derivation of eqs (8) and (8') remains the same yielding a RHP for one variable. This problem is much easier to tackle and a sufficient condition for the existence of solutions was given: $\mathrm{Re}G > 0$ on $|\lambda_1| = 1$. Using similar arguments as above one can derive a linear singular integral equation for $\psi(\lambda)$. In what follows we present it in a slightly different form – as used in the SD case [8] – capable of generating infinitely many new solutions from a given one. Assuming that a $\psi_o(\lambda)$ solution of (6) with g_o is known we look for the new solutions in the form $\psi(\lambda) = \chi(\lambda)\,\psi_o$, this yields the following RHP for χ

$$g\,\chi^{+-1}(-\bar{\lambda}^{-1})g_o^{-1} = \chi(\lambda)\,\psi_o(\lambda)\,G_o(\lambda, \lambda y_1+\bar{y}_2, \lambda y_2-\bar{y}_1, \lambda^3 z_1+\bar{z}_2, \lambda^3 z_2-\bar{z}_1)\,\psi_o^{-1}(\lambda), \qquad (13)$$

here $g = \psi(0)$ is the new solution of (5) . Representing $\chi(\lambda)$ as

$$\chi(\lambda) = I + \int_{\partial D_1} \frac{\sigma(t)}{t-\lambda}\, dt , \qquad (14)$$

we obtain the following singular integral equation for $\sigma(t)$

$$\frac{1}{\pi i}\left[P \int_{\partial D_1} \frac{\sigma(t)}{t-\lambda}\, dt + I \right]\left(\psi_o G_o \psi_o^{-1} - I\right) + \sigma(\lambda)\left(\psi_o G_o \psi_o^{-1} + I\right) = 0 , \quad \lambda \in \partial D_1 \qquad (15)$$

This equation corresponds to the regular RHP / det $\chi \neq 0$ / which obviously does not give all solutions of (6) . However, with the aid of the RHP with zeroes / i.e. allowing det χ =0 at some points within its domain of analyticity / one can get all solutions.

As it was shown for the first time in Ref. [1] the SD equations possess an infinite parameter invariance group. In an analogous way one can establish the existence of a similar – but much larger – group for our system (4,5) . Here we construct the infinitesimal action of this group by solving our integral equation (15) for infinitesimal G_o; $G_o = I + v$ / v infinitesimal /. In this case we have

$$\sigma(t) = -\frac{1}{2\pi i}\, \psi_o(t) v(t) \psi_o^{-1}(t) , \qquad (16)$$

which, with the aid of (14), defines the new solution, ψ . Defining the potentials $G_o^{(m,n)}$ – that are generalizations of the infinitely many conserved quantities found in SD case – as

$$\frac{s - t\, \psi_o^{-1}(s)\, \psi_o(t)}{s - t} = \sum_{m,n=0}^{\infty} t^m s^n\, G_o^{(m,n)}$$

/ and similarly for ψ / we obtain that the infinitesimal transformations act on the potentials as

$$G^{(k,o)} = G_o^{(k,o)} - \sum_j G_o^{(j,o)} v^{(k-j)} + \sum_{j,n} G_o^{(j,o)} v^{(-j-n)} G_o^{(k,n)} , \qquad (17)$$

where $v(t) = \sum_{k=\infty}^{+\infty} t^k v^{(k)}$. The transformation on $G^{(k,n)}$ can be derived in a similar way. It is easy to show that these transformations constitute an infinite dimensional Lie algebra.

To implement our construction in practice there are two ways: either one uses upper triangular G's / which are straightforward generalizations of the Atiyah-Ward [5] ansätze / or one chooses $G_o \equiv 1$ and looks for meromorphic χ / which is the generalization of the procedure applied in [8] to the SD equations/.

As it was mentioned earlier all solutions of (1) solve the sourceless Yang-Mills equations. Nevertheless, in general these are only special solutions e.g. in the case of SU(2) they are necessarily selfdual, antiselfdual or abelian ones. The necessary condition for A_μ to satisfy the field equations is the existence of a power series in $w^i = \frac{1}{2}(a^i - b^i)$ up to second order satisfying (1) [6,7].

The main advantage of our construction is that it can be readily extended for solutions satisfying (1) up to a given order in w's. In the most interesting case, when A_μ is defined only up to second order, the corresponding linear system is easily constructed and is defined only up to third order. This leads to an RHP also defined up to third order in w. The resulting system of RHP's is of course more complicated than the original one, (15) , but this is the linearization of the second order Yang-Mills equations.

The detailed exposition of this procedure would exceed the bounds of this letter and is going to be the subject of a forthcoming publication.

A very important consequence of the linearizability is that an infinite parameter invariance group also exists for SU(N) non self-dual gauge fields satisfying the field equations. Although, the infinitesimal action of this group is rather complicated, in principle, there is no difficulty to obtain it from our formulae (17) .

In our opinion the existence of such an intricate structure / the huge dynamical symmetry group / may provide alternative ways to develop new nonperturbative quantization techniques for gauge theories.

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