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ONE CAN HAVE NONINTEGER TOPOLOGICAL CHARGE

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In a recent paper [1] a new finite action solution of the SU(2) self-duality equations /SDE/ was found with fractional Pontryagin number 3/2. This implies that there are hitherto unsuspected contributions to the functional integral which might be of physical importance.

Our paper was severely criticized, and it was claimed that our solution was wrong [2]. Although the criticism of Ref. [2] is based on simply misunderstanding our paper we find it worthwhile to explain why it is not relevant to our solution and we try to elucidate some subtle points concerning the possibility of fractional Ponryagin number.

Here we reiterate the main points concerning the solution for details we refer to [1].

The crucial point was that we solved the SDE in $R^4 \setminus S^2$, that is we left out a two dimensional sphere of R^4. We covered $R^4 \setminus S^2$ with coordinate patches P_i / $i = 1,2$ / and the gauge field, A_μ, is given as

$$A_\mu^{(i)} = \sigma_{\mu\nu} \partial^\nu \ln \varrho_i \tag{1}$$

With the assumption of four dimensional axial symmetry $\varrho_i = \varrho_i(r,z)$ / $r = \sqrt{x^2 + y^2 + t^2}$ / ϱ_i can readily be written as

$$\varrho_i = \frac{1}{r} \operatorname{Re} G_i(r + iz) \tag{2}$$

where $G_i(r + iz)$ is an analytic function in P_i; however, outside P_i it has a cut. The cut of G_i starts from the branch singular at the origin. After continuing back the solution / in a distributional sense / to the origin, they found that the Yang-Mills equations were satisfied only if one introduced a pointlike source. In our case we found that there is no source term.

We remark that our solution can obviously be extended to $S^4 \setminus S^2$ but not to the whole S^4. Since we work on a manifold with boundary, it appears quite likely that the η invariant would make the index of the Dirac operator an integer number[5] .

Finally we mention that solutions with arbitrary real topological charge greater than 1/2 exist.

In conclusion, we showed that although our solution does not represent a connection of a principal SU(2) fibre bundle over R^4 only over $R^4 \setminus S^2$, it solves the sourceless Yang-Mills field equations, and there is no a priori reason not to take into account in the semiclassical approximation.

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"point" / which is in our case S^2 /. This S^2 is not movable by gauge transformations, and our solution has a singularity there. We solved the SDE leaving out this singular surface, however, we were able to demonstrate that the source-free Yang-Mills equations / and the SDE of course / are satisfied even on this S^2 in a distributional sense.

The problem raised in [2] was that our solution is not a connection over R^4, and it is not possible to continue $A_\mu^{(i)}$ -s back to S^2 in such a way as to obtain a connection over R^4. This is of course true, however, as it was already stressed in [1] our solution is a connection over $\underline{R^4 \setminus S^2}$, and it was never claimed to be extendible to a connection over R^4. There is no surprise here, since it was known for some time that in this case only integer topological charge is permitted [3] .

Since the singularity of our solution on S^2 is not a gauge artifact it represents a new, non-instanton type configuration. The rather interesting property that it has finite action, and it solves the SDE and the source-free Yang-Mills equations in $\underline{R^4}$ makes one wonder if these new solutions would be important in the evaluation of the functional integral in the semiclassical approximation.

Physically one is interested in finding classical field configurations which are saddle points / finite action / even if these solutions are not connections over R^4.

It is worth pointing out that our case is analogous in a sense with the well known Wu-Yang monopole [4] . The Wu-Yang monopole solution does not give rise to an SO(3) principal fiber bundle over R^3 / only over $R^3 \setminus \{0\}$ /, as it is

ABSTRACT

It is argued that the recently found fractionally charged self-dual solution is relevant.

АННОТАЦИЯ

Показано что недавно найденное само-дуальное решение с дробным топологическим зарядом может иметь значение.

KIVONAT

Megmutatjuk, hogy a nemrég talált tört töltésü önduális megoldás fizikailag releváns.

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