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ABSTRACT

In this paper the G_E and G_M form factors of the proton and neutron are calculated in the geometrodynamical model of hadrons. Asymptotic behaviour and gross features are correctly reproduced, there are deviations from experiments in fine details.

АННОТАЦИЯ

В статье приведены расчеты формфакторов G_E и G_M протонов и нейтронов в геометродинамической модели адронов. Модель правильно отражает асимптотическое поведение и основные характеристики, однако, наблюдаются некоторые незначительные расхождения с экспериментальными данными.

KIVONAT

Ebben a cikkben a proton és neutron G_E és G_M formfaktorai vannak kiszámolva a hadronok geometrodinamikai modelljében. Az aszimptotikus viselkedést és a főbb tulajdonságokat a modell helyesen visszaadja, de a mérési eredményektől finomabb eltérések vannak.

1. INTRODUCTION

The geometrodynamical model is a bag-type model for hadrons, invented and elaborated by G. Preparata and his coworkers [1]. The basic idea - explained in more detail in the next section - is that simple geometrical approach is enough to determine the wave-functions of hadrons, built from quark degree of freedom.

The model has been succesfull in explaining the meson [2] and baryon spectrum [3], as well as certain dynamical properties too [4]. When describing the baryons, the interesting result was obtained that the three quarks in it are in a quark-diquark formation in the center-of-mass system. This reduces the inner degrees of freedom.

In this paper we have extend that model to find the current matrix elements of nucleons. Specifying it to the electromagnetic current, the formfactors G_E and G_M can be obtained for the nucleons. It is a fairly difficult question how to get conserved current matrix elements in models; for the case of geometrodynamical model it was discussed in [5]. In our case, using an approximate form of the wave function, the current is conserved. That approximate form is very suitable for calculation, hence we do not treat the question of conservation in general.

The physical picture for current interaction is very simple: the current hits one of the quarks, the other two exchange momentum via exchanging a meson in their t-channel, and get rearranged into quark-diquark formation.

The picture is similar in other formfactor calculations, too. Farrar and Jackson [6], calculated the pion formfactor by solving the light-cone pion Bethe-Salpeter equation to leading-log accuracy, taken the kernel from QCD. They obtained

$$F_{\pi}(q^2) \xrightarrow[q^2 \to \infty]{} \frac{1}{Q^2 \ln(Q^2)} .$$

Similar analysis was done by Lepage and Brodsky [7] for the baryon. Up to logarithmic correction the $(Q^2)^{-2}$ behaviour was reproduced. For the low--energy region the result depended on the assumption how the effective wave function of the 3-quark system looked like. Trying two different Ansätze for that, the curves fall faster than the experimental one. The character of our results is very similar, let alone that in our model there is no room for different assumptions.

To compare our results with experiment, we used the excellent review paper of Höhler at al [8].

In sect 2. we summarise briefly how the baryon wave-function looks like in the model, in sect 3. the current matrix element is discussed and the results are presented.

2. THE MODEL

The baryon wavefunction is denoted by

$$\Psi_{\alpha a,\beta b,\gamma c}(P;x_1,x_2,x_3) = e^{iPX}\ \Psi_{\alpha a,\beta b,\gamma c}(P,x,y) \tag{2.1.}$$

where a, b and c are internal flavour indices, α, β and γ are Dirac indices, x_i are the quark coordinates, and

$$X = \frac{1}{3}(x_1+x_2+x_3) \qquad x = \frac{1}{\sqrt{2}}(x_2-x_3) \qquad y = \sqrt{\tfrac{2}{3}}\left(x_1 - \frac{x_2+x_3}{2}\right). \tag{2.2.}$$

The principles, on which the geometrodynamical approach is based, are the following:

i) Confinement

$$\Psi_{\alpha a,\beta b,\gamma c}(P;x,y) = 0 \quad \text{for} \quad x,y \notin R^8(P;x,y) \tag{2.3.}$$

where $R^8(P;x,y)$ is a compact eight-dimensional space-time region with boundary $B^8(P;x,y)$

ii) Continuity

For $x,y \in B^8(P;x,y)$ $\Psi(P;x,y)$ must be continous, i.e.

$$\Psi_{\alpha a,\beta b,\gamma c}(P;x,y) = 0 \text{ for } x,y \in B^8(P;x,y) \tag{2.4.}$$

More precisely, continuity is required only for suitable scalar functions appearing in a Lorentz-covariant decomposition of the wavefunctions. See ref. [4] for the details.

iii) Wave equation

for $x,y \in R^8(P;x,y)$ Ψ obeys the simple differential equation

$$D_1 D_2 D_3 \Psi = 0 \qquad (2.5.)$$

where $D_i = (i\not\partial_i + m_i)$ is the Dirac operator, acting on the i-th coordinate.

iv) Approximate freedom

The "distance" of Ψ from the "free solution" $\Psi^{(o)}(P;x,y)$, determined by the free equations

$$D_i \Psi^{(o)} = 0 \qquad i = 1,2,3 \qquad (2.6.)$$

is minimum, the distance is calculated by an appropriate definition of the norm of the wavefunctions [2].

The solution of equs 2.3-2.5 was examined in [3]. The wavefunction factorizes into a scalar-part and a spin-part.

$$\Psi = \tilde{\Phi}\ |B\rangle \qquad (2.7a.)$$

The scalar-part has the form

$$\tilde{\Phi}(P;x,y) = \int d\xi \tilde{\Phi}(P;y,\xi)\delta^4(\xi P-x) \qquad (2.7b.)$$

The Fourier-transform of $\tilde{\Phi}(P;y,\xi)$ is

$$\Phi(P;q,\eta) = \int d^4x d\xi e^{-iqx} e^{-i\eta\xi} \tilde{\Phi}(P;y,\xi) \qquad (2.8.)$$

where

$$\begin{aligned} p_1 &= \tfrac{1}{3}P + \sqrt{\tfrac{2}{3}}\, q \\ p_2 &= \tfrac{1}{3}P - \sqrt{\tfrac{1}{6}}\, q - \sqrt{\tfrac{1}{2}}\, p \\ p_3 &= \tfrac{1}{3}P - \sqrt{\tfrac{1}{6}}\, q + \sqrt{\tfrac{1}{2}}\, p \end{aligned} \qquad (2.9.)$$

are the quark momenta. Due to the quark-diquark structure, both x and p are proportional to P, $x = \xi P$, $p = \eta P$. The quantities ξ and η are scalars in the CM-system their value coincide with x_0 for ξ, and with p_0 for η.

In the CM,

$$\tilde{\Phi}_{CM}(P;q,p_o) = \frac{\omega^2}{8} \cos \frac{R + C_1}{\frac{\omega^2}{16} - C_1^2} \cos \frac{R + C_2}{\frac{\omega^2}{16} - C_2^2}\, f(|\underline{q}|) \qquad f(|\underline{q}|) = \frac{J_{1/2}\left(\frac{\pi|\underline{q}|}{a}\right)}{\frac{\pi}{a}|\underline{q}|(|\underline{q}|^2 - a^2)}$$

$$C_1 = -E_2 + \frac{3\omega}{4} + \frac{M}{3} - \frac{1}{\sqrt{6}} q_o - \frac{1}{\sqrt{2}} p_o \qquad a = \frac{\sqrt{\pi}}{2}, \quad m = 0.1, \quad E_1 = \sqrt{\frac{2}{3}a^2 + m^2}$$

$$C_2 = -E_3 + \frac{3\omega}{4} + \frac{M}{3} - \frac{1}{\sqrt{6}} q_o + \frac{1}{\sqrt{2}} p_o \qquad E_2 = E_3 = \sqrt{\frac{a^2}{6} + m^2}, \quad \omega = E_1 + E_2 + E_3 - M$$

$$R_t = \frac{2\pi}{\omega} \tag{2.10.}$$

Next we turn to the spin part.

In the non-relativistic SU(6) model the baryon has the wavefunction

$$|B\rangle = \varphi_S^{SU(3)} \chi_S^{SU(2)} + \varphi_A^{SU(3)} \chi_A^{SU(2)} \tag{2.11.}$$

where φ_S, φ_A are the symmetric/antisymmetric combinations of the SU(3) part and χ_S, χ_a are the appropriate spin part:

$$\begin{aligned}
\varphi_S &= \frac{1}{\sqrt{6}}(2ppn - pnp - npp) \\
\varphi_A &= \frac{1}{\sqrt{2}}(pnp - npp) \\
\chi_S &= \frac{1}{\sqrt{6}}(2\alpha\alpha\beta - \alpha\beta\alpha - \beta\alpha\alpha) = \frac{1}{\sqrt{6}}(2{\uparrow\uparrow\downarrow} - {\uparrow\downarrow\uparrow} - {\downarrow\uparrow\uparrow}). \\
\chi_A &= \frac{1}{\sqrt{2}}(\alpha\beta\alpha - \beta\alpha\alpha) = \frac{1}{\sqrt{2}}({\uparrow\downarrow\uparrow} - {\downarrow\uparrow\uparrow})
\end{aligned} \tag{2.12.}$$

where p, n are the proton and neutron quarks, α and β are the spin up and down functions.

In our case the only difference is (in the CM frame) that instead of the static spinors, Dirac spinors are used with the appropriate momentum variable e.g. (for instance χ_A has the following form):

$$\chi_A = \frac{1}{\sqrt{2}}[U_\uparrow(p_1)U_\downarrow(p_2)U_\uparrow(p_3) - U_\downarrow(p_1)U_\uparrow(p_2)U_\uparrow(p_3)] \tag{2.13.}$$

The wave function in arbitrary system can be obtained by a Lorentz transformation Λ, which satisfies the equations:

$$\Lambda P = P'$$
$$\Lambda p_i = p_i' \qquad (2.14.)$$

Due to alignement (i.e. quark-diquark structure) in CM, no Wigner rotation appears in (2.13.).

3. CALCULATION OF THE FORMFACTORS

As it was mentioned in the Introduction, the physical picture of current--hadron interaction is that the current interacts with one of the quarks, and the other two interacts between themselves to get rearranged into quark-diquark formation.

The kinematics is given according to Fig.1.

$$\begin{aligned} P &= (M,0,0,0) \\ Q &= (-t/2M, 0, 0, -\frac{1}{2M}\sqrt{t(t+4M^2)}\,) \\ P' &= P - Q \qquad\qquad t = Q^2 \end{aligned} \qquad (3.1.)$$

The incoming hadrons is characterised by the set P, q, η. the outgoing by P', q', η' according to equs (2.9). If the first quark is hit by the current, in the configuration space one has to evaluate the integral

$$\langle \underline{P} | J_\mu(Z) | \underline{P}' \rangle_1 = \int \prod_{i=1}^{3} d^4x_i\, d^4x_i'\, \overline{\Psi}_{\alpha\beta\gamma}(P, x_i)\, \Psi_{\alpha'\beta'\gamma'}(\underline{P}', x_i') \qquad (3.2.)$$
$$e_1 [\gamma_\mu]_{\alpha\alpha'}\, \delta^4(z-x_1)\, \delta^4(x_1-x_1')\, \Delta_{\beta\gamma,\beta'\gamma'}(x_2,x_3,x_2',x_3')\, \delta^4(x_2+x_3-x_2'-x_3')$$

where e_1 is the charge of the first quark, and

$\Delta_{\beta\gamma,\beta'\gamma'}(x_2,x_3,x_2',x_3')$ takes care of the quark interaction. Inserting (2.1.) into it and using eq. (2.7.)

$$\langle \underline{P}|J_\mu(Z)|\underline{P}'\rangle_1 = e^{-iQZ} \int d^4q\,d\eta\,d\eta'\; \bar{\Psi}_{\alpha\beta\gamma}(q,\eta)e_1[\gamma\mu]_{\alpha\alpha'}\Psi_{\alpha'\beta'\gamma'}(q+\sqrt{\tfrac{2}{3}}\,Q,\eta') \cdot \tag{3.3.}$$

$$\Delta_{\beta\gamma,\beta'\gamma'}(\eta P-\eta' P')$$

where $\Psi(q,\eta)$ is given by equs (2.7.).

We have to sum (3.3.) for all three configurations. It is not difficult to see that

$$\langle \underline{P}\uparrow|J_o(t)|\underline{P}'\uparrow\rangle = \sqrt{\tfrac{1}{2}(t+4M^2)}\;\; G_E(t) \tag{3.4a.}$$

$$\langle \underline{P}\uparrow|J_1(t)|\underline{P}'\downarrow\rangle = -\sqrt{\tfrac{t}{2}}\, G_M(t) \tag{3.4b.}$$

where G_E and G_M are the electromagnetic form factors.

Next we have to fix $\Delta(\eta P-\eta' P')$. The first natural guess would be

$$\Delta(\eta P-\eta' P') = \delta^4(\eta P-\eta' P') \tag{3.5a.}$$

i.e. that the non-hit quarks propagate freely. However, it is easy to see, that it leads to $P = P'$ which is nonsense. So we have to allow meson exchanges, e.g.

$$\Delta_{\beta\gamma,\beta'\gamma'}(\eta P-\eta' P') = \frac{\delta_{\beta\beta'}\delta_{\gamma\gamma'}}{[\eta P-\eta' P']^2+i\epsilon} \tag{3.5b.}$$

for scalar meson exchange. Mass term could have been allowed in the denominator.

This propagator has a pole in variable t, its imaginary part is just $\delta[(\eta P-\eta' P')^2]$. Using equs (3.4.) we are going to calculate the imaginary part of G_E, G_M and they will be recovered by using dispersion relation.

$$G_{E,M} = \frac{1}{\pi} \int \frac{\mathrm{Im}G(t')}{t'-t_o} dt \tag{3.6.}$$

Here this is just a mathematical trick, it has nothing to do with the analytical properties of the form factors.

Strictly speaking, eq. (3.3.) is valid only if the current hits the single quark, and the diquarks rearrange among themselves. However it can be proven that quark-diquark rearrangement (i.e. when the single quark becomes a member of the diquark system after the interaction) is very much suppressed.

The current (3.3.) is not conserved as it stands. This can be remedied in the same way as in [5] for the meson current in this model. However, the space part of the wave-function, (2.10.), $f(|\underline{q}|)$ can be approximated by $\delta(q^2-a')$.

As can be calculated, in this approximation the current is conserved. This approximation considerably simplifies the calculation, so we use it in this paper.

It is fairly easy to obtain the high t behaviour of (3.3.). A t^{-1} factor comes from the propagator (3.5.), and another t^{-1} from the space-part-function of the outgoing baryon. The spin-part of the outgoing baryons at first sight contibutes with $\sqrt{t^3}$ as each spinor in the kinematical configuration (3.1.) corries $\sqrt{t}$, however for those two, wich are coupled to scalar, this $\sqrt{t^2}$ cancels out, and the net behaviour of the spin part is $\sqrt{t}$. Comparing this to (3.4.), we obtain

$$G_E(t) \sim G_M(t) \sim t^{-2} \quad \text{for high } t.$$

This behaviour is confirmed by experiments.

We have started this chapter with the physical picture of the current-hadron interaction. However, it is quite conceivable, that the current interacts via vector-meson dominated term too, as in Fig. 2, and not only the direct term of Fig. 1 as allowed.

In the framework of the geometrodynamics, the meson wave-function which appears here, depends on the variable $Q(p_1-p_1')$which is fixed to a number due to the δ-functions in the wave-functions. So, the t-dependence will not be altered by adding this type of contributions.

The overall normalisation is fixed at $t = 0$ by the requirement that it should give the total charge, which is the sum of quark charges. If the quark masses were not equal, to recover the quarks charge additivity would be quite a problem.

The rest is numerical calculation. It is fairly tiresome because the integrand is oscillating, and most of the multidimensional integration procedures fail. We have succeded at last with the routine DIVONNE (D105 in CERNLIB), which performs a multidimensional Monte-Carlo integration with intervallum adjustment.

Let alone the propagator (3.5.) we tried pseudoscalar propagator too, but that yielded funny small-t behaviour. We have not tried vector-meson nominator, due to technical (i.e. numerical) complication. However, we consider (3.5.) as some "effective propagator" for several types of meson exchanges.

The results for proton in the low-energy region is presented in Table 1 and Fig. 3,4. As can be seen, they fall faster then the experimental data.

The similar calculation can be repeated for neutron too. Roughly it is true that

$$G_M^{(n)} = -{}^2/_3\, G_M^{(p)}$$

the factor between them varyes with q^2. $G_E^{(n)}(q^2)$ is fairly flat as a function of q^2, (the high q^2 behaviour was the same) but in the small-q region $(0.1<q^2<0.4)$ its value was too big to be accepted.

In our wiev this shows that the model using the kernel (3.5b.) reproduces the gross features, but further improvement would be necessary to get fived details. However, due to enormous technical difficulty, we do not think that it is possible and feasible.

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Q^2 $[\mathrm{GeV}]^2$	G_E^P	G_M^P
0.017	0.94	2.60
0.120	0.69	1.95
0.146	0.62	1.64
0.195	0.52	1.51
0.311	0.34	1.08
0.393	0.31	0.82
0.584	0.20	0.59
0.780	0.12	0.34
0.990	0.11	0.25
1.170	0.08	0.20
1.360	0.05	0.16
1.560	0.025	0.11
1.750	0.014	0.09
2.00	0.010	0.06
2.40	0.008	0.02

Table 1

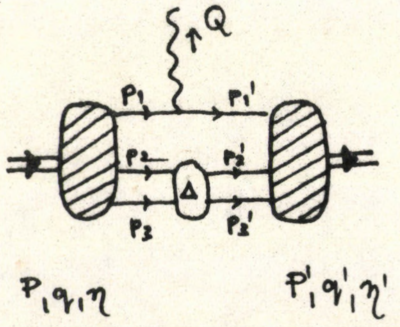


Fig. 1.

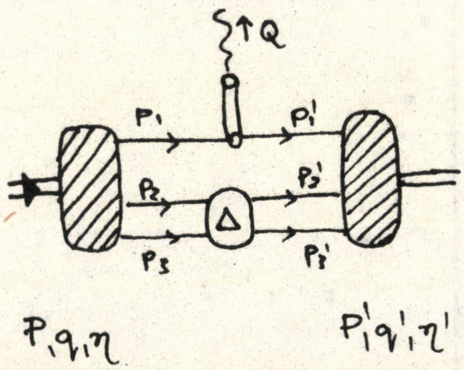
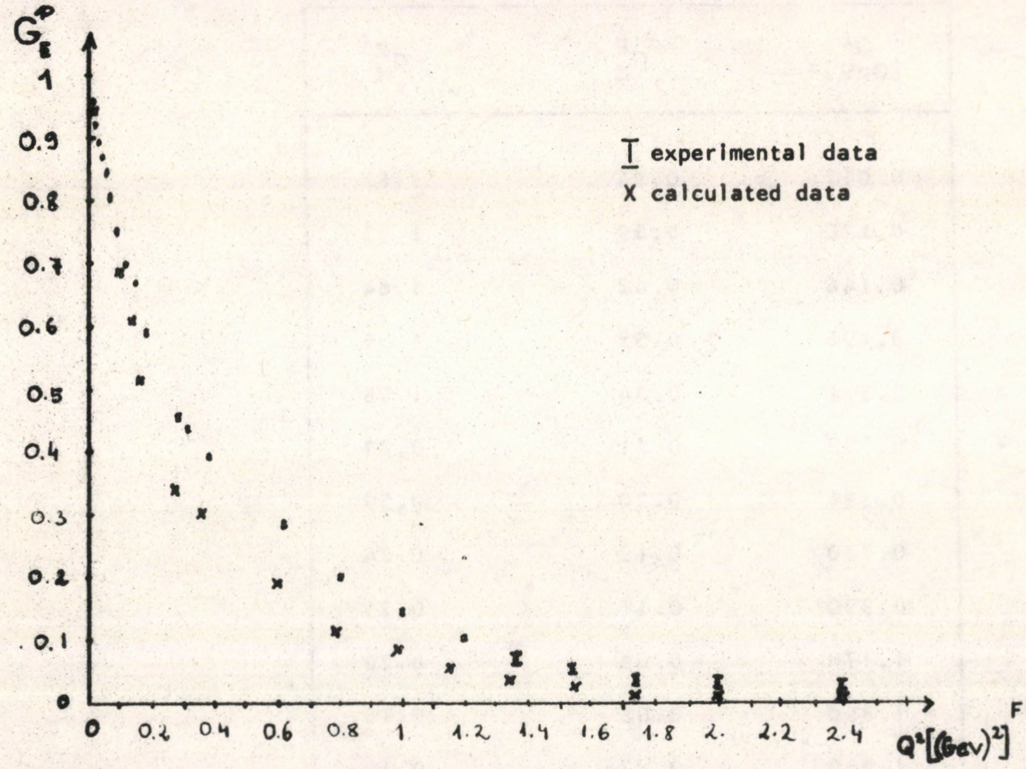
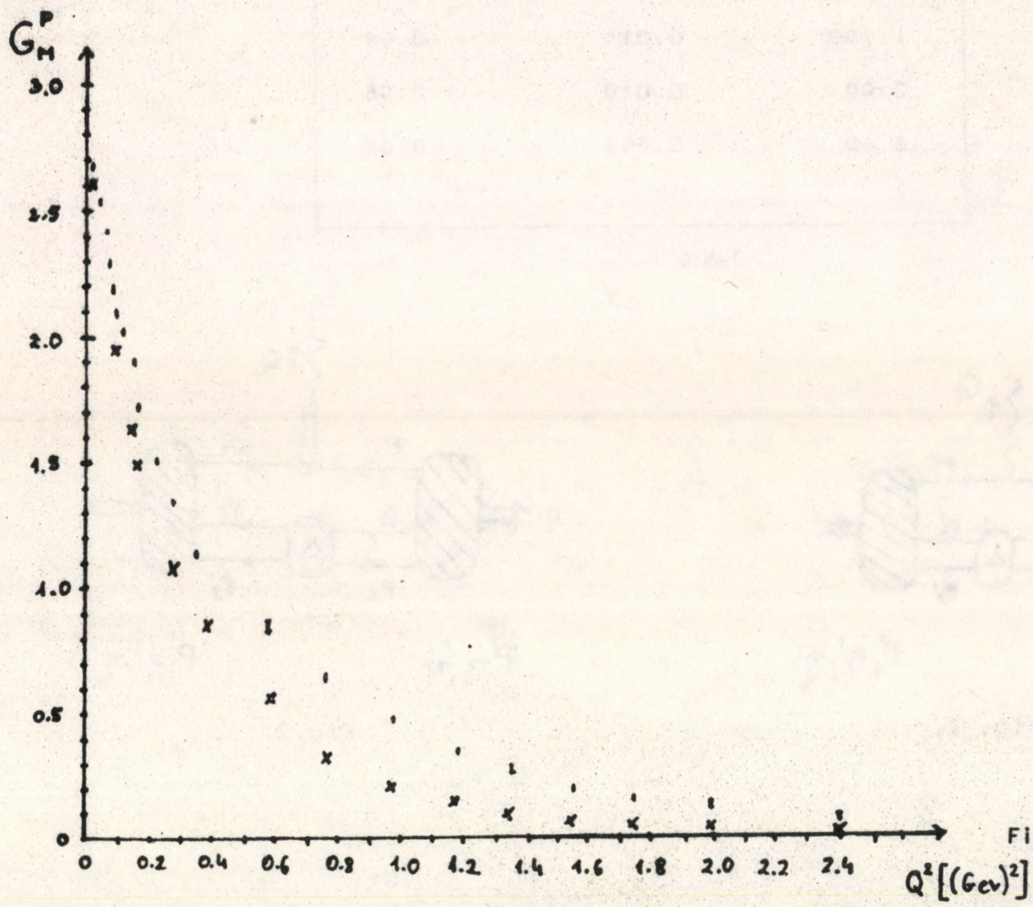


Fig.2.



G_E^p
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
I experimental data
X calculated data
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2 2.2 2.4
$Q^2[(Gev)^2]$

Fig.3.



G_M^p
3.0
2.5
2.0
1.5
1.0
0.5
0
0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0 2.2 2.4
$Q^2[(Gev)^2]$

Fig.4.

6J.188

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Szakmai lektor: Tóth Kálmán
Nyelvi lektor: Sebestyén Ákos
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Budapest, 1981. szeptember hó