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# DYNAMIC RENORMALIZATION GROUP TREATMENT OF A BOSE GAS MODEL

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ABSTRACT

Dynamic normalization group analysis of the complex time-dependent Ginzburg-Landau model with infinitely many component order parameter is given, with emphasis on the limiting case, representing a Bose gas model, when the kinetic coefficient is treated as pure imaginary. The fact that dynamical scaling behaviour holds for the Bose gas model in a region of small frequencies (containing the critical mode) with boundary proportional to the wavenumber is shown to be the consequence of that that a well-behaved fixed point can be achieved in this region.

АННОТАЦИЯ

Методом динамической группы ренормировок исследована комплексная, зависящая от времени модель Гинзбурга-Ландау, в которой число компонент параметров порядка бесконечное. Отдельно рассмотрен предельный случай, соответствующий модели Бозе-газа, когда кинетический коэффициент является чисто мнимым. В модели Бозе-газа поведение динамического подобия наблюдается в области малых частот /критическая мода находится в этой области/, граница которой пропорциональна волновому числу. Показано, что этот факт является следствием того, что в этой области можно найти фиксированную точку хорошего поведения.

KIVONAT

Megadjuk a végtelen rendparaméter-komponensü komplex, időfüggő Ginzburg-Landau-modell dinamikai renormálási csoport analizisét külön vizsgálva azt a határesetet, amikor a kinetikus együttható tisztán imaginárius, ami egy Bose-gáz modellnek felel meg. Kimutatjuk, hogy az a tény, miszerint a Bose-gáz modellben dinamikai skála-viselkedés egy kis-frekvenciás tartományban található (mely tartalmazza a kritikus módust), s melynek határvonala arányos a hullámszámmal, annak a következménye, hogy ebben a tartományban jól viselkedő fixpont érhető el.

## I. INTRODUCTION AND SUMMARY

The time-dependent Ginzburg-Landau (TDGL) model (or model A in the usual classification [1]) describing a simple relaxational dynamics has played an important role in understanding critical dynamics. Studying this model in the limit when the number of components of the order parameter goes to infinity has proved to be a useful theoretical laboratory in connection with several aspects of critical phenomena [2-9]. Our purpose in this paper is to further extend the investigations in this limit especially from the point of view of the properties of the dynamic renormalization group (DRG).

We consider the relaxational model for a complex, non-conserved, m-component order parameter field and take the kinetic coefficient L also complex. In this framework a classical field description for the m-component Bose system can be obtained ($m = 1$ corresponds to liquid $^4$He) by treating L as pure imaginary (see e.g. [1]). It is important, however, that the model with $\mathrm{Re}\,L = 0$ and those for which $\mathrm{Re}\,L \neq 0$ do not belong to the same universality class and exhibit quite different critical dynamics. The reason behind this lies in the fact that in the former case there exist a number of conserved quantities in the system and this is not the case if $\mathrm{Re}\,L \neq 0$. While this statement holds independently from whether m is finite or we have $1/m = 0$, the behaviour changes appreciably in this limiting case, especially when $\mathrm{Re}\,L = 0$. Although we are interested here in the properties of the $1/m = 0$ systems, it is worth summarizing also the main features characterizing the situation for finite m for the sake of comparison. Let us consider first the case $\mathrm{Re}\,L \neq 0$, $1/m \neq 0$. Then by the repeated application of the DRG a fixed point is reached with a zero fixed point value for $\mathrm{Im}\,L$ [10] and the model exhibits the same critical behaviour as model A of Halperin, Hohenberg and Ma [11]. On the other hand when $\mathrm{Re}\,L = 0$, $1/m \neq 0$, it is expected that by integrating out the field variables with large wave-numbers one arrives at phenomenological models with mode-coupling terms among the order parameter and the densities of the conserved quantities.*

*Although this procedure has been demonstrated in detail only in case of a lattice dynamical model by Bausch and Halpering [12] the conclusion as sketched above can be expected to apply also here [1, 13].

In the case of the multicomponent Bose system ($m \geq 2$) examples for such conserved quantities are the infinitesimal generators of the symmetry group U(m) [14].

Let us turn now to the case $1/m = 0$. Taking Re $L \neq 0$ the only difference as contrasted with the situation with finite m is that Im L becomes a marginal parameter so the temporal oscillations during the relaxational process are present also in the critical region. When L is treated as purely imaginary, however, the dynamic critical behaviour changes in a significant way in comparison with the finite-m case. The reason behind this dissimilarity lies in the fact that the collision-dominated hydrodynamic region (and its continuation into the critical one) shrinks to zero for $1/m = 0$ [15].(Note that we consider wave-numbers and frequencies of order unity and not of $\mathcal{O}(1/m)$ as in refs. [15, 16, 17].) Therefore one must realize that no information concerning the critical behaviour taking place just in this collision dominated region of an m-component Bose system with finite m can be deduced from the results obtained in the limit $1/m = 0$. In spite of this fact, however, the $1/m = 0$, Re $L = 0$ model deserves attention, having interesting features from several points of view including the unusualy properties of the DRG. We shall call this model the Bose-gas model in the following.

The Bose gas model can be obtained also by starting from the quantum-mechanical Hamiltonian for an m-component Bose gas and taking the limit $m \to \infty$ as well as the classical (small frequency) limit of the Matsubara diagrams. In this way critical properties, including the dynamical ones, of the model have been determined in refs [18, 19]. The most notable feature found there is that the longitudinal correlation function obeys dynamical scaling only in that frequency region where the critical mode lies. This region terminates for a given wave number near the frequency of a second, non-critical excitation branch proportional to the first power of the wave-number. Similar conclusion applies for the four-point correlation function below as well as above the critical temperature.

In this paper we are going to present the RG background of this behaviour. For this purpose besides the Bose gas model (Re $L = 0$) the case Re $L \neq 0$ will also be discussed since comparison of their corresponding features helps elucidating the problem in question.

Similarly to the model with a purely real kinetic coefficient studied previously [7, 8, 9] it turns out that the couplings local in space and time form an invariant subset of the parameter space and their transformation under DRG can be followed in a global way applying the path probability formalism. In this manner the transformations and the fixed point values of an infinite number of static and dynamic parameters are given. It is shown that the fixed point values of the dynamic parameters tend to infinity when taking the Bose gas limit (Re L = 0). This indicates at least some restrictions on the dynamical scaling properties in the model in agreement with the results of the direct solution [18, 19] discussed above. To clarify the nature of this singularity we study next with the help of a perturbation method the transformation of some frequency- and wave-number-dependent couplings. It is found that due to the singular nature of the bubble diagram for small frequency and wave-number transfer in the limit Re L = 0 a finite well-behaved fixed point function can be achieved only in the region $|\omega|\Lambda^{-2}/\mathrm{Im}\, L \ll k\Lambda^{-1}$ where $\omega$ and k denote the frequency and wave-number, respectively and $\Lambda$ stands for the cut-off in momentum space. Then it follows that dynamical scaling is also restricted to the above region in the framework of the Bose gas model, a result in accord with that of refs [18, 19].

From the point of view of the RG technique it is of importance that such singular behaviour does not prevent its application and it can correctly account for the consequences of the singular behaviour. It would be interesting to see how the field theoretic technique based on the Callan-Symanzik equation could describe such a behaviour. For this purpose it would be necessary to extend investigations made for the two-point function in ref. [10] to the four-point one in the 1/m = 0 limit.

The organization of the paper is as follows. In Section II the model is introduced. The limit 1/m = 0 is discussed in Section III along with the presentation of a number of explicit results. The RG procedure is outlined in Section IV The transformation and the fixed point values of the parameters local in space and time are given in Section V using the path probability formalism. Section VI contains the RG treatment of frequency and wave-number-dependent couplings and the discussion of the dynamic scaling properties of the Bose gas model. Some of the mathematical details are relegated to the Appendix.

## II. THE MODEL

We consider a d-dimensional system of volume unity which in the critical regime can be characterized by an m-component complex slowly varying order parameter field: $\Phi(x,t) = \{\phi_j(x,t) \mid j = 1,2,\dots,m\}$ with momentum cut-off $\Lambda$. We shall use the notation for complex numbers z: Re $z = z^{(1)}$, Im $z = z^{(2)}$, i.e. $\phi_j = \phi_j^{(1)} + i\phi_j^{(2)}$. The dynamics of the system is specified by a TDGL equation with a complex kinetic coefficient L. Only the case of non-conserved order parameter will be considered. In coordinate representation the equation of motion reads

$$\dot{\phi}_j(x,t) = -L(-a\nabla^2 + r(|\phi|^2))\phi_j(x,t) + \xi_j(x,t), \tag{2.1}$$

$$|\phi|^2 \equiv \frac{1}{2}\sum_{l=1}^{m} |\phi_l|^2, \tag{2.2}$$

where r stands for a real function and a is a real number. The complex noise $\xi$ is assumed to be a Gaussian white noise with zero mean value and correlation functions as

$$\begin{aligned} &\langle \xi_j^{(1)}(x,t)\xi_{j'}^{(1)}(x',t')\rangle = 2L^{(1)}\delta(x-x')\delta(t-t')\delta_{j,j'}, \\ &\langle \xi_j^{(2)}(x,t)\xi_{j'}^{(2)}(x',t')\rangle = 2L^{(1)}\delta(x-x')\delta(t-t')\delta_{j,j'}. \end{aligned} \tag{2.3}$$

$\xi_j^{(1)}$ and $\xi_j^{(2)}$ are independent random variables. The parameters specifying r can be defined for example by the power series

$$r(|\phi|^2) = \sum_{\alpha=1}^{\infty} u_{2\alpha}(2|\phi|^2)^{\alpha-1}. \tag{2.4}$$

It is assumed that

$$u_2 = T - T_0, \tag{2.5}$$

where T is the temperature of the system, while $T_0$ denotes its critical value in the mean-field approximation. To keep terms of powers up to infinity in (2.4) is required by the RG treatment (see Section V.).

The stationary probability distribution of the process (2.1) reached for $t \to \infty$, $P_{eq}\{\phi_j^{(1)},\phi_j^{(2)}\}$, can be written as

$$P_{eq}\{\phi_j^{(1)},\phi_j^{(2)}\} \propto \exp(-F), \tag{2.6}$$

where F is given by the Ginzburg-Landau form

$$F = \int d^d x \left\{ a|\nabla\phi|^2 + U(|\phi|^2) \right\} \tag{2.7}$$

with

$$|\nabla\phi|^2 \equiv \frac{1}{2} \sum_{l=1}^{m} |\nabla\phi_l|^2 \tag{2.8}$$

and

$$U(|\phi|^2) = \int_0^{|\phi|^2} r(x)dx = \sum_{\alpha=1}^{\infty} \frac{u_{2\alpha}}{2\alpha}(2|\phi|^2)^{\alpha}. \tag{2.9}$$

Thus F is interpreted as the free energy functional of the system in units of kT. It can easily be seen that (2.6)-(2.9) represent indeed the time-independent solution of the Fokker-Planck equation associated to the process (2.1):

$$\dot{P} = \sum_{j=1}^{m} \int d^d x \left[ \frac{\delta}{\delta\phi_j^{(1)}} \left\{ (L(-a\Delta\phi_j + r(|\phi|^2)\phi_j))^{(1)} P \right\} + \right.$$

$$\left. + \frac{\delta}{\delta\phi_j^{(2)}} \left\{ (L(-a\Delta\phi_j + r(|\phi|^2)\phi_j))^{(2)} P \right\} + L^{(1)} \left\{ \frac{\delta^2 P}{\delta\phi_j^{(1)2}} + \frac{\delta^2 P}{\delta\phi_j^{(2)2}} \right\} \right]. \tag{2.10}$$

The large-m case is defined by taking the limit $m \to \infty$. In order to ensure the existence of this limit (e.g. in (2.9)) we assume $u_{2\alpha}$ to be of order $m^{1-\alpha}$. For the dimensionality of the system $2 < d < 4$ w . be assumed.

In the next Section we present the solution of (2.1) with an arbitrary function $r(|\phi|^2)$ in the large-m limit.

## III. SOLUTION FOR $1/m = 0$

### Symmetric phase

The simplifying feature of the large-m limit appears in that, since m is large and $|\phi|^2$ is a sum of m terms, that the relative fluctuations of $|\phi|^2$ are small [4]. Therefore $r(|\phi|^2)$ in (2.1) can be replaced by $r(N)$, where N denotes the average value of $|\phi|^2$ in equilibrium. Thus we arrive at a linear equation of motion, which in terms of the Fourier components $\phi_{j,k}(t)$ and $\xi_{j,k}(t)$ reads:

$$\dot{\phi}_{j,k}(t) = -L\alpha_k \phi_{j,k}(t) + \xi_{j,k}(t) \tag{3.1}$$

with

$$\alpha_k = (ak^2 + r(N)) \tag{3.2}$$

where N is to be calculated self-consistently. In this Section we shall set a = 1 (see also Section V.).

One can easily check that the transition probability of this linear process is given as follows:

$$P(\{\phi_{j,k}\}, t|\{\phi^o_{j,k}\}, 0) \propto \propto \exp\left\{- \frac{\sum_{k,j} \alpha_k |\phi_{j,k} - \phi^o_{j,k} e^{-L\alpha_k t}|^2}{2(1 - e^{-2L^{(1)}\alpha_k t})}\right\}. \tag{3.3}$$

Consequently the average of $\phi_{j,k}$ reads

$$\langle\phi_{j,k}(t)\rangle = \phi^o_{j,k}\, e^{-L^{(1)}\alpha_k t}\, e^{-iL^{(2)}\alpha_k t} \tag{3.4}$$

which exhibits a temporal oscillation with frequency $L^{(2)}\alpha_k$ during its relaxation to the equilibrium value.

The equilibrium distribution is generated by taking the limit $t \to \infty$ of expression (3.3):

$$P_{eq}(\{\phi_{j,k}\}) \propto \exp\{- \frac{1}{2} \sum_{k,j} \alpha_k |\phi_{j,k}|^2\} \tag{3.5}$$

and is of course, independent of L. As a consequence of (3.5) and (3.2) the equal time correlation function in equilibrium is obtained as

$$\langle|\phi_{j,k}|^2\rangle = \frac{2}{k^2 + r(N)}. \tag{3.6}$$

For the (time dependent) correlation function in the equilibrium state one gets

$$C(k,t) \equiv \langle \bar{\Phi}_{j,k}(t)\Phi_{j,k}(0)\rangle = \frac{2}{k^2+r(N)}\begin{cases} \exp(-\bar{L}\alpha_k t), t>0, \\ \exp(L\alpha_k t), t<0, \end{cases} \tag{3.7}$$

where and in the following complex conugation is denoted by a bar. Note the oscillatory behaviour of C(k,t) too. From (3.6) the self-consistency equation

$$N = \frac{1}{2}\sum_{j,k} \langle |\Phi_{j,k}|^2\rangle = m\sum_k \frac{1}{k^2+r(N)} = m\int_0^{\Lambda} \frac{1}{k^2+r(N)}\,\frac{d^dk}{(2\pi)^d} \tag{3.8}$$

is found. At the critical point the relaxation rate of $\Phi_{j,o}$ vanishes. Consequently, if we denote $r(|\Phi|^2)$ and N at the critical point by $r_c(|\Phi|^2)$ and $N_c$, respectively, it follows from (3.1) and (3.2) that $r_c(N_c) = 0$ should be fulfilled. This makes straightforward to calculate $N_c$ from (3.8) [4]:

$$N_c = m\,\frac{K_d\Lambda^{d-2}}{d-2}, \tag{3.9}$$

where $K_d(2\pi)^d$ is the area of the d-dimensional unit sphere. The condition $r_c(N_c) = 0$ together with (2.4), (2.5) determines the critical temperature:

$$T_c = T_o - \sum_{\alpha=2}^{\infty} u_{2\alpha}(2N_c)^{\alpha-1}. \tag{3.10}$$

The solution of the self-consistency equation (3.6) for T close to $T_c$ is given as

$$N - N_c = \frac{T_c - T}{v(N_c)} \tag{3.11}$$

where the notation

$$v(|\Phi|^2) \equiv \frac{dr(|\Phi|^2)}{d|\Phi|^2} \tag{3.12}$$

has been introduced. The static critical behaviour of the system is the same as that of the usual TDGL model in the spherical model limit, therefore the critical indices $\nu$ and $\eta$ are 0 and 1/(d-2), respectively [4], while the dynamic exponent z turns out to be 2 for any $L^{(1)}$ as it follows from (3.7).

Symmetry-breaking phase

The order parameter of the system is the average of the filed variable, in general a complex m-component vector. However, we can always choose the order parameter to point in the direction of the j=1 axis and to be real by making use of the isotropy of the system in the component space and the gauge invariance of the free energy (2.7), respectively. We introduce a constant external field, h, coupled to the j=1 component. Then the equation of motion is as follows

$$\dot{\phi}_j = -L(-a\nabla^2 + r(|\phi|^2))\phi_j + Lh\delta_{j,1} + \xi_j. \tag{3.13}$$

We separate the order parameter $M \equiv \langle\phi_1(x,t)\rangle$ by writting

$$\phi_j(x,t) = \phi'_j(x,t) + M\delta_{j,1}. \tag{3.14}$$

It will turn out that M is of order $m^{1/2}$, therefore when calculating $|\phi|^2$, defined by (2.2), the term $M(\phi'_1 + \overline{\phi}'_1)$ can be neglected as compared to terms of order m, and thus we can use the approximate relation

$$|\phi|^2 = |\phi'|^2 + M^2/2. \tag{3.15}$$

If N' denotes the average value of $|\phi'|^2$, it follows from (3.15) that the average value of $|\phi|^2$ is given as

$$N = N' + M^2/2. \tag{3.16}$$

Finally we use the fact, that $|\phi'|^2$ can be replaced by N' in the large-m limit. After these steps we arrive at an equation of motion of $\phi'_j$ for components $j \geq 2$, the Fourier transform of which is of the same form as (3.1), (3.2) but now N is defined by (3.16). Then similarly as in the previous subsection we obtain the self-consistency equation:

$$N' = m \int_o^{\Lambda} \frac{1}{k^2 + r(N' + M^2/2)} \frac{d^dk}{(2\pi)^d}. \tag{3.17}$$

Furthermore from the equation of miton of $\phi'_1$ the following condition is found for a stationary solution

$$r(N' + M^2/2) = h/M \tag{3.18}$$

which is the equation of state of the system. It is easy to check that (3.17) and (3.18) in the critical region yield the exponents $\beta = 1/2$, $\delta = (d+2)/(d-2)$ known for the spherical model [4].

As a consequence of (3.18) one obtains for the correlation function of $\Phi'_{j,k}$ $(j \geq 2)$ in the equilibrium state:

$$\langle \Phi'_{j,k}(t)\Phi'_{j,k}(0)\rangle = \frac{2}{k^2+h/M}\begin{cases} \exp(-\bar{L}(k^2+h/M)t), & t>0, \\ \exp(L(k^2+h/M)t), & t>0. \end{cases} \tag{3.19}$$

Also (3.19) exhibits an oscillatory behaviour. The singular nature of (3.19) when k and h/M go to zero is a manifestation of the Goldstone theorem.

The evaluation of the quantities characterizing the longitudinal (j=1) component is much more complicated. For convenience we shall discuss only the properties of the response function

$$G(k,\omega) = \frac{\langle \Phi_{1,k,\omega}\rangle}{h_{k,\omega}}, \tag{3.20}$$

where $h_{k,\omega}$ is the Fourier componetn of the real external field h(x,t) coupled to the j=1 component. It has been calculated for the Bose gas model in [18,19] and for the TDGL model with a complex kinetic coefficient in [20]. Here we quote the result taking into account the slight modifications due to the fact that our model (see (2.1)) contains an infinite power series in $|\Phi|^2$. We obtain

$$G(k,\omega) = \frac{(-i\omega/\bar{L}+k^2)(1+mv(N)\pi(k,\omega))}{(-i\omega/\bar{L}+k^2)(-i\omega/L+k^2)(1+mv(N)\pi(k,\omega)) - (i\omega L^{(1)}/|L|^2 - k^2)v(N)M^2} \tag{3.21}$$

where N and the function v have been defined by (3.16) and (3.12), respectively, furthermore $\pi(k,\omega)$, the contribution of the bubble diagram for small k values, is given as [20]:

$$\pi(k,\omega) = k^{-\varepsilon}K_d B(\varepsilon/2,d/2)/(2-\varepsilon)\Big\{\frac{L}{2L^{(1)}}\Big(\frac{L-i\omega/k^2}{2L^{(1)}}\Big)^{2-\frac{\varepsilon}{2}} F\Big(\frac{\varepsilon}{2},1-\frac{\varepsilon}{2},2-\frac{\varepsilon}{2},\frac{L}{2L^{(1)}(1-i\omega/(k^2L))}\Big) -$$

$$-\frac{\bar{L}}{2L^{(1)}}\Big(\frac{\bar{L}-i\omega/k^2}{2L^{(1)}}\Big)^{2-\frac{\varepsilon}{2}} F\Big(\frac{\varepsilon}{2},1-\frac{\varepsilon}{2},2-\frac{\varepsilon}{2},\frac{\bar{L}}{2L^{(1)}(1-i\omega/(k^2L))}\Big)\Big\}, \tag{3.22}$$

where B and F denote the beta and the hypergeometric functions, respectively; $\varepsilon \equiv 4-d$ and $K_d(2\pi)^d$ is the area of the d-dimensional unit sphere. In the limit $L^{(1)} \to 0$ it goes over to [19]

$$\pi(k,\omega) = k^{-\varepsilon}4^{1-d}\pi^{\frac{1-d}{2}}\Gamma\Big(\frac{3-d}{2}\Big)\frac{e^{-i\pi(d-3)}}{\sin(d-3)\pi}^{-1}\Big\{\Big(\frac{\omega}{L^{(2)}k^2}+1\Big)^{d-3} - \Big(\frac{\omega}{L^{(2)}k^2}-1\Big)^{d-3}\Big\}. \tag{3.23}$$

Investigating the poles of G in the Bose gas model it was found in [18,19] that the critical modes beeing proportional to $k^2$ comes from the region where $mv(N)\pi(k,\omega)$ is much more larger than unity. A non-critical mode has also been found, the frequency of which is proportional to k. Whether it can be interpreted as a sound excitation or an overdamped mode depends on the strength of the bare coupling constant. It turned out that the response function $G(k,\omega)$ obeyed dynamical scaling around the excitation branch of the critical mode in a region between the $\omega = 0$ line and the excitation branch of the non-critical mode, that is besides the usual conditions $|\omega\Lambda^{-2}/L| << 1$, $k\Lambda^{-1} << 1$ also $|\omega|\Lambda^{-2} << L^{(2)}_{(1)}k\Lambda^{-1}$ should be fulfilled. On the contrary, in models with finite values of $L^{(1)}$ there is no restriction on the ratio of the frequency and the wave number in the asymptotic region for dynamical scaling to hold.

Before turning to the renormalization group analysis we note that instead of (2.4) an other representation of $r(|\phi|^2)$ turns out to be more convenient in the large-m limit, namely the power series

$$r(|\phi|^2) = \sum_{\alpha=1}^{\infty} U_{2\alpha,2}[2(|\phi|^2 - N_c)]^{\alpha-1}, \tag{3.24}$$

where $N_c$ is given by (3.9). The introduction of the second index of $U_{2\alpha,2}$ is here entirely formal, but in the next Section we shall introduce an even broader parameter space the elements of which will be the parameters $U_{2\alpha,2\beta}$ $(\beta \gtrsim 1)$ (see (4.8)). The present notation indicates the relation of the set of parameters defined by (3.24) to the broader space. The connection between $U_{2\alpha,2}$ and the parameters specified by (2.4) is given by

$$U_{2\alpha,2} = u_{2\alpha} + \sum_{\beta=1}^{\infty} u_{2(\alpha+\beta)} \binom{\alpha+\beta-1}{\beta} (2N_c)^{\beta}, \tag{3.25}$$

as can be checked easily. In the language of a digaram technique this means, of course, an appropriate resummation of diagrams. For $\alpha = 1$ one obtains

$$U_{2,2} = u_2 + \sum_{\beta=2}^{\infty} u_{2\beta}(2N_c)^{\beta-1} = T - T_c \tag{3.26}$$

where the second equality follows from (2.5) and (3.10).

## IV. RENORMALIZATION GROUP (RG) PROCEDURE

In order to describe the dynamic renormalization group it is convenient to use the response field formalism [21-24, 7-9] and then the transformation is to be carried out on the path probability functional W=exp J. For the action associated to equation (2.1) we obtain in the large-m limit

$$J = J_o + \int dt \int d^d x \left[ \sum_{j=1}^{m} \tfrac{1}{2}(L(-\tilde{\Phi}_j \phi_j + K) + c.c.) r(|\phi|^2) \right]. \tag{4.1}$$

where c.c. denotes complex conjugation and $\tilde{\Phi}_j$ represents the m-component complex response field, furthermore

$$J_o = \int dt \int d^d x \left[ \sum_{j=1}^{m} \{ L^{(1)} |\tilde{\Phi}_j|^2 - \tfrac{1}{2}(\tilde{\Phi}_j(\dot{\phi}_j - aL\nabla^2 \phi_j) + c.c.) \} \right], \tag{4.2}$$

and

$$K \equiv K_d \int_0^{\Lambda} k^{d-1} dk. \tag{4.3}$$

When calculating averages by means of the path probability $W\{\tilde{\Phi}_j, \phi_j\}$ integration is to be performed over $\phi_j^{(1)}$, $\phi_j^{(2)}$ and $i\tilde{\Phi}_j^{(1)}$ and $i\tilde{\Phi}_j^{(2)}$. Note that the dependence upon $\tilde{\Phi}_j$ in $J-J_o$ appears only through the (real) combination

$$\varphi(x,t) = \frac{1}{2} \sum_{j=1}^{m} (L(-\tilde{\Phi}_j \phi_j + K) + c.c.). \tag{4.4}$$

The dynamic RG transformation is defined by integrating the path probability over field variables with wave numbers in the shell $\Lambda/b < k < \Lambda$ and by rescaling of the remaining variables. The new action is determined by the equation.

$$\exp J' = \int \prod_{j, \frac{\Lambda}{b} < k < \Lambda, \omega} d\phi^{(1)}_{j,k\omega} d\phi^{(2)}_{j,k\omega} \, d(i\tilde{\Phi}^{(1)}_{j,k\omega}) d(i\tilde{\Phi}^{(2)}_{j,k\omega}) \tag{4.5}$$

$$\exp J \Big|_{\substack{\phi(x,t) \to b^{1-\eta/2-d/2} \phi(x/b, t/b^z) \\ \tilde{\Phi}(x,t) \to b^{-1+\eta/2-d/2} \tilde{\Phi}(x/b, t/b^z)}} .$$

Here the quantities with subscript $k,\omega$ stand for the Fourier components of the field variables. The quantities $\eta$ and $z$ are the static correlation function exponent and the dynamic critical exponent, respectively.

Before turning to explicit calculations let us discuss first the structure of the parameter space. If we start with (4.1), after the dynamic RG transformation an infinite number of new couplings arise in the new action which are non-local in space and time. The non-local ($k$ and $\omega$ dependent) couplings can be treated only by perturbative methods. However, the couplings which are local in space and time transform among themselves similarly as in the case of the usual TDGL model with a real kinetic coefficient [7,8]. Even this part of the parameter space contains an infinite number of parameters in the large-m limit. They are specified by the following action

$$J = J_o + \int dt \int d^dx \, L^{(1)} Y(|\phi|^2, \varphi/L^{(1)}) \tag{4.6}$$

where $Y$ is a real valued function and $J_o$ and $\varphi$ are defined by (4.2) and (4.4), respectively. Note that (4.6) corresponds to a general equation of motion the vertices of which are delta-correlated random variables with non-Gaussian distribution (see also [8]). We then define the parameters $U_{2\alpha,2\beta}$ associated to $Y$ by the power series

$$L^{(1)} Y(|\phi|^2), \varphi/L^{(1)} = \sum_{\alpha=1}^{\infty} \sum_{1\leq\beta\leq\alpha} U_{2\alpha,2\beta} \varphi^{\beta} [2(|\phi|^2 - N_c)]^{\alpha-\beta} \quad . \tag{4.7}$$

Conversely $U_{2\alpha,2\beta}$ is given as

$$U_{2\alpha,2\beta} = \frac{1}{(L^{(1)})^{\beta-1}} \frac{1}{\alpha! 2^{\alpha-\beta}} \binom{\alpha}{\beta} Y_{\alpha-\beta,\beta}(N_c, 0), \tag{4.8}$$

where the notation

$$Y_{ij}(z_1, z_2) \equiv \frac{\partial^{i+j} Y(z_1, z_2)}{\partial z_1{}^i \partial z_2{}^j} \tag{4.9}$$

has been introduced. The physical significance of the parameters $U_{2\alpha,2\beta}$ is given by the fact, that, similarly as in the case of the large-m limit of the usual TDGL model [8] $U_{2\alpha,2\beta}$ for $\beta > 1$ are those parameters which are directly related to the cumulants of random vertices appearing in the equation of motion. Some of the parameters $U_{2\alpha,2\beta}$ are related to frequency dependent ones via fluctuation-dissipation theorems in the parameter space (see for example (6.3)). It corresponds to the usual Langevin equation if one takes

$$Y(|\phi|^2, \varphi/L^{(1)}) = (\varphi/L^{(1)}) r(|\phi|^2) \tag{4.10}$$

as can be seen from (4.1).

The transformation of the local parameters specified by (3.6) can be treated in a global way using the simplifying features of the large-m limit. In the next Section we discuss this non-perturbative method.

## V. TRANSFORMATION OF THE PARAMETERS LOCAL IN SPACE AND TIME

In order to perform the multiple integral (4.5) the fields are decomposed into two parts

$$\Phi_j \to \Phi_j + \hat{\Phi}_j , \qquad \tilde{\Phi}_j \to \tilde{\Phi}_j + \hat{\tilde{\Phi}}_j , \tag{5.1}$$

where $\Phi_j$ and $\tilde{\Phi}_j$ on the right hand sides involve only wave numbers smaller tahn $\Lambda/b$, while $\hat{\Phi}_j$ and $\hat{\tilde{\Phi}}_j$ contain the large wave number components. In the large-m limit cross terms like $\sum_j \tilde{\Phi}_j \hat{\Phi}_j$ are negligible as compared to $\sum_j \tilde{\Phi}_j \Phi_j$. Consequently we can write

$$|\Phi|^2 \to |\Phi|^2 + |\hat{\Phi}|^2 , \qquad \varphi \to \varphi + \hat{\varphi} . \tag{5.2}$$

Since $|\hat{\Phi}|^2$ and $\hat{\varphi}$ are sums of m terms and m is large the relative deviations of them from $\langle|\hat{\Phi}|^2\rangle_b$ and $\langle\hat{\varphi}\rangle_b$, respectively are small, where $\langle\ldots\rangle_b$ denotes the average over field variables with wave numbers between $\Lambda/b$ and $\Lambda$. Thus $Y(|\Phi|^2 + |\hat{\Phi}|^2, \varphi + \hat{\varphi})$ in (4.5) can be replaced by the first few terms of its Taylor series expanded in powers of $\hat{\varphi} - \langle\hat{\varphi}\rangle_b$ and $|\hat{\Phi}|^2 - \langle|\hat{\Phi}|^2\rangle_b$ reducing the multiple integral to Gaussian integrations. The calculation is a straightforward generalization of that followed in [7,8] therefore we shall skip the intermediate steps and turn directly to the recursions. We obtain

$$a' = ab^{-\eta} , \qquad L' = Lb^{z-2+\eta} \tag{5.3}$$

indicating that a finite fixed point can be achieved only if

$$\eta = 0 , \qquad z = 2. \tag{5.4}$$

This means that both a, $L^{(1)}$ and $L^{(2)}$ are marginal parameters. Therefore we can put a = 1 but we do not fix the value of $L^{(1)}$ since the limit $L^{(1)} \to 0$ will be interesting for us.

Furthermore $Y(|\Phi|^2, 0)$ can be proved to be a constant after the transformation which plays no role and will not be regarded as a parameter. Thus $Y_{0,1}(|\Phi|^2, \varphi/L^{(1)})$ defined by (4.9) specifies all the parameters besides a and L. Its recursion couples to that of $Y_{1,0}$. Starting with (4.10) we find:

$$Y'_{0,1}(|\Phi|^2, \varphi/L^{(1)}) = b^2 r(b^{2-d} Q + N_c), \tag{5.5}$$

$$Y'_{1,0}(|\Phi|^2, \varphi/L^{(1)}) = b^{4-d} v(b^{2-d} Q + N_c), \tag{5.6}$$

where v and $N_c$ have been defined by (3.12) and (3.9), respectively, furthermore

$$R = \varphi/L^{(1)} - m \int\limits_q^{>} \{(q^2 + Y'_{0,1})/S - 1\}, \tag{5.7}$$

$$Q = |\phi|^2 - N_c + m \int\limits_q^{>} \{1/S - 1/q^2\}, \tag{5.8}$$

$$S = [(q^2 + Y'_{0,1})^2 - 2Y'_{1,0}]^{1/2}, \tag{5.9}$$

where the notation

$$\int\limits_q^{>} \equiv K_d \int\limits_{\Lambda}^{\Lambda b} dq\, q^{d-1} \tag{5.10}$$

has been introduced. It follows from (5.5), (5.6) that $Y'_{0,1}$ and $Y'_{1,0}$ can approach a finite fixed point expression for $b \to \infty$ only if Q and R tend to zero in this limit. Then the fixed point expressions $Y^*_{0,1}$, $Y^*_{1,0}$ are determined by

$$Q^* = 0 \qquad R^* = 0, \tag{5.11}$$

where $Q^*$ and $R^*$ are given by (5.8) and (5.7), respectively with the only change that $Y'_{0,1}$ and $Y'_{1,0}$ are to be replaced by $Y^*_{0,1}$ and $Y^*_{1,0}$. Furthermore from (5.6) a necessary condition for the existence of this fixed point is found, namely

$$r_c(N_c) \equiv U_{2,2} = 0, \tag{5.12}$$

i.e. $U_{2,2}$ must vanish at $T_c$ in accordance with the results of Seciton II.

Note that in the special case of a real L the relations (5.5)-(5.10) coincide with the recursions of the usual TDGL model with real order parameter in the many-component limit [7,8].

It is easy to deduce from (5.5)-(5.10) the recursions for the parameters $U_{2\alpha,2\beta}$ defined by (4.7). Here we give as examples the first few of them:

$$U'_{2,2} \equiv \Lambda^2 \tilde{U}'_{2,2} = b^2 r(I_b{}^{(1)}), \tag{5.13}$$

$$U'_{4,2} \equiv \Lambda^{4-d} \tilde{U}'_{4,2} = \frac{b^{4-d} v(I_b{}^{(1)})/2}{1 + b^{4-d} m v(I_b{}^{(1)}) I_b{}^{(2)}} \tag{5.14}$$

$$U'_{4,4} \equiv \Lambda^{2-d} \tilde{U}'_{4,4} \frac{1}{L^{(1)}} = \frac{2}{L^{(1)}} m\, I_b{}^{(3)} U'^{\,2}_{4,2}, \tag{5.15}$$

where $\tilde{U}'_{2,2}$, $\tilde{U}'_{4,2}$ and $\tilde{U}'_{4,4}$ are dimensionless parameters, the function v is defined by (3.12), and

$$I_b^{(1)} = b^{2-d} Q(N_c, 0) + N_c, \tag{5.16}$$

$$I_b^{(2)} = \int_q^{>} \frac{1}{(q^2 + U'_{2,2})^2}, \tag{5.17}$$

$$I_b^{(3)} = \int_q^{>} \frac{1}{(q^2 + U'_{2,2})^3}. \tag{5.18}$$

At $T_c$, of course, $I_b^{(1)} = N_c$ and $U'_{2,2} = 0$.

The fixed point values are generated by taking the limit $b \to \infty$ at $T_c$. For the above parameters we obtain

$$\tilde{U}^*_{2,2} = 0, \tag{5.19}$$

$$\tilde{U}^*_{4,2} = \frac{4-d}{2mK_d}, \tag{5.20}$$

$$\tilde{U}^*_{4,4} = \frac{(4-d)^2}{2mK_d(6-d)}. \tag{5.21}$$

An important feature of equations (5.5)-(5.10) is that at $\varphi = 0$ for any finite value of $L^{(1)}$ they describe the transformation of the static parameters. Indeed for $\varphi = 0$ the function $Y'_{1,0} \equiv 0$ and in fact for $Y'_{0,1}(|\phi|^2, 0)$ we recover the expression first obtained by Ma in the framework of static critical phenomena [4,5].

The limit, however, when $L^{(1)} \to 0$ must be taken with care. As one can see from (5.5)-(5.7) the static parameters of the system are recovered only if we take the limit $\varphi \to 0$ first and the limit $L^{(1)} \to 0$ only afterwards. At the same time the dynamic parameters $U_{2\alpha,2\beta}$ with $\beta > 1$, generated by the RG exhibit singular behaviour: they blow up for $L^{(1)} \to 0$. (See for example $U'_{4,4}$ given by (5.15).) This reflects a non-analytic behaviour of the model reached for $L^{(1)} \to 0$, i.e. of the Bose gas model. In the next Section we discuss also non-local couplings in space and time in order to get more insight into the nature and origin of this singular behaviour.

## VI. TRANSFORMATION OF PARAMETERS NON-LOCAL IN SPACE AND TIME

As we have mentioned previously the space and time (or equivalently k and $\omega$) dependences of the couplings generated by the RG transformation can be investigated only by means of perturbative methods. The perturbation evaluation of the multiple integral (4.5) is to be performed in a similar way as in the case of the usual TDGL model [8]. We do not discuss here the technical details but give the recursions for the two simplest k and $\omega$ dependent couplings, namely for $U'_{4,2}(k,\omega)$ and $U'_{4,4}(k,\omega)$. The results are plausible generalizations of (5.14) and (5.15). We obtain

$$U'_{4,2}(k,\omega) \equiv \Lambda^{4-d}\tilde{U}'_{4,2}(k,\omega) = \frac{b^{4-d}v(I_b^{(1)})/2}{1 + b^{4-d}mv(I_b^{(1)})I_b^{(2)}(k,\omega)} \tag{6.1}$$

where $I_b^{(1)}$ is given by (5.12) and

$$I_b^{(2)}(k,\omega) = \int_{\Lambda<q<\Lambda b} \frac{d^dq}{(2\pi)^d} \frac{1}{(q^2 + U'_{2,2})((\underset{\sim}{k}-\underset{\sim}{q})^2 + U'_{2,2})}$$

$$\frac{L(q^2 + U'_{2,2}) + \bar{L}((\underset{\sim}{k}-\underset{\sim}{q})^2 + U'_{2,2})}{-i\omega + L(q^2 + U'_{2,2}) + \bar{L}((\underset{\sim}{k}-\underset{\sim}{q})^2 + U'_{2,2})} \; . \tag{6.2}$$

The variables k and $\omega$ in (6.1) and (6.2) denote the rescaled wave number and frequency, respectively. The meaning of the quantities $I_b^{(1)}$ and $I_b^{(2)}$ can be given in the language of diagrams as follows: $I_b^{(1)} - b^{2-d}N_c$ and $b^{4-d}\,I_b^{(2)}$ denote the contributions of the Hartree loop and the bubble graph, respectively, in which the integration over wave-numbers runs in the interval $(\Lambda/b,\Lambda)$.

The coupling $U'_{4,4}(k,\omega)$ can be expressed by the formula

$$U'_{4,4}(k,\omega) = -\frac{2}{\omega}\,\mathrm{Im}U'_{4,2}(k,\omega) \tag{6.3}$$

representing a fluctuation-dissipation theorem. Thus $U'_{4,2}(k,\omega)$ determines $U'_{4,4}(k,\omega)$ uniquely.

In order to calculate $U'_{4,2}(k,\omega)$ one must know $I_b^{(2)}(k,\omega)$. After performing the integration over the angle variables in (5.2) we can write

$$I_b^{(2)}(k,\omega) = I_b^{(S)}(k) + I_b^{(D)}(k,\omega), \tag{6.4}$$

where $I_b^{(S)}(k)$ represents the contribution of the static bubble diagram (and consequently does not depend on L), while the dynamic part reads

$$I_b^{(D)}(k,\omega) = i\omega \int_q^{>} f\left(\frac{q}{k}, \frac{k^2}{U'_{2,2}}, s\right) q^{-6} . \tag{6.5}$$

Here the notation (5.10) has been used, $U'_{2,2}$ is given by (5.12) and f stands for an infinite series of hypergeometric functions. Explicit expressions are given in the Appendix. It will play a crucial role in determining the analytic properties of the function $I_b^{(2)}(k,\omega)$, that f depends on the frequency and the kinetic coefficient only through the combination:

$$s = \frac{2kq\bar{L}}{i\omega - 2L^{(1)}(q^2 + U'_{2,2}) - \bar{L}k^2} . \tag{6.6}$$

As far as $L^{(1)}$ is finite s is a small quantity near the origin of the k, $\omega$ plane and as a consequence of it (6.1) can be expanded in a power series as

$$\tilde{U}'_{4,2}(k,\omega) = \tilde{U}'_{4,2} + \tilde{C}'_{1,0}(k^2/\Lambda^2) + \tilde{C}'_{0,1}(i\omega/\Lambda^2 L^{(1)}) + \ldots , \tag{6.7}$$

where $\tilde{U}'_{4,2} \equiv \tilde{U}'_{4,2}(0,0)$ is given by (5.13) and the dimensionless quantities $\tilde{C}'_{1,0}, \tilde{C}'_{0,1}$ read

$$\tilde{C}'_{1,0} = 2m(\tilde{U}'_{4,2})^2 \int_1^b \frac{K_d q^{d-1}}{(q^2 + \tilde{U}'_{2,2})^3}\left(1 - \frac{4q^2}{d(q^2 + \tilde{U}'_{2,2})}\right) dq \tag{6.8}$$

and

$$\tilde{C}'_{0,1} = -\tfrac{1}{2}\tilde{U}'_{4,4} . \tag{6.9}$$

Equation (6.9) follows from the fluctuation-dissipation theorem (6.3). $\tilde{U}'_{2,2}$ and $\tilde{U}'_{4,4}$ have been determined by (5.12) and (5.14), respectively. Note that $L^{(2)}$ does not appear in (6.7)-(6.9).

In the limit $L^{(1)} \to 0$ s becomes a singular function of k and

$$\lim_{L^{(1)} \to 0} s = \frac{-2L^{(2)}k}{\omega + L^{(2)}k^2} q , \tag{6.10}$$

whose value at the origin depends on whether k or $\omega$ goes first to zero. This causes a similar singularity in $U'_{4,2}(k,\omega)$, namely $\lim_{L^{(1)} \to 0} U'_{4,2}(0,\omega)$ is increasing as $b \to \infty$, while $U'_{4,2}(k,0)$, which is a static quantity (and therefore independent of $L^{(1)}$) approaches a finite expression for $b \to \infty$. All that indicates that the constant part of the four-point vertex can not be defined uniquely when $L^{(1)} \to 0$ in contrast to the case of a finite $L^{(1)}$. (In some sense this fact is reflected in the singular behaviour we have found in the transformation of the parameters being local in space and time: there the limit $\varphi \to 0$ and $L^{(1)} \to 0$ were not interchangeable.)

A more detailed investigation of $U'_{4,2}(k,\omega)$ in the limit $L^{(1)} \to 0$ shows that we must distinguish between two regions where the properties of this function are entirely different. Region I corresponds to $|\omega|\Lambda^{-2} < L^{(2)}k\Lambda^{-1}$, while region II corresponds to $|\omega|\Lambda^{-2} > L^{(2)}k\Lambda^{-1}$. In contrast to the unusual features of the transformation in the latter regime, $U'_{4,2}(k,\omega)$ approaches a finite fixed point in region I. Here it can be expanded in powers of $k^2/\Lambda$, $\omega/\Lambda^2L^{(2)}$ and $(\omega/\Lambda^2L^{(2)})/(k/\Lambda)$. For the dimensionless coupling $\tilde{U}'_{4,2}(k,\omega)$ one obtains in region I:

$$\tilde{U}'_{4,2}(k,\omega) = \tilde{U}'^{(I)}_{4,2} + \tilde{C}'^{(I)}_{1,0}(k^2/\Lambda^2) + \tilde{C}'^{(I)}_{0,1}(\omega/\Lambda^2L^{(2)}) + \\ + \tilde{C}'^{(I)}_{-\frac{1}{2},1}\, i(\omega/\Lambda^2L^{(2)})/(k/\Lambda) + \ldots \tag{6.11}$$

where $\tilde{U}'^{(I)}_{4,2}$ and $\tilde{C}'^{(I)}_{1,0}$ coincide with the corresponding coefficients of the case of a finite $L^{(1)}$, i.e.

$$\tilde{U}'^{(I)}_{4,2} = \tilde{U}'_{4,2}, \qquad \tilde{C}'^{(I)}_{1,0} = \tilde{C}'_{1,0}, \tag{6.12}$$

where $\tilde{U}'_{4,2}$ and $\tilde{C}'_{1,0}$ are defined by (5.13) and (6.8), respectively, while $\tilde{C}'^{(I)}_{0,1}$ and $\tilde{C}'^{(I)}_{-\frac{1}{2},1}$ are

$$\tilde{C}'^{(I)}_{0,1} = 2m(\tilde{U}'_{4,2})^2 \int_1^b \frac{K_d q^{d-1}}{(q^2+\tilde{U}'_{2,2})^2}\left(\frac{d-2}{4}\frac{1}{q^2} - \frac{1}{q^2+\tilde{U}'_{2,2}}\right)dq\,, \tag{6.13}$$

$$\tilde{C}'^{(I)}_{-\frac{1}{2},1} = -\frac{m}{2}(\tilde{U}'_{4,2})^2 \int_1^b \frac{K_{d-1}q^{d-2}}{(q^2+\tilde{U}'_{2,2})^2}\,dq\,. \tag{6.14}$$

A novel feature of (6.11) lies in the presence of the third term, a trace of the singular behaviour, giving, however, only a small contribution in region I. Note also that $\tilde{C}'^{(I)}_{0,1}$ is a real number as it is expected in the Bose gas model.

The fluctuation-dissipation (6.3) remains valid also in the limit $L^{(1)} \to 0$, consequently we obtian in the region I:

$$U'_{4,4}(k,\omega) = -(\Lambda^{2-d}/L^{(2)})\,2(\Lambda/k)\tilde{C}'^{(I)}_{-\frac{1}{2},1} \tag{6.15}$$

indicating that the $k \to 0$ limit of $U'_{4,4}(k,0)$ is singular in accordance with our previous finding when having treated the parameters local in space and time. Then $U'_{4,4} \equiv U'_{4,4}(0,0)$ turned out to be proportional to $1/L^{(1)}$ (see (5.13)) which was also singular for $L^{(1)} \to 0$.

All parameters defined by (6.11) have finite fixed point values, for example:

$$\tilde{C}^{*\,(I)}_{1,0} = \tilde{C}^{*}_{1,0} = -\frac{(4-d)^3}{2mK_d d(6-d)}\,, \tag{6.16}$$

$$\tilde{C}^{*\,(I)}_{0,1} = -\frac{(4-d)^3}{8mK_d}\,, \tag{6.17}$$

$$\tilde{C}^{*\,(I)}_{-\frac{1}{2},1} = -\frac{(4-d)^2 K_{d-1}}{8mK_d^2(5-d)}\,, \tag{6.18}$$

and the deviations from them turn out to be small for large values of b. This means that the recursion of the parameters associated to region I can be linearized and consequently dynamic scaling is fulfilled in this region.

The result can be interpreted as the RG background of the characteristic features of the explicit solution of the Bose gas model found in[18,19] and discussed also at the end of Section III in the present paper.

## APPENDIX

## EVALUATION OF THE DYNAMIC PART OF THE BUBBLE DIAGRAM

The dynamic part of the bubble diagram is given by

$$I_b^{(D)}(k,\omega) =$$

$$= \int_{\Lambda<q<\Lambda b} i\omega\left[(q^2+U'_{2,2})((\underset{\sim}{k}-\underset{\sim}{q})^2+U'_{2,2})(-i\omega+L(q^2+U'_{2,2})+\bar{L}((\underset{\sim}{k}-\underset{\sim}{q})^2+U'_{2,2})\right]^{-1} \frac{d^dq}{(2\pi)^d} .$$

It should be noted that the precise definition of $I_b^{(D)}$ involves the restriction $\Lambda<|\underset{\sim}{k}-\underset{\sim}{q}|<\Lambda b$ too. This would lead, however, to unnecessary complications which are unphysical and are consequences of the sharp cut-off, so we disregard it. By means of the identity

$$\int_0^\pi \cos^l\vartheta \sin^{d-2}\vartheta (1 + x\cos\vartheta)^{-1} d\vartheta =$$

$$= \begin{cases} B(\frac{l+1}{2}, \frac{d-1}{2}) F(1, \frac{l+1}{2}, \frac{d+l}{2}, x^2), & l \text{ even} \\ -xB(\frac{l+2}{2}, \frac{d-1}{2}) F(1, \frac{l+2}{2}, \frac{d+l+1}{2}, x^2), & l \text{ odd} \end{cases}$$

where B denotes the beta function and F stands for the usual hypergeometric function, we obtain after integrating over angle variables:

$$I_b^{(D)}(k,\omega) = \int_q^{>} \frac{-i\omega s}{2kq\bar{L}} \frac{F(1,1/2,d/2,s^2)}{(q^2+U'_{2,2})(q^2+k^2+U'_{2,2})} +$$

$$+ \int_q^{>} \frac{-i\omega s}{2kq\bar{L}} \sum_{l=1}^{\infty} \frac{\Gamma(\frac{2l+1}{2})\Gamma(\frac{d}{2})}{\Gamma(\frac{2l+d}{2})\Gamma(\frac{1}{2})} \frac{2^{2l-1}k^{2l-1}q^{2l-1}}{(q^2+k^2+U'_{2,2})^{2l}(q^2+u'_{2,2})} .$$

$$. F(1, \frac{2l+1}{2}, \frac{d+2l}{2}, s^2)\left(\frac{2kq}{q^2+k^2+U'_{2,2}} - s\right) ,$$

where the notation (5.10) has been used and s has been defined by (6.6). $\Gamma$ denotes the gamma function. This formula specifies the function f appearing in (6.5).

If $L^{(1)}$ is finite and k and $\omega$ are small ($k\ll\Lambda, \omega\ll\Lambda^2L^{(2)}$) we obtain (6.7).

In the limiting case of $L^{(1)} \to 0$ $|s|$ becomes large in region I, therefore we have to use the asymptotic expressions of the hypergeometric functions. For the first few terms (6.11) is found.

## REFERENCES

[1] P.C.Hohenberg and B.I.Halperin, Rev.Mod.Phys. 49, 435 (1977)

[2] H.E.Stanley, Phys.Rev. 176, 718 (1968)

[3] F.J.Wegner and A.Houghton, Phys.Rev. A8, 401 (1973)

[4] S.Ma, Rev.Mod.Phys. 45, 589 (1973)

[5] S.Ma, J.Math.Phys. 15, 1866 (1974)

[6] S.Ma, Phys.Rev. A10, 1818 (1974)

[7] P.Szépfalusy and T.Tél, J.Phys. A 12, 2141 (1979)

[8] P.Szépfalusy and T.Tél, Z.Physik B 36, 343 (1980)

[9] P.Szépfalusy and T.Tél, Z.Physik B 39, 249 (1980)

[10] C. De Dominicis, E.Brézin and J.Zinn-Justin, Phys.Rev. B12, 4945 (1975)

[11] B.I.Halperin, P.C.Hohenberg and S.Ma, Phys.Rev. B10, 139 (1974)

[12] R.Bausch and B.I.Halperin, Phys.Rev. B18, 190 (1978)

[13] B.I.Halperin, P.C.Hohenberg and E.D.Siggia, Phys.Rev. B13, 1299 (1975)

[14] B.I.Halperin, Phys.Rev. B11, 178 (1975)

[15] S.Ma and L.Senbetu, Phys.Rev. A10, 2401 (1974)

[16] J.Sak, Phys.Rev. B14, 2932 (1976)

[17] G.S.Grest, Phys.Rev. B15, 2753 (1977)

[18] P.Szépfalusy and I.Kondor, Ann. of Phys. (N.Y.) 82, 1 (1972)

[19] P.Szépfalusy and I.Kondor, in Proceedings of the International School of Physics "Enrico Fermi", Varenna, LIX Course, eds. Müller K.A., Rigamonti A., pp. 806-818, North-Holland, 1976

[20] T.Tél, Diploma work (Eötvös University), 1975 (in Hungarian)

[21] P.C.Martin, E.D.Siggia and H.A.Rose, Phys.Rev. A8, 423 (1973)

[22] C. De Dominicis, J.Physique C 1, 247 (1976)

[23] H.K.Janssen, Z.Physik B 23, 377 (1976)

[24] R.Bausch, H.K.Janssen and H.Wagner, Z.Physik B 24, 113 (1976)

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