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ABSTRACT

The nucleons taking part in heavy ion reaction are considered as a three-fluid system. The first and second components correspond to the nucleons of the target and the projectile, while the thermalised nucleons produced in the course of the collision belong to the third component. Making use of the Boltzmann-equation, hydrodynamical equations are derived which yield also the anisotropy of the momentum distribution. The equation of state for anisotropic nuclear matter is derived from a field theoretical model in the mean field approximation.

АННОТАЦИЯ

Нуклоны, участвующие в реакциях тяжелых ионов, рассматриваются как трехкомпонентная жидкость. Первая и вторая компоненты содержат нуклоны мишени и бомбардирующего ядра, а третья - нуклоны, которые рассеиваются в процессе столкновения. На основе уравнения Больцмана получаются гидродинамические уравнения, которые учитывают анизотропию импульсного распределения. Выводится уравнение состояния анизотропного ядерного вещества из теоретико-полевой модели в приближении среднего поля.

KIVONAT

A nehéz-ion reakcióban résztvevő nukleonokat három komponensü folyadéknak tekintjük. Az első és második komponens a target illetve a bombázó mag nukleonjait, mig a harmadik az ütközés során termalizálódó nukleonokat tartalmazza. A Boltzmann-egyenlet alapján olyan hidrodinamikai egyenleteket származtatunk le, amelyek anizotróp impulzus eloszlásra vezetnek. Az anizotróp maganyag állapotegyenletét átlag tér közelitésben, térelméleti modellből származtatjuk.

1. Introduction

The anisotropy of the momentum distribution of the nucleons is one of the most characteristic features of heavy ion reactions. The anisotropy is determined by the relative momentum of the colliding nuclei. In the course of the collision the anisotropy is decreasing. It does not disapper completely, however, since a global thermodinamical equilibrium is not reached until the final stage of the collision process. Since this residual anisotropy has an essential influence on the momentum distribution of the reaction products, it seems to be desirable to take into account the development of the anisotropy. The conventional hydrodynamical models of heavy ion reactions[1-2]) are formulated in terms of equations expressing the conservation of the particle number, the momentum and the energy. These equations determine only four of the parameters of the momentum distribution, namely the three components of the flow velocity and the temperature. The parameter characterising the anisotropy of the momentum distribution remains indeterminate. It seems to be desirable to construct a generalization of the hydrodynamical model, in which also the anisotropy is determined. In the present work the outline of such an anisotropic hydrodynamical model will be given. The model is based on the Boltzmann transport theory. This means that the basic assumptions of the Boltzmann theory are regarded to be valid. The validity of these assumptions, however, is not granted in the case of heavy ion reactions. The errors introduced by these assumptions, will be compensated in some extent by applying a proper equation of state. The basic equations of the anisotropic hydrodynamical model will be obtained from the Boltzmann equation by taking the moments of the distribution function. These equations will be generalized to a many-fluid system. The anisotropic pressure tensor occuring in the equations, will be obtained from Walecka's mean field theory of nuclear matter[5]). Finally the outline of a simplified three-fluid model will be given. For the sake of simplicity our considerations will be presented in non-relativistic framework. The relativistic generalization can be carried out without serious difficulties.

2. The equations of the anisotropic hydrodynamical model

We begin our considerations with the Boltzmann transport equation[3]), given by

$$(\partial_t + \sum_i v_i \partial_i) f = C, \qquad (i = x, y, z), \tag{1}$$

where the one-particle function $f(\vec{r}\ \vec{v}\ t)$ describes the space and velocity distribution of the nucleons. The right hand side of this equation gives the changing rate of the distribution function produced by nucleon-nucleon scattering :

$$C = \tfrac{1}{2}\int d^3v_1 d^3v' d^3v_1' \,[f(\vec{v}')f(\vec{v}_1')\, w(\vec{v}'\vec{v}_1' | \vec{v}\vec{v}_1) - f(\vec{v})f(\vec{v}_1) w(\vec{v}\vec{v}_1, |\vec{v}'\vec{v}_1')], \tag{2}$$

where w denotes the transition probability of the nucleon-nucleon collision.

Following the usual procedure, we multiply the Boltzmann equation with m, mv_i and $\frac{1}{2}mv_jv_k$ and integrate over the velocity space to obtain:

$$\partial_t\rho + \sum_i \partial_i \rho u_i = 0, \tag{3}$$

$$\partial_t \rho u_j + \sum_i \partial_i (\rho u_i u_j + P_{ij}) = 0, \tag{4}$$

$$\partial_t(\tfrac{1}{2}P_{jk} + \tfrac{1}{2}\rho u_j u_k) + \sum_i \partial_i (u_i(\tfrac{1}{2}P_{jk} + \tfrac{1}{2}\rho u_j u_k) + Q_{ijk} + \tfrac{1}{2}u_j P_{ik} + \tfrac{1}{2}u_k P_{ij}) = \frac{m}{2}\int C v_j v_k \, d^3v \tag{5}$$

where the mass density, $\rho(\vec{r}t)$ and flow velocity $u_i(\vec{r}t)$ are defined by the relations:

$$\rho = m\int f d^3v, \tag{6}$$

$$u_i = \frac{m}{\rho}\int f v_i d^3v. \tag{7}$$

The pressure tensor $P_{ij}(\vec{r}t)$ and the heat flux tensor $Q_{ijk}(\vec{r}t)$ are defined as follows:

$$P_{ij} = m\int f c_i c_j d^3v, \tag{8}$$

$$Q_{ijk} = \tfrac{1}{2}m\int f c_i c_j c_k \, d^3v, \tag{9}$$

where

$$c_i = v_i - u_i. \tag{10}$$

The right hand sides of the equations (3) and (4) vanish, because of the particle number and momentum conservation. These are the usual equations of continuity and momentum. The trace of the tensor equation (5) yields the energy equation:

$$\partial_t(\varepsilon + \tfrac{1}{2}\rho\vec{u}^2) + \sum_i \partial_i (\varepsilon + \tfrac{1}{2}\rho\vec{u}^2) + q_i + \sum_i u_j P_{ij}) = 0, \tag{11}$$

where the energy density $\varepsilon(\vec{r}t)$ and the heat flux vector $q_i(\vec{r}t)$ are defined as:

$$\varepsilon = \frac{m}{2}\int f\vec{c}^2 d^3v, \tag{12}$$

$$q_i = \frac{m}{2}\int f c_i \vec{c}^2 d^3v . \tag{13}$$

The right hand side of equation (11) vanishes because of the energy conservation. The remaining linearly independent elements of the tensor equation (5) form a traceless, symmetric tensor equation. The five independent quantities determined by this equation are the elements of the anisotropy density tensor. The collision integrals on the right hand side do not vanish now since there is no conservation law for the anisotropy. The anisotropy changes due to the collisions. In the general case one has to keep all elements of the anisotropy density tensor. In the case of symmetric,central collision of heavy ions, however, the anisotropy density is characterized by a single quantity. Therefore in such a situation the usual set of hydrodynamical equations have to be supplemented only by one more equation:

$$\partial_t [a_{jj} + \tfrac{1}{2}\rho(u_j^2 - \tfrac{1}{3}\vec{u}^2)] + \sum_i \partial_i [u_i(a_{jj} + \tfrac{1}{2}\rho(u_j^2 - \tfrac{1}{3}\vec{u}^2)) +$$

$$+(Q_{ijj} - \tfrac{1}{3}q_i) + (u_j P_{ij} - \tfrac{1}{3}\sum_k u_k P_{ik})] = \tfrac{m}{2}\int C v_j^2 d^3v. \qquad (14)$$

The anisotropy density $a_{jj}(\vec{r}t)$ is defined as:

$$a_{jj} = \tfrac{1}{2}P_{jj} - \tfrac{1}{3}\varepsilon \qquad (15)$$

The index j corresponds to the coodinate axis along which the heavy ions collide.

For the illustration of the discussion above, let us assume a Maxwell--Boltzmann type distribution function for the nucleons:

$$f = \frac{\rho}{m}\left(\frac{m}{2\pi\theta}\right)^{3/2} \left(\frac{1}{\alpha}\right)^{1/2} e^{-(\vec{c}^2+(\frac{1}{\alpha}-1)c_z^2)\frac{m}{2\theta}} ,$$

$$(\theta = k_B T), \qquad (16)$$

where α is the anisotropy parameter along the z axis. In this case the energy density, the heat flux,the pressure tensor and the anisotropy density are given by the following expressions:

$$\varepsilon = \tfrac{1}{2}\frac{\rho}{m}\ \theta(2+\alpha), \qquad (17)$$

$$q_i = 0, \qquad (18)$$

$$P_{ij} = \frac{\rho}{m}\theta\delta_{ij}(1+(\alpha-1)\delta_{iz}), \qquad (19)$$

$$a_{zz} = \frac{1}{3}\frac{\rho}{m}\theta(\alpha-1). \qquad (20)$$

These equations show, that all physical quantities depend on the anisotropy parameter α. One observes also that the quantity a_{zz} is really a measure of the anisotropy, because it vanishes for α=1, that is for an isotropic velocity distribution.

3. The three-fluid model

The traditional hydrodynamical models of the heavy ion reactions based on the one- or two-fluid assumption[1-2] do not provide an adequate description for the thermalisation process taking place during the collision and the production of hadrons different from the nucleons. Therefore it seems to be desirable to construct a model which is able to accomodate the description of these processes. The first steps along this line were taken by Montvay and Zimányi working out the hadron-chemistry model[4]). To describe the thermalising process the nucleon distribution function f is splitted into three components:

$$f = \sum_s f^s \qquad (s = 1,2,3) \tag{21}$$

and an anisotropic three-fluid hydrodynamical model is constructed. It is assumed that the nucleons of the target and the projectile belong to the distribution s = 1 and s = 2, respectively, while the third distribution (s = 3) is populated by the nucleons scattered out from the two previous distributions. If hadrons different from nucleons are produced in the collision process then additional components must be introduced. However, this possibility is not discussed in this paper. Due to the decomposition of the distribution function f the Boltzmann-equation can be written in the form of a coupled set of equations:

$$(\partial_t + \sum_i v_i \partial_i) f^s = C^s, \tag{22}$$

where

$$C^s = \tfrac{1}{2}\sum_{rs'r'} \int d^3v_1 d^3v' d^3v'_1 [f^{s'}(\vec{v}')f^{r'}(\vec{v}'_1)w^{s'r'}_{sr}(\vec{v}'\vec{v}'_1|\vec{v}\vec{v}_1) - f^s(\vec{v})f^r(\vec{v}_1)w^{sr}_{s'r'}(\vec{v}\vec{v}_1|\vec{v}'\vec{v}'_1)] \tag{23}$$

where the transition probability for two nucleons from the distributions (sr) into the distributions (s'r') is denoted by $w^{sr}_{s'r'}$. The definition of this transition probability would be unique if the particles of the different distributions were distinguishable. However, this is not the case. The transition probabilities could be defined uniquely also if the distributions were extended only on non-overlapping regions of the velocity space. In our case this criterion is fulfilled only approximately and this fact should be taken into account properly computing the collision integrals. Multiplying the equation (22) by m, mv and $\frac{1}{2}m\, v_i v_k$, and integrating over the velocity space, the equations of the anisotropic three-fluid hydrodynamics are obtained in the following form:

$$\partial_t \rho^s + \sum_i \partial_i \rho^s u^s_i = m \int C^s d^3v, \tag{24}$$

$$\partial_t \rho^s u^s_j + \sum_i \partial_i (\rho^s u^s_i u^s_j + P^s_{ij}) = m\int C^s v_j d^3v, \tag{25}$$

$$\partial_t (\tfrac{1}{2}P^s_{jk} + \tfrac{1}{2}\rho^s u^s_j u^s_k) + \sum_i \partial_i [u^s_i(\tfrac{1}{2}P^s_{jk} + \tfrac{1}{2}\rho^s u^s_j u^s_k) +$$

$$+ Q^s_{ijk} + \tfrac{1}{2}u^s_j P^s_{ik} + \tfrac{1}{2}u^s_k P^s_{ij}] = \frac{m}{2}\int C^s v_j v_k d^3v\ , \tag{26}$$

Taking the trace of the tensor equation (26) one get the energy equations for each component. The remaining linearly independent equations form a traceless, symmetric tensor equation for the anisotropy density. Due to the nucleon-nucleon collisions nucleon, momentum and energy exchange is taking place among the various components. Consequently the collision integrals standing on the right hand side of the equations do not vanish. These collison integrals represent the couplings among the components. If a summation is performed on the component index s then, of course, the overall continuity, momentum and energy equations are obtained with vanishing right hand side, except for the equations of the anisotropy density. The anisotropy of the colliding two nuclei has its maximum before the collision. The anisotropy decreases in the framework of this model via two mechanisms: nucleons are scattered from the target and projectile components into the third one, on the other hand the anisotropy of the third component is decreasing further due to subsequent nucleon-nucleon collisions.

4. Equation of state for anisotropic nuclear matter

In order to specify completely the equations of the anisotropic hydrodynamical model, discussed in the previous sections, in addition to the transition probabilities, the pressure tensor $P_{jk}(\vec{r}t)$ and the heat flux $q_j(\vec{r}t)$ must be provided. If these quantities are given then the equations can be solved for the density $\rho(\vec{r}t)$, flow velocity $u_i(\vec{r}t)$, energy density $\varepsilon(\vec{r}t)$ and anisotropy density $a_{jk}(\vec{r}t)$. From these functions the experimental quantities can be computed. The transition probabilities $w^{sr}_{s'r'}$ can be expressed in terms of the nucleon-nucleon differential cross section, measured as the function of energy. The pressure tensor P_{jk} and the heat flux q_j depend on the energy density ε and on the anisotropy density a_{jk} therefore a model of the nuclear matter is needed which is able to produce P_{jk} and q_j in the function of ε and a_{jk}. For this purpose the mean field theory of Walecka offers an excellent possibility. This theory in its original formulation is a relativistic, renormalisable field theory, in which the interaction among nucleons (Ψ) is mediated by isosinglet scalar (Φ) and vector (V_λ) mesons. The coupled field equations are given by

$$(\partial_\mu\partial_\mu - m_s^2)\,\Phi = -\frac{g_s}{c^2}\,\bar{\Psi}\Psi\,, \tag{27}$$

$$\partial_\nu F_{\mu\nu} + m_v^2\, V_\mu = i g_v \bar{\Psi}\gamma_\mu\Psi, \tag{28}$$

$$[\gamma_\mu(\partial_\mu - \frac{i g_v}{\hbar c} V_\mu) + (m - \frac{g_s}{\hbar c}\Phi)]\,\Psi = 0, \tag{29}$$

where the field strength $F_{\mu\nu}$ is defined by

$$F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu \tag{30}$$

and the inverse Compton wave length of the scalar meson, vector meson and nucleon is denoted by m_s, m_v and m, respectively. With the appropriate choice of the coupling constants g_s and g_v a nucleon-nucleon potential can be derived, which is rather similar to the phenomenological soft-core potentials, except for the one-pion tail. In the mean field approximation the meson fields are replaced by their average values which are constant for homogeneous nuclear matter:

$$\Phi \rightarrow \langle\Phi\rangle \equiv \Phi_0\,, \tag{31}$$

$$V_\lambda \rightarrow \langle V_\lambda\rangle \equiv i\delta_{\lambda 4}\, V_0. \tag{32}$$

By means of the field equations the average fields Φ_o and V_o can be expressed in terms of the densities:

$$\Phi_o = \frac{g_s}{m_s^2 c^2} \langle \bar{\Psi} \Psi \rangle = \frac{g_s}{m_s^2 c^2} \rho_s , \qquad (33)$$

$$V_o = \frac{g_v}{m_v^{\;2}} \langle \bar{\Psi} \gamma_4 \Psi \rangle = \frac{g_v}{m_v^{\;2}} \rho . \qquad (34)$$

By substituting these classical fields into the Dirac-equation an exactly soluble model is obtained. The Hamiltonian density in this mean field approximation can be expressed as follows:

$$H = g_v V_o \rho + \frac{\hbar c}{\Omega} \sum_{\vec{k}\lambda} (\vec{k}^2 + m^{*2})^{\frac{1}{2}} (a^{\dagger}_{\vec{k}\lambda} a_{\vec{k}\lambda} + b^{\dagger}_{\vec{k}\lambda} b_{\vec{k}\lambda}) + \frac{m_s^2 c^2}{2} \Phi_o^2 - \frac{m_v^2}{2} V_o^2 , \qquad (35)$$

where Ω stands for the normalization volume, the effective mass m^* is given by

$$m^* = m - \frac{g_s}{\hbar c} \Phi_o , \qquad (36)$$

the creation and annihilation operator for nucleon and antinucleon with wave number $\vec{k}$ and spin-isospin quantum numbers λ is denoted by $a^{\dagger}_{\vec{k}\lambda}$, $b^{\dagger}_{\vec{k}\lambda}$ and $a_{\vec{k}\lambda}$, $b_{\vec{k}\lambda}$, respectively. Formally this Hamiltonian density corresponds to a non-interacting nucleon-antinucleon system, The interaction via the scalar mesons is reflected by the effective mass m^*, while the interaction via the vector mesons gives rise to a constant energy shift of the one-particle energies expressed by the first term of the Hamiltonian density. Since formally we have a non-interacting system, the nucleon and antinucleon assemblies can be described by Fermi-Dirac distributions:

$$n(T\alpha) = (\exp((\vec{k}^2 + (1/\alpha - 1)k_z^2 + m^{*2})^{\frac{1}{2}} \frac{\hbar c}{\theta} - \nu) + 1)^{-1} , \qquad (37)$$

$$\bar{n}(T\alpha) = (\exp((\vec{k}^2 + (1/\alpha - 1)k_z^2 + m^{*2})^{\frac{1}{2}} \frac{\hbar c}{\theta} + \nu) + 1)^{-1} , \qquad (38)$$

where the anisotropy parameter is denoted by α and the quantity ν is related to the chemical potential. It is worth while to note that at the available energies of heavy ion reactions the contribution of the antinucleons is negligible. Taking the expection value of the energy-momentum tensor of the system by the help of the distributions given above the energy density and the pressure tensor can be obtained as the function of the temperature and the anisotropy parameter:

$$\varepsilon = \varepsilon (\rho\, T\, \alpha), \qquad (39)$$

$$P_{ij} = P_{ij} (\rho\, T\, \alpha). \qquad (40)$$

By eliminating the density ρ the desired equation of state for anisotropic nuclear matter is obtained[7-8]:

$$P_{ij} = P_{ij} (\varepsilon\, T\, \alpha). \qquad (41)$$

5. A simplified model

The solution of the equations of the anisotropic hydrodynamic model in its full complexity seems to be a hopelessly difficult task, since 30 coupled partial differential equations should be solved. The task is further complicated by the fact that the calculation of the pressure and the heat flux is very complicated in the general case. For the sake of the applicability of the model a series of simplifying assumptions should be introduced.

First of all the task is simplified in a great extent if the treatment is restricted for central collision of identical nuclei. In this case, as it was pointed out earlier, only one remains relevant out of the five equations for the anisotropy density. On the other hand, the calculation of the pressure and the heat flux can be performed along the lines discussed in the previous section. The second assumption concerns the form of the distribution functions f^s. The explicit parametrization of the distribution function is inevitably needed for the computation of the collision integrals. Since the hydrodynamical equations determine only some of the moments of the distribution function, it is not meaningful to introduce more independent parameters as the number of the equations. Keeping in mind this requirement, the simplest form is assumed for the distribution functions:

$$f^s = \frac{\rho^s}{m}\left(\frac{m}{2\pi\,\theta^s}\right)^{3/2}\left(\frac{1}{\alpha^s}\right)^{\frac{1}{2}} e^{-\left(\vec{c}^{s^2}+\left(\frac{1}{\alpha^s}-1\right)c_z^{s^2}\right)\frac{m}{2\theta^s}} . \qquad (42)$$

The distributions $s = 1,2$ are considered to be isotropic in their own frame of reference:

$$\alpha^s = 1, \qquad (s = 1,2). \qquad (43)$$

This choice, on one hand facilitates the calculation of the collision integrals by making use of the saddle point method, on the other hand the error caused by the third assumption, to be introduced below, is compensated in some extent. The third, rather drastic, assumption is the following: in the course of the collision the velocity distribution remains unchanged both for the target and for the projectile. The nucleons scattered out from the distributions of the target and the projectile increase the number of the nucleons of the third distribution but there is no rescattering from the third distribution into the other ones. This assumption is very well justified in the first stage of the collision process when the number of nucleons in the third distribution is almost zero, however, it is rather questionable in the later stages. Therefore to describe the target and projectile nucleons by Maxwell-Boltzmann distributions, instead of Fermi-Dirac ones, seems to be much more appropriate on the time average of the whole collison process. This third assumption can be formulated quantitatively in the following way: all of the transition probabilities $w^{sr}_{s'r'}$ are zero except for the cases when $(sr) = (12), (13), (23)$ and $(s'r') = (33)$, furthermore both the temperature and the flow velocity for the components $s = 1$ and $s=2$ are constant:

$$\vec{u}^s = \text{const}, \quad T^s = \text{const}, \quad (s = 1,2). \qquad (44)$$

On the price of these simplifying assumptions only 8 independent equations remain out of the 30 ones needed for the description of the general case. The difficulties associated with the calculation of the collision integrals and also the errors implied by these calculations can be diminished if the momentum and energy equations are summed up for the index s. In this way the rigth hand side should vanish exactly and therefore it is not necessary to calculate some of the collision integrals. The details of the model outlined above and the results of calculations along these lines will be published in the near future.

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