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THE SCALES OF EUCLIDIAN AND HAMILTONIAN LATTICE QCD

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ABSTRACT

Our earlier result on $\Lambda^{MOM}_{F.g.}/\Lambda^{latt.}_{Eucl.}$ is confirmed by recalculating this ratio using the background field method. The relation between the scales of Hamiltonian and Euclidian SU(N) lattice gauge theory is also determined. We obtained

$$\Lambda_H^{latt.}/\Lambda_E^{latt.} = 0.968\, e^{-\frac{0.5495}{N^2}} = \begin{matrix} 0.91, & N=3 \\ 0.84, & N=2. \end{matrix}$$

It is in strong disagreement with the numbers previously used in the literature. It is argued that the strong coupling expansions for the string tension should be carefully reanalyzed.

АННОТАЦИЯ

Была определена связь $\Lambda_E^{latt.}/\Lambda_{MOM}$ и $\Lambda_H^{latt.}/\Lambda_E^{latt.}$ в квантовой хромодинамике на решетке. Результаты:

$$\Lambda_E^{latt.}/\Lambda_{MOM} = 83.4 \quad SU(3)$$

$$\Lambda_H^{latt.}/\Lambda_E^{latt.} = 0.91 \quad SU(3)$$

KIVONAT

Meghatároztuk $\Lambda_E^{latt.}/\Lambda_{MOM}$ és $\Lambda_H^{latt.}/\Lambda_E^{latt.}$ arányokat rács QCD-ben.

Eredményeink:

$$\Lambda_E^{latt.}/\Lambda_{MOM} = 83.4 \quad SU(3)$$

$$\Lambda_H^{latt.}/\Lambda_E^{latt.} = 0.91 \quad SU(3)$$

INTRODUCTION

Dimensional quantities obtained in QCD in different regularization schemes can be compared if the relation between the correspondig Λ parameters is known. The available perturbative and non-perturbative results can be unified by calculating the relation between the scales of Euclidian lattice QCD $(\Lambda_E^{latt.})$, of the Hamiltonian lattice formulation $(\Lambda_H^{latt.})$ and of the continuum formulation $(\Lambda^{MOM}_{Feynman\ gauge}$ for instance$)$.

Calculating the two- and three- point functions at the one loop level on an Euclidian lattice the ratio $\Lambda^{MOM}_{F.g.}/\Lambda_E^{latt.}$ has been determined by us [1]. The procedure was rather involved therefore the result required an independent confirmation. The errors in direct Monte-Carlo simulations were too large to be conclusive in this respect [2]. In a recent paper [3] Dashen and Gross recalculated this ratio in a simpler way using the background field method [4]. Their final result was in slight (5%) disagreement with our numbers.

Subsequently Gross determined $\Lambda_H^{latt.}/\Lambda_E^{latt.}$ [5]. This ratio is the bridge between the Hamiltonian strong coupling results [6-8] v.s. Euclidian MC simulations [9-13] and strong coupling expansions [14]. Using this number the string tension extracted from Hamiltonian strong coupling expansions has been compared with the Euclidian results [7,8]. The numbers were found to be consistent.

In order to resolve the discrepancy concerning $\Lambda^{MOM}/\Lambda_E^{latt.}$, we have calculated this ratio again, using the background field technique of Ref. [3]. We have found errors in [3]. Correcting them, we reproduced our original numbers. These points and some general questions concerning the background field method will be discused in Section I.

We have also determined $\Lambda_H^{latt.}/\Lambda_E^{latt.}$ (Sect. II). Our result is in complete disagreement with that obtained previously by Gross [5]. As we do not know any details of his calculation, we could not find the reason. We note that the MC simulations in SU(2) by Kuti, Polónyi and Szlachányi [12] indicated $\Lambda_H^{latt.}/\Lambda_E^{latt.} \approx 1$, which is consistent with our number.

If our result is correct, it would necessitate a reanalysis of the Hamiltonian strong coupling expansions of the string tension. We believe however, that a critical reanalysis of the strong coupling results is necessary anyhow both in the Hamiltonian and in the Euclidian formulations. Some arguments will be given in Section III.

SECTION I: DETERMINATION OF $\Lambda^{MOM}_{F.g.}/\Lambda^{latt.}_{E}$

Let us consider the action $S(A_i)$ of the fields A_i. Let us shift the field A_i by a classical background field W_i: $A_i = W_i + \alpha_i$, and expand the action in terms of the quantum fields α_i. Consider the terms quadratic in α: $S_2(\alpha, W)$. It is a trivial combinatorics to show that all irreducible tree and one-loop diagrams are correctly generated by $S_{eff}(W)$ defined as

$$e^{-S(W)}\int D\alpha\, e^{-S_2(\alpha,W)} = d\cdot e^{-S_{eff}(W)} . \tag{1}$$

Let us denote the effective actions of an SU(N) gauge theory in a given gauge by $S^{(1)}_{eff}(W^a_\mu)$ and $S^{(2)}_{eff}(W^a_\mu)$ in schemes (1) and (2) respectively. We shall use the background gauge. As we shall see, in the background gauge $S_{eff}(W^a_\mu)$ is a gauge invariant functional of W^a_μ. On the other hand, it is assured by renormalizability that $\Delta S_{eff}(W^a_\mu) = S^{(1)}_{eff}(W^a_\mu) - S^{(2)}_{eff}(W^a_\mu)$ is local. Therefore, in the background gauge ΔS_{eff} is a local, gauge invariant expression of the background fields, and in the infinite cut-off limit it should have the form

$$\Delta S_{eff}(W^a_\mu) = \frac{1}{4}\int d^4x \sum_{\substack{\mu,\nu\\ a}} (F^a_{\mu\nu})^2 \cdot \left[\frac{1}{g^2_{(1)}} - \frac{1}{g^2_{(2)}} + c\right]. \tag{2}$$

The condition $\Delta S_{eff}(W^a_\mu) = 0$ gives the relation between the coupling constants of schemes (1) and (2) ($z_3 = 1$ in this gauge), giving the relation between the Λ parameters

$$\Lambda^{(1)}/\Lambda^{(2)} = e^{-\frac{1}{2\beta_o}\left(\frac{1}{g^2_{(1)}} - \frac{1}{g^2_{(2)}}\right)} = e^{\frac{1}{2\beta_o}\cdot c} . \tag{3}$$

We shall follow the notation of Ref. [3]. The lattice gauge variable is parametrized as*

$$U_{x,x+\mu} = e^{iga\alpha_\mu(x)}\, U^{(o)}_{x,x+\mu},\quad U^{(o)}_{x,x+\mu} = e^{iaW_\mu(x)} , \tag{4}$$

where a is the lattice distance. The Wilson action is expanded up to second order in the quantum fields $\alpha_\mu = \alpha^a_\mu T^a$. It is to be completed by the gauge fixing term and by the ghost action. The corresponding equations in Ref. [3] contain several misprints therefore we thought it useful to give these equations here.

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*This parametrization is different from that $A_\mu = W_\mu + \alpha_\mu$ discussed before, but the coefficient of the corresponding extra terms in the quadratic action is zero if the background field satisfies the classical equations of motions.

The gauge fixing term in the background gauge is

$$S_{gf} = a^4 \sum_x Tr\left(\sum_\mu \bar{D}_\mu^{(o)} \alpha_\mu(x)\right)^2, \tag{5}$$

where

$$\begin{aligned} \bar{D}_\mu^{(o)} f(x) &= \frac{1}{a}\left(U^{+(o)}_{x-\mu,x}\, f(x-\mu)\, U^{(o)}_{x-\mu,x} - f(x)\right), \\ D_\mu^{(o)} f(x) &= \frac{1}{a}\left(U^{(o)}_{x,x+\mu}\, f(x+\mu)\, U^{+(o)}_{x,x+\mu} - f(x)\right). \end{aligned} \tag{6}$$

The ghost action has the form

$$S_{gh} = a^4 \cdot 2 \sum_x \sum_\mu Tr\left[\left(D_\mu^{(o)}\phi(x)\right)^+ \left(D_\mu\phi(x)\right)\right] \tag{7}$$

where D_μ is the covariant derivative with $U^{(o)} \to U$ in Eq.(6) and ϕ is the ghost field. Combining all the terms, the complete action quadratic in the quantum fields has the following form*

$$S_2(\alpha,\phi;W) = S_{sc} + S_T + S_A + S_B + S_{gh} + S_T' ,$$

where

$$S_{sc} = a^4 \sum_x \sum_{\mu,\nu} Tr\,(D_\mu^{(o)}\alpha_\nu\, D_\mu^{(o)}\alpha_\nu) , \tag{8a}$$

$$S_T = -a^4 \sum_x \sum_{\substack{\mu,\nu \\ a,b}} \frac{1}{16N} a^4 \,(F^a_{x,\mu\nu}\, F^a_{x,\mu\nu})\, (\Delta_\mu\alpha_\nu^b - \Delta_\nu\alpha_\mu^b)(\Delta_\mu\alpha_\nu^b - \Delta_\nu\alpha_\mu^b), \tag{8b}$$

$$S_A = a^4 \sum_x \sum_{\mu,\nu} \frac{1}{2} Tr\,(A_{x,\mu\nu}\, F_{x,\mu\nu}), \tag{8c}$$

where

$$A_{x,\mu\nu} = -2i\{2[\alpha_\nu,\alpha_\mu]_- + a[\alpha_\nu, D_\nu^{(o)}\alpha_\mu]_- + a[D_\mu^{(o)}\alpha_\nu,\alpha_\mu]_- - \frac{1}{2} a^2 [D_\nu^{(o)}\alpha_\mu, D_\mu^{(o)}\alpha_\nu]_-\},$$

$$S_B = a^4 \sum_x \sum_{\mu,\nu} \frac{1}{2} Tr\,(B_{x,\mu\nu}\, F_{x,\mu\nu}), \tag{8d}$$

$$B_{x,\mu\nu} = -i\,\{a[\alpha_\nu, D_\mu^{(o)}\alpha_\nu]_- + a[D_\nu^{(o)}\alpha_\mu,\alpha_\mu]_-\},$$

$$S_{gh} = a^4 \sum_x \sum_\mu 2\, Tr\,[(D_\mu^{(o)}\phi)^+ (D_\mu^{(o)}\phi)] , \tag{8e}$$

$$S_T' = -a^4 \sum_x \sum_{\mu,\nu} a^2 Tr\{F_{x,\mu\nu}(aD_\mu^{(o)}\alpha_\nu + \alpha_\nu)[aD_\nu^{(o)}\alpha_\mu + \alpha_\mu, F_{x,\mu\nu}]_-\}. \tag{8f}$$

$D_\mu^{(o)}$ and $F_{x,\mu\nu}$ are built up from the background fields W_μ. S_2 is invariant under the following transformation

$$U^{(o)}_{x,x+\mu} \to V(x)\, U^{(o)}_{x,x+\mu}\, V^+(x+\mu) \tag{9a}$$

*S_T' is absent in Ref.[3]. However, it does not contribute to the final result.

$$\alpha_\nu(x) \to V(x)\ \alpha_\nu(x)\ V^+(x) \qquad \text{(no inhomogenous term)} \tag{9b}$$

$$\phi(x) \to V(x)\ \phi(x)\ V^+(x) \tag{9c}$$

where $V(x) \in SU(N)$. Eq. (9b) and (9c) can be considered as a change of the integration variables in the functional integral. Therefore $S_{eff}(W)$ is invariant under Eq. (9a), that is in the background gauge $S_{eff}(W)$ is a gauge invariant functional of the background fields.

According to Eq.(2), it is enough to search for terms proportional to $F^2_{x,\mu\nu}$ in the effective action. (We did not check the cancellation of all the unwanted terms. We have done it carefully in Ref.[1] using a different method.) We obtained the following results:

$$S^{Eucl.latt.}_{eff} = \int d^4x \sum_{\substack{\mu,\nu\\a}} \left(F^a_{\mu\nu}(x)\right)^2 \left\{\frac{1}{4g^2_E} - N\left[\frac{N^2-1}{32N^2} + \frac{11}{12}\frac{1}{(2\pi)^4}\int_S d^4k \frac{1}{(\hat{k}\hat{k}^*)^2} - \right.\right.$$
$$- \frac{5}{48}\frac{1}{(2\pi)^4}a^2\int_S d^4k\frac{1}{(\hat{k}\hat{k}^*)} + \frac{1}{24}\frac{1}{(2\pi)^4}a^2\int_S d^4k\frac{(\hat{k}^*_1+\hat{k}^*_1)(\hat{k}^*_2+\hat{k}^*_2)}{(\hat{k}\hat{k}^*)^2} - \tag{10}$$
$$\left.\left.- \frac{1}{16}\frac{1}{(2\pi)^4}a^2\int_S d^4k\frac{(\hat{k}^*_1-\hat{k}_1)^2}{(\hat{k}\hat{k}^*)^2}\right]\right\},$$

where

$$\hat{k}_\mu = \frac{1}{a}(e^{-ik_\mu a}-1),\quad \hat{k}^*_\mu = \frac{1}{a}(e^{ik_\mu a}-1),\quad (\hat{k}\hat{k}^*) = \sum_\mu \hat{k}_\mu\hat{k}^*_\mu,$$
$$\int_S d^4k = \iiiint_{-\pi/a}^{\pi/a} d^4k. \tag{11}$$

The integral $\int_S d^4k \frac{1}{(\hat{k}\hat{k}^*)^2}$ is infrared divergent. It is meant (here and in the following) that the difference between the result of two schemes is taken. The equations are written separately only for the reader's convenience. Let us take the continuum theory in the Pauli-Villars scheme. It is easy to show that

$$S^{PV}_{eff} = \int d^4x \sum_{\substack{\mu,\nu\\a}} \left(F^a_{\mu\nu}(x)\right)^2\left\{\frac{1}{4g^2_{PV}} - N\left[\frac{11}{12}\frac{1}{(2\pi)^4}\int_{PV} d^4k\frac{1}{(k^2)^2}\right]\right\}. \tag{12}$$

The finite difference between the infrared integrals is given by

$$\frac{1}{(2\pi)^4}\int_S d^4k\frac{1}{(\hat{k}\hat{k}^*)^2} - \frac{1}{(2\pi)^4}\int_{PV} d^4k\frac{1}{(k^2)^2} =$$
$$= 0.015847 + \frac{1}{16\pi^2}\left(\ln\frac{\pi^2}{a^2m^2} - 1\right), \tag{13}$$

where m is the PV regulator mass. Calculating the remaining finite integrals we obtained:

$$S_{eff}^{Eucl.latt.} - S_{eff}^{PV} = \int d^4x \sum_{\substack{\mu,\nu \\ a}} (F^a_{\mu\nu})^2 \left\{ \frac{1}{4g_E^2} - \frac{1}{4g_{PV}^2} - \frac{11N}{96\pi^2} \left[-\ln(a\cdot m) - \frac{3\pi^2}{11N^2} + 3.7053 \right] \right\}. \tag{14}$$

This is slightly different from the result in Eq.(4.10) of Ref.[3]. The difference comes from the value of $\frac{1}{2} \ln \frac{Z_a^{sc}}{Z_m^{sc}}$ (in the notation of Ref.[3]), which is correctly

$$\int d^4x \sum_{\substack{\mu,\nu \\ a}} (F^a_{\mu\nu})^2 \frac{11N}{96\pi^2} \left[0.0131 + \frac{1}{11} \ln(ma) \right].$$

Our result implies

$$\Lambda_{PV}/\Lambda_E^{latt.} = 40.66\, e^{-\frac{3\pi^2}{11N^2}}. \tag{15}$$

Finally, Λ_{PV} should be connected with $\Lambda^{MOM}_{F.g.}$. Celmaster and Gonsalves obtained [15]*:

$$\Lambda^{MOM}_{F.g.}/\Lambda_{DR} = 7.692, \tag{16}$$

therefore we need the relation between the scales of PV regularization and dimensional regularization with minimal subtraction. In Ref. [3] this ratio has been taken over from 't Hooft [16], but this number is incorrect. Using the same background field method applied before it can be shown that

$$\Lambda_{PV}/\Lambda_{DR} = e^{\left(\frac{1}{2} \ln 4\pi - \frac{\gamma_{Euler}}{2} + \frac{6}{132}\right)}. \tag{17}$$

In deriving this result one should remember the triviality: $\sum_\mu \delta_{\mu\mu} = n \neq 4$ in n dimensions. In the language of Ref. [16], the difference comes from two sources. In order to arrive to Eq.(13.5) of Ref.[16] one starts from

$$\int d^4k \frac{k_\mu k_\nu k_\rho k_\sigma}{(k^2+\mu_o^2)^4} = (g_{\mu\nu}g_{\rho\sigma}+g_{\mu\rho}g_{\nu\sigma}+g_{\mu\sigma}g_{\nu\rho}) \frac{1}{24}\left(1-\frac{5}{12}(n-4)\right) \int d^4k \frac{(k^2)^2}{(k^2+\mu_o^2)^4}. \tag{18}$$

*We have checked this number and agree.

The term $-\frac{5}{12}(n-4)\int d^4k \frac{(k^2)^2}{(k^2+\mu_o^2)^4}$ gives a finite contribution, which just cancels the term -5/12 in Eq.(13.7) of [16]. On the other hand in n dimensions the number of gauge field components is n, while the number of real ghost fields is always 2. Therefore, the usual simplifying argument saying that in the background gauge the ghost contribution just cancels one half of the contribution from $\int \sum_{\mu,\nu} Tr(D_\mu \alpha_\nu D_\mu \alpha_\nu)$ is not exactly true, but there is an extra contribution*. This is the source of the term 6/132 in Eq.(17).

(We note, that the correct connecting factor in Eq.(17) resolves the appearant discrepancy between the results of Ref.s [16] and [17]. Shore calculated the one loop corrections around an instanton directly in n dimensions using dimensional regularization. His result is in agreement with that derived in the Pauli-Villars scheme by 't Hooft [16], if the connecting factor of Eq.(17) is used.)

Combining Eq.s (15), (16) and (17) one obtains

$$\Lambda^{MOM}_{F.g.}/\Lambda^{latt.}_{E} = 112.5\, e^{-\frac{3\pi^2}{11N^2}} = \begin{matrix} 83.4, & N=3, \\ 57.4, & N=2, \end{matrix} \qquad (19)$$

in agreement with the result of Ref.[1] within the accuracy of the calculation.

SECTION II: DETERMINATION OF $\Lambda^{latt.}_{H}/\Lambda^{latt.}_{E}$

Consider Wilson's action on a symmetric (hypercubic) lattice with lattice distance a. If a is small (g_E is small) we know the relation between a and g_E: it is given by the equation $\Lambda^{latt.}_E = const.$

Let us fix the lattice distance along the directions 1, 2, 3 ($a_1=a_2=a_3=a$, a is small, but fixed) and decrease it along the fourth direction: $\frac{a}{a_4} \equiv \xi \to \infty$. How should we change the coupling constant in order to keep the physics unchanged? We must allow the couplings to be different for plaquettes lying in the $i,4$ or in the i,k planes:

$$S = \beta_t \sum_x \sum_i L_{i4} + \beta_s \sum_x \sum_{i>k} L_{ki}, \qquad (20)$$

where

$$L_{\mu\nu} = Tr\,(1-U_{x,x+\mu}\, U_{x+\mu,x+\mu+\nu}\, U^+_{x+\nu,x+\mu+\nu}\, U^+_{x,x+\nu}) + h.c.\ ,$$
$$U_{x,x+\mu} = e^{ia_\mu A_{x,\mu}}. \qquad (21)$$

*This point has been observed also in a recent paper by Weisz [18].

By expanding the matrix $U_{x,x+\mu}$ in terms of the gauge field variables $A_{x,\mu}$, one can find the classical value of β_t and β_s: $\beta_t = \frac{1}{g^2}\xi$, $\beta_s = \frac{1}{g^2}\frac{1}{\xi}$. In the quantum theory

$$\beta_t = \frac{1}{g_t^2}\xi, \quad \beta_s = \frac{1}{g_s^2}\frac{1}{\xi}, \tag{22}$$

where $g_t^2 \neq g_s^2$ if $\xi \neq 1$. By tuning two different coupling constants, an Euclidian invariant quantum theory can be defined, which is equivalent to the theory on the symmetric lattice with lattice distance a and coupling constant g_E (a is small).

Our problem is to find the relation between g_t^2, g_s^2 and g_E^2. Classically $g_t^2 = g_E^2$, $g_s^2 = g_E^2$. For small g_E^2 we have

$$\begin{aligned} \frac{1}{g_t^2} &= \frac{1}{g_E^2} + c_t + O(g_E^2), \\ \frac{1}{g_s^2} &= \frac{1}{g_E^2} + c_s + O(g_E^2). \end{aligned} \tag{23}$$

These relations are gauge independent. In a Monte Carlo simulation they could have been determined even without fixing the gauge.

Let us consider the action in Eq.(20) in the $A_o = 0$ gauge. It has been shown by Creutz [19]* that in the $\xi\to\infty$ limit this theory can be described by the following transfer matrix

$$T = c\, e^{-a_4 H} \tag{24}$$

where

$$H = \sqrt{\frac{g_t^2}{g_s^2}} \cdot \frac{1}{2a} g_H^2 \left\{ \sum_{links} \vec{E}_\ell^2 + \frac{2}{g_H^4} \sum_x \sum_{i>k} L_{ki} \right\}, \quad g_H^2 = g_t \cdot g_s. \tag{25}$$

Apart from the overall factor of $\sqrt{\frac{g_t^2}{g_s^2}}$, H is just the Kogut-Susskind Hamiltonian. In the continuum limit $(a\to 0)$

$$\sqrt{\frac{g_t^2}{g_s^2}} = 1 + O(g_E^2). \tag{26}$$

In calculating the spectrum of H the factor $\sqrt{\frac{g_t^2}{g_s^2}}$ can be replaced by 1. (Similar $O(g_E^2)$ corrections are neglected in the definition of the Λ parameter, for instance.) In perturbative calculations however, this factor is important to restore Lorentz invariance**.

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*In Creutz's paper $g_t^2=g_s^2$ was taken, but it is trivial to correct his final result.

**The presence of this factor has also been observed by J. Shigemitsu et al [8] recently.

$\Lambda_H^{latt.}$ is built up from a and g_H in the usual way.
Therefore we get

$$\Lambda_H^{latt.}/\Lambda_E^{latt.} = e^{-\frac{1}{2\beta_o}\left(\frac{1}{g_H^2}-\frac{1}{g_E^2}\right)} = e^{-\frac{1}{2\beta_o}\frac{c_t+c_s}{2}} \tag{27}$$

In determining c_t and c_s we follow the method described in the previous Section. The gauge variable is parametrized as

$$U_{x,x+\mu} = e^{iga_\mu\alpha_\mu}\, U^{(o)}_{x,x+\mu},\quad U^{(o)}_{x,x+\mu} = e^{ia_\mu W_\mu}. \tag{28}$$

(no sum over μ)

As we neglect $O(g^2)$ terms, it is irrelevant which g occurs in the exponent in Eq.(28): $\frac{g^2}{g_E^2} = \frac{g_t^2}{g_E^2} = \frac{g_s^2}{g_E^2} = 1$ in this order. The covariant derivative is defined as

$$D_\mu f(x) = \frac{1}{a_\mu}\left(U_{x,x+\mu}\, f(x+\mu)\, U^+_{x,x+\mu} - f(x)\right). \tag{29}$$

We shall choose the background gauge again:

$$S_{gf} = a^3 a_4 \sum_x Tr\left(\sum_\mu \bar{D}^{(o)}_\mu \alpha_\mu\right)^2. \tag{30}$$

The quadratic action is essentially the same as given by Eq.(8), with the only modification that a is replaced by the appropriate a_μ everywhere. In the background gauge the effective action is gauge invariant as before. We are interested in the $\xi\to\infty$ limit. In momentum space the integration is over an asymetric region:

$$\int_{AS} d^4k = \int_{-\pi/a_4}^{\pi/a_4} dk_4 \iiint_{-\pi/a}^{\pi/a} d^3k. \tag{31}$$

Let us summarize the results. In the following expressions I_1, I_2, I_3 and I_4 are simple integrals specified later and determined numerically. The contributions from S_T, S_A, S_B and $S_{sc}+S_{ghost}$ are given as follows

S_T:

$$-N\int d^4x \sum_{i,k} (F^a_{ik})^2\, \frac{N^2-1}{24N^2}\, I_1, \tag{32a}$$

S_A:

$$\begin{aligned} &-N\int d^4x \sum_i \left[(F^a_{i4})^2 + (F^a_{4i})^2\right]\cdot\left[\frac{1}{(2\pi)^4}\int_{AS} d^4k\, \frac{1}{(\hat{k}\hat{k}^*)^2} - \frac{1}{96} I_2\right] - \\ &-N\int d^4x \sum_{i,k} (F^a_{ik})^2\left[\frac{1}{(2\pi)^4}\int_{AS} d^4k\, \frac{1}{(\hat{k}\hat{k}^*)^2} - \frac{1}{48} I_2 + \frac{1}{32} I_3\right], \end{aligned} \tag{32b}$$

S_B:

$$-N\int d^4x \sum_i [(F^a_{i4})^2 + (F^a_{4i})^2] \cdot \frac{1}{64} I_2 - \tag{32c}$$
$$-N\int d^4x \sum_{i,k} (F^a_{ik})^2 \frac{1}{128} I_4 ,$$

$S_{sc}+S_{ghost}$

$$-N\int d^4x \sum_i [(F^a_{i4})^2 + (F^a_{4i})^2][\frac{1}{576} I_2 - \frac{1}{12}\frac{1}{(2\pi)^4}\int_{AS} d^4k \frac{1}{(\hat{k}\hat{k}^*)^2} - \tag{32d}$$
$$-N\int d^4x \sum_{i,k} (F^a_{ik})^2 [\frac{1}{288} I_2 - \frac{1}{96} I_3 - \frac{1}{12}\frac{1}{(2\pi)^4}\int_{AS} d^4k \frac{1}{(\hat{k}\hat{k}^*)^2}] .$$

There is no relevant one loop contribution from S_T' and from the cross terms. Collecting everything one obtains:

$$S^{Asymm.latt.}_{eff} = \int d^4x \sum_i \left[(F^a_{i4})^2+(F^a_{4i})^2\right]\left\{\frac{1}{4g_t^2} - N\left[\frac{11}{12}\frac{1}{(2\pi)^4}\int_{AS} d^4k \frac{1}{(\hat{k}\hat{k}^*)^2} + \frac{1}{144} I_2\right]\right\}+ \tag{33}$$
$$+\int d^4x \sum_{i,k} (F^a_{ik})^2\left\{\frac{1}{4g_s^2} - N\left[\frac{N^2-1}{24N^2} I_1 + \frac{11}{12}\frac{1}{(2\pi)^4}\int_{AS} d^4k \frac{1}{(\hat{k}\hat{k}^*)^2} - \frac{5}{288} I_2 + \frac{1}{48} I_3 + \frac{1}{128} I_4\right]\right\}.$$

$S^{Symm.latt.}_{eff}$ has been determined before. It can be written as

$$S^{Symm.latt.}_{eff} = \int d^4x \sum_{\mu,\nu} (F^a_{\mu\nu})^2\left\{\frac{1}{4g_E^2} - N\left[\frac{N^2-1}{32N^2} + \frac{11}{12}\frac{1}{(2\pi)^4}\int_S \frac{1}{(\hat{k}\hat{k}^*)^2} - 0.010246\right]\right\}. \tag{34}$$

Eq.s (33) and (34) give immediately the relation between g_t^2, g_s^2 and g_E^2. The finite difference of the infrared divergent integrals is given by:

$$\frac{1}{(2\pi)^4}\int_{AS} d^4k \frac{1}{(\hat{k}\hat{k}^*)^2} - \frac{1}{(2\pi)^4}\int_S d^4k \frac{1}{(\hat{k}\hat{k}^*)^2} = -0.001774. \tag{35}$$

Let us give now the definition of the integrals $I_1,\ldots I_4$ and their numerical values:

$$I_1 = (\frac{2}{\pi})^3 \iiint_0^{\pi/2} d^3x \, (\sum_i \sin^2 x_i)^{1/2} = 1.19379,$$
$$I_2 = (\frac{2}{\pi})^3 \iiint_0^{\pi/2} d^3x \, (\sum_i \sin^2 x_i)^{-1/2} = 0.91070,$$
$$I_3 = (\frac{2}{\pi})^3 \iiint_0^{\pi/2} d^3x \, \sin^2 x_1 \cdot \sin^2 x_2 \, (\sum_i \sin^2 x_i)^{-3/2} = 0.10459, \tag{36}$$
$$I_4 = (\frac{2}{\pi})^3 \iiint_0^{\pi/2} d^3x \, \sin^2 2x_1 \cdot (\sum_i \sin^2 x_i)^{-3/2} = 0.45930.$$

Using these numbers one obtains:

$$\frac{1}{4g_t^2} = \frac{1}{4g_E^2} + N[-0.01631 + \frac{1}{32N^2}],$$

$$\frac{1}{4g_s^2} = \frac{1}{4g_E^2} + N[+0.01707 - \frac{0.59173}{N^2}],$$

which gives

$$\Lambda_H^{latt.}/\Lambda_E^{latt.} = 0.968\ e^{-\frac{0.5495}{N^2}} = \begin{matrix} 0.91, & N=3, \\ 0.84, & N=2. \end{matrix} \tag{37}$$

SECTION III: REMARKS ON THE STRONG COUPLING ANALYSIS' OF THE STRING TENSION

Gross obtained $\Lambda_H^{latt.}/\Lambda_E^{latt.} = 3.01$ for N=3 [5]. Using this number consistency was found between the Hamiltonian and Euclidian results for the string tension. Our result is $\Lambda_H^{latt.}/\Lambda_E^{latt.} = 0.91$.

An obvious possibility is that we are in error. However independently of our result, it is hard to believe the numbers extracted from the strong coupling expansions [20].

- The expected presence of the roughening transition [21-23] prevents a straightforward Padé analysis or other extrapolation methods. On the other hand the series itself does not seem to be convergent in the relevant region.
- The β function derived from a 6^{th} order Hamiltonian series tends to match onto the weak coupling curve. This matching breaks down for $g \lesssim 1.05$ $(y \equiv \frac{\sqrt{2}}{g^2} \gtrsim 1.28)$, where the high order terms begin overhelming the low order terms [7]. On the other hand the string tension is extracted from the region $1.35 \lesssim y \lesssim 1.55$.
- From the i^{th} order Hamiltonian expansion $\sqrt{T} = C^{(i)}\ \Lambda_H^{latt.}$, where $C^{(i)}$ is decreasing with i; $C^{(6)} \approx 180$, $C^{(\infty)}$ is estimated to be $\sim 69\pm15$ [7]. Therefore $C^{(6)}$ and $C^{(\infty)}$ are significantly different. However, in the Euclidian case no such procedure was used [14] and the 12^{th} order result itself was claimed to be consistent with the MC result. Which of the procedures is correct?

After completing our calculation on $\Lambda_{F.g.}^{MOM}/\Lambda_E^{latt.}$ we received papers considering this ratio [18,24,25]. Kawai et al. [24] recalculated the two- and three-point functions and obtained a result identical to our original numbers. Weisz [18] observed one of the errors in Ref.[3]. Both Kawai et al. and Weisz completed the result by adding the contribution of fermions. Iwasaki [25] presented rather different arguments, which we can not agree with.

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