KFKI-1981-01

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ON THE QUANTIZATION OF SPACETIME

Hungarian Academy of Sciences

CENTRAL RESEARCH INSTITUTE FOR PHYSICS

BUDAPEST

2017

KFKI-1981-01

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Publishing House of the Hungarian Academy of Sciences
H-1363 Budapest, P.O.B. 24, Hungary

To appear in an abriged form in Proceedings of 4th Conference on Quantum Theory and the Structures of Time and Space, Tutzing 1980.

HU ISSN 0368 5330
ISBN 963 371 777 9

*Work partially supported by funds from a Research Corporation Grant administered through Loyola University, New Orleans, and presented at the 4th conference on the Quantum Theory and the Structure of Time and Space, Tutzing, July 7 - 11, 1980.

ABSTRACT

A program of quantization of relativistic local field theories in terms of Hilbert modules over non-commutative C^*-algebras is outlined. The spacetime of the considered systems should become a "quantum" one representable by a Hilbert space. Two suggestions are given that how this quantum spacetime might be determined.

АННОТАЦИЯ

Намечается программа квантования локальных теорий полей с помощью Гилбертовых модулей над некоммутативными C^*-алгебрами. Пространство-временная структура рассматриваемых моделей станет квантованной, которая репрезентируется Гилбертовым пространством. Дается два предложения для возможного определения этой квантованной пространство-временной структуры.

KIVONAT

Körvonalazzuk a relativisztikus lokális térelméletek kvantálásának programját nem kommutativ C^*-algebrák fölötti Hilber modulusok segitségével. A tekintett rendszerek téridő-modellje "kvantáltá" válik, amit egy Hilbert-tér reprezentál. Két javaslatot adunk ennek a kvantum-téridőnek a lehetséges meghatározására.

INTRODUCTION

It is a serious problem of quantum theory to find a sound analytical formalism, satisfactory physically as well as mathematically, for handling field theoretic systems. Another, theoretically very important, unsolved problem of physics is the unification of quantum theory and (general) relativity. In our opinion the two problemes must be in a deep connection. Our proposal for the former problem is the investigation of topological module structures over *-algebras (in particular over C*algebras) [1]. The main goal of this paper is to show a possible way that the latter problem might be connected to the former in this new mathematical framework of quantum theory. For this purpose we first summarize our results, and outline a program that we call a "C*-module quantization" of local field theories.

Along the quantum logic approach of quantum theory [2, 3, 4, 5] we investigated in [6, 7] propositional systems for local field theories, using the new technique of lattice-valued logics. The uncertainties of principle in measuring processes made on field theoretic systems can be formulated intrinsically in these lattice-valued propositional systems. One can characterize the classical and quantum, non-relativistic and relativistic cases of physical theories in a natural way in these logics.

The favorite candidate for realization of these lattice-valued logics is a Hilbert module $\mathcal{H}_A$ over a C*-algebra A. This fact suggests that we may use a Hilbert module $\mathcal{H}_A$ for describing a local quantum field theoretic system (instead of a Hilbert space). In [6, 8] we developed the Hilbert-module formalism of non-relativistic quantum field theory. In this case $\mathcal{H}_A$ can be constructed over the commutative C*-algebra $C_o(\mathbb{R}^3)$ of the bounded compex valued continuous functions which vanish at infinity on the configuration space $\mathbb{R}^3$ of the system.

The local propositional system of a relativistic quantum system is representable with a Hilbert module $\mathcal{H}_A$ over a non-commutative C*-algebra A. Thus we should know the theory of Hilbert modules over non-commutative C*-algebras to be able to develop the Hilbert module formalism of a relativistic quantum field theoretic system.

The first physical problem in this direction is the determination of A. The lattice of self-adjoint projectors of A represents the lattice ℓ of values of local propositions and thus A can be generated by ℓ. Following from

Einstein causality ℓ must have a deep connection with the causal structure of spacetime $\mathbb{M}^4$ [7].

If one starts with the causal structure of $\mathbb{M}^4$ to solve this problem then it can be shown (see in [9]) that the closed sets of the Alexandrov topology of spacetime constitutes a complete orthomodular, irreducible and atomic lattice $\ell(\mathbb{M}^4)$, where the atoms are the points of $\mathbb{M}^4$. But this lattice does not satisfy the covering law [5, 10], and thus we cannot apply Piron's results to realize $\ell(\mathbb{M}^4)$, and there are other complications with this lattice $\ell(\mathbb{M}^4)$, too.

We get an another more intuitive starting point, which is also suggested by recent works of Marlow [11, 12, 13], to solve the problem if we recognize that the real object is a "quantum spacetime" $Q\mathbb{M}^4$ (representable with a separable, as we should prefer, Hilbert space $H(\mathbb{M}^4)$), rather than a classical one, which should reduce to the classical spacetime $\mathbb{M}^4$ in a certain classical limit.

Along this line of thought we proceed with the following, physically plausible, consideration: The arbitrarily small cells of spacetime $\mathbb{M}^4$ cannot represent a physically observable reality (like the arbitrarily small cells of phasespace represent no measurable reality in quantum mechanics). There is a Heisenberg type uncertainty relation.

$$\Delta x_o \ \Delta x_1 \geq \frac{1}{2} \hbar$$

for the coordinates of a point (event) $x_\mu \in \mathbb{M}^2$ (considering now for simplicity a two-dimensional spacetime) with a new Planck constant $\hbar$ characteristic for the spacetime (from a simple consideration $\hbar \sim 10^{-37}$ cms). The timelike and spacelike components of an event x_μ cannot be measured with an arbitrary precicion. Thus we are led to consider the components of x_μ as non-commutative quantities $\hat{x}_o$, $\hat{x}_1$ satisfying the commutation relation

$$[\hat{x}_\mu, \hat{x}_\nu] \subset -i\hbar\hat{A}_{\mu\nu}, \quad \hat{A}_{\mu\nu} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

The corresponding Hilbert space $H(\mathbb{M}^2)$ should represent the quantum spacetime $Q\mathbb{M}^2$ which we looked for, and the algebra of bounded operators on $H(\mathbb{M}^2)$ should give the non-commutative C^*-algebra A in $\mathcal{R}_A$.

Some remarks: The idea that the spacetime coordinates would have to be <u>operators</u> in a semantically consistent quantum theory of fields was emphasized by von Weizsäcker [14, 15]. Thus the detailed elaboration of the above "canonical quantisation" of spacetime would be a kind of realization of this idea.

It is a well known idea of Heisenberg that quantum uncertainties should emerge in the spacetime metric in small spacetime regions but the spacetime points remain unchanged, i.e., the null cones should become "smeared out" in the quantum area. Then the relation of cause and effect, the causality becomes indefinite.

In the twistor theory of Penrose [16, 17], spacetime points arise as secondary concepts corresponding to linear sets in twistor space, and they, rather than the null cones should become "smeared out" on the passage to a quantized relativity theory.

It is straightforward to see that both the spacetime points and null cones should become "smeared out" in the above "canonically quantized" spacetime. Thus this quantization of spacetime would involve both ideas mentioned above and would take a step forward relative to each, simply unifying them. We could say in connection with causality that this is a macroscopic concept and microscopically is unobservable (in about $\hbar$ sized regions).

Finally, it seems so that this quantum spacetime QM^4 provides a kind of a super relativity principle suggested by M. Davis, who established a relativity principle of quantum measurements is [18], because the geometrical symmetry group of QM^4 is a kind of "blend" of the Lorentz and unitary groups.

We would like to note that we choose a pragmatic and positivistic attitude to find first a mathematically consistent formalism for the relativistic quantum field theory and the hard problems of a complete interpretation is left for further research.

2. A "QUANTUM LOGIC" APPROACH OF LOCAL FIELD THEORIES

To solve the acute difficulties of quantum field theory (divergences and others), at least partly, we think that one should find, first of all, a mathematically well defined kinematical picture in which we can then implement the dynamical principles. In such a kinematical picture, the uncertainties of measuring processes made on physical fields (or on observables) must be reflected intrinsically in the resulting mathematical structures. The uncertainties of principle consist of the following two components:

1) The measurings of two different observables on a (small) region of the physical space Ω (or, ideally, at a point $x \in \Omega$) can disturb each other. (In non-relativistic quantum mechanics this is the only considered case).

2) The measurings of observables in two <u>different regions</u> of the physical space Ω (or, ideally, at two different points $x_1, x_2 \in \Omega$) can disturb each other. (This is characteristic for a relativistic quantum field theory).

<u>Einstein causality</u> (or local commutativity) which refers to the second case restricts the possible regions (or points) to the light like or timelike separated ones.

Our starting point in [6, 7] was the quantum logic approach to axiomatic quantum theory [2, 3, 4, 5]. First we considered the classical local field theory. The space of the all possible values of local observables (the observation space of Birkhoff and von Neumann [2] of a local physical system $P(\Omega)$ is a submodule of $R^N(\Omega)$, where $R(\Omega)$is a real valued function algebra on Ω.

The experimental local propositions (briefly, local propositions or only propositions) of $P(\Omega)$ correspond to the subsets (of certain type) of the observation space $R^N(\Omega)$. From a brief consideration it is found that we preserve <u>all</u> information of the measurements in the simplest way if we allow a new "logical" value (called true-false value) for local propositions $S^N(\Omega)$ (see in [7]).

Now one could say that $S^N(\Omega)$ is

x) false if its value is the empty set ϕ,

xx) true-false if its value is a subset ω of Ω; $S^N(\Omega)$ is true on ω and false on $C\omega$.

xxx) true if its value is the whole set Ω.

We get the result that the local propositions of a classical physical system $P(\Omega)$ take their values in the power set $P(\Omega)$ and we are led to consider the system of propositions of $P(\Omega)$ as a "Boolean-valued logic". (In connection with this see the Boolean valued models of set theory in [19, 20]).

Following this conclusion we supposed (keeping in mind the two components 1) and 2) of the uncertainties of measurements) that the set $\ell=\{a, b, c, \ldots\}$ of values of the local propositions of a general physical system $P(\Omega)$ is a <u>complete orthomodular lattice</u> (CROC in terminology of Piron [5]) with union $\vee$, intersection $\wedge$ and orthocomplementation $'$, and every experiment was called a <u>local proposition,</u> which leads to an alternative of which the terms are the elements of a CROC ℓ. Let $\mathcal{L} = \{A, B, C, \ldots\}$ be the system of local propositions of a general $P(\Omega)$. Guided by practical reasons we imposed on $\mathcal{L}$, following Gudder [4] and Piron [5], such axioms that $\mathcal{L}$ became a CROC with union $\cup$, intersection $\cap$ and orthocomplementation $^-$, and with maximal element $\mathbf{1}$ and minimal element Θ. Then the notion of lattice (CROC)-valued logic was defined operationally introducing the mathematical concept of a value function which assigns to a local proposition $A \in \mathcal{L}$ its value a in ℓ. A value function v is a unitary c-morphism from $\mathcal{L}$ onto ℓ such that the condition $A \perp B \Leftrightarrow v(A) \perp v(B)$ is fulfilled. Thus a triple $(\mathcal{L}, \ell, V)$ is called a <u>CROC-valued logic</u> if $\mathcal{L}$ and ℓ are CROCs and V is the non empty class of value functions from $\mathcal{L}$ onto ℓ such that $\forall\, a \in \ell\ \exists\, A^a \in \mathcal{L}\ v(A^a) = a\ \forall\, v \in V$. It is clear that the element A^a is unique.

The definition shows that only those CROCs can occur as a propositional lattice $\mathcal{L}$ in a CROC-valued logic $(\mathcal{L}, \ell, V)$ which contain a sublattice isomorphic to ℓ, and this expresses the lattice-valuedness of the local propositions.

From practical reasons it was imposed on $\mathcal{L}$ the atomicity axiom with the covering law [5] (but the role of this axiom is not vital). Then a CROC-valued logic $(\mathcal{L}, \ell, V)$ is called a <u>CROC-valued propositional system</u> whenever $\mathcal{L}$ and ℓ are propositional systems, and classical if $\mathcal{L}$ and ℓ are distributive (Boolean).

The decompositions of CROC-valued logics into irreducible and weakly irreducible ones (see in [6, 7]) shows that:

1) Every CROC-valued propositional system is the direct union of irreducible CROC-valued propositional systems, and a CROC-valued propositional

system is irreducible if its center pair $(C,c) = (\{\Theta, \mathbb{1}\}, \{0, 1\})$. Furthermore, a CROC-valued logic is the direct union of irreducible CROC-valued logis if its center pair (C,c) is atomic.

2) Every CROC-valued logic $(\mathcal{L},\ell,V)$ is the direct union of weakly irreducible CROC-valued logics:

$(\mathcal{L},\ell,V) = \bigcup_\alpha (\mathcal{L}_\alpha,\ell,V_\alpha)$, where $C_\alpha \cong c$, $\forall\ \alpha$ (C_α is the center of $\mathcal{L}_\alpha$), and a CROC-valued logic is weakly irreducible if $C \cong c$ in its center pair (C, c).

Also for the realization of CROC-valued propositional systems, respectively, CROC-velued logics it is enough to know the realization of irreducible CROC-valued propositional systems, respectively, of weakly irreducible CROC--valued logics. (The results under 2) suggest that we may drop the axiom of atomicity in a more general consideration).

The above results allow us to classify the CROC-valued logics [7] and to characterize the different cases (classical and quantum, non-relativistic and relativistic) of local field theories. The two types of non-compatibility relative to measurement processes (1, and 2, above) can describe intrinsically in the CROC-valued propositional systems.

a) The local propositional systems of the classical (both non-relativistic and relativistic) local physical systems are classical CROC-valued propositional systems (if there exists such a propositional system).

b) The local propositional systems of non-reltivistic local quantum systems are CROC-valued propositional systems (if they exist) with <u>distributive</u> value lattices.

c) The local propositional systems of relativistic local quantum systems are CROC-valued propositional systems (if they exist) with <u>non-distributive</u> value lattices (see in [6, 7]).

The states, symmetries and (local) observables can be defined in a similar way to the corresponding definitions of Piron [5] with some modifications (mainly in the relativistic cases) [7]. Comparing with the algebraic approach we have shown that the postulates of isotony and local commutativity of Haag and Kastler [21] are satisfied by our propositional systems and the spacetime covariance can be implemented in natural way in these propositional systems.

On the other hand it is clear that one could control with the determination of the algebraic structure of local algebras generated by the CROC-valued propositional systems (or CROC-valued logics) the other postulates of Haag and Kastler in which they postulate the C^*-algebraic properties and structure of the algebra of local observables. Some results in [6, 8] suggest that the requirement of the C^*-algebraic postulate for the algebra of local observables on a physical system reduces the range of this approach essentially to the <u>non-relativistic</u> quantum field theoretic cases (apart from the cases of free fields).

Finally, let us give examples for CROC-valued logics.

Let Γ be a set then $(\mathcal{P}(\mathcal{P}(\Gamma)), \mathcal{P}(\Gamma), V)$ is a classical CROC-valued propositional system, where the elements of V are generated by the real valued functions $\Gamma \to \mathbb{R}$ (if Γ is countable) [7].

2) Let $H_A = (H,A)$ be a faithful AW*-module of Kaplansky [22] with a commutative AW*-algebra A generated by a complete lattice $\ell(A)$ of its projectors. Then $(\mathcal{P}(H_A), \ell(A), V)$ is a weakly irreducible CROC-valued logic, where $\mathcal{P}(H_A)$ denotes the set of all AW*-submodules of H_A [7].

3) Our conjecture as a general example for a weakly irreducible CROC--valued logic with a non-distributive ℓ is the following: Let $H_A = (H,A)$ be a faithful Hilbert module over the non-commutative C*-algebra A generated by a CROC $\ell(A)$ of projectors. Then $(\mathcal{P}(H_A), \ell(A), V)$ is a weakly irreducible CROC-valued logic, where $\mathcal{P}(H_A)$ denotes the set of all closed submodules of H_A [7].

As a generalization of these examples we may expect that all weakly irreducible lattice-valued logics are representable in this way via a Hilbert module $(\mathcal{H},A,\phi)$, where $\mathcal{H}$ is also a left module over the C*-algebra A and ϕ is definite Hermitian form, constructed on $(\mathcal{H},A)$, taking values in A.

3. A C*-MODULE QUANTIZATION OF LOCAL FIELD THEORIES

The above example (example 2)) suggests that we use a Hilbert module $H_A = (H,A,\phi)$, in the description of a local quantum system $P(\Omega)$ in a similar way as a Hilbert space is used in the Hilbert space formulation of a quantum mechanical system.

We developed a quantization formalism of non-relativistic local field theoretic systems in [6, 8] in terms of a Hilbert module, with an introduction to the theory of Hilbert modules as topological modules. In these cases the Hilbert modules $(\mathcal{H}_A,\phi)$, the "state modules", can be constructed over the commutative C*-algebra $C_o(\mathbb{R}^3)$ of all complex valued continuous functions which vanish at infinity on the configuration space $\mathbb{R}^3$ of the system (or over $C(\Omega)$ if the system under consideration is concentrated on a compact subset Ω of $\mathbb{R}^3$). $\mathbb{R}^3$ is equipped with the usual Euclidean distance topology and with the geometrical symmetry group $\mathbb{E}^3$. ϕ is a definite Hermitian form from $\mathcal{H}_A \times \mathcal{H}_A$ to $A = C_o(\mathbb{R}^3)$ and generates an A-valued norm $\phi(X,X)^{1/2} = ||X||_A$ on $\mathcal{H}_A$, in which $\mathcal{H}_A$ is complete.

The local and global aspects of the considered systems are studied in detail in [6, 8]. The local states are represented by the elements of $\mathcal{H}_A$ (with A-norm 1 on a compact subset Ω in $\mathbb{R}^3$ for bound states of the system), The local observables are represented by self-adjoint (module) operators on $\mathcal{H}_A$ and the local symmetries are represented by (A-) unitary operators on $\mathcal{H}_A$ (an A-unitary operator leaves the form ϕ invariant). The global aspects of the system (global states, observables and symmetries) are generated by the corresponding local aspects via the "states" of the C*-algebra $A = C_o(\mathbb{R}^3)$ (or $C(\Omega)$), which are generated by regular Borel measures on $\mathbb{R}^3$ (or on Ω), as it is well known from a theorem of Segal [23]. (We note that one may interpret these states of A as the states (possible preparations) of the measuring apparatuses (see in [8])). This procedure of the description of the quantized system suggests the name "C*-module" quantization; this approach would be a kind of "blend" of the C*-algebraic and Hilbert space approaches of quantum theory.

It is an interesting fact that many unitarily inequivalent Hilbert spaces, representing the global states, observables and symmetries of the system, correspond to the same Hilbert module $\mathcal{H}_A$ of local states (Hilbert spaces belonging to inequivalent regular Borel measures on $\mathbb{R}^3$). Also it seems that the Hilbert module which describes the system locally is an invariant concept under local canonical transformations (represented by A-unitary operators) while it incorporates the globally inequivalent representation spaces of the system.

Let us see briefly the concrete quantization procedure of a Lagrangian local non-relativistic field theory using a Hilbert module as a "state module".

a) The system is given classically by its Lagrangian density:

$$\mathcal{L} = \mathcal{L}(\varphi(x,t),\ \nabla\varphi(x,t),\ \dot{\varphi}(x,t),\ t)$$

where $\varphi(x,t) : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}$. The energy density and energy are

$$\mathcal{H}(\varphi,\pi) = \pi\dot{\varphi} - \mathcal{L}(\varphi,\pi),\ H = \int_{\mathbb{R}^3} \mathcal{H}(\varphi,\pi)d^3x.$$

($\pi = \frac{\partial\mathcal{L}}{\partial\dot{\varphi}}$) (for the sake of simplicity here we neglect the contribution of $\nabla\varphi$ in $\mathcal{L}$).

b) Now the "state module" is (H_A,ϕ), where $A = C_o(\mathbb{R}^3)$ and $\phi : H_A \times H_A \to C_o(\mathbb{R}^3)$ is a definite Hermitian form, and H_A has an ω-sequence of orthogonal elements (if the system is concentrated on a (large) compact subset Ω of $\mathbb{R}^3$ then this sequence is orthonormal). The infinitesimal time evaluation of the system is determined by the Schrödinger equation of the form:

$$i\hbar \frac{\partial\Phi(\varphi,t)}{\partial t} = \hat{\mathcal{H}}(\hat{\varphi},\hat{\pi})\Phi(\varphi,t)$$

where $\Phi(\varphi,t) \in H_A$, $\hat{\mathcal{H}} : H_A \to H_A$ is an A-linear (module) operator such that $\hat{\mathcal{H}} = \hat{\mathcal{H}}^+$ (self adjoint) and $\hat{\varphi},\hat{\pi} : H_A \to H_A$ are A-linear operators such that they satisfy the C.C.R.:

$$[\hat{\pi},\hat{\varphi}] \subset -i\hbar\, \mathbf{1}.$$

The form of $\hat{\mathcal{H}}$ is given by its classical counterpart $\mathcal{H}(\varphi,\pi)$. If $H_A = L^2[R(\Omega),\delta\varphi]$ is the Hilbert module of square integrable functions on $R(\mathbb{R}^3)$ to $C(\mathbb{R}^3)$ then

$$\hat{\varphi} = \varphi\cdot \quad \text{and} \quad \hat{\pi} = -\, i\hbar \frac{\delta}{\delta\varphi}\ .$$

If the "state vector" $\Phi(\varphi,t)$ satisfies the condition

$$\int_{\mathbb{R}^3} \phi(\Phi,\Phi)^{1/2}\, d^3x = \int_{\mathbb{R}^3} ||\Phi||_A\, d^3x = N$$

then we could say that $\Phi(\varphi,t)$ represents N "particles" in H_A.

This formalism is the natural generalization of the usual quantum mechanical one and its locality mature is striking. If we consider the system on $\bar{\Omega}$ (on the completion of the union of all compact subsets of $\mathbb{R}^3$) then the algebra of the (quasi-) local bounded observables of the system $P(\bar{\Omega})$ is given by $\mathfrak{B}(H_A)$ (the set of bounded module operators on H_A) which is a C*-algebra with center isomorphic to $A = C(\bar{\Omega})$. Thus the connection with the Haag-Kastler field theory is clear.

Further advantages: we can formulate the interaction via the Lagrangian formalism, and the concrete computations of local and global quantities can be carried through without divergences.

On the other hand the analysis is [6, 8] suggests that most of the mathematical concepts and methods used until now are applicable only for the relatively simple non-relativistic quantum systems.

In the more realistic case of relativistic local quantum field theory one should look for new mathematical methods and techniques.

By the way, we note in connection with the theory of Hilbert modules that many interesting connections exist between the topological module structures (essentially these are the natural generalization of topological vector space structures of functional analysis) and other mathematical structures, for example the direct integral and direct product spaces of functional analysis and even the differential geometric structures: a vector bundle is also a topological module over a commutative ring, the cross-sections of the bundle, the vector fields, are the elements of the module. Thus e.g., a Hilbert bundle is also a special Hilbert module. (H_A above gives the direct integral Hilbert space $H = \int_{R^3} H_x$ where H_x is a Hilbert space at the point $x \in \mathbb{R}^3$ with the same ω-sequence of basic elements as H_A has. Furthermore H_A is a Hilbert bundle with fiber H_x at the point x, too).

The next step along this line of thought would be the generalization of the above formalism to the local relativistic quantum field theory. We should like to outline a program now in this direction.

I, The classical configuration space of the system $P(\Omega)$ is $\Omega \subseteq \mathbb{M}^4$ (considering now only flat spacetime), this system is given classically by the Lagrangian density

$$\mathcal{L} = \mathcal{L}(\varphi(x),\ \partial_\mu\varphi(x), x)$$

where $x \in \Omega,\ \varphi(x):\Omega \to \mathbb{R}$ or $\mathbb{C}$.

For simplicity, let us consider only scalar fields. Now the energy-momentum tensor field and the energy-momentum 4-vector are

$$\mathcal{T}_{\mu\nu}(\varphi,\pi_\mu) = \pi_\mu \partial_\nu \varphi - g_{\mu\nu}\mathcal{L}(\varphi,\pi_\mu),$$

$$P_\mu = \int_\sigma \mathcal{T}_{\mu\nu} d\sigma^\nu .$$

II, Quantization: What is the "state module" (H_A,ϕ)? Following, more or less directly, from the second type of uncertainties in measurings, mentioned in Sec. 2., A cannot be $C(\Omega)$, also a commutative C*-algebra, now. A is a non-commutative C*-algebra and we should replace $\Omega \subseteq \mathbb{M}^4$ with a "non-commutative" one (spacetime).

We note that the simple notion that a C*-algebra is just "$C(\Omega)$ for a non-commutative Ω" is used quite fruitfully in the theory of C*-algebras [24].

Also we should replace $\mathbb{M}^4$ with a "quantum spacetime $\mathbb{QM}^4$". Thus our first problem is the determination of $\mathbb{QM}^4$, and by this, A.

We give some suggestion in the next two sections how this problem might be handled.

a) Once we have $\mathbb{QM}^4$ and A then the next problem is the determination of scalars, vectors, tensors of the geometrical symmetry group of $\mathbb{QM}^4$, denoted by $\hat{\mathcal{P}}(\hat{\mathcal{P}}$ is the Poincaré group of $\mathbb{QM}^4$).

Then we should translate all classical concepts and quantities (and the field equations, too), such as, e.g., $\mathcal{L}(\varphi,\partial_\mu\varphi)$ and $\mathcal{T}_{\mu\nu}(\varphi,\pi_\mu)$ into this new language:

$$\mathcal{L}(\varphi,\partial_\mu\varphi) \to \hat{\mathcal{L}}(\hat{\varphi},\hat{\partial}_\mu\hat{\varphi}),$$

$$\mathcal{T}_{\mu\nu}(\varphi,\pi_\mu) \to \hat{\mathcal{T}}_{\mu\nu}(\hat{\varphi},\hat{\pi}_\mu).$$

This would be a "first quantization".

b) If we have A, then we construct (H_A,ϕ) over A as a left module with an ω-sequence of basic elements, and we represent $\hat{\mathcal{P}}_+^\uparrow$ on H_A. The local "spacetime evolution" of the system would be determined by the Schrödinger equation of the form (this is my conjecture):

$$i\hbar\hat{\delta}(dx)\Phi(\hat{\varphi},x) = \hat{\hat{\mathcal{T}}}(\hat{\varphi},\hat{\pi})\Phi(\hat{\varphi},x), \quad x \in \mathbb{QM}^4$$

where $\hat{\delta}(d\,x)$ denotes the infinitesimal generator of the "spacetime" translation on $\mathbb{QM}^4$. Some classical field theoretic argument suggests the following covariant form:

$$i\hbar\hat{g}_{(\mu}\hat{\delta}_{\nu)}\Phi^\nu(\hat{\varphi},x) = \hat{\hat{\mathcal{T}}}_{\mu\nu}(\hat{\varphi},\hat{\pi}_\mu)\Phi^\nu(\hat{\varphi},x)$$

where $x \in QM^4$, $\Phi^{\nu}(\hat{\varphi},x) \in H_A$, $\hat{\hat{\mathcal{T}}}_{\mu\nu}: H_A \to H_A$ and selfadjoint, $\hat{\hat{\pi}}_{\mu}$, $\hat{\hat{\varphi}}: H_A \to H_A$ such that they satisfy the C.C.R.:

$$[\hat{\hat{\pi}}_{\mu},\hat{\hat{\varphi}}] \subset -i\hbar\hat{g}_{\mu}$$

where $\hat{g}_{\mu}$ is in a connection with the conventional propagator. The form of $\hat{\hat{\mathcal{T}}}_{\mu\nu}$ is given by its first quantized counterpart $\hat{\mathcal{T}}_{\mu\nu}(\hat{\varphi},\hat{\pi}_{\mu})$. If H_A is represented by an appropriately generalized "function module" $\mathcal{L}^2[R(QM^4),\delta\hat{\varphi}]$ then we might have

$$\hat{\hat{\varphi}} = \hat{\varphi}\cdot \quad \text{and} \quad \hat{\hat{\pi}}_{\mu} = -i\hbar\hat{g}_{\mu}\frac{\delta}{\delta\hat{\varphi}} .$$

If the "state vector" $\Phi^{\nu}(\hat{\varphi},x)$ satisfies, for example, the condition of the form

$$\int_{\sigma} \hat{g}_{\nu}\, \phi(\Phi_{\gamma},\Phi^{\gamma})d\sigma^{\nu} = \int_{V} \phi(\Phi_{\gamma},\Phi^{\gamma})d^3x = N$$

where σ is a "spacelike" hyperplane in QM^4, then we might say that $\Phi^{\gamma}(\hat{\varphi},x)$ represents N "particle" in H_A. This would be a "second quantization" with clear locality nature in this approach.

4. SPACETIME STRUCTURE FROM LATTICE-VALUED PROPOSITIONAL SYSTEMS

In this section and the next one we should like to give suggestions for the possible ways out of the problem of the determinations of QM^4 and A. First we give an abstract way for handling the problem, while in the next section a more physical suggestion will be given.

In the lattice-valued propositional system $(\mathcal{L},\ell,V)$ of a realativistic local system, the conceptual uncertainty of second type in measuring processes (see in Sec. 2.) is expressed by the non-distributive character of ℓ. Einstein causality determines the distributive sublattices of ℓ, thus the structure of ℓ is in a deep connection with the causal structure of the spacetime $\mathbb{M}^4$.

If one considers the causal structures of $\mathbb{M}^4$ then one can show that the closed sets [defined by $(J^+(p) \cap J^-(q)) \cup (J^+(q) \cap J^-(p))$] of the Alexandrov topology of the spacetime constitute a complete orthomodular and atomic lattice $\ell(\mathbb{M}^4)$, where the atoms are the points of $\mathbb{M}^4$ (see [9] for the proof and definition of the union and intersection and orthocomplementation in $\ell(\mathbb{M}^4)$). The center of $\ell(\mathbb{M}^4)$ consists of the empty set and the whole $\mathbb{M}^4$, thus $\ell(\mathbb{M}^4)$ is an irreducible atomic CROC. But the covering law [5] is not satisfied by $\ell(\mathbb{M}^4)$. Hence $\ell(\mathbb{M}^4)$ is not a propositional system and Piron's theorem is not applicable to realize $\ell(\mathbb{M}^4)$, i.e. $\ell(\mathbb{M}^4)$ is not realizeable by the set of all closed subspaces of a generalized Hilbert space. An another complication with this lattice $\ell(\mathbb{M}^4)$ is that the light like separated regions of $\mathbb{M}^4$ correspond to compatible elements of $\ell(\mathbb{M}^4)$ thus massless fields would exclude from the quantized theory if $\ell(\mathbb{M}^4)$ is used as a lattice of values of local propositions.

We could approach the problem, abstractly, from an axiomatic point of view. Let the local physical system be given by a lattice-valued propositional system. Also we start with abstract, physically justifiable axioms and the corresponding structures (now these are the lattice-valued propositional systems (logics) $(\mathcal{L},\ell,V)$), and we look for the corresponding physical (phenomenologically observable) systems with their configuration spaces (spacetimes) together. We saw in [7] that the lattice ℓ is in direct connection with the usual physical spaces $\mathbb{R}^3$, respectively, $\mathbb{M}^4$. We may expect that ℓ determines the physicsal space of the system corresponding to the abstract $(\mathcal{L},\ell,V)$. Thus we are led to the following question: What kind of "spacetimes" follow from lattice-valued propositional systems i.e., from the different, abstractly determined ℓ' s?

Now, given ℓ in a concrete $(\mathcal{L},\ell,V)$ with its abstract structure and with its symmetry group $\mathcal{G}_\ell$. Then the mathematical space that realizes ℓ with its "power structure" (power-set for a Boolean propositional system or $\mathcal{P}(H)$ for an irreducible propositional system, for example) gives the corresponding physical space (configuration space or spacetime) and the symmetry group generated by $\mathcal{G}_\ell$ gives the geometrical symmetry group of this physical space.

We can divide into two parts of the problem:

1) ℓ is distributive, i.e., the corresponding $(\mathcal{L},\ell,V)$ describes a non-relativistic quantum system. In this case, ℓ should determine the configuration space of the system. The time is handled separately as an absolute one.

2) ℓ is non-distributive, i.e., the corresponding $(\mathcal{L},\ell,V)$ should describe a relativistic quantum system. In this case, ℓ should determine the "spacetime" of the system.

We now consider briefly the case 2, and we suppose that $(\mathcal{L},\ell,V)$ is a CROC-valued propositional system. In these cases ℓ is a propositional system and, if it has at least four atoms, representable via a (generalized) Hilbert space $H(\Omega)$, following from Piron's result, where Ω is the set of the atoms of ℓ.

(When Ω is countable, $H(\Omega)$ is separable). Now the symmetry group $\mathcal{G}_\ell$ of ℓ generates the unitary group of $H(\Omega)$ [5]. We get that in a wide class of lattice-valued propositional systems the "spacetime" is represented by a <u>Hilbert space</u> $H(\Omega)$ and the corresponding symmetry group is the unitary group of $H(\Omega)$. This fact suggests that we consider the "spacetime" represented by $H(\Omega)$ as a <u>"quantum spacetime"</u>. Physically observable quantities of this q-spacetime are represented by self adjoint operators on $H(\Omega)$. Thus the observable time and space coordinates should be represented by <u>operators</u> on $H(\Omega)$ and the spectrum of this time operator should determine a <u>chronology</u> for the corresponding observer (cf. [11]).

For these "quantum spacetimes" to be realistic and phenomenologically allowable, they should satisfy the "correspondence principle" that they reproduce the classical spacetime $\mathbb{M}^4$ in a classical limit; e.g., the lattice of closed subspaces of $H(\Omega)$ reproduces the causal lattice $\ell(\mathbb{M}^4)$ of $\mathbb{M}^4$ in a limit (the distributive sublattices of $\mathcal{P}(H(\Omega))$ determine spacelike hyperplanes in $\mathbb{M}^4$).

In these cases the determination of the non-commutative C^*-algebra A in H_A is very simple; it is the von Neumann algebra generated by the projectors of $H(\Omega)$.

5. A "CANONICAL QUANTIZATION" OF SPACETIME

We suggest now a more physical way for the determination of $Q\mathbb{M}^4$ and A. We start with the following physically plausible consideration: The arbitrary small cells of spacetime $\mathbb{M}^4$ cannot represent a physically observable reality (like the arbitrarily small cells of phasespace represent no measurable reality in quantum mechanics). The spacetime events, the elements of $\mathbb{M}^4$, do not correspond to measurable objects. Thus Heisenberg type uncertainty relations of the form

$$\Delta x_o \Delta x_i \geq \frac{1}{2}\hbar, \quad i = 1, 2, 3 \tag{5.1}$$

should hold for the coordinates of a spacetime event $x_\mu \in \mathbb{M}^4$ with a new Planck constant $\hbar$ characteristic for spacetime. The timelike coordinate of x_μ together with its spacelike coordinates cannot be measured with arbitrary precision, while separatly can be measured! The great empirical success of non-relativistic quantum mechanics which presupposes the Euclidean space geometry without further assumptions could justify the assumption that the spacelike coordinates of an event can be measured with arbitrary precision. (We note here that this assumption should be dropped if an elementary length does exist in nature). In connection with the magnitude of $\hbar$ we get from a simple consideration that $\hbar \sim 10^{-37}$ cmsec, considering the characteristic distances $\Delta x \sim 10^{-13}$ cm in nuclei and the characteristic time of resonances $\Delta t \sim 10^{-24}$ sec.

Thus we are led to consider the coordinates of a spacetime fourvector x_μ as non-commuting quantities

$$x_\mu \to \hat{x}_\mu = (\hat{x}_o, \hat{x}_1, \hat{x}_2, \hat{x}_3) \tag{5.2}$$

which satisfy the commutation relation

$$[\hat{x}_\mu, \hat{x}_\nu] \subset -i\hbar\hat{A}_{\mu\nu} \tag{5.3}$$

$$\text{where} \qquad \hat{A}_{\mu\nu} = \begin{pmatrix} 0 & 1 & 1 & 1 \\ -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} . \tag{5.4}$$

This means that we may consider the coordinates of a four-vector x_μ as self-adjoint operators on a Hilbert space $H(\mathbb{M}^4)$ with identity operator 1. It follows at once that the 4-dimensional vector space $\mathbb{M}^4$ becomes a "non commutative" vector space $Q\mathbb{M}^4$:

$$\mathbb{M}^4 \to Q\mathbb{M}^4 : = \{\hat{\underline{x}} \mid \hat{\underline{x}} = (vx)_\mu e^\mu, \; \langle e_\nu, e^\mu \rangle = \delta^\mu_\nu,$$

$v_\mu \in \mathbb{R}$, $\hat{x}_\mu$'s are selfadjoint operators on $H(\mathbb{M}^4)$, satisfying (5.3)} with the scalar product:

$$< \hat{\underline{x}}, \hat{\underline{x}} > = < \hat{x}_\mu e_\mu, \; \hat{x}^\nu e^\nu > = \hat{x}_\mu \hat{x}^\mu .$$

Now the pseudo metric of $\mathbb{M}^4$ becomes an operator equation:

$$s \to \hat{s} = \hat{g}_{\mu\nu} (\hat{x}^\mu - \hat{y}^\mu)(\hat{x}^\nu - \hat{y}^\nu) \tag{5.5}$$

where

$$\hat{g}_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} . \tag{5.6}$$

The equation (5.5) should determine the metrical properties of $Q\mathbb{M}^4$ via its eigenvalue problem; (the spacelike, light like and timelike regions of $Q\mathbb{M}^4$).

Now the transformations leaving the equations (5.3) and (5.5) invariant should give the elements of geometrical symmetry group $\hat{\mathcal{P}}$ of $Q\mathbb{M}^4$ (the Poincaré group of the "quantum spacetime"). The transformations leaving the following indefinite form

$$(\hat{x}^0)^2 - (\hat{x}^1)^2 - (\hat{x}^2)^2 - (\hat{x}^3)^2$$

and (5.3) invariant should define the "Lorentz" subgroup $\hat{L}$ of $\hat{\mathcal{P}}$. Such a transformation has the following general form:

$$\hat{x}'^\mu = \hat{\Lambda}^\mu_\nu \, \hat{x}^\nu = \Lambda^\mu_\nu \, \hat{U} x^\nu \hat{U}^{-1}, \; \hat{U}\hat{U}^{-1} = 1, \tag{5.7}$$

where Λ^μ_ν's are real numbers transforming $\hat{x}^\nu$ as a vector, while $\hat{U}$ is a unitary operator on $H(\mathbb{M}^4)$ transforming $\hat{x}^\nu$ as an operator on $H(\mathbb{M}^4)$. Hence we have the following two conditions determining the elements of $\hat{L}$:

$$\hat{g}_{\mu\nu}\hat{x}^{\mu}\hat{x}^{\mu} = \hat{g}_{\mu\nu}\hat{x}'^{\gamma}\hat{x}'^{\nu} = \hat{g}_{\mu\nu}\hat{\Lambda}^{\mu}_{\gamma}\hat{x}^{\gamma}\hat{\Lambda}^{\nu}_{\alpha}\hat{x}^{\alpha} = \hat{g}_{\gamma\alpha}\hat{x}^{\gamma}\hat{x}^{\alpha}, \tag{5.8}$$

$$[\hat{x}'^{\mu},\hat{x}'^{\nu}] = [\hat{\Lambda}^{\mu}_{\gamma}\hat{x}^{\gamma},\hat{\Lambda}^{\nu}_{\alpha}\hat{x}^{\alpha}] = [\hat{x}^{\mu},\hat{x}^{\nu}]. \tag{5.9}$$

A simple computation gives from (5.8) and (5.9):

$$\hat{g}_{\gamma\alpha} = \hat{\Lambda}^{T\mu}_{\gamma}\hat{g}_{\mu\nu}\hat{\Lambda}^{\nu}_{\alpha},\ \hat{g} = \hat{\Lambda}^{T}\hat{g}\hat{\Lambda}, \tag{5.10}$$

$$\hat{A}^{\mu\nu} = \hat{\Lambda}^{\mu}_{\gamma}\hat{A}^{\gamma\alpha}\hat{\Lambda}^{T\nu}_{\alpha},\ \hat{A} = \hat{\Lambda}\hat{A}\hat{\Lambda}^{T}. \tag{5.11}$$

We see that only those Lorentz matrices $\hat{\Lambda}$ are the elements of $\hat{L}$ which leave the anti-symmetric matrice $\hat{A}$ invariant. This leads to a breaking of the Lorentz invariance in the "quantum spacetime" QM^4. (Such a symmetry breaking would not appear in a 2-dimensional QM^2 because in this case $\hat{A}$ is the anti--symmetric tensor $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$).

At the present stage of our work we do not know the immediate consequences of this symmetry breaking. Nevertheless, we see that the geometrical symmetry "group" of QM^4 is a kind of blend of the Lorentz and unitary groups and thus we might get in this way a "superrelativity" principle suggested earlier by M. Davis [18].

Finally, let us give a representation for $\hat{x}_{\mu}$.

Let $H(\mathbb{M}^4)$ be the Hilbert space $L^2(\mathbb{R}^3)$ then the following operators* satisfy the C.R. (5.3)

$$\hat{x}_o = -i\hbar\left(\frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2} + \frac{\partial}{\partial x_3}\right), \qquad \hat{x}_1 = x_i\cdot, \qquad \hat{x}_2 = x_2\cdot, \qquad \hat{x}_3 = x_3\cdot$$

Also in this approach the separable Hilbert space $L^2(\mathbb{R}^3)$ represents the "quantum spacetime" $H(\mathbb{M}^4)$; the unit vectors in $L^2(\mathbb{R}^3)$ might be interpreted as the "events of the quantum spacetime". The time operator $\hat{x}_o$ has a continuous spectrum which defines a chronology for the corresponding observer. It is worthwhile to note that the time operator $\hat{x}_o$ has a discrete spectrum in a finite universe (in space) (as is well known from the practice of q. m.).

This quantum spacetime would satisfy easily the "correspondence principle" mentioned in the foregoing section, because QM^4 is reduced to $\mathbb{M}^4$ if we take the "limit $\hbar \to 0$".

We note that the detail elaboration of the above ideas is the task of the further research.

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* This representation is due to Prof. T. Nagy.

6. ON SOME CONSEQUENCES

We now list some immediate consequences of the above "canonical" quantization of spacetime.

a) Relativistic mechanics on QM^4: We could consider the relativistic point mechanics on QM^4 such that we replace the spacetime coordinates x_μ with operators $\hat{x}_\mu$ in the spacetime dependent classical formulas. For example, the energy-momentum four vector of a particle classically is defined by the formula:

$$P_\mu = - \frac{\partial S(x)}{\partial x^\mu}$$

where $S(x)$ is the spacetime dependent action of the particle, and thus the corresponding operator is

$$\hat{P}_\mu = - \frac{\partial \hat{S}(\hat{x})}{\partial \hat{x}^\mu} = - \hat{\partial}_\mu \hat{S}(\hat{x}).$$

Now the Hamilton-Jacobi equation becomes the operator equation

$$\hat{\partial}_\mu \hat{S}(\hat{x})\hat{\partial}^\mu \hat{S}(\hat{x}) - \hat{m}^2 = \Theta$$

for a free particle and

$$(\hat{\partial}_\mu \hat{S}(\hat{x}) - e\hat{A}_\mu(\hat{x}))(\hat{\partial}^\mu \hat{S}(\hat{x}) - e\hat{A}^\mu(\hat{x})) - \hat{m}^2 = \Theta$$

for a particle moving in an external field $A_\mu(x)$. If $\hat{S}(\hat{x})$ is known (e.g., from its classical counterpart; the reason for such a choice might be that $\hat{S}(\hat{x})$ is classical for large scale comparison with $\hbar$) then this equation provides a nontrivial eigenvalue equation for the mass of the particle. When $\hat{m}$ would have a discrete spectrum we might interpret the corresponding states as "spacetime excitations".

In this way we may get a kind of "relativistic quantummechanics".

b) Field theory on QM^4: As we mentioned in Sec. 3 along the C*-module quantization program of local field theories, we may translate all classical concepts such as Lagrangian density, the principle of least action, field equations etc.

$$\mathcal{L}(\varphi, \partial_\mu \varphi) \to \hat{\mathcal{L}}(\hat{\varphi}, \hat{\partial}_\mu \hat{\varphi})$$

$$0 = \delta S = \delta \int d^4x \mathcal{L}(\varphi, \partial_\mu \varphi) \to \Theta = \hat{\delta} \hat{S} = \hat{\delta} \int d^4\hat{x} \hat{\mathcal{L}}(\hat{\varphi}, \hat{\partial}_\mu \hat{\varphi})$$

$$0 = \frac{\delta\mathcal{L}}{\delta\varphi} - \partial_\mu \frac{\delta\mathcal{L}}{\delta\partial_\mu\varphi} \to \frac{\delta\hat{\hat{\mathcal{L}}}}{\hat{\hat{\delta\varphi}}} - \hat{\partial}_\mu \frac{\delta\hat{\hat{\mathcal{L}}}}{\hat{\hat{\delta\partial_\mu}}\hat{\varphi}} = \Theta$$

using QM^4 instead of the classical M^4.

In our C*-module quantization program of relativistic local fields this procedure would mean the "first quantization".

c) Quantum gravity: We recommend QM^4 for local coordinatization of the general "quantum" spacetime manifold. In the development of a "quantum" differential geometry (Riemannian geometry) the first steps have been taken by A. R. Marlow in his recet works [13, 25]. We hope that we can arrive not too long at a more concrete "Riemannian geometry" of quantum spacetime with the use of the above suggested concrete "quantum" Minkowski space QM^4. It can be seen that the Einstein equations can be translated into this "quantum" language once we have the quantum Riemannian geometry. The main problem would be in this direction the determination of the antisymmetric quantity $\hat{A}_{\mu\nu}$ in the curved quantum spacetime, which should determine the commutation properties of local coordinates.

7. REMARKS AND CONNECTIONS WITH OTHER APPROACHES

α) The lattice-valued quantum logic was introduced by us simply generalizing the usual two-valued quantum logic approach of Birkhoff and v. Neumann and Gudder and Piron, following the conclusion that the experimental local propositions of a classical local field theory assume truth values in a Boolean algebra. Nowadays G. Takeuti [26] introduced a quantum set theory generalizing the Boolean-valued models of Scott and Solovay [19, 20] to L-valued Models where L is the lattice of closed subspaces of a Hilber space H. He has shown that a) a reasonable set theory holds in L-valued models and b) as a special result, the real numbers in an L-valued model correspond to the self-adjoint operators on H. Now is is clear that 1) the quantum set theory of Takeuti is in a close connection with the lattice-valued quantum logic $(\mathcal{L},\ell,V)$ of us. For, if $\ell = L$ then $(\mathcal{L},\ell,V)$ provides a special example for L-valued models; the value functions in V are elements in the universe $V^{(\ell)}$ of the corresponding L-valued model. Furthermore 2) the functional analysis based on quantum set theory may provide the theory of modules over operator algebras (*-algebras) and thus we could develop the theory of topological modules over C*-algebras (and so the theory of Hilbert modules) in an elegant and satisfactory way using the mathematics based on quantum set theory (and we could say similar remark about the quantum differential geometry).

We note that D. Finkelstein introduced in this conference a new quantum set theory different from that of Takeuti; he quantizes the Frége bracket while Takeuti quantizes the characteristic function.

β) M. Davis in [18] interpreting in a possible way the model and result of Takeuti (result b) above) established a relativity principle in quantum mechanics introducing Boolean reference frames in it; he used the complete Boolean subalgebras of L as reference "frames". The quantization of a classical theory means in his context the application of appropriate Boolean valuations to the formulas ("sentences") of the theory. Now it can easily be seen that the conceptual framework of quantum theory suggested by M. Davis is in a straightforward connection with the approach to quantum theory proposed independently by us (and summarized in Sed. 2. and 3.). Davis suggested two directions for further research. One of these is that there should be a relativity theory combining special relativity and quantum theory in which the underlying group combines the Lorentz and the unitary groups. We already mentioned in Sec. 5 that the quantum spacetime QM^4 might provide such a relativity theory; Boolean reference frames (complete Boolean algebras) correspond to spacelike hyperplanes in QM^4. The second suggestion of Davis concerns quantum field theory (and quantum gravity). The elaboration of our lattice--valued quantum logical and C*-module quantizational approaches to field theory might be a kind of realization of this suggestion.

γ) In connection with the quantum spacetime of microworld, we want to mention that the S-matrice picture of Heisenberg suggests such a quantum spacetime representable with a Hilbert space $H(M^4)$, too; the incoming states on a spacelike hyperplane at $t = -\infty$ provide a basis for $H(M^4)$ and the (renormalized) propagator function (determining the transition probability) is proportional with the scalar produce of $H(M^4)$. The asymtotic completeness ($H_{in} = H = H_{out}$) in this context simply means that the physical system has a unique (quantum) spacetime. As a final remark we hope that we can report before long about the further progress.

ACKNOWLEDGMENTS

I would like to express my gratitude to Prof. T. Nagy and Dr. P. Forgács for fruitful discussions about the above themes, and I would like to thank Mrs. Germaine Murray for typing the manuscript.

July, 1980

REFERENCES

[1] M. Banai, preprint, KFKI-1978-20.

[2] G. Birkhoff, and J. von Neumann, Ann. Math. 37, 1936, 823-843.

[3] A. Mackey, The Mathematical Foundations of Quantum Mechanics, Benjamin, 1963, New York.

[4] S. Gudder, Axiomatic Quantum Mechanics and Generalized Probability Theory, Probabilistic Methods in Applied Mathematics, 1970, vol. 2. 53-129.

[5] C. Piron, Foundations of Quantum Physics, W. A. Benjamin, Inc., Reading, Mass. 1976.

[6] M. Banai, Thesis, University Loránd Eötvös, Budapest, 1980.

[7] M. Banai, Propositional Systems in Local Field Theories, preprint, KFKI-1980-42.

[8] M. Banai, A New Algebraic Approach to Non-Relativistic Quantum Theory, (in preparation).

[9] W. Cegta and A. Z. Jadczyk, Comm. Math. Phys. 57, 213-217 (1977).

[10] Private communication from W. Cegta.

[11] A.R. Marlow, An Axiomatic General Relativistic Quantum Theory, in Quantum Theory and Gravitation, ed. A.R. Marlow, Academic Press, 1980.

[12] A.R. Marlow, Spacetime Structure from Quantum Logic, Proceedings, Workshop on Quantum Logic, Erice, Dec. 2-9, 1979.

[13] A.R. Marlow, Empirical Topology: Topologies From Partially Ordered Sets, to be published, Int. J. Theor. Phys.

[14] C.F. von Weirzäcker, Geometrie und Physik, in Physical Reality and Mathematical Description, eds. Ch. P. Euz, F. Mehra, D. Reidel Publ. Co., Dordrecht - Holland, 1974.

[15] C.F. von Weizsäcker, Classical and Quantum Descriptions, in The Physicist's Conception of Nature, ed. F. Mehra, D. Reidel Publ. Co., Dordrecht - Holland, 1973.

[16] R. Penrose, M.A.H. Mac Callum, Twistor Theory: An Approach to the Quantization of Fields and Space-time, Phys. Rep. C6, No. 4. (1972) 241-316.

[17] R. Penrose, in Quantum Gravity-An Oxford Symposium, eds. C.F. Isham, R. Penrose, D.W. Sciama, Claredon, Oxford, 1975.

[18] M. Davis, Int. J. Theor. Phys. 16, No. 11, (1977), 867-874.

[19] Th. Jech. Lecture Notes in Set Theory, Lecture Notes in Mathematics, 217, Springer-Verlag, Berlin, 1971.

[20] G. Takeuti, W.M. Zaring, Axiomatic Set Theory, Springer, New York, 1973.

[21] R. Haag, D. Kastler, J. Math. Phys., 5(7), (1964), 848-861.

[22] I. Kaplansky, Amer, J. Math., 75, (1963), 839-858.

[23] I.E. Segal, Ann. Math, 48, (1947) 930-948.

[24] See, e.g., E.G. Effros, in C*-algebras and Applications to Physics, Proceedings, Los Angeles 1977, Lecture Notes in Math., Springer-Verlag, Berlin, 1978.

[25] A.R. Marlow, Categories and General Relativistic Quantum Theory, manuscript (talk on the Conference on Quantum Theory and the Structures of Time and Space, Tutzing, July 7-11, 1980).

[26] G. Takeuti, Quantum Set Theory, in Current Issues in Quantum Logic, proceedings of Erice Workshop on Quantum Logic, Dec. 2-9, 1979 (eds. E. Beltrametti, van Fraassen).

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Kiadja a Központi Fizikai Kutató Intézet
Felelős kiadó: Szegő Károly
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Nyelvi lektor: Forgács Péter
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Készült a KFKI sokszorosító üzemében
Felelős vezető: Nagy Károly
Budapest, 1981. január hó

MTA KFKI