KFKI-1980-124

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Hungarian Academy of Sciences

CENTRAL
RESEARCH
INSTITUTE FOR
PHYSICS

BUDAPEST

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HU ISSN 0368 5330
ISBN 963 371 769 8

ABSTRACT

Within the folding model approach to the microscopic optical model potential the nonlocal kernels are generated for projectile energies at 45 MeV and 95 MeV. Single particle ground state densities are used which simultaneously reproduce charge densities from electron scattering. To display important ingredients the nonlocal potential, for further use, exact phase equivalent forms are generated and used to evaluate differential cross sections with a local OMP code.

АННОТАЦИЯ

В рамках модели-фольдинг микроскопическим методом был определен нелокальный оптический потенциал при энергиях 45 и 90 МэВ. При этом использованная плотность одночастичных состояний репродуцирует и плотность зарядов, изулеченную из рассеяния электронов. Для определения важнейших свойств потенциала вычислен нелокальный и эквивалентный по фазе локальный вид потенциала. Определены дифференциальные сечения рассеяния нуклонов с использованием этого потенциала.

KIVONAT

A folding modell keretei között mikroszkopikus optikai potenciált, a megfelelő nonlokális kernelt állitottunk elő 45 és 90 MeV energiákon. A felhasznált egy részecske állapot sürüségek egyidejüleg reprodukálják az elektron szórásból nyert töltéssürüséget. A potenciál fontos sajátságainak kimutatása céljából a nonlokális és további, konkrét reakcióanalizisekben való felhasználásra az exakt fázis equivalens lokális formáit számitottuk ki. Lokális OMP code alapján megadtuk a differenciális hatáskeresztmetszetet.

1. Introduction

Recently great efforts have been made to compute quantitatively the complex optical model when starting from a realistic free internucleon potential[1]. These efforts resulted in nonlocal potentials, which were transformed into local equivalent potentials to make the results more transparent and easier comparable with phenomenological OMP. Thereby the definition and construction of exact phase equivalent potentials was most rewarding[2]. It permitted to understand the phenomenologically observed energy dependence and gave evidence for a ℓ-dependent repulsive core, which is beyond standard ℓ-independent Saxon-Woods parameterzations. Detailed analyses and applications made clear that the predicted ℓ-dependence may only be discernable for projectile energies above 100 MeV. Its effect is manifest in a backangle rise of the angular distribution. The results outlined in this note are generated with theoretical ingredients as described in another papers[1,2,3].

2. Theoretical background

The sutdy of interacting nucleons in infinitely extended nuclear matter is well established and approximate treatments for finite nuclei seem justified. Methods developed by Brückner and Bether /BB/ have thereby been widely applied and the theory with calculational procedures for the understanding of nucleon--nucleus elastic scattering starting from a realistic NN force is on from ground[1,2,3].

The approach pursued in our studies is based on the evaluation of the effective internucleon t-matrix from the free NN interaction. The real and imaginary optical model for nucleons we calculate to first order in the effective NN interaction with an improved version of the local density approximation /LDA/ in a fording approach with single particle target densities. The model relies on the quite general approach to generate in first approximation the OMP as á sum of a direct term and a nonlocal exchange term

$$U(\vec{r},\vec{r}';E) = \delta(\vec{r}-\vec{r}')\sum_n \int \phi_n^*(\vec{x})\, t_D(|\vec{r}-\vec{r}'|;E)\,\phi_n(\vec{x})\,dx^3 + \sum_n \phi_n^*(\vec{r})\, t_E(|\vec{r}-\vec{r}'|;E)\,\phi_n(\vec{r}') \qquad /1/$$

The coordinates $\vec{r}$ and $\vec{r}'$ refer to projectile coordinates with the summation of single particle wave functions we represent the best possible /Hartree-Fock/ particle densities - diagonal and mixed densities - for protons and neutrons. The basic ingredients of the LDA enters here in the choice of t_D and t_E which are mixtures of direct and exchange effective NN interactions. In principle it should be calculated in the finite system with its full structural details. The hypothesis is made that this effective interaction can be approximated by the one corresponding to the local density and energy dependent situation in nuclear matter. This effective interaction is our version of LDA.

The interaction contains automatically real and imaginary parts and the correct features of the finite range of the interaction. This is important since the ranges are different for real and imaginary parts and the relative spin and isospin channels. Any other approximation inherent in the nuclear matter approach in computing the effective interactions remains unaltered to previous calculations.

The stationary Schroedinger equation

$$\Delta\psi(\vec{r},\vec{k}) + (k^2 - V_D)\psi(\vec{r},\vec{k}) = \int U(\vec{r},\vec{r}';E)\psi(\vec{r}',\vec{k})\,d\vec{r} \qquad /2/$$

for the single particle OMP scattering solutions is most easily solved in the standard partical wave decomposition, where the numerical problem is reduced to an ordinary second order integrodifferential equation, viz.

$$\left[\frac{d^2}{dr^2} - \frac{L(L+1)}{r^2} + k^2 - V_D(r)\right] u_{LJ}(r) = \int_0^\infty \omega_L(r,r')\, u_{LJ}(r')\,dr' \qquad /3/$$

The diagonal potential v_D contains the standard homogenous charged sphere Coulomb potential and the spin orbit potential which we kapt in a local form[3].
The multipole decomposition of the nonlocal OMP is formally obtained for a rotational invariant symmetric potential from

$$U(\vec{r},\vec{r}') = U_D(r)\,\delta(\vec{r}-\vec{r}') + U_E(\vec{r},\vec{r}')$$
$$= \sum_L \omega_L(r,r')/_{rr'}\,(Y_L(\hat{r})\cdot Y_L(\hat{r}')) \qquad /4/$$
$$= \sum_L \frac{2L+1}{4\pi}\,\omega_L(r,r')P_L(\hat{r},\hat{r}')/_{rr'}$$

The local direct potential is subsummized into the nonlocal knockout exchange potential which represents the proper source of nonlocality. The energy dependence in /1/ results from the small energy dependence of the effective interaction. The multipole decomposition, eq. /4/ is technically straight forward but is numerically quite involved due to required energy and density interpolation of numerically stored effective interactions.

The folding integral for the direct potential is simple and was generated with a Gauss-Legendre integration routine when performing the radial and angular integrations.

$$U_D(r) = 2\pi \int_0^\infty \int_{-1}^{1} dr'\,dx \sum_{l_j,\tau} S^{\tau}_{l_j}\,\phi^2_{l_j,\tau}\left(\sqrt{r^2+r'^2+2rr'x'}\right) \qquad /5/$$
$$t_D(r', k_F(r), \bar{E}(r))$$

with S^{τ}_{lj} specifying the occupation number in the single particle orbit /lj/ for protons/neutrons /τ/. The radial wave functions $\phi_{lj,\tau}$/r/ are solutions of a Frahn-Lemmer type nonlocal bound state potential with parameters V_o=-72 MeV, r=1.2 $A^{1/3}$ fm,

a=0.65 fm, and the range of nonlocality =0.8 fm. Coulomb and spin orbit potentials are kept in the standard local form with V_{ls}=7 MeV, R_{ls}=1.1 $A^{1/3}$ fm, a_{ls}=0.65 fm.
The exchange potential is best directly generated in its multipole decomposition

$$\sum_{ST,n} \psi_n(\vec{r})\,\psi_n(\vec{r}')\,t_E^{ST}(|\vec{r}-\vec{r}'|, k_F((\vec{r}+\vec{r}')/2), E(\tfrac{\vec{r}+\vec{r}'}{2}))$$

$$= \frac{1}{r r'} \sum \phi_{lj,\tau}(r)\,\phi_{lj,\tau}(r')\,t_E^{ST}(|\vec{r}-\vec{r}'|, k_F, E) \quad /6/$$

$$Y_{lj,m}(\hat{r})\,Y_{lj,m}(\hat{r})\,S_{lj}^{\tau}$$

In the limit of no-spin/isospinflip this expression assumes the form

$$u_E(\vec{r},\vec{r}') = \frac{1}{r r'} \sum \phi_{lj,\tau}(r)\,\phi_{lj,\tau}(r')\,S_{lj}^{\tau} \langle l\,1/2\,\mu\,\nu\,|\,j m\rangle^2$$
$$(-)^{S+T+1} \langle 1/2\;1/2\;\nu\lambda\,|\,S m_S\rangle^2 \langle 1/2\;1/2\;\tau\varepsilon\,|\,T m_T\rangle^2 \quad /7/$$
$$Y_{l\mu}(\hat{r})\,Y_{l\mu}^{*}(\hat{r}')\,t^{ST}(|\vec{r}-\vec{r}'|, k_F, E)$$

Together with a multipole expansion of the effective interaction

$$t_E^{ST}(|\vec{r}-\vec{r}'|, k_F, E) = \sum_{\lambda} t_{\lambda}^{ST}(r,r')\,(Y_{\lambda}(\hat{r})\cdot Y_{\lambda}(\hat{r}')) \quad /8/$$

we obtain

$$\omega_L^E(r,r') = \sum_{lj,\tau,\lambda} \phi_{lj,\tau}(r)\,\phi_{lj,\tau}(r')\,\langle l\,\lambda\,0\,0\,|\,L\,0\rangle^2$$
$$S_{lj}^{t}\,\frac{\hat{\lambda}}{2\pi\hat{L}}\Big\{\frac{1}{4}(t_{\lambda}^{01} - 3t_{\lambda}^{11})\,\delta_{p,\tau} + \frac{1}{16}(3t_{\lambda}^{10} + t_{\lambda}^{01} \quad /9/$$
$$- t_{\lambda}^{00} - 3t_{\lambda}^{11})(1-\delta_{p\tau})$$

The isospin /proton/neutron/ of the projectile enters through the index p /for projectile/ on the Kronecker symbol $\delta_{p,\tau}$ and accounts for like or unlike projectile and target nucleons. As already mentioned, the required multipoles $t_\lambda^{ST}/r,r'/$ are computed from tabulated values $t^{ST}/s,k_F,E/$. To eliminate possible errors in interpolations we apply a double transformation and obtain

$$t_\lambda^{ST}(r,r') = 8\int_0^\infty k\,dk\, j_\lambda(kr) j_\lambda(kr') \int_0^\infty r\,dr\, \sin(kr)\, t^{ST}(|\vec{r}-\vec{r}'|) \qquad /10/$$

The multipole expansion of the local direct potential is straight forward and yields an L independent purely diagonal one:

$$\omega_L^D(r,r) = U_D(r)\delta(r-r') \qquad /11/$$

Since standard phenomenological OMP analysis uses local potentials it appears desireable to further delineate properties in the language of local potentials. We therefore outline the salient features of the transformation of a Schroedinger equation with a nonlocal potential eq. /3/ to a Schroedinger equation with a local potential

$$\left[\frac{d^2}{dr^2} - \frac{L(L+1)}{r^2} + k^2 - V_D(r)\right] U_{Lj}(r) = V_{eq}\, U_{Lj}(r) \qquad /12/$$

The local potential $V_{eq}/r/$ is said to be equivalent to the nonlocal kernel $\omega_L/r,r'/$ if it can be completely specified in terms of eq./3/ and its solutions and if it analytically reproduces observable features as a function of energy. The transformation for the nonlocal equation is described in ref. 2,3.

3. Results

^{24}Mg is known as rotational nucleus with relative large ground state deformations. Since the numerical results are intended to be applied in DWBA /p,d/ calculations we neglect the deformation and generate spherical OMP's. The ground state density for protons and neutrons are described within an independent particle model. It is shown in fig. 1 where we compare it with the charge density extracted from electron scattering experiments. Nucleon charge form factors are included, recoil effects are neglected. The shell model parameters are taken from literature[4] V_o=-62.09 MeV; r_o=1.33 fm; a_o=0.72 fm; U_{ls}=10 MeV; r_{ls}=1.1 fm; a_{ls}=0.6 fm.
Occupation numbers were chosen S_{1j}=j with the exception $S_{1d5/2}$=3.4; $S_{2s1/2}$=0.6.

The folding model uses the energy and density dependent t-matrices with fully complex central and non central components. The effective interaction has been generated within the self-consistent Brückner-Bethe theory of infinite nuclear matter with the Hamada-Johnston Potential as input[1].

The nonlocal potentials are computed for ^{24}Mg /p,p/; 95 MeV ^{23}Na /p,p/; 45 MeV and ^{23}Mg /n,n/; 45 MeV. Derived therefrom we generated phase equivalent potentials for the relevant partial waves. The spin orbit potentials are local, Table /4-6/. The transition from local to nonlocal solutions requires multiplication of wave functions with the damping function /Perey effect/. We restrict our representation of this damping function to L=o, since $\ell > o$ damping functions differ only by 10 to 15 % were wave functions are practically negligible small. For completeness the table values are graphically represented in figs. /2-8/.
The angular distribution for elastic scattering of 95 MeV protons on ^{24}Mg is shown in fig. 9, which is predicted by the

calculated ℓ-dependent microscopic local equivalent optical potential. A comparison with experimental data will show larger deviation as we are accustomed from phenomenological fits. This should not discourage to adjust the strength within 10-15 %. In particular the spin orbit potential may by chosen phenomenologically since the microscopic potential is the least accurate. Microscopic potentials in general are designed to reproduce global features and not detailed fits.

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Figure captions

Fig. 1 Ground state charge and matter densities

Fig. 2 ℓ-dependent real central potentials of 95 MeV protons for ^{24}Mg

Fig. 3 ℓ-dependent imaginary central potentials of 95 MeV protons for ^{24}Mg

Fig. 4a 95 MeV proton local spin orbit potentials for ^{24}Mg generated with expressions developed in ref. 5

Fig. 4b The real part of the proton damping function (Perey effect) for ^{24}Mg at 95 MeV

Fig. 5 As fig. 2 for ^{23}Mg and 45 MeV neutrons

Fig. 6 As fig. 2 for ^{23}Na and 45 MeV protons

Fig. 7 As fig. 3 for ^{23}Na + p and ^{24}Mg + n at 45 MeV

Fig. 8 As fig. 4a-b for ^{23}Na + p and ^{24}Mg + n at 45 MeV

Fig. 9 95 MeV elastic scattering angular distribution for ^{24}Mg predicted by potentials shown in figs. 2-4

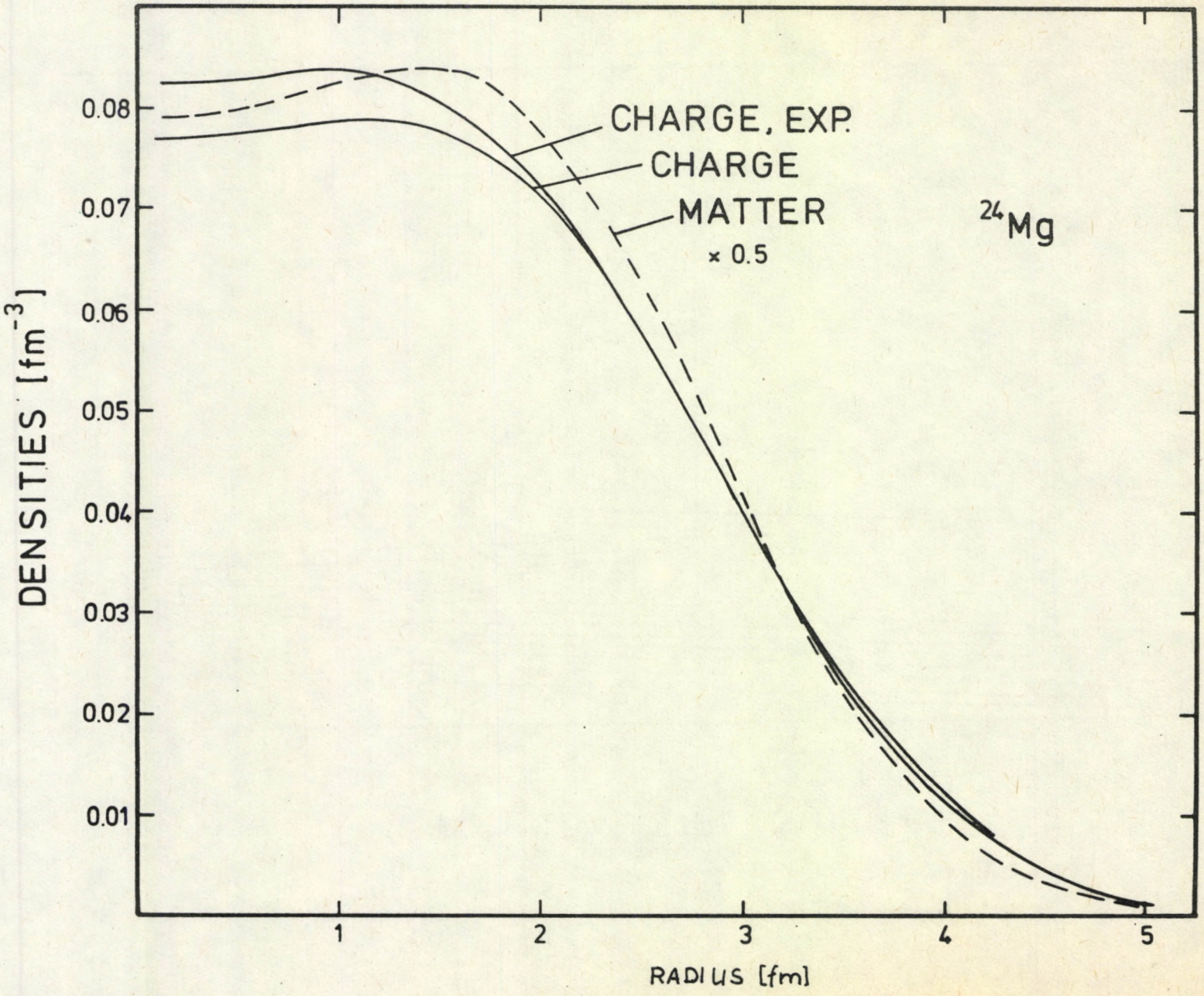

CHARGE, EXP.
CHARGE
MATTER
× 0.5
^{24}Mg
DENSITIES [fm^{-3}]
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
1
2
3
4
5
RADIUS [fm]

Fig. 1

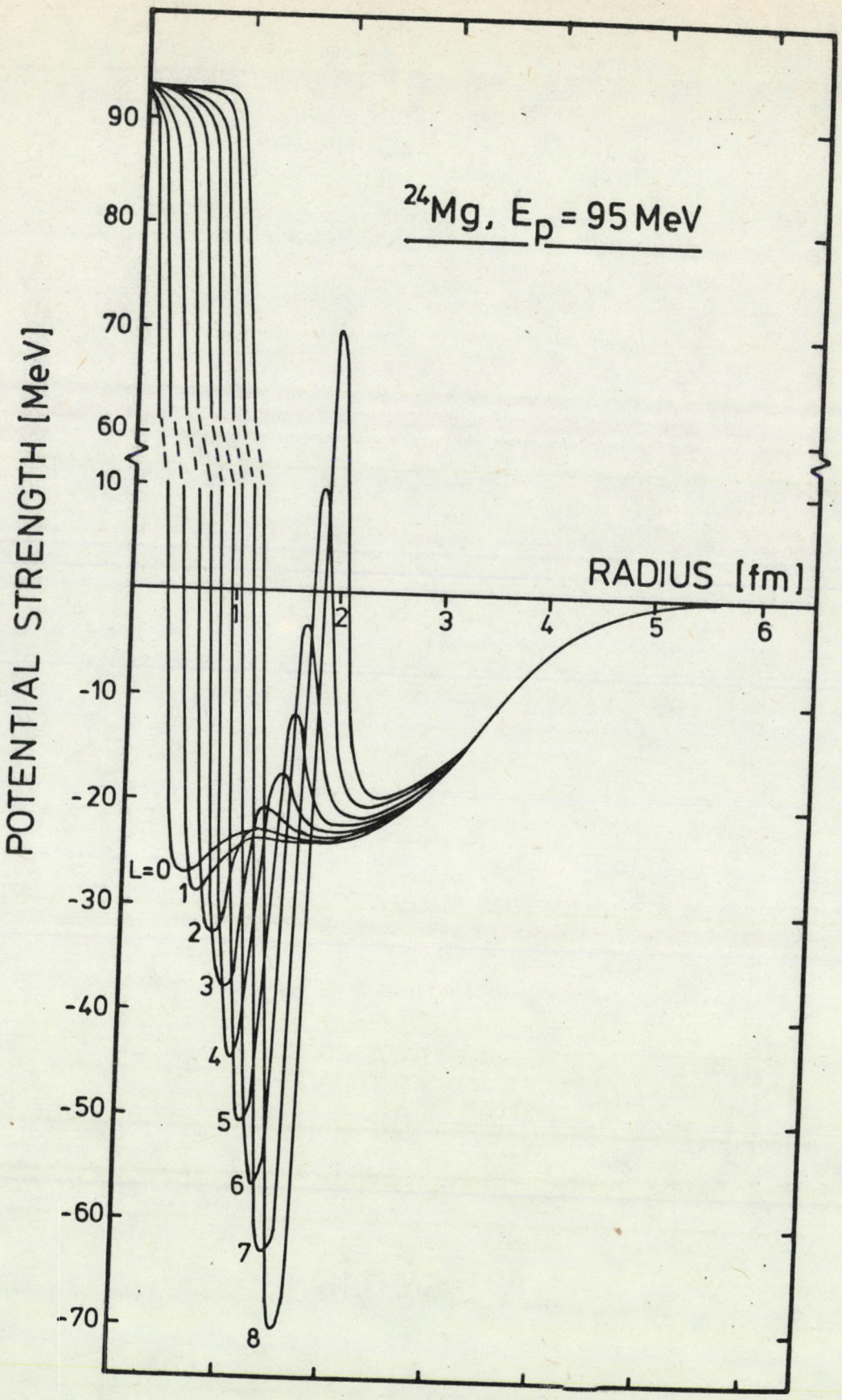

^{24}Mg, E_p = 95 MeV
POTENTIAL STRENGTH [MeV]
RADIUS [fm]
90
80
70
60
10
-10
-20
-30
-40
-50
-60
-70
1
2
3
4
5
6
L=0
1
2
3
4
5
6
7
8

Fig. 2

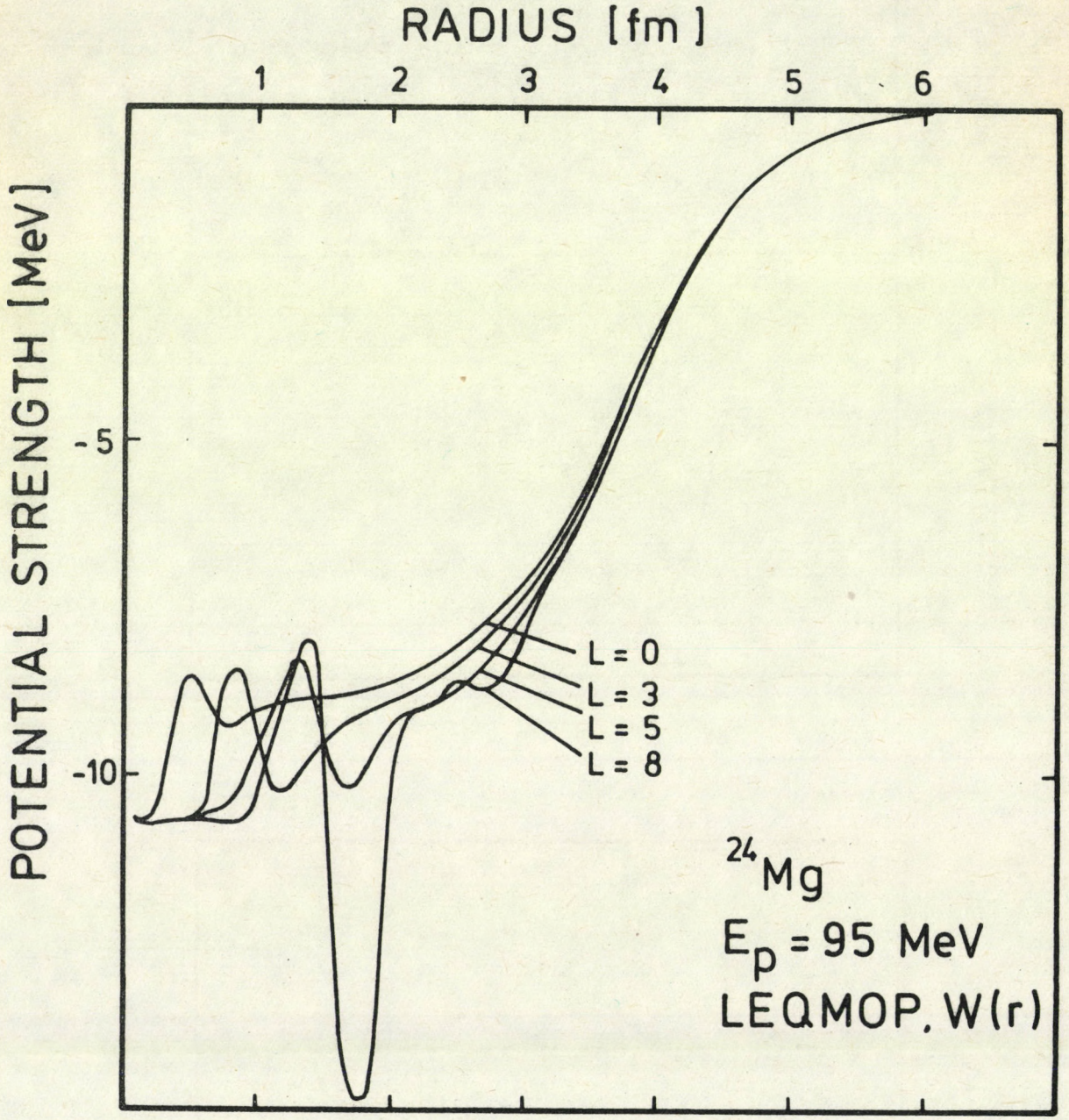
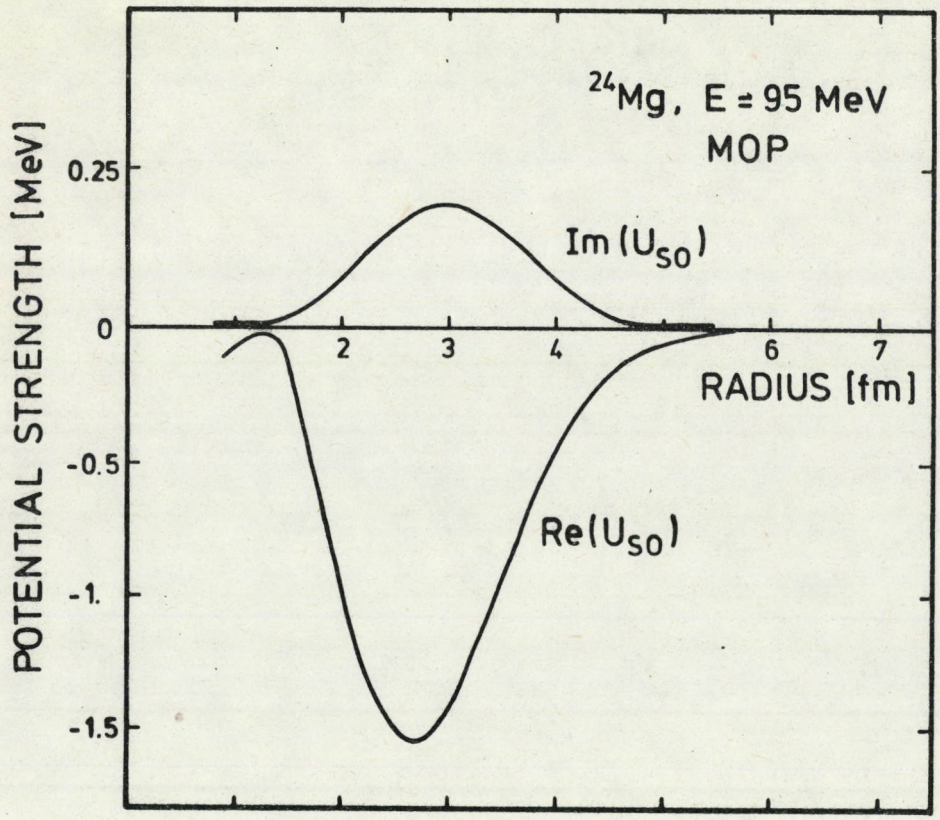

RADIUS [fm]
1
2
3
4
5
6
POTENTIAL STRENGTH [MeV]
-5
-10
L = 0
L = 3
L = 5
L = 8
^{24}Mg
E_p = 95 MeV
LEQMOP, W(r)

Fig. 3


^{24}Mg, E = 95 MeV
MOP
Im (U_{SO})
Re (U_{SO})
POTENTIAL STRENGTH [MeV]
0.25
0
-0.5
-1.
-1.5
2
3
4
6
7
RADIUS [fm]

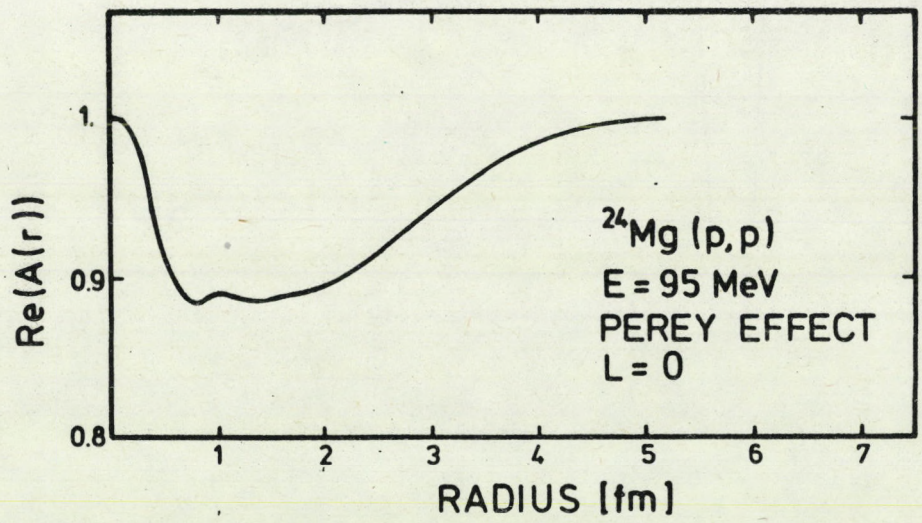

Re(A(r))
1.
0.9
0.8
^{24}Mg (p,p)
E = 95 MeV
PEREY EFFECT
L = 0
1
2
3
4
5
6
7
RADIUS [fm]

Fig. 4

Fig. 5

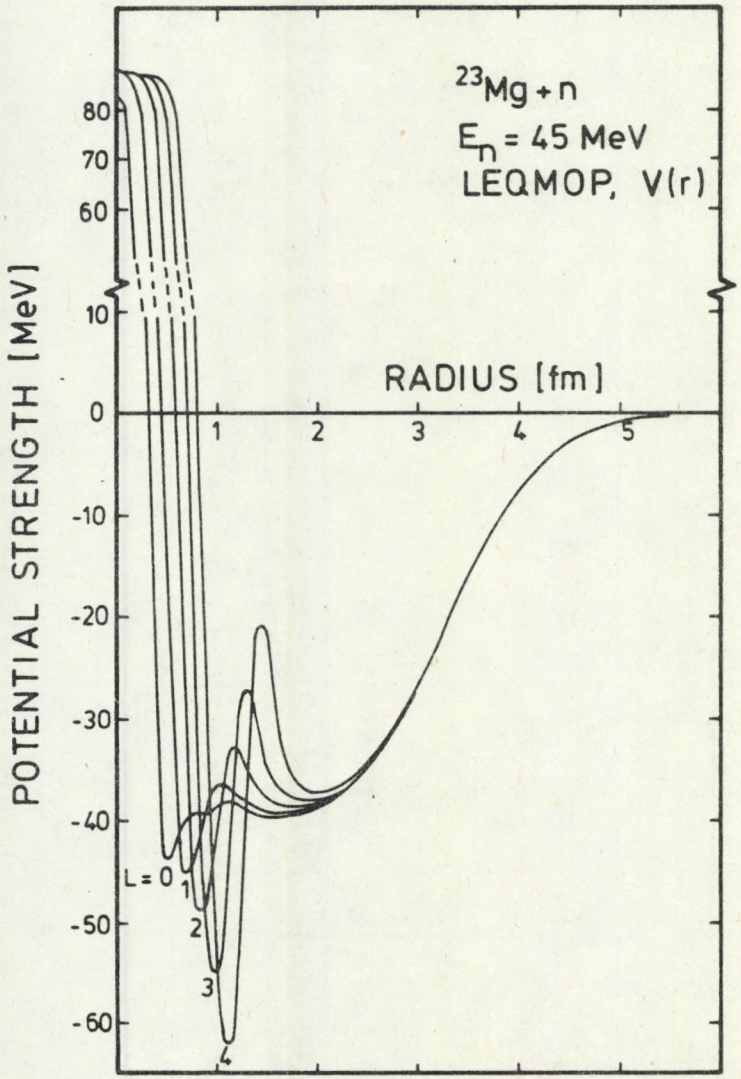
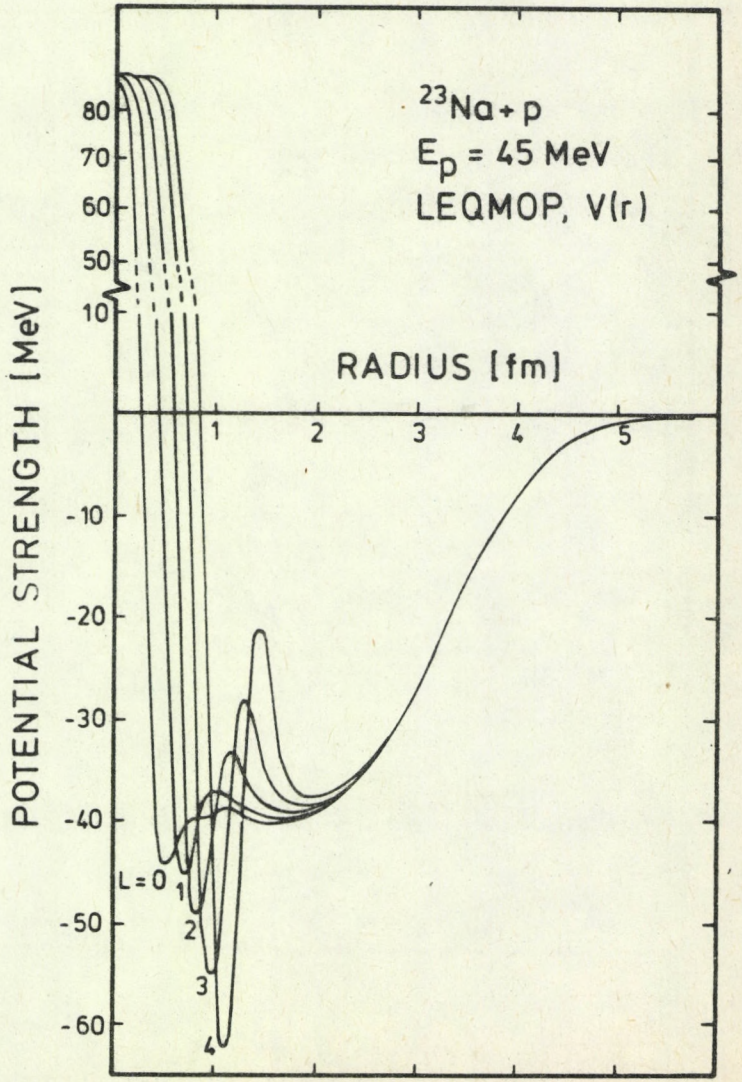

^{23}Mg + n
E_n = 45 MeV
LEQMOP, V(r)
RADIUS [fm]
POTENTIAL STRENGTH [MeV]
80
70
60
10
0
-10
-20
-30
-40
-50
-60
1
2
3
4
5
L = 0
1
2
3
4

Fig. 6


^{23}Na + p
E_p = 45 MeV
LEQMOP, V(r)
RADIUS [fm]
POTENTIAL STRENGTH [MeV]
80
70
60
50
10
-10
-20
-30
-40
-50
-60
1
2
3
4
5
L = 0
1
2
3
4

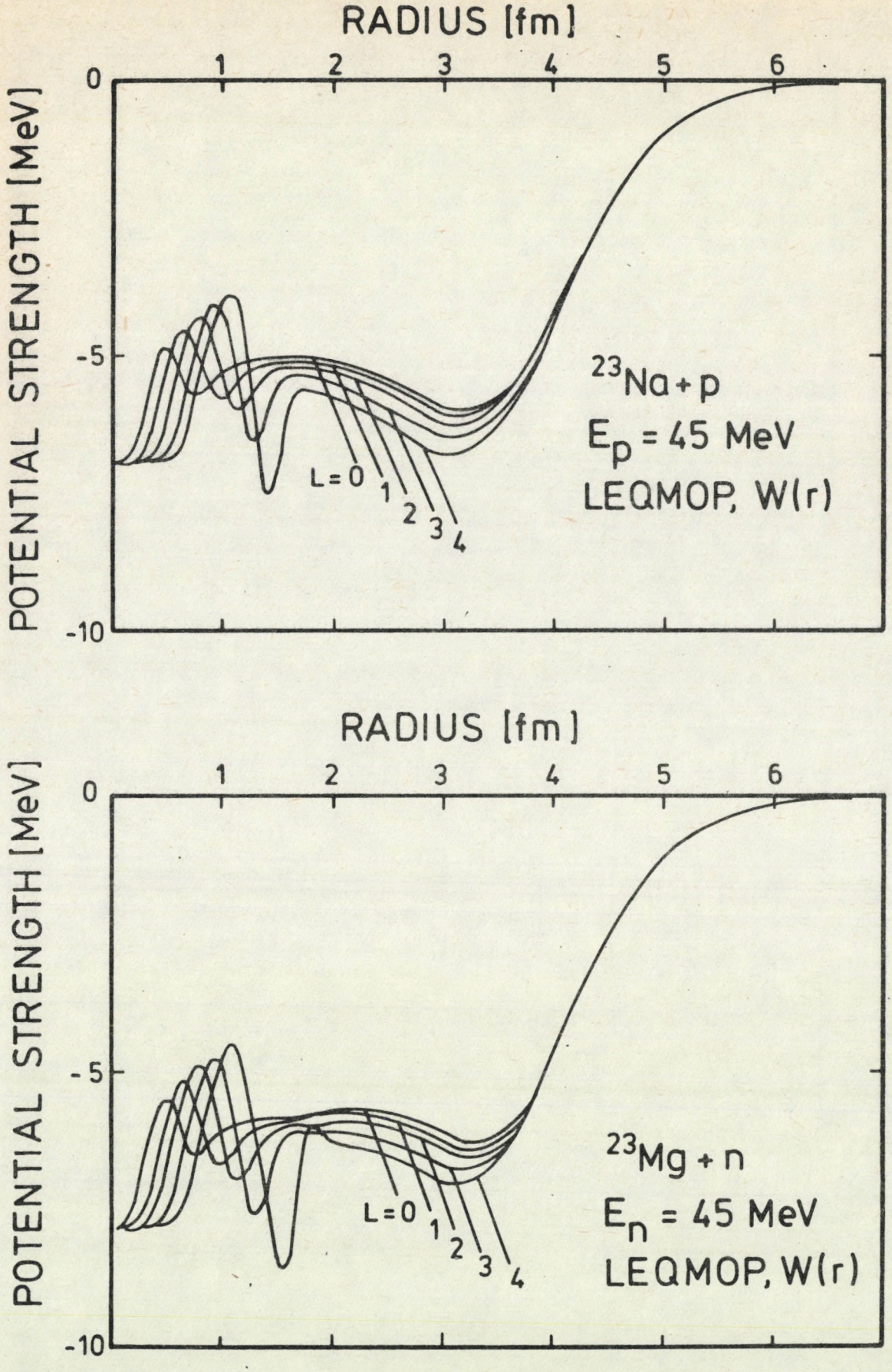
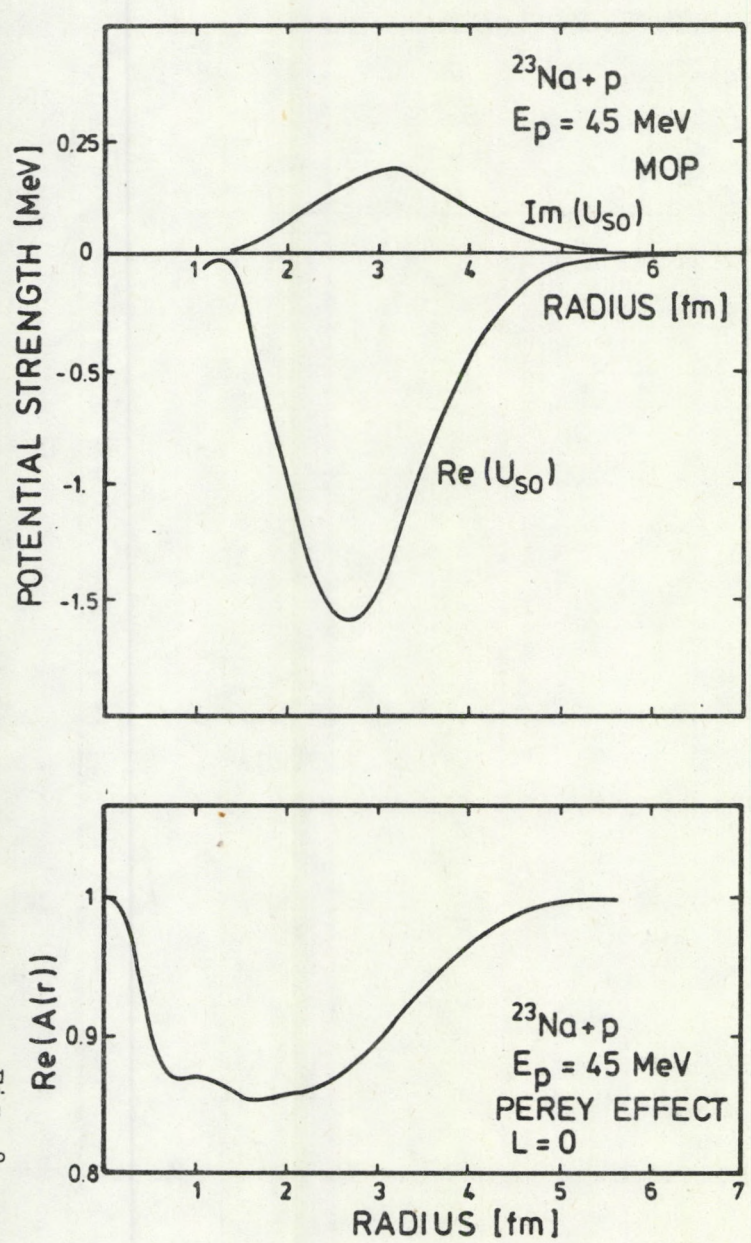
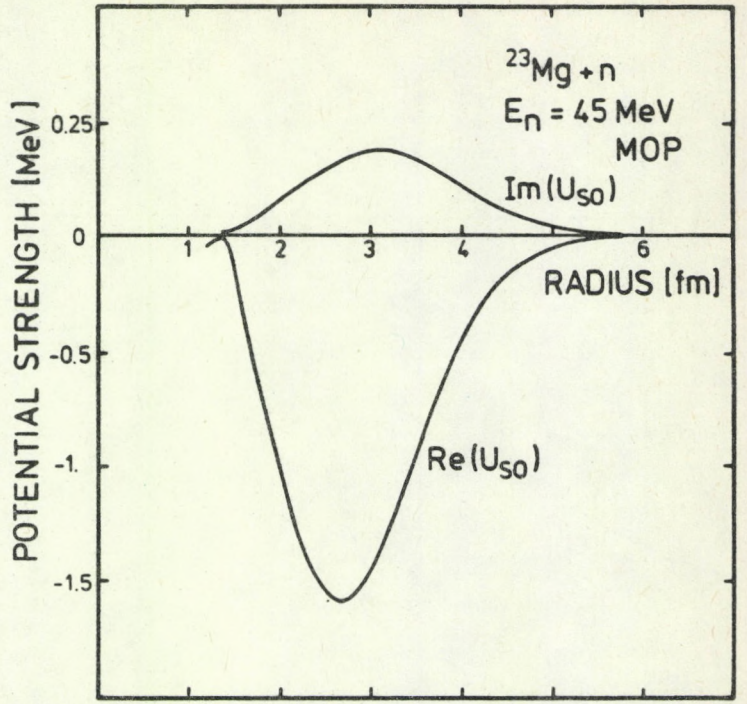

RADIUS [fm]
1
2
3
4
5
6
0
-5
-10
POTENTIAL STRENGTH [MeV]
L=0
1
2
3
4
^{23}Na + p
E_p = 45 MeV
LEQMOP, W(r)
RADIUS [fm]
1
2
3
4
5
6
0
-5
-10
POTENTIAL STRENGTH [MeV]
L=0
1
2
3
4
^{23}Mg + n
E_n = 45 MeV
LEQMOP, W(r)

Fig. 7


^{23}Na + p
E_p = 45 MeV
MOP
Im (U_{SO})
Re (U_{SO})
POTENTIAL STRENGTH [MeV]
RADIUS [fm]
0.25
0
-0.5
-1.
-1.5
1
2
3
4
6

^{23}Na + p
E_p = 45 MeV
PEREY EFFECT
L = 0
Re(A(r))
RADIUS [fm]
1
0.9
0.8
1
2
3
4
5
6
7


^{23}Mg + n
E_n = 45 MeV
MOP
Im (U_{SO})
Re (U_{SO})
POTENTIAL STRENGTH [MeV]
RADIUS [fm]
0.25
0
-0.5
-1.
-1.5
1
2
3
4
6

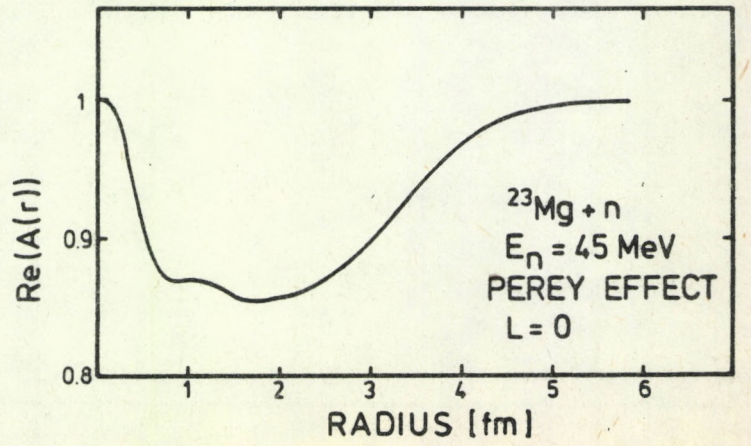

^{23}Mg + n
E_n = 45 MeV
PEREY EFFECT
L = 0
Re(A(r))
RADIUS [fm]
1
0.9
0.8
1
2
3
4
5
6

Fig. 8

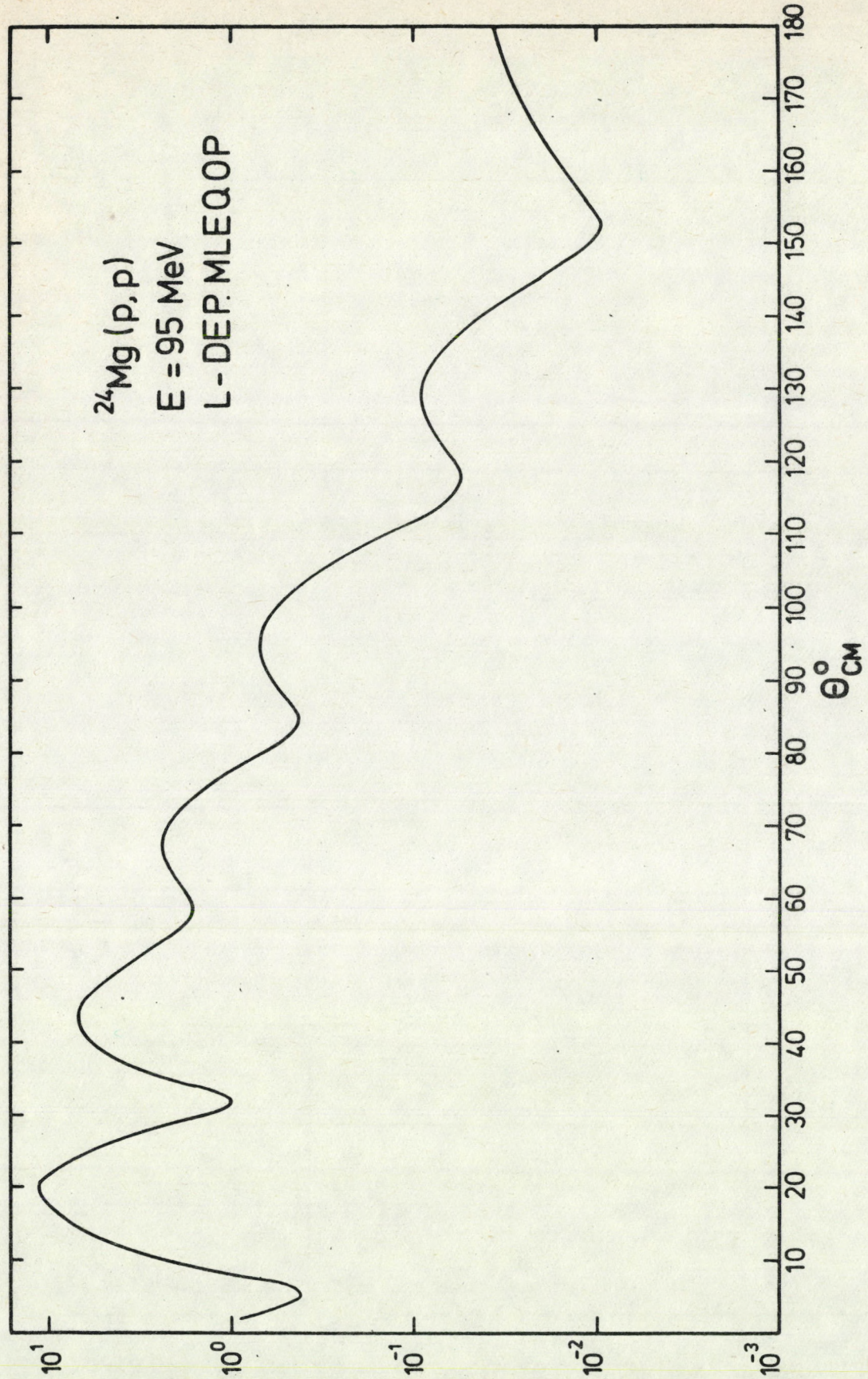
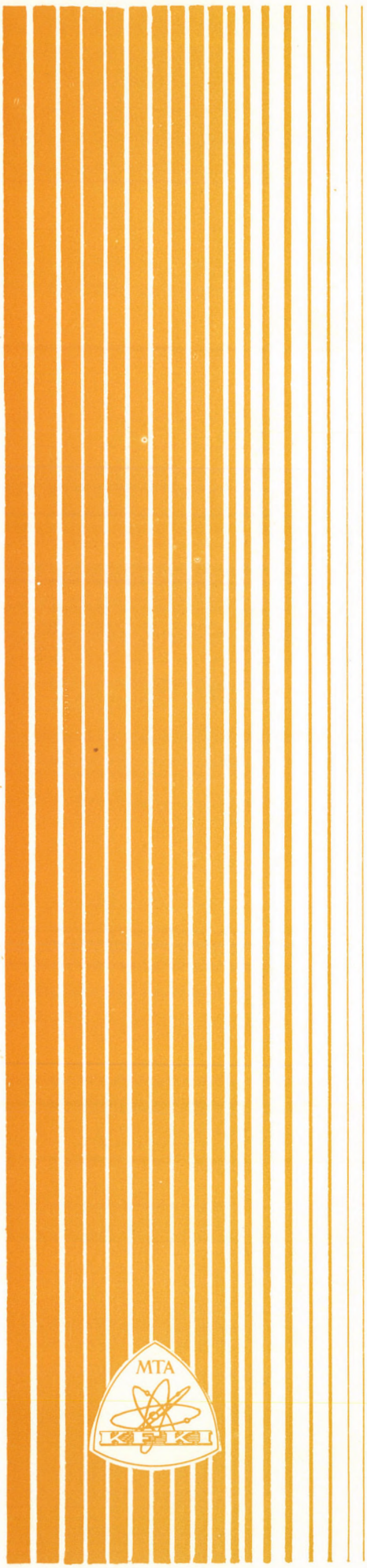

^{24}Mg (p, p)
E = 95 MeV
L - DEP. MLEQOP
σ/σ$_R$
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180
θ°$_{CM}$

Fig. 9



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Felelős kiadó: Szegő Károly
Szakmai lektor: Doleschall Pál
Nyelvi lektor: Kluge Gyula
Példányszám: 430 Törzsszám: 80-744
Készült a KFKI sokszorosító üzemében
Felelős vezető: Nagy Károly
Budapest, 1980. december hó