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N-BODY METHODS IN THE THEORY OF NUCLEAR REACTIONS

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Gy. Bencze

Central Research Institute for Physics
H-1525 Budapest 114, P.O.B. 49, Hungary

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1. Introduction

In the last decade multiparticle scattering theory has developed into a separate branch of mathematical physics. Unfortunately the increasing activity and rapid development in the field of N-particle scattering theory have made almost no impact on nuclear reaction theory. Traditional nuclear reaction theory /following Kowalski "traditional" is used to mean pre-Faddeev not necessarily chronologically but in spirit/ bases its considerations on the multiparticle Schrodinger equation and emphasises the importance of numerical results as well as description of experimental data even if a large degree of phenomenology is involved. Such an attitude is justified to some extent by the success of optical model, DWBA, CCBA, etc. However, in such circumstances it is easy to overlook the limitations and possible inconsistencies of the basic formalism involved. Multiparticle scattering theory on the other hand offers exact treatment and full understanding of the dynamics at the price of a relatively complicated formalism and enormous numerical difficulties in some of the applications. Of course Faddeev theory has also had spectacular success in describing the three-nucleon system and by now realistic four-nucleon calculations have also become feasible. Due to the increasing amount of numerical work, however N-particle equations are clearly not practical for treating complex nuclear systems. Nevertheless the application of concepts and methods developed by N-particle scattering theory may contribute to our better understanding of nuclear reaction dynamics.

2. Traditional methods

Until Faddeev's work only the two-body problem could be solved exactly. Thus traditional nuclear reaction theory is necessarily based on two-body methods even if multiparticle processes are studied [1]. The implications of this basic limitation are not always fully recognised.

Feshbach's elegant projection operator formalism [2] provides a rigorous justification for the optical model of elastic and inelastic scattering. This theory of the optical potential can also be extended with some difficulty to account for exchange effects. However, Feshbach's formalism is only formally exact since in order to construct the optical potential one has to solve in principle the full multiparticle scattering problem.

The treatment of rearrangement processes already presents some nontrivial difficulties. Since even the simplest case of rearrangement involves at least three constituents or nuclear clusters, two-body methods obviously cannot describe the dynamics in a consequent manner. The standard procedures, e.g. DWBA, CCBA treat the interactions responsible for the rearrangement by perturbation theory with a suitable combination of optical model and two-potential formalism. Various attempts to extend the coupled channels method for rearrangement processes in the frame of traditional theory suffer from serious mathematical and conceptual difficulties, as is discussed in detail by Levin [3].

The resonating group method (RGM) is different in spirit since it is based on variational formalism [4]. As a result it is flexible enough to treat all two cluster channels, rearrangement and identical particle effects without invoking perturbation theory. RGM in its present form cannot treat three- or more cluster channels and some formal properties /e.g. non-orthogonality effects/ reflect the basic limitations of the method. Hovewer, if the known asymptotic form of three-body scattering wave functions is made use of, it is in principle possible to extend RGM to treat three-body channels as well [5].

3. N-particle scattering theory

These exist various exact formulations of N-particle scattering which are based on integral equations and determine transition operators or components of the scattering operator of the system.

Details of N-particle scattering theory can be found in recent review articles [6,7]. In the last year three important developments occured which will be briefly discussed:

- the general algebraic theory of identical particle scattering
- Merkuriev's theory of the Coulomb three body problem
- Combinatorial Hamiltonian Unitary Connected Kernel /CHUCK/ theory.

The general algebraic theory of identical particle scattering developed by Bencze and Redish [8] can be applied to a large class of N-particle scattering formalisms and results in considerable simplification, i.e. the maximal reduction of the number of coupled integral equations. Thus the theory can be formulated in such a way as to involve only quantities relevant to physical processes [9].

The presence of long range Coulomb interactions in nuclear systems leads to essential mathematical difficulties. As a consenquence the Faddeev and other N-particle approaches cannot be applied. Since in this case the asymptotic condition has to be modified [10], correspondingly scattering theory has to be reformulated as well. The problem of three charged particles has been recently solved by Merkuriev [11] by introducing suitably modified Faddeev equations. Unfortunately this theory is rather complicated and so far the nontrivial problem of generalising it to an arbitrary number of particles has not been considered. Thus for partical applications simpler approximation schemes have to be introduced which accomodate the modified asymptotic condition [12].

As the number of particles increases due to the large number of variables as well as coupled equations exact N-particle equations become impractical for describing multiparticle collision processes. On the other hand in actual physical processes the mechanism of the collision can be often described in terms of only a small number of nuclear clusters, i.e. the system is dominated by just a few channels. In such a case one intuitively expects to be able to give a simplified treatment in terms of the dominant channels or reaction mechanisms. The CHUCK theory of Polyzou and Redish [13] has been developed to handle such situations and combines the flexibility of the projection operator method with an exact multiparticle scattering formalism.

Due to the above new developments it is now feasible to attack some of the problems of nuclear reaction theory which cannot be systematically treated by traditional methods.

4. Effective two-body problems

The simplest effective two-body problem is the elastic /and inelastic/ scattering of two nuclear particles. Their interaction, the optical potential is given by a formal expression in Feshbach's formalism. N-particle scattering theory on the other hand yields an explicit expression which can be evaluated by quadratures provided the subsystem properties are known. Indeed the elastic channel transition operator can be shown either by elimination or channel decoupling technique [14] to satisfy an L-S type equation. By restriction to the channel subspace of the appropriate internal states of both projectile and target one obtains the optical potential explicitly. With a suitable rearrangement of the N-particle equations [15] the folding potential is obtained as a first approximation and the contribution of various processes to the imaginary part can be systematically studied. However, there is a lot of practical problems to be solved before microscopic optical potentials can be actually calculated.

In traditional theory the coupled channel method cannot be systematically extended to rearrangement processes. In N-particle scattering theory this presents no problem. Given a set of minimally coupled [6,7] N-particle equations the "pole" -or "bound state approximation" in the kernel immediately yields a set of coupled effective two-body equations which are suitable for numerical calculations. This approach has been extensively studied by Levin and collaborators [16] . By making use of the new developments identical particle as well as Coulomb effects can be easy included and one arrives at an alternative to the RGM method with the important difference that non-orthogonality terms are absent.

5. Effective few-body problems

A large number of multiparticle collision processes can be intuively described as effective few-body problems. If there are final states which involve more than two clusters two-body methods are clearly not sufficient. However, one still expects to be able to treat these processes with few (<N)-body methods. Recently the RGM approach was extended by Schmid [17] to obtain effective few-body Schrodinger equation with effective interactions.

Starting with exact N-particle formalism there are two ways of arriving at effective few-body problems. The "dominant partition method" of Dixon and Redish [18] reduces the N-particle BRS-equations to a set of few-particle BRS-equations with effective /few-body/ interactions by a suitable elimination or truncation procedure. This remarkable property has been demonstrated so far only for the BRS-formalism.

CHUCK theory on the other hand offers a flexible way of keeping all the dominant channels irrespectively of their cluster structure. As a result the few-body equations can be classified according to the number of vector variables rather than the cluster structure of channels. In this approach the "spectroscopic factor problem" does not occur [19].

6. Conclusions

The above brief summary gave only a glimpse of the conceptual clarity and promising new techniques N-particle methods can offer. However, there is clearly very much to be done in order to develop practical methods of applications. In any case, as it was pointed out by Faddeev at the 1979

Dubna meeting, it is time N-particel scattering theory was applied to nuclear reactions so as not to become an isolated discipline alien to real physical problems.

So far only techniques relevant to so called direct reactions have been discussed. If there is a large number of degrees of freedom involved in the collision process usually statistical considerations are used. These are based on the assumption that the number of particles is large. However, in real nuclear systems the number of reaction channels rather than that of the particles is very large. So that statistical or stochastic methods are best to be applied to the set of channels rather than particles themselves. This idea has been elaborated by Baz and collaborators [20] and applied to heavy ion reactions with promising results. Such considerations can be easily extended to exact N-particle equations with the advantage that there channel structure is very suitably displayed.

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