# ON THE IDEAL BEHAVIOUR OF THE EQUIMOLAR $\mathrm{H}_{2} \mathrm{O}-\mathrm{D}_{2} \mathrm{O}$ MIXTURE 

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# ON THE IDEAL BEHAVIOUR OF THE EQUIMOLAR $\mathrm{H}_{2} \mathrm{O}-\mathrm{D}_{2} \mathrm{O}$ MIXTURE 

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#### Abstract

New high precision vapour pressure data on equimolar $\mathrm{H}_{2} \mathrm{O}-\mathrm{D}_{2} \mathrm{O}$ mixtures are analysed. The results show that the $\mathrm{H}_{2} \mathrm{O}-\mathrm{HDO}-\mathrm{D}_{2} \mathrm{O}$ liquid mixture can be considered as an ideal solution within the limits of the presently available data. The present investigation supports the earlier conclusion that the law of the geometric mean for the vapour pressure isotope effect in the series $\mathrm{H}_{2} \mathrm{O}$, HDO and $\mathrm{D}_{2} \mathrm{O}$ is not obeyed.


## АННОТАЦИЯ

Были проанализированы новые высокоточные данные по давлению пара эквимолеш кулярной смеси $\mathrm{H}_{2} \mathrm{O}-\mathrm{D}_{2} \mathrm{O}$. Результаты показывают, что жидкая смесь $\mathrm{H}_{2} \mathrm{O}-\mathrm{HDO}-\mathrm{D}_{2}$ б может считаться идеальной в пределах точности измерений экслеримеитальных данных. Исследования подтверждают сделанное ранее предположение о том, что к давлению пара молекул $\mathrm{H}_{2} \mathrm{O}$, HDO и $\mathrm{D}_{2} \mathrm{O}$ не приложимо правило средней геометрической величины.

## KIVONAT

Ekvimoláris $\mathrm{H}_{2} \mathrm{O}-\mathrm{D}_{2} \mathrm{O}$ elegyek gốznyomására rendelkezésre álló uj, nagypontosságu adatokat analizáltuk. Az eredmények azt mutatják, hogy a $\mathrm{H}_{2} \mathrm{O}$ - HDO-- $D_{2}$ O folyadék elegy ideálisnak tekinthetỗ a kisérleti adatok mérési pontosságán belül. A vizsgálatok alátámasztják azt a korábbi következtetést is, mely szerint $\mathrm{a}_{2} \mathrm{O}$, HDO és $\mathrm{D}_{2} \mathrm{O}$ molekulák gỗznyomására nem érvényes a geometriai közép szabálya.

## INTRODUCTION

Isotopic mixtures have for some while been considered as the best examples of ideal solutions therefore the study of the deviation of such solutions from the ideal behaviour seems to be especially interesting. Precise vapour pressure measurements on isotopic mixtures with known isotopic composition represent one of the most convenient ways to carry out such investigations. The results of the recent determination of the vapour pressures of equimolar solutions of $\mathrm{C}_{6} \mathrm{H}_{6}$ and $\mathrm{C}_{6} \mathrm{D}_{6}$ and of $\mathrm{C}_{6} \mathrm{H}_{12}$ and $\mathrm{C}_{6} \mathrm{D}_{12}$ have shown that these mixtures deviate from ideality [1]. The excess thermodynamic properties of these solutions were interpreted in terms of the molar volume isotope effect and the pressure dependence of the molecular vibrational frequencies [2,3].

The different experimental techniques which have been used for determining the eventual nonideal behaviour of $\mathrm{H}_{2} \mathrm{O}-\mathrm{D}_{2} \mathrm{O}$ liquid mixtures were summarized by Phutela and Fenby [4] and Van Hook [5]. These techniques include the measurement of the vapour pressures of $\mathrm{H}_{2} \mathrm{O}-\mathrm{D}_{2} \mathrm{O}$ mixtures [6-8], ebulliometric measurements [9] and the determination of the freezing points of the mixtures [10]. The latter data have been considered in detail by Van Hook [11] who concluded that the $\mathrm{H}_{2} \mathrm{O}-\mathrm{D}_{2} \mathrm{O}$ solutions do not deviate from the ideal behaviour and by using this assumption the ratio

$$
\begin{equation*}
r=\ln \left(\frac{\mathrm{p}_{\mathrm{H}_{2} \mathrm{O}}}{\mathrm{p}_{\mathrm{D}_{2} \mathrm{O}}}\right) / \ln \left(\frac{\mathrm{p}_{\mathrm{H}_{2} \mathrm{O}}}{\mathrm{p}_{\mathrm{HDO}}}\right) \tag{1}
\end{equation*}
$$

was evaluated from the literature data on vapour pressure isotope effects [13-15] and found to be 1.91士0.03 [12]. The extension of the data base to the boiling points [9] and freezing points [10] of mixtures of the isotopic waters resulted in a constant, temperature independent, $r$ value ( $1.91 \pm 0.02$ ) between 0 and $200{ }^{\circ} \mathrm{C}$ over a wide HOD concentration range [5].

The $\mathrm{H}_{2} \mathrm{O}-\mathrm{D}_{2} \mathrm{O}$ liquid mixtures should be treated as three component systems due to the equilibrium

$$
\begin{equation*}
\mathrm{H}_{2} \mathrm{O}(\mathrm{liq})+\mathrm{D}_{2} \mathrm{O}(\mathrm{liq})=2 \mathrm{HDO}(\mathrm{liq}) . \tag{2}
\end{equation*}
$$

Phutela and Fenby [4], by using vapour pressure isotope effect data for HDO and $\mathrm{D}_{2} \mathrm{O}$ and assuming the $\mathrm{H}_{2} \mathrm{O}-\mathrm{HDO}-\mathrm{D}_{2} \mathrm{O}$ liquid mixtures to be ideal, evaluated deviations from Raoult's law. The negative deviations found mean that the vapour pressure of the mixture is higher if it is considered as a two component system instead of a three component system.

New high precision vapour pressure data on equimolar $\mathrm{H}_{2} \mathrm{O}-\mathrm{D}_{2} \mathrm{O}$ mixtures obtained in this laboratory for a wide temperature range [16] make it possible to reinvestigate the problem of the ideal behaviour of this solution.

In this paper the following definition of an ideal mixture is used: it is a mixture in which the chemical potential of component $i$ is given by the equation

$$
\begin{equation*}
\mu_{i}\left(p, T, x_{i}\right)=\mu_{i}^{O}(p, T)+R T \ln x_{i} \tag{3}
\end{equation*}
$$

where $\mu_{i}^{O}(p, T)$ is the chemical potential of the pure liquid component $i$ at the same pressure and temperature as the mixture being studied and $x_{i}$ is the mole fraction of component $i$ in the liquid mixture [17]. (This definition leads to Raoult's law if the vapour is a perfect gas and the molar volumes of the liquids are negligibly small.)

## DISCUSSION

The details of the experimental determination of the vapour pressure of the equimolar $\mathrm{H}_{2} \mathrm{O}-\mathrm{D}_{2} \mathrm{O}$ mixture are given elsewhere [16]. A summary of the experimental data obtained is given in table $I$.

Let us consider a liquid mixture at temperature $T$, formed from 0.5 mol of $\mathrm{H}_{2} \mathrm{O}$ and 0.5 mol of $\mathrm{D}_{2} \mathrm{O}$; then the equilibrium mixture will contain 2 s mol of HDO, ( $\mathrm{O} .5-\mathrm{s}$ ) mol of $\mathrm{D}_{2} \mathrm{O}$ and (0.5-s) mol of $\mathrm{H}_{2} \mathrm{O}$. The value of $s$ can be obtained from the equilibrium constant of Eq. (2) ( $\mathrm{K}_{\text {liq }}$ ). The vapour pressure of an

Table I
The values of $\ln \left(\mathrm{p}_{\mathrm{H}_{2} \mathrm{O}} / \mathrm{p}_{\text {mixture }}\right)$ and $\ln \left(\mathrm{p}_{\mathrm{H}_{2} \mathrm{O}} / \mathrm{p}_{\mathrm{D}_{2} \mathrm{O}}\right)$ calculated from the least-squares fit equations of the experimental data [16]

| $\begin{gathered} \left.\mathrm{o}^{\mathrm{o}} \mathrm{C}\right) \end{gathered}$ | $\ln \frac{\mathrm{p}_{\mathrm{H}_{2} \mathrm{O}}}{\mathrm{p}_{\mathrm{D}_{2} \mathrm{O}}}$ | $\ln \frac{\mathrm{p}_{\mathrm{H}_{2} \mathrm{O}}}{\mathrm{p}_{\text {mixture }}}$ | $\begin{gathered} \mathrm{p}_{\mathrm{H}_{2} \mathrm{O}}[18] \\ (\mathrm{kPa}) \end{gathered}$ | $\begin{gathered} \mathrm{p}_{\mathrm{H}_{2} \mathrm{O}^{-\mathrm{p}_{\text {mixture }}}}^{(\mathrm{kPa})} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $0.20362^{\text {c }}$ ) | $0.10196^{\text {d }}$ | $\begin{gathered} 0.6106 \\ (4.580) \end{gathered}$ | $\begin{array}{r} 0.0592 \\ (0.444) \end{array}$ |
| 5 | 0.19041 | 0.09536 | $\begin{array}{r} 0.8718 \\ (6.539) \end{array}$ | $\begin{array}{r} 0.0793 \\ (0.595) \end{array}$ |
| 10 | 0.17809 | 0.08921 | $\begin{array}{r} 1.2271 \\ (9.204) \end{array}$ | $\begin{gathered} 0.105 \\ (0.785) \end{gathered}$ |
| 15 | 0.16659 | 0.08346 | $\begin{array}{r} 1.7041 \\ (12.782) \\ \hline \end{array}$ | $\begin{array}{r} 0.1365 \\ (1.024) \\ \hline \end{array}$ |
| 20 | 0.15585 | 0.07810 | $\begin{array}{r} 2.3370 \\ (17.529) \\ \hline \end{array}$ | $\begin{array}{r} 0.1756 \\ (1.317) \\ \hline \end{array}$ |
| 25 | 0.14582 | 0.07309 | $\begin{array}{r} 3.1667 \\ (23.752) \\ \hline \end{array}$ | $\begin{array}{r} 0.2232 \\ (1.674) \\ \hline \end{array}$ |
| 30 | 0.13643 | 0.06841 | $\begin{array}{r} 4.2426 \\ (31.822) \end{array}$ | $\begin{aligned} & 0.2805 \\ & (2.104) \end{aligned}$ |
| 35 | 0.12766 | 0.06404 | $\begin{array}{r} 5.6231 \\ (42.177) \end{array}$ | $\begin{array}{r} 0.3488 \\ (2.616) \end{array}$ |
| 40 | 0.11944 | 0.05994 | $\begin{array}{r} 7.3771 \\ (55.333) \\ \hline \end{array}$ | $\begin{array}{r} 0.4292 \\ (3.219) \\ \hline \end{array}$ |
| 45 | 0.11174 | 0.05610 | $\begin{array}{r} 9.5848 \\ (71.892) \end{array}$ | $\begin{array}{r} 0.5229 \\ (3.922) \\ \hline \end{array}$ |
| 50 | 0.10453 | 0.05251 | $\begin{array}{r} 12.3387 \\ (92.548) \\ \hline \end{array}$ | $\begin{array}{r} 0.6312 \\ (4.735) \\ \hline \end{array}$ |
| 55 | 0.09778 | 0.04915 | $\begin{array}{r} 15.7452 \\ (118.099) \\ \hline \end{array}$ | $\begin{array}{r} 0.7551 \\ (5.664) \\ \hline \end{array}$ |
| 60 | 0.09144 | 0.04600 | $\begin{array}{r} 19.9252 \\ (149.451) \\ \hline \end{array}$ | $\begin{array}{r} 0.8957 \\ (6.718) \\ \hline \end{array}$ |
| 65 | 0.08550 | 0.04304 | $\begin{array}{r} 25.0150 \\ (187.628) \end{array}$ | $\begin{gathered} 1.054 \\ (7.904) \\ \hline \end{gathered}$ |
| 70 | 0.07993 | 0.04027 | $\begin{array}{r} 31.1681 \\ (233.780) \end{array}$ | $\begin{gathered} 1.230 \\ (9.227) \\ \hline \end{gathered}$ |
| 75 | 0.07469 | 0.03767 | $\begin{array}{r} 38.5552 \\ (289.188) \end{array}$ | $\begin{array}{r} 1.425 \\ (10.69) \end{array}$ |
| 80 | 0.06978 | 0.03523 | $\begin{array}{r} 47.3658 \\ (355.273) \end{array}$ | $\begin{array}{r} 1.640 \\ (12.30) \\ \hline \end{array}$ |
| 85 | $0.06517{ }_{5}$ | 0.03294 | $\begin{array}{r} 57.8086 \\ (433.600) \end{array}$ | $\begin{array}{r} 1.873 \\ (14.05) \end{array}$ |
| 90 | 0.06085 | 0.03079 | $\begin{array}{r} 70.1128 \\ (525.889) \end{array}$ | $\begin{array}{r} 2.126 \\ (15.95) \\ \hline \end{array}$ |

a) $\mathrm{P}_{\mathrm{H}_{2} \mathrm{O}}, \mathrm{p}_{\mathrm{D}_{2} \mathrm{O}}$ and $\mathrm{p}_{\text {mixture }}$ represent vapour pressures of $\mathrm{H}_{2} \mathrm{O}, \mathrm{D}_{2} \mathrm{O}$ and equimolar $\mathrm{H}_{2} \mathrm{O}-\mathrm{D}_{2} \mathrm{O}$ mixture, respectively.
b) The values in parentheses are given in mmHg .
c) The standard deviation of the calculated values is $\pm 3 \times 10^{-5}$.
d) The standard deviation of the calculated values is $\pm 2 \times 10^{-5}$.
ideal liquid mixture (p) in equilibrium with a vapour phase which can be considered as an ideal mixture of imperfect gases is given by [17]

$$
\begin{align*}
p & =x_{H_{2} \mathrm{O}} \mathrm{p}_{\mathrm{H}_{2} \mathrm{O}} \exp \frac{\left(\mathrm{p}-\mathrm{p}_{\mathrm{H}_{2} \mathrm{O}}\right)\left(\mathrm{v}_{\mathrm{H}_{2} \mathrm{O}^{-\mathrm{B}_{2} \mathrm{O}}}\right)}{R T}+ \\
& +x_{\mathrm{HDO}} \mathrm{p}_{\mathrm{HDO}} \exp \frac{\left(\mathrm{p}-\mathrm{p}_{\mathrm{HDO}}\right)\left(\mathrm{v}_{\mathrm{HDO}}-\mathrm{B}_{\mathrm{HDO}}\right)}{R T}+  \tag{4}\\
& +x_{\mathrm{D}_{2} \mathrm{O}} \mathrm{p}_{\mathrm{D}_{2} \mathrm{O}} \exp \frac{\left(\mathrm{p}-\mathrm{p}_{\mathrm{D}_{2} \mathrm{O}}\right)\left(\mathrm{v}_{\mathrm{D}_{2} \mathrm{O}^{-\mathrm{B}_{\mathrm{D}_{2} \mathrm{O}}}}\right)}{R T}
\end{align*}
$$

where in the case of equimolar $\mathrm{H}_{2} \mathrm{O}-\mathrm{D}_{2} \mathrm{O}$ mixture $\mathrm{x}_{\mathrm{H}_{2} \mathrm{O}}=\mathrm{x}_{\mathrm{D}_{2} \mathrm{O}}=0.5-\mathrm{s}$, $x_{\text {HDO }}=2 s, p_{i}$ and $v_{i}$ are the respective vapour pressures and molar volumes of the pure liquids i at temperature $T$; and $B_{i}$ is the second virial coefficient.

In order to estimate the contributions of different terms in Eq. (4) to the vapour phase nonideality, calculations were carried out for $t=75{ }^{\circ} \mathrm{C}$ where the effect of the nonideal behaviour of the vapour phase might be expected to be significant. The data employed in the calculations are collected in Table II.

The second virial coefficient data need some comments. The isotope effect on the second virial coefficient of the $D_{2} O$ vapour has been reported by Kell, McLaurin and Whalley [21] for the temperature range 150 to $500{ }^{\circ} \mathrm{C}$. The data show a smooth decrease in
 $450{ }^{\circ} \mathrm{C}$. Gupta, Jain and Nanda [22] carried out calculations on ${ }^{B_{D_{2}}} \mathrm{O}$ using $\mathrm{p}-\mathrm{T}$ data along the saturation line for the temperature range $70-160{ }^{\circ} \mathrm{C}$. The values were found to be consistently more negative than those for ordinary water and $\left|\Delta \mathrm{B} / \mathrm{B}_{\mathrm{H}_{2} \mathrm{O}}\right|$ was found to decrease from $12.4 \%$ at $70^{\circ} \mathrm{C}$ to about $2 \%$ at $160{ }^{\circ} \mathrm{C}$. They concluded that the principle of corresponding states with respect to the second virial coefficients for the two waters is obeyed if $\mathrm{V}_{\mathrm{CH}_{2}} \mathrm{O}$ and $\mathrm{T}_{\mathrm{CH}_{2} \mathrm{O}}$ ' and $0.983 \mathrm{~V}_{\mathrm{CD}_{2} \mathrm{O}}$ and $1.017 \mathrm{~T}_{\mathrm{CD}_{2} \mathrm{O}}$ - where $\mathrm{V}_{\mathrm{C}}$ and $\mathrm{T}_{\mathrm{C}}$ are the critical volume and temperature, respectively - are used as reducing parameters. On the other hand Lagutkin and Dergachev [23] arrived at the conclusion (using a different set of $p-v-T$ data in their calculation) that the law of corresponding states is obeyed with the reduced variables $\mathrm{V} / \mathrm{V}_{\mathrm{C}}$ and $\mathrm{T} / \mathrm{T}_{\mathrm{C}}$. However, the use of the latter authors, results is made difficult by the discrepancies between the data shown on their graph and those obtained by using the equation formulated for the description of the data.

Input data used in the estimation of the effect of the nonideality of the vapour phase
$t=75^{\circ} \mathrm{C}$

$\left.\mathrm{x}_{\mathrm{D}_{2} \mathrm{O}}=\mathrm{x}_{\mathrm{H}_{2} \mathrm{O}}=0.25304 ; \quad \mathrm{x}_{\mathrm{HDO}}=0.49392^{\mathrm{b}}\right)$
$\mathrm{V}_{\mathrm{H}_{2} \mathrm{O}}=18.475 \mathrm{~cm}^{3} \mathrm{~mol}^{-1} ; \quad \mathrm{V}_{\mathrm{D}_{2} \mathrm{O}}=18.513 \mathrm{~cm}^{3} \mathrm{~mol}^{-1}[19]$
$M_{H_{0}}=18.0106 \mathrm{gmol}^{-1}, \quad M^{2}=20.0231 \mathrm{gmol}^{-1} \quad 1$
$B_{H_{2} \mathrm{O}}=-599 \mathrm{~cm}^{3} \mathrm{~mol}^{-1}{ }^{\mathrm{c})} ; \quad-540 \mathrm{~cm}^{3} \mathrm{~mol}^{-1 \mathrm{~d})} ; \quad-578 \mathrm{~cm}^{3} \mathrm{~mol}^{-1}[22]$
$\frac{\mathrm{B}_{\mathrm{H}_{2} \mathrm{O}}-{ }^{\mathrm{B}_{\mathrm{D}_{2} \mathrm{O}}}}{\mathrm{B}_{\mathrm{H}_{2} \mathrm{O}}} \times 100=11.2$ [22]
a) This value was obtained by using $r=1.91$.
b) $K_{l_{\text {iq }}}=3.81$ (see in the text).
c) Calculated from the equation $B\left(\mathrm{~cm}^{3} \mathrm{~g}^{-1}\right)=1.89-\frac{2641.6}{\mathrm{~T}} \times \exp \frac{1.858 \times 10^{5}}{\mathrm{~T}^{2}}$ given in [20].
c) Extrapolated value obtained from the data given in Table II of [21].
e) The values in parentheses are given in $m m H g$.

If the equimolar liquid mixture of $\mathrm{H}_{2} \mathrm{O}$ and $\mathrm{D}_{2} \mathrm{O}$ obeyed Raoult's law the vapour pressure of the mixture could be given as

$$
\begin{equation*}
\mathrm{p}=\mathrm{x}_{\mathrm{H}_{2} \mathrm{O}} \times \mathrm{p}_{\mathrm{H}_{2} \mathrm{O}}+\mathrm{x}_{\mathrm{D}_{2} \mathrm{O}}+\mathrm{p}_{\mathrm{D}_{2} \mathrm{O}}+\mathrm{x}_{\mathrm{HDO}} \times \mathrm{p}_{\mathrm{HDO}} . \tag{5}
\end{equation*}
$$

Since the differences between Eqs. (4) and (5) are small, we can, in very good approximation, substitute p from Eq. (5) into the nonideal vapour phase correction terms in Eq. (4). The partial vapour pressures obtained from Eq. (4) are then compared with those given by Raoult's law /Eq. (5)/ and the results are summarized in $T a b l e I I I$. It can be seen that the corrections due to the nonideality of the vapour phase can be completely neglected since the precision of the determination of the vapour pressure difference between the equimolar $\mathrm{H}_{2} \mathrm{O}-\mathrm{D}_{2} \mathrm{O}$ mixture and the pure $\mathrm{H}_{2} \mathrm{O}$ is about $\pm 1.3-2.7 \mathrm{~Pa}( \pm 0.01-0.02 \mathrm{mmHg})$. Therefore Eq. (5) can be used instead of Eq. (4) in the present case and any deviation from the ideal solution behaviour can be observed as a deviation from Raoult's law.

Next, the effect of the equilibrium constant for Eq. (2) will be investigated. The available literature data for $\mathrm{K}_{1 i q}$ and $\mathrm{K}_{\text {gas }}$ at different temperatures are collected in Table IV. The vapour pressure difference between the pure $\mathrm{H}_{2} \mathrm{O}$ and the equimolar $\mathrm{H}_{2} \mathrm{O}-\mathrm{D}_{2} \mathrm{O}$ mixture $(\Delta \mathrm{p})$ is given by

$$
\begin{equation*}
\Delta \mathrm{p}=\mathrm{p}_{\mathrm{H}_{2} \mathrm{O}}-\mathrm{p}=\mathrm{p}_{\mathrm{H}_{2} \mathrm{O}}\left[1-2 \mathrm{~s}\left(\frac{\mathrm{p}_{\mathrm{D}_{2} \mathrm{O}}}{\mathrm{p}_{\mathrm{H}_{2} \mathrm{O}}}\right)^{1 / \mathrm{r}}-(0.5-\mathrm{s})\left(\frac{\mathrm{p}_{\mathrm{D}_{2} \mathrm{O}}}{\mathrm{p}_{\mathrm{H}_{2} \mathrm{O}}}-1\right)\right] \tag{6}
\end{equation*}
$$

where the values for $s$ are obtained from $K_{\text {liq }}$. The calculations with different values of $\mathrm{K}_{\text {liq }}$ and with the data given in Table I were performed for $t=75^{\circ} \mathrm{C}$ and the results are shown in Table $V$. The values obtained for $\Delta p$ demonstrate that they are insensitive to the value selected for $\mathrm{K}_{\text {liq }}$ - at least within the limits of experimental error.

Equation (6) can be written as

$$
\begin{equation*}
\Delta \mathrm{p}=\mathrm{x}_{\mathrm{D}_{2} \mathrm{O}}\left(\mathrm{p}_{\mathrm{H}_{2}} \mathrm{O}^{-\mathrm{p}_{\mathrm{D}_{2} \mathrm{O}}}\right)+\mathrm{x}_{\mathrm{HDO}}\left(\mathrm{p}_{\mathrm{H}_{2} \mathrm{O}}-\mathrm{p}_{\mathrm{HDO}}\right) \tag{7}
\end{equation*}
$$

which is convenient for estimating the influence of the precision of the measurement on the calculations.

Comparison of partial vapour pressures $p_{i}{ }^{*}$ and $p_{i}{ }^{* *}$ obtained with and without nonideal vapour phase correction terms, respectively

| Assumption |  | $\mathrm{p}_{\mathrm{H}_{2}}^{*} \mathrm{O}^{-} \mathrm{p}_{\mathrm{H}_{2} \mathrm{O}} \mathrm{O}$ | $\mathrm{p}_{\mathrm{D}_{2}}^{*} \mathrm{O}^{-\mathrm{p}_{\mathrm{D}_{2}}^{* *} \mathrm{O}}$ | $\mathrm{p}_{\mathrm{HDO}}^{*}-\mathrm{p}_{\mathrm{HDO}}^{* *}$ | $\Sigma p_{i}^{*}-\Sigma p_{i}^{* *}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | (Pa) |  |  |  |
| $\begin{aligned} & \mathrm{B}_{\mathrm{HDO}}=\mathrm{B}_{\mathrm{H}_{2} \mathrm{O}}= \\ & -599 \mathrm{~cm}^{3} \mathrm{~mol}^{-1} \end{aligned}$ | $\begin{aligned} & \mathrm{v}_{\mathrm{HDO}}=\mathrm{V}_{\mathrm{H}_{2} \mathrm{O}} \\ & \mathrm{v}_{\mathrm{HDO}}=\mathrm{V}_{\mathrm{D}_{2} \mathrm{O}} \end{aligned}$ | $\begin{aligned} & \left.-2.99^{-2.02)_{4}}\right)^{b} \\ & (-2.99 \\ & \left(-0.022_{4}\right) \end{aligned}$ | $\begin{aligned} & 2.88 \\ & \left(0.021_{6}\right) \\ & 2.88 \\ & \left(0.021_{6}\right) \end{aligned}$ | $\begin{gathered} 0.1_{9} \\ \left(0.001_{4}\right) \\ 0.1_{9} \\ \left(0.001_{4}\right) \end{gathered}$ | $\begin{aligned} & 0.08 \\ & \left(0.000_{6}\right) \\ & 0.0{ }_{8} \\ & \left(0.000_{6}\right) \end{aligned}$ |
| $\mathrm{B}_{\mathrm{HDO}}=1.056 \mathrm{~B}_{\mathrm{H}_{2} \mathrm{O}}$ | $\mathrm{V}_{\mathrm{HDO}}=\mathrm{V}_{\mathrm{H}_{2} \mathrm{O}}$ | $\begin{aligned} & -2.99 \\ & \left(-0.022_{4}\right) \end{aligned}$ | $\begin{aligned} & 2.88 \\ & \left(0.021_{6}\right) \end{aligned}$ | $\begin{gathered} 0.20 \\ \left(0.00 i_{5}\right) \end{gathered}$ | $\begin{aligned} & 0.0_{9} \\ & \left(0.000_{7}\right) \end{aligned}$ |
| $\begin{gathered} { }_{\mathrm{H}_{2} \mathrm{O}}=\mathrm{B}_{\mathrm{D}_{2} \mathrm{O}}=\mathrm{B}_{\mathrm{HDO}}= \\ -599 \mathrm{~cm}^{3} \mathrm{~mol}^{-1} \end{gathered}$ |  | $\begin{aligned} & -2.99 \\ & \left(-0.022_{4}\right) \end{aligned}$ | $\begin{aligned} & 2.6_{0} \\ & \left(0.019_{5}\right) \end{aligned}$ | $\begin{aligned} & 0.1_{9} \\ & \left(0.001_{4}\right) \end{aligned}$ | $\begin{gathered} 0.20 \\ \left(-0.001_{5}\right) \end{gathered}$ |
| $\begin{aligned} & \mathrm{B}_{\mathrm{HDO}}=\mathrm{B}_{\mathrm{H}_{2} \mathrm{O}}= \\ & -54 \mathrm{o} \mathrm{~cm} \mathrm{cmol}^{-1} \end{aligned}$ |  | $\begin{aligned} & -2.6_{9} \\ & \left(-0.02 \mathrm{O}_{2}\right) \end{aligned}$ | $\begin{aligned} & 2.6_{0} \\ & \left(0.019_{5}\right) \end{aligned}$ | $\begin{aligned} & 0.1_{6} \\ & \left(0.001_{2}\right) \end{aligned}$ | $\begin{aligned} & 0.0_{7} \\ & \left(0.000_{5}\right) \end{aligned}$ |

a) $p_{i, E q .(4)}^{*}=x_{i} p_{i} \exp \frac{\left(p-p_{i}\right)\left(v_{i}-B_{i}\right)}{R T} ; p_{i, E q \cdot(5)}^{* *}=x_{i} p_{i}$
b) The values in parentheses are given in mmHg.

The equilibrium constant of the reaction $\mathrm{H}_{2} \mathrm{O}+\mathrm{D}_{2} \mathrm{O} \rightleftharpoons 2 \mathrm{HDO}$

| $\begin{gathered} \mathrm{t} \\ \left({ }^{\circ} \mathrm{C}\right) \end{gathered}$ | $\mathrm{K}_{\text {gas }}$ |  | $\mathrm{K}_{11 \text { q }}$ |
| :---: | :---: | :---: | :---: |
| 0 | $\begin{aligned} & 3.74 \pm 0.02 \\ & 3.83 \\ & 3.75 \pm 0.08 \end{aligned}$ | $\begin{aligned} & {[24]^{a)}} \\ & {[26-28]^{b)}} \\ & {[25]^{a)}} \end{aligned}$ | $3.78 \pm 0.03[12]^{\text {d) }}$ |
| 20 |  |  | $3.94 \pm 0.12[29]^{\text {c) }}$ |
| 25 | $\begin{aligned} & 3.76 \pm 0.02 \\ & 3.85 \\ & 3.74 \pm 0.07 \end{aligned}$ | $\begin{aligned} & {[24]} \\ & {[26-28]} \\ & {[25]} \end{aligned}$ | $\begin{aligned} & 3.78 \pm 0.03[12] \\ & 3.85 \pm 0.03[30] \mathrm{e}) \end{aligned}$ |
| 75 | $3.80 \pm 0.04$ | [24] | $3.81 \pm 0.02$ [12] |
|  | 3.90 | [26-28] |  |

a) Mass spectrometric determination.
b) Theoretical calculation from spectroscopic data.
c) Measured by NMR technique.
d) Calculated by using $K_{\text {gas }}[24]$ and vapour pressure isotope effect data.
e) From measuring the fractionation factor for 1,3,5-trimethoxy benzene dissolved in water.

The influence of the value of the equilibrium constant ( $\mathrm{K}_{1 \mathrm{iq}}$ ) on the $\Delta p$ calculation

| $\mathrm{K}_{1 \text { iq }}^{\mathrm{a})}$ | $\Delta \mathrm{p}^{\mathrm{b})}$ <br> $(\mathrm{kPa})$ |
| :---: | :---: |
| 3.5 | 1.4316 <br> $(10.738)^{\mathrm{c})}$ |
| 3.7 | 1.4321 |
|  | $(10.742)$ |
| 3.8 | 1.4325 |
| 3.9 | $(10.745)$ |
|  | 1.4328 |
|  | $(10.747)$ |
|  | 1.4331 |
|  | $(10.749)$ |

a) $\mathrm{K}_{\text {liq }}=\frac{(2 \mathrm{~s})^{2}}{(0.5-\mathrm{s})^{2}}$
b) $t=75^{\circ} \mathrm{C} ; \mathrm{r}=1.91$
c) The values in parentheses are given in mmHg .

Since

 difference measurements is $\pm \delta^{*}$ then the corresponding uncertainties in $\mathrm{p}_{\mathrm{HDO}}$ and in $\Delta \mathrm{p}$ will be $\pm \delta^{*} / \mathrm{r}$ and $\sim \pm \delta^{*} / 2$, respectively. In other words, even if the vapour pressure differences between the equimolar $\mathrm{H}_{2} \mathrm{O}-\mathrm{D}_{2} \mathrm{O}$ mixture and $\mathrm{H}_{2} \mathrm{O}$, and between the pure $\mathrm{H}_{2} \mathrm{O}$ and $\mathrm{D}_{2} \mathrm{O}$ are determined on the same apparatus any systematic errors present will only partly cancel each other when the measured values of $\Delta p$ are compared with those calculated from the right hand side of Eq. (6). On the other hand it is still advisable to employ $\mathrm{p}_{\mathrm{H}_{2}} \mathrm{O}^{-\mathrm{p}_{\mathrm{D}_{2} \mathrm{O}}}$ values obtained on the same apparatus otherwise the errors arising from the calibration of different equipment might considerably shift the calculated $\Delta p$ values and render the investigation of the validity of Raoult's law more difficult.

Although in the subsequent calculations a smoothed value of $r$, which was obtained by Van Hook [12] from experimental $\ln \left(\mathrm{p}_{\mathrm{H}_{2} \mathrm{O}} / \mathrm{p}_{\mathrm{D}_{2} \mathrm{O}}\right)$ [14] and $\ln \left(\mathrm{p}_{\mathrm{H}_{2} \mathrm{O}} / \mathrm{p}_{\mathrm{HDO}}\right)$ [13] values, will be used and thus the error assigned to it are smaller than those of the individual $\ln \left(\mathrm{p}_{\mathrm{H}_{2} \mathrm{O}} / \mathrm{p}_{\mathrm{HDO}}\right)$ values, it is interesting to consider the importance attaching to the precision of the determination of $\mathrm{p}_{\mathrm{HDO}}$.

Taking Eq.(7) into consideration and that

$$
\begin{equation*}
\ln \frac{\mathrm{p}_{\mathrm{HDO}}}{\mathrm{p}_{\mathrm{H}_{2} \mathrm{O}}} \cong \frac{\mathrm{p}_{\mathrm{HDO}^{-}} \mathrm{p}_{\mathrm{H}_{2} \mathrm{O}}}{\mathrm{p}_{\mathrm{H}_{2} \mathrm{O}}} \tag{9}
\end{equation*}
$$

and assuming that the error in $\ln \left(\mathrm{p}_{\mathrm{H}_{2} \mathrm{O}} / \mathrm{p}_{\mathrm{HDO}}\right)$ is $\pm \delta$ then the error in the calculated $\Delta \mathrm{p}$ value is $\pm \delta \times \mathrm{X}_{\mathrm{HDO} \times \mathrm{p}_{\mathrm{H}_{2} \mathrm{O}} \text {. Since the }}$ error of Majoube's measurement of the vapour pressure of HDO [13] is $\pm 0.001$ in $\mathrm{p}_{\mathrm{H}_{2} \mathrm{O}} / \mathrm{p}_{\mathrm{HDO}}$ for the temperature range $0-100{ }^{\circ} \mathrm{C}$ the uncertainty in the calculated $\Delta \mathrm{p}$ at $75^{\circ} \mathrm{C}$ can be given as.
$0.5 \times 38530 \mathrm{~Pa} \times( \pm 0.001)= \pm 1.9 \mathrm{~Pa}( \pm 0.14 \mathrm{mmHg})$; see also Fig.(1). This is considerably larger than the precision of the $\mathrm{p}_{\mathrm{H}_{2}} \mathrm{O}^{-\mathrm{p}_{2}} \mathrm{O}$ difference measurement.

The calculation of $\mathrm{p}_{\mathrm{H}_{2}} \mathrm{O}^{-\mathrm{p}}$ values were carried out for the temperatures of 20,50 and $75^{\circ} \mathrm{C}$ by using Eq. (6) and $\mathrm{K}_{11 \mathrm{q}}=3.8$. The calculated and observed $\Delta \mathrm{p}$ values are compared in Fig. 1. The error bars attached to the measured data points correspond to about three standard deviations calculated from the fitting equations of paper [16] by using the law of propagation of errors [31,32].


Fig. 1
Comparison of calculated and observed $\Delta p\left(p_{\mathrm{H}_{2}} \mathrm{O}^{-p_{m i x t u r e}}\right)$ values: O calculated with $r=1.91$, the diameter is proportional to the error $\pm 1 \times 10^{-4}$ in $\ln \left(p_{H_{2}} \mathrm{O} / p_{D_{2} O}\right)$, the error bars correspond to an error of $\pm 0.03$ in $r$; ealculated with $r=2$; $\Delta$ calculated by using directly measured $\ln \left(p_{H D O} / p_{H_{2}} O\right)$ data [13], the error bars correspond to $\pm 0.001$ in $p_{H_{2}} / p_{H D O} ; \times$ measured values with error bars corresponding to $\pm 1 \times 10^{-4}\left(0.6 \times 10^{-4}\right.$ at $\left.75^{\circ} \mathrm{C}\right)$ in In ( $\mathrm{p}_{\mathrm{H}}^{2} \mathrm{O} / p_{\text {mixture }}$ )

## CONCLUSION

It can be seen from Fig. 1 that there are no significant differences between the measured and calculated $\Delta p$ values, i.e. the vapour pressure of the equimolar mixture can be evaluated by using Eq. (4) or Eq. (5), which implies that the equimolar $\mathrm{H}_{2} \mathrm{O}-\mathrm{HDO}-$ $-D_{2} \mathrm{O}$ liquid mixture does not deviate from the ideal behaviour within the limits of the presently available data. It is also clear that before any small deviation from the ideality can be detected the precision of the vapour pressure determination of HDO must be considerably improved. The values calculated with r=2 lie far beyond the range of the experimental results which indicates - in agreement with the conclusion of other investigators [15] - that the law of the geometric mean for the vapour pressure isotope effect in the series $\mathrm{H}_{2} \mathrm{O}$, HDO and $\mathrm{D}_{2} \mathrm{O}$ is not obeyed.

## REFERENCES

[1] G. Jákli, P. Tzias, W.A. Van Hook, J. Chem. Phys. 68, 3177 (1978)
[2] G. Jancsó, W.A. Van Hook, J. Chem. Phys. 68, 3191 (1978)
[3] G. Jancsó, W.A. Van Hook, Physica A 91, 619 (1978)
[4] R.C. Phutela, D.V. Fenby, Aust. J. Chem. 32, 197 (1979)
[5] W.A. Van Hook, J. Phys. Chem. 76, 3040 (1972)
[6] I.E. Puddington, Can. J. Chem. 27, 1 (1949)
[7] R.L. Combs, J.M. Googin, H.A. Smith, J. Phys. Chem. 58, 1000 (1954)
[8] I. Kiss, G. Jákli, H. Illy, Acta Chim. Acad. Sci. Hung. 47, 379 (1966)
[9] K. Zieborak, Z. Phys. Chem. 231, 248 (1966)
[10] V.K. LaMer, W.N. Baker, J. Am. Chem. Soc. 56, 2641 (1934)
[11] W.A. Van Hook, J. Phys. Chem. 72, 1234 (1968)
[12] W.A. Van Hook, J.C.S. Chem. Comm. 1972, 479
[13] M. Majoube, J. Chim. Phys. 68, 1423 (1968)
[14] J. Pupezin, G. Jákli, G. Jancsó, W.A. Van Hook, J. Phys. Chem. 76, 743 (1972)
[15] G. Jancsó, W.A. Van Hook, Chem. Rev. 74, 689 (1974)
[16] Gy. Jákli, H. Illy, KFKI Report 80-15 (1980)
[17] J.S. Rowlinson, "Liquids and Liquid Mixtures", Butterworths Sci. Publ., London, 1959
[18] J.A. Goff, in "Humidity and Moisture", A. Wexler, Ed., Reinhold, New York, 1963, Vol. 3, p. 289
[19] G.S. Kell, J. Chem. Eng. Data 12, 66 (1967)
[20] F.G. Keyes, L.B. Smith, H.T. Gerry, Proc. Am. Acad. Arts Sci. 70, 319 (1936)
[21] G.S. Kell, G.E. McLaurin, E. Whalley, J. Chem. Phys. 48, 3805 (1968)
[22] R.N. Gupta, P.C. Jain, V.S. Nanda, Indian J. Pure Appl. Phys. 11, 308 (1973)
[23] O.D. Lagutkin, B.V. Dergachev, Izv. Vyssh. Uchebn. Zaved. Energ. 19, 144 (1976)
[24] L. Friedman, V.J. Shiner, Jr., J. Chem. Phys. 44, 4639 (1966)
[25] J.W. Pyper, R.S. Newbury, G.W. Barton, J. Chem. Phys. 46, 2253 (1967)
[26] M. Wolfsberg, J. Chem. Phys. 50, 1484 (1969); Adv. Chem. Ser. 89, 185 (1969)
[27] J.R. Hulston, J. Chem. Phys. 50, 1483 (1969)
[28] M. Wolfsberg, A.A. Massa, J.W. Pyper, J. Chem. Phys. 53, 3138 (1970)
[29] V. Gold, C. Tomlinson, J.C.S. Chem. Comm. 1970, 472
[30] A.J. Kresge, Y. Chiang, J. Chem. Phys. 49, 1439 (1968)
[31] P.R. Bevington, "Data Reduction and Error Analysis for the Physical Sciences", McGraw-Hill, New York, 1969, Chap. 4
[32] A.A. Clifford, "Multivariate Error Analysis", Appl. Sci. Publ., London, 1973

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