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BUDAPEST

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POSSIBLE EFFECT OF FREE ELECTRON AND ION DENSITIES ON THE RESULTS OF LAMB SHIFT MEASUREMENTS

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ABSTRACT

Corrections to the Lamb shift occurring in plasma surroundings are discussed.

АННОТАЦИЯ

Были определены поправки смещения Лемба в окружении плазмы.

KIVONAT

A Lamb eltolódás plazma környezetben keletkező korrekcióit tárgyaljuk.

In the last ten years several publications have appeared detailing Lamb shift measurements in hydrogenics up to $Z = 18$[1,2]. The experimantal techniques utilized suggest that the correction caused by the microphysical surroundings, i.e. by real photons, free electrons and ions being present be taken into account. The temperature dependent part of the Lamb shift was calculated in the case of black body radiation[3,4]. In the above mentioned experiments[1,2] the photon spectrum is not like a black body one, but is reminiscent rather of the spectra radiated by plasmas. In view of this, we discuss corrections to the Lamb shift occurring in plasma surroundings, especially in low density plasmas.

To get the effect of the photons we use the nonrelativistic Lamb shift correction formula of Walsh[3] obtained for the case of black body radiation

$$\Delta E_m = - \frac{\alpha}{4\pi^2}\left(\frac{\hbar}{m}\right)^2 \int \sum_n (\underline{p}_{nm}\underline{e}_k)^2 \left[\frac{1}{E_n - E_m + \hbar\omega} + \frac{1}{E_n - E_m - \hbar\omega}\right] * \\ * N(\omega) k dk d\Omega_k \quad , \tag{1}$$

where ω denotes the frequency and k the wave number of the photons, $N(\omega)$ is the photon occupation number, $\underline{e}_k$ is the polarization vector of the photons and $d\Omega_k$ is the unit solid angle in the k space. The energies of the stationary states of the hydrogen-like systems are denoted by E_n and the energy of the investigated state is E_m. The $\underline{p}_{nm}$ is the matrix element of the $-i\hbar\underline{\nabla}$ operator and α is the fine structure constant. Instead of the formula of the black body radiation:

$$N(\omega) = (e^{\hbar\omega/KT} - 1)^{-1}$$

we substitute the occupation number of the plasma radiation $N_p(\omega)$

into (1). The $N_p(\omega)$ is obtained from the spectral energy densities of the transverse and longitudinal waves ($u_{\omega T}$ and $u_{\omega L}$) using the following identity

$$\int u_\omega d\omega = \frac{1}{(2\pi)^3}\int \hbar\omega N_p(\omega)\, d^3\underline{k} \tag{2}$$

and the dispersion relations of the transverse and longitudinal photons. The low frequency limit of the plasma radiation gives the dominant effect and we use the spectral energy density formulae of this extreme case[5]. The transverse and longitudinal photon numbers are

$$N_{pT}=\frac{KT}{E_n-E_m}\frac{1}{X} \tag{3}$$

and

$$N_{pL}=\frac{KT}{(E_n-E_m)^2}\frac{1}{X^2}\hbar\omega_p \tag{4}$$

where ω_p is the plasma frequency, T the electron temperature, K the Boltzmann constant and $X = \hbar\omega/(E_n-E_m)$. Putting (3) and (4) into (1), using the $k_T dk_T=\frac{1}{c^2}\omega d\omega$ and the $k_L dk_L=\frac{1}{3u^2}\omega d\omega$ identities obtained from the dispersion relations[5] (where $u^2 = KT/m$) and the formulae

$$\int (\underline{p}_{nm}\underline{e}_k)^2 d\Omega_k=\begin{cases}\frac{8\pi}{3}|\underline{p}_{nm}|^2 & \text{for transverse photons}\\ \frac{4\pi}{3}|\underline{p}_{nm}|^2 & \text{for longitudinal photons}\end{cases} \tag{5}$$

we obtain the shift caused by the transverse and longitudinal photons present in the plasma radiation

$$\Delta E_{mT}= -\frac{2}{3\pi}\alpha\frac{1}{(mc)^2}\sum_n |p_{nm}|^2 KT\int_0^\infty \frac{2}{1-X^2}\,dX \tag{6}$$

$$\Delta E_{mL}= -\frac{\alpha}{9\pi}\frac{1}{(mu)^2}\sum_n |p_{nm}|^2 \frac{KT}{E_n-E_m}\hbar\omega_p\, I(Y) \tag{7}$$

where

$$I(Y) = \int_Y^{\infty} \frac{2dX}{X(1 - X^2)} \tag{8}$$

with $Y = \hbar\omega_p/(E_n - E_m)$. The choice of Y as the lower limit in integral (8) expresses the fact that all photons having frequency $\omega < \omega_p$ are absorbed. The integral in (6) gives zero thus $\Delta E_{mT} = 0$ and the shift is caused solely by the longitudinal photons. In low density plasmas the relation $Y \ll 1$ holds, therefore $I(Y)$ can be written approximately as $I = -2 \ln Y$. Using the identity $\underline{p}_{nm} = -im(E_n - E_m)\underline{x}_{nm}/\hbar$, the level shift of the state m is

$$\Delta E_m = \frac{\alpha}{9\pi}\hbar\omega_p \sum_n \frac{m(E_n - E_m)}{\hbar^2}|\underline{x}_{nm}|^2 \; 2 \ln\left|\frac{\hbar\omega_p}{E_n - E_m}\right| \tag{9}$$

where $\underline{x}_{nm}$ is the dipole matrix element. The concrete forms of the level shifts in the $2S$ and $2P$ states are

$$\Delta E_{2S} = \frac{\alpha}{36\pi}\hbar\omega_p \sum_{n>2} \frac{1}{3} \left|R_{20}^{n1}\right|^2 \left(\frac{n^2-4}{n^2}\right) \ln \frac{4n^2}{n^2-4} + B \tag{10}$$

$$\Delta E_{2P} = \frac{\alpha}{36\pi}\hbar\omega_p \sum_{n>2} \left(\frac{1}{3}\left|R_{21}^{n0}\right|^2 + \frac{2}{3}\left|R_{21}^{n2}\right|^2\right)\frac{n^2-4}{4n^2} \ln \frac{4n^2}{n^2-4} + B - \frac{\alpha}{36\pi}\hbar\omega_p \left|R_{21}^{10}\right|^2 \ln\frac{4}{3} \tag{11}$$

where

$$B = -\frac{1}{3\pi}\alpha\hbar\omega_p \ln \frac{RZ^2}{\hbar\omega_p}, \tag{12}$$

n is the principal quantum number and the quantities R_{20}^{n1}, R_{21}^{n0} and R_{21}^{n2} are expressed in atomic units[6]. The energy shift difference $\delta_1 = \Delta E_{2S} - \Delta E_{2P}$ is

$$\delta_1 = \frac{2^{13}}{3^{11}}\frac{\alpha}{\pi}\hbar\omega_p \left\{3^7 \sum_{n>2} \frac{n^5(n-2)^{2n-6}}{(n+2)^{2n+6}} (-21n^4 - 24n^2 + 48) \ln\frac{4n^2}{n^2-4} + \ln\frac{4}{3}\right\} \tag{13}$$

and it has no Z dependence.

Another effect of the electron density is a shielding of the Coulomb potential of the nucleus. The Debye-Hückel potential for an electron in the shielded field of an ion of charge Ze is

$$V(r) = - Ze^2 \frac{e^{-r/d}}{r} , \tag{14}$$

where d is the Debye length,

$$d = (KT/4\pi n_e e^2)^{1/2} , \tag{15}$$

n_e and T are the density and temperature of the free electron gas, respectively. The energy levels of pure Coulomb potential are shifted because of this shielding. A useful approximate formula of these level shift is obtained by Smith[7] in the form of a power series in a_o/d (a_o is the Bohr radius)

$$\Delta E_{n\ell} = R(2\frac{a_o}{d} - \frac{1}{2}(\frac{a_o}{d})^2 \frac{1}{Z}\left[3n^2 - \ell(\ell+1)\right] + \text{higher order terms }) . \tag{16}$$

The level shift difference between the $2S$ and $2P$ states caused by the shielding is

$$\delta_2 = - \frac{R}{Z} (\frac{a_o}{d})^2 . \tag{17}$$

The Lamb shift and its change by the Stark effect is discussed by Bethe and Salpeter for the case of weak external fields[8]. Their result for the $2S_{1/2} - 2P_{1/2}$ splitting is

$$\Delta E(2S_{1/2} - 2P_{1/2}) = \frac{1}{2} L\left[1 \pm (\frac{1 + 4(n^2-1)(nm)^2 \underline{E}^2 e^2 a_o^2}{L^2})^{1/2}\right] , \tag{18}$$

where $\underline{E}$ is an external electrostatic field, e is the unit charge, n and m are the principal and the magnetic quantum numbers respectively, and L is the Lamb splitting without external field. In our case $\underline{E}$ is the sum of two terms

$$\underline{E} = \underline{E}_{ext} + \underline{E}_{ion} \quad , \tag{19}$$

where $\underline{E}_{ext}$ is the applied external field and $\underline{E}_{ion} = Z_p e \underline{r}_p / r_p^3$ is the electrostatic field produced by a perturber ion of $Z_p e$ charge. The total change of the Lamb shift caused by all the ions present can be obtained by taking the average of (18) with the ion-distribution function, normalized to unity, giving the probability of finding a perturber ion at a distance r_p from the hydrogen-like ion[9]

$$dP(\frac{r_p}{\rho}) = e^{-(r_p/\rho)^3} \, d(\frac{r_p}{\rho})^3 \quad , \tag{20}$$

where the quantity ρ is defined by

$$\frac{4\pi}{3} n_p = \rho^{-3} \quad , \tag{21}$$

and n_p is the density of the perturber ions. The cross term from the average vanishes because of symmetry considerations, and the Lamb splitting takes the form

$$\Delta E(2S_{1/2}-2P_{1/2}) = \frac{1}{2}L\left[1 \pm (1 + \frac{1}{2}F + \frac{1}{2}b\int_0^\infty x^{-2/3} e^{-x} dx)\right] \tag{22}$$

where $x = r/\rho$,

$$b = 16(n^2-1)(nm)^2 Z_p^2 R^2 (\frac{a_o}{\rho})^4 / L^2, \tag{23}$$

where $R = me^4/2\hbar^2$ and

$$F = 4(n^2-1)(nm)^2 \underline{E}_{ext}^2 e^2 a_o^2 / L^2 \ . \tag{24}$$

Thus the change of the $2S_{1/2} - 2P_{1/2}$ splitting due to the ion cloud is

$$\delta_3 = \frac{1}{2} b \, \Gamma(\frac{1}{3}) L \ . \tag{25}$$

The total correction to the Lamb shift occurring in plasma surroundings is

$$\delta = \delta_1 + \delta_2 + \delta_3 \quad . \tag{26}$$

Putting $\omega_p = (4\pi n_e e^2/m)^{1/2}$ into Eq.(13) and evaluating the formula numerically one gets

$$\delta_1 = -5.858 \sqrt{n_e(\mathrm{cm}^{-3})} \cdot 10^{-5} \quad \mathrm{MHz}. \tag{27}$$

Similarly from formula (17) one can obtain

$$\delta_2 = -1.667 \cdot 10^{-13} n_e/(ZKT) \quad \mathrm{MHz} \ , \tag{28}$$

where n_e and KT are expressed in cm^{-3} and eV units, respectively. If $KT = 0.025$ eV, corresponding to room temperature, then $Z\delta_2$ becomes equal to δ_1 at $n_e \cong 10^{14}\mathrm{cm}^{-3}$ electron density. It follows from (27) and (28) that $|\delta_1| \gg |Z\delta_2|$ if $n_e < 10^{10}\mathrm{cm}^{-3}$ (supposing $KT = 0.025$ eV). Even low electron densities produce δ_1 corrections for the Lamb shift which are in the order of the experimental errors, e.g. $n_e = 1.66 \cdot 10^5\ \mathrm{cm}^{-3}$ gives $\delta_1 = -\ 0.02$ MHz, which is equal to the experimental error of the Lamb shift measurement of Andrews and Newton[10]. Because of the strong Z dependence of L the relative Stark effect change δ_3/L, decreases for hydrogenics of increasing Z, as can be seen from Eqs.(23) and (25), thus we only deal with the $Z = 1$ case. Using the experimental value $L = 1057.862$ MHz[10] and Eqs.(21), (23) and (25) with $Z_p = 1$ one gets

$$\delta_3/L = 9.877 \cdot 10^{-18}\ n_p^{4/3} \quad . \tag{29}$$

Computing the density n_p in the 21 keV proton beam used in the above mentioned experiment[10], and substituting it into (29) we obtain the numerical values of δ_3/L at different beam current densities i, expressed in $\mu A/mm^2$ units (see Table 1).

The above calculations show that the most important correction to the Lamb shift is likely to arise from the electron density, therefore we suggest that it be taken into account and that it be included in the systematic corrections when evaluating the Lamb shift measurements.

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Table 1.

The dependence of the relative Stark effect change δ_3/L, on the proton beam current density i.

i ($\mu A/mm^2$)	δ_3/L
1	$4.52 \cdot 10^{-7}$
2	$1.14 \cdot 10^{-6}$
5	$3.86 \cdot 10^{-6}$
10	$9.73 \cdot 10^{-6}$
20	$2.45 \cdot 10^{-5}$
50	$8.32 \cdot 10^{-5}$

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