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ENERGY LOSS IN THE SOLAR SYSTEM AND MODULATION OF COSMIC RADIATION

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ENERGY LOSS IN THE SOLAR SYSTEM AND MODULATION OF COSMIC RADIATION

J. Kóta
Central Research Institute for Physics
H-1525 Budapest, P.O.B. 49, Hungary

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ABSTRACT

Penetration of galactic cosmic rays into the solar system is investigated by considering the time reversed propagation process. Reformulated transport equation is set up and equations of moments are deduced. By applying the simplest approximation, the force-field solution is reproduced. Discussed are two effects pointed out to cause deviations /in opposite directions/ from the force-field solution. The resulting radial anisotropy is estimated, too.

АННОТАЦИЯ

Исследуется проникновение галактических космических лучей в солнечную систему с помощью прослеживания время-обращенного процесса распространения частиц. Составляется уравнение транспорта и выводятся уравнения моментов. При самом простом приближении получается решение силового поля. Обсуждаются два эффекта, которые дают отклонение в разные стороны от решения силового поля. Оценивается значение возникшей анизотропии.

KIVONAT

A részecsketerjedés idő-tükrözött folyamatának nyomonkövetésével vizsgáljuk a galaktikus kozmikus sugárzás behatolását a naprendszerbe. A transzport-egyenlet felirása után momentum-egyenleteket vezetünk le. A legegyszerübb közelités az ismert erőtér-megoldást adja. Két hatás tárgyalunk, amelyek eltérést okoznak - ellenkező irányban - az erőtér megoldástól. Becslést adunk a fellépő anizotrópia mértékére is.

1. INTRODUCTION

The steady-state transport of cosmic rays in the solar system is governed by the equation /Parker 1965, Gleeson and Axford 1967/

$$\mathrm{div}(\vec{V}\cdot F - \kappa \ \mathrm{grad}\ F) = \frac{1}{3p^2}\ \mathrm{div}\ \vec{V}\cdot \frac{\partial}{\partial p}(p^3 F) \qquad /1/$$

where $\vec{V}(\vec{r})$ is the solar wind speed; $\kappa(\vec{r},p)$ is the diffusion coefficient; p is the cosmic ray momentum; and F(r,p) is the mean distribution function /averaged over pitch angle/ in term of momentum. Eq. /1/ has been extensively studied under the assumption of spherical symmetry and separable diffusion coefficient. Gleeson and Axford /1968/ have found a simple approximate solution known as force-field solution. Gleeson and Urch /1973/ studied the validity of the force-field approximation by solving /1/ numerically. Analytic solutions of /1/ have also been obtained for a wide class of κ /see Webb and Gleeson 1973, 1977/.

In this work, we endeavour to directly exploit Liouville's theorem stating that distribution function is conserved on trajectories of motion. To obtain the galactic momentum /p*/ distribution of particles of detected momentum, p, the time reversed motion should be inspected i.e. antiparticles of momentum, p will be released from the site of observation. The distribution of their momentum at the exit from the modulation region, G(p*) yields

$$F(p) = \int G(p^*)\ F_g(p^*)\ dp^* \qquad /2/$$

with F_g denoting the unmodulated galactic distribution.

The two approaches to obtain F are obviously equivalent thus the same mathematical difficulties emerge in both cases. Although, in some cases being clumsier than tackling Eg. /1/, the technique of time-reversal to be presented here also has some attractive features

/i/ using it may appear advantageous if we ask what part of the galactic spectrum is observed at the earth at momentum, p, or if Monte-Carlo calculation is made /in the latter case it is ensured that only particles reaching the earth should be considered/.

/ii/ Approximate equations of moments can be deduced which enable the moments of $G(p^*)$ to be estimated in a simple way.

2. TRANSPORT EQUATION, EQUATIONS OF MOMENTS

The transport equation for particles released from r_o with momentum, p_o, in random directions is

$$\frac{\partial g}{\partial p} + \operatorname{div}\left[-\kappa \cdot \operatorname{grad} g/\dot{p} - \vec{V}(g/\dot{p})\right] = \delta(p-p_o)\cdot\delta^3(\vec{r}-\vec{r}_o) \qquad /3/$$

with

$$g(\vec{R},p) = 0$$

i.e. free escape is allowed at the boundary of the modulating region $/\vec{r}=\vec{R}/$. $g(\vec{r}_o,p_o,\vec{r},p)$ $/r_o$ and p_o appearing here as parameters only/ is the particle density, while $\dot{p}$ stands for adiabatic deceleration /Parker 1965/

$$\dot{p} = \frac{p}{3}\operatorname{div} V \qquad /4/$$

Note that at time-reversal the <u>solar wind velocity reverses,</u> too. Now, the distribution of momentum at the exit, $G(\vec{r}_o,p_o,p)$, can be evaluated from

$$\text{either} \quad G=-\frac{\partial}{\partial p}\int g\cdot dV \ ; \qquad \text{or} \qquad G=-\int \kappa\cdot\operatorname{grad}\,(g/\dot{p})\,d\vec{F} \qquad /5/$$

/integrating over the volume/ surface of the solar cavity/ and hence G yields the modulated spectrum /Eq.2./.

For comparison the main features of the two approaches are summarized below

	Conventional	Time-reversed
	particle	anti-particle
solar wind:	outward	inward
boundary cond.:	$F(\vec{R},p)=F_g(p)$	$g(\vec{R},p)=0;$ $g(p_o)=\delta^3(\vec{r}-\vec{r}_o)$
route to F:	solve /1/	solve /3/, then $g \to G$ /5/, $G \to F$ /2/

It should be emphasized that the two routes are equivalent and Equations /1/ and /3/ can be transformed to the same form, however, with different boundary conditions.

2.1 Equations of moments.

Let it be assumed that the usual conditions of the force-field approximation are met, i.e. the diffusion coefficient is separable and diffusion plays the dominant role in particle propagation

$$\kappa(\vec{r},p) = \kappa_1(\vec{r})\cdot\kappa_2(p)\cdot\beta \quad \text{and} \quad (V\cdot r)/\kappa \ll 1 \qquad /6/$$

β being particle speed/speed of light. Similarly as in force-field approximation /see Gleeson and Urch 1973/, introduce ϕ with

$$\int_p^{p^*} \frac{\kappa_2' \beta'}{p'} dp' = \phi; \qquad p^*=p^*(p,\phi). \qquad /7/$$

Instead of having, as in force-field approximation, unique values, p^* and ϕ will have distributions, $G(p^*)$ and $H(\phi)$, respectively. From Eq. /7/ it follows

$$H(\phi) = G(p^*)\cdot\frac{p^*}{\kappa_2(p^*)\beta^*} . \qquad /8/$$

By exploiting conditions /7/, the convective term of Eq. /3/ can be neglected and equations can be deduced for the moments of the distribution $H(\phi)$ /in detail will be published elsewhere/

$$\mathrm{div}(-\kappa_1\ \mathrm{grad}\langle\phi^n\rangle) = \frac{n}{3}\langle\phi^{n-1}\rangle\ \mathrm{div}\ V, \qquad /9/$$

where

$$\langle\phi^n\rangle = \int_0^\infty H(\phi)\cdot\phi^n\cdot d\phi\ .$$

All the relations obtained so far are valid, under the assumption made, in general and are not restricted to spherical symmetry. Now, in order to be able to carry out calculations, we apply Eq. /9/ for spherically symmetric case and evaluate the mean value of ϕ. It is found that

$$\langle\phi\rangle = \int_r^R \frac{V(r')}{3\kappa_1(r')}\,dr' = \phi_o(r) \qquad /10/$$

ϕ_o denoting the well-known modulation parameter. This clearly shows: what force-field approximation contains is replacing the convolution /2/ with the value of the galactic distribution taken ap p^* corresponding to the mean of ϕ, i.e.

$$F(r,p) = \int_0^\infty H(\phi)\cdot F_g(p^*(p,\phi))d\phi \Rightarrow F_g(p^*(p,\langle\phi(r)\rangle)) \qquad /11/$$

In what follows the applicability of this approximation will be examined.

3. STUDY OF THE FORCE-FIELD APPROXIMATION

The force-field solution is approximate in two ways:

/i/ the accuracy of the approximation /11/ depends on the shape of the galactic spectrum, F_g.

/ii/ Convective term has been neglected in Eq. /3/. Including it will result in modifying Eq. /9/ and cause a shift of the mean value of ϕ.

In order to estimate the resulting deviations from both effects, we turn to the Taylor-expansion of F around the force field solution /i.e. around $\phi=\phi_o$/

$$F-F_{ff} = \frac{\partial F_{ff}}{\partial \phi} < \phi - \phi_o > + \frac{1}{2} \frac{\partial^2 F_{ff}}{\partial \phi^2} < (\phi-\phi_o)^2 > + \ldots \qquad /12/$$

F_{ff} denoting the force-field solution.

3.1 The effect of spectral shape.

The value of $<(\phi-\phi_o)^2>$ can easily be evaluated from Eq. /9/

$$\phi_2 = \frac{1}{2} < (\phi-\phi_o)^2 > = \int_r^R \frac{dy}{y^2 \kappa_1} \int_o^y dx\, x^2 \frac{V^2}{9\kappa_1} \qquad /13/$$

ϕ_2 gives an immediate estimation for the width of the distribution of loss of momentum /$\Delta p \approx p\Delta\phi/\kappa_2\beta$/. The results obtained in this way are in general agreement of those of Urch and Gleeson /1973/ who made numerical investigations. Adopting the parameters used by Urch and Gleeson /1972/, we have ϕ_2=0.025 M^{-2}. This implies that the distribution is relatively narrow at solar minimum when M is large, and it is broad at solar maximum.

It is interesting to note that the ratio ϕ_2/ϕ_o^2 is minimal if $\kappa_1 \alpha r$. Thus a lower limit can be set for ϕ_2

$$\phi_2 \geq \phi_o^2 / |2 \cdot \ln(R/r)| \qquad /14/$$

The derivatives of F_{ff} can be obtained by using relation /7/.

Hence we arrive at

$$\frac{F-F_{ff}}{F_{ff}} \approx \left\{\frac{\partial^2 \ln F_{ff}}{\partial \ln p^2}+\left(\frac{\partial \ln F_{ff}}{\partial \ln p}\right)\cdot\left(\frac{\partial \ln F_{ff}}{\partial \ln p}-1+\beta^2-\frac{\partial \ln \kappa_2}{\partial \ln p}\right)\right\} \frac{\phi_2}{(\kappa_2\beta)^2} \qquad /15/$$

Since the spectrum steeply decreases with increasing momentum, the effect discussed here tends to result in higher cosmic ray density than predicted from force-field approximation. /Note that $\partial \ln F/\partial \ln p=-2+(\partial \ln j/\partial \ln p)$, j being the differential intensity in term of kinetic energy/.

Figure 1. shows the correction from this effect for both electrons and protons for the year 1970.

3.2 The effect of convection.

To demonstrate the role of convection a qualitative picture can be drawn. The average effect of diffusion is $r\cdot dr/dt \approx 3\kappa$. Combining this with inward convection and adiabatic deceleration, we have

$$\frac{p}{3}\,\mathrm{div}\cdot V\cdot\frac{dr}{dp} = \frac{3\cdot\kappa}{r} - V \qquad /16/$$

implying that, if the condition $Vr/\kappa<1$ is not met, convection forces particles to spend longer time in the modulating region leading to larger loss of momentum. At low energies this effect, because of their lower speed, is stronger for protons than electrons giving rise to the exclusion of low energy galactic protons from the inner solar system. /Here, we briefly mention that, for convection dominant propagation of solar particles, Eq. /16/ immadiately gives p^3r^2V=const. reproducing the results of Fisk and Axford /1969/ and Gleeson /1971//.

Inclusion of convection results in the modified form of Eq. /9/

$$\mathrm{div}\left[-\kappa_1\ \mathrm{grad}\langle\phi^n\rangle\right] + \frac{V}{\kappa_2\beta}\ \mathrm{grad}\langle\phi^n\rangle = \frac{n}{3}\ \mathrm{div}\ V\langle\phi^{n-1}\rangle \qquad /17/$$

leading to

$$\langle\phi\rangle = \phi_o + \phi_1 \quad ; \qquad \phi_1 = \frac{3\phi_2}{\kappa_2\beta} \qquad /18/$$

It should be noted, however, that Eqs. /17/ and /18/ are approximative only as far as the change of $\kappa_2\beta$ is ignored in the convective term of /17/. Nevertheless, /18/ gives useful estimation

$$\frac{F-F_{ff}}{F_{ff}} \approx 3\ \frac{\partial \ln F_{ff}}{\partial \ln p}\ \frac{\phi_2}{(\kappa_2\beta)^2}\ . \qquad /19/$$

The convection, as also expected from /16/, gives rise to lower intensity than predicted from force-field. This is shown in Figure 1, too.

3.3. Combined effect, anisotropy.

The total effect of spectral shape and convection is on the basis of /15/ and /19/:

$$\frac{\Delta F}{F} = (F-F_{ff})/F_{ff} \approx -\ \frac{\partial\mu}{\partial \ln p} + (\mu+2)\left(\mu-1+(1-\beta^2)+\frac{\partial \ln\kappa_2}{\partial \ln p}\right)\frac{\phi_2}{(\kappa_2\beta)^2}\ . \qquad /20/$$

For larger $\Delta F/F$, /20/ is extended by putting $\Delta F/F \Rightarrow \ln F/F_{ff}$. The ratio of F to F_{ff} is shown in Figure 1. together with that obtained from the numerical solution of Gleeson and Urch /1973/. Values of μ are determined as the negative exponent

$$\mu = -\ \frac{\partial \ln\ j_T}{\partial \ln T}\cdot\frac{E_o+2T}{E_o+T} \qquad /21/$$

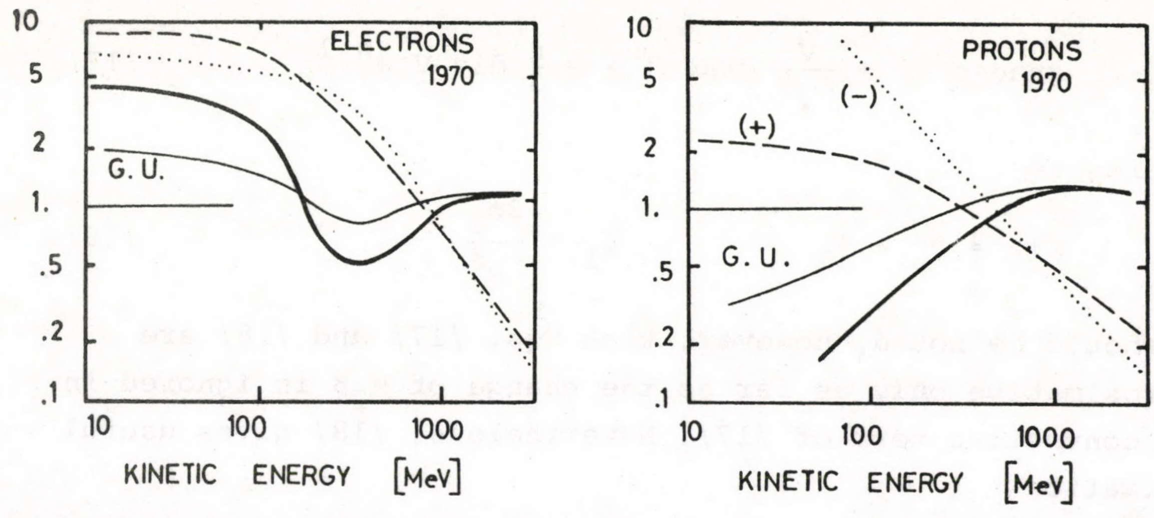

Fig.1.: Magnitude of modifying effects: /a/ increasing effect of spectral shape /dashed line/ and /b/ decreasing effect of convection /dotted line/. Full line shows the resulting ratio of cosmic ray intensity to that obtained from force-field. Solid line /G.U./ indicates the ratio obtained by numerical investigation of Gleeson and Urch /1973/.

Inspection of Fig.1 shows that the predicted ratios describe the general character of those obtained by Gleeson and Urch /1973/, although the present calculation seems to overestimate the deviations from unity.

Anisotropy. In the present model, in contrast to the force-field solution, gradients of ϕ_2 and ϕ_1 give rise to radial streaming, too. The calculated anisotropy

$$\xi_r \approx -3\kappa_1 \cdot \kappa_2 \cdot F_{ff}^{-1}(F-F_{ff}) \cdot \mathrm{grad}(\ln\phi_2)/c \qquad /22/$$

is directed outward if $F>F_{ff}$, and inward if $F<F_{ff}$. Its amplitude, however, remains below 0.03 per cent.

4. SUMMARY

A technique based on time-reversal and determining the moments of energy loss has been presented. Deviations from force-field solution are qualitatively well described. To achieve quantitative agreement further refinement and considering higher moments are needed.

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