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ABSTRACT

A computational method has been devised for optimizing holograms of high dynamic range. As an example the Fourier hologram of a slit is given and application to holographic correlation is shown.

АННОТАЦИЯ

Разработан вычислительный метод для оптимизации записи голограмм, регистрирующих световые распределения, меняющиеся в широких пределях. Применение метода демонстрируется на примере одномерной Фурье-голограммы щели и оптимизации согласованного фильтра голографического коррелятора.

KIVONAT

Számitási módszert dolgoztunk ki nagy dinamikáju jelek holografikus rögzitésének optimalizálására. A módszer illusztrálására bemutatjuk egy rés egydimenziós Fourier-hologramjának optimalizálását és az eljárás alkalmazását holografikus korrelátor szürőjének elkészitéséhez.

I. INTRODUCTION

It is well known, that holographic recording materials have a relatively narrow linear range. Nearly linear recording can be achieved by increasing the reference-to-object beam ratio. In this case, however, the diffraction efficiency strongly decreases. Additional difficulties may arise if the intensity distribution of the object beam varies to a large degree, e.g. in the case of Fourier-holograms. In the present paper we describe a computation method for optimizing the exposure **parameters** of such holograms. Several authors [1-4] base their calculations on the transmittance v.s. exposure curve. Gibin et. al. [5] utilize curves proposed by Upatnieks et. al. [6]. Their method, however, only allows optimization of holograms recorded with a plane reference wave. Our method is based on the Lin-curves of holographic materials [7] and can be used for an arbitrary recording and reconstructing distribution. We applied the method in the case of correlation signals.

II. THEORY

The Lin-curves represent the diffraction efficiency as a function of the average exposure energy and visibility /see (14) in [7]/:

$$\sqrt{\eta} = S E_o V \tag{1}$$

where the average exposure energy is

$$E_o = \left(|s|^2 + |r|^2\right) \cdot \tau \tag{2}$$

and the visibility is

$$V = \frac{2 \cdot s \cdot r}{|s|^2 + |r|^2} \tag{3}$$

τ denotes the exposure time, **s** and **r** are the complex amplitudes of the object and reference plane waves, **respectively.** For ideal recording materials the S-factor depends only on the properties of the material and the developing process. For real recording materials, however, S is not a const. but varies with the exposure parameters, so $\sqrt{\eta}$ is a nonlinear function of E_o and V :

$$\sqrt{\eta}(E_o, V) = S(E_o, V) \cdot E_o \cdot V \tag{4}$$

The $\sqrt{\eta}(E_o, V)$ function can be obtained experimentally by producing a series of holograms, recording plane waves with several E_o and V values and measuring the diffraction efficiency of each hologram. The $\sqrt{\eta}$-curves for amplitude holograms of the holographic plate AGFA-GAEVERT 10E75 /handled with standard developing process/ can be seen in fig. 1.

In practice the distribution of the object beam is not uniform, especially in the case of Fourier holograms, where the intensity of the beam to be recorded can change several orders of magnitude and the reference beam distribution is usually Gaussian instead of uniform. The nonoptimal choice of the exposure parameters can lead to serious nonlinear distortions and to significant decrease of the diffraction efficiency. For this reason it is very important to optimize correctly the intensity ratio of the beams and the exposure time.

Let the complex amplitude distributions of the signal and reference waves be

$$s(x_H, y_H) = |s(x_H, y_H)| \cdot e^{i\alpha_s(x_H, y_H)} \tag{5}$$

$$r(x_H, y_H) = |r(x_H, y_H)| \cdot e^{i\alpha_r(x_H, y_H)} \tag{6}$$

where (x_H, y_H) are the coordinates of the hologram plane. In this case E_0 and V are functions of (x_H, y_H):

$$E_0(x_H, y_H) = \left(|s(x_H, y_H)|^2 + |r(x_H, y_H)|^2 \right) \cdot \tau \tag{7}$$

$$V(x_H, y_H) = \frac{2 \cdot |s(x_H, y_H)| \cdot |r(x_H, y_H)|}{|s(x_H, y_H)|^2 + |r(x_H, y_H)|^2} \tag{8}$$

For any given $s(x_H, y_H)$ and $r(x_H, y_H)$ light distributions and τ values the hologram can be described as a $\sqrt{\eta}(x_H, y_H)$ hologram transfer function. This function can be computed at each (x_H, y_H) point by means of the measured $\sqrt{\eta}(E_0, V)$ curves. The E_0 and V values at every (x_H, y_H) point are calculated according to (7) and (8).

The recostructing process can be considered as follows: Let the complex amplitude distribution of the reconstructing beam be

$$r'(x_H, y_H) = |r'(x_H, y_H)| \cdot e^{i\alpha_{r'}(x_H, y_H)} \tag{9}$$

The beam reconstructed from the hologram is

$$s'(x_H, y_H) = |r'(x_H, y_H)| \cdot \sqrt{\eta}(x_H, y_H) \cdot e^{i\alpha_s(x_H, y_H) + i[\alpha_{r'}(x_H, y_H) - \alpha_r(x_H, y_H)]} \tag{10}$$

/Only the proper diffraction order is considered./ Formula (10) is valid for light distributions satisfying the condition

$$|\alpha_{r'} - \alpha_r| \leqslant \alpha_{Bragg} \tag{11}$$

at each (x_H, y_H) point. α_{Bragg} denotes the angle limit beyond which the volume natúre of the hologram must be considered.

The reconstructed beam, after passing through an optical system reaches the detector plane. Consequently, in the (x_D, y_D)

detector plane the intensity of the $s_{out}(x_D, y_D)$ output distribution can be observed.

Let P_{out} denote the total output power:

$$P_{out} = \iint |s_{out}(x_D, y_D)|^2 \, dx_D \, dy_D \qquad (12)$$

The normalized intensity distribution of the output signal is

$$i(x_D, y_D) = \frac{|s_{out}(x_D, y_D)|^2}{P_{out}} \qquad (13)$$

We define the influence of nonlinear distortion of the hologram by the following expression:

$$\Delta = \sqrt{\iint \left[i(x_D, y_D) - i_{id}(x_D, y_D)\right]^2 dx_D \, dy_D} \qquad (14)$$

where $i_{id}(x_D, y_D)$ describes the desired normalized output intensity distribution which could be achieved in the case of an ideal linear hologram. Repeating the calculations for several values of reference and object beam intensities and exposure times we can choose the optimal combination corresponding to the highest P_{out} by the lowest Δ values. In most cases we can define the optimum as the maximum of the P_{out}/Δ ratio.

III. EXAMPLE: ONE-DIMENSIONAL FOURIER HOLOGRAM OF A SLIT

The method described in section II. was used for the numerical analysis of an optical system. This system produces the output image by two Fourier transforms in x-dimension and two successive projections in y-dimension by means of proper combination of cylindrical and spherical lenses. The hologram is in the Fourier plane /which is identical with the first image plane in the y-dimension/. Similar systems are described e.g. in [8]. A rectangular slit is placed in the input plane. The object beam to be recorded is

$$s(x_H, y_H) = s_0 \frac{\sin \frac{\pi d}{\lambda f} x_H}{\frac{\pi d}{\lambda f} x_H} \mathrm{rect}\left\{ \frac{y_H}{h} \right\} \qquad (15)$$

where d and h denote the width and height of the slit, λ is the wavelength of the laser light and f is the focal length of the spherical lens. A plane wave was used as a reference beam in recording and reconstructing process. The intensity of the beam and the exposure energy can be characterized by the intensity and energy values at the origin:

$$J_s(0,0) = s_0^2 \qquad (16)$$

$$E_0(0,0) = \left[J_r + J_s(0,0) \right] \cdot \tau \qquad (17)$$

where J_r is the intensity of the reference wave. The data of the numerical calculations were: d =0,33 mm; h =1 mm, λ =632.8 nm, the intensity of the reconstructing reference beam $J_{r'} = |r'|^2 = 1 \frac{mW}{mm^2}$ and the Lin-characteristics of the AGFA-GAEVERT 10E75 holographic plate have been used /see fig. 1/.

The results of calculating P_{out} and Δ v.s. exposure parameters are shown in figs. 2 and 3.

In our method the P_{out} -curves enable us to determine the absolute output power for any given $J_{r'}$ reconstructing reference intensity.

As can be seen in fig. 3 the nonlinear distortion, even in the case of the simple slit, is a complicated function of recording parameters, underlineing the necessity of optimizing.

The numerical values of the nonlinear distortion depend obviously on the definition of Δ. To illustrate the real effect of the nonlinear distortion the output intensity distributions along the x-axis are shown in fig. 4. The vertical scales of the distributions are different, thus only the shapes should be compared. From figs. 2 and 3 it is evident, that the maximum output power and minimal nonlinear distortion conditions cannot be satisfied simultaneously. The optimal exposure parameters can be determined according to P_{out}/Δ -curves of fig. 5.

As the calculations show, relatively small /10-20%/ deviations from the optimal exposure parameters cause serious losses in diffraction efficiency and significant increase in distortion.

IV. APPLICATION

The method described was used for making the matched filter of a holographic correlator [9]. The filter was the Fourier hologram of a circular hole. Fig. 6 shows the calculated and the measured intensity distribution of the autocorrelation signal of the hole along the line $(x_D, 0)$ of the detector plane. The good agreement shows the correctness of the optimization of the exposure parameters corresponding to the maximum value of the P_{out}/Δ ratio. Some of the data in the conventional correlation measurement: the diameter of the hole was 1.5 mm, the focal length of Fourier lenses = 300 mm. A scanning photomultiplier was used as a detector. A 5 μ-pinhole was placed in front of the multiplier to achieve necessary resolution in the detector plane.

CONCLUSIONS

The described optimization method is suitable for choosing the optimal holographic parameters for signals having a large dynamic range by simple numerical calculations. We can also obtain the absolute intensity distribution of the reconstructed beam and thus determine the necessary laser power and suitable detector.

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FIGURE CAPTIONS

Fig. 1 Lin-curves of AGFA GAEVERT 10E75 plates for amplitude holograms.

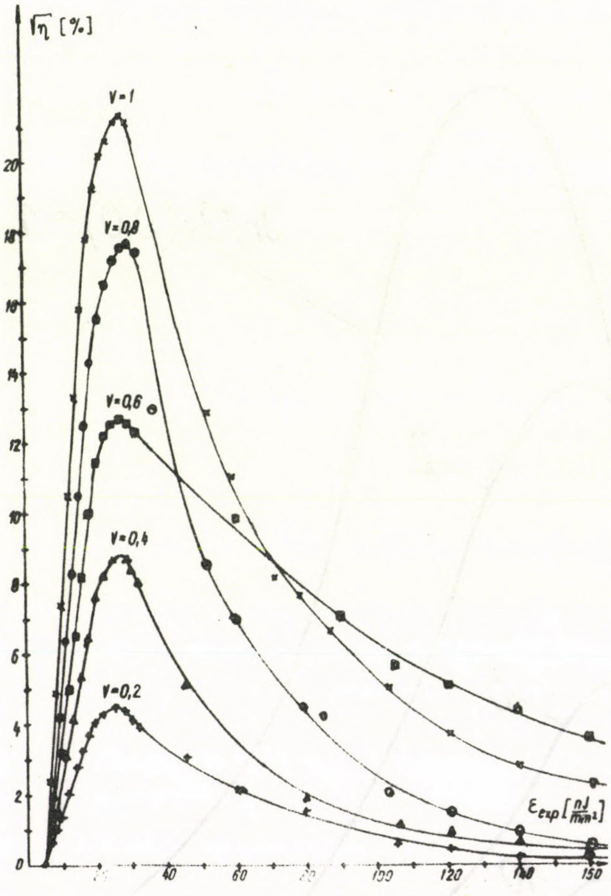
Fig. 2 The output power of the reconstructed image v.s. $E_0(x_H=0, y_H=0)$ with reference to object beam intensity ratio as a parameter.

Fig. 3 The nonlinear distortion as a function of exposure parameters.

Fig. 4 Calculated shapes of the reconstructed images. /The vertical scales are different./

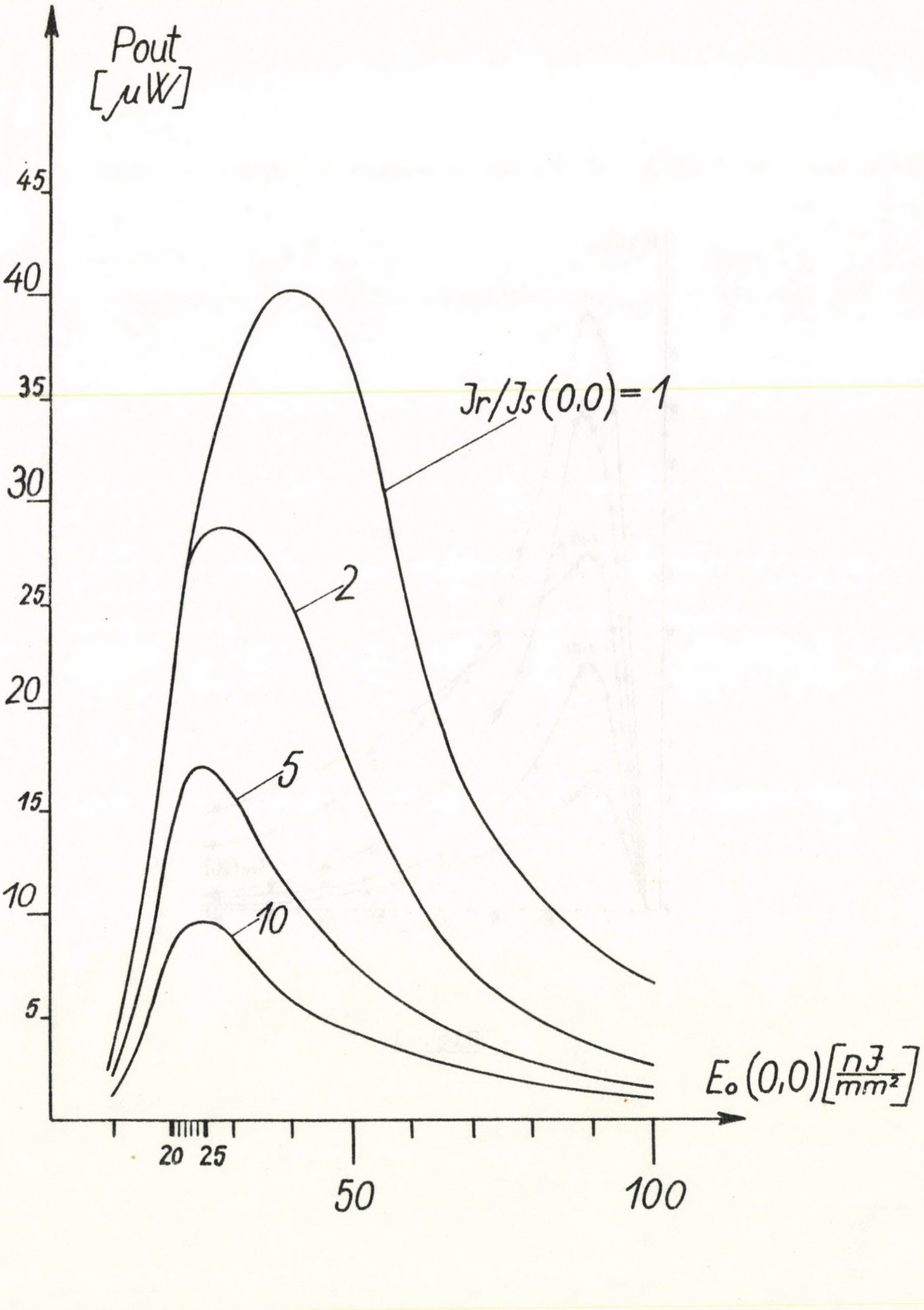
Fig. 5 P_{out}/Δ as a function of exposure parameters.

Fig. 6 Calculated and measured intensity distribution of the autocorrelation signal.



$\sqrt{\eta}$ [%]
V=1
V=0,8
V=0,6
V=0,4
V=0,2
ε_{exp} [$\frac{nJ}{mm^2}$]
0
2
4
6
8
10
12
14
16
18
20
40
60
100
120
150

Fig. 1



Pout
[μW]
45
40
35
30
25
20
15
10
5
Jr/Js(0,0) = 1
2
5
10
$E_o(0,0)\left[\frac{nJ}{mm^2}\right]$
20 25
50
100

Fig. 2

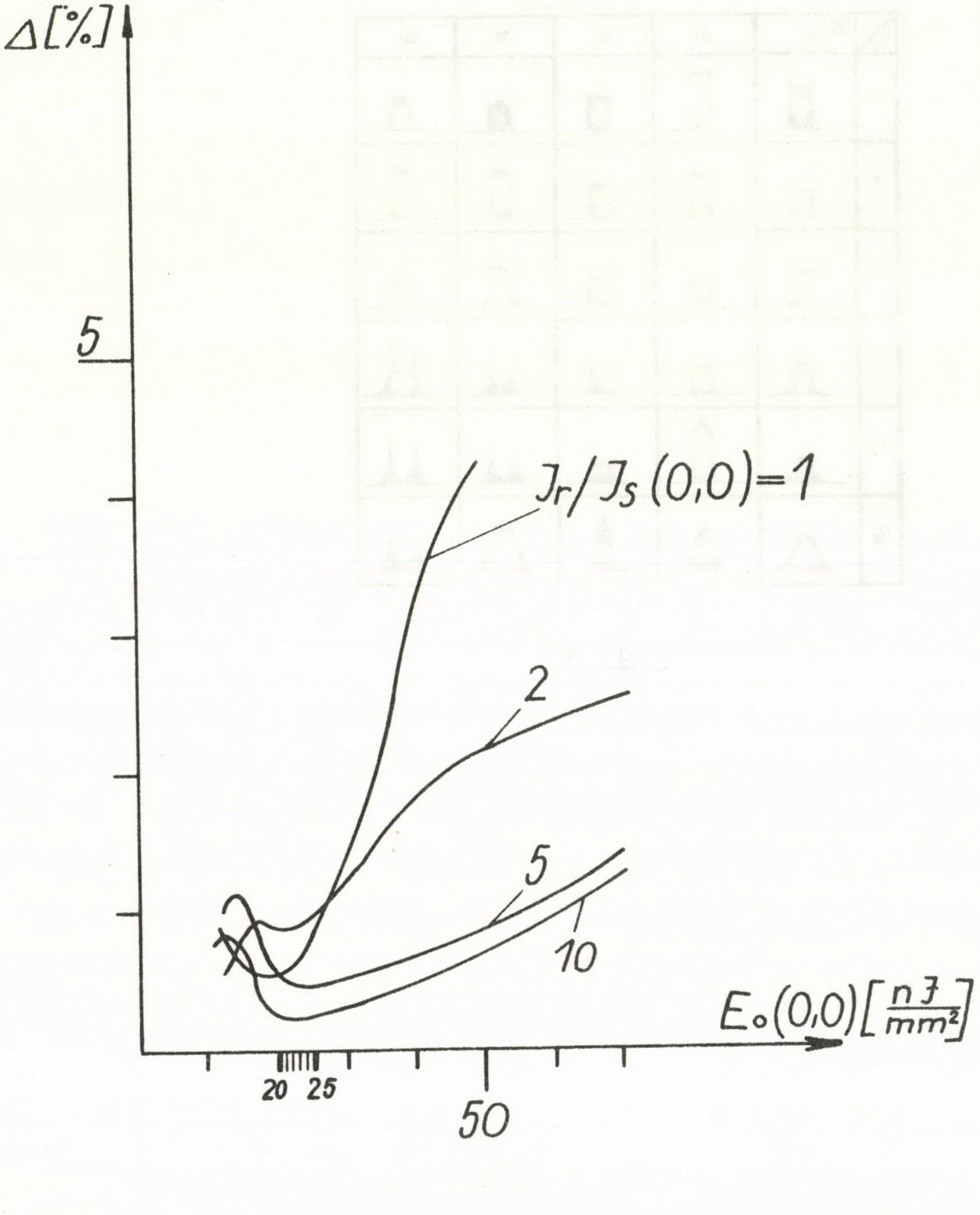


Fig. 3

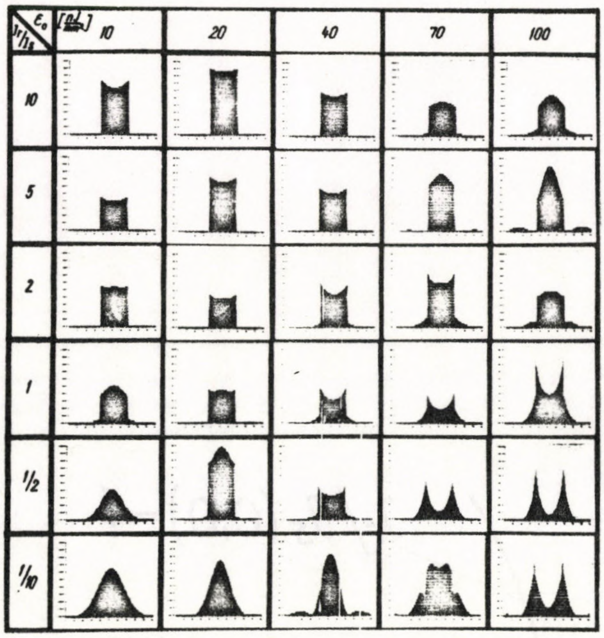


Fig. 4

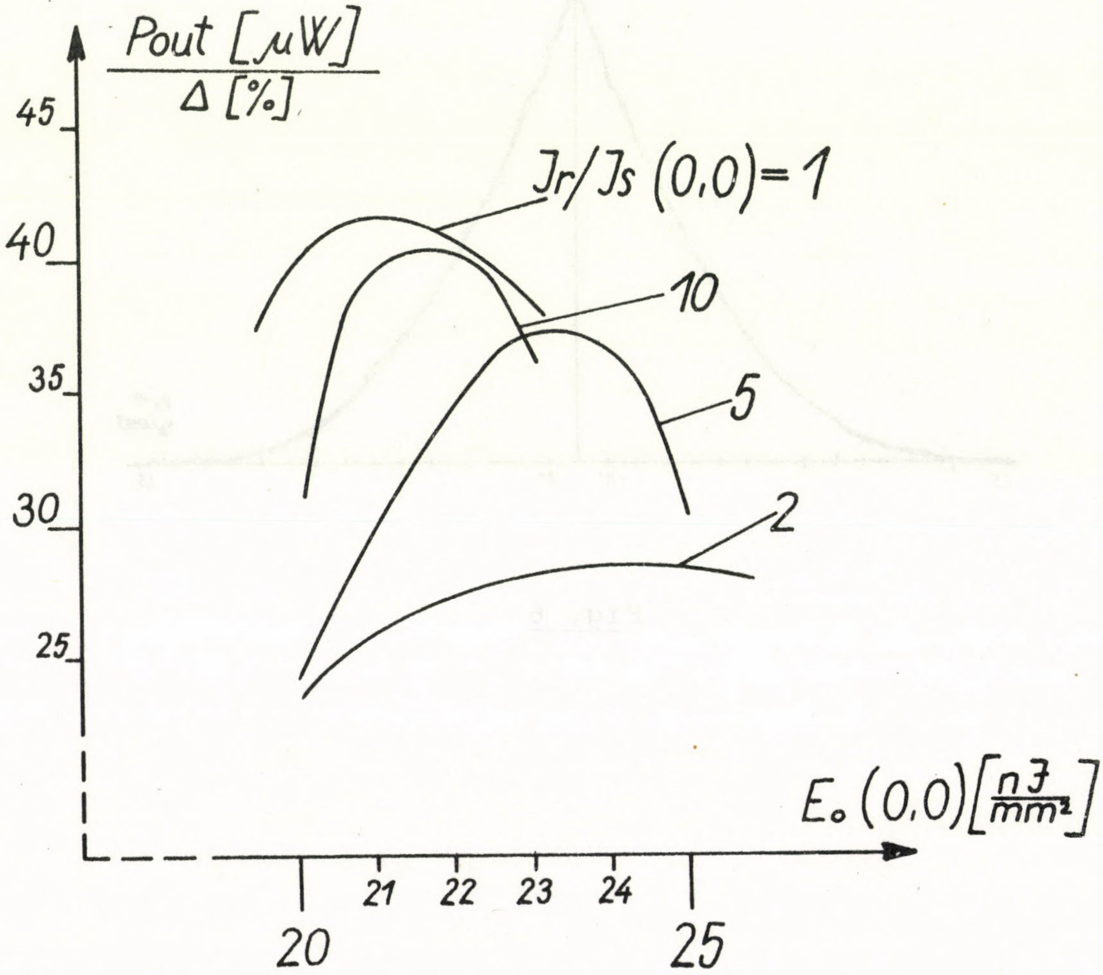


Fig. 5

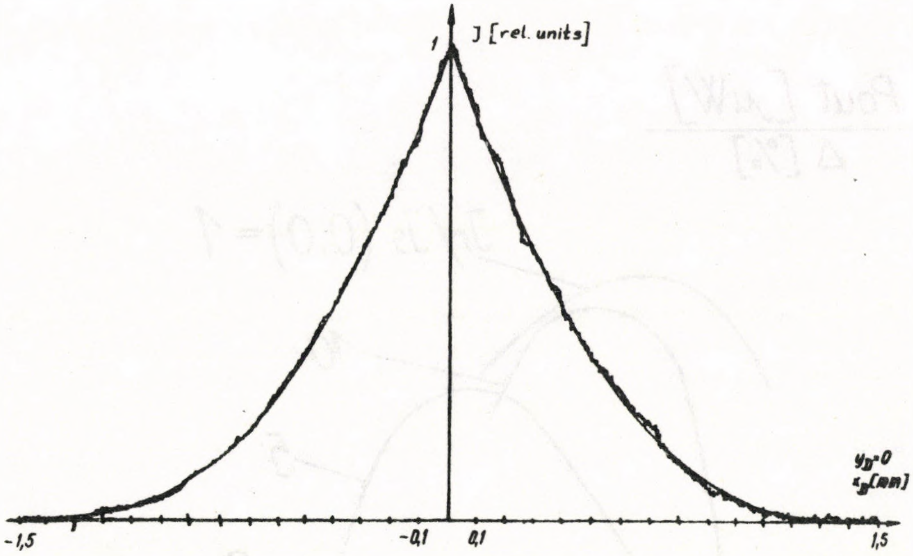


Fig. 6

62.146

Kiadja a Központi Fizikai Kutató Intézet
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