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HADRONIC CURRENTS IN THE INFINITE MOMENTUM FRAME

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ABSTRACT

The problem of the transformation properties of hadronic currents in the infinite momentum frame is investigated. A general method is given to deal with the problem, which is based upon the concept of group contraction. The two-dimensional aspects of the IMF description are studied in detail, and the current matrix elements in a three-dimensional Poincaré covariant theory are reduced to those of a two-dimensional one. It is explicitly shown that the covariance group of the two-dimensional theory may either be a "non-relativistic" /Galilei/ group, or a "relativistic" /Poincaré/ one depending on the value of a parameter reminiscent of the light velocity in the three-dimensional theory. The value of this parameter cannot be determined by kinematical arguments. Our results offer a natural generalization of models which assume the Galilean symmetry in the infinite momentum frame.

АННОТАЦИЯ

Изучена проблема трансформационных свойств адронных токов в системе координат бесконечного импульса. Предложен общий метод, основанный на концепции стягивания группы, для двухмерного варианта описания в этой системе. Показано, что группа ковариантности двухмерной теории может быть как Галилеевская, так и Пуанкаре в зависимости от значения некоторого параметра. Величину этого параметра невозможно определить на основе кинематических аргументов. Наши результаты дают возможность естественно обобщить модели основанные на галилеевской симметрии в системе координат бесконечного импульса.

KIVONAT

A hadronáramok végtelen impulzusu koordinátarendszerbeli transzformációs tulajdonságainak problémáját vizsgáljuk. A kérdés tárgyalására általános módszert adunk, amely a csoportkontrakció koncepciójára épül. A végtelen impulzusu leirás kétdimenziós aspektusait tanulmányozzuk, és a háromdimenziós, Poincaré-kovariáns elméletekben fellépő áram mátrix-elemeket kétdimenziós elmélet mátrixelemeire redukáljuk. Részletesen megmutatjuk, hogy a kétdimenziós elmélet kovariancia-csoportja lehet akár nem relativisztikus /Galilei-/, akár relativisztikus /Poincaré-/ csoport, attól függően, hogy mennyi egy - a háromdimenziós elméletbeli fénysebességgel rokon jelentésü - paraméter értéke. Ez a paraméterérték nem határozható meg kinematikai vizsgálatokkal. Eredményeink természetes általánositását teszik lehetővé azoknak a modelleknek, amelyek a végtelen impulzusu koordinátarendszerben a Galilei-szimmetriát tételezték fel.

1. INTRODUCTION

In this paper we are going to describe a detailed programme for the derivation of the transformation properties of local interaction currents in the infinite momentum frame /IMF/. Taking the infinite momentum limit /IML/ of the current matrix elements has been a standard method for the calculation of various dynamical quantities, but little attention has been paid to the question of what happens to the group-theoretical structure of the Poincaré covariant theory in this limit. Even less effort has been made to explicitly using the group-theoretical properties in the IMF, although this would certainly increase the power of the infinite momentum methods. Nevertheless, it is clear from the early papers on this subject, that in the IML the Poincaré symmetry group contracts into an "iso-Poincaré group" [1] /a Poincaré group isomorphic with the symmetry group in the ordinary reference frame /ORF//, or into some of its subgroups [2,3]. Since the mathematical procedure of group contraction is ambiguous [4], the actual symmetry group in the IMF must be chosen on physical grounds.

The only contraction scheme, which so far seems to have been applied is the one, which results in a Galilei subgroup of the Poincaré group. This seems to fit with the IML of the old-fashioned perturbation series [5,6]. It seems to be justified also by the observed scaling behaviour in deep-inelastic scattering [7]. On the other hand, we argued in a recent paper [3], that the old-fashioned perturbation series in the IML is not necessarily interpreted as the perturbation series for a non-relativistic theory in three dimensions /in two space + one time dimensions/. Alternatively, it can also be interpreted as the non-covariant perturbation series for a "relativistic theory" in three dimensions. This observation inspired us to a systematic investigation of the transformation properties of local interaction currents in the IMF. By exploring the detailed structure of such physical quantities like current matrix elements, we hope to find new approximations, and new models of physical processes, when the various contraction schemes are used in the IML.

In what follows we give a detailed derivation of the transformation properties of "transverse" currents [7] in the IMF. Our method will be based upon the contraction of the Poincaré group representations defined on the matrix elements of the currents. The crucial steps of the method are, firstly, the contraction of the Poincaré group, and secondly, the "contraction" of the representation space by means of appropriately defined integrals of the matrix elements. Only after performing both parts of the programme we can deduce the transformation properties of the "current" in the IMF. In Sect. II. we summarize some notions and concepts needed in the subsequent parts of the paper, and formulate our programme for the deduction of the transformation properties we are looking for. In Sect. III. we describe the contraction schemes of the Poincaré group which we shall be interested in. In Sect. IV. we construct the representation spaces for the groups obtained in the course of the contraction. In Sect. V. the new current operators are deduced, and their properties are discussed in detail in Sect. VI. A short summary of the paper is given and remarks concerning possible applications are made in Sect.VII.

2. FORMULATION OF THE IMF PROBLEM

In general, we shall be engaged in the properties of a scalar current $s(x) \equiv s(x^o, \underline{x}_\perp, x^3)$, but the methods to be presented can easily be adapted to the case of more complicated quantities, such as vector and tensor currents. We shall assume that we are given all states, ϕ, of a physical system, the physical observables being the matrix elements of $s(x)$ between such states:

$$f(x) \equiv f(x^o, \underline{x}_\perp, x^3) \equiv (\phi_\beta, s(x)\phi_\alpha). \qquad /II.1/$$

On the set of states ϕ a unitary, irreducible representation of the Poincaré group is given:

$$U(a,\Lambda)\phi = \phi' , \qquad /II.2/$$

$$U(a_1,\Lambda_1)U(a_2,\Lambda_2)\phi = U(a_2,\Lambda_2)(U(a_1\Lambda_1)\phi) = U(a_1\Lambda_2 + a_2,\Lambda_1\Lambda_2)\phi.$$

The relation between matrix elements, given in two different Lorentz reference frames, comes from the principle of relativistic covariance, which says:

$$(U(a,\Lambda)\phi_\beta, s(x)U(a,\Lambda)\phi_\alpha) = (\phi_\beta, s(x\Lambda + a)\phi_\alpha). \qquad /II.3/$$

The relations /II.1-3/ can be converted into a representation of the Poincaré group on all functions $f(x)$, which are all matrix elements of $s(x)$ between the states ϕ:

$$T(a,\Lambda)f(x) = f(x\Lambda + a), \qquad /II.4/$$

or, in the infinitesimal form:

$$M^{\mu\nu}f(x) = -i\left[x^{\mu}\frac{\partial}{\partial x_{\nu}} - x^{\nu}\frac{\partial}{\partial x_{\mu}}\right] f(x), \qquad /II.5/$$

$$P^{\mu}f(x) = -i\frac{\partial}{\partial x_{\mu}} f(x). \qquad /II.6/$$

This representation corresponds to a scalar current and the relations /II.4-6/ are to be compared with the usual operator relations:

$$U^{-1}(a,\Lambda)s(x)U(a,\Lambda) = s(x\Lambda + a), \qquad /II.4'/$$

$$\left[M^{\mu\nu}, s(x)\right] = -i\left[x^{\mu}\frac{\partial}{\partial x_{\nu}} - x^{\nu}\frac{\partial}{\partial x_{\mu}}\right]s(x) \qquad /II.5'/$$

$$\left[P^{\mu}, s(x)\right] = -i\frac{\partial}{\partial x_{\mu}} s(x). \qquad /II.6'/$$

The detailed properties, like unitarity, etc., of the representation /II.4-6/ depend very much on the operator $s(x)$. These properties will not be important in this paper. But we shall need the following properties of the functions $f(x)$:

i. They are infinitely differentiable with respect to any of the variables;

ii. Series like

$$T(e^{-i\alpha G_k})f(x) \equiv \sum_{n=o}^{\infty} \frac{1}{n}(-i\alpha)^n \quad G_k^n f(x) \qquad /II.7/$$

converge, G_k being any of the infinitesimal generators. These properties assure that infinitesimal and finite group elements can be used on an equal footing.

We shall split the representation space to "transitivity sets" proceeding in the following manner. We consider two arbitrary, but fixed physical states ϕ_{α}, ϕ_{β}, and define the set of functions $f_{\alpha\beta}(x;(a,\Lambda))$ of x by

$$f_{\alpha\beta}\ (x;(a,\Lambda)) \equiv (\phi_{\beta}, s(x\Lambda + a)\phi_{\alpha}), \qquad /II.8/$$

where all functions of the set are listed by means of all elements (a,Λ) of the Poincaré group. Due to the properties i. and ii. described above, all functions of such a set can be represented by Taylor series, the infinitesimal relations /II.5,6/ being specifically applied to the function

$$f_{\alpha\beta}(x) = (\phi_\beta,\ s(x)\phi_\alpha). \qquad /II.9/$$

It is the consequence of relativistic covariance that the same set /II.8/ arises from the functions

$$f'_{\alpha\beta}(x;(a,\Lambda)) \equiv (\phi'_\beta\ s(x\Lambda + a)\phi'_\alpha), \qquad /II.10/$$

where $\phi'_\alpha = U(a',\Lambda')\phi_\alpha$, $\phi'_\beta = U(a',\Lambda')\phi_\beta$, and (a',Λ') is fixed.
The relation between the functions /II.8/ and /II.10/ is as follows:

$$f'_{\alpha\beta}(x;(a,\Lambda)) = \qquad /II.11/$$
$$= T(a',\Lambda')f_{\alpha\beta}(x;(a',\Lambda')^{-1}(a,\Lambda)(a',\Lambda')),$$

for every (a,Λ). The corresponding representations of the Poincaré group can also be simply related:

$$T'(a_1,\Lambda_1)f'_{\alpha\beta}(x;(a,\Lambda)) = \qquad /II.12/$$
$$= T(a',\Lambda')T((a',\Lambda')^{-1}(a_1,\Lambda_1)(a',\Lambda'))f_{\alpha\beta}(x;(a',\Lambda')^{-1}(a,\Lambda)(a',\Lambda')),$$

for every (a,Λ) and (a_1,Λ_1). Especially, if we choose a Z-boost, $\exp(-i\xi N_3)$, for (a',Λ'), the primed functions $f'_{\alpha\beta}$ and operators T' give the description of the physical system, in comparison with the unprimed ones, $f_{\alpha\beta}$ and T, respectively, in a moving reference frame. In the limit $\xi\to\infty$ we obtain the description in the IMF.

In order to be able to specify the symmetry properties of the theory in the IMF one must discuss the following problems:

<u>Problem I.:</u> We are to describe the group arising from the limit

$$\lim_{\xi\to\infty}\left[e^{i\xi N_3}\ (a,\Lambda)e^{-i\xi N_3}\right] \equiv (a,\Lambda)_\infty\ ; \qquad /II.13/$$

<u>Problem II.:</u> We must calculate all the functions

$$\lim_{\xi\to\infty} T(e^{-i\xi N_3})\ T(e^{i\xi N_3}(a,\Lambda)e^{-i\xi N_3})f_{\alpha\beta}(x) \equiv \qquad /II.14/$$
$$\equiv f^\infty_{\alpha\beta}(x;(a,\Lambda)_\infty);$$

<u>Problem III.:</u> Finally, we must interpret the functions $f^{\infty}_{\alpha\beta}(x;(a,\Lambda)_{\infty})$ as the matrix elements of an IMF current $s^{\infty}(x)$ between states ϕ^{∞}, all having well-defined transformation properties with respect to the group $(a,\Lambda)_{\infty}$.

Problem I. is, actually, a contraction problem for the Poincaré group. It has several solutions, the limit gives either the Poincaré group itself, or one of its subgroups [4]. /Strictly speaking, these groups are isomorphic to the original Poincaré group or its subgroups./ Contraction into the Poincaré group has been described in ref. 1., and into some of its subgroups in ref. 3. No a priori reason can be given for choosing one or another solution of the contraction problem. Only some physical hints may inspire one to make a definite choice. In this paper we look for such solutions of Problem 1. that /II.13/ leads to contraction of the Poincaré group into certain subgroups. This choice is motivated by refs. 2. and 7. The subgroups will be Galilei and Poincaré subgroups which transform two "space" coordinates and a non-relativistic or relativistic "time"coordinate, respectively. /In what follows we use the terminology of ref. 3. and call 3-Poincaré group the one described by the formulas /II.4-6/, its contractions will be called 2-Galilei and 2--Poincaré groups, respectively./ After coming to this decision on Problem I. it is clear, that the four-dimensional homogeneous space x of the 3-Poincaré tranformations is to be reduced to some three-dimensional one. One may hope to achieve this by integrating the function $f_{\alpha\beta}(x;(a,\Lambda))$ over one of their variables and reformulating Problems II. and III. in terms of these integrated functions. For a convenient choice of the integration variable we change x^{o} and x^{3} by τ and ζ:

$$\zeta = \frac{1}{2}(x^3-x^o), \qquad \tau = (x^o+x^3), \qquad /II.15/$$

and, for the functions $f'_{\alpha\beta}(x^o, \underline{x}_{\perp}, x^3;(a,\Lambda))$ we use the notation $g'_{\alpha\beta}(\tau,\underline{x}_{\perp},\zeta;(a,\Lambda)) \equiv g'_{\alpha\beta}(x;(a,\Lambda))$. We shall be interested in the functions

$$g'_{\alpha\beta}(\tau,\underline{x}_{\perp};(a,\Lambda)) \equiv \int_{-\infty}^{\infty} g'_{\alpha\beta}(x,(a,\Lambda))d\zeta \qquad /II.16/$$

in the IMF, that is, for $\xi\to\infty$. We must evaluate the functions

$$g^{\infty}_{\alpha\beta}(\tau,\underline{x}_{\perp};(a,\Lambda)_{\infty}) \equiv \lim_{\xi\to\infty}\int_{-\infty}^{\infty} T(e^{-i\xi N_3})T(e^{i\xi N_3}(a,\Lambda)e^{-i\xi N_3})g_{\alpha\beta}(x)d\zeta \qquad /II.17/$$

and deduce the transformation rules in the IMF for the "transverse" current $\int s(x)d\zeta$.

Before concluding this section we make an important remark concerning the solutions of Problem II. In general, the solution of Problem II. yields different representation spaces and, therefore, different representations of the group $(a,\Lambda)_\infty$, if $f_{\alpha\beta}(x)$ is changed to some $f_{\tilde{\alpha}\tilde{\beta}}(x)$, $f_{\tilde{\alpha}\tilde{\beta}}(x)$ being the matrix element of $s(x)$ between the states $U(\tilde{a},\tilde{\Lambda})\phi_\alpha$, and $U(\tilde{a},\tilde{\Lambda})\phi_\beta$, with $(\tilde{a},\tilde{\Lambda})$ fixed. Problem II. for the function $f_{\tilde{\alpha}\tilde{\beta}}(x)$ would mean the calculation of

$$\lim_{\xi\to\infty} T(e^{-i\xi N_3})T(e^{i\xi N_3}(a,\Lambda)e^{-i\xi N_3})T(\tilde{a},\tilde{\Lambda})f_{\alpha\beta}(x). \qquad /II.18/$$

Since $(\tilde{a},\tilde{\Lambda})$ is a fixed element of the 3-Poincaré group, in the limit $\xi\to\infty$ it becomes, in general, a foreign object from the point of view of the contracted group $(a,\Lambda)_\infty$.

Finally, for the reader's convenience, we write down here the action of the 3-Poincaré generators on the functions $g_{\alpha\beta}(x)$:

$$(M_1-N_2)g(x) = i(\tau\frac{\partial}{\partial x^2} - x^2\frac{\partial}{\partial\zeta})g(x),$$

$$(M_1+N_2)g(x) = 2i(\zeta\frac{\partial}{\partial x^2} - x^1\frac{\partial}{\partial\tau})g(x),$$

$$(M_2-N_1)g(x) = 2i(x^1\frac{\partial}{\partial\tau} - \zeta\frac{\partial}{\partial x^1})g(x),$$

$$(M_2+N_1)g(x) = i(x^1\frac{\delta}{\delta\zeta} - \tau\frac{\delta}{\delta x^1})g(x),$$

$$M_3\, g(x) = i(x^2\frac{\partial}{\partial x^1} - x^1\frac{\delta}{\partial x^2})g(x), \qquad /II.19/$$

$$N_3\, g(x) = i(\zeta\frac{\partial}{\partial\zeta} - \tau\frac{\partial}{\partial\tau})g(x),$$

$$(P_o+P_3)g(x) = -2i\frac{\partial}{\partial\tau}g(x), \qquad (P_o-P_3)g(x) = i\frac{\partial}{\partial\zeta}g(x),$$

$$\underline{P}_\perp g(x) = -i\frac{\partial}{\partial\underline{x}_\perp}g(x).$$

3. CONTRACTIONS OF THE 3-POINCARÉ GROUP

In order to make this paper self-contained as much as possible we give a short summary of those contractions of the 3-Poincaré group which we are interested in. /For more details see also ref.3./

The two-dimensional Galilean description in the IMF stems from the following connection between the generators of spacetime transformations in the limiting and ordinary reference frames, O and O', respectively |2,3|:

$$S_1 = \lambda \lim_{\xi\to\infty} \{e^{-\xi} U(\xi)(M_2' + N_1')\, U^{-1}(\xi)\}, \qquad /III.1.a/$$

$$S_2 = -\lambda \lim \{e^{-\xi} U(\xi)(M_1' - N_2')\, U^{-1}(\xi)\}, \qquad /III.1.b/$$

$$M_3 = \lim \{U(\xi)\, M_3' U^{-1}(\xi)\}, \qquad /III.1.c/$$

$$H^G = \frac{1}{2\lambda} \lim \{e^{\xi} U(\xi)(P_o' + P_3')\, U^{-1}(\xi)\}, \qquad /III.1.d/$$

$$\mu_o = \lambda \lim \{e^{-\xi} U(\xi)(P_o' - P_3')\, U^{-1}(\xi)\}, \qquad /III.1.e/$$

$$\lim \{e^{-\xi} U(\xi)\, N_3' U^{-1}(\xi)\} = \lim \{U(\xi)(M_1' + N_2')\, U^{-1}(\xi)\} = 0, \qquad /III.1.f/$$

$$\lim \{U(\xi)(M_2' - N_1')\, U^{-1}(\xi)\} = 0. \qquad /III.1.g/$$

The symbol $U(\xi)$ denotes a z-boost, $U(\xi) = e^{i\xi N_3'}$, λ is an arbitrary positive number. These relations give a mapping of the 3-Poincaré algebra

$$\begin{aligned} &[M_i', M_j'] = -[N_j', N_j'] = i\varepsilon_{ijk} M_k', \\ &[M_i', N_j'] = i\varepsilon_{ijk} N_k', \\ &[M_{\mu\nu}', P_\rho'] = i(g_{\nu\rho} P_\mu' - g_{\mu\rho} P_\nu') \end{aligned} \qquad /III.2/$$

onto the 2-Galilei algebra, its elements being S_i, P_i, $(i = 1,2)$, M_3, H^G and μ_o:

$$[S_i\ ,\ S_j] = 0, \qquad [M_3\ ,\ S_i] = i\varepsilon_{ij}S_j,$$

$$[P_i\ ,\ P_j] = 0, \qquad [H^G\ ,\ P_i] = 0,$$

$$[S_i\ ,\ H^G] = iP_i, \qquad [S_i\ ,\ P_j] = i\mu_o\delta_{ij}, \qquad /III.3/$$

$$[M_3\ ,\ H^G] = 0, \qquad [M_3\ ,\ P_i] = i\varepsilon_{ij}P_j,$$

$$[S_i\ ,\ \mu_o] = [M_3\ ,\ \mu_o] = [P_i\ ,\ \mu_o] = [H^G\ ,\ \mu_o] = 0.$$

The mapping /III.1/ can also be expressed as follows:

$$\begin{vmatrix} H^G \\ P_1 \\ P_2 \\ \mu_o \end{vmatrix} = \begin{vmatrix} \frac{1}{2\lambda} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \lambda \end{vmatrix} \begin{vmatrix} P'_o + P'_3 \\ P'_1 \\ P'_2 \\ P'_o - P'_3 \end{vmatrix}, \qquad /III.4.a/$$

$$\begin{vmatrix} S_1 \\ S_2 \\ M_3 \\ 0 \\ 0 \\ 0 \end{vmatrix} = \lim_{\xi\to\infty} \begin{vmatrix} \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & -\lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{-\xi} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{-\xi} & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{-\xi} \end{vmatrix} \begin{vmatrix} M'_2 + N'_1 \\ M'_1 - N'_2 \\ M'_3 \\ N'_3 \\ M'_2 - N'_1 \\ M'_1 + N'_2 \end{vmatrix} \qquad /III.4.b/$$

This is obviously a contraction of the 3-Poinçaré algebra [4]. Now the question arises if other contractions of the 3-Poincaré algebra may also be of interest. As it was shown in ref. 3. it is natural to consider, for example, the following contraction:

$$\begin{pmatrix} K_1 \\ K_2 \\ M_3 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \lim_{\xi\to\infty} \begin{pmatrix} \lambda & 0 & 0 & 0 & -\frac{1}{4\lambda c^2} & 0 \\ 0 & -\lambda & 0 & 0 & 0 & \frac{1}{4\lambda c^2} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{-\xi} & 0 & 0 \\ 0 & 0 & 0 & 0 & -e^{-\xi} & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{-\xi} \end{pmatrix} \begin{pmatrix} M_2' + N_1' \\ M_1' - N_2' \\ M_3' \\ N_3' \\ M_2' - N_1' \\ M_1' + N_2' \end{pmatrix} , \qquad /III.5.a/$$

$$\begin{pmatrix} H^P \\ P_1 \\ P_2 \\ \mu \end{pmatrix} = \begin{pmatrix} \frac{1}{4\lambda} & 0 & 0 & \lambda c^2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{1}{4\lambda c^2} & 0 & 0 & \lambda \end{pmatrix} \begin{pmatrix} P_o' + P_3' \\ P_1' \\ P_2' \\ P_o' - P_3' \end{pmatrix} , \qquad /III.5.b/$$

The "infinite momentum limit" character of this mapping becomes more obvious if one rewrites /III.5/ in terms of z-boosts:

$$K_1 = \lim_{\xi\to\infty} \{U(\xi)[\lambda e^{-\xi}(M_2'+N_1') - \frac{1}{4\lambda c^2} e^{\xi}(M_2'-N_1')] U^{-1}(\xi)\} ; \qquad /III.6.a/$$

$$K_2 = \lim \{U(\xi)[- e^{-\xi}(M_1'-N_2') + \frac{1}{4\lambda c^2} e^{\xi}(M_1'+N_2')] U^{-1}(\xi)\} ; \qquad /III.6.b/$$

$$H^P = \lim \{U(\xi) [\frac{1}{4\lambda} e^{\xi} (P_o'+P_3') + \lambda c^2 e^{-\xi}(P_o'-P_3')] U^{-1}(\xi)\} ; \qquad /III.6.c/$$

$$\mu = \lim \{U(\xi) [- \frac{1}{4\lambda c^2} e^{\xi}(P_o'+P_3') + \lambda e^{-\xi}(P_o'-P_3')] U^{-1}(\xi)\} ; \qquad /III.6.d/$$

In /III. 5,6/ the letters λ, and c denote arbitrary positive numbers. This contraction is closely related with the previous one, as it can be seen from the following commutators:

$$[K_1 , K_2] = -i\frac{1}{c^2} M_3, \qquad [M_3 , K_i] = i\varepsilon_{ij}K_j,$$

$$[H^P , P_i] = 0, \qquad [P_i , P_j] = 0,$$

$$[M_3 , H^P] = 0, \qquad [M_3 , P_i] = i\varepsilon_{ij}P_j , \qquad /III.7/$$

$$[K_i , H^P] = iP_i, \qquad [K_i , P_j] = i\frac{1}{c^2} \delta_{ij}H^P,$$

$$[\mu , H^P] = [\mu , P_i] = [\mu, M_3] = [\mu , K_i] = 0.$$

This is a 2-Poincaré algebra, its elements being the generators of inhomogeneous Lorentz transformations of two spacelike and one timelike coordinates. The parameter c plays the same mathematical role as light velocity does in the usual 3+1 dimensional case. When c goes to infinity the algebra /III.7/ contracts into the non-relativistic one /III.3./.

In the definitions /III. 4,5/ we have an arbitrary positive number λ which, in contrast with the parameter c, does not appear at all in the commutators /III.3/ and /III.7/. Obviously, for the different values of λ the mappings /III.4/ and /III.5/ yield different /but isomorphic/ 2-Galilei and 2-Pioncaré subalgebras of the 3-Poincaré algebra, respectively. We are going to assume that 3-Poincaré covariant theories become 2-Poincaré covariant ones in the IMF with some given value of the parameter c. Its value is to be determined phenomenologically. For the 2-Galilei covariant case $c=\infty$. It seems natural to postulate, on the other hand, that all contractions corresponding to the various values of λ are physically equivalent, that is, none of the predictions of the theory in the IMF must depend on λ.

For a comparison with Susskind's treatment of the 2-Galilei symmetry in the IMF we mention that he chooses $\lambda=1$, but preserves the N_3 generator [2,3] . Thus the symmetry group in the IMF becomes a 2-Galilei group extended with dilatations. Since the dilatations correspond to changing the value of λ, dilatation invariance of the theory corresponds just to the postulate we formulated above. Our formulation has the advantage that it can be generalized without difficulty to the 2-Poincaré case, while the N_3 generator cannot be added to the 2-Poincaré generators to form a closed algebra.

There are, obviously, further ambiguities in choosing the matrices on the right hand side of /III.4/ and /III.5/. Let us denote by A any of the matrices in /III.4.a,b./ /or /III.5.a,b//. The algebra /III.3/ /or /III.7// remains unchanged if A_1AA_2 is substituted for A, where A_2 is any 3-Poincaré transformation of the generators $M'_{\mu\nu}$, P'_μ, and A_1 is any 2-Galilei /or 2-Poincaré/ transformation of the generators S_i, M_3, H^G, P_i /or, K_i, M_3, H^P, P_i/. We shall require that the IMF theory be independent of the choice of A_1 and A_2. Various choices for A_1 correspond to various frames of reference for the IMF theory, which we are going to assume to be covariant with respect to coordinate transformations by 2-Galilei /or, 2-Poincaré/ matrices. Therefore, in what follows, we take $A_1 = 1$. Similar independence on A_2 should also be required, but this is slightly spoiled by the integration of the matrix elements over ζ. We come back to this point in the next section.

4. THE REPRESENTATION SPACES IN THE IMF

In this section we deal with the solution of Problem II. and construct those functions which form the representation spaces for the contracted groups. This task, in its original form /II.14/ means the calculation of such limits:

$$\lim_{\xi\to\infty} T(e^{-i\xi N_3})g_{\alpha\beta}(\tau,\underline{x}_\perp,\zeta) = \lim_{\xi\to\infty} g_{\alpha\beta}(\tau e^{-\xi},\ \underline{x}_\perp,\zeta e^{\xi}), \qquad /IV.1/$$

and we need much more detailed properties of the current matrix elements than we have used so far. To overcome this problem one may use Susskind's proposal [2] for integrating over the variable ζ and calculating by means of the rule

$$\int_{-\infty}^{\infty} \lim g_{\alpha\beta}\ (\tau e^{-\xi},\ \underline{x}_\perp,\ e^{\xi})d\xi = \lim e^{-\xi}\int g_{\alpha\beta}\ (\tau e^{-\xi},\ \underline{x}_\perp,\zeta)d\zeta, \qquad /IV.2/$$

but then one faces the problem that the factor $e^{-\xi}$ makes zero the functions we are looking for. One may use certain "physical" arguments [2] to eliminate the factor $e^{-\xi}$ from /IV.2/ and may conclude, that in the IMF the correspondence

$$\int_{-\infty}^{\infty} g_{\alpha\beta}(\tau,\ \underline{x}_\perp,\zeta)\ d\zeta \Rightarrow \int_{\infty}^{\infty} g_{\alpha\beta}((0,\ \underline{x}_\perp,\zeta)d\zeta \qquad /IV.3/$$

is valid. One must notice, however, that /IV.3/ is part of the mapping

$$\int_{-\infty}^{\infty} g_{\alpha\beta}(x;(a,\Lambda))d\zeta \Rightarrow \int_{-\infty}^{\infty} g_{\alpha\beta}(x;(a,\Lambda)_\infty)d\zeta \qquad /IV.3'/$$

we have to specify when we solve Problem II. It is this mapping which really determines the symmetry properties of the theory in the IMF.

As a first step towards specifying the mapping /IV.3'/ we deal with integrals of the following type:

$$\int_{-\infty}^{\infty} G g_{\alpha\beta}(x) d\zeta , \qquad /IV.4/$$

where G is an arbitrary polynomial of the 3-Poincaré generators /II.19/. In practice, /IV.4/ means such expressions:

$$\int D(\tau, \underline{x}_{\perp}; \frac{\partial}{\partial \tau}, \frac{\partial}{\partial \underline{x}_{\perp}}) \zeta^{k} \frac{\partial^{\ell}}{\partial \zeta^{\ell}} g_{\alpha\beta}(\tau, \underline{x}, \zeta) d\zeta , \qquad /IV.5/$$

where k,l = 0, 1, 2, ..., and D is some polynomial of its arguments. In order to obtain tractable formulas we must assume that the derivatives with respect to τ, and $\underline{x}_{\perp}$ are interchangeable with the integral in /IV.5/. This means, that the integral

$$\int \zeta^{k} \frac{\partial^{\ell}}{\partial \zeta^{\ell}} g_{\alpha\beta}(x) d\zeta \qquad /IV.6/$$

must exist for every k, l = 0, 1, 2, ..., . Then it follows that

$$\left| \zeta^{k} \frac{\partial^{\ell}}{\partial \zeta^{\ell}} g_{\alpha\beta}(x) \right| \rightarrow 0, \quad \text{if} \quad |\zeta| \rightarrow \infty. \qquad /IV.7/$$

In general, this condition is not fulfilled even if the function $g_{\alpha\beta}(x)$ is a matrix element of the current s(x) between normalizable states ϕ_{α}, ϕ_{β}. Since the variable ζ was arbitrarily chosen as an integration variable, the strong asymptotic behaviour /IV.7/ must be required also for the dependences on τ and $\underline{x}_{\perp}$. But this class of functions is mapped onto itself by Fourier transformation, therefore, if

$$\hat{g}_{\alpha\beta}(q) = \int_{\infty}^{\infty} g_{\alpha\beta}(x) e^{ixq} d^4x, \qquad /IV.8/$$

then it follows that

$$(q^2)^n g_{\alpha\beta}(q) \rightarrow 0 \qquad /IV.9/$$

for any n=0, 1, 2, ..., if $|q^2| \rightarrow \infty$. If, especially, $g_{\alpha\beta}(x)$ is a matrix element of s(x) between normalizable superpositions of momentum eigenstates, the condition /IV.9/ means, in general, that the form factor $F((k-k')^2) = \langle k'|s(0)|k\rangle$ decreases faster than any inverse power of $(k-k')^2$, if $|(k-k')^2| \rightarrow \infty$. If we do not want such an unduly restricted theory, we must accept that, in general, the integrals /IV.5/ diverge, and we must decide on the meaning we are going to attribute to them. This is, in fact, a reformulation of the problem of the mapping /IV.3/.

With special attention to the purpose of describing functions in the IMF we define /IV.5/ as follows:

$$\int_{-\infty}^{\infty} D\,(\tau,\ \underline{x}_{\perp};\frac{\partial}{\partial\tau},\ \frac{\partial}{\partial\underline{x}_{\perp}})\ \zeta^{k}\ \frac{\partial^{\ell}}{\partial\zeta^{\ell}}\ g_{\alpha\beta}\ (x)\ d\zeta\ = \qquad /IV.10/$$

$$= D(\tau,\ \underline{x}_{\perp};\ \frac{\partial}{\partial\tau},\ \frac{\partial}{\partial\underline{x}_{\perp}})\ g_{\alpha\beta}\ (\tau,\ \underline{x}_{\perp})$$

for k=0, l=0, and zero otherwise. The quantity $g_{\alpha\beta}\ (\tau,\ \underline{x}_{\perp})$ is the canonical distribution theoretical value of the integral of $g_{\alpha\beta}\ (x)$:

$$\int_{-\infty}^{\infty} g_{\alpha\beta}\ (\tau,\ \underline{x}_{\perp},\zeta)\ d\zeta\ \equiv\ g_{\alpha\beta}\ (\tau,\ \underline{x}_{\perp}).$$

First of all, it follows from this definition, that

$$\lim_{\xi\to\infty}\ T(e^{-i\xi N_3})g_{\alpha\beta}\ (\tau,\ \underline{x}_{\perp},\ \zeta)\ d\zeta\ =\ g_{\alpha\beta}\ (0,\ \underline{x}_{\perp}). \qquad /10.a/$$

This is the function in the IMF which corresponds to the unit element of the group $(a,\Lambda)_{\infty}$, independently of the actual group contraction scheme we want to choose. /Notice that in /IV.10.a/ we arrived at a function of only two variables./

In order to construct the other functions of a transitivity set one must calculate the action of the generators of the group $(a,\underline{\Lambda})_{\infty}$ on $g_{\alpha\beta}(0,\underline{x}_{\perp})$. In the 2-Galilei case we proceed by using /II.17/, /II.19/, /III.1/, /III.10/ and obtain:

$$S_i g_{\alpha\beta}\ (0,\ \underline{x}_{\perp})\ =\ 0, \qquad i = 1,\ 2,$$

$$M_3 g_{\alpha\beta}\ (0,\ \underline{x}_{\perp})\ =\ i(x^{2}\frac{\partial}{\partial x^{1}}\ -\ x^{1}\ \frac{\partial}{\partial x^{2}})\ g_{\alpha\beta}\ (0,\ \underline{x}_{\perp}), \qquad /IV.11/$$

$$\underline{P}_{\perp} g_{\alpha\beta}\ (0,\ \underline{x}_{\perp})\ =\ -i\frac{\partial}{\partial\underline{x}_{\perp}}\ g_{\alpha\beta}\ (0,\ \underline{x}_{\perp}),$$

$$H^{G} g_{\alpha\beta}\ (0,\ \underline{x}_{\perp})\ =\ -\ \frac{i}{\lambda}\ \cdot\frac{\partial}{\partial\tau}\ g_{\alpha\beta}\ (0,\ \underline{x}_{\perp}).$$

By means of these relations one can easily construct all the functions $g^{\infty}_{\alpha\beta}\ (\tau,\ \underline{x}_{\perp};(a,\ \Lambda)_{\infty})$:

$$g^{\infty}_{\alpha\beta}(\tau, \underline{x}_{\perp}; (a,\Lambda)_{\infty}) \equiv g^{G}_{\alpha\beta}(0, \underline{x}_{\perp};(\ell, L)) \equiv g_{\alpha\beta}(\tfrac{1}{\lambda}\ell_t, \underline{x}_{\perp}R + \underline{\ell}_{\perp}), \qquad /IV.12/$$

where (ℓ, L) denotes a general, six-parameter element of the 2-Galilei group. Its homogeneous part $L \equiv (R, \underline{v}_{\perp})$ involves rotations and Galilei boosts in the two-dimensional plane $\underline{x}_{\perp} = (x^1, x^2)$. Its inhomogeneous part $\ell=(\ell_t, \underline{\ell}_{\perp})$ corresponds to "time" and space translations. Altogether the transformation rule is

$$(t_o, \underline{x}_{\perp})(\ell, L) = (t_o + \ell_t, \underline{x}_{\perp}R + t_o\underline{v}_{\perp} + \underline{\ell}_{\perp}), \qquad /IV.13/$$

The functions /IV.12/ depend on the two variables $\underline{x}_{\perp}$, and look like the functions of the variables $(t_o, \underline{x}_{\perp})$

$$f^{G}_{\alpha\beta}(t_o, \underline{x}_{\perp};(\ell, L)) \equiv e^{it_oH^G} g^{G}_{\alpha\beta}(0, \underline{x}_{\perp};(\ell, L)) \qquad /IV.14/$$

for zero value of the non-relativistic "time" t_o. It follows from the construction of the functions /IV.14/ that a scalar representation of the 2-Galilei group can be defined on all of them by the rule:

$$T^G(\ell', L')f^{G}_{\alpha\beta}(t_o, \underline{x}_{\perp};(\ell, L)) = f^{G}_{\alpha\beta}(t_o,\underline{x}_{\perp}; (\ell', L')(\ell, L)) =$$
$$= f^{G}_{\alpha\beta}(t_o + \ell'_t, \underline{x}_{\perp}R' + t_o\underline{v}'_{\perp} + \underline{\ell}'_{\perp}; (\ell,L)). \qquad /IV.15/$$

This procedure can also be repeated when $(a,\Lambda)_{\infty}$ is the 2-Poincaré group. Equations /IV.11/ remain unchanged in the case of M_3 and $\underline{P}_{\perp}$, and the action of the generators $\underline{K}_{\perp} = (K_1, K_2)$, H^P on $g_{\alpha\beta}(0, \underline{x}_{\perp})$ reads as follows:

$$\underline{K}_{\perp}g_{\alpha\beta}(0, \underline{x}_{\perp}) = -i\,\frac{1}{2\lambda c^2}\,\underline{x}_{\perp}\,\frac{\partial}{\partial\tau}\,g_{\alpha\beta}(0, \underline{x}_{\perp}),$$
$$H^P g_{\alpha\beta}(0, \underline{x}_{\perp}) = -i\,\frac{1}{2\lambda}\,\frac{\partial}{\partial\tau}\,g_{\alpha\beta}(0, \underline{x}_{\perp}). \qquad /IV.16/$$

Instead of eq. /IV.12/ now the functions

$$g^{\infty}_{\alpha\beta}(\tau, \underline{x}_{\perp}; (a, \Lambda)_{\infty}) \equiv g^{P}_{\alpha\beta}(0, \underline{x}_{\perp}; (\underset{\sim}{a}, \underset{\sim}{\Lambda})) \equiv g_{\alpha\beta}(\hat{\underset{\sim}{x}}\underset{\sim}{\Lambda} + \hat{\underset{\sim}{a}}) \qquad /IV.17/$$

appear, where $(\underset{\sim}{a},\underset{\sim}{\Lambda})$ denotes a general element of the 2-Poincaré group, the symbol $\underset{\sim}{\Lambda}$ being also used for the 3x3 matrix of the homogeneous 2-Lorentz transformations, and the three-vector $\underset{\sim}{a}=(a_o,\underline{a}_{\perp})$ refers to the translations of the 1+2 dimensional Minkowski spacetime $\underset{\sim}{x} = (t,\underline{x}_{\perp})$. In /IV.17/ the following notations are also used:

$$\hat{\underset{\sim}{x}} = (0,\ \underline{x}_{\perp}), \qquad \hat{\underset{\sim}{a}} = (\frac{1}{2\lambda}\, a_o,\ \underline{a}_{\perp}).$$

Again, the two-variable functions /IV.17/ can be provided with "time dependence" by means of the definition:

$$f^{P}_{\alpha\beta}\ (t,\ \underline{x}_{\perp};(\underset{\sim}{a},\underset{\sim}{\Lambda})) \equiv e^{itH^{P}}\ g^{P}_{\alpha\beta}\ (0,\ \underline{x}_{\perp};(\underset{\sim}{a},\underset{\sim}{\Lambda})). \qquad /IV.18/$$

On the functions /IV.18/ it is easy to give the action of the 2-Poincaré group:

$$T^{P}(\underset{\sim}{a}',\Lambda')\ f^{P}_{\alpha\beta}\ \ (\underset{\sim}{x};(\underset{\sim}{a},\underset{\sim}{\Lambda})) = f^{P}_{\alpha\beta}(\underset{\sim}{x};(\underset{\sim}{a}',\underset{\sim}{\Lambda}')(\underset{\sim}{a},\underset{\sim}{\Lambda})) = \qquad /IV.19/$$

$$= f^{P}_{\alpha\beta}\ (\underset{\sim}{x}\underset{\sim}{\Lambda}' + \underset{\sim}{a}';\ (\underset{\sim}{a},\underset{\sim}{\Lambda})).$$

Only the last point, Problem III., of our programme remains, namely, to convert the equations /IV.15/ and /IV.19/ into transformation rules for the transverse current $\int s(x)d\zeta$ in the IMF. For this purpose, in the next section we investigate the limit of the matrix elements between momentum eigenstates. The discussion will be performed in the 2-Poincaré case, its connection with the 2-Galilei case will briefly be touched.

We finish this section with a discussion of the freedom in choosing the matrix A_2, mentioned in the previous section. Let us assume, that A_2 corresponds to a 3-Lorentz group element Λ_2. /Obviously, the unit matrix was used in place of A_2 in the present section./ In this case our procedure for solving the contraction problem gets modified by that both in Problems I. and II. the z-boost $\exp(-i\xi N_3)$ must be changed to $\Lambda_2 e^{-i\xi N_3}\Lambda_2^{-1}$. The calculations described in the present section must be repeated using the variables

$$(\tau',\ \underline{x}'_{\perp},\ \zeta') = (\tau,\ \underline{x}_{\perp},\zeta)(\Lambda_2)$$

instead of the ones $(\tau,\ \underline{x}_{\perp},\ \zeta)$. Especially, the variable ζ' must be integrated over in order to be able to define the limit $\xi\to\infty$. Only under this supplementary condition can one postulate that in the IMF physics be independent of A_2.

5. THE MATRIX ELEMENTS BETWEEN MOMENTUM EIGENSTATES

In this section we solve Problem III. and convert eqs. /IV.15/ and /IV.19/ into operator relations similar to eqs. /II.4-6/. For this end we investigate the IML of the matrix elements between momentum eigenstates:

$$\langle p'|s(x)|p\rangle = \frac{1}{(2\Pi)^4} e^{ix(p'-p)} F(m', m, (p-p')^2), \qquad /V.1/$$

where $p^2=m^2$, $p'^2=m'^2$. The right hand side of /V.1/ follows from the transformation properties of the scalar current s(x) and the spinless states $|m,p\rangle$, $|m',p'\rangle$. The current s(x), is Hermitian, $s(x) = s^+(x)$, therefore

$$F^*(m', m, (p-p')^2) = F(m, m', (p-p')^2), \qquad /V.2/$$

where the asterisk denotes complex conjugation.

The matrix elements with all possible values of p and p' play the role of basis functions in the function space of all matrix elements of s(x) between arbitrary physical states. The functions /V.1/ have the important property that they can be grouped into transitivity sets, as defined in Sect. II. We shall exploit this in the following manner. We enumerate all matrix elements $\langle p'|s(x)|p\rangle$, with arbitrary four-momenta p and p', by writing

$$\langle m',p'|s(x)|m,p\rangle = \langle m',\hat{p}'|U^{-1}(\Lambda)s(x)U(\Lambda)|m,\hat{p}\rangle =$$
$$= T(\Lambda)\langle m',\hat{p}'|s(x)|m,\hat{p}\rangle, \qquad /V.3/$$

where $\hat{p}$ and $\hat{p}'$ are some fixed four-momenta,

$$\hat{p} = (\hat{p}_+, \hat{p}_{\underline{\perp}}, \hat{p}_\perp), \qquad \hat{p}' = (\hat{p}'_+, \hat{p}'_{\underline{\perp}}\hat{p}'_\perp), \qquad /V.4/$$

and Λ is the 3-Lorentz transformation:

$$\Lambda = \exp(-i)\{\lambda\alpha_1 e^{-\xi}(M_2 + N_1) - \lambda\alpha_2 e^{-\xi}(M_1 - N_2) + \alpha_3 M_3 -$$
$$- \frac{1}{4\lambda c^2}\alpha_1 e^{\xi}(M_2 - N_1) + \frac{1}{4\lambda c^2}\alpha_2 e^{\xi}(M_1 + N_2)\} . \qquad /V.5/$$

We shall investigate the IMF image of the transitivity sets represented by /V.3/ for arbitrary fixed $\lambda > 0$, $c^2 > 0$, $\hat{p}$ and $\hat{p}'$, when the parameters α_1, α_2, α_3 run over the ranges

$$-\infty < \alpha_1 < \infty, \qquad -\infty < \alpha_2 < \infty, \qquad 0 \leqslant \alpha_3 \leqslant 2\Pi. \qquad /V.6/$$

All matrix elements of s(x) between momentum eigenstates are obtained, if all these sets are taken for every

$$0 < \hat{p}_{\underline{\ }} < \infty, \qquad 0 < \hat{p}'_{\underline{\ }} < \infty, \qquad 0 \leqslant \hat{p}'_\perp < \infty, \qquad /V.7/$$

while $\hat{p}_\perp \geqslant 0$ is fixed.

Applying the contraction prescription /III. 5,6/ to the 3-Lorentz transformations /V. 5,6/ one obtains

$$\lim_{\xi\to\infty} (e^{-i\xi N_3}\, \Lambda\, e^{i\xi N_3}) \equiv \underset{\sim}{\Lambda} = \exp(-i)\{\alpha_1 K_1 + \alpha_2 K_2 + \alpha_3 M_3\}. \qquad /V.8/$$

That is, the elements of the above transitivity sets are enumerated by means of the elements of the 2-Lorentz group in the IMF. The second step towards the description of the transitivity sets in the IMF is the calculation of the following quantity:

$$T(e^{itH^P}) \lim_{\xi\to\infty} \int T(e^{-i\xi N_3}) \langle m', \hat{p}' | s(x) | m, \hat{p}\rangle\, d\zeta =$$

$$= \frac{1}{(2\Pi)^3} \exp i[t(\hat{h}-\hat{h}') + \underline{x}_\perp(\hat{\underline{p}}_\perp - \hat{\underline{p}}'_\perp)]\; F(m', m, (p-p')^2)\,\delta(\hat{p}_- - \hat{p}'_-) \equiv \qquad /V.9/$$

$$\equiv f_{\hat{p}\hat{p}'}(t, \underline{x}_\perp).$$

The right hand side of /V.9/ can be found by applying the prescriptions of the previous section to the function /V.1/, and by introducing the notations

$$\hat{h} = \frac{1}{4\lambda}\hat{p}_+ + \lambda c^2 \hat{p}_-\,, \qquad \hat{h}' = \frac{1}{4\lambda}\hat{p}'_+ + \lambda c^2 \hat{p}'_-\,. \qquad /V.10/$$

If, furthermore, we introduce $\underset{\sim}{x} = (t, \underline{x}_\perp)$, $\underset{\sim}{\hat{k}} = (\hat{h}, \hat{\underline{p}}_\perp)$, $\underset{\sim}{\hat{k}}' = (\hat{h}', \hat{\underline{p}}'_\perp)$, $t(\hat{h}-\hat{h}') + \underline{x}_\perp(\hat{\underline{p}}_\perp - \hat{\underline{p}}'_\perp) = \underset{\sim}{x}(\underset{\sim}{\hat{k}}-\underset{\sim}{\hat{k}}')$, then we may write for the transitivity sets in the IMF:

$$T(\underset{\sim}{\Lambda})\, f_{\hat{p}\hat{p}'}(t, \underline{x}_\perp) = \frac{1}{(2\Pi)^3} e^{i\underset{\sim}{x}\underset{\sim}{\Lambda}(\underset{\sim}{\hat{k}}-\underset{\sim}{\hat{k}}')} F(m', m; (p-p')^2)\,\delta(\hat{p}_- - \hat{p}'_-). \qquad /V.11/$$

In eq. /V.11/ we have the typical exponential $\exp i\, \underset{\sim}{x}\underset{\sim}{\Lambda}(\underset{\sim}{\hat{k}}-\underset{\sim}{\hat{k}}')$ multiplied by a function of the momenta, which is a constant for the whole transitivity set. The next task is to express this function in terms of 2-Lorentz invariants. The 3-Lorentz invariants are automatically 2-Lorentz invariants, thus, in fact, we must only deal with $\delta(\hat{p}_- - \hat{p}'_-)$.

The 2-Lorentz transformations leave invariant the quantities

$$\underset{\sim}{\hat{k}}^2 = \frac{1}{c^2}\hat{h}^2 - \hat{\underline{p}}_\perp^2\,, \qquad \underset{\sim}{\hat{k}}\underset{\sim}{\hat{k}}' = \frac{1}{c^2}\hat{h}\hat{h}' - \hat{\underline{p}}_\perp\hat{\underline{p}}'_\perp\,, \qquad /V.12/$$

and we know from the mass-shell conditions, that

$$\underset{\sim}{\hat{k}}^2 = m^2 + \mu^2 c^2\,, \qquad \underset{\sim}{\hat{k}}'^2 = m'^2 + \mu'^2 c^2\,, \qquad /V.13/$$

where

$$\mu = \lambda \hat{p}_{-} - \frac{1}{4\lambda c^2}\hat{p}_{+}, \qquad \mu' = \mu \hat{p}'_{-} - \frac{1}{4\lambda c^2}\hat{p}'_{+} . \qquad /V.14/$$

Obviously, μ and μ' are 2-Lorentz invariant. From /V.10/ and /V.14/ we have

$$2\lambda\hat{p}_{-} = \mu + \frac{1}{c^2}\hat{h} \equiv \mu + \frac{1}{c}\sqrt{\hat{\underset{\sim}{k}}^2}\cosh\frac{\alpha}{c},$$

$$2\lambda\hat{p}'_{-} = \mu' + \frac{1}{c^2}\hat{h}' \equiv \mu' + \frac{1}{c}\sqrt{\hat{\underset{\sim}{k}}'^2}\cosh\frac{\alpha'}{c}, \qquad /V.15/$$

which is already written in terms of IMF quantitites, but is still not manifestly invariant. To find invariant forms for $\cosh\frac{\alpha}{c}$ and $\cosh\frac{\alpha'}{c}$ we introduce an auxiliary three-momentum $\hat{\underset{\sim}{q}}$, which has space-components $\hat{q}_{\perp}$ parallel with $\hat{p}_{\perp}$ /for simplicity we may choose both of them in the y-direction/:

$$\hat{\underset{\sim}{q}} = (0,\ 0,\ (-\hat{q}_{\perp}^2)^{1/2}). \qquad /V.16/$$

The momentum $\hat{\underset{\sim}{q}}$ is spacelike from the point of view of the 2-Lorentz group, $\hat{\underset{\sim}{q}}^2 < 0$. By a straightforward calculation one obtains:

$$\cosh\frac{\alpha}{c} = \left[1 - \frac{(\hat{q}_{\perp}\hat{\underset{\sim}{k}})^2}{\hat{q}_{\perp}^2\hat{\underset{\sim}{k}}^2}\right]^{1/2},$$

$$\cosh\frac{\alpha}{c}\cosh\frac{\alpha'}{c} = \frac{\hat{\underset{\sim}{k}}\hat{\underset{\sim}{k}}'}{\sqrt{\hat{\underset{\sim}{k}}^2\hat{\underset{\sim}{k}}'^2}} - \frac{q\underset{\sim}{k}'}{\sqrt{-q_{\perp}^2 k'^2}}\sinh\frac{\alpha}{c}, \qquad /V.17/$$

and we define

$$\sinh\frac{\alpha}{c} \geqslant 0, \qquad \sinh\frac{\alpha'}{c} \geqslant 0. \qquad /V.18/$$

Now we can deduce from eqs. /V.8-18/ the following result:
The matrix elements of the "transverse" scalar current $\int s(x)d\xi$ between all momentum eigenstates $|m,p\rangle$, $|m',p'\rangle$ correspond to the matrix elements of a set of "current operators" $s(x;q)$ between all momentum eigenstates $|m,\mu,\underset{\sim}{k}\rangle$, $|m',\mu',\underset{\sim}{k}'\rangle$, $\underset{\sim}{k}^2 = m^2 + \mu^2c^2$, $\underset{\sim}{k}'^2 = m'^2 + \mu'^2c^2$, $-\infty<\mu<\infty$, $-\infty<\mu'<\infty$. The states $|m,\mu,\underset{\sim}{k}\rangle$ are basis functions for the representation of the 2-Poincaré group:

$$U(\underset{\sim}{a},\underset{\sim}{\Lambda})|\mu,\underset{\sim}{k}\rangle = e^{i\underset{\sim}{a}\underset{\sim}{k}}|\mu,\underset{\sim}{\Lambda k}\rangle . \qquad /V.19/$$

The set of operators $s(x;q)$ is listed by means of the spacelike momenta $\underset{\sim}{q} = (\pm c(q_{\perp}^2 + \underset{\sim}{q}^2)^{1/2}, q_{\perp})$, $q_{\perp}^2 + \underset{\sim}{q}^2 > 0$. The 2-Poincaré transformations act on

$s(\underset{\sim}{x};\underset{\sim}{q})$ according to the rule:

$$U^{-1}(\underset{\sim}{a},\underset{\sim}{\Lambda})\ s(\underset{\sim}{x};\underset{\sim}{q})\ U(\underset{\sim}{a},\underset{\sim}{\Lambda}) = s(\underset{\sim}{x}\underset{\sim}{\Lambda} + \underset{\sim}{a};\underset{\sim}{\Lambda}^{-1}\underset{\sim}{q}). \qquad /V.20/$$

The matrix elements of $s(\underset{\sim}{x};\underset{\sim}{q})$ between the above momentum eigenstates read as follows:

$$\langle m',\mu';\underset{\sim}{k}'|s(\underset{\sim}{x};\underset{\sim}{q})|m,\mu;\underset{\sim}{k}\rangle = \qquad /V.21/$$

$$= (2\Pi)^3\ e^{i\underset{\sim}{x}(\underset{\sim}{k}'-\underset{\sim}{k})} f(m',m;\mu',\mu;(\underset{\sim}{k}'-\underset{\sim}{k})^2;\ \underset{\sim}{q}\underset{\sim}{k}',\ \underset{\sim}{q}\underset{\sim}{k},\ \underset{\sim}{q}^2),$$

where

$$f(m',m;\ \mu',\mu;\ (\underset{\sim}{k}'-\underset{\sim}{k})^2\ ;\ \underset{\sim}{q}\underset{\sim}{k}',\ \underset{\sim}{q}\underset{\sim}{k},\ \underset{\sim}{q}^2) = \qquad /V.22/$$

$$= 4\Pi F(m',m;(\underset{\sim}{k}'-\underset{\sim}{k})^2 - (\mu'-\mu)^2c^2)\delta(\mu+\frac{1}{c}\sqrt{\underset{\sim}{k}^2}\cosh\frac{\alpha}{c} - \mu' - \frac{1}{c}\sqrt{\underset{\sim}{k}'^2}\cosh\frac{\alpha'}{c}),$$

$$\cosh\frac{\alpha}{c} = \left[1 - \frac{(\underset{\sim}{q}\underset{\sim}{k})^2}{\underset{\sim}{q}^2\underset{\sim}{k}^2}\right]^{1/2}, \qquad \sinh\frac{\alpha}{c} > 0,$$

$$/V.23/$$

$$\cosh\frac{\alpha}{c}\ \cosh\frac{\alpha'}{c} = \frac{\underset{\sim}{k}\ \underset{\sim}{k}'}{\sqrt{\underset{\sim}{k}^2\underset{\sim}{k}'^2}} - \frac{\underset{\sim}{q}\ \underset{\sim}{k}'}{\sqrt{-\underset{\sim}{q}^2\underset{\sim}{k}'^2}}\sinh\frac{\alpha}{c}, \qquad \sinh\frac{\alpha'}{c} > 0.$$

The correspondence between the matrix elements belonging to a transitivity set in the ordinary reference frame and those in the IMF is the following:

$$\langle m',\mu';\underset{\sim}{k}'|s(\underset{\sim}{x};\underset{\sim}{q})|m,\mu;\underset{\sim}{k}\rangle = \frac{1}{2\lambda}\lim_{\xi\to\infty}\int T(e^{-i\xi N_3}\ \Lambda\ e^{i\xi N_3}) \times \qquad /V.24/$$

$$\times\ T(e^{-i\xi N_3}\ e^{it\left[\frac{1}{4\lambda}e^{\xi}P_+ + \lambda c^2 e^{-\xi}P_-\right]}\ e^{i\xi N_3})\ T(e^{-i\xi N_3})\ \langle\hat{m}',\hat{p}'|s(x)|m,\hat{p}\rangle d\zeta.$$

In /V.24/ the 3-Lorentz transformation Λ is given by /V.5,6/, the integral over ζ is to be evaluated by the rule described in Sect.IV. The momenta $\underset{\sim}{k},\underset{\sim}{k}',\underset{\sim}{q}$ and the 2-Lorentz invariants μ,μ' are to be calculated by using /V.8,10,14-18/, and $\underset{\sim}{k} = \underset{\sim}{\Lambda}\hat{\underset{\sim}{k}}$, $\underset{\sim}{k}' = \underset{\sim}{\Lambda}\hat{\underset{\sim}{k}}'$, $\underset{\sim}{q} = \underset{\sim}{\Lambda}\hat{\underset{\sim}{q}}$.

6. MISCELLANEOUS RESULTS AND DISCUSSION

In this section we discuss certain features of the results presented in the previous sections.

a. The operators $s(\underset{\sim}{x};\underset{\sim}{q})$ are not Hermitian, $s^{+}(\underset{\sim}{x};\underset{\sim}{q}) \neq s(\underset{\sim}{x};\underset{\sim}{q})$, that is,

$$\langle m',\mu';\underset{\sim}{k}'|s(\underset{\sim}{x};\underset{\sim}{q})|m,\mu;k\rangle^{*} \neq \langle m,\mu;\underset{\sim}{k}|s(\underset{\sim}{x};\underset{\sim}{q})|m',\mu';\underset{\sim}{k}'\rangle .$$

This is the consequence of the non-symmetric role of the momenta $\underset{\sim}{k}$ and $\underset{\sim}{k}'$ in /V.23/. Indeed, in /V.17/ the alternative choice $\hat{q}_{\perp}||\hat{p}'_{\perp}$ would also be possible, when the definitions

$$\cosh\frac{\overline{\alpha}'}{c} = \left[1 - \frac{(\hat{q}\,\hat{\underset{\sim}{k}}')^{2}}{\hat{q}^{2}\hat{\underset{\sim}{k}}'^{2}}\right]^{1/2}, \qquad \sinh\frac{\overline{\alpha}'}{c} > 0, \qquad /VI.1/$$

$$\cosh\frac{\overline{\alpha}'}{c}\cosh\frac{\overline{\alpha}}{c} = \frac{\hat{\underset{\sim}{k}}\,\hat{\underset{\sim}{k}}'}{\sqrt{\hat{\underset{\sim}{k}}^{2}\hat{\underset{\sim}{k}}'^{2}}} - \frac{\hat{\underset{\sim}{q}}\,\hat{\underset{\sim}{k}}}{\sqrt{-\hat{q}^{2}\hat{\underset{\sim}{k}}^{2}}}\sinh\frac{\overline{\alpha}'}{c}, \qquad \sinh\frac{\overline{\alpha}}{c} > 0$$

arise, and we have

$$2\lambda\hat{p}_{-} = \mu + \frac{1}{c}\sqrt{\hat{\underset{\sim}{k}}^{2}}\cosh\frac{\overline{\alpha}}{c}, \qquad 2\lambda\hat{p}'_{-} = \mu' + \frac{1}{c}\sqrt{\hat{\underset{\sim}{k}}'^{2}}\cosh\frac{\overline{\alpha}'}{c}. \qquad /VI.2/$$

This choice would obviously yield some operators $\overline{s}(\underset{\sim}{x};\underset{\sim}{q})$, different from $s(\underset{\sim}{x};\underset{\sim}{q})$, their matrix elements being

$$\langle m,\mu;\underset{\sim}{k}|\overline{s}(\underset{\sim}{x};\underset{\sim}{q})|m',\mu'\underset{\sim}{k}'\rangle = \qquad /VI.3/$$

$$= (2\Pi)^{3}\, e^{i\underset{\sim}{x}(\underset{\sim}{k}-\underset{\sim}{k}')}\overline{f}(m,m';\mu,\mu';(\underset{\sim}{k}-\underset{\sim}{k}')^{2}, \underset{\sim}{q}\underset{\sim}{k}, \underset{\sim}{q}\underset{\sim}{k}', \underset{\sim}{q}^{2}),$$

where

$$4\Pi F(m,m';(\underset{\sim}{k}-\underset{\sim}{k}')^{2} - (\mu-\mu')^{2}c^{2})\,\delta(\mu+\frac{1}{c}\sqrt{\underset{\sim}{k}^{2}}\cosh\frac{\overline{\alpha}}{c} - \mu' - \frac{1}{c}\sqrt{k'^{2}}\cosh\frac{\overline{\alpha}'}{c}) = \qquad /VI.4/$$

$$= \overline{f}(m,m';\mu,\mu';(\underset{\sim}{k}-\underset{\sim}{k}')^{2}, \underset{\sim}{q}\underset{\sim}{k}, \underset{\sim}{q}\underset{\sim}{k}', \underset{\sim}{q}^{2}),$$

and $\cosh\frac{\overline{\alpha}}{c}$, $\cosh\frac{\overline{\alpha}'}{c}$ are given by formulas like /VI.1/ except that $\underset{\sim}{k}$, $\underset{\sim}{k}'$ and $\underset{\sim}{q}$ must be substituted for $\hat{\underset{\sim}{k}}$, $\hat{\underset{\sim}{k}}'$ and $\hat{q}$, respectively. The comparison of eqs. /VI.1-4/ and /V.21-23/ shows, together with /V.2/, that

$$\langle m',\mu';\underset{\sim}{k}'|s(\underset{\sim}{x};\underset{\sim}{q})|m,\mu;k\rangle^{*} = \langle m,\mu;\underset{\sim}{k}|\overline{s}(\underset{\sim}{x};\underset{\sim}{q})|m',\mu';\underset{\sim}{k}'\rangle ,$$

that is,

$$\overline{s}(\underset{\sim}{x};\underset{\sim}{q}) = s^{+}(\underset{\sim}{x};\underset{\sim}{q}) \neq s(\underset{\sim}{x};\underset{\sim}{q}).$$

b. The presence of the auxiliary momentum $\underset{\sim}{q}$ in $s(\underset{\sim}{x};\underset{\sim}{q})$ is the most surprising result of the previous sections, and needs some explanation. We stress, that it is related with the remark we made in Sect. II. concerning the IML of the transitivity sets. Namely, that while the two sets $(\phi_\alpha, s(x\Lambda + a)\phi_\beta)$ and $(\phi_{\tilde\alpha}, s(x\Lambda + a)\phi_{\tilde\beta})$ are the same in the ordinary reference frame, if $\phi_{\tilde\alpha} = U(\tilde a,\tilde\Lambda)\phi_\alpha$, $\phi_{\tilde\beta} = U(\tilde a,\tilde\Lambda)\phi_\beta$, they may be completely different in the IMF. Indeed, it is easy to see, that the operators $s(\underset{\sim}{x};\underset{\sim}{q})$ with various $\underset{\sim}{q}$ are related with the various choices of the fixed transverse momentum $\hat{p}_\perp$ /c.f./V.7// for the transitivity sets $\langle m',\hat{p}'|s(x\Lambda+a)|m,\hat{p}\rangle$. Similarly, the matrix elements /V.21/ of $s(\underset{\sim}{x};\underset{\sim}{q})$ form a one-parameter set from the point of view of $\underset{\sim}{q}$, since they depend only upon $\dfrac{\underset{\sim}{q}}{(-\underset{\sim}{q}^2)^{1/2}}$, and are independent of the angle between $\underline{q}_\perp$ and $\underline{k}_\perp$.

c. It is an obvious arbitrariness in the definition of the operators $s(\underset{\sim}{x};\underset{\sim}{q})$ that $\underset{\sim}{q}^2 < 0$ was chosen. There is no a priori reason for this choice, $\underset{\sim}{q}^2 \geqslant 0$ would also be possible. It seems to us, that, /at least for $\underset{\sim}{q}^2 > 0$/, an alternative form of IMF physics emerges in this case. We are going to discuss it in more details in a forthcoming paper. We only mention here, that for $c\to\infty$, that is, in the "nonrelativistic" limit, eq, /V.21/ yields

$$\lim_{c\to\infty} \langle m',\mu';\underset{\sim}{k}'|s(\underset{\sim}{x},\underset{\sim}{q})|m,\mu;\underset{\sim}{k}\rangle =$$

$$= (2\Pi)^3 \exp i\{t\cdot(\frac{m'^2+k_\perp'^2}{2\mu'} - \frac{m^2+k_\perp^2}{2\mu}) + \underline{x}_\perp(\underline{k}_\perp' - \underline{k}_\perp)\} \times \qquad /VI.5/$$

$$\times\ 2\Pi F(m',m; -(\underline{k}_\perp' - \underline{k}_\perp)^2)\delta(\mu-\mu'),$$

if $\mu > 0$, $\mu' > 0$. /The variables m, m', μ, μ', $\underline{k}_\perp$, $\underline{k}_\perp'$, $\underline{q}_\perp$ and $\underset{\sim}{q}^2$ are kept fixed, when the limit $c\to\infty$ is evaluated./ The right hand side of eq. /VI.5/ coincides with Susskind's result in the Galilean case of contraction [7]. Therefore, the results presented in Sect. V. can be considered as the "relativistic" generalization of the Galilean matrix elements. /A more complete discussion of the limit $c\to\infty$ when either of μ and μ' is negative will be discussed elsewhere./

d. In the contraction schemes we had the parameter $\lambda > 0$, which was postulated to be arbitrary. From this a specific scaling property of the matrix elements follows. Namely,

$$\langle m',\mu';\underset{\sim}{k}'|s(\underset{\sim}{x};\underset{\sim}{q})|m,\mu;\underset{\sim}{k}\rangle = \lambda'\langle m',\mu_1';\underset{\sim}{k}_1'|s(\underset{\sim}{x}_1;\underset{\sim}{q})|m,\mu_1;\underset{\sim}{k}_1\rangle \qquad /VI.6/$$

if the following set of relations fulfills:

$$\langle m',\mu';\underset{\sim}{k}'|s(\underset{\sim}{x};\underset{\sim}{q})|m,\mu;\underset{\sim}{k}\rangle = \langle m',\mu';\hat{\underset{\sim}{k}}'|s(\underset{\sim}{x}\underset{\sim}{\Lambda};\hat{\underset{\sim}{q}})|m,\mu;\hat{\underset{\sim}{k}}\rangle ,$$

$$\langle m',\mu_1';\ \underset{\sim}{k}_1'|s(\underset{\sim}{x}_1;\underset{\sim}{q})|m,\mu_1;\ \underset{\sim}{k}_1\rangle = \langle m',\mu_1';\ \hat{\underset{\sim}{k}}_1'|s(\underset{\sim}{x}_1\underset{\sim}{\Lambda};\hat{\underset{\sim}{q}})|m,\mu_1;\ \hat{\underset{\sim}{k}}_1\rangle$$

the coordinates and momenta, respectively, being related by

$$\hat{\underset{\sim}{q}} = (0,\ 0,\ \sqrt{-\underset{\sim}{q}^2}),$$

$$\hat{\underset{\sim}{k}} = (c\sqrt{m^2 + \mu^2c^2 + \hat{k}_\perp^2},\ 0,\ \hat{k}_\perp), \qquad \hat{\underset{\sim}{k}}_1 = (c\sqrt{m^2 + \mu_1^2c^2 + \hat{k}_\perp^2},\ 0,\ \hat{k}_\perp),$$

$$\hat{\underset{\sim}{k}}' = (c\sqrt{m'^2 + \mu'^2c^2 + \hat{k}_\perp'^2},\ \hat{\underline{k}}_\perp'), \qquad \hat{\underset{\sim}{k}}_1' = (c\sqrt{m'^2 + \mu_1'^2c^2 + \hat{k}_\perp'^2},\ \hat{\underline{k}}_\perp'),$$

$$\underset{\sim}{x}\underset{\sim}{\Lambda} = (t',\ \underline{x}_\perp'), \qquad \underset{\sim}{x}_1\underset{\sim}{\Lambda} = (t_1',\ \underline{x}_\perp'), \qquad \underset{\sim}{q} = \underset{\sim}{\Lambda}\hat{\underset{\sim}{q}},$$

$$\underset{\sim}{k}_1' = \underset{\sim}{\Lambda}\hat{\underset{\sim}{k}}_1', \qquad \underset{\sim}{k}_1 = \underset{\sim}{\Lambda}\hat{\underset{\sim}{k}}_1, \qquad \underset{\sim}{k} = \underset{\sim}{\Lambda}\hat{\underset{\sim}{k}}, \qquad \underset{\sim}{k}' = \underset{\sim}{\Lambda}\hat{\underset{\sim}{k}}',$$

$$t' = \lambda' t_1',$$

$$\lambda'(\mu + \frac{1}{c}\sqrt{m^2 + \mu^2c^2 + \hat{k}_\perp^2}) = \mu_1 + \frac{1}{c}\sqrt{m^2 + \mu_1^2c^2 + \hat{k}_\perp^2}\ ,$$

$$\lambda'(\mu' + \frac{1}{c}\sqrt{m'^2 + \mu'^2c^2 + \hat{k}_\perp'^2}) = \mu_1' + \frac{1}{c}\sqrt{m'^2 + \mu_1'^2c^2 + \hat{k}_\perp'^2}\ .$$

e. So far it is not clear how to calculate the matrix elements for vacuum transitions. We complete eqs. /V.21-23/ by defining

$$\langle 0|s(\underset{\sim}{x};\underset{\sim}{q})|m,\mu,\underset{\sim}{k}\rangle = \lim_{\kappa\to 0} \langle \kappa m',\kappa\mu',\kappa\underset{\sim}{k}'|s(\underset{\sim}{x};\underset{\sim}{q})|m,\mu;\underset{\sim}{k}\rangle.$$

It follows, that

$$\langle 0|s(\underset{\sim}{x};\underset{\sim}{q})|m,\mu;\underset{\sim}{k}\rangle = 0.$$

Evidently, the statement, common in the literature, that the absence of vacuum transitions indicates the non-relativistic symmetry of the theory in the IMF, is unjustified.

f. In this paper we investigated the properties of a scalar current in the IMF. The same arguments can also be used for vector currents, $j_\mu(x)$, apart from that in place of $\int s(x)d\zeta$ the quantities

$$\int\left[\frac{1}{4\lambda}e^{\xi}(j_o(x)+j_3(x))+\lambda c^2 e^{-\xi}(j_o(x)-j_3(x))\right]d\zeta,\quad \int j_{\perp}(x)\,d\zeta, \qquad /VI.7/$$

and

$$\int\left[\lambda e^{-\xi}(j_o(x)-j_3(x))-\frac{1}{4\lambda c^2}e^{\xi}(j_o(x)+j_3(x))\right]d\zeta \qquad /VI.8/$$

are suitable for the transformation into the IMF. As a result, one arrives at a "vector current" $\underset{\sim}{j}(\underset{\sim}{x};\underset{\sim}{q})$ and a "scalar current" $s(\underset{\sim}{x};\underset{\sim}{q})$, corresponding to /VI.7/ and /VI.8/, respectively.

7. SUMMARY AND OUTLOOK

In this paper the transformation properties of "semi-integrated" quantities, such as $\int s(x)d\zeta$, in the IMF are systematically derived. Our guide in the course of the investigation has been, that it is actually a group contraction problem which must be solved. Having in mind that the group contraction is non-unique, we have dealt with a one-parameter family of contractions. The parameter, denoted by c, distinguishes Galilei and Poincaré type transformation groups of 2+1 dimensional Euclidean and Minkowski spacetimes, respectively. The actual value of c seems to be of dynamical origin.

In order to derive the transformation properties of $\int s(x)d\zeta$ in the IMF we have described the contraction of the 3-Poincaré group together with the contraction of its representation on the /generalized/ function space of all matrix elements of $\int s(x)d\zeta$. In this function space we introduced "transitivity sets", obtaining thereby a mapping between the basis functions /the matrix elements between momentum eigenstates/ of this space, and the elements of the 3-Poincaré group. The contraction of the representation space has been performed essentially by requiring a similar mapping between the elements of the contracted group and the basis functions of the contracted space. As a result, the operators $s(\underset{\sim}{x};\underset{\sim}{q})$ have appeared in the IMF. They are defined via their matrix elements, /V.21/.

In practical applications one usually needs the IML of one given matrix element, which can always be considered as the initial element of a transitivity set /belonging to the unit element of the actual symmetry group/. Therefore, it is justified to make the identification:

$$\frac{1}{2\lambda}\lim_{\xi\to\infty} 2\Pi e\ \exp i\left[\frac{1}{2}(x^o+x^3)(p'_+-p_+)+\underset{\sim}{x}_{\perp}(\underset{\sim}{p}'_{\perp}-\underset{\sim}{p}_{\perp})\right]F(m',m;(p-p')^2)\,\delta(p'_--p_-) = \qquad /VII.1/$$

$$=\langle m',\mu',\underset{\sim}{k}|s(\underset{\sim}{x};\underset{\sim}{q})|m,\mu,\underset{\sim}{k}\rangle\ ,$$

if $p_- = \frac{1}{2\lambda}(\mu + \frac{1}{c}\sqrt{\underset{\sim}{k}^2}\cosh\frac{\alpha}{c})e^{\xi}$, $\quad p'_- = \frac{1}{2}(\mu' + \frac{1}{c}\sqrt{\underset{\sim}{k}'^2}\cosh\frac{\alpha'}{c})e^{\xi}$,

$$x^o + x^3 = \frac{1}{2\lambda}te^{\xi}.$$

Equation /VII.1/ forms the bridge between the conventional and our methods for calculating the IML of a matrix element.

The value of our enlarged framework for the IML calculations, and its physical content can be learned only from applications to explicit problems, which is out of the scope of this paper. A rather natural idea is, that series expansion of quantities, calculated in the 2-Poincaré framework, with respect to powers of $\frac{1}{c^2}$ reproduces the old results in the zeroth order, and gives "corrections" to them in the higher orders. Such a programme will be performed for deep-inelastic scattering in a separate paper.

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