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L. MIHÁLY
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RENORMALIZATION GROUP TREATMENT OF THREE-DIMENSIONAL ORDERING IN A SYSTEM OF WEAKLY COUPLED LINEAR CHAINS

L. Mihály and J. Sólyom
Central Research Institute for Physics
H-1525 Budapest, P.O.B. 49. Hungary

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ABSTRACT

The earlier renormalization group treatment of a one-dimensional Fermi gas model is extended to a system of weakly coupled chains. The coupling is a Coulomb type interaction between electrons on different chains, no interchain hopping is allowed. This system may have a phase transition to a charge density wave state. The charge density waves on the neighbouring chains are in phase or opposite phase depending on the sign of the interchain coupling. No phase transition to a superconducting or antiferromagnetic phase is obtained in the present approximation.

АННОТАЦИЯ

Обобщен метод группы ренормировок, разработанный для исследования модели одномерного газа фермионов, для изучения слабо-связанных нитей. Связь осуществляется кулоновским взаимодействием электронов, однако перескок электронов не допускается. В такой системе может иметь место фазовый переход в диэлектрическое состояние. Волна плотности заряда на соседних нитях имеет одинаковую или противоположенную фазу в зависимости от знака связи между нитями. В исследованном приближении не возникает фазовый переход ни в сверхпроводящее ни в антиферромагнитное состояния.

KIVONAT

Az egydimenziós fermion gáz modellre kidolgozott renormálási csoportos tárgyalást általánositjuk a gyengén csatolt láncok esetére. A csatolás a különböző láncokon levő elektronok közötti Coulomb tipusu csatolás, az elektronok láncok közötti átugrása nincs megengedve. A rendszerben lehetséges egy fázisátalakulás egy sürüséghullámmal rendelkező állapotba. A sürüséghullám a szomszédos láncokon azonos vagy ellentétes fázisban van, a láncok közötti csatolás előjelétől függően. Az adott közelitésben sem szupravezető, sem antiferromágneses állapotba való átmenet nem adódik.

1. INTRODUCTION

The recent interest in the behaviour of quasi one-dimensional systems is due to a large extent to the discovery of high conductivity of some TCNQ salts[1]. These salts undergo a phase transition to an insulating state at low temperatures. This transition could be interpreted as a Peierls transition, although the situation is not cleared up yet. Similar behaviour has been observed in KCP[2].

The Peierls transition is a typically one-dimensional effect. The abovementioned systems are in fact formed of one-dimensional chains of atoms, the interchain distance being much larger than the distance between atoms in the same chain. Therefore at first sight the interchain coupling can be neglected and the system can be considered as strictly one-dimensional.

There were many attempts to account for the behaviour of these systems starting basically from two different models. In one approach[3-5] the electron-phonon coupling is considered either in mean field approximation or the fluctuations are also taken into account by using the Ginzburg-Landau functional. In another approach[6-10] only the electron-electron interaction is studied and the electron-phonon interaction is treated as an effective electron-electron coupling. Both electron-electron and electron-ion interactions were considered simultaneously by Levin et al.[11, 12] There is a wide variety of mathematical methods which have been applied and exact results have been obtained in special cases only[10].

Since it is known that there is no phase transition at any finite temperature in strictly one-dimensional systems with short range interaction, the experimental findigs can be accounted for only if interchain coupling is also considered. Indeed, almost all of the abovementioned approaches were extended to the quasi-one-dimensional case. Considering the electron-phonon system it is natural to assume that the electronic spectrum is one-dimensional, the phonon spectrum is, however, nearly isotropically three-dimensional and therefore induces a three-dimensional coupling. Such a system was studied by Rice and Strässler[13], Gutfreund et al[14] and Bjelis et al[15]. Another

possible extension is to suppose a nonplanar Fermi surface[16]. The work by Lee et al[5] was extended by Dieterich[17] to investigate a system of weakly coupled linear chains.

The problem of electron-electron interaction has also been extended to include interchain couplings. Gorkov and Dzyaloshinsky[18] considered both Coulomb type interaction and hopping between the chains. The exact solution of Luther and Emery[10] was used by Klemm and Gutfreund[19] to calculate the effect of interchain coupling in mean field approximation. Another approximate treatment of the interchain coupling was given by Schuster[20] extending the equation of motion method.

Our model for the system of weakly coupled linear chains falls into this last category, where only electron-electron interaction is considered. The approximation scheme corresponds to an extension of the treatment by Gorkov and Dzyaloshinsky[18] taking into account, however, not only the parquet diagrams but next to leading logarithmic corrections as well. This can be achieved by using the renormalization group method as developed by Menyhárd and Sólyom[7,8] for the strictly one-dimensional case. In this treatment, however, only the Coulomb type interaction will be considered. No hopping will be allowed between the chains.

In Section 2 we describe our model and define the generalized susceptibilities in terms of which the behaviour of the system will be investigated. The renormalization group treatment of the model is briefly presented in Section 3, and the Lie equations of the group are solved in Section 4. A self-consistent solution is obtained which shows a phase transition to a charge density wave state. The results are discussed in Section 5.

2. THE MODEL

The system to be considered consists of a set of weakly interacting linear chains, which are strictly one-dimensional. The electrons can travel along the chains only and no interchain hopping is permitted. Therefore the electrons will be labelled by an index i which refers to the i^{th} chain. The coupling between the chains is due to a three-dimensional Coulomb type electron-electron interaction. It can also be considered as a phonon or polaron mediated effective coupling between electrons on different chains.

The Hamiltonian of such a system can be written generally in the form

$$H = H_o + H_{int}, \qquad /1/$$

$$H_o = \sum_{i,k,\alpha} \varepsilon_k \, C^+_{ik\alpha} \, C_{ik\alpha}, \qquad /2/$$

$$H_{int} = \frac{1}{L} \sum_{ij\ell m} \sum_{\substack{k_1 k_2 k_3 \\ \alpha\beta}} g_{ij\ell m}(k_1,k_2,k_3) C^+_{ik_1\alpha} \, C^+_{jk_2\beta} \, C_{\ell k_3\beta} \, C_{mk_1+k_2-k_3\alpha} \qquad /3/$$

where $C^+_{ik\alpha}$ ($C_{ik\alpha}$) is the creation /annihilation/ operator of an electron on the i^{th} chain with momentum k and spin α, $g_{ij\ell m}(k_1,k_2,k_3)$ is the coupling constant and L is the length of the chains. All the momenta in these expressions have components along the chains only and the kinetic energy of the electrons ε_k depends also only on this parallel momentum component.

Instead of this quite general form of interaction we will use a more restricted form by assuming that the interaction cannot transfer electrons from one chain to another and also spin exchange processes will be neglected. This amounts to taking i=m and j=ℓ. Supposing furthermore that the interaction is important only for electrons around the Fermi surface within an energy range of width $2\omega_D$ and taking into account the one-dimensional character of the energy spectrum, the electrons can be classified into two classes. The group of electrons with momentum around $+k_F$ and the other group consisting of electrons with momentum around $-k_F$ will be distinguished by denoting the corresponding operators by a_k and b_k, respectively. Two types of interaction will be considered, with and without exchange of electrons between the two groups. The corresponding momentum transfer parallel to the chain direction is nearly $2k_F$ or O.

Under these assumptions the interaction part of the Hamiltonian can be written - by a straightforward generalisation of the Hamiltonian of Ref. 7 - in the form

$$H_{int} = \frac{1}{L} \sum_{ij} \sum_{\substack{k_1,k_2,k_3 \\ \alpha\beta}} (g_{1ij} \, a^+_{ik_1\alpha} \, b^+_{jk_2\beta} \, a_{jk_3\beta} \, b_{ik_1+k_2-k_3\alpha} +$$

$$+ g_{2ij} \, a^+_{ik_1\alpha} \, b^+_{jk_2\beta} \, b_{jk_3\beta} \, a_{ik_1+k_2-k_3\alpha}), \qquad /4/$$

where g_{1ij} and g_{2ij} are the coupling constants of the backward and forward scattering, respectively. They depend on the distance between the chains i and j. These interactions are shown diagrammatically in Fig. 1, where the continuous and dotted lines denote the two types of electrons. Fig. 2. shows some other possible interaction processes which are neglected in this treatment. They include the Umklapp processes and spin exchange scattering.

In order to investigate the possible phase transitions in this system the temperature dependence of three generalized susceptibilities was studied. The appearance of charge density waves, spin density waves and Cooper pairs is related to the divergence in the charge density, spin density and Cooper pair fluctuations, respectively. The temperature dependent response functions can be obtained in turn by the analytic continuation to the upper ω halfplane of the correlation functions

$$N(k,q,\omega_\nu) = -\int_0^{1/T} d\tau e^{i\omega_\nu\tau} \sum_{ij} e^{iq(R_i-R_j)} \cdot$$

$$\cdot < T_\tau \{ \sum_\alpha \int \frac{dp}{2\pi} C^+_{ip\alpha}(t)\, C_{ip+k\alpha}(t) \sum_\beta \int \frac{dp'}{2\pi} C^+_{jp',\beta}(o) C_{jp',-k\beta}(o) \} >, \qquad /5/$$

$$\chi(k,q,\omega_\nu) = -\int_0^{1/T} d\tau e^{i\omega_\nu\tau} \sum_{ij} e^{iq(R_i-R_j)}$$

$$\cdot < T_\tau \{ \int \frac{dp}{2\pi} C^+_{ip\uparrow}(t)\, C_{ip+k\downarrow}(t) \int \frac{dp'}{2\pi} C^+_{jp',\uparrow}(o)\, C_{jp',-k\downarrow}(o) \} >, \qquad /6/$$

$$\Delta(\omega_\nu) = -\int_0^{1/T} d\tau e^{i\omega_\nu\tau} \sum_{ij} < T_\tau \{ \int \frac{dp}{2\pi} C_{ip\uparrow}(t)\, C_{i-p\downarrow}(t) \cdot$$

$$\cdot \int \frac{dp'}{2\pi} C^+_{j-p',\uparrow}(o)\, C^+_{jp',\downarrow}(o) \} > . \qquad /7/$$

Here $\omega_\nu = 2\pi\nu T$, k denotes the momentum component parallel to the chains, and q is the perpendicular component. The vector R_i is also perpendicular to the direction of the chains and gives the position of the i^{th} chain.

The singularity in N and χ will first appear at $k=2k_F$ and therefore only this special value will be considered. The frequency variable will be fixed for similar reasons to $\omega=0$ after analytic continuation. The remaining variables are the temperature and the perpendicular momentum component q. Depending on whether the divergence shows up first at q=0 or at a finite q=Q /probably at the zone boundary/ the ordering between the chains will be in phase or in opposite phase.

3. RENORMALIZATION GROUP TREATMENT

The interacting Fermi gas model of a single one-dimensional chain has been previously studied by Menyhárd and Sólyom[7,8] using the renormalization group approach. It was shown that this method enables us to consider successively leading and next to leading logarithmic corrections, the first order renormalization being equivalent to the parquet approximation. The second order renormalization led to the disappearance of the unphysical pole of the parquet approximation and gave qualitatively good results for the low temperature behaviour of the system.

The parquet approximation has been used by Gorkov and Dzyaloshinsky[18] to study the behaviour of a system of weakly coupled chains. They have shown that in this approximation without interchain hopping there are two possible solutions of the problem. One is the so-called "standing pole", the other is the "moving pole" solution. The first one is a straightforward generalization of the solution of the parquet equations for the strictly one-dimensional case. Since the pole in this latter case is an artifact of the approximation and disappears in second order renormalization, it is expected that the physical case will correspond to a "moving pole" solution. Because we consider part of the non-parquet diagrams as well, the physical picture which arises from this calculation is different and we hope that it gives a better insight into the physics of quasi one-dimensional systems.

Similarly to Ref. 7, the renormalization of the coupling constants g_{1ij} and g_{2ij} is determined from the requirement that the dimensionless Green's function d and the dimensionless vertices $\tilde{\Gamma}_{1ij}$, $\tilde{\Gamma}_{2ij}$, defined by the relations

$$G = G^{(o)}\, d, \qquad /8/$$

$$\Gamma^{(ij\ell m)}_{\alpha\beta\gamma\delta} = g_{1ij}\, \tilde{\Gamma}_{1ij}\, \delta_{\alpha\gamma}\, \delta_{\beta\delta}\, \delta_{i\ell}\, \delta_{jm} - g_{2ij}\, \tilde{\Gamma}_{2ij}\, \delta_{\alpha\delta}\, \delta_{\beta\gamma}\, \delta_{im}\, \delta_{j\ell}, \qquad /9/$$

be multiplicatively renormalizable under a scaling of the cut-off energy ω_D. Denoting by x any energy-like variable, such as ω, vk or T (k_B=1), it is supposed that

$$d\left(\frac{x}{\omega'_D}, g'_{1ij}, g'_{2ij}\right) = z\, d\left(\frac{x}{\omega_D}, g_{1ij}, g_{2ij}\right), \qquad /10/$$

$$\tilde{\Gamma}_{1ij}\left(\frac{x}{\omega'_D}, g'_{1ij}, g'_{2ij}\right) = z^{-1}_{1ij}\, \tilde{\Gamma}_{1ij}\left(\frac{x}{\omega_D}, g_{1ij}, g_{2ij}\right), \qquad /11/$$

$$\tilde{\Gamma}_{2ij}\left(\frac{x}{\omega_D'}, g_{1ij}', g_{2ij}'\right) = z_{2ij}^{-1}\,\tilde{\Gamma}_{2ij}\left(\frac{x}{\omega_D}, g_{1ij}, g_{2ij}\right), \qquad /12/$$

$$g_{1ij}' = g_{1ij}\, z^{-2}\, z_{1ij}, \qquad /13/$$

$$g_{2ij}' = g_{2ij}\, z^{-2}\, z_{2ij}. \qquad /14/$$

These relations have been checked in perturbation theory up to second order and it is easy to show that they can be satisfied with multiplicative factors z, z_{1ij} and z_{2ij} which depend on the scaling factor ω_D'/ω_D and the bare coupling constants but do not depend on x. The new couplings are called invariant couplings. The Lie equations of the group are the differential forms of these scaling equations.

Introducing the dimensionless couplings by the definition

$$\gamma_{1ij} = \frac{g_{1ij}'}{2\pi v}, \qquad \gamma_{2ij} = \frac{g_{2ij}'}{2\pi v} \qquad /15/$$

where v is the Fermi valocity, the Lie equations for the invariant couplings can be written - after a straightforward perturbational calculation of the Green's function and vertex - in the form

$$\frac{\partial\gamma_{1ij}}{\partial\xi} = 2\left\{\sum_k \gamma_{1ik}\,\gamma_{1kj} + \gamma_{1ij}\,(\gamma_{2ij} - \gamma_{2ii}) + \gamma_{1ij}\left[\gamma_{1ii}(\gamma_{2ij}-\gamma_{2ii}) + \right.\right.$$

$$\left.\left. + \sum_k(\gamma_{ik}^2 + \gamma_{2ik}^2 - \gamma_{2ik}\,\gamma_{2kj})\right] + \ldots\right\}, \qquad /16/$$

$$\frac{\partial\gamma_{2ij}}{\partial\xi} = \gamma_{1ij}^2(1+\gamma_{1ii}) + 2\sum_k \gamma_{1ik}^2(\gamma_{2ij} - \gamma_{2kj}) + \ldots, \qquad /17/$$

where $\xi = \ln(\omega_D'/\omega_D)$. The second order terms on the right hand sides of eqs. /16/ and /17/ are the same as in the analogous equations derived by Gorkov and Dzyaloshinsky[18] for the vertex in parquet approximation. The additional third order terms in our equations come from the next to leading logarithmic corrections. It should also be mentioned that only in parquet approximation are the invariant coupling and vertex equal. In our approximation the invariant coupling contains also corrections from self-energy diagrams.

The invariant couplings as defined here have no temperature dependence. We can, however, formally replace $\xi = \ell n(\omega_D'/\omega_D)$ by $\xi = \ell n(T/\omega_D)$ and define a temperature dependent invariant coupling. The reason is that when the temperature dependence of the response functions is studied, the argument of the invariant coupling, ω_D'/ω_D should be replaced by T/ω_D in the appropriate Lie equation. It is in this sense that we will speak about the temperature dependence of the invariant coupling.

Eqs. /16/ and /17/ form an infinite set of equations which is generally unsoluble. Even if the bare coupling is limited to nearest neighbour chains, the renormalization procedure generates an effective coupling between chains at arbitrary distance, and it becomes impossible to follow the change of all the couplings. It is expected, however, that near T_c, where the three-dimensional ordering sets in, the range of the effective coupling goes to infinity and the invariant coupling becomes independent from the distance. With this assumption we can get a self-consistent solution of the problem, as it will be shown.

Assuming that the invariant couplings are independent of the distance, $\gamma_{1ij} = \gamma_1$ and $\gamma_{2ij} = \gamma_2$, eqs. /16/ and /17/ simplify to

$$\frac{\partial \gamma_1}{\partial \xi} = 2N(\gamma_1^2 + \gamma_1^3 + \ldots) \qquad /18/$$

$$\frac{\partial \gamma_2}{\partial \xi} = \gamma_1^2 + \gamma_1^3 + \ldots , \qquad /19/$$

or

$$\frac{\partial \gamma_2}{\partial \gamma_1} = \frac{1}{2N} \qquad /20/$$

where N is the number of chains. This shows that in this special case the coupling constant of backward scattering γ_1 is renormalized much stronger than that of the forward scattering. Even in a less extreme case eqs. /16/ and /17/ show that starting from a weak coupling case, the backward scattering renormalization is stronger because mediated couplings via a third chain will also contribute.

We will therefore assume that the forward scattering can be neglected compared to the backward scattering and the Lie equation for the invariant coupling γ_{1ij} can be written as

$$\frac{\partial \gamma_{1ij}}{\partial \xi} = 2 \sum_k \gamma_{1ik} \gamma_{1kj} + 2\gamma_{1ij} \sum_k \gamma_{1ik}^2 + \ldots \qquad /21/$$

It is more convenient to work in Fourier space with

$$\gamma_1(q) = \sum_i \gamma_{1ij} \, e^{-iq(R_i - R_j)} \qquad /22/$$

where the vector q is perpendicular to the chain direction. In the Fourier transformed form the Lie equation reads

$$\frac{\partial \gamma_1(q)}{\partial \xi} = 2\{\gamma_1^2(q) + \gamma_1(q) \frac{1}{N} \sum_{q'} \gamma_1^2(q') + \ldots\} \qquad /23/$$

The aim of the renormalization group treatment is to study the response functions N, χ and Δ of eqs. /5/ - /7/, using their expressions in terms of the invariant coupling. Similarly to the strictly one-dimensional case, Lie equations can be derived for the auxiliary quantities $\bar{N}$, $\bar{\chi}$ and $\bar{\Delta}$ only, which are related to the response functions by

$$\bar{N}(q,\xi) = \pi v \frac{\partial N(q,\xi)}{\partial \xi} \qquad /24/$$

$$\bar{\chi}(q,\xi) = 2\pi v \frac{\partial \chi(q,\xi)}{\partial \xi} \qquad /25/$$

$$\bar{\Delta}(\xi) = - \pi v \frac{\partial \Delta(\xi)}{\partial \xi} \qquad /26/$$

A straightforward perturbational calculation of the response functions leads, after having neglected the forward scattering γ_{2ij}, to the following Lie equations:

$$\frac{\partial \ell n \bar{N}(q,\xi)}{\partial \xi} = 4\gamma_1(q) + 2 \frac{1}{N} \sum_{q'} \gamma_1^2(q') + \ldots \qquad /27/$$

$$\frac{\partial \ell n \bar{\chi}(q,\xi)}{\partial \xi} = 2 \frac{1}{N} \sum_{q'} \gamma_1^2 (q') + \ldots \qquad /28/$$

$$\frac{\partial \ell n \bar{\Delta}(\xi)}{\partial \xi} = 2 \frac{1}{N} \sum_{q'} \gamma_1(q') + 2 \frac{1}{N} \sum_{q'} \gamma_1^2(q') + \ldots \qquad /29/$$

Knowing the temperature dependence of the invariant coupling, the integration of these equations gives the temperature dependence of the response functions.

4. SELF-CONSISTENT SOLUTION

First the invariant coupling has to be determined from eq. /23/. We will try to find a solution for $\gamma_1(q)$ supposing a form $\gamma_1(q)=Jf(q)$. It is expected physically that the range of the interaction and therefore the q dependence of the invariant coupling varies with temperature in the course of renormalization. This means that both J and f(q) have to depend on ξ. The Lie equation /23/ can be separated into two equations

$$\frac{\partial J(\xi)}{\partial \xi} = 2J^3(\xi) \frac{1}{N} \sum_{q'} f^2(q',\xi) + J(\xi)h(\xi), \qquad /30/$$

$$\frac{\partial f(q,\xi)}{\partial \xi} = 2J(\xi)f^2(q,\xi) - h(\xi)f(q,\xi), \qquad /31/$$

where $h(\xi)$ is an arbitrary functions of ξ. Without loss of generality the terms with $h(\xi)$ can be neglected. They correspond in fact to a renormalization of J and f(q).
Introducing the quantities

$$J'(\xi) = J(\xi)g(\xi) \qquad \text{and} \qquad f'(q,\xi) = f(g,\xi)/g(\xi) \qquad /32/$$

where $g(\xi)$ is related to $h(\xi)$ by

$$\frac{1}{g(\xi)} \frac{dg(\xi)}{d\xi} = -h(\xi), \qquad /33/$$

these quantities obey equations

$$\frac{dJ'(\xi)}{d\xi} = 2J'^3(\xi) \frac{1}{N} \sum_{q'} f'^2(q',\xi), \qquad /34/$$

$$\frac{\partial f'(q,\xi)}{\partial \xi} = 2J'(\xi)f'^2(q,\xi), \qquad /35/$$

and $\gamma_1(q) = J'f'(q)$. So we will work with eq. /34/ and /35/, neglecting the prime on J and f(q).

These equations can be written in a more convenient form if a new function $\alpha(\xi)$ is introduced by the definition

$$\frac{d\alpha(\xi)}{d\xi} = -2J(\xi). \qquad /36/$$

Then we get from eqs. /34/ and /35/

$$\frac{dJ(\alpha)}{d\alpha} = -J^2(\alpha)\frac{1}{N}\sum_{q'} f^2(q';\alpha), \qquad /37/$$

$$\frac{\partial f(q,\alpha)}{\partial \alpha} = -f^2(q,\alpha). \qquad /38/$$

The solution of eq. /38/ can be written in the form

$$f(q,\alpha) = \frac{1}{w(q)+\alpha(\xi)}, \qquad /39/$$

where $w(q)$ is independent of ξ and is determined by the bare couplings. Let us suppose that the dimensionless bare coupling $\gamma_1^o(q)$ has a minimum at $q=Q$ /if there are several local minima we take the absolute minimum/ and denote the minimum value by J_o ($J_o = \gamma_1^o(Q)$). Then the bare coupling can be written as

$$\gamma_1^o(q) = \frac{J_o}{w(q)+1} \qquad /40/$$

It follows from the above choice of J_o that $w(Q)=0$ and $w(q)\geq 0$ around $q=Q$. Comparing this form with eq. /39/ and keeping in mind that the renormalization procedure starts from the bare value of the coupling at $\xi=0$, the boundary conditions for the solution of the Lie equations are given as

$$J(\xi=0) = J_o, \qquad \alpha(\xi=0) = 1. \qquad /41/$$

Starting from any bare coupling, i.e. knowing J_o and $w(q)$ and using eq. /39/ for $f(q,\alpha)$, eq. /37/ can be solved to obtain J in terms of α. Once the dependence of J on α is known, the dependence of α on ξ and thereby the dependence of J on ξ can also be obtained from eq. /36/.

Looking at eqs. /36/ - /38/ the following general remarks can be made. If J_o is positive, which implies that $(w(q)+1)^{-1}$ is also positive, since J_o is the minimum value of $\gamma_1^o(q)$, the renormalization procedure leads to a decrease of both J and $f(q)$ as a function of ξ as ξ goes from zero to negative values /as the temperature decreases/. Our assumption that the coupling constant of the backward scattering is strongly enhanced is not valid.

In this case, however, the parquet approximation gives good results, since the coupling is small. This situation is not interesting physically since at low temperatures it leads to a decoupling of the chains.

The physically more interesting case is when $J_o < 0$. We have to distinguish two regions in q space, the region where $(w(q)+1)^{-1} > 0$, i.e. the region around the minimum at $q=Q$, and the other region where $(w(q)+1)^{-1}<0$. In the first region the renormalization procedure leads to an increase of $f(q,\xi)$ with decreasing temperature, while in the other region $f(q,\xi)$ decreases in absolute value. Since J itself decreases in absolute value during the renormalization, a divergence in the invariant coupling or a strong enhancement, which may lead to a phase transition, can only be expected for q near to Q.

We will now consider some special cases for the bare couplings, i.e. for w(q). A mathematically particularly simple and physically meaningful form for w(q) is

$$w(q) = \frac{q^2}{\kappa^2} \qquad /42/$$

This corresponds to a bare coupling which decreases with distance /for a one-dimensional array of the chains this falling-off is exponential/. The solution for $J(\alpha)$ is

$$J(\alpha) = \begin{cases} \dfrac{J_o}{1+J_o I_d \left(\frac{d}{2} - 1\right)^{-1} \left(\alpha^{d/2-1} - 1\right)} & \text{if } d\neq 2 \\ \dfrac{J_o}{1+J_o I_2 \ell n\alpha} & \text{if } d=2 \end{cases} \qquad /43/$$

where

$$I_d = \left(\frac{\kappa a}{2\pi}\right)^d \int_o^\infty \frac{d^d x}{(x^2+1)^2} \qquad /44/$$

d is the dimensionality of the lattice formed by the chains /d=2 for a real three-dimensional system/ and a is the lattice constant.

The invariant coupling $\gamma_1(q) = Jf(q)$ can be obtained from eqs. /43/ and /39/ in terms of α. In order to determine the dependence on ξ we use eq. /36/. The relation between α and ξ is obtained in the form

$$\xi = \begin{cases} -\alpha \frac{1}{2}\left[\frac{1}{J_o} - \frac{I_d \alpha^{d/2-1}}{d/2} + I_d \frac{\alpha^{d/2-1}-1}{d/2-1}\right] + \xi_c & \text{for } d\neq 2 \\ -\alpha \frac{1}{2}\left[\frac{1}{J_o} - I_2 + I_2 \ell n \alpha\right] + \xi_c & \text{for } d=2 \end{cases} \qquad /45/$$

where

$$\xi_c = \frac{1}{2}\left(\frac{1}{J_o} - \frac{2}{d} I_d\right). \qquad /46/$$

For $J_o<0$ the renormalization leads to a decrease of α from $\alpha_o=1$ to $\alpha=0$, which is reached at $\xi=\xi_c$. The corresponding temperature is denoted by T_c,

$$\xi_c = \ell n(Tc/\omega D) \qquad T_c = \omega_D \exp \frac{1}{2}\left(\frac{1}{J_o} - \frac{2}{d} I_d\right). \qquad /47/$$

T_c is the transition temperature in this model. When α reaches zero, $f(q)$ becomes infinity at $q=0$. It is true that at the same time J decreases in absolute value, but the invariant coupling $\gamma_1(q) = Jf(q)$ itself is divergent at $\xi=\xi_c$. This is easily seen if we write eq. /43/ for J in terms of $\xi-\xi_c$ in the form

$$J = \frac{\alpha}{-2(\xi-\xi_c)+I_d \frac{2}{d} \alpha^{d/2}}, \qquad /48/$$

and therefore

$$\gamma_1(q) = \frac{1}{-2(\xi-\xi_c)+I_d \frac{2}{d} \alpha^{d/2}} \cdot \frac{1}{\frac{w(q)}{\alpha}+1}. \qquad /49/$$

From eq. /45/ it is seen that for $\alpha << 1$

$$\xi-\xi_c \propto \begin{cases} \alpha & \text{for } d > 2 \\ \alpha \ell n \alpha & \text{for } d = 2 \\ \alpha^{d/2} & \text{for } d < 2 \end{cases} \qquad /50/$$

and therefore $\alpha^{d/2}$ goes linearly with $\xi-\xi_c$ or faster and near ξ_c we can write

$$\gamma_1(q) \propto \frac{1}{2(\xi_c-\xi)} \cdot \frac{1}{\frac{q^2}{\kappa^2(\xi)}+1}, \qquad /51/$$

where $\kappa^2(\xi) = \kappa^2\alpha(\xi)$. This shows that the invariant coupling diverges at T_c and its range, which is given by the inverse of $\kappa(\xi)$, goes to infinity.

The behaviour of the model for d=1 is shown in Fig. 3. Starting from an exponentially falling-off attractive interchain coupling, the renormalization leads to an increase of the range of the coupling and finally at T_c the effective interchain coupling becomes independent of the distance between the chains. It should be mentioned here that γ_{1ij} itself does not diverge, it is only $\gamma_1(q=0) = \sum_i \gamma_{1ij}$ which is divergent.

This tendency manifests itself under more general conditions as well. As it was mentioned already, an enhancement of the invariant coupling can be expected in that region of q space where $\gamma_1(q)$ is negative. It follows from our definition of J_o and $w(q)$ that in this region $w(q) \geq 0$. $\gamma_1^o(q)$ has a minimum at q=Q and here $w(Q)=0$. In the same way as above it is easily seen that α is renormalized down to zero and the invariant coupling diverges at q=Q when the temperature is scaled down to T_c.

To illustrate this behaviour, we consider now the cases where the bare couplings are attractive or repulsive nearest neighbour couplings. The intrachain coupling is neglected for the moment. Fig. 4. and 5. show the bare couplings in real space and in Fourier space for d=1 and also schematically the invariant coupling near $\xi=\xi_c$. For attractive nearest neighbour coupling the minimum of $\gamma_1^o(q)$ is at q=0. The renormalization leads to a long range attractive coupling between the chains. On the other hand in the case of repulsive nearest neighbour coupling the minimum of $\gamma_1^o(q)$ is at the zone boundary $Q = \pm \pi/a$. The divergence of the invariant coupling appears at this value of q, and this leads to a long range alternating coupling between the chains.

The intrachain coupling can be taken into account very easily since it only means an additional constant in $\gamma_1^o(q)$. If the intrachain coupling is repulsive ($\gamma_{1ii}^o > 0$) and larger than the sum of the first neighbour interchain couplings, $\gamma_1^o(q)$ is always positive and the invariant coupling decreases. For attractive intrachain coupling the above described consideration holds and the type of ordering is determined by the nearest neighbour coupling.

Knowing the behaviour of the invariant couplings we turn to the study of the response functions. Near the transition point $1/N \sum_q \gamma_1(q)$ can be neglected compared to $\gamma_1(q)$ near q=Q and compared to $1/N \sum_q \gamma_1^2(q)$. We see therefore from eqs. /27/ - /29/ that the behaviour of $\chi(Q)$ and Λ will be similar, while N(Q) will behave differently.

It follows from these equations and from eq. /30/ that

$$\frac{\partial \ell n \bar{\chi}}{\partial J} = \frac{\partial \ell n \bar{\Delta}}{\partial J} = \frac{1}{J} \qquad /52/$$

from which we get

$$\bar{\chi}(\xi) \propto \bar{\Delta}(\xi) \propto J(\xi) \qquad /53/$$

It was shown that the renormalization leads to a decrease of $J(\xi)$ when ξ goes to ξ_c, so $\bar{\chi}$ and $\bar{\Delta}$ are not singular. The response function $\chi(\xi)$ and $\Delta(\xi)$ can also be obtained using the definitions of $\bar{\chi}(\xi)$ and $\bar{\Delta}(\xi)$ in eqs. /24/ and /25/. Combining the equations

$$\frac{\partial \chi(\xi)}{\partial \xi} \propto \frac{\partial \Delta(\xi)}{\partial \xi} \propto J(\xi) \qquad /54/$$

and eq. /36/, we get

$$\chi(\xi) \propto \Delta(\xi) \propto \alpha(\xi) \qquad /55/$$

Since $\alpha(\xi)$ is renormalized down to zero when ξ goes to ξ_c $\chi(\xi)$ and $\Delta(\xi)$ are non-singular, indicating that the transition is not of magnetic or superconducting type.

On the other hand the density-density response function obeys the equation

$$\frac{\partial \ell n \bar{N}(q,\xi)}{\partial \xi} = 4J(\xi)f(q,\xi) + 2J^2(\xi)\frac{1}{N}\sum_{q'} f^2(q',\xi) + \ldots \qquad /56/$$

$$= \frac{2}{f(q,\xi)} \frac{\partial f(q,\xi)}{\partial \xi} + \frac{1}{J(\xi)} \frac{\partial J(\xi)}{\partial \xi} + \ldots = \frac{\partial}{\partial \xi} \ell n J(\xi) f^2(q,\xi) + \ldots$$

From this we get

$$\bar{N}(q,\xi) \propto J(\xi) f^2(q,\xi) \qquad /57/$$

and the response function itself is

$$N(q,\xi) \propto f(q,\xi) \qquad /58/$$

As we have seen $f(q,\xi)$ diverges at $q=Q$ and this is an indication of the formation of charge density waves with perpendicular momentum Q, the parallel momentum component being $2K_F$. If $Q=0$, the charge density waves on the neighbouring chains will be in phase, while if Q is at the zone boundary, the charge density waves are in opposite phase on the neighbouring chains.

5. DISCUSSION AND CONCLUSIONS

In this paper we have investigated the behaviour of a system of weakly coupled linear chains. It is supposed that the coupling is due to electron--electron interaction. The interchain hopping of electrons is not allowed and therefore the electronic spectrum is strictly one-dimensional. The second order renormalization of the renormalization group approach is applied to study the temperature dependence of the invariant coupling and the character of the three-dimensional ordering. The most important results of the calculation is the possibility of the occurrence of a charge density wave state. According to our results no magnetic or superconducting ordering is allowed in this model.

The physical picture which arises from our calculation is as follows. Starting from a short range interchain coupling far from the transition point, an effective coupling develops between the chains as the temperature is lowered and the range of the effective interaction increases. At the transition temperature all the chains are equally coupled independent of the distance between them, i.e. the range of the interaction diverges at T_c. In our approximation the interchain coupling itself does not diverge, it is only its Fourier transform which diverges at a given value of the perpendicular momentum.

Instead of giving a full solution of the second order renormalization equations, a self-consistent solution has been found supposing that the backward scattering is strongly enhanced due to the renormalization and the small momentum transfer interaction remains small relative to the backward scattering. Due to this assumption only the asymptotic behaviour of the system near T_c can be studied. The exact trajectories in the space of couplings cannot be determined to a given set of bare couplings. Nevertheless the following general conclusions can be drawn. Supposing first neighbour coupling only and assuming that the interchain coupling with large momentum transfer is stronger than the small momentum transfer interaction, an attractive coupling between the chains leads to the formation of charge density waves which are in the same phase for all the chains. On the other hand a repulsive interaction gives rise to a situation where the phase of the charge density waves on the neighbouring chains are shifted by π. A strong interchain repulsion can modify the situation and above a critical strength no phase transition will occur.

The numerical values for the temperature dependence of the coherence length perpendicular to the chains and of the response functions should not be taken seriously, since the dimensionless coupling constants tend to unity and the higher order corrections are not negligible. We feel that this self-consistent solution is physically acceptable and clearly shows how the ordered phase sets in.

Our results can be compared to the calculations of Gorkov and Dzyaloshinsky[18] and that of Klemm and Gutfreund[19] who have treated the same model but in a different approximation. Gorkov and Dzyaloshinsky have used the parquet approximation, which corresponds to taking the second order terms only in eqs. /16/ and /17/. They have found that the interchain coupling is a relevant perturbation and the one-dimensional fixed point solution is not stable. Their solution for the system of weakly coupled chains shows up a Peierls type transition if no interchain hopping is taken into account. With interchain hopping superconducting state is also possible. Similar result has been obtained by Klemm and Gutfreund. They have treated the intrachain coupling exactly and the interchain coupling as small perturbation in mean field approximation. The conclusion is that the nearest neighbour interchain electron-electron scattering gives rise to a phase transition of the charge density wave type. Superconducting type phase transition can only occur if hopping is also considered. These results are in agreement with our finding, that in our approximation as well, only charge density wave state can appear if the motion of electrons is restricted to a single chain.

Our starting Hamiltonian is a straightforward generalization of the strictly one-dimensional Fermi gas model[7]. Another generalization has been treated by Menyhárd[21] assigning a new component index to the electons. This component index can be interpreted as a chain index thus it is of some interest to compare the results. In Ref. 21 the interaction between electrons having different component indices is constant and of $o(1/n)$, where n is the number of components. In our language this means that the interchain coupling is of infinite range. For temperatures near T_c a procedure which starts from a bare coupling which is already of infinite range is justified. In agreement with our results, taking only the type of interaction shown on Fig. 1.b of Ref. 21 a CDW type phase transition is obtained in the $n\to\infty$ limit.

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FIGURE CAPTIONS

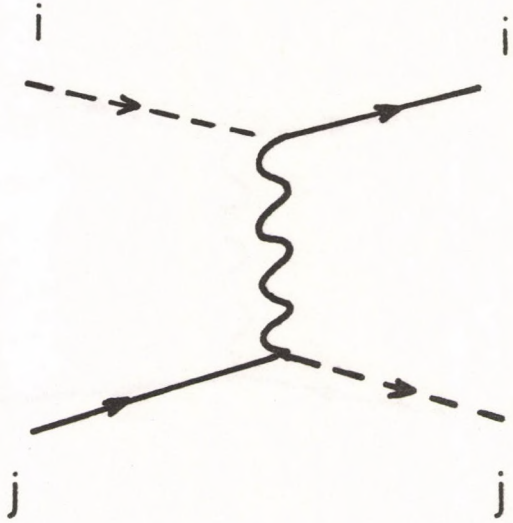
Fig. 1. Diagrammatic representation of the large momentum transfer g_{ij} and small momentum transfer g_{2ij} interactions. The dotted /solid/ lines correspond to electrons with momentum around $-k_F$ $(+k_F)$.

Fig. 2. Diagrammatic representation of interchain hopping and Umklapp processes.

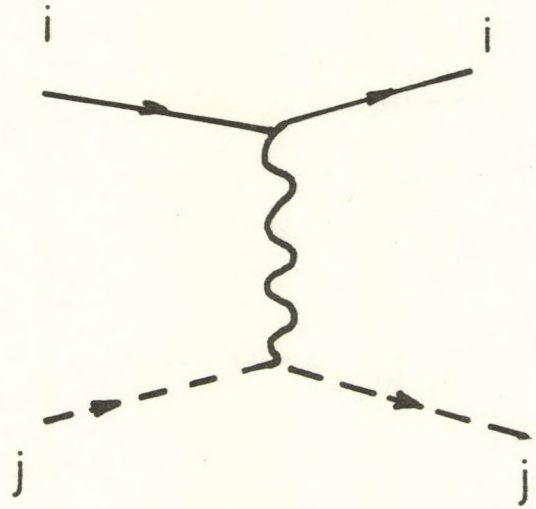
Fig. 3. Schematic representation of the bare and renormalized couplings in real and Fourier space for exponentially decreasing interchain interaction.

Fig. 4. Schematic representation of the bare and renormalized couplings in real and Fourier space for attractive nearest neighbour interaction.

Fig. 5. Schematic representation of the bare and renormalized couplings in real and Fourier space for repulsive nearest neighbour interaction.

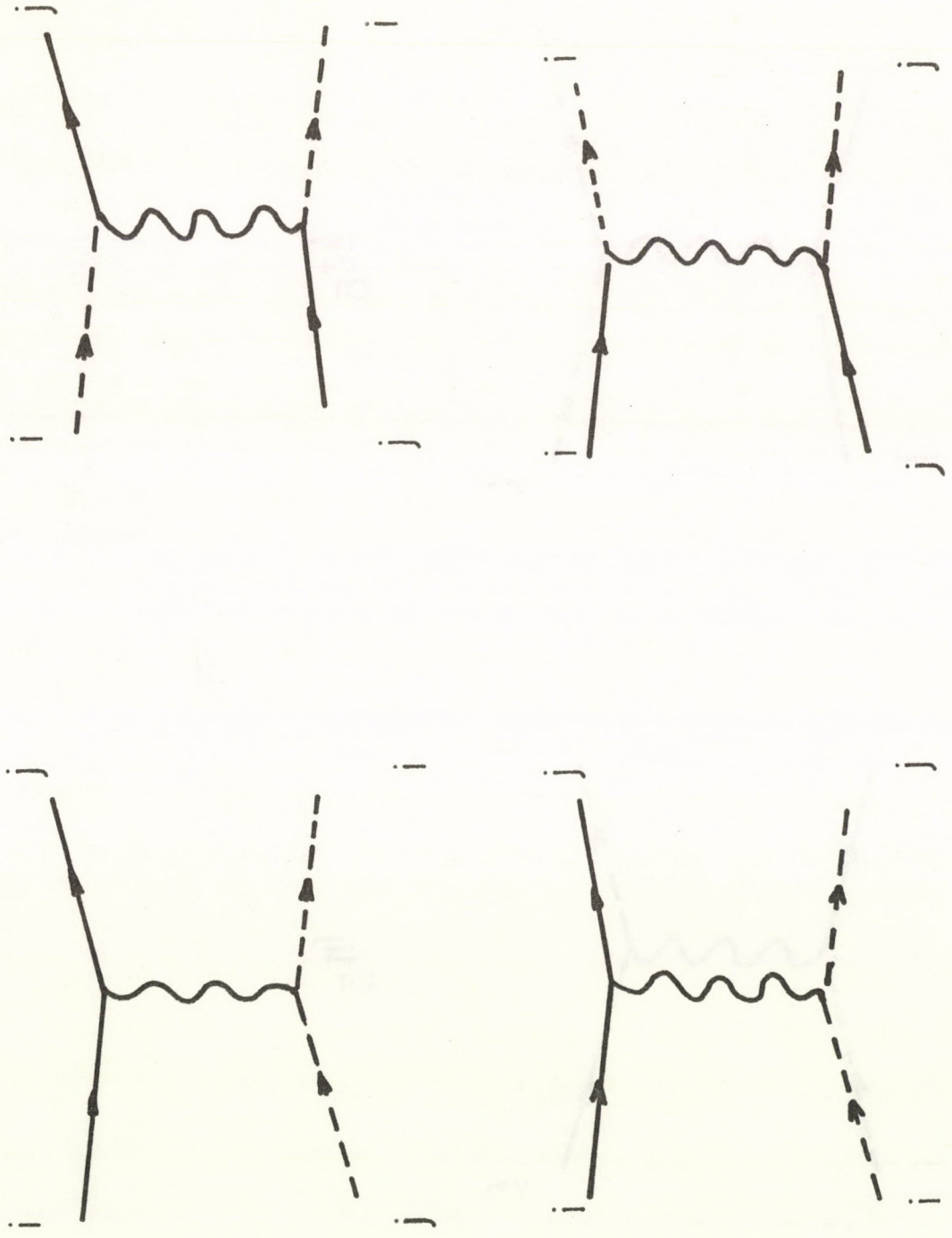


g_{1ij}



g_{2ij}

Fig. 1.



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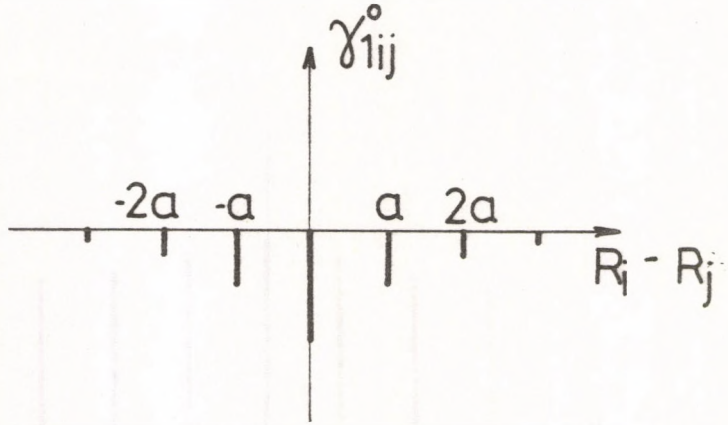
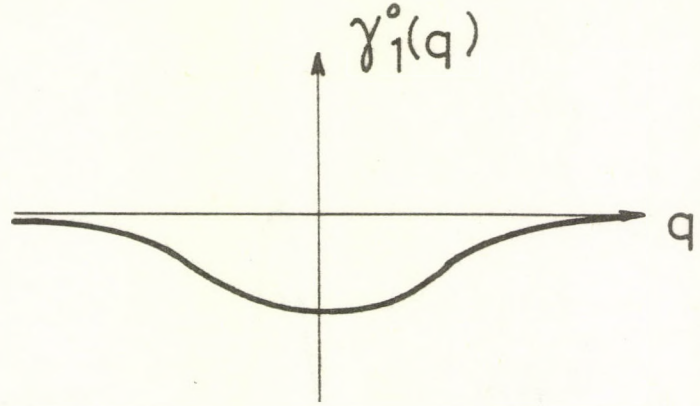
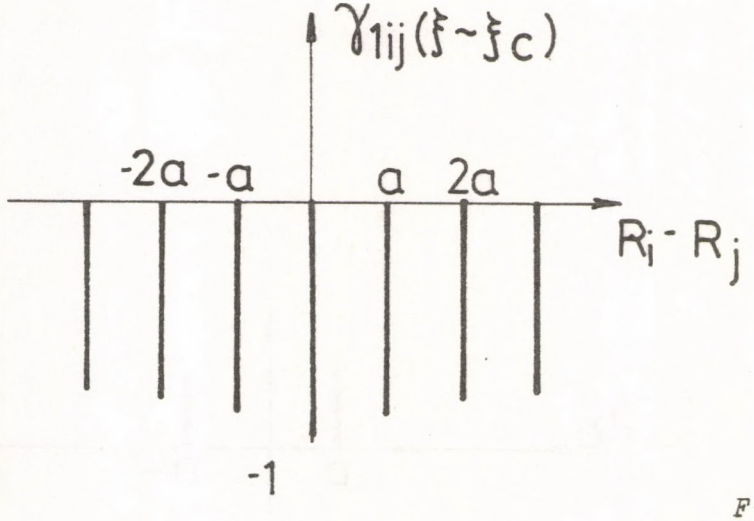

γ°_{1ij}
-2a
-a
a
2a
$R_i - R_j$


$\gamma^{\circ}_{1}(q)$
q


$\gamma_{1ij}(\xi \sim \xi_c)$
-2a
-a
a
2a
$R_i - R_j$
-1

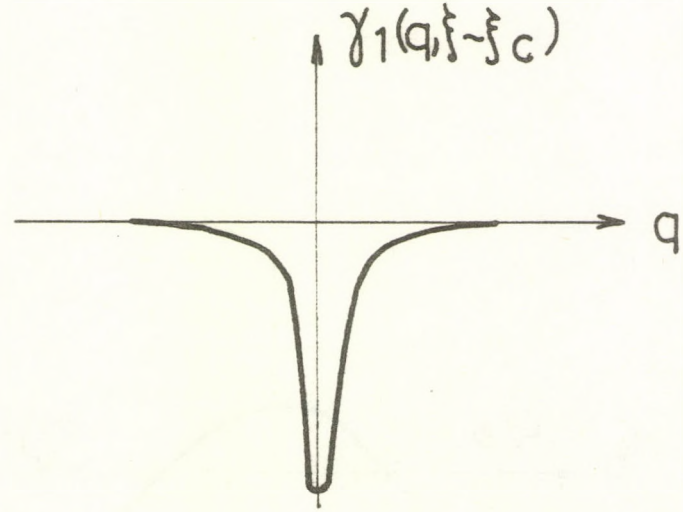
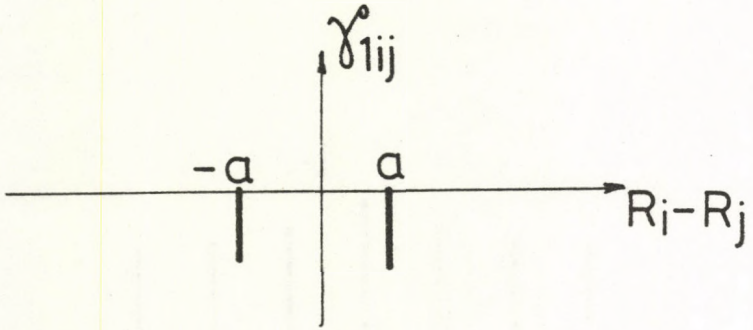
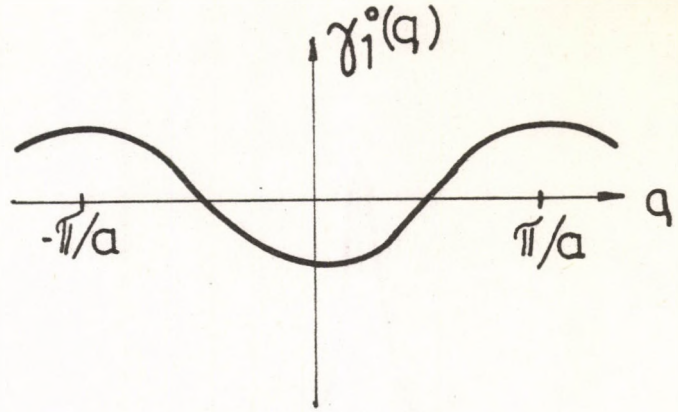
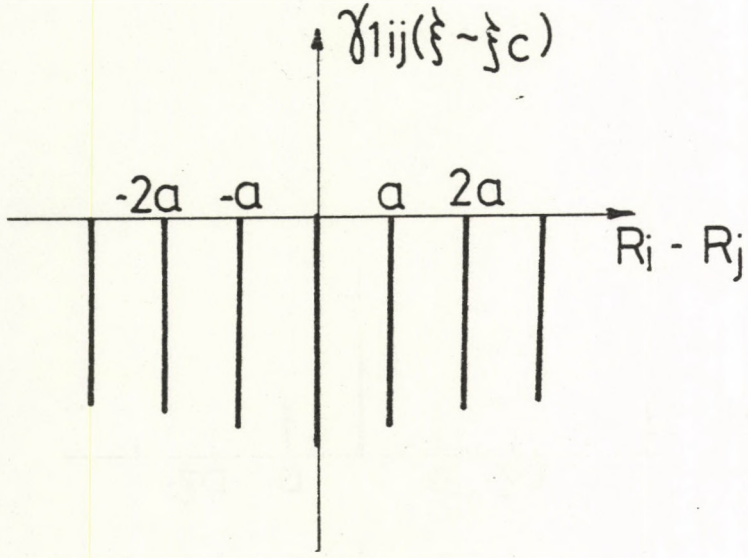

$\gamma_1(q, \xi \sim \xi_c)$
q

Fig. 3.


γ_{1ij}
-a
a
$R_i - R_j$


$\gamma_1^\circ(q)$
$-\pi/a$
π/a
q


$\gamma_{1ij}(\xi \sim \xi_c)$
-2a
-a
a
2a
$R_i - R_j$

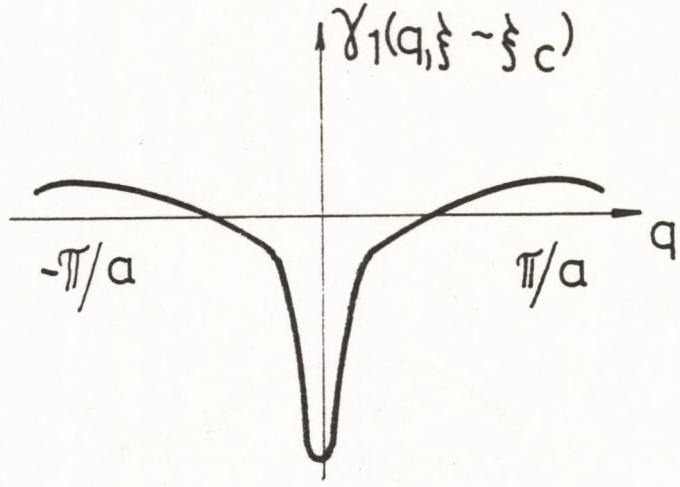

$\gamma_1(q, \xi \sim \xi_c)$
$-\pi/a$
π/a
q

Fig. 4.

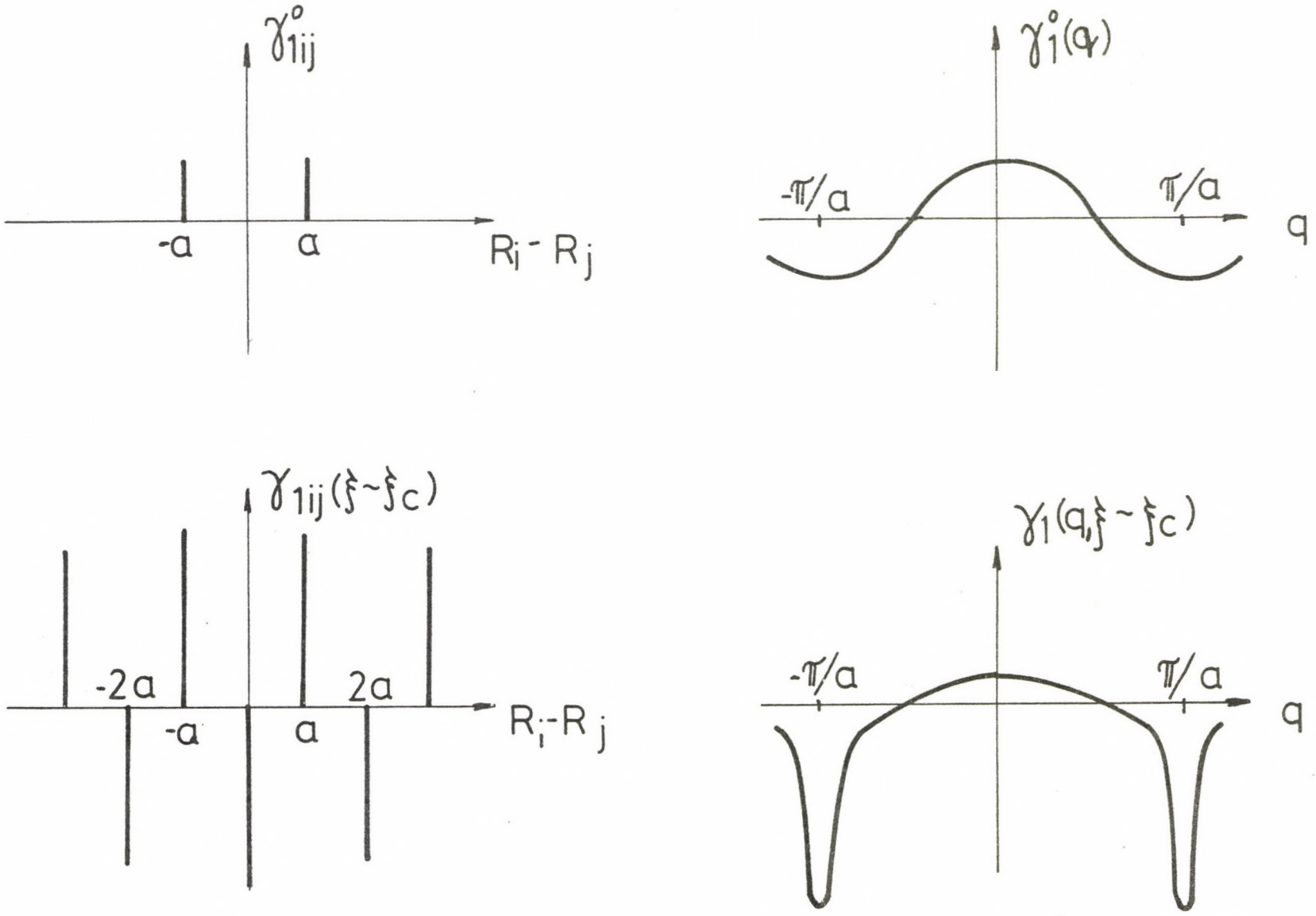
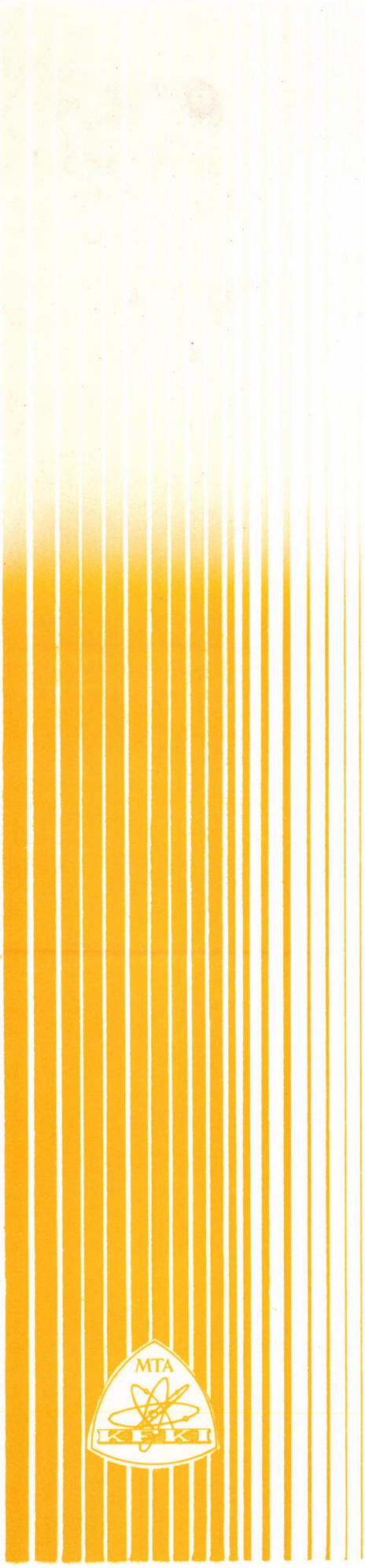

γ°_{1ij}
-a
a
$R_i - R_j$
$\gamma^{\circ}_1(q)$
$-\pi/a$
π/a
q
$\gamma_{1ij}(\xi \sim \xi_c)$
-2a
-a
a
2a
$R_i - R_j$
$\gamma_1(q, \xi \sim \xi_c)$
$-\pi/a$
π/a
q

Fig. 5.



Kiadja a Központi Fizikai Kutató Intézet
Felelős kiadó: Siklós Tivadar, a KFKI Szilárdtestkutatási Tudományos Tanácsának szekcióelnöke
Szakmai lektor: Menyhárd Nóra
Nyelvi lektor: Menyhárd Nóra
Példányszám: 290 Törzsszám: 76-17
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