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WALL FIELD INTERPRETATION OF MAGNETIC BUBBLE BEHAVIOUR

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Резюме

Расчет статических и динамических параметров магнитных цилиндрических доменов является довольно трудным, вследствие сложно выражаемой энергии размагничивания. Во многих случаях расчеты могут быть облегчены, если известны величины напряженности поля для случая доменов, форма которых определяется зависимостью $r=r_o + \sum_{n=1} r_n \cdot \cos(n\varphi)$, и записанные с их помощью уравнения равновесия, результатами решения которых являются желаемые параметры пузырьков(доменов) $-r_n, v$ и т.д.

KIVONAT

Hengeralaku mágneses domének statikus és dinamikus paramétereinek számitása igen nehézkes a lemágnesező energia bonyolult kifejezése következtében. Számos esetben a számitások könnyebben elvégezhetők a falra ható térerősségek ismeretében. Az $r = r_o + \sum_{n=1} r_n \cdot \cos(n\varphi)$ összefüggés által meghatározott alaku doménok esetére ismertetjük a térerősségek kifejezéseit és a velük felirható egyensulyi egyenleteket, melyek a kivánt buborékparamétereket - r_n, v stb. - eredményezik.

ABSTRACT

The computation of the static and dynamic parameters of cylindrical magnetic domains is rather difficult due to the complex expression of the demagnetising energy. The fields acting on the wall of a cylindrical magnetic domain are discussed for the domain shape defined by $r = r_0 + \sum_{n=1} r_n \cdot \cos n\varphi$. Results have shown that both the demagnetising field and the wall energy field vary by the function $\cos(n\varphi)$ along the perimeter of the wall. Using the expressions of the fields acting on the domain wall, the stability condition is determined by the balance of the fields, which yields such parameters as r_n, v, etc. Results are presented in a number of cases using this wall field formulation method.

INTRODUCTION

Since Thiele[1],[2] introduced his theory of cylindrical magnetic domains there are now essentially two methods for describing the behaviour of "magnetic bubbles" :

Thiele's method where the bubble endeavours to reach the minimum energy state and after computing every energy term the minimalization of the total energy results in such parameters as r_o, r_n, v, etc. ;

the second method being the balance of the forces or fields acting on the wall yields the main parameters by which a bubble is characterised.

This latter method introduced by Bobeck[3] is restricted to the circular cylindrical case only. In the case of general cylindrical shape this method has only been worked out for the computer study of bubble domains[4].

Assuming small deviations from the circular cylindrical shape we can derive the analytical forms of the wall fields thereby enabling to be calculated the bubble parameters without the need for a computer. Not only is this computing method descriptive, it is sometimes simple than the first one.

In the following we show how the wall fields can be determined and we apply them in order to evaluate some characteristic bubble parameters.

THE COMPUTATION OF WALL FIELDS

The following fields must be taken into account when computing the stability of a bubble in an infinite platelet:

H_e the external magnetic field which is directed parallel to the wall and tends to decrease the volume of the bubble

H_d the demagnetising field originating from the magnetic free charges on the surface of the platelet. Its parallel component to the wall that tends to increase the volume of the bubble must be taken into consideration.

H_w the wall energy field which tends to decrease the bubble volume at every point normal to the wall.

H_v the viscose damping / force / field opposing the normal movement of the wall.

These fields are computed by the following formulas:

$$H_e = H_z$$

where H_z is the external magnetic field component lying in the plane of the wall normal to the surface.

$$H_d(\alpha) = \frac{2M_s}{h} \int_0^{2\pi} \left(\sqrt{y^2 + h^2} - y \right) d\varphi \qquad (1)$$

where M_s is the saturation magnetisation of the magnetic platelet

h is the thickness of the platelet

α the independent variable of the cylindrical coordinate system

y, φ the meanings of these variables are shown in Fig. 1

$$H_w(\alpha) = \frac{2\pi \cdot M_s \cdot l}{r} = \frac{\sigma_w}{2 \cdot r \cdot M_s} \qquad (2)$$

where σ_w the wall energy density

l the characteristic length / $l = \sigma_w / 4 M_s^2$ /

r the radius of curvature of the wall at the angle α

$$H_v = \frac{1}{\mu} v_n \qquad (3)$$

where μ the mobility of a plane wall

v_n the normal velocity component of the moving plane wall

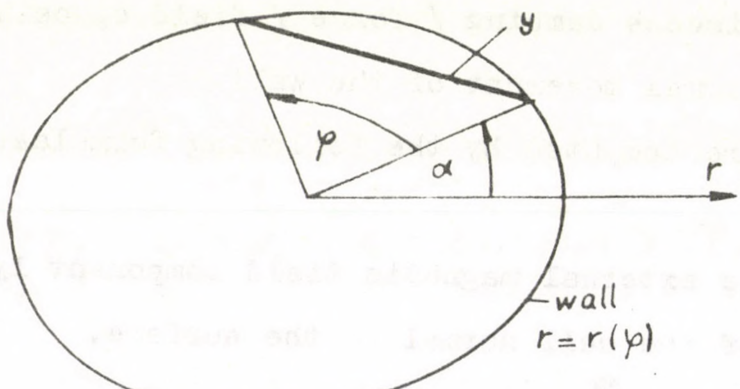

Fig. 1
Interpretation of the parameters used in the expression of the demagnetising field

Among these fields the demagnetising field(1) gives rise to difficulties because the integral in (1) is not elementary even in the case of a circular cylindrical shape. To solve this problem we supposed small perturbations from the circular cylindrical shape, in accordance with

Thiele [1] , and the shape as described in the following way :

$$r = r_o + \sum_{n=1} r_n \cdot \cos n\varphi \tag{4}$$

Substituting (4) into (1) and (2) we get the formulas:

$$H_w = \frac{\sigma_w}{2 M_s \cdot r_o \left[1 - \sum (n^2-1) \frac{r_n}{r_o} \cos n\varphi\right]} \tag{5}$$

$$H_d = \frac{2 M_s}{h} \int_0^{2\pi} \left(\sqrt{y_o^2 + A^2 + h^2} - y_o \cdot \sqrt{1 + \frac{A^2}{y_o^2}} \right) d\varphi \tag{6}$$

where

$$y_o = \sqrt{2 r_o^2 (1 - \cos\varphi)}$$

$$A^2 = \sum_{n=1} 2 r_o r_n \left[\cos n\alpha + \cos(n\alpha + n\varphi) - \cos \cdot n(\alpha+\varphi) \cos\varphi - \cos n\alpha \cos\varphi\right] \tag{7}$$

These formulas are then transformed so that the fields existing only in the noncircular case would be separated. We thereby obtain

$$H_w = H_{wo} + \Delta H_w = \frac{\sigma_w}{2 \cdot M_s \cdot r_o} + \frac{\sigma_w}{2 \cdot M_s \cdot r_o} \sum_{n=1} (n^2-1) \frac{r_n}{r_o} \cos n\varphi \tag{8}$$

$$H_d = H_{do} + \Delta H_d = \frac{2 M_s}{h} \int_0^{2\pi} \left(\sqrt{y_o^2 + h^2} - y_o \right) d\varphi + \sum_{n=1} \sqrt{2} \frac{M_s}{h} r_n \cdot \cos(n\alpha) \cdot B_n \tag{9}$$

Expressions (8) and (9) show that in perturbing the circular shape by $r_n \cos(n\varphi)$, the additional wall fields vary also by the function $\cos(n\varphi)$.

The values of some B_n which depend only on r_o and h are plotted in Fig. 2 . It can be seen in the figure that there is no large difference among the B_n values in the range of the optimum device conditions / $0{,}5 < a < 0{,}7$ /.

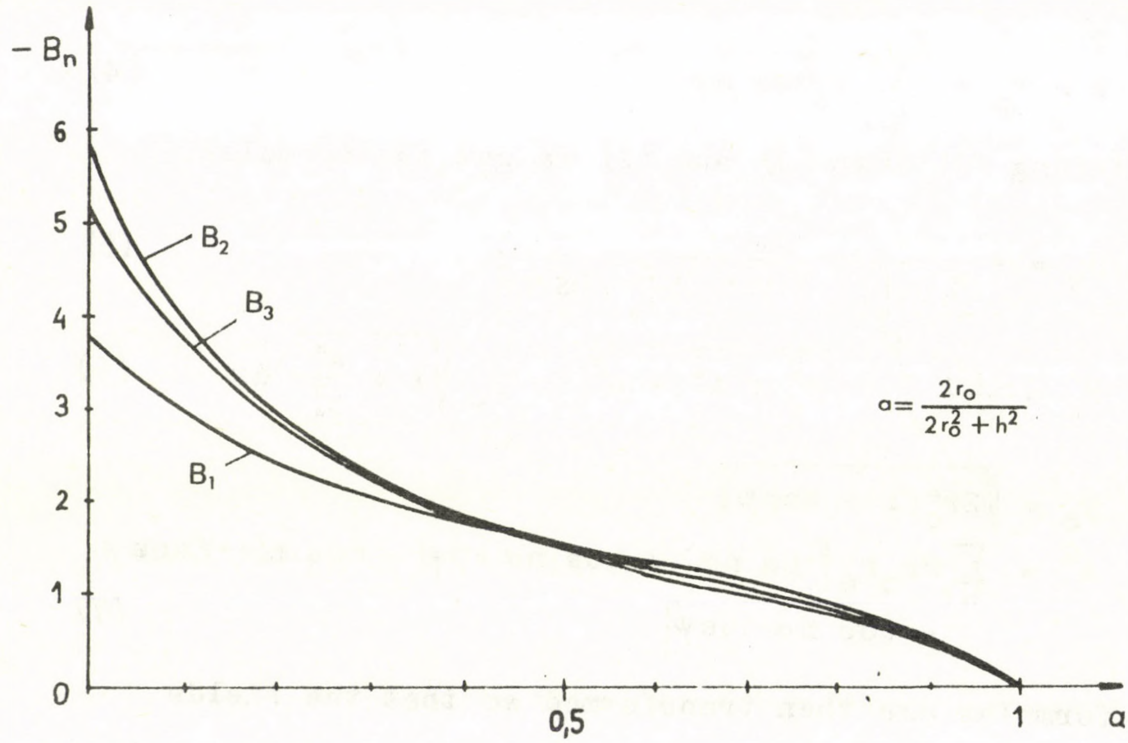

Fig. 2
The dependence of B_n versus a

STABILITY CALCULATIONS OF BUBBLE WITH MEANS OF WALL FIELDS

Disposing of the fields acting on the wall in the form of Fourier expansions, the stability condition can be formulated: The fields acting on the wall of the bubble must result in a zero resultant field. This formulation is valid for the moving case too, if the damping field is also taken into account. This formulation of the stability has the following advantages :

1. r_o is determined by the fields independent of φ
2. translation / velocity / is determined by the fields depending on cos φ

3. bubble deformation is determined by the fields depending on $\cos 2\varphi$, $\cos 3\varphi$, $\cos 4\varphi$ .....

The first statement is trivial, the validity of the others can be proved by integrating the fields along the perimeter into any direction :

$$\int_{\alpha}^{\alpha+2\pi} \cos n\varphi \cos\varphi \, d\varphi \begin{cases} \neq 0 & \text{if } n=1 \quad \text{translation} \\ = 0 & \text{if } n\neq 1 \quad \text{deformation} \end{cases} \tag{10}$$

Case a. Homogeneous external magnetic field, zero coercivity and isotropic wall energy density.

In this case bubbles with noncircular shape cannot exist as shown by the sign of H_d and H_w. The diameter of the bubble can be computed using the expressions of H_{do}, H_{wo}, H_e, that is from eguation (11)

$$H_{do} + H_{wo} + H_e = 0 \tag{11}$$

or by equation (12) derived by Thiele [1]

$$\frac{\ell}{h} + \frac{d}{h} \cdot \frac{H_e}{4 M_s} - F\left(\frac{d}{h}\right) = 0 \tag{12}$$

Case b. Homogeneous magnetic field, nonzero coercivity and isotropic wall energy density.

We will not examine the trivial case when there is no shape deformation, and the bubble changes only its diameter. The stability condition for the deformed state:

$$H_c \geqslant \sum_n \left(\sqrt{2} \frac{M_s}{h} \cdot r_n \cdot B_n + (n^2 - 1) \frac{\sigma_w}{2 M_s r_o} \cdot \frac{r_n}{r_o} \right) \tag{13}$$

This expression shows that the deformation / in the sense of the ratio of the axes / is the largest where n=2. For this case from (13)

$$\frac{\Delta r_2}{r_o} \leq \frac{H_c}{4\pi M_s} \cdot \frac{1}{\frac{\sqrt{2}}{4\pi} \cdot \frac{r_o}{h} \cdot B_2 + \frac{3}{2} \frac{\ell}{r_o}} \tag{14}$$

Case c. Homogeneous external magnetic field gradient, zero coercivity .

There are two forces or fields acting on the moving bubble if we ignore the fields of the static stability state / H_e, H_{wo}, H_{do} /. The field originated from the field gradient and the viscose damping field have the following dependence along the perimeter of the wall:

$$H_{grad} = H' \cdot r_o \cos\varphi$$

$$H_v = \frac{1}{\mu} v \cdot \cos\varphi$$

We see that there is no field which tends to deform the bubble. From the stability condition

$$H_{grad} = H_v$$

we get

$$v = \frac{1}{2} \cdot H' \cdot 2r_o \tag{15}$$

Case d. Homogeneous external magnetic field gradient, nonzero coercivity.

The fields acting on the bubble are :

$$H_{grad} = H' \cdot r_o \cdot \cos\varphi$$

$$H_v = \frac{1}{\mu}\cdot v\cdot\cos\varphi$$

$$H_c = \begin{cases} -H_c & \text{if } -\frac{\pi}{2}<\varphi<\frac{\pi}{2} \\ +H_c & \text{if } -\frac{\pi}{2}>\varphi>\frac{\pi}{2} \end{cases}$$

Assuming the bubble is sufficiently rigid so there is no shape deformation and integrating the fields along the perimeter yields the velocity of the bubble

$$\sum\int_0^{2\pi} H_x r d\varphi = \int_0^{2\pi} H'\cdot r_o\cos^2\varphi - \frac{1}{\mu}v\cos^2\varphi - |H_c\cos\varphi|\cdot r\cdot d\varphi = 0 \qquad (16)$$

which results in

$$v = \frac{1}{2}\mu\left(H'\cdot 2\cdot r_o - \frac{8}{\pi}H_c\right) \qquad (17)$$

If we do not assume the rigidity of the bubble we get the same result. We can take H_c into consideration by its Fourier expansion

$$H_c(\varphi) = \frac{4}{\pi}H_c\left(\cos\varphi + \frac{1}{3}\cos(3\varphi) + \frac{1}{5}\cos(5\varphi) + \ldots\right. \qquad (18)$$

and (10) shows that only the $\cos\varphi$ term computing the translation condition has to be taken into consideration, so from the balance of fields

$$\sum H(\cos\varphi) = H'\cdot r_o\cos\varphi - H_c\frac{4}{\pi}\cos\varphi - \frac{1}{\mu}v\cdot\cos\varphi = 0$$

we get the same velocity as in (17).

In the case of inhomogeneous field gradient, the field gradient must be expanded into Fourier serie and the computational procedure can be done in the same way.

Case e. Homogeneous field gradient, nonzero coercovity, anisotropic wall energy density.

Della Torre [5] has shown that in the case of anisotropic wall energy density the bubble boundary has the following dependence

$$r = r_o + r_2 \cdot \cos 2\varphi \qquad (19)$$

for those conditions given in his paper.

We will show that in this case also it is easy to calculate the dynamic parameters of the bubble by means of the fields. If the field gradient is extended along the easy direction then the balance of the fields :

$$\sum \int H_x r \cdot d\varphi = \int_0^{2\pi} \Big(H' \cdot r \cdot \cos\varphi \cdot \cos(\varphi+\beta) - \frac{1}{\mu} v \cdot \cos^2(\varphi+\beta) - H_c \cdot \cos(\varphi+\beta)\Big) r \cdot d\varphi \qquad (20)$$

where β is the angle between the tangent line of the perimeter and the normal of radius r .

(20) yields

$$v = \frac{1}{2}\mu \left[H' \cdot 2 \cdot r_o - \frac{8}{\pi} H_c \left(1 - \frac{r_2}{r_o}\right)\right] \cdot \left(1 + \frac{3}{2} \cdot \frac{r_2}{r_o}\right) \qquad (21)$$

and with the same procedure for the motion along the hard direction

$$v = \frac{1}{2}\mu \left[H' \cdot 2 \cdot r_o - \frac{8}{\pi} H_c \left(1 + \frac{r_2}{r_o}\right)\right] \left(1 - \frac{3}{2} \cdot \frac{r_2}{r_o}\right) \qquad (22)$$

which results are in agreement with those of Wanas [6].

CONCLUSIONS

It is concluded that for the calculation of the static and dynamic parameters of magnetic bubbles this wall field formulation method can also be used resulting in simplicity of computation in the dynamic cases c,d,e.

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