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SELF-CONSISTENT CHARGE EXCHANGE POTENTIAL

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1. INTRODUCTION

The charge exchange part of the optical potential has been the object of a number of investigations and by now its existence can be considered well established. On the other hand it is usually assumed that the common origin of both the optical potential and the shell model potential is the self-consistent part of the nucleon-nucleon interaction. It is clear that this point of view can be extended to a charge exchange optical potential only if the shell model potential has charge exchange part as well. Otherwise self-consistency would certainly be destroyed.

The violation of self-consistency can be seen also in the following way. The existence of the charge exchange potential is usually deduced on the basis of isospin invariance, with which one can show that the potential is proportional to the difference between the neutron and proton optical potentials. In some zero'th order approximation, in which one neglects the energy dependence of the potentials, the neutron and proton potentials are those of the shell model. The latter are always assumed to have been derived in a manner in which the possibility that there is a charge exchange part of the self-consistent potential is explicitly excluded. This is always the case whenever orbitals of definite charge are involved independently of how the potentials have actually been obtained. An argument in which both the existence and nonexistence of the charge exchange potential is assumed cannot be considered self-consistent.

In the following a self-consistent scheme for the interpretation of a potential with charge exchange will be described. Though free from the aforementioned inconsistency, the scheme shares the deficiency common to all self-consistent procedures: it violates symmetries obeyed by the exact solutions of the problem. In particular the potential which can be obtained by this method does not have that simple connection with the neutron and proton potentials which is required by the isospin invariance.

2. MULTICHANNEL SELF-CONSISTENCY

In order to treat the charge exchange and charge conserving parts of the potentials on equal footing, a self-consistent procedure is needed in which all states of an isobaric multiplet appear in a symmetrical fashion. For an analogy one can refer to the theory of superconductivity, in which states with different number of particles have to be coupled to each other in a symmetrical way. This aspect of the superconductivity is especially clearly seen in the approach developed by Gorkov [1]. This theory is based on the Green function method and exploits all the Green functions which can be constructed from the states involved.

Following this line we introduce all single-particle Green functions which can be constructed from the states of a given isobaric multiplet

$$\mathcal{G}^{\tau_o}_{\sigma_1\sigma_2}(x_1t_1;x_2t_2) = -i\langle\tau_o-\sigma_1\left|\Theta\left(\psi_{\sigma_1}(x_1,t_1)\,\psi^+_{\sigma_2}(x_2,t_2)\right)\right|\tau_o-\sigma_2\rangle$$

Here Greek indices assuming values ±1/2 are used to distinguish between neutrons and protons. Space-spin coordinates are denoted by x. The symbol Θ means time ordering. The states $|T_o\rangle$ are those of an isobaric multiplet of isospin T and nucleon number A.

The Green functions $\mathcal{G}$ depend separately on both their time arguments. This is a consequence of the non-degeneracy of the states $|T_o\rangle$, which obey the Schrödinger equation

$$\mathcal{H}|T_o\rangle = \left(E - \Delta_c\cdot T_o\right)|T_o\rangle$$

As a result

$$\mathcal{G}^{\tau_o}_{\sigma_1\sigma_2}\left(x_1,t_1+\tau;\ x_2,t_2+\tau\right) = e^{i\Delta_c(\sigma_1-\sigma_2)\tau}\,\mathcal{G}^{\tau_o}_{\sigma_1\sigma_2}(x_1t_1;x_2t_2)$$

It is, therefore, convenient to use a Green function defined as

$$G^{\tau_o}_{\sigma_1\sigma_2}\left(x_1,x_2;\ t_1-t_2\right) = e^{i\Delta_c(\sigma_2-\sigma_1)t_1}\,\mathcal{G}^{\tau_o}_{\sigma_1\sigma_2}\left(x_1t_1,x_2t_2\right) \qquad /1/$$

which depends only on the difference between the time arguments.

The Green functions $\mathcal{G}$ obey the equation of motion [2]

$$i\,\frac{\partial\mathcal{G}^{\tau_o}(x_1t_1;x_2t_2)}{\partial t_1} - \Delta_c\cdot\sigma_1\,\mathcal{G}^{\tau_o}_{\sigma_1\sigma_2}\left(x_1t_1,x_2t_2\right) +$$

$$+ \sum_{\sigma_3} \int dx_3 \, \langle x_1 | H^{\tau_o}_{\sigma_1\sigma_3} | x_3 \rangle \, G^{\tau_o}_{\sigma_3\sigma_2}(x_3 t_1; x_2 t_2) = \delta_{\sigma_1\sigma_2} \, \delta(x_1, x_2) \, \delta(t_1 - t_2) -$$

$$- \sum_{\sigma_3} \int dx_3 \, \langle x_1 | V^{\tau_o}_{\sigma_1\sigma_3} | x_3 \rangle \, G^{\tau_o}_{\sigma_3\sigma_2}(x_3 t_1; x_2 t_2) +$$

$$+ i \int dx_3 \, dx_4 \, dx_5 \, \langle x_1 x_3 | U | x_5 x_4 \rangle \, K^{\tau_o}_{\sigma_1\sigma_2}(x_5, x_4, x_3, x_2; t_1, t_2) \qquad /2/$$

where the integration over x includes summation over spin.

In the derivation of this equation it has been assumed that the nuclear Hamiltonian contains the Coulomb interaction in the form $-\Delta_c . T_o$, where T_o is the third component of the isospin operator. The nucleon-nucleon potential U is the most general isospin invariant interaction containing both direct and exchange contributions [3]

$$\langle x_1 x_3 | U | x_5 x_4 \rangle = \langle x_1 x_3 | U^{o} | x_5 x_4 \rangle - \langle x_1 x_3 | U^{\tau} | x_4 x_5 \rangle$$

The two-particle Green function is defined as

$$K^{\tau_o}_{\sigma_1\sigma_2}(x_5, x_4, x_3, x_2; t_1, t_2) =$$

$$= \sum_{\sigma_3} \langle \tau_o - \sigma_1 | \Theta\Big(\psi_{\sigma_1}(x_5 t_5) \, \psi_{\sigma_3}(x_4 t_4) \, \psi^{+}_{\sigma_3}(x_3 t_3) \, \psi^{+}_{\sigma_2}(x_2, t_2)\Big) | \tau_o - \sigma_2 \rangle$$

with

$$t_1 \gg t_3 \gg t_5 \gg t_4 \qquad /3/$$

where the relation $a \gg b$ means $b = a - 0 = a^{-}$.

The single-particle Hamiltonian H contains the kinetic energy and the single-particle potential V. Equation /2/ is, therefore, independent of V, since the terms containing V cancel each other. Hence the V is completely arbitrary. The reason for including it into /2/ will be discussed later.

As a consequence of the presence of K equation /2/ is not an equation for $\mathcal{G}$. It is well known that the simplest possibility of turning it into an equation for $\mathcal{G}$ consists in approximating K by the antisymmetrized product of $\mathcal{G}$-s, which in our case is

$$K^{\tau_o}_{\sigma_1\sigma_2}(x_5,x_4,x_3,x_2;\,t_1,t_2) = \sum_{\sigma_3} \mathcal{G}^{\tau_o}_{\sigma_1\sigma_3}(x_5t_5;\,x_3t_3)\,\mathcal{G}^{\tau_o}_{\sigma_3\sigma_2}(x_4t_4;\,x_2t_2) -$$

$$-\left(\sum_{\sigma_3} \mathcal{G}^{\tau_o-\sigma_1+\sigma_3}_{\sigma_3\sigma_3}(x_4t_4;\,x_3t_3)\right)\mathcal{G}^{\tau_o}_{\sigma_1\sigma_2}(x_5t_5;\,x_2t_2)$$

This expression has to be substituted into /2/. At the same time /1/ can be used to replace $\mathcal{G}$ by G and /3/ to simplify time arguments. In this way one obtains

$$i\,\frac{\partial G^{\tau_o}_{\sigma_1\sigma_2}(x_1,x_2;t)}{\partial t} - \sum_{\sigma_3}\int dx_3\,\langle x_1|H^{\tau_o}_{\sigma_1\sigma_3}|x_3\rangle\, G^{\tau_o}_{\sigma_3\sigma_2}(x_3,x_2;t) = \delta_{\sigma_1\sigma_2}\,\delta(x_1,x_2)\,\delta(t) -$$

$$-\sum_{\sigma_3}\int dx_3\,\langle x_1|V^{\tau_o}_{\sigma_1\sigma_3}|x_3\rangle\, G^{\tau_o}_{\sigma_3\sigma_2}(x_3,x_2;t) + i\int dx_3\,dx_4\,dx_5\,\langle x_1x_3|U|x_5x_4\rangle\,\cdot$$

$$\cdot\left\{\sum_{\sigma_3} G^{\tau_o}_{\sigma_1\sigma_3}(x_5,x_3;0^-)\,G^{\tau_o}_{\sigma_3\sigma_2}(x_4,x_2;t) - \left(\sum_{\sigma_3} G^{\tau_o-\sigma_1+\sigma_3}_{\sigma_3\sigma_3}(x_1,x_3;0^-)\right) G^{\tau_o}_{\sigma_1\sigma_2}(x_5,x_2;t)\right\}$$

These equations are in principle suitable for a multichannel self-consistent calculation. However, instead of discussing them in this form, we take a further step by making on the right hand side the approximation

$$\sum_{\sigma_3} G^{\tau_o-\sigma_1+\sigma_3}_{\sigma_3\sigma_3}(x_4,x_3;0^-) \cong \sum_{\sigma_3} G^{\tau_o}_{\sigma_3\sigma_3}(x_4,x_3;0^-) \qquad /4/$$

which is probably justified in the case of a sufficiently large system. As a result the equations for the different τ_o-s decouple and for each τ_o one has

$$i\,\frac{\partial \hat{G}(x_1,x_2;t)}{\partial t} - \int dx_3\,\langle x_1|\hat{H}|x_3\rangle\,\hat{G}(x_3,x_2;t) =$$

$$= \hat{\delta}(x_1,x_2)\ \delta(t) - \int dx_3\ \langle x_1|\hat{V}|x_3\rangle\ \hat{G}(x_3,x_2;t) +$$

$$+ i\int dx_3\ dx_4\ dx_5\ \langle x_1x_3|U|x_5x_4\rangle \left\{ \hat{G}(x_5,x_3;0^-)\ \hat{G}(x_4,x_2;t) - \right.$$

$$\left. - \left(\mathrm{Sp}\ \hat{G}(x_4,x_3;0^-)\right) \cdot \hat{G}\left(x_5,x_2;t\right) \right\} \qquad /5/$$

where $\hat{G}$ etc. are 2x2 matrices and the index τ_0 has been omitted.

The single particle Hamiltonian in /5/ is understood to contain, besides the kinetic and potential energies, the term

$$-\alpha - \underline{\beta}\ \hat{\underline{t}} \qquad /6/$$

where the components of $\hat{\underline{t}}$ are the single-particle isospin operators. The constant α will be used to insure the correct nucleon number A, while the constant isovector $\underline{\beta}$ serves to fix the matrix elements of the isospin operator. The corresponding subsidiary conditions are easily seen to be

$$-\frac{i}{2}\left\{\int dx\ G_{\frac{1}{2}\frac{1}{2}}(x,x;0^-) - \int dx\ G_{-\frac{1}{2},-\frac{1}{2}}(x,x;0^-)\right\} = \tau_0 \cong T_0 \qquad /7/$$

$$-i\int dx\ G_{\pm\frac{1}{2},\mp\frac{1}{2}}(x,x;0^-) = \sqrt{\left(T+T_0+\frac{1}{2}\right)\left(T-T_0+\frac{1}{2}\right)} \qquad /8/$$

$$-i\left\{\int dx\ G_{\frac{1}{2}\frac{1}{2}}(x,x;0^-) + \int dx\ G_{-\frac{1}{2},-\frac{1}{2}}(x,x;0^-)\right\} = A - 1 \cong A \qquad /9/$$

where the approximations made are in the same spirit as in /4/.

It has been mentioned that the single-particle potential V can be chosen arbitrarily since equation /5/ in fact does not depend on V. However, this equation is nonlinear and possesses a multitude of different solutions. In the course of actually solving it the auxiliary quantity V can be expressed in different ways through the Green functions and the nucleon-nucleon potential. Different choices of V will generally lead to different solutions, since they imply differences in the available states which are to be filled in some standard manner. In the following, two possible choices of V will be discussed.

3. THE HARTREE-FOCK EQUATIONS

It is possible to choose V so as to make the second and third terms on the right hand side of /5/ cancel each other:

$$\langle x_1|\hat{V}|x_2\rangle = -i\ \hat{I}\int dx_3\ dx_4\ \langle x_1x_3|U|x_2x_4\rangle\ \mathrm{Sp}\ \hat{G}\left(x_4,x_3;0^-\right) +$$

$$+\ i\int dx_3\ dx_4\ \langle x_1x_3|U|x_4x_2\rangle\ \hat{G}\left(x_4,x_3;0^-\right) \qquad /10/$$

Since this matrix contains off-diagonal elements the eigenstates of $\hat{H}$ will be the two-component charge-mixed states

$$\psi_q(x) = \begin{pmatrix} \psi_{q,\frac{1}{2}}(x) \\ \\ \psi_{q,-\frac{1}{2}}(x) \end{pmatrix}$$

In this "q-representation" G is diagonal

$$G_{q_1q_2}(t) = \delta_{q_1q_2}\ G_{q_1}(t)$$

and /10/ takes the form

$$\langle q_1|V|q_2\rangle = \sum_{q\leq A}\left(\langle q_1q_2|U|q_2q\rangle - \langle q_1q|U|qq_2\rangle\right) \qquad /11/$$

with

$$\langle q_1q_2|U|q_3q_4\rangle = \int dx_1\ dx_2\ dx_3\ dx_4\ \Big\{\langle x_1x_2|U^o|x_3x_4\rangle\Big(\psi^+_{q_1}(x_1)\,\psi_{q_3}(x_3)\ \ \psi^+_{q_2}(x_2)\psi_{q_4}(x_4)\Big) +$$

$$+\ \langle x_1x_2|U^\tau|x_3x_4\rangle\Big(\psi^+_{q_1}(x_1)\ \ \psi_{q_4}(x_4)\Big)\Big(\psi^+_{q_2}(x_2)\ \ \psi_{q_3}(x_3)\Big)$$

Equation /11/ shows that V coincides with the Hartree-Fock potential for a system of A nucleons when charge-mixed orbitals are allowed. Since the subsidiary condition /9/ is automatically fulfilled the α can be set equal to zero. The remaining subsidiary conditions are

$$\frac{1}{2}\sum_{q\leq A}\left\{\int dx\,\psi_{q,\frac{1}{2}}(x)\,\psi_{q,\frac{1}{2}}(x) - \int dx\,\psi_{q,-\frac{1}{2}}(x)\,\psi_{q,-\frac{1}{2}}(x)\right\} = T_o$$

$$\sum_{q\leq A}\int dx\,\overset{*}{\psi}_{q,\frac{1}{2}}(x)\,\psi_{q,-\frac{1}{2}}(x) = \sqrt{\left(T+T_o+\frac{1}{2}\right)\left(T-T_o+\frac{1}{2}\right)}$$

The meaning of β_o is the difference between the neutron and proton chemical potentials

$$\beta_o = \mu^n - \mu^p$$

On the stability line this is compensated by $-\Delta_c$. The constants $\beta_\pm$ determine the weight of the neutron and proton component in ψ_q in the asymptotic region.

It should be noted that a nonzero value for the off-diagonal elements of $\hat{V}$ can be obtained only if the iteration begins with charge-mixed states. This is a consequence of the charge conserving character of U.

There is an obvious difficulty inherent to the scheme described above. In a Hartree-Fock procedure with well distinguished neutrons and protons the double sequence of orbitals usually remains the same when a nucleon is added to the system. In other words, the quantum numbers of the first unfilled orbital of an even-even nucleus determine the quantum numbers of the next odd nucleus. This is certainly not the case for a potential well with a charge exchange part. If, for example, for ^{208}Pb the first unfilled state had quantum numbers $9/2^+$ then the next odd nucleus might be ^{209}Pb but not ^{209}Bi, for which the ground state is $9/2^-$, though both ^{209}Pb and ^{209}Bi are obtained with the addition of a single nucleus to the system. The subsidiary conditions are of course different in the two cases, but this example shows that the filling up of the single sequence of orbitals in the Hartree-Fock procedure with charge-mixed states must be irregular. It may even occur that to obtain the ground state some of the first A states have to be left unfilled.

4. MODIFIED HARTREE-FOCK METHOD

Another possibility for choosing $\hat{V}$ is

$$\langle x_1|V_{\sigma_1\sigma_2}|x_2\rangle = \delta_{\sigma_1\sigma_2}\,\langle x_1|V_{\sigma_1}|x_2\rangle$$

with

$$\langle x_1|V_{\sigma_1}|x_2\rangle = -i\int dx_3\, dx_4\, \langle x_1x_3|U|x_2x_4\rangle\, Sp\, \hat{G}\left(x_4,x_3;0^-\right) +$$

$$+ i\int dx_3\, dx_4\, \langle x_1x_3|U|x_4x_2\rangle\, G_{\sigma_1\sigma_1}\left(x_4,x_3;0^-\right) \qquad /12/$$

The equation /5/ takes the form

$$i\,\frac{\partial\hat{G}\left(x_1,x_2;t\right)}{\partial t} - dx_3\,\langle x_1|\hat{H}|x_3\rangle\,\hat{G}\left(x_3,x_2;t\right) =$$

$$= \hat{\delta}\left(x_1,x_2\right)\,\delta(t) + \int dx_3\,\langle x_1|\hat{\Sigma}|x_3\rangle\,\hat{G}\left(x_3,x_2;t\right) \qquad /13/$$

Here $\langle x_1|\hat{\Sigma}|x_3\rangle$ is the off-diagonal part of the matrix $i\int dx_4\, dx_5\, \langle x_1x_4|U|x_5\, x_3\rangle\, \hat{G}\left(x_5,x_4;0\right) + \left(\beta_-\hat{t}_+ + \beta_+\hat{t}_-\right)\,\delta\left(x_1,x_3\right)$ and $\hat{H}$ contains only the diagonal part of /6/. Therefore $\hat{H}$ is diagonal in charge. The eigenstates of H for protons are $|\alpha\rangle$, $|\beta\rangle$ ... while those for neutrons are $|a\rangle$, $|b\rangle$... and the corresponding eigenvalues are ε_α, ε_β, ε_a, ε_b ...

The Green function $\hat{G}^o$ constructed from these states satisfies the equation

$$i\,\frac{\partial\hat{G}^o(x_1,x_2;t)}{\partial t} - \int dx_3\,\langle x_1|\hat{H}|x_3\rangle\,\hat{G}^o(x_3,x_2;t) = \hat{\delta}(x_1,x_2)\,\delta(t)$$

Our aim will be to express $\hat{G}$ and $\hat{\Sigma}$ in terms of $\hat{G}^o$, which will be assumed to be known.

In the representation of the eigenstates of $\hat{H}$ the Green function $\hat{G}^o$ is diagonal and the Fourier transform

$$G^o(\omega) = \int_{-\infty}^{\infty} G(t)\, e^{i\omega t}\, dt \qquad /14/$$

of its diagonal elements has the form

$$G^o_\alpha(\omega) = \frac{1}{\omega-\varepsilon_\alpha+i\eta_\alpha}$$

for proton states and

$$G^{o}_{a}(\omega) = \frac{1}{\omega-\varepsilon_{a}+i\eta_{a}}$$

for neutron states. The sign of the infinitesimal quantities has to be negative for filled and positive for unfilled orbitals. Therefore, our modified scheme differs from the Hartree-Fock procedure of the previous section solely in the selection of the filled and unfilled states. Here an auxiliary set of states of definite charge have to be filled in a given manner, the charge-mixed states being introduced only afterwards, whereas in the previous section orbitals with already mixed charge had to be filled. Nevertheless, the equation /5/ to be solved is the same in both cases! The role of the arbitrary V is clearly seen: together with the filling convention it serves to select particular solutions to /5/.

In the representation of the eigenstates of $\hat{H}$

$$\Sigma_{\alpha a} = i\langle\alpha b|U|\beta a\rangle\, G_{\beta b}(t=0) - \beta\langle\alpha|a\rangle$$

$$\Sigma_{a\alpha} = i\langle a\beta|U|b\alpha\rangle\, G_{b\beta}(t=0) - \overset{*}{\beta}\,\langle a|\alpha\rangle \qquad /15/$$

with $\beta = \frac{1}{\sqrt{2}}\beta_{-}$. Summation over indices occuring twice is understood. Introducing the Fourier transform of G according to /14/, equation /13/ can be transformed into

$$G_{\alpha\alpha'}(\omega) = G^{o}_{\alpha}(\omega)\,\delta_{\alpha\alpha'} + G^{o}_{\alpha}(\omega)\Sigma_{\alpha a}\, G_{a\alpha}(\omega) \qquad /16/$$

$$G_{aa'}(\omega) = G^{o}_{a}(\omega)\,\delta_{aa'} + G^{o}_{a}(\omega)\Sigma_{a\alpha}\, G_{\alpha a'}(\omega) \qquad /17/$$

$$G_{\alpha a}(\omega) = G^{o}_{\alpha}(\omega)\,\Sigma_{\alpha b}\, G_{ba}(\omega) \qquad /18/$$

$$G_{a\alpha}(\omega) = G^{o}_{a}(\omega)\,\Sigma_{a\beta}\, G_{\beta\alpha}(\omega) \qquad /19/$$

The solution consists of the following steps: First substitute /18/ and /19/ into /16/ and /17/ to obtain

$$G_{\alpha\alpha'}(\omega) = G^{o}_{\alpha}(\omega)\,\delta_{\alpha\alpha'} + G^{o}_{\alpha}(\omega)\,\Sigma_{\alpha a}\, G^{o}_{a}(\omega)\,\Sigma_{a\beta}\, G_{\beta\alpha'}(\omega)$$

$$/20/$$

$$G_{aa'}(\omega) = G^{o}_{a}(\omega)\,\delta_{aa'} + G^{o}_{a}(\omega)\,\Sigma_{a\alpha}\, G^{o}_{\alpha}(\omega)\,\Sigma_{\alpha\beta}\, G_{ba'}(\omega)$$

The first equation may be solved for the proton Green function $G_{\alpha\alpha}$, and the second for the neutron Green function G_{aa}, in terms of the Σ-s. Their substitution into /18/ and /19/ leads to an expression for the anomalous Green functions $G_{\alpha a}$ and $G_{a\alpha}$ through the Σ-s. The inverse Fourier transform of the anomalous Green functions have to be substituted into /15/. The equation thus obtained determines the self-energy parts Σ. Finally the proton and neutron Green functions expressed through these Σ-s have to be substituted into /12/ to check self-consistency.

The subsidiary conditions are of the form

$$-\frac{i}{2}\left\{\sum_{a} G_{aa}(0^{-}) - \sum_{\alpha} G_{\alpha\alpha}(0^{-})\right\} = T_{0}$$

$$-i \sum_{a\alpha} \langle a|\alpha\rangle\, G_{\alpha a}(0) = \sqrt{\left(T+T_{0}+\frac{1}{2}xT-T_{0}+\frac{1}{2}\right)}$$

$$-i \sum_{a} G_{aa}\, 0^{-} + \sum_{\alpha} G_{\alpha\alpha}\, 0^{-} = A$$

The distributions of the neutrons and protons

$$\rho_{a}^{n} = -i\, G_{aa}(t=0^{-})$$

$$\rho_{\alpha}^{p} = -i\, G_{\alpha\alpha}(t=0^{-})$$

will generally not be sharp. The stationary states of the system are those which diagonalize $\hat{G}$. These correspond to the ψ_q of the Hartree-Fock and also have mixed charge. However, the distribution of the nucleons over these states is not necessarily sharp.

As a special example consider an infinite system of spinless neutrons and protons. In order to simulate the situation inside a large but finite nucleus positive values for the difference between the neutron and proton chemical potentials β_0 will be allowed. In this respect the system considered differs from nuclear matter as it is usually understood.

Now, if it is meaningful to consider propagation of nucleons in the presence of charge exchange in an infinite system then it has to be realized that charge exchange and isospin are related to each other in a complicated manner. Indeed, as the isospin is a quantum number which labels the irreducible representations of an SU(2) group it is certainly inapplicable to an infinite system. If, nevertheless, charge, exchange appeared in it, then this would have to be described solely in terms of local quantities instead of isospin.

In an infinite system the Green functions are all diagonal in momentum representation:

$$G(\underline{p},\underline{p}';\omega) = (2\pi)^3\ \delta(\underline{p}-\underline{p}')\ G(\underline{p};\omega)$$

$$\langle\underline{p}|\Sigma|\underline{p}'\rangle = (2\pi)^3\ \delta(\underline{p}-\underline{p}')\ \Sigma(\underline{p})$$

From /15/ one obtains

$$\Sigma_{\sigma\bar{\sigma}}(\underline{p}) = i\int\frac{d^3p'}{(2\pi)^3}\ U(\underline{p},\underline{p}')\ G_{\sigma\bar{\sigma}}\left(\underline{p};\ t=0\right) -\beta \qquad /21/$$

where

$$\langle\underline{p},p'|U|\underline{p}',\underline{p}''\rangle = \ 2\pi\ ^3\ \delta\ \underline{p}-\underline{p}''\ \ U\ \underline{p},\underline{p}'$$

and $\sigma+\bar{\sigma} = 0$. In the following instead of ±1/2 the labels n and ρ will be used.

For G^o we can take

$$G^o_\sigma(\underline{p};\omega) = \frac{1}{\omega-\varepsilon_\sigma+i\eta\varepsilon_\sigma}$$

Here ε_n and ε_p are the neutron and proton single-particle energies normalized to zero at the corresponding chemical potentials. Both depend on the momentum, but it will be assumed that their difference is equal to the constant β_o. This is the case when, for example, in the vicinity of the two chemical potentials and between them both ε_n and ε_p can be approximated by the same effective mass.

Using /20/ we can write e.g. the proton Green function as

$$G_p(\underline{p};\omega) = \frac{\omega - \varepsilon_p}{(\omega-\varepsilon_n)(\omega-\varepsilon_p)-|\Sigma|^2}$$

where

$$|\Sigma(\underline{p})|^2 = \Sigma_{np}(\underline{p})\ \Sigma_{pn}(\underline{p})$$

However, it is convenient to write G_p in a different form:

$$G_p(\underline{p};\omega) = \frac{v^2}{\omega-\omega_n + i\eta\omega_n} + \frac{u^2}{\omega-\omega_p + i\eta\omega_p}$$

where

$$\omega_n = \frac{1}{2}\left[\varepsilon_n + \varepsilon_p - \sqrt{\beta_o^2 + 4|\Sigma|^2}\right]$$
$$\omega_p = \frac{1}{2}\left[\varepsilon_n + \varepsilon_p + \sqrt{\beta_o^2 + 4|\Sigma|^2}\right] \qquad /22/$$

and

$$v^2 = \frac{\sqrt{\beta_o^2 + 4|\Sigma|^2} - \beta_o}{2\sqrt{\beta_o^2 + 4|\Sigma|^2}}$$

$$v^2 + u^2 = 1$$

The sign of the infinitesimal imaginary term in the denominators of G_p has been chosen so as to make the analytic properties of G_p appropriate for a causal Green function.

For G_n we obtain in an analogous manner

$$G_n(\underline{p};\omega) = \frac{v^2}{\omega-\omega_p + i\eta\omega_p} + \frac{u^2}{\omega-\omega_n + i\eta\omega_n}$$

The proton and neutron densities are

$$\rho_p(\underline{p}) = \begin{cases} 1 \\ v^2 \\ 0 \end{cases} ; \qquad \rho_u(\underline{p}) = \begin{cases} 1 & \omega^n < 0, \ \omega^p < 0 \\ u^2 \quad \text{for} & \omega^n < 0, \ \omega^p > 0 \\ 0 & \omega^n > 0, \ \omega^p > 0 \end{cases} \qquad /23/$$

The neutron and proton Fermi momenta p_n and p_p can be defined as the solutions of the equations $\omega_n(p_n) = 0$, $\omega_p(p_p) = 0$.

From /22/ it is seen that $p_n > p_p$. The relations /23/ show that for $p < p_p$ both neutron and proton states are filled, while for $p > p_n$ both are empty. States with momenta between p_p and p_n are occupied by a single nucleus which is partly proton and partly neutron. If the vacuum is defined to be the state in which neutron and proton levels up to p_I and p_p are occupied, then in the ground state the levels with momenta between p_p and p_n contain proton particle - neutron hole pairs with the probability $v^2(p)$.

The anomalous Green functions can be determined from /18/ and /19/. For example

$$G_{pn}(\underline{p};\omega) = \frac{\Sigma_{pn}(\underline{p})}{\left(\omega-\omega_p + i\eta\omega_p\right)\left(\omega-\omega_n + i\eta\omega_n\right)}$$

From this

$$G_{pn}\left(\underline{p};t=0\right) = \begin{cases} -i\,\dfrac{\Sigma_{pn}}{\sqrt{\beta_0^2 + 4|\Sigma|^2}} & \text{for} \quad p_p < p < p_n \\ 0 & \text{otherwise} \end{cases}$$

Substituting /24/ into /21/ we arrive at the equation for $\Sigma(\underline{p})$.

The subsidiary conditions /7/ and /9/ can be written as

$$\int d^3p \; \rho_n(\underline{p}) = n$$

$$\int d^3p \; \rho_p(\underline{p}) = z$$

where n and z are the neutron and proton densities. However it is not clear how the subsidiary condition /8/ should be applied to an infinite system. It may be simply omitted by putting $\beta = 0$. Or, it will be noticed that the right hand sides of /7/ and /9/ are proportional to the volume Ω, while the right hand side of /8/ is proportional to the square root of the volume. Since the replacement of $G(\underline{p},\underline{p}';0)$ by $G(\underline{p};0)$ in the subsidiary conditions is equivalent to the multiplication by Ω^{-1} it is reasonable to put the right hand side of /8/ equal to zero. Then

$$\int d^3p \; G_{np}\left(\underline{p};t=0\right) = 0$$

61.887

5. SUMMARY

In a Fermi system every type of Bose excitation has associated components in the ground state correlations and vice versa. In the case of the vibrations the features of the excitation were the first to be understood and the corresponding ground state correlation was introduced afterwards. In the case of superconductivity it was the ground state correlation which was emphasized first and the corresponding excitations /the so-called "pairing vibrations"/ were conceived only later. From this point of view the present work can be viewed as a suggestion to associate the ground state correlation corresponding to the analogue state with the charge mixing in single-particle orbitals.

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ABSTRACT

It is argued that the charge exchange optical potential has to be determined through a multichannel self-consistent procedure which can be simplified to a two-channel problem. Two modifications of the simplified version are discussed. The first is identical to the Hartree-Fock method with charge-mixed orbitals, the second resembles the treatment of superconductivity.

РЕЗЮМЕ

Предлагается многоканальный самосогласованный метод определения обменной части оптического потенциала, который затем приводится к двухканальной проблеме. В статье рассматривается два варианта упрощенной схемы. Первый вариант соответствует методу Хартри-Фока для орбит со смешанным зарядом, а другой - аналогичен теории сверхпроводимости.

KIVONAT

A dolgozatban a töltéskicserélő optikai potenciál meghatározására egy sokcsatornás self-konzisztens módszert javasolunk, amely egy kétcsatornás problémára redukálható. Az egyszerüsitett probléma két változatát tárgyaljuk. Az első a Hartree-Fock módszerrel azonos kevert töltésü pályák esetén, a másik pedig a szupravezetés tárgyalásával analóg.

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