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C VIOLATION IN η DECAY?

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The recent experimental results on the charge asymmetry in η decay have been compared with a theoretical model of C violation. Five asymmetry parameters of the $\eta\to\pi^+\pi^0\pi^-$ energy distribution have been defined and their measured values have been reproduced by the model within one Standard deviation. The implication of the experimental upper limit on the $\eta\to\pi^0 e^+ e^-$ decay has also been taken into account. Order of magnitude estimates indicate that the model is not in contradiction with the experimental results on the K_L decay.

I. INTRODUCTION AND MAIN RESULTS

The superweak theory of CP violation [1] would certainly be the simplest solution of the CP puzzle. The known experimental results in the K_L decay seem to be consistent with the prediction of this theory. There exists, however, a number of other models of CP violation, which predict nearly the same results for the K_L decay as the superweak model, and due to the large statistical error in the measured value of the η_{oo} parameter[x] one cannot choose between all these models on the basis of the available K_L data.

In 1968 new experimental results have been published on the $\eta\to\pi^+\pi^o\pi^-$ decay, which indicate that C violation may be present in this process, too [3] . The measured number of $\eta\to\pi^+\pi^o\pi^-$ events in each sextant of the Dalitz plot are quoted in Fig.1. The corresponding experimental values of the five asymmetry parameters,

$$\Delta_1 = \frac{N_1 - N_6}{N_1 + N_6}, \qquad \Delta_2 = \frac{N_2 - N_5}{N_2 + N_5}, \qquad \Delta_3 = \frac{N_3 - N_4}{N_3 + N_4}, \tag{1}$$

$$\Delta = \frac{N_1+N_2+N_3-N_4-N_5-N_6}{N_1+N_2+N_3+N_4+N_5+N_6} \quad \text{and} \quad \bar{\Delta} = \frac{N_1-N_2+N_3-N_4+N_5-N_6}{N_1+N_2+N_3+N_4+N_5+N_6}$$

are given in the second column of our Tables. It can be seen that while the value of the so called sextant asymmetry $\bar{\Delta}$ and that of the partial asymmetry Δ_1 is consistent with zero whitin one standard deviation, the values of Δ_2 and Δ_3 differ from zero by more than 2, and the value of the right--left asymmetry Δ by 3 standard deviations. Of course only more precise measurements can clarify whether these asymmetries are the manifestation of the CP violation, or they are due to statistical or systematical errors. Nevertheless, we thought that an attempt to find a theoretical model which can explain these and other correlated data may be of interest.

In Part II of the paper the experimental results on the η decay are compared with the predictions of a theoretical model in which C violation is introduced by means of the strangeness and parity conserving $\eta\rho\pi$

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[x] For the description of CP violation in K^o decays we use the well known notation and phase convention of T.D. Lee and C.S. Wu [2].

vertex [4]. The line of thought is similar to that of B.Barrett et al [5] and M. Veltman et al [6] , but due to the new experimental results on the $\eta\to\pi^+\pi^o\pi^-$ assymetry [3] , on the energy dependence of the $\eta\to\pi^+\pi^o\pi^-$ decay [7], on the full and partial widths of the η meson and on the upper limit of the $\eta\to\pi^o e^+e^-$ decay [8] , a more quantitative analysis of the $\Delta I=0$ and $\Delta I=2$ isotopic spin transitions became possible. We shall see that all the available experimental results can be reproduced within one standard deviation by a $\Delta I=2$ coupling $\eta\rho\pi$ with coupling constant $g_2 \approx 10^{-2}$. The experimental data are rather insensitive to the strength of the $\Delta I=0$ coupling; its coupling constant may vary within the limits $0 \lesssim g_o \lesssim 50 g_2$. This is due to the well known "centrifugal barrier" effect, which strongly damps the contribution of the $\Delta I=0$ channel [5] . We shall also see that a pure $\Delta I=0$ transition is not consistent with the experimental data. In Part III some further theoretical aspects of this analysis will be discussed. We stress that the possibility of choosing $g_o >> g_2$ supports the compelling idea [6] that the genuine C violation is given by the strangeness, isospin and parity conserving medium strong $\eta\rho\pi$ coupling, and that the $\Delta I=2$ impurity in the $\eta\rho\pi$ coupling arises only as a radiative correction.We are then logically forced to allow for a C violating, $\Delta I=1$ impurity of similar strength, which may be described by the $\eta\omega\pi$ coupling. It will be shown that the introduction of this coupling does not affect our results for the η decay. Finally we shall see in the Appendix on the basis of very crude estimation that our model probably does not contradict the favorised experimental result

$$|\varepsilon'|<|\varepsilon| \approx 2\cdot 10^{-3}$$

of the K_L decays.

II. THE η-DECAY

It is easy to see that in the $\eta\to\pi^+\pi^o\pi^-$ decay the $\Delta I=0,2$ transitions are C violating, the $\Delta I=1,3$ transitions are C conserving. In order to describe the C violation the Hamiltonian

$$H_{\not{C}} = H_o + H_2 \tag{2}$$

will be introduced, where

$$H_o = g_o\left[\rho^+_\mu(\pi^-\partial_\mu\eta-\eta\partial_\mu\pi^-)+\rho^o_\mu(\pi^o\partial_\mu\eta-\eta\partial_\mu\pi^o)+\rho^-_\mu(\pi^+\partial_\mu\eta-\eta\partial_\mu\pi^+)\right] , \tag{3}$$

$$H_2 = g_2\left[-\rho^+_\mu(\pi^-\partial_\mu\eta-\eta\partial_\mu\pi^-)+2\rho^o_\mu(\pi^o\partial_\mu\eta-\eta\partial_\mu\pi^o)-\rho^-_\mu(\pi^+\partial_\mu\eta-\eta\partial_\mu\pi^+)\right] \quad . \qquad (4)$$

H_o and H_2 are irreducible $I=0$ and $I=2$ tensor operators which produce pure $0 \to 0$ and $0 \to 2$ isotopic spin transitions, respectively. /In the papers [5] and [6] the coupling constants used do not correspond to pure $\Delta I=0$ and $\Delta I=2$ transitions./ The C violating $\eta\to\pi^+\pi^o\pi^-$ amplitude is of course produced via the $\eta\to\rho\pi\to\pi\pi\pi$ chain with the help of the strong interaction

$$H_\rho = G_\rho\left[\rho^+_\mu(\pi^o\partial_\mu\pi^- - \pi^-\partial_\mu\pi^o)+\rho^o_\mu(\pi^-\partial_\mu\pi^+ - \pi^+\partial_\mu\pi^-)+\rho^-_\mu(\pi^+\partial_\mu\pi^o - \pi^o\partial_\mu\pi^+)\right] . \qquad (5)$$

The C violating amplitude $A_{\not C}$ is then easily calculated to be

$$A_{\not C} = A_o + A_2 = ig_oG_\rho\left[\frac{s-t}{M^2-u}+\frac{t-u}{M^2-s}+\frac{u-s}{M^2-t}\right] +$$
$$+ ig_2G_\rho\left[-\frac{s-t}{M^2-u}+2\frac{t-u}{M^2-s}-\frac{u-s}{M^2-t}\right] . \qquad (6)$$

In (6) the usual Mandelstam variables $s=(p_+ + p_-)^2$, $t=(p_- + p_o)^2$, $u=(p_+ + p_o)^2$ has been introduced /p_+, p_- and p_o denote the four-momenta of the π^+, π^- and π^o mesons/. In the denominators a term taking account the width of the ρ-meson should be added; it can be seen however that its contribution is less than $3^o/oo$ in the whole physical region, and therefore this term can be safely neglected [5] .

The charge asymmetry in the $\eta\to\pi^+\pi^o\pi^-$ decay arises from the interference of the C violating amplitude with the C conserving one. The theoretical description of the C conserving $\eta\to\pi^+\pi^o\pi^-$ decay is subject to many uncertainities, summarized in a recent paper by D.G. Sutherland [9] . In our work we have opted for the semi-phenological C conserving amplitude A_C given in [5] :

$$A_C = a\left[\frac{1}{1 - ia_o\frac{1}{2}\sqrt{s-4\mu^2}} + b\left(\frac{s-u}{M^2-t}+\frac{s-t}{M^2-u}\right)\right] . \qquad (7)$$

In equ. (7) the $\Delta I=3$ transition has been neglected because the decay is assumed to be induced by an electromagnetic process of second order. In the remaining $I=1$ 3π state two-pion final state interactions has been taken into account. Namely, the first term in (7) describes the interactions of two pions in the $L=0$, $I=0$ channel in the scattering length approximation. a_o stands for the scattering length, μ denotes the pion mass. The $I=2$ interaction is known to be small and has been neglected. The second term in (7) gives the L=1, I=1 two-pion final state interaction through the ρ meson pole approximation. The effect of the ρ width has been again neglected. The constants* a and b are real if CPT invariance holds. CPT conservation will be assumed throughout this work.

We have now at our disposal the full $\eta \to \pi^+\pi^0\pi^-$ amplitude

$$A = A_C + A_{\not{C}} \quad , \tag{8}$$

which contains two strong interaction parameters a_o and G_ρ, two C conserving η decay parameters a and b and two C violating coupling constants g_o and g_2. We are interested in the possible values of the latter two quantities. Of course the uncertainities in the values of the other four parameters will influence them. Fortunately it is not necessary to treate all the six parameters at the same time. Indeed, it is experimentally known /see our Tables/ that the amount of the C violation in the η decay is at most a few percent. This allows us to neglect $A_{\not{C}}$ when fixing the parameters of A_C. In A_C it is reasonable to treat a_o as an external parameter to be taken from the strong interaction physics. The value of a_o is not well known, so we have made all calculations for three different values of a_o, namely for 0,2, for 0,5 and for 1,0 /in μ^{-1} units/. For each given value of a_o the parameter b has been determined from a best fit of A_C to the experimental energy distribution of the π^0 meson [7]. For a given value of a_o the value of b has an uncertainity $\lesssim$ 5%. With a_o and b known, $|a|$ has been calculated from the measured width of the η [8]. The error in the value of a for a given pair of values of a_o and b equals approximately 15 %.

Let us now turn to the calculation of the five asymmetry parameters Δ_i /we define $\Delta_4 \equiv \Delta$, $\Delta_5 \equiv \bar{\Delta}$ /. They are given by expressions of the form

$$\Delta_i = -2 \frac{\int_{S_i} \rho d\rho d\theta \ \mathrm{Im}A_C \ \mathrm{Im}A_{\not{C}}}{\int_{F_i} \rho d\rho d\theta \left[|A_C|^2 + |A_{\not{C}}|^2 \right]} \ , \tag{9}$$

*Notice that we denote by b the b/a of B.Barret et al.

where S_i and F_i are appropriate domains of integration for the well known Dalitz variables ρ and θ. In the denominator the term $|A_{\not C}|^2$ can be neglected, and then inspecting the formulae (6) and (7) it is easy to see that the Δ_i-s can be written in the form

$$\Delta_i = \gamma_0 \; n_i^{(0)} + \gamma_2 \; n_i^{(2)} \qquad /i = 1,\dots,5/, \tag{10}$$

where the reduced coupling constants γ_0 and γ_2 are defined as follows:

$$\gamma_0 \equiv \frac{G_\rho}{a} g_0 \quad , \quad \gamma_2 \equiv -\frac{G_\rho}{a} g_2 \tag{11}$$

The quantities $n_i^{(0)}$, $n_i^{(2)}$ are complicated functions of a_0 and b , but they do not depend on a, $G\rho$, g_0 and g_2. Therefore they could be calculated on a computer with high accuracy for any fixed pair of a_0 and b , the error in b being small. We have now five linear equations in γ_0 and γ_2 to determine the best values of these coupling constants. This calculation has been carried out under various conditions.

Before discussing the results, we have still to investigate the implication of the measured upper limit of the $\eta\to\pi^0 e^+ e^-$ decay on the allowed values of γ_0 and γ_2.

In lowest order the decay $\eta\to\pi^0 e^+ e^-$ takes place via the $\eta\to\pi^0\rho^0\to\pi^0\gamma\to\pi^0 e^+ e^-$ chain. Taking into account the well known relation

$$f_\rho = e\, G_\rho^{-1} \tag{12}$$

for the $\rho-\gamma$ coupling constant f_ρ , a straighforward calculation gives

$$\Gamma(\eta\to\pi^0 e^+ e^-) = 42\ \text{eV} \left(\frac{g_0 + 2g_2}{G\rho} \right)^2 \tag{13}$$

From the measured total η width and from the upper limit on the $\eta\to\pi^0 e^+ e^-$ decay [8] one gets

$$\frac{\Gamma(\eta\to\pi^0 e^+ e^-)}{\Gamma(\eta\to a\ell\ell)} = \frac{42\ \text{eV}}{2630\ \text{eV}} \; \frac{(g_0 + 2g_2)^2}{G_\rho^2} < 10^{-4} \quad , \tag{14}$$

i.e.

$$|g_0 + 2g_2| \le 0{,}08\ |G_\rho| \quad , \tag{15}$$

or

$$|\gamma_o + 2\gamma_2| < 0{,}08 \frac{G_\rho^2}{|a|} \quad . \qquad (16)$$

From the experimental width of the ρ^o meson one finds $G_\rho = 5{,}14 \pm 0{,}13$ [10] , and then

$$|g_o + 2g_2| < 0{,}41 \quad , \qquad (17)$$

$$|\gamma_o + 2\gamma_2| < \frac{2{,}1}{|a|} \quad . \qquad (18)$$

Let us now discuss the possible solutions of the equations (10). For definiteness we shall refer first to the case $a_o = 0{,}5$, given in Table II. /The other two cases will be briefly discussed at the end of this paragraph./

By assuming $a_o = 0{,}5$ one finds $b = 1{,}95$ and $|a| = 0.395$ from the energy distribution and width of the $\eta\to\pi^+\pi^o\pi^-$ decay. We have calculated the quantities $n_i^{(0)}$ and $n_i^{(2)}$ with these values of a_o and b to be

$$\begin{aligned}
n_1^{(0)} &= 0{,}001550 & n_1^{(2)} &= 0{,}9462 \\
n_2^{(0)} &= -0{,}001398 & n_2^{(2)} &= 1{,}3663 \\
n_3^{(0)} &= 0{,}001056 & n_3^{(2)} &= 0{,}4933 \\
n_4^{(0)} &= 0{,}000352 & n_4^{(2)} &= 0{,}8436 \\
n_5^{(0)} &= 0{,}001243 & n_5^{(2)} &= -0{,}0272
\end{aligned} \qquad (19)$$

The errors of these quantities /coming from the 5 % uncertainity in b and from the computational error/ have been neglected, because they are much less than the errors in the experimental values of the Δ_i-s.

Introducing the values quoted in equ. (19) into equ. (10) we have looked for the best fitting values of γ_o and γ_2 under different conditions:

1/ Pure $\Delta I = 0$ C ciolation, i.e. $\gamma_2 = 0$. The results are given in column 3 of Table II. The "best" fit $\gamma_o = 20$ badly violates the $\eta\to\pi^o e^+ e^-$ limit $|\gamma_o| < 5{,}3$. Also, the parameters Δ_2, Δ_3 and Δ are poorly reproduced. Thus the pure $\Delta I = 0$ C violation is not acceptable on the basis of the available experimental results.

2/ Pure $\Delta I = 2$ C violation, i.e. $\gamma_o = 0$. The results are quoted in column 4 of Table II. The value $\gamma_2 = 0{,}0979 \pm 0{,}0163$ is fully consistant with the upper limit $|2\gamma_2| < 4{,}2$ coming from the $\eta \to \pi^0 e^+ e^-$ decay. All the five asymmetry parameters Δ_i are reproduced within one standard deviation. Thus the pure $\Delta I = 2$ C violation is fully acceptable.

3/ Mixed C violation, i.e. γ_0 and γ_2 both different from zero.

From (19) we see that $n_i^{(2)} >> n_i^{(0)}$, namely,

$$n_i^{(2)} \geqslant 500 \; n_i^{(0)} \tag{20}$$

in all but the last case, in which*

$$n_5^{(2)} \approx 25 \; n_5^{(0)} \tag{21}$$

From these relations we learn that γ_0 should be considerably larger than γ_2 in order to produce a discernible contribution to the asymmetry parameters. Therefore the cases $|\gamma_0| \leqslant |\gamma_2|$ will lead practically to the same values as the pure $\Delta I = 2$ case, i.e. to fully acceptable results. It is worth to investigate the possibility $\gamma_0 >> \gamma_2$ in some details. This possibility has been summarized in columns 5 and 6 of Table II, where the consequences of the assumptions $\gamma_0 = 25\gamma_2$ and $\gamma_0 = 50\gamma_2$ are considered. As expected from equ. (20) and (21) , even now the $\Delta I = 2$ coupling governes the asymmetry. We see, indeed, that the best fitting value of γ_2 remains almost the same as for the pure $\Delta I = 2$ case, and the asymmetry parameters $\Delta_1, \Delta_2, \Delta_3$ and Δ change only by 10 -20 %. Only $\bar{\Delta}$ changes appreciably when γ_0 increases: it goes up linearly with γ_0/γ_2 being practically constant/, but this effect is somewhat masked by the fact that $\bar{\Delta}$ passes through zero and remains small for the region $0 \leqslant \gamma_0 \leqslant 50\gamma_2$. Anyhow, again all the asymmetry parameters are reproduced within one standard deviation in this domain of the coupling constants.

In the last column of Table II the best independent fit of γ_0 and γ_2 to the asymmetry parameters is presented. It can be seen that in this case we get $\gamma_0 \approx 160\gamma_2$ and the value of γ_2 is still very close to the pure $\Delta I = 2$ case. It is easy to see that a large value of γ_0 is needed in order to get closer to the experimental mean value of $\bar{\Delta}$. It can be seen, however, that this large value of γ_0 violate already the $\eta \to \pi^0 e^+ e^-$ limit $|\gamma_0 + 2\gamma_2| < 5{,}3$. This limit is bypassed when $\gamma_0 \approx 54\gamma_2$. So we can predict that according to our model the true value of $\bar{\Delta}$, should be less then 0,7°/oo

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*The damping of the coefficients of γ_0 by rapport to those of γ_2 is a well known consequence of the peculiar symmetry properties of the $\pi^+\pi^0\pi^-$ state with I=0 [5]. The fact that this damping is less pronounced for the sextant asymmetry $\bar{\Delta} \equiv \Delta_5$ can also be understood on symmetry grounds.

instead of the quoted experimental value $4{,}4^{o}/oo$. We also predict that the true value of Δ, should be 2-3 times larger than the quoted experimental mean value. These predictions are statistically in agreement with the experimental results.

Thus on the basis of our model and of the data on the $\eta\to\pi^+\pi^o\pi^-$ and $\eta\to\pi^o e^+ e^-$ decays we have found that for $a_o = 0{,}5$ the best value of γ_2 is calculable with great accuracy, while any value of γ_o between $\gamma_o \approx 0$ and $\gamma_o \approx 54\gamma_2$ is appropriate to reproduce the asymmetry parameters within one standard deviation and to fulfil the requirement of the $\eta\to\pi^o e^+ e^-$ limit at the same time. The most probable value is just the upper limit $\gamma_o \approx 54\gamma_2$ because this is closest to the best fit with independent* γ_o and γ_2.

We would like to stress that the parameters a and G_ρ do not influence the calculation of the best values of γ_o and γ_2 coming from the fit to the asymmetry parameters. They enter however the formula /16/ for the $\eta\to\pi^o e^+ e^-$ limit. This limit is thus subject to the uncertainities in the values of a and G_ρ which are of 15% and 3% respectively. The best values of g_o and g_2 also depend on a and G_ρ, as shown by (11). The ratio of g_o and g_2 is, however, seen to be equal to that of γ_o and γ_2, and thus the relative strength of the C violating $\Delta I = 0$ and $\Delta I = 2$ couplings turns out to be independent of a and $G\rho$.

Let us now turn to the cases $a_o = 0{,}2$ and $a_o = 1{,}0$ presented in Table I and Table III, respectively. First of all we remark that with these scattering length assumptions it is again possible to find a good value of b from the energy spectrum of the $\eta\to\pi^+\pi^o\pi^-$ decay. /It is practically impossible to calculate both a_o and b from the spectrum, the data being statistically poor for a two parameter fit./ The analysis proceeds then on the same lines as for the $a_o = 0{,}5$ case and the results for the asymmetry parameters are very similar, the best fits being $\gamma_o \approx 22\,\gamma_2$ with $\gamma_2 = 0{,}24 \pm 0{,}03$ for $a_o = 0{,}2$ and $\gamma_o \approx 93\gamma_2$ with $\gamma_2 = 0{,}049 \pm 0{,}009$ for $a_o = 1{,}0$.

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*Let us briefly comment on the signs of our parameters. With $a_o = +0{,}5$ b turns out to be positive and the numbers n_i as given in 19 . If $a_o \to -a_o$ then $b \to b$, but the n_i change sign, their absolute values remaining close to the previous values. This shows that $\gamma_2 \to \gamma_2' \approx -\gamma_2$ when $a_o \to -a_o$. Concerning γ_o, for a given value of $|\gamma_o|$ the case $\gamma_o\gamma_2 > 0$ is statistically preferred to the case $\gamma_o\gamma_2 < 0$, but for small $|\gamma_o|$ the last possibility is also tolerated. The agreement with the measured value of $\bar{\Delta}$ becomes then worse than in the pure $\Delta I = 2$ case. Thus the sign of γ_2 and γ_o is positive for $a_o = +0{,}5$. The sign of g_2 and g_o depends then on the unknown sign of G_ρ and a, as seen from formula (11).

We have also calculated the $\Gamma(\eta\to\pi^0\pi^0\pi^0) : \Gamma(\eta\to\pi^+\pi^0\pi^-)$ branching ratio. The difference in the masses of the $\pi^\pm$ and π^0 mesons has been taken into account. The results for a_o=0,2; 0,5 and 1,0 are 1,39 1,36 and 1,34 respectively. These values are in good agreement with the experimental result 1,24 ± 0,18 given in [8].

III. THEORETICAL INTERPRETATION

A compelling theoretical interpretation is offered by our result $\gamma_o \approx 50\gamma_2$ / we again return to the $a_o = 0{,}5$ case for definiteness/. Namely, one can suppose that the genuine C violating interaction is described by a strangeness, isospin and parity conserving Hamiltonian H_o given in (3) with a rather big* coupling constant $g_o \approx 0{,}4$. Radiative corrections will then naturally add small $\Delta I = 1$ and $\Delta I = 2$ impurities. We might describe the $\Delta I = 2$ correction by the effective Hamiltonian H_2 given in (4) with $g_2 \approx g_o/50 \approx 0{,}008$ and we might introduce a C violating Hamiltonian H_1, in order to take into account also the $\Delta I = 1$ impurity:

$$H_1 = g_1 \omega_\mu (\pi^0 \partial_\mu \eta - \eta \partial_\mu \pi^0) . \tag{22}$$

We suppose g_1 to be of the same order of magnitude as g_2, both being radiative corrections. H_1 does not contribute to the $\eta\to\pi^+\pi^0\pi^-$ decay, but it does contribute to the $\eta\to\pi^0 e^+ e^-$ decay, consequently in formulae (13) to (18) one has

$$g_o + 2g_2 + \frac{f\omega}{f_\rho} g_1 \tag{23}$$

instead of $g_o + 2g_2$. Here f_ω stands for the ω-γ coupling constant. Since $f_\omega \approx 3f_\rho$ [10], it can be seen that with $g_o >> g_2 \approx g_1$ we get practically the same upper limit for the allowed value of the ratio $g_o : g_2$ as without H_1.

* Notice, however, that the value of the "small" electron charge is 0,3.

This model of C violation is compelling because of its simplicity: the genuine C violating Hamiltonian conserves all* the quantum numbers of the strong interaction but C. It will be shown in the Appendix by means of order of magnitude estimates that this model does not contradict the experimental data on the K_L decay either. Unfortunately no precise calculation of $|\varepsilon'/\varepsilon|$, of $\arg \varepsilon$ and of the neutron dipole moment is possible in our model, and in this respect the superweak theory with its clear-cut predictions is certainly more compelling. On the other hand, the fact that a simple model reproduces the observed values of the various asymmetry parameters of the $\eta\to\pi^+\pi^0\pi^-$ Dalitz plot may be considered as an indication that these measured asymmetry values are not purely due to statistical errors or background effects [11]. Clearly more precise measurements are needed to decide the issue.

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* Evidently T is also violated if CPT holds.

APPENDIX

The parameters ε and ε' of the K^o system are defined as follows [2]:

$$\varepsilon = \frac{<K^o|\Lambda|K^o> - <\bar{K}^o|\Lambda|K^o>}{(\gamma_L - \gamma_S) - 2i\,(m_L - m_S)} \tag{A.1}$$

$$\varepsilon = \frac{i}{\sqrt{2}}\,\frac{\mathrm{Im}A_2}{A_o}\,e^{i(\delta_2 - \delta_o)} \tag{A.2}$$

where

$$<a|\Lambda|b> = i<a|H + H\,\frac{1}{E_a - H^o + i\varepsilon}\,H \; +$$

$$+ H\,\frac{1}{E_a - H^o + i\varepsilon}\,H\,\frac{1}{E_a - H^o + i\varepsilon}\,H + \ldots|b>. \tag{A.3}$$

In (A.3) H^o stands for the Hamiltonian of the strong interactions, while

$$H = H_{CP} + H_{\not{C}\not{P}} \quad , \tag{A.4}$$

where H_{CP} denotes the CP conserving weak interaction, and $H_{\not{C}\not{P}}$ in our case reads

$$H_{\not{C}\not{P}} = H_{\not{C}} = H_o + H_1 + H_2 \quad , \tag{A.5}$$

with H_o , H_2 and H_2 given in equ. (3), (22) and (4) , respectively.

$\varepsilon = 0$ if $H_{\not{C}\not{P}} = 0$. On the other hand our $H_{\not{C}}$ conserves the strangeness, and we need at least $H_{CP} \cdot H_{CP}$ in order to get from K^o to $\bar{K}^o$. Thus the first non vanishing contribution to ε comes from terms of the form

$$<K^o|H_{CP}|n><n|H_{\not{C}}|n'><n'|H_{CP}|\bar{K}^o> \quad .$$

At first sight one may be inclined to say that for the intermediate states 2π, 3π, $\pi\ell\nu$ one may put

$$| < K^0 \,|H_{CP}|\, n > | \approx \sqrt{\gamma_S} \quad ;$$

furthermore,

$$| < n|\, H_{\not{C}} \,|\, n' > | \approx g_0$$

if $g_0 >> g_2$. /The damping for g_0 now is not at work, the 3π state being mainly $I = 1$ in the K^0 decay./ Then one might conclude that

$$|\, \varepsilon \,| \approx \left| \frac{\sqrt{\gamma_S}\;\; g_0 \;\; \sqrt{\gamma_S}}{\gamma_S} \right| \approx g_0$$

In our model, however, the otherwise dominating 2π , 3π, $\pi\ell\nu$ intermediate states lead to forbidden transitions due to the known selection rules for H_{CP} and for our $H_{\not{C}}$. Therefore only such transitions are possible*, for which it is experimentally known that

$$| < K^0 \,|\, H_{CP}|\, n > | \approx \beta \sqrt{\gamma_S} \;,\; \beta < 10^{-1} \quad ;$$

so we conclude that

$$|\, \varepsilon \,| \approx \beta^2 g_0 \quad ,$$

a result which is consistent with our result $g_0 \approx 0{,}4$ and with the data $|\, \varepsilon \,| \approx 2 \cdot 10^{-3}$. Let us now turn to ε' . The first non vanishing contribution to $\mathrm{Im}\, A_2$ is given by matrix elements of the type

$$< (2\pi)_{I = 2} \,|\, H_{\not{C}} \,|\, n > < n \,|\, H_{CP}|\, K^0 > \quad .$$

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*Indeed, $<2\pi|H_{\not{C}}|2\pi>=0$ because $H_{\not{C}}$ is odd under C; $<3\pi|H_{\not{C}}|2\pi>=0$ because $H_{\not{C}}$ conserves parity; $<3\pi|H_{\not{C}}|3\pi>$ is very small because in the 3π states of the K^0 decay we have perdominatly I=1, and hence they are almost pure C eigenstates with equal eigenvalues. The $<2\pi|H_{\not{C}}|\pi\ell\nu>$ and $<\pi\ell\nu|H_{\not{C}}|2\pi>$ matrix elements are zero because $H_{\not{C}}$ does not contain lepton operators. Finally, $<\pi^-\ell^+\nu|H_{\not{C}}|\pi^-\ell^+\nu>$ and $<\pi^+\ell^-\bar{\nu}|H_{\not{C}}|\pi^+\ell^-\nu>$ may arise only if the $\Delta S=\Delta Q$ rule is violated in the K^0 or in the $\bar{K}^0$ decay. The $\Delta S=\Delta Q$ allowed $K_{\ell 3}$ decays lead to $<\pi^-\ell^+\nu\,|H_{\not{C}}|\pi^+\ell^-\bar{\nu}>=0$ and to $<\pi^+\ell^-\bar{\nu}|H_{\not{C}}|\pi^-\ell^+\nu>=0$ because $H_{\not{C}}$ does not contain lepton operators.

Here again $|n\rangle$ cannot be 2π, 3π or $\pi\ell\nu$ state. Moreover, we see that if we respect the $\Delta I = 1/2$ rule in the K^o decay, then only g_1 or g_2 may be active in $H_{\not{C}}$. Thus we get

$$|\varepsilon'| \approx g_{1,2}\,\beta \quad \text{or} \quad |\varepsilon'| \approx g_o\,\beta'$$

with $\beta' \ll \beta$. Since in our model we have $g_o \approx 50\, g_2$ and $g_1 \approx g_2$, in both cases it is conceivable* that $|\varepsilon'| < \varepsilon$.

*In all these estimates divergent integrals are thought to be cut off and off shell matrix elements are supposed to behave nicely.

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FIGURE AND TABLE CAPTION

Figure 1 : The experimental $\eta\to\pi^+\pi^0\pi^-$ Dalitz plot [3] .

Table I. : Best fitting values of the reduced coupling constants γ_0 and γ_2 to the measured $\eta\to\pi^+\pi^0\pi^-$ asymmetry parameters for $a_0 = 0,2$, $\eta\to\pi^0 e^+ e^-$ limit: $|\gamma_0 + 2\gamma_2| \leq 5,6$

Table II. : Best fitting values of the reduced coupling constants γ_0 and γ_2 to the measured $\eta\to\pi^+\pi^0\pi^-$ asymmetry parameters for $a_0 = 0,5$, $\eta\to\pi^0 e^+ e^-$ limit: $|\gamma_0 + 2\gamma_2| \leq 5,3$

Table III.: Best fitting values of the reduced coupling constants γ_0 and γ_2 to the measured $\eta\to\pi^+\pi^0\pi^-$ asymmetry parameters for $a_0 = 1,0$, $\eta\to\pi^0 e^+ e^-$ limit: $|\gamma_0 + 2\gamma_2| \leq 4,6$

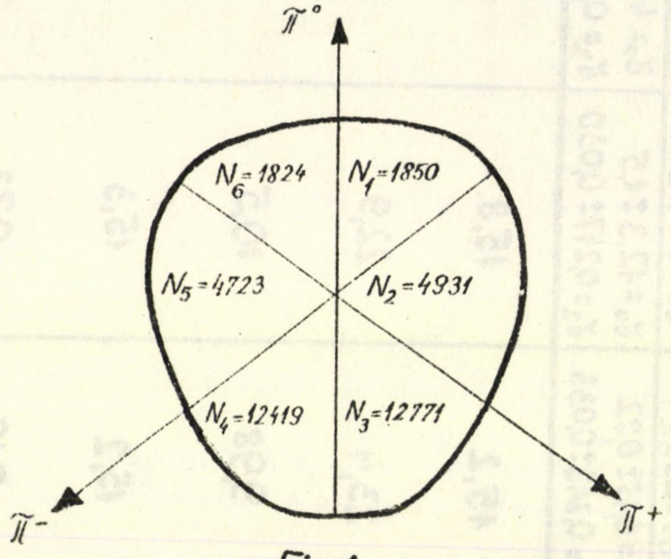

Fig.1

TABLE I. $a_0 = 0{,}2$ $b = 1{,}65$ $a = 0{,}375$ $g_{0,2} = 0{,}073\,\gamma_{0,2}$

Asymmetry parameters	Experiment [3] $10^{-3}\times$	Theory				
		$\gamma_2 = 0$	$\gamma_0 = 0$	$\gamma_0 = 25\gamma_2$	$\gamma_0 = 50\gamma_2$	γ_0 and γ_2 indep.
		$\gamma_0 = 54 \pm 86$ $\gamma_2 = 0$	$\gamma_0 = 0$ $\gamma_2 = 0{,}245 \pm 0{,}036$	$\gamma_0 = 6{,}45 \pm 0{,}82$ $\gamma_2 = 0{,}245 \pm 0{,}033$	$\gamma_0 = 12{,}3 \pm 1{,}5$ $\gamma_2 = 0{,}247 \pm 0{,}030$	$\gamma_0 = 40{,}7 \pm 13$ $\gamma_2 = 0{,}241 \pm 0{,}023$
Δ_1	$7{,}06 \pm 16$	4.66	14,6	15,2	15,8	18,0
Δ_2	$21{,}6 \pm 9$	-5,12	23,9	23,4	22,9	19,5
Δ_3	$13{,}9 \pm 6{,}4$	4,07	9,44	9,98	10,5	12,5
Δ	$15{,}2 \pm 5$	1,23	14,9	15,2	15,3	15,7
$\bar{\Delta}$	$4{,}4 \pm 5$	4,51	-0,35	0,19	0,73	3,2

TABLE II. $a_0 = 0,5$ $b = 1,95$ $a = 0,395$ $g_{0,2} = 0,077\, \gamma_{0,2}$

Asymmetry parameters	Experiment [3] $10^{-3}\times$	Theory				
		$\gamma_2 = 0$	$\gamma_0 = 0$	$\gamma_0 = 25\gamma_2$	$\gamma_0 = 50\gamma_2$	γ_0 and γ_2 indept.
		$\gamma_0 = 20 \pm 34$ $\gamma_2 = 0$	$\gamma_0 = 0$ $\gamma_2 = 0,0979 \pm 0,01$[illegible]	$\gamma_0 = 2,74 \pm 0,39$ $\gamma_2 = 0,0984 \pm 0,0150$	$\gamma_0 = 4,93 \pm 0,69$ $\gamma_2 = 0,0986 \pm 0,0138$	$\gamma_0 = 16,2 \pm 9,0$ $\gamma_2 = 0,0565 \pm 0,0131$
Δ_1	7,06±16	5,61	16,5	17,3	18,0	20,8
Δ_2	24,6±9	-5,06	23,9	23,4	22,8	19,5
Δ_3	13,9±6,4	3,82	8,62	9,13	9,62	11,5
Δ	15,2±5	1,27	14,7	15,0	15,2	15,5
$\bar{\Delta}$	4,4±5	4,50	-0,47	0,068	0,62	3,1

TABLE III. $a_0 = 1{,}0$ $b = 2{,}25$ $a = 0{,}459$ $g_{0,2} = 0{,}039\,\gamma_{0,2}$

Asymmetry parameters	Experiment [3] $10^{-3}\times$	Theory				
		$\gamma_2 = 0$	$\gamma_0 = 0$	$\gamma_0 = 25\gamma_2$	$\gamma_0 = 50\gamma_2$	γ_0 and γ_2 indept.
		$\gamma_0 = 9{,}1 \pm 13$ $\gamma_2 = 0$	$\gamma_0 = 0$ $\gamma_2 = 0{,}0409 \pm 0{,}0092$	$\gamma_0 = 1{,}23 \pm 0{,}22$ $\gamma_2 = 0{,}0492 \pm 0{,}0088$	$\gamma_0 = 2{,}47 \pm 0{,}41$ $\gamma_2 = 0{,}0494 \pm 0{,}0082$	$\gamma_0 = 8{,}22 \pm 5{,}84$ $\gamma_2 = 0{,}0488 \pm 0{,}00825$
Δ_1	$7{,}05 \pm 15$	5,88	18,4	19,3	20,1	23,5
Δ_2	$24{,}6 \pm 9$	−4,56	23,8	23,4	22,8	19,6
Δ_3	$13{,}9 \pm 6{,}4$	2,91	7,54	7,98	8,4	10,1
Δ	$15{,}2 \pm 5$	4,04	14,5	14,7	14,9	15,3
$\bar{\Delta}$	$4{,}4 \pm 5$	3,93	−0,61	−0,082	0,45	2,9