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EXPERIMENTAL DETERMINATION OF THE RATIO β^{eff}/l ON THE ZR-1 CRITICAL ASSEMBLY

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S u m m a r y

The parameter β^{eff}/l has been experimentally determined on the light water moderated, enriched uranium fuelled, heterogeneous critical assembly ZR-1 using both the Feynman and the Rossi methods. It was found that the Rossi method can be used in the case of thermal reactors with hydrogeneous moderator and enriched fuel provided the average neutron lifetime is not larger than 10^{-4} sec. Actually, the latter of the two methods proved to be faster and more reliable and even the effect of the delayed neutrons does not disturb the evaluation of the data measured.

EXPERIMENTAL DETERMINATION OF THE RATIO β^{eff}/l ON THE ZR-1 CRITICAL ASSEMBLY

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1. Introduction

There are several methods available for the experimental determination of the ratio β^{eff}/l [x] which is the characteristic parameter of the dynamic behaviour of a reactor. On the whole, these procedures can be divided into two groups:

a/ perturbation methods
b/ statistical methods.

The perturbation methods are based on the analysis of phenomena observed upon perturbation of the steady state of a reactor. One alternative is to determine the transfer function describing the dynamical features of the reactor, including the ratio β^{eff}/l in question from the effect of the reactivity change induced periodically or by a single pulse. The other one is to analyse the time decay of the neutron flux in a subcritical reactor into which a neutron pulse was injected and to determine the largest relaxation constant $\omega_6 = \alpha$, the reciprocal of which is called the prompt period. The ratio β^{eff}/l is then computed from the value of α provided the value of the negative reactivity $\gamma = \frac{1-k}{k\,\beta^{eff}}$ is known.

The statistical methods deal with the time dependent fluctuations of the neutron flux in nearly critical /or critical/ steadily operating reactors. The short-term fluctuations are determined by the highest relaxation constant from which the ratio β^{eff}/l can be computed.

In the following the results obtained by the application of two statistical methods, namely the Feynman and the Rossi methods, will be discussed.

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x β^{eff} is the effective fraction of delayed neutrons, while $l = \tau/k$ is the ratio of average neutron lifetime to multiplication factor.

2. Theory

2.1 The relaxation constants describing the dynamic behaviour of a reactor are known to be given by the roots $\omega_0 \leq \omega_1 \leq \ldots \leq \omega_6$ of the equation

$$\frac{\beta^{eff}}{\ell}\left(1+\gamma\right)-\omega=\frac{\beta^{eff}}{\ell}\sum_{i=1}^{6}\frac{\lambda_i \beta_i}{\beta(\lambda_i-\omega)} \qquad /2.1/$$

where γ is the negative reactivity expressed in \$, while λ_i is the decay constant for nuclei emitting delayed neutrons of type i . It can be shown that for $\gamma > 0$ the value of ω_6 is given with good approximation as

$$\frac{\beta^{eff}}{\ell}\left(1+\gamma\right)-\omega_6=0 \qquad /2.2/$$

In the following the notation $\omega_6=\alpha$ will be used. It is apparent from Eq. /2.2/ that the value of the relaxation constant extrapolated to $\gamma=0$ gives actually the value of β^{eff}/ℓ .

A change in the value of γ would imply in principle a change in the ratio β^{eff}/ℓ as well. This means that one has to choose a procedure leaving the ratio β^{eff}/ℓ unaffected by, or hardly sensitive to any reactivity change. This requirement can be met if the change in reactivity is brought about by the uniform distribution of an additional absorbent. It is of interest to note that the ratio β^{eff}/ℓ varies only slightly with the reactivity for quite wide a range of reactivity values, so that the ratio determined for a subcritical state can be considered as valid for the supercritical one too.

In the present experiments two methods were used to determine the value of α :

a/ a direct /Feynman/ method and
b/ a correlation /Rossi/ method.

2.2 a/ In principle the Feynman method /Hoffmann,1949, Feynman et al., 1956, Clouet et al., 1959, Bennett, 1960, Albrecht, 1962/ is based on the determination of the variance of the number of neutrons counted during the time Δt by a highly sensitive detector placed into a steadily operating subcritical /near critical/ reactor containing a neutron source. The variance can be expressed /Pál, 1962/ by

$$D(\Delta t)=N\Delta t\left\{1+\varepsilon\sum_{j=0}^{6}D_j\psi(\omega_j\Delta t)\right\}=N\Delta t(1+\varepsilon\phi)\,, \qquad /2.3/$$

where N is the expected number of neutrons recorded per unit time, ε is the detector efficiency, while

$$\psi(\omega_j \Delta t) = 1 - \frac{1-e^{-\omega_j \Delta t}}{\omega_j \Delta t} \; . \qquad /2.4/$$

The amplitudes D_j depend, in addition to the reactivity, on the dynamic parameters of the reactor and on the variance of the number of neutrons per fission as well./For the full expression see Pál, 1962./

One of the authors /Pál, 1963/ has thorougly investigated the contribution of the effect of delayed neutrons to the value of the variance under various conditions. It has been found that by an appropriate choice of sufficiently small measuring intervals, the terms due to the delayed neutrons could be neglected. In this case we find

$$D(\Delta t) \sim N\Delta t \left[1 + \varepsilon D_6 \, \psi(\omega_6 \Delta t)\right] \qquad /2.5/$$

which enables one to use the simple relation

$$\frac{D}{M} = 1 + \varepsilon \, \frac{\nu_{00} - \nu_0}{\nu \ell \alpha^2} \left(1 - \frac{1-e^{-\alpha \Delta t}}{\alpha \Delta t}\right) \qquad /2.6/$$

for determining the value of α . In the expression /2.6/ ν_{00} is the second moment of the number of prompt neutrons produced per fission, while ν_0 is their expected value.

The ratio $R = \frac{D}{M}$ can be estimated from the number of counts $m_1, m_2, \dots, m_n$ recorded in the intervals Δt separated from each other by a given waiting time Θ . The following expressions can be used for the estimate $\tilde{R}$ of the ratio R :

$$\tilde{R} = \frac{1}{n\tilde{M}} \sum_{i=1}^{n} (m_i - \tilde{M})^2 \quad \text{and} \quad \tilde{M} = \frac{1}{n} \sum_{i=1}^{n} m_i \qquad /2.7/$$

The variance of the estimate $\tilde{R}$ can be calculated with help of the formula:

$$\langle (\delta \tilde{R})^2 \rangle \sim \frac{2\tilde{R}}{n} \left(1 + \frac{1}{2} \frac{\tilde{R}}{\tilde{M}}\right) + O\left(\tilde{M}^{-3/2}\right) \; . \qquad /2.8/$$

The expressions /2.7/ cannot be used but in the case when the correlation between the numbers of counts recorded in the successive intervals Δt separated by the waiting time Θ , is negligibly small. In order to choose the most appropriate waiting time we have carried out investigations /Pál, 1963/ for determining the dependence of the correlation on the waiting time.

Let $\tilde{R}_{ik}$ be the relative variance for the interval Δt_{ik} and reactivity γ_i . Assuming the distribution of $\tilde{R}_{ik}$ to be approximately normal, the parameters

$$\frac{\beta^{eff}}{\ell} = a_1 \qquad \text{and} \qquad \varepsilon \frac{\nu_{00} - \nu_0}{\nu \ell} = a_2 \tag{2.9}$$

can be estimated from the expression

$$Q = -\frac{1}{2} \sum_{i=1}^{S} \sum_{k=1}^{S_i} w_{ik} \left[\tilde{R}_{ik} - 1 - a_2 \alpha_i^{-2} \psi(\alpha_i \Delta t_{ik}) \right]^2 , \tag{2.10}$$

where the weighting factor w_{ik} is defined as

$$w_{ik} = \frac{1}{\langle (\delta \tilde{R}_{ik})^2 \rangle} \tag{2.11}$$

and

$$\alpha_i = a_1 (1 + \gamma_i) . \tag{2.12}$$

The estimates of the parameters a_1 and a_2 are given by the roots $\tilde{a}_1$ and $\tilde{a}_2$ of the equations

$$\frac{\partial Q}{\partial a_\mu} = F_\mu = 0 \qquad (\mu = 1,2) \tag{2.13}$$

minimizing the expression of Q. The r-th iterative of the root $\tilde{a}_\mu$ can be computed from the relation

$$\tilde{a}_\mu(r) = \tilde{a}_\mu(r-1) + \sum_{\mu'=1}^{2} V_{\mu\mu'} \left[\underline{\tilde{a}}(r-1) \right] F_{\mu'} \left[\underline{\tilde{a}}(r-1) \right] , \qquad (\mu = 1,2) \tag{2.14}$$

where $V_{\mu\mu'}$ is the corresponding element of the inverse of the matrix $\underline{S}$ with the elements

$$S_{\mu\mu'} = \left\langle \frac{\partial F_\mu}{\partial a_{\mu'}} \right\rangle \tag{2.15}$$

and

$$\underline{\tilde{a}} = (\tilde{a}_1 , \tilde{a}_2) .$$

If reasonable initial values $\tilde{a}_\mu(0)$ $(\mu = 1,2)$ required to start the iteration are assumed, a few steps lead to satisfactory results. The variance of the parameter $a_1 = \frac{\beta^{eff}}{\ell}$, we are interested in, is given by the formula

$$\langle (\delta \tilde{a}_1)^2 \rangle = -V_{11} . \tag{2.16}$$

2.3 b/ The Rossi method /Orndoff, 1957; Ingram et al., 1959/ is based on the fact that the consecutive neutron counting events are usually not independent. The correlation between two events is the stronger the closer they are in time. Let $C(t)dt$ be the conditional probability that a count in the time interval $(t, t+dt)$ follows a count recorded at time $t=0$. $C(t)$ is, in fact, the expected number of counts per unit time at time $t>0$, if a count has occurred at time $t=0$. It was shown by one of the authors /Pál, 1963/ that

$$C(t) = N + \frac{1}{2}\varepsilon\sum_{j=0}^{6}\omega_j D_j e^{-\omega_j t} \quad . \qquad /2.17/$$

Since for small values of t the variation in $e^{-\omega_6 t}$ is large compared to that in the other exponential functions, it can be written that

$$C(t) \sim b_1 + b_2 e^{-b_3 t} \quad , \qquad /2.18/$$

where

$$b_1 = N + \frac{1}{2}\varepsilon\sum_{j=0}^{5}\omega_j D_j \quad , \quad b_2 = \frac{1}{2}\varepsilon\,\omega_6 D_6 \quad , \quad b_3 = \omega_6 = \alpha \quad . \qquad /2.19/$$

The time analyser used in our measurement is triggered by a signal of the neutron detector. The probability that at least one count falls after the triggering pulse into the i -th channel of width Δt beginning at the time $t_{i-1} = \Delta t(i-1)$ is given by

$$p_i = 1 - e^{-B_i} \qquad /2.20/$$

where

$$B_i = \int_{(i-1)\Delta t}^{i\Delta t} C(t')\,dt' \quad . \qquad /2.21/$$

At the end of an analysing cycle the analyser comes to a standstill and a new triggering count is necessary to start the next cycle. Thus the analysing cycles follow one another at random.

Let the analyser operate in a way that in any individual analysing cycle no count or only a single count is recorded in any of the channels. Assuming now the total number of analysing cycles to be Z , the probability of finding n_1 counts in the first channel, n_2 in the second channel, $\ldots, n_S$ in the S -st channel is given by

$$P_z(\underline{b}, n_1, \ldots, n_s) = \prod_{i=1}^{s} \binom{z}{n_i} \left(1 - e^{-B_i}\right)^{n_i} e^{-(z-n_i)B_i} \quad . \qquad /2.22/$$

The estimates of the parameters b_1, b_2, b_3 can be obtained by determining the roots $\tilde{b}_1, \tilde{b}_2, \tilde{b}_3$ of the equations

$$\frac{\partial}{\partial b_\mu} \ln P_z(\underline{b}, \underline{n}) = F_\mu(\underline{b}, \underline{n}) = 0 \quad . \qquad /2.23/$$

$$(\mu = 1, 2, 3)$$

They are readily evaluated by iteration. The variance giving the error of the estimated values can be computed simultaneously /Pál, 1963/

3. The critical assembly ZR-1 and the measuring apparatus

3.1 The heterogeneous critical assembly, called ZR-1, is a light water moderated and reflected enriched uranium system. Cylindrical fuel elements /ЭК-10/, 10 mm in diameter, and of 500 mm effective length with aluminium cladding of 1,5 mm, and filled with a mixture of UO_2 and magnesium are used. Each fuel element contains from 79 to 80 g uranium enriched in U-235 to 10%. A full description of the ZR-1 assembly has been already given /Gyimesi et al., 1961/.

In our experiments a square lattice of 17,5 mm spacing was used. The whole experimental setup including fuel elements, control and safety rods as well as the counter detector can be seen in Fig.1. A CHM-5 type BF_3 counter serves as detector. The position of the latter was chosen in order to prevent the takeup of too much reactivity that might lead to a scram should the detector be inadvertently removed. In the arrangement shown in the figure the detector compensates about 0,35 ¢ reactivity.

The Po-Be neutron source about 2C was introduced into an aluminium tube along the vertical axis of the core. The position of the source within the tube could be easily varied.

3.2 The units used in both kinds of experiments are the low noise preamplifier mounted close to the neutron counter, the amplifier and the discriminator.

The output pulses from the discriminator are analysed in the Feynman method by an automatic counter of special design. The pulses due to the neutron counts are passed to the pulse counter S through the gate G kept open for a preset time interval Δt determined by the timing unit T . The number of pulses accumulated during time Δt is read out and recorded on a tape by the printing device P . The block diagram of the measuring apparatus is shown in Fig.2. The time sequence of the various operations can be seen in Fig.3. The measuring intervals Δt and the waiting time Θ which may be chosen independently of one another are determined by the timer T . The latter is a quartz-controlled device which enables to vary both Δt and Θ in a range from 10^{-4} to 10^{2} sec. The leading edge of the output pulse of the timer T /see Fig.3/ opens the gate G and the pulses of the neutron detector arriving during time Δt are counted by the counter S . The trailing edge representing the end of the interval Δt , closes the gate G and acts on the printing device P which records the number of pulses stored by the counter and sets the latter to zero. This operation is performed during the waiting time Θ . By the time of the next opening of the gate G the counter S is empty and ready for storing the pulses so the whole process repeats itself automatically. The number of repetitions is recorded by a separate counter.

3.3 The 20+4 channel time analyser /AIDA/ used for the Rossi experiments is particularly suitable for analysing decay processes since the channel widths can be varied in a wide range $(\Delta t = (1,2,4)10^{n} \mu sec\ n=1-6)$/Szlávik,1962/. The analyser has several modes of operation. In our experiments one of the pulses to be analysed was chosen to serve as start signal. The pulses arriving later are counted by the successively opening channels. Right after the operation of the 20-th channel, the control and selection circuits of the analyser are restored during an interval of $t_d = 50$ μsec to the initial position. Pulses which may arrive during this interval are prevented by a gating circuit from entering any of the channels. The first pulse arriving after the interval t_d triggers the next analysing cycle. The time sequence of the analysing process is shown in Fig.4. The channels are switched and their widths are set by means of the 1 Mc/sec frequency quartz-controlled timing device. A separate counter is used to record the number of cycles per run. Under our experimental conditions it was advantageous to set the analyser to that mode of operation in which during one cycle not more than one count per channel is recorded. In this case if the pulse rate is high a saturation effect can be observed. The saturation effect was taken into account in the evaluation of the measured data.

4. Measurements and results

4.1 Both for the Feynman and the Rossi methods the negative reactivity in the ZR-1 assembly was set to values varying in a fairly wide range. Since the values obtained using the control rod were not satisfactorily reproducible, calibrated absorbent rods were used instead. These small rods were calibrated by measuring the change in the asymptotic period of the reactor in supercritical state after the insertion of each rod into a given place of the core.

Several rods of this kind were then inserted into the slightly supercritical reactor to obtain different values of negative reactivity. /In some cases calibrated fuel elements on the core boundary were used for varying the reactivity of the system./ The initial supercritical state could be reproduced accurately, since it was set by the complete withdrawal of the safety and control rods from the reactor. The reactivity characterizing the supercritical state could be calculated from the value of the asymptotic period measured with great precision by the automatic counter used in the Feynman experiment.

The assembly being nearly critical, particular care had to be taken to prevent any disturbing effect occurring in the reactor during the measurement lasting several hours /e.g. the value of the temperature as well as the position of the neighbouring objects acting as possible neutron reflectors had to be kept unaltered etc/. This was particularly important when the Feynman method was used, since at each reactivity the measurements needed 12 to 16 hours.

4.2 a/ In the Feynman experiments measurements were taken at four different values of reactivity. The time intervals covered at each reactivity are listed in Table I. The number of runs per interval was 2.10^3 and the waiting time between the measuring intervals was set at $\Theta = 3$ sec. In this case the correlation between the numbers of counts in consecutive time intervals can be neglected.

$\gamma = 0,834$	0.570	0,464	0,133
Δt in msecs			
4	2	2	2
7	4	4	4
10	6	6	6
20	8	8	8
30	10	10	10
50	15	15	14
70	20	20	18
90	30	30	22
110	50	40	26
-	70	50	30

Table I.
The set of measuring intervals

Since with the decrease of γ neutron multiplication becomes excessive, the position of the Po-Be neutron source was chosen so that the effect of detector dead time should be negligible. For calculating the ratios $\tilde{R}_{ik}$ and $\langle(\delta\tilde{R}_{ik})^2\rangle$ the measured values were fed into a Bull-Gamma 3 computer. The values of $\tilde{R}_{ik}$ versus measuring time are plotted in Fig.5. The effect of delayed neutrons clearly manifests itself on the curve with $\gamma = 0,133$. The ratio $\frac{\beta^{eff}}{l} = a_1$ was calculated from those $\tilde{R}_{ik}$ values only which lie in the figure within the dashed line contour. This ratio was determined by making use of the iteration formula /2.14/ using for the computations our Ural-I type electronic computer. As a result we obtained

$$\frac{\beta^{eff}}{\ell} = 140,4 \pm 6,6 \ \ sec^{-1}.$$

The values of α obtained at various reactivites are listed in Table II.

γ= 0,834	0,570	0,464	0,133
α= 265,2±12,5	235,4±11,1	209,3±9,9	160,6±7,6

Table II.

The dependence of the values of α on the reactivity

4.3 b/ Owing to the relatively long neutron lifetimes / $\ell = 10^{-3}$ - 10^{-4}sec/ it is rather difficult to apply the Rossi method in the case of thermal reactors. Nevertheless, in small thermal reactors with hydrogeneous moderator and enriched uranium fuel the value of ℓ seems to be short enough to prevent the overlap of the decay term $b_2 e^{-b_3 t}$ under suitable experimental conditions by the term b_1 in/2.17/.Favourable conditions obviously prevail if the neutrons in the reactor are due to a few reaction chains only. For this reason investigations were carried out to see in what measure does the exponential term dominate in /2.17/ at various source strengths for $\gamma = 0,358$. It is apparent from Fig.6. that for low source strengths the individual neutron chains do not overlap each other appreciably and the exponential law of decay can be well observed. On each curve the number of counts per unit time is indicated for various source strengths. The channel width was chosen to be $\Delta t = (1 \pm 0,0001).10^{-3}$ sec.

Runs were made at 7 different values of negative reactivity varying the position of the source in order to keep the number of counts per sec below 80. In this case the number of counts showing some correlation exceeds the background counts in the first channel by a factor of about 3. The value of $b_3 = \alpha$ was determined from the equations 2.22 using the Ural-I type computer. Knowing thus the values $\tilde{b}_1$, $\tilde{b}_2$ and $\tilde{b}_3$ the corrected value of the number of counts per cycle is obtained from the formula

$$n_i^{corr} = -\ell n\left(1 - \frac{n_i}{z}\right) - \tilde{b}_1 \Delta t \ .$$

This number is seen to decay exponentially in terms of the increasing channel number. The results are shown in Fig.7.

The variation in the value of α with γ is plotted in Fig.8.The prompt relaxation constant α_0 extrapolated to $\gamma = 0$, gives the parameter β^{eff}/ℓ. The ratio was computed from the values of α using the method of least squares and was found to be

$$\frac{\beta^{eff}}{\ell} = 150{,}65 \pm 0{,}65 \quad sec^{-1}$$

which is significantly higher than the value obtained by the Feynman method.

5. Conclusion

In our investigations the Rossi method proved to be the more satisfactory of the two methods. It permits quicker and more accurate measurements, further the effect of delayed neutrons is negligible for analysing times of the order of 2.10^{-2} sec. The Feynman method, on the other hand, seems to necessitate the use of measuring intervals above 2.10^{-2} sec too. Therefore the neglect of the effect of the delayed neutrons is responsible for the somewhat lower value of the ratio β^{eff}/ℓ determined by the Feynman method.

Finally, it can be stated that the Rossi method is a satisfactory one for the determination of the parameter β^{eff}/ℓ in the case of reactors with hydrogeneous moderator and enriched uranium fuel.

R e f e r e n c e s

Albrecht,R.W., Nucl.Sci.Engng. 14, 153. /1962/
Bennett,E.F., Nucl.Sci.Engng. 8, 53. /1960/
Clouet d'Orval C., Deilgat,E., Labbe,J., Molbert,M., and Tachon,J., Étude de la fluctuation de la population de neutrons sur une pile homogene de plutonium, Note E.C./S. No. 28./1959/
Feynman,R.P., de Hoffmann F., and Serber,R., J.Nucl.Energy,3,64./1956/
Gyimesi Z., et al. International Reactor Conference, Bucharest,/1961/
de Hoffmann F., The Science and Engineering of Nuclear Power Vol.II., Addison Wesley Press, Cambridge, Mass./1949/
Ingram,G., McCulloch D.B., and Sanders,J.E., J.Nucl.Energy A10,22,/1959/
Orndoff,J.D., Nucl.Sci.Engng.2,450,/1957/
Pál,L., Acta Phys.Hung.14,345; 356; 369. /1962/
Pál L., Preprint /1963/
Szlávik F., and Pálmai I., Kernenergie 6, /1963/

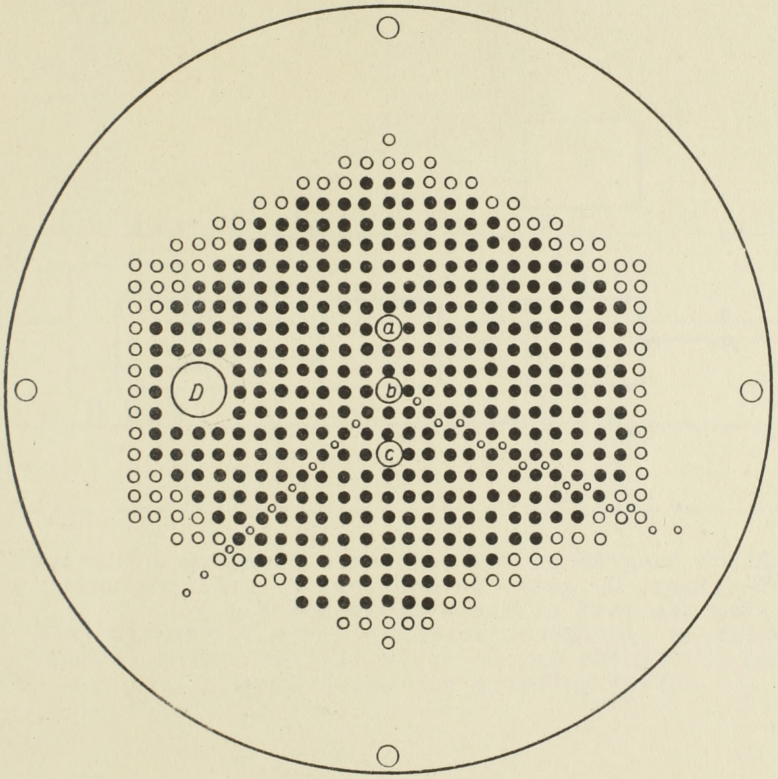

Fig.1 Horizontal section of the core in the ZR-1 critical assembly. The fuel elements are represented by dark circles. D- neutron detector, a- safety rod, b- tube holding neutron source, c- control rod

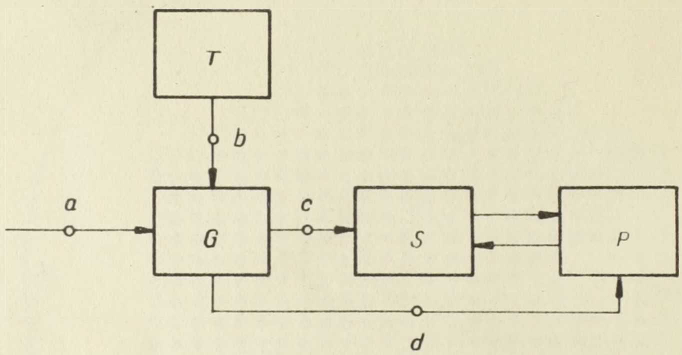

Fig.2 Block diagram of the automatic counting apparatus.
T- timer, G- gate, S- counter, P- printing unit
/For the rest of the symbols see Fig.3/

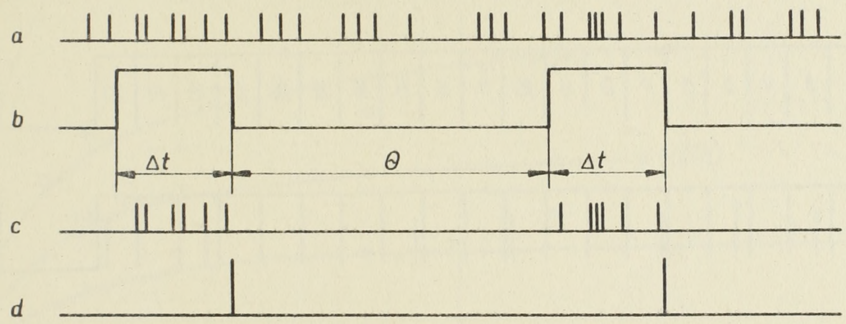

Fig.3 Time diagram of the automatical counting. a- detector pulses, b- timing pulses, c- stored pulses, d- pulses controlling the printing device

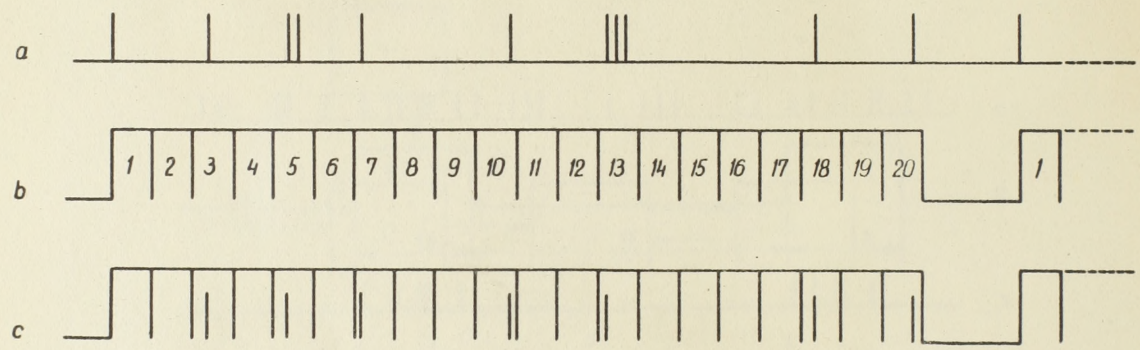

Fig.4 Time diagram of an analysing cycle. a- detector pulses, b- storing channels, c- stored pulses

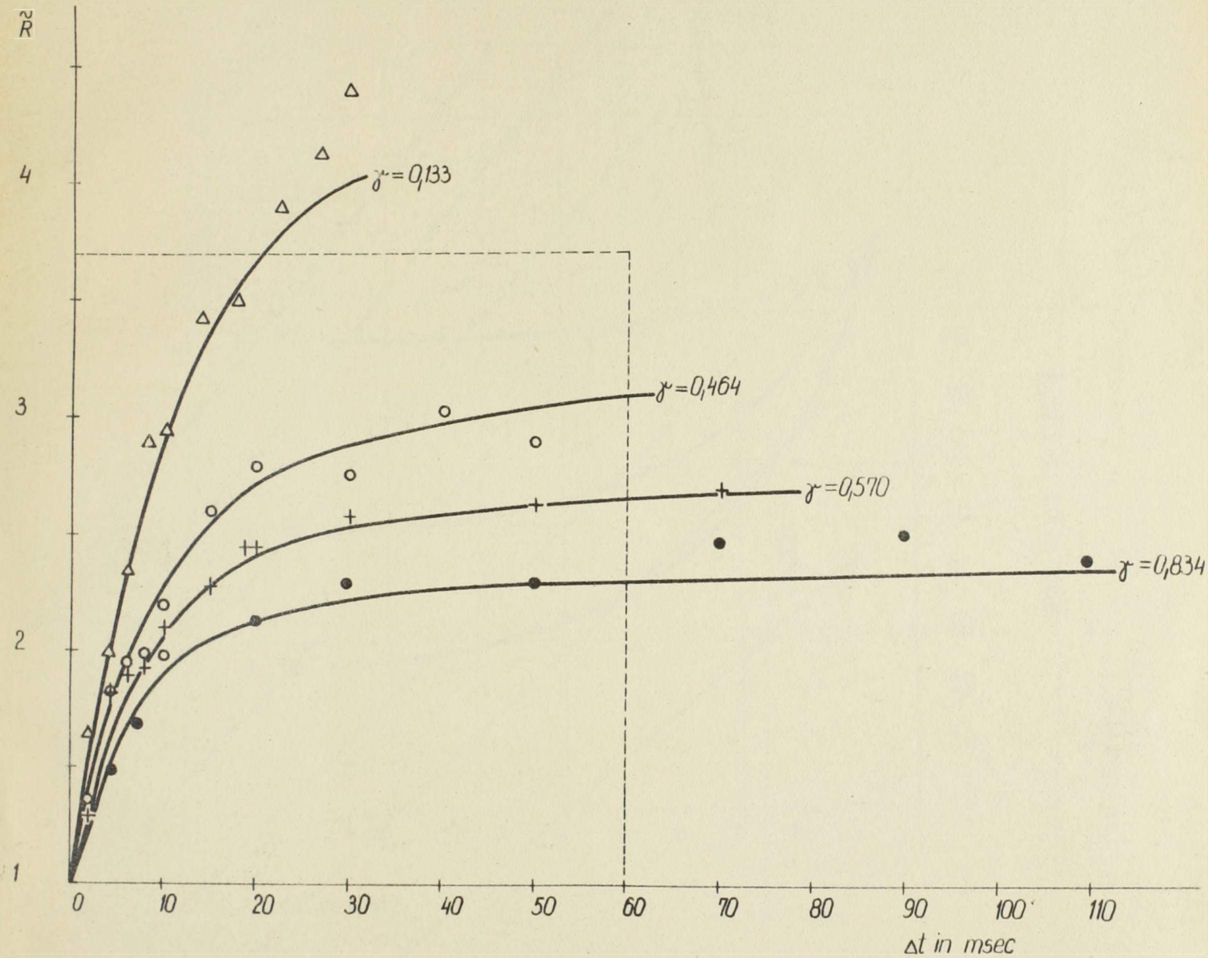

Fig.5 Relative variance of the number of counts versus counting time at various negative reactivities. /For simplicity of illustration the statistical error in the values of $\tilde{R}_{ik}$ is not indicated./

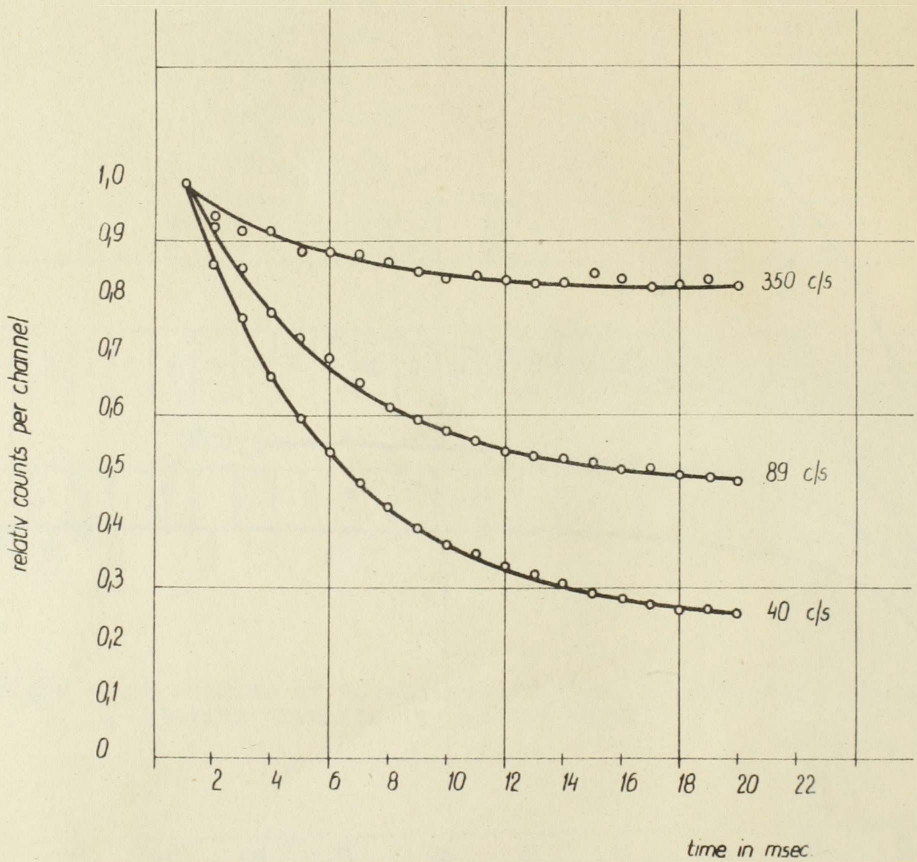

Fig.6 Decay curves for various source strengths./Increasing source strengths are seen to obscur the exponential law of decay./

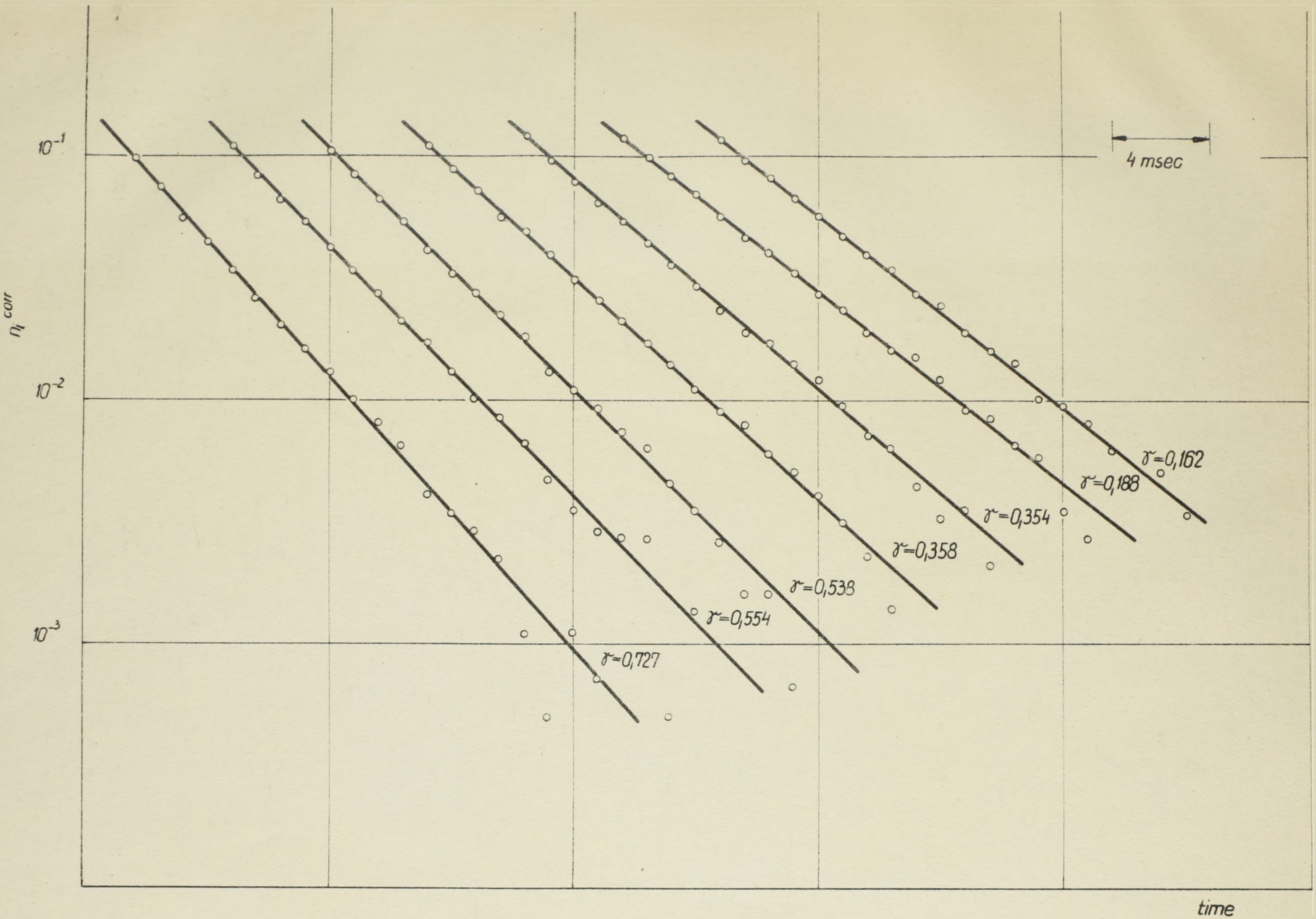

Fig.7 Expected number of counts in a channel per analysing cycle vs delay time at various negative reactivities. For simplicity of illustration the initial point of each decay curve has been shifted in time.

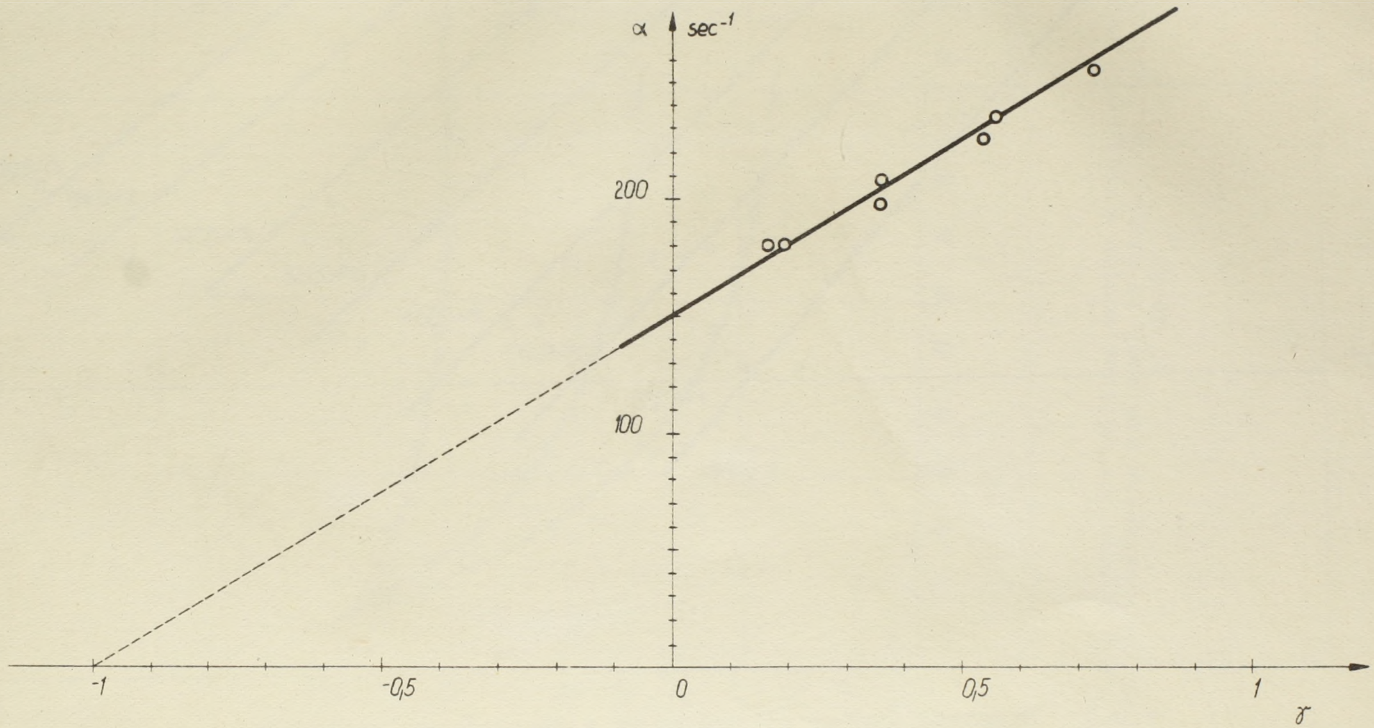

Fig.8 Determination of the ratio β^{eff}/l from the γ dependence of the parameter α.