KFKI
2/1969

MILLISTRONG C VIOLATION AND η DECAY

A.Frenkel*
and
G.Marx
Institute for Theoretical Physics of the Roland Eötvös University
Budapest

*HUNGARIAN ACADEMY OF SCIENCES
CENTRAL RESEARCH INSTITUTE FOR PHYSICS

BUDAPEST

Millistrong C Violation and η Decay *

by

A.Frenkel

Central Research Institute for Physics of the Hungarian Academy of Sciences

and

G.Marx

Institute for Theoretical Physics of the Roland Eötvös University

Budapest

The isospin properties of the /assumed/ millistrong C violation have been investigated. The consequences of a $\Delta I = 0$ and $\Delta I = 2$ selection rule are discussed for the $\eta \to \pi^+\pi^0\pi^-$ and $\eta \to \pi^0 e^+ e^-$ decays. If the observed charge asymmetry of the $\eta \to \pi^+\pi^0\pi^-$ is realistic, the C violating 3π final state is mainly an $I = 2$ eigenstate, produced by a $\Delta I = 0$ coupling with a constant $g_2 \simeq 10^{-2}$. Since the $I = 0$ eigenstate is here very much suppressed by the centrifugal barrier, the $\Delta I = 0$ coupling may be much stronger, e.g. $g_0 > 10^{-1}$, limited only by the negative results of the $\eta \to \pi^0 e^+ e^-$ experiments. So the $\Delta I = 0$ dominance of the millistrong C violation, suggested by the indication $|\varepsilon| \gg |\varepsilon'|$ in the K_L^0 decays, is by no means in contradiction with the observed $\Delta I = 2$ character of the $\eta \to \pi^+\pi^0\pi^-$ asymmetry. The importance of more accurate experiments is emphasized.

1. Possible Models of CP Violation

The CP violation has been discovered in the decays of the K_L^0 meson / $K_L^0 \to \pi^+\pi^-, \pi^0\pi^0, e\pi\nu, \mu\pi\nu$ /. Assuming a CPT symmetry, in the case of a complete phenomenological analysis of the K^0 system two pieces of information may be extracted concerning the nature of the CP violation: the complex parameters ε and ε'. These are defined as follows:

$$2\left[m_L - m_S - \frac{i}{2}\left(\frac{1}{\tau_L} - \frac{1}{\tau_S}\right)\right]\varepsilon = \langle K^0|H'+\ldots|\bar{K}^0\rangle - \langle \bar{K}^0|H'+\ldots|K^0\rangle \qquad /1/$$

$$\sqrt{2}\,|\varepsilon'| = \left| \mathrm{Im}\, \frac{\langle \pi\pi, I=2|H'+\ldots|K^0\rangle}{\langle \pi\pi, I=0|H'+\ldots|K^0\rangle} \right| . \qquad /2/$$

Here H' is the sum of the weak and of the CP violating Hamiltonians, $m_{L,S}$ and $\tau_{L,S}$ are the masses and life times of the neutral $K_{L,S}^0$ mesons.

* To be published in Acta Physica Hungarica. ITP-Report N°250, Budapest.

The actual values of ε and ε' are still uncertain, but according to the most popular guess [1]

$$|\varepsilon| = 2.10^{-3} >> |\varepsilon'| \qquad /3/$$

/The published experimental data [1] seem to be still in conflict with each other, so the possibility $|\varepsilon| \approx |\varepsilon'|$ is not yet completely excluded./ From the equ. /1/ we can deduce the following alternative conclusions:

I. If the CP violating Hamiltonian is characterized by the strangeness selection rule $\Delta S = 2$,then the right hand side of equ. /1/ may be interpreted as a first order expression, consequently $(f_w \sin\Theta)^2\varepsilon \approx g$. /Here g is the coupling constant in the CP violating Hamiltonian, f_w is the weak coupling constant and Θ is the Cabibbo angle./ In this case the CP violation is about 10^4 times weaker than a second order weak interaction. /Superweak CP violation./

II. If $\Delta S = 1$, one gets $\Delta S = 2$ on the right hand side of equ. /1/ as the result of an interplay between weak and CP violating interactions, consequently $(f_w \sin\Theta)^2\varepsilon \approx g\, f_w \sin\Theta$. In this case the CP violating coupling may be about 1000 times weaker than the weak interaction. /Milliweak CP violation, according to the nomenclature of L.B. Okun./

III. If $\Delta S = 0$ for the C violating Hamiltonian, one reads from equ. /1/ that $(f_w \sin\Theta)^2\varepsilon \approx g (f_w \sin\Theta)^2$. The C violating coupling is now of the order of $|\varepsilon| \approx 2.10^{-3}$. /Millistrong C violation./

IV. If $\Delta S = 0$ but the emission of one photon is associated to the C violating vertex, this photon must be reabsorbed by a conventional electromagnetic interaction, so from equ. /1/ one arrives at the estimation $(f_w \sin\Theta)^2\varepsilon \approx e\, g (f_w \sin\Theta)^2$, which may be solved by writing $g = e$ because $e^2 = 4\pi \,.\, 137^{-1}$. /Electromagnetic C violation./

V. If $\Delta S = 1$, but the CP violation is associated to the emission of one photon, equ. /1/ gives $(f_w \sin\Theta)^2\varepsilon \approx e\, g (f_w \sin\Theta)$, which may be solved by writing $g = e f_w \sin\Theta$. /Weak electromagnetic CP violation./

The estimation of g is evidently only a very rough one. The values of ε and ε' are sensitive not only to the S character but also to the I character of the CP violating Hamiltonian. The most interesting possibilities are listed on the Table I. In each columne we have indicated the most crucial tests of type of CP violation under consideration. The most important implications are the following:

a/ Very strict restrictions are offered by the electric dipole moment of the neutron. By dimensional arguments the electromagnetic model of C violation predicts $d_n \approx f_w M_n^{-1} = 10^{-19}$ cm. A more detailed calculation suggest 10^{-21} cm in the same model [11]. The millistrong prediction is something like $d_n \approx |\varepsilon| f_w M_n^{-1} = 10^{-22}$ cm. The most recent experimental value is $d_n = (2 \pm 2) 10^{-23}$ cm [1]. This apparently rules out the possibility of an electromagnetic C violation with $g = e$.

b/ The experimental upper limit on the C violating decay $\eta \to \pi^o e^+ e^-$ is [13]

$$B\left(\eta \to \pi^o e^+ e^-\right) = \Gamma\left(\eta \to \pi^o e^+ e^-\right) : \Gamma(\eta) < 10^{-4} . \qquad /4/$$

This is in conflict with the prediction of electromagnetic C violation with $\Delta I = 1$.

c/ All the decays $\eta \to 3\pi$ $(I_f = 0,2)$, $\omega \to 3\pi$ $(I_f = 1,3)$, $\rho \to 3\pi$ $(I_f = 1,3)$ do violate the C symmetry. An interference between the C conserving and C violating amplitudes can produce a charge asymmetry

$$\Delta = \frac{N\left(E_+ > E_-\right) - N\left(E_- > E_+\right)}{N\left(E_+ > E_-\right) + N\left(E_- > E_+\right)} \qquad /5/$$

which is of the first order in the ratio of the C violating and C conserving coupling constants g/G. The most appropriate situation is offered by the $\eta \to \pi^+ \pi^o \pi^-$ decay, where the background may be /more or less/ eliminated by making use of the small η width,* and where the C violation is competing with a C-symmetric, but I-asymmetric transition. As a matter of fact, from the analysis of 36800 $\eta \to \pi^+ \pi^o \pi^-$ decays an asymmetry

$$\Delta = \left(1,52 \pm 0,5\right) \% \qquad /6/$$

has been found [14], which differs from zero by three standard deviations.

d/ The different I and S selection rules predict different inequalities /or approximate equalities/ for ε and ε'. If we accept the result /3/ as a realistic one, only three possibilities of the Table I must be kept in mind:

1/ Superweak CP violation $(\Delta S = 2)$,

2/ Milliweak CP violation $(\Delta S = 1$, $\Delta I = 1/2)$,

3/ Millistrong C violation $(\Delta S = 0$, $\Delta I = 0)$.

* Recently it has been pointed out by H.Yuta and S.Okubo [15], that in the experiment [14] the asymmetry may be caused by a 10 % background effect. This worning must be taken into account in further experimental work. In our paper we arbitrarily suppose that the bulk of the asymmetry is due to the C violation [1].

If we consider the electromagnetic model of C violation as excluded by the experiments, an $\eta\to\pi^+\pi^0\pi^-$ charge asymmetry of the order of % or ‰ may be given only by the millistrong models $\Delta I = 0$ or $\Delta I = 2$. The symmetry properties of the $\pi^+\pi^0\pi^-$ final state may be visualized on the Dalitz plot /Fig.1./. The N_i denote the number of $\eta\to\pi^+\pi^0\pi^-$ events in the i-th sextant of the Dalitz domain. The measured values are [14]:

$$N_1 = 1850 \qquad N_4 = 12419$$
$$N_2 = 4931 \qquad N_5 = 4723 \qquad /7/$$
$$N_3 = 12771 \qquad N_6 = 1824$$

The asymmetry Δ is defined according to equ. /5/ as

$$\Delta = \frac{N_1 + N_2 + N_3 - N_4 - N_5 - N_6}{N_1 + N_2 + N_3 + N_4 + N_5 + N_6} \qquad /8/$$

and the numbers /7/ give the asymmetry value quoted under /6/. It has been emphasized by M.Nauenberg, that in the case of a pure $I = 0$ final state the odd-even " sextant asymmetry"

$$\bar{\Delta} = \frac{N_1 - N_2 + N_3 - N_4 + N_5 - N_6}{N_1 + N_2 + N_3 + N_4 + N_5 + N_6} \qquad /9/$$

is three times bigger, than the simple "right-left" asymmetry /Fig.2./. The experimental values /7/ give, however,

$$\bar{\Delta} = (0{,}44 \pm 0{,}5)\ \% \qquad /10/$$

which seems to be smaller than Δ given in /6/ and is consistent with zero within one standard deviation. The experimental indication $\Delta > \bar{\Delta}$ /instead of $\Delta = \bar{\Delta}/3$/ suggests that not the $I = 0$, but the $I = 2$ final state governes the C violating transition.

An apparent contradiction arises: both the $K_L^0 \to \pi\pi$ decay and the $\eta\to\pi^+\pi^0\pi^-$ charge asymmetry can be explained by the millistrong model of C violation, but the experimental indication $|\varepsilon| >> |\varepsilon'|$ suggests the $\Delta I = 0$ version, the result $\Delta > \bar{\Delta}$ suggests the $\Delta I = 2$ version of this model. The aim of the present paper is to give a more detailed discussion of the charge asymmetry in the $\eta\to\pi^+\pi^0\pi^-$ decay, in order to clarify this discrepancy.

2. The $\eta\to\pi^+\pi^o\pi^-$ Decay.

In analyzing the $\eta\to\pi^+\pi^o\pi^-$ decay we shall write T_+, T_o, T_- for the pion kinetic energies, p_+, p_o, p_- for the pion four momenta. In the rest system of the η meson we evidently have:

$$T_+ + T_o + T_- = m - 3\mu = Q \ .$$

/ We shall denote the masses of the π, η and ρ mesons by μ, m and M. For simplicity we shall take $\mu : m : M = 1 : 4 : 5{,}55$ and we shall forget about electromagnetic mass splittings./ We shall make use also of the Mandelstam variables s, t, u which may be expressed in the rest system of the η meson as follows:

$$\begin{aligned} s &\equiv (p_+ + p_-)^2 = (m-\mu)^2 - 2mT_o \ , \\ t &\equiv (p_- + p_o)^2 = (m-\mu)^2 - 2mT_+ \ , \\ u &\equiv (p_+ + p_o)^2 = (m-\mu)^2 - 2mT_- \ . \end{aligned} \qquad /11/$$

The decay may be analyzed conveniently on the Dalitz diagram /Fig.1./. Let us introduce the Dalitz variables ρ and Θ according to the formulas

$$\begin{aligned} T_+ &= \frac{Q}{3}\left(1 - \frac{1}{2}\rho\cos\Theta - \frac{\sqrt{3}}{2}\rho\sin\Theta\right), \\ T_o &= \frac{Q}{3}\left(1 + \rho\cos\Theta\right), \\ T_- &= \frac{Q}{3}\left(1 - \frac{1}{2}\rho\cos\Theta + \frac{\sqrt{3}}{2}\rho\sin\Theta\right). \end{aligned} \qquad /12/$$

In terms of Dalitz variables the decay probability can be written as follows:

$$\Gamma(\eta\to\pi^+\pi^o\pi^-) = (4\pi)^{-3}\,\frac{Q^2}{6\sqrt{3}\,m}\int_0^{2\pi} d\Theta \int_0^{R(\Theta)} d\rho\,\rho\,|A\ \rho,\Theta\ |^2 \ . \qquad /13/$$

The 30% - 100% experimental errors of the measured asymmetries /6/ and /10/ make a very accurate calculation unnecessary. We put simply $R(\Theta) = 1$, allowing an inaccuracy of 20% on the boundary of the Dalitz plot.*

The dominating decay mode is the $\Delta I = 1$ channel, which can be described by the C conserving vertex

$$H_1 = \frac{1}{2}G_1\eta\pi^o(\underline{\pi}\ \underline{\pi}) = G_1\ \eta\left(\pi^+\pi^o\pi^- + \frac{1}{2}\pi^o\pi^o\pi^o\right) . \qquad /14/$$

---

* See the footnote on page 6.

If we had only this coupling among η and 3π, the perturbation theory would give $A_1(\rho,\Theta)=G_1=$ const for the invariant amplitude. The observed $\eta\to\pi^+\pi^o\pi^-$ width [13]

$$\Gamma(\eta\to\pi^+\pi^o\pi^-) = (0{,}237 \pm 0{,}017)\Gamma(\eta) = (0{,}237\pm0{,}017)(2{,}3\pm0{,}5)\text{kev} \qquad /15/$$

may be obtained if we put

$$|G_1| = 0{,}32 \pm 0{,}04 . \qquad /16/$$

The observed value of $\Gamma(\eta\to\pi^o\pi^o\pi^o)$ indicates a small $I = 3$ impurity in the final state, but the corresponding coupling constant is very small and may be taken to zero within three standard deviations. So in our analysis we shall forget about the $\Delta I = 3$ possibility.

The C conserving $\Delta I = 1$ transition amplitude $A_1(\rho,\Theta)$ is influenced by the final state interactions of the pions. The incomplete knowledge of this modification is perhaps the weakest point in the analysis of the $\eta\to3\pi$ decay. The observed enhancement in the lower parts of the Dalitz diagram may be simply described by a formula linear in T_o [17]:

$$|A_1(\rho,\Theta)| = |G_1| \frac{1+\beta\rho\cos\Theta}{\left(1+\frac{1}{4}\beta^2\right)^{1/2}} , \qquad /17/$$

with

$$\beta = -0{,}55 \pm 0{,}02 . \qquad /18/$$

The imaginary part of $A_1(\rho,\Theta)$ may be obtained only from specific assumptions concerning the final state interaction. B.Barret et al. [18] have suggested the following formula:

$$A_1(\rho,\Theta) = \frac{a}{1-ia_o\,q(s)} + b\left[\frac{s-u}{D\,t} + \frac{s-t}{D\,u}\right] . \qquad /19/$$

This takes into account a $\pi\pi$ interaction in the $I = 0, L= 0$ state, described by the scattering length a_o and a $\pi\pi$ interaction in the

---

We have checked that the conclusions of the present paper remain unchanged if the electromagnetic mass differences and the exact boundary of the Dalitz domain are taken into account. Some numerical values are of course changed, sometimes by a factor of 2.

I=1, L=1 state, described by the ρ meson pole. Here $q(s)=\frac{1}{2}(s-4\mu^2)^{1/2}$ and $D(s)$ comes from the ρ propagator, being $D(s)=M^2-s-iM^{-1}\Gamma(\rho)q^3(s)\,s^{-1/2}$. The empirical constants a and b are real if the CPT symmetry holds.

The recent analysis [19] suggests a value

$$\mu a_o = 0,2 \; . \qquad /20/$$

It is shown in the Appendix I, that the formula [19] may be well approximated with a linear expression, if we make use of the numerical values /18/ and /20/. So we shall use the following expression for the C conserving amplitude:

$$A_1\,(\rho,\Theta) = G_1\, e^{i\alpha}\, \frac{1 + \beta\,\rho\,\cos\Theta}{\left(1 + \frac{1}{4}\,\beta^2\right)^{1/2}} \qquad /21/$$

with

$$\alpha = \mathrm{arc\ tg}\left[a_o\left(\frac{1}{4}\,m^2 - \frac{2}{3}\,mQ - \frac{3}{4}\,\mu^2\right)^{1/2}\right] = 8^o 40' \qquad /22/$$

coming from the fit of /21/ to /19/. The possible consequences of the non-linear terms in /21/and of the variation of the scattering length a_o are discussed in the Appendix II.

Let us now turn to the C violating part of the invariant amplitude $A(\rho,\Theta)$ coming from the I = 0 and I = 2 final states. B. Barret et al [18] have used the following expression:

$$A_o(\rho,\Theta) + A_2(\rho,\Theta) = ic\left[\frac{t-u}{D\,s} + \frac{u-s}{D\,t} + \frac{s-t}{D\,u}\right] + id\,\frac{t-u}{D\,s} \; . \qquad /23/$$

In the present work we assume that the C violating transitions $\eta\to\pi\,\pi\,\pi$ /I=0,2/are dominated by the ρ meson /Fig.3/ and the C violation is due to the $\eta\rho\pi$ vertex [4]:

$$H_o = g_o\left[\rho^+_\mu\left(\pi^-\partial_\mu\eta - \eta\partial_\mu\pi^-\right) + \rho^o_\mu\left(\pi^o\partial_\mu\eta - \eta\partial_\mu\pi^o\right) + \rho^-_\mu\left(\pi^+\partial_\mu\eta - \eta\partial_\mu\pi^+\right)\right], \qquad /24/$$

$$H_2 = g_2\left[-\rho^+_\mu\left(\pi^-\partial_\mu\eta - \eta\partial_\mu\pi^-\right) + 2\rho^o_\mu\left(\pi^o\partial_\mu\eta - \eta\partial_\mu\pi^o\right) - \rho^-_\mu\left(\pi^+\partial_\mu\eta - \eta\partial_\mu\pi^+\right)\right]. \qquad /25/$$

The C symmetric $\rho\pi\pi$ vertex is the known strong interaction

$$H_\rho = G_\rho \left[\rho_\mu^+ \left(\pi^o \partial_\mu \pi^- - \pi^- \partial_\mu \pi^o\right) + \rho_\mu^o \left(\pi^- \partial_\mu \pi^+ - \pi^+ \partial_\mu \pi^-\right) + \rho_\mu^- \left(\pi^+ \partial_\mu \pi^o - \pi^o \partial_\mu \pi^+\right)\right]. \qquad /26/$$

The observed ρ width $\Gamma(\rho) = 130 \pm 20$ MeV [13] gives

$$|G_\rho| = 5{,}6 \pm 0{,}4 \qquad /27/$$

The interplay of the couplings /24/ and /25/ with /26/ results in

$$A_o(\rho,\Theta) + A_1(\rho,\Theta) = i\left(g_o - g_2\right) G_\rho \left[\frac{t-u}{M^2-s} - \frac{u-s}{M^2-t} - \frac{s-t}{M^2-u}\right] + i\, 3g_2\, G_\rho \frac{t-u}{M^2-s} \;. \qquad /28/$$

/The real part coming from the ρ width may be neglected compared to the imaginary contribution of $A_1(\rho,\Theta)$ coming from the final state interactions./ Comparing the expression /28/ with the formula /23/ one can see that the parameters c and d do not simply describe the $\Delta I = 0$ and $\Delta I = 2$ transitions, but they are combinations of g_o and g_2. In order to avoide any misunderstanding we shall analyze the η asymmetry in terms of the pure isospin constants g_o and g_2 instead of the usual c and d.

By performing a power expansion according to the new variables $Y_{+,o,-} = 3Q^{-1}T_{+,o,-} - 1$ /see Appendix I/ and by introducing the small centrifugal barrier parameter

$$k = \frac{mQ/3}{M^2 - (m-\mu)^2 + \frac{2mQ}{3}} = 0{,}054 \;, \qquad /29/$$

one arrives at the formula

$$A_o(\rho,\Theta) + A_2(\rho,\Theta) = i\, 6\sqrt{3}\, G_\rho \left[g_2 \frac{k\,\rho \sin\Theta}{1+2k\rho\cos\Theta} + \left(g_o - g_2\right) k^3 \rho^3 \sin 3\Theta + \ldots\right],$$

which may be written also in the more convenient form

$$A_o(\rho,\Theta) + A_2(\rho,\Theta) = i\, 6\sqrt{3}\, G_\rho \Big[g_2 \left(k\rho + k^3\rho^3\right)\sin\Theta - g_2\, k^2\rho^2 \sin 2\Theta + g_o\, k^3\rho^3 \sin 3\Theta + O(k^4)\Big]. \qquad /30/$$

We have now everything ready to construct the theoretical distribution on the Dalitz circle. Let us calculate the Θ distribution $f(\Theta)$ up to first powers of g_o/G_1, g_2/G_1 and up to k^3. Evidently,

$$f(\Theta) = \frac{1}{G_1^2\pi} \int_o^1 \left| A_1(\rho,\Theta) + A_o(\rho,\Theta) + A_2(\rho,\Theta)\right|^2 \rho\, d\rho \ .$$

By substituting the formulas /21/ and /30/ one gets the following expression:

$$2\pi f(\Theta) = 1 - \left(1+\frac{1}{4}\beta^2\right)^{-1} \left\{ -\frac{4}{3}\beta\cos\Theta - \frac{1}{4}\beta^2\cos 2\Theta \right\} +$$

$$+ \frac{24\sqrt{3}\sin\alpha}{\left(1+\frac{1}{4}\beta^2\right)^{1/2}} \frac{G_\rho}{G_1} \left\{ g_2\left(\frac{k}{3} - \frac{\beta k^2}{10} + \frac{k^3}{5}\right)\sin\Theta - \right.$$

$$- \left[g_2\left(-\frac{k\beta}{8} + \frac{k^2}{4} - \frac{k^3\beta}{12}\right) - g_o\frac{k^3\beta}{12}\right]\sin 2\Theta +$$

$$\left. + \left[-g_2\frac{k^2\beta}{10} + g_o\frac{k^3}{5}\right]\sin 3\Theta + g_o\frac{k^3\beta}{12}\sin 4\Theta + O(k^4)\right\} =$$

$$= 2\pi \left| \sum_n x_n \cos n\Theta + \sum_n y_n \sin n\Theta \right| \ . \qquad /31/$$

The charge asymmetry is given by the $\sin n\Theta$ terms. The asymmetry parameters, defined in /8/ and /9/ may be obtained from $f(\Theta)$:

$$\Delta = -\int_o^\pi \left[f(\Theta) - f(-\Theta)\right] d\Theta = -4\left(y_1 + \frac{1}{3}y_3\right) ,$$

$$\bar{\Delta} = -\int_o^{\pi/3} \left[f(\Theta) - f(-\Theta)\right] d\Theta + \int_{\pi/3}^{2\pi/3} \left[f(\Theta) - f(-\Theta)\right] d\Theta - \int_{2\pi/3}^{\pi} \left[f(\Theta) - f(-\Theta)\right] d\Theta = -4y_3 ,$$

i.e.

$$\Delta = -\frac{16\sqrt{3}\sin\alpha}{5\pi\left(1+\frac{1}{4}\beta^2\right)^{1/2}} k^3 \frac{G_\rho}{G_1}\left[g_o + g_2\left(\frac{5}{k^2} - \frac{2\beta}{k} + 3\right)\right], \qquad /32/$$

$$\bar{\Delta} = -\frac{16\sqrt{3}\sin\alpha}{5\pi\left(1+\frac{1}{4}\beta^2\right)^{1/2}} k^3 \frac{G_\rho}{G_1}\left[3g_o - g_2\frac{3}{2}\frac{\beta}{k}\right] \qquad /33/$$

Substituting the numerical values quoted in the equations /16/,/18/, /22/, /27/ and /29/ we have finally

$$\Delta = 7 \cdot 10^{-4} \left(g_o + 1740\ g_2\right) \ , \qquad /34/$$

$$\bar{\Delta} = 7 \cdot 10^{-4}\left(3g_o + 15\ g_2\right) \ ; \qquad /35/$$

for simplicity we take the sign of the product $\left(-\ G_\rho\ G_1^{-1}\ \sin\alpha\right)$ as positive. If it turns out to be negative, we have to change the sign of g_o and of g_2.

Remembering that the centrifugal barrier parameter k,introduced in equ. /29/ is small, we arrive at the following conclusions:

a/ In case of a pure $\Delta I = 0$ charge asymmetry /i.e. with $g_2 = 0$/ one would have $\bar{\Delta}=3\Delta$. The sextant asymmetry $\bar{\Delta}$ has been originally introduced by M. Nauenberg [16] just as a good tool to demonstrate the presence of a pure $\Delta I = 0$ asymmetry in the $\eta\to\pi^+\pi^o\pi^-$ decay. However, one cannot live with a pure $\Delta I = 0$ C violation : radiative corrections introduce a $\Delta I = 1$ and a $\Delta I = 2$ impurity necessarily, and from equ. /34/ we see that in Δ the contribution of the $\Delta I = 2$ chanell will dominate even if $g_2 \approx g_o/137$. The experimental indication $\Delta > \bar{\Delta}$ then becomes understandable. Let us also remark that to reproduce the observed order of magnitude for the Δ asymmetry with $g_2 = 0$, one has to take $g_o \approx 20$. Such a strong C violating $\eta\rho\pi$ coupling would lead to very large charge asymmetries in many strong and electromagnetic transitions, where the centrifugal barrier effect, specific for the $\eta\to\pi^+\pi^o\pi^-$ decay, is absent. Thus we conclude that we are forced to reject the $g_2=0$ solution.

b/ If $g_2 \neq 0$, the situation becomes more involved. The first thing to note is that the experimental value $\Delta \approx 10^{-2}$ can be reproduced if $g_2 \approx 10^{-2}$, which is a reasonable "millistrong" value. At the same time g_o may be zero, but this is by no means a necessity. Equ. /34/ clearly shows that even if $g_o \approx 100\ g_2$, the contribution of g_o will be practically undetectable in Δ . Let us also look at the sextant asymmetry $\bar{\Delta}$. It is important to remark that while in Δ the coefficient of g_2 is almost entirely given by the constant $5k^{-2} \approx 1700$, in $\bar{\Delta}$ this coefficient depends also on β i.e. it is sensitive to the energy dependence of the C conserving matrix element. In Appendix II we show that the value of this coefficient is influenced also by the small quadratic term of the C conserving matrix element, neglected in our formulae given in the text. Thus the contribution of

g_2 to $\bar{\Delta}$ cannot be firmly established, and the calculated value "15" in equ. /35/ can be used only for orientation. It just shows that with $g_2 \simeq 10^{-2}$, $g_o < 100\, g_2$, $\bar{\Delta}$ is expected to be much smaller than Δ, a result which is fully consistent whith the measured value $\bar{\Delta} = 0{,}44 \pm 0{,}5\ \%$

The basic experimental informations $\Delta \simeq 10^{-2}$ and $\Delta > \bar{\Delta}$ given in equ. /6/ and /10/ are just at the limit of the statistical significance. It would be of great importance to ascertain these experimental results. Indeed, let us suppose that the solution $|\varepsilon| >> |\varepsilon'|$ holds for the K_L^o decays. Then the CP violation may be superweak /$\Delta s = 2$/ or millistrong /$\Delta s = 0$/, but in the K_L decays this cannot be decided anymore. The experiment crucis may then be the $\eta \to \pi^+\pi^o\pi^-$ decay. If the value $\Delta \simeq 10^{-2}$ turns out to be realistic*, then the CP violation is millistrong. We have just seen that the results $\Delta \simeq 10^{-2}$, $\Delta > \bar{\Delta}$ are fully compatible with $|\varepsilon| >> |\varepsilon'|$ since the dominance of the $\Delta I = 2$ chanell in the $\eta \to \pi^+\pi^o\pi^-$ decay does not necessarily mean that $g_2 >> g_o$. The possibility $g_o >> g_2$ is also open, due to the strong damping of the $\Delta I = 0$ channel by the centrifugal barrier effect.

Let us point out that useful supplementary information on the mechanism of the C violation in the $\eta \to \pi^+\pi^o\pi^-$ decay can be extracted from the study of the partial asymmetry parameters Δ_1, Δ_2, Δ_3. These parameters have also been considered by M. Nauenberg for the pure $\Delta I = 0$ case. They are defined as follows / see Fig.1/:

$$\Delta_1 = \frac{N_1 - N_6}{N_1 + N_6} \quad ; \quad \Delta_2 = \frac{N_2 - N_5}{N_2 + N_5} \quad ; \quad \Delta_3 = \frac{N_3 - N_4}{N_3 + N_4} \qquad /36/$$

Starting from our distribution function $f(\Theta)$ given in equ./31/ the theoretical expressions for these asymmetry parameters can be easily calculated. The comparison with the experimental values, taken from /7/ are presented in table II. We see that the available experimental data on the partial asymmetries support our conclusions, based on the study of the Δ and $\bar{\Delta}$ parameters alone. The influence of the quadratic energy terms in the C conserving matrix element on the partial asymmetries is discussed in Appendix II.

Let us end this discussion with the remark that the available data on the $\eta \to \pi^+\pi^o\pi^-$ charge asymmetry do not give an effective upper limit for the coupling constant g_o. As noted above, such a limit may come from C violating effects in strong or electromagnetic processes. It turns out that the working limit is given by the $\eta \to \pi^o e^+e^-$ decay, which we shall discuss presently.

* see footnote on page 3.

3. The $\eta\to\pi^{o}e^{+}e^{-}$ Decay.

It was emphasized already in 1965 [20], that the millistrong model of the C violation predicted a strict correlation between the branching ratio of the $\eta\to\pi^{o}e^{+}e^{-}$ decay and the charge asymmetry in the $\eta\to\pi^{+}\pi^{o}\pi^{-}$ decay, if the ρ pole dominance was assumed in both reactions. In the light of the accurate upper limit /4/ it looks to be worthwhile to reinvestigate this question.

In the ρ dominance model /Fig.4/ we can express the $\eta\to\pi^{o}e^{+}e^{-}$ decay probability in terms of the coupling constants g_o and g_2 If the ρ-γ coupling is given by $f = e\, G_{\rho}^{-1}$, we have [18]:

$$\Gamma\left(\eta \to \pi^{o}e^{+}e^{-}\right) = 42 \text{ eV} \left(\frac{g_o + 2g_2}{G_{\rho}}\right)^2 \qquad /37/$$

i.e.

$$B\left(\eta\to\pi^{o}e^{+}e^{-}\right) = \Gamma(\eta\to\pi^{o}e^{+}e^{-}) : \Gamma(\eta) = 2.10^{-3}\left(\frac{g_o + 2g_2}{G_1\, G_{\rho}}\right)^2 . \qquad /38/$$

We can express g_o and g_2 through Δ and $\bar{\Delta}$ using the equations /32/ and /33/. Introducing these expressions into /38/ we see that B is independent of G_1. Using the known numerical values of the other parameters, we get

$$B\left(\eta\to\pi^{o}e^{+}e^{-}\right) = 142\left(\bar{\Delta} - 0{,}00525\Delta\right)^2 \qquad /39/$$

The experimental limit quoted in equ. /4/ puts the restriction

$$|\bar{\Delta} - 0{,}00525\Delta| < 10^{-3} \qquad /40/$$

on the charge asymmetries Δ and $\bar{\Delta}$ consequently /4/ and /6/ predict $\bar{\Delta}$ to be smaller than 10^{-3}. This is a much more stringent restriction than the experimental limit /10/. It must be emphasized that the uncertainity of the "15" value of formula /35/ influences only the "0,00525" on the left hand side of equ. /40/ and this uncertainity does not affect the restriction on $\bar{\Delta}$ seriously.

If we are ready to use the η width quoted in /15/, the formula /38/ gives

$$|g_o + 2g_2| = 40\left[B\left(\eta\to\pi^{o}e^{+}e^{-}\right)\right]^{1/2}, \qquad /41/$$

and using the limit /4/ we get

$$|g_o + 2g_2| < 0,4 \; . \qquad /42/$$

It can be seen now that g_2 is restricted by the $\eta\to\pi^+\pi^o\pi^-$ result /6/ due to the formula /34/ ; g_o on the other hand, is restricted by the formula/42/ much stronger than by the result/10/ through the formula /35/ . All these relations are visualized in the Fig. 5; it can be seen that the experimental limits $1\% < \Delta < 2\%$ /one standard deviation/, $0 < \bar{\Delta} < 1\%$ /one standard deviation/, and $B < 10^{-4}$ allow essentially the following possibilities:

$$-0,1 < g_o < +0,4 \quad ; \quad 0,008 < g_2 < 0,016 \; . \qquad /43/$$

If we use the more liberal limits $0,5\% < \Delta < 2,5\%$ /two standard deviation/, $B < 2.10^{-4}$ /the value given by Bazin et al [21] with 90 % confidence/ the possibilities are wider*:

$$-0,6 < g_o < +0,6 \quad ; \quad 0,004 < g_2 < 0,02 \quad ; \qquad /44/$$

the experimental $\bar{\Delta}$ value does not give any further restriction if taken with two standard deviation.

Let us note at this point that in the $\eta\to\pi^o e^+e^-$ decay a $\Delta I = 1$ C violating $\eta\omega\pi$ coupling may interfere with the $\Delta I = 0$ and $\Delta I = 2$ couplings. The $\Delta I = 1$ way of C violation is briefly dicussed in Appendix III.

We turn now to Table II, where the experimental values of the Δ, $\bar{\Delta}$ Δ_1, Δ_2 and Δ_3 asymmetry parameters are compared with the calculated ones for some specific choices of the g_o, g_2 coupling constants. For simplicity we have always choosen g_o and g_2 to reproduce the observed mean value 1,52 % of the Δ asymmetry parameter. It can be seen from Table II that the choice $g_2=0$ contradicts both the $\eta\to\pi^o e^+e^-$ limit /given in parenthesis at the top of each column/, and the measured value of the other asymmetry parameters. On the other hand the cases $g_2\neq 0$ are seen to be in agreement with the $\eta\to\pi^+\pi^o\pi^-$ results, and up to $g_o\approx 30g_2$ also with the $\eta\to\pi^o e^+e^-$ limit. We have seen from Fig.5. that if Δ is also allowed to change, then $g_o\approx 50g_2$ /one standard deviation for Δ/ and even $g_o\approx 100g_2$ /two standard deviation for Δ/ is possible.

This means that the measured C violating effects in the $\eta\to\pi^+\pi^o\pi^-$ decays allow a pure $\Delta I = 2$ C violation which is millistrong indeed, being

---

* Let us mention that in the electromagnetic model of C violation [21] equ. /4/ gives the restriction $g_o < 0,1e$, from which $\Delta < 0,17\%$ follows, in contradiction with the experimental result /6/.

$$g_2 : G_\rho \approx 5.10^{-3} . \qquad /45/$$

In this case an $|\varepsilon| \approx |\varepsilon'|$ solution would be expected in the K_L^o decays. A more interesting possibility is offered by the solution $g_o >> g_2$. In this case a comparatively strong iso-symmetric C violation(g_o) is accompanied by a much weaker isospin asymmetric C violation (g_2). The latter may pearhaps be interpreted as an electromagnetic correction to the isosymmetric C violation. In this case the $|\varepsilon|>>|\varepsilon'|$ property of the K_L^o decay can be explained by the dominance of the isosymmetric C violation. In the $\eta\to\pi^+\pi^o\pi^-$ decay the $\Delta I = 0$ transition is very much suppressed by the centrifugal barrier / $\bar{\Delta} \approx 10^{-3}$ is predicted/, and as a radiative correction the $\Delta I = 2$ transition comes to daylight in the right-left asymmetry $\Delta \approx 10^{-2}$.

An interesting possibility /not yet understood/ is offered by the following choice:

$$g_o = G_1 = e = 0,3 \quad ; \quad \frac{2g_o}{137} < g_2 < \frac{10g_o}{137} .$$

This reproduces the observed values of $\Gamma(\eta\to\pi^+\pi^o\pi^-)$ and of the five asymmetry parameters, respects the $\eta\to\pi^o e^+e^-$ limit and explains $|\varepsilon|>>|\varepsilon'|$ at the same time.

The "big g_o" possibility implies some problems. As first, one has to find out how $g_o \approx 0,1$ leads to $|\varepsilon| = 2.10^{-3}$*. As second, one has to check whether the big coupling constant g_o is compatible with the restrictive upper limit 4.10^{-23} cm of the neutron dipole moment or not. As third, one has to look for processes where a $g_o > 0,1$ coupling can manifest itself directly. E.g. $\Gamma(\rho^+\to\pi^+\eta) : \Gamma(\rho) > 10^{-3}$ is predicted. $N\bar{N}$ annihilation may also be sensitive to a big g_o. Up to now the lowest limit for g_o is given by the $\eta\to\pi^o e^+e^-$ experiment. It would be desirable to raise the accuracy of this experiment which offers the best way to test the $\Delta I = 0$ model.

Finally let us formulate an important negative conclusion: <u>the available experimental data on the $\eta\to\pi^+\pi^o\pi^-$ charge asymmetry and on the $\eta\to\pi^o e^+e^-$ limit do not contain any serious restriction for the $|\varepsilon| : |\varepsilon'|$ ratio.</u>

* The parameter ε can be represented also by the formula

$$\varepsilon.\langle 2\pi, I=0|Hw|K_S^o\rangle = \sum_n \frac{\langle 2\pi, I=0|H^{\Delta I=0}|n\rangle \langle n|H_w|K_L^o\rangle}{m_L - E_n - i\varepsilon}$$

For the most direct /on mass shell/ state $|n\rangle = |3\pi, I=1\rangle$ the right hand side is vanishing. In order to get an ε with the isosymmetric millistrong C violation, one is forced to choose states far away from the mass shell $E_n >> m_L$. This may be a possible explanation why ε turns out to be considerably smaller than g_o. On the other hand if $g_o >> \varepsilon$, then the inequality $|\varepsilon|>>|\varepsilon'|$ cannot be expected to hold very strongly: a $\Delta I = 1$ C violating vertex /see appendix III/ may give non negligible contributions both to ε and to ε'.

Appendix I. The C Állowed $\eta\to\pi^+\pi^o\pi^-$ Decay.

We shall now establish the numerical values of the real constans a and b in the C conserving matrix element A_1, which we rewrite from equ. /19/:

$$A_1 = \frac{a}{1 - ia_o\, q(s)} + b\left(\frac{s-u}{D(t)} + \frac{s-t}{D(u)}\right) ,$$

/I.1/

$$q(s) = \frac{1}{2}\sqrt{s - 4\mu^2} \quad , \quad D(s) = M^2 - s - i\,\frac{\Gamma(\rho)\; q^3(s)}{M\sqrt{s}} \; ,$$

The ratio b: a will be determined by the experimentally established energy dependence of the $\eta\to\pi^+\pi^o\pi^-$ decay [17], and then $|a|$ will be found from the known $\Gamma(\eta\to\pi^+\pi^o\pi^-)$ width. The imaginary part of the D(s) function is less then $3^o/oo$ of the real part, and will be neglected. For the $\pi\,\pi$ scattering length a_o we shall use the numerical value $a_o\mu = 0{,}2$ [19]. The influence of the variation of a_o on the asymmetry formulae will be discussed in Appendix II.

In [17] 7170 $\eta\to\pi^+\pi^o\pi^-$ decays has been used to find the energy dependence of a purely phenomenological C conserving $\eta\to\pi^+\pi^o\pi^-$ matrix element M, defined as follows:

$$M = 1 + \beta Y_o + \gamma Y_o^2 + \delta Y_+\, Y_- \; . \qquad /I.2/$$

The dimensionless energy variables Y are related to the kinetic energies of the pions in the following manner:

$$Y_{+,o,-} = \frac{3}{Q}\, T_{+,o,-} - 1 \; . \qquad /I.3/$$

In terms of the polar coordinates ρ,Θ of the Dalitz plot the Y-s read:

$$Y_o = \rho\,\cos\Theta \quad , \quad Y_\pm = -\,\frac{1}{2}\,\rho\left(\cos\Theta \pm \sqrt{3}\,\sin\Theta\right) . \qquad /I.4/$$

These variables are commonly used because it is expected that the non-linear terms turn then up with small coeffitients in the amplitude M.

The matrix element M is introduced in [17] only for the study of the energy dependence, and its normalisation and overall phase are arbitrary. This means that our matrix element A_1 may be written as follows:

$$A_1 = h\,M \quad ,$$

where h is a complex number, independent of the energies of the pions. Let us put A_1 into the form /I,5/. The second term in A_1 is easily expanded into power series of the Y-s:

$$\frac{s-u}{M^2-t} + \frac{s-t}{M^2-u} = 2k\left(\frac{Y^- - Y^o}{1+kY^+} + \frac{Y^+ - Y^o}{1+kY^-}\right) =$$

$$= -6k\,Y_o - 4k^2\left(Y_o^2 + 2Y_+\,Y_-\right) + O\,k^3 \quad ; \qquad /I.6/$$

/ k is the centrifugal barrier parameter, defined in /29/ /.
In the first term of A_1 the function q/s/ cannot be conveniently expanded. A fairly good approximation to it is provided by a parabola, fitted to the points $q_{max} = 1{,}12\mu$ $q_o = 0{,}765\mu$ and $q_{min} = 0$. These values corresponds to $Y_{omin} = -1, Y_o = 0$ and $Y_{o\,max} = 0{,}875$ respectively. In this approximation we have

$$q(s) = q(Y_o) = \left(0{,}765 - 0{,}629Y_o - 0{,}274Y_o^2\right)\mu \quad . \qquad /I.7/$$

Writing now

$$\frac{1}{1 - ia_oq(s)} = \frac{1}{1 + a_o^2q^2(s)}\;1 + ia_oq(s) \quad , \qquad /I.8/$$

expanding the first factor into power series of Y_o and using /I,7/ in the second, we find from /I,6/ and /I,8/ the desired expression for A_1:

$$A_1 = \frac{a}{1 + a_o^2q_o^2}\left(1 + ia_oq_o\right)\left(1 + \beta Y_o + \gamma Y_o^2 + \delta Y_+Y_-\right) , \qquad /I.9/$$

where

$$\beta = E - Fa_o\mu 0,48 - iF0,629 - 6\,\frac{b}{a}k\left(1 - ia_oq_o\right),$$

$$\gamma = E^2 - EFa_o\mu 0,48 - Fa_o\mu 0,189 - iF\left(E0,629 + 0,247\right) - 4\,\frac{b}{a}\,k^2\left(1 - ia_oq_o\right),$$

$$\delta = -8\,\frac{b}{a}\;k^2\left(1 - ia_oq_o\right); \qquad /I.10/$$

$$E \equiv \frac{1}{6}\,\frac{a_o^2\,Q\mu}{1 + a_o^2q_o^2}, \quad F \equiv \frac{a_o\,\mu}{1 + a_o^2q_o^2},$$

$$q_o \equiv q\left(Y_o = 0\right) = \left(\frac{1}{4}\,m^2 - \frac{2}{3}\,mQ - \frac{3}{4}\,\mu^2\right)^{1/2} = 0,765\mu\ . \qquad /I.11/$$

Let us now remind the reader that in [17] the authors used several working assumption to find the coeffitients β, γ, δ in /I,2/. Namely, they have tried to fit the experimental energy distribution with: a/ $\gamma = \delta = 0$, β arbitrary complex number b/ β, γ and δ arbitrary real numbers and c/ β, γ and δ arbitrary complex numbers. It turned out that only the real part of $\beta \equiv \beta_1 + i\beta_2$ has a stable value in all cases:

$$\beta_1 = -0,55 \pm 0,02\ , \qquad /I.12/$$

the mean values of all the other coeffitients change violently from case to case, and due to their very large statistical errors are always consistent with zero.

Taking into account this state of affairs, we shall use only the value of β_1 in our calculation. From /I,10/ and /I,12/ we have

$$E - Fa_o\mu\ 0,48 - 6k\,\frac{a}{b} = \beta_1 = -0,55\ . \qquad /I.13/$$

With $a_o\mu = 0,2$ we find

$$\frac{b}{a} = 1,65. \qquad /I.14/$$

Introducing these values of a_o and of $\frac{b}{a}$ into /I,10/ and then into /I,9/, we arrive at the desired result:

$$A_1 = \frac{a}{1,023}\left(1 + i0,153\right)\Big[1 + \left(-0,55 - i0,037\right)Y_o +$$

$$+ \left(-0,028 - i0,053\right)Y_o^2 + \left(-0,0416 + i0,0063\right)Y_+\,Y_-\Big]\ . \qquad /I.15/$$

We see that $|\beta_1|$ is much bigger than the other coeffitients. This result is consistent with, but does not follow from the experimental analysis [17]. If we keep only $\beta = \beta_1 = -0,55$ and introduce the notation

$$\frac{G_1 \, e^{i\alpha}}{\left(1 + \frac{1}{4}\,\beta^2\right)^{1/2}} = \frac{a}{1 + a_o^2 q_o^2}\left(1 + i a_o \, q_o\right), \qquad /I.16/$$

we arrive at the matrix element, given in equ. /21/. The value of $|a|$, i.e. the value of $|G_1|$ is then found from the $\Gamma\left(\eta\to\pi^+\pi^o\pi^-\right)$ width/see equ. /16/ /:

$$|a| = 0,31 \quad ; \quad |G_1| = 0,32 \quad . \qquad /I.17/$$

Appendix II. The Asymmetry Parameters of the $\eta\to\pi^+\pi^o\pi^-$ Decay in the Quadratic Approximation.

In §2 we neglected the imaginary part of the Y_o term and all the quadratic energy terms of the C conserving matrix element /I,9/. We have also choosen a value for the $\pi\pi$ scattering length /$a_o = 0,2\ \mu^{-1}$ /which, even if it is correct, may be too low to describe the I = 0 final state interaction in the whole physical domain $4\mu^2 \leqslant s \leqslant (m-\mu)^2 = 9\mu^2$ of the $\pi\ \pi$ system.

Below we shall investigate the effect of the neglected terms and of the variation of a_o on the charge asymmetry parameters of the $\eta\to\pi^+\pi^o\pi^-$ decay. We shall see that no dramatic changes are produced if those terms are taken into account and if a_o goes up to $1\mu^{-1}$. This justifies a posteriori the use of the simplified matrix element /21/. Sure, these results are model dependent. Nevertheless, they illustrate the influence of a possible 10% complexity and nonlinearity of the C conserving matrix element on the calculated values of the charge asymmetry parameters.

The full amplitude of the $\eta\to\pi^+\pi^o\pi^-$ decay is given by the expression

$$A_1 + \left(A_o + A_2\right) \equiv A_c + A_{\not c} \qquad /II.1/$$

where $A_1 = A_c$ is the C conserving matrix element /I,1/ or /I,9/, and $A_0+A_2 = A_{\not c}$ is the C violating matrix element given in equ. /30/. Let us denote the real and the imaginary part of A by $A^{/1/}$ and $A^{/2/}$ respectively. We also introduce the cumulative notation δ for the asymmetry parameters Δ, $\bar{\Delta}$, Δ_1, Δ_2, Δ_3 defined in equ. /8/, /9/ and /36/. The general expression for δ is easily seen to be

$$\delta = -2 \frac{\int_\delta d\Theta \int_0^{R(\Theta)} d\rho\, \rho\, A_c^{/2/}(\rho,\Theta)\, A_{\not c}^{/2/}(\rho,\Theta)}{\int_{s_\delta} d\Theta \int_0^{R(\Theta)} d\rho\, \rho \left(|A_c\ \rho,\Theta|^2 + |A_{\not c}\ \rho,\Theta|^2\right)} , \qquad /II.2/$$

where the symbols $\int_\delta$ and $\int_{s_\delta}$ mean:

$$\int_\Delta = \int_{s_\Delta} = \int_0^\pi ,$$

$$\int_{\bar{\Delta}} = \int_0^{\pi/3} - \int_{\pi/3}^{2\pi/3} + \int_{2\pi/3}^{\pi} , \qquad \int_{s_{\bar{\Delta}}} = \int_0^\pi , \qquad /II.3/$$

$$\int_{\Delta_j} = \int_{s_{\Delta_j}} = \int_{\frac{\pi}{3}(j-1)}^{\frac{\pi}{3}j} , \qquad j = 1,2,3 .$$

Let us first discuss the integral

$$\int_0^{R(\Theta)} d\rho\, A_c^{/2/}(\rho,\Theta)\, A_{\not c}^{/2/}(\rho,\Theta).$$

We again neglect the very small contribution of the ρ width. Then we find:

$$A_c^{/2/} = \mathrm{Im} \frac{a}{1 - ia_0 q} = a \frac{a_0 q}{1 + a_0^2 q^2} =$$

$$= a \frac{a_0 q_0}{1 + a_0 q_0} \left(1 + c_1 Y_0 + c_2 Y_0^2 +\right) \dots , \qquad /II.4/$$

where

$$c_1 = E - 0{,}82$$

$$c_2 = E^2 - 0{,}82E - 0{,}357 \qquad /II.5./$$

For the definition of E and q_o see equ. /I,11/

Let us remark that the dependence of c_1 and c_2 on a_o is weak. Indeed,

if	$a_o\mu = 0{,}2$	then	$c_1 = -0{,}794$,	$c_2 = -0{,}377$,
if	$a_o\mu = 0{,}6$	then	$c_1 = -0{,}623$,	$c_2 = -0{,}46$,
and if	$a_o\mu = 1$	then	$c_1 = -0{,}4$,	$c_2 = -0{,}525$.

Now the ratio of the contribution of the C violating coupling constants g_o and g_2 to the asymmetry parameters is determined by the product

$$\left(1 + c_1 Y_o + c_2 Y_o^2\right) A_\phi^{/2/} , \qquad /II.6/$$

and we may expect that the ratio $g_2 : g_o$ is only weakly influenced by a 500% change in a_o.

The coeffitients c_1 and c_2 can also be written in the form

$$c_1 = \beta_1 + \frac{\beta_2}{a_o q_o} , \qquad c_2 = \gamma_1 + \frac{\gamma_2}{a_o q_o} \qquad /II.7/$$

with $\beta = \beta_1 + i\beta_2$, $\gamma = \gamma_1 + i\gamma_2$ defined in Appendix I. The linear approximation used in §2 with $a_o\mu = 0{,}2$ corresponds to the substitution of $c_1 = -0{,}55$ instead of $c_1 = -0{,}794$ and of $c_2 = 0$ instead of $c_2 = -0{,}377$.

From /II,4/ and /30/ we find in the $R(\Theta) = 1$ approximation*

$$\int_0^1 d\rho \, \rho \, A_c^{/2/} A_\phi^{/2/} = 6\sqrt{3} \, a \, G_\rho \frac{a_o q_o}{1 + a_o q_o} \sum_{n=1}^{5} u_n \sin n\Theta , \qquad /II.8/$$

---

* see footnote on page 6.

where

$$u_1 = g_2 \left[\frac{k}{3} + \frac{k^3}{5} - \frac{c_1 k^2}{10} \right] + g_o \frac{c_2 k^3}{28} ,$$

$$u_2 = g_2 \left[- \frac{k^2}{4} + \frac{1}{2} c_1 \left(\frac{k}{4} + \frac{k^3}{6} \right) - \frac{c_2 k^2}{12} \right] + g_o \frac{c_1 k^3}{13} ,$$

$$u_3 = g_2 \left[- \frac{c_1 k^2}{10} + \frac{1}{4} c_2 \left(\frac{k}{5} + \frac{k^3}{7} \right) \right] + g_o \left[\frac{k^3}{5} + \frac{c_2 k^3}{14} \right] ,$$

$$u_4 = -g_2 \frac{c_2 k^2}{24} + g_o \frac{c_1 k^3}{12} ,$$

$$u_5 = g_o \frac{c_2 k^3}{28} . \qquad /II.9/$$

Let us now look at the denominator in equ. /II,2/. The contribution of $|A_{\not{c}}|^2$ is very small in comparison with $|A_c|^2$ and can be neglected. This means that the denominator do not influence the ratio of the contribution of g_o and g_2 to the asymmetry parameters, but only modifies their common multiplicative factor. This allows us to use the linear approximation for A_c in the integral of the denominator. A straightforward calculation leads to the formulae

$$\Delta = -12\sqrt{3}\, a_o q_o \frac{G_\rho}{a} \frac{2u_1 + \frac{2}{3} u_3 + \frac{2}{5} u_5}{\frac{\pi}{2}\left(1 + \frac{\beta_1^2}{4}\right)}$$

$$\bar{\Delta} = -12\sqrt{3}\, a_o q_o \frac{G_\rho}{a} \frac{2u_3}{\frac{\pi}{2}\left(1 + \frac{\beta_1^2}{4}\right)} .$$

$$\Delta_1 = -12\sqrt{3}\, a_o q_o \frac{G_\rho}{a} \frac{\frac{1}{2}u_1 + \frac{3}{4}u_2 + \frac{2}{3}u_3 - \frac{3}{8}u_4 + \frac{1}{10}u_5}{\frac{\pi}{6}\left(1 + \frac{\beta_1^2}{4}\right) + \frac{\beta_1}{\sqrt{3}} + \frac{\beta_1^2\sqrt{3}}{32}} \qquad /II.10/$$

$$\Delta_2 = -12\sqrt{3}\, a_o q_o \frac{G_\rho}{a} \frac{u_1 - \frac{2}{3} u_3 + \frac{1}{5} u_5}{\frac{\pi}{6}\left(1 + \frac{\beta_1^2}{4}\right) - \frac{\beta_1^2\sqrt{3}}{16}}\,,$$

$$\Delta_3 = -12\sqrt{3}\, a_o q_o \frac{G_\rho}{a} \frac{\frac{1}{2}u_1 - \frac{3}{4}u_2 + \frac{2}{3}u_3 - \frac{3}{8}u_4 + \frac{1}{10}u_5}{\frac{\pi}{6}\left(1 + \frac{\beta_1^2}{4}\right) - \frac{\beta_1}{\sqrt{3}} + \frac{\beta_1^2\sqrt{3}}{32}} \qquad /II.10/$$

Let us denote by δ^1 an asymmetry parameter in the linear approximation $/c_1=-0{,}55,\ c_2=0/$ and by δ^q the same parameter in the quadratic approximation. With $a_o\mu = 0{,}2$. We get the following numerical results /if $a_o \frac{G_\rho}{a} < 0$/:

$$\Delta^1 = 7.10^{-4}\,[\,g_o + 1740 g_2\,]\,, \quad \Delta^q = 7.10^{-4}\,[\,0{,}62 g_o + 1720 g_2\,]\,,$$

$$\bar{\Delta}^1 = 7.10^{-4}\,[3 g_o + 15{,}3 g_2\,]\,, \quad \bar{\Delta}^q = 7.10^{-4}\,[\,2{,}6 g_o - 47{,}1 g_2\,]\,,$$

$$\Delta_1^1 = 7.10^{-4}\,[4 g_o + 1760 g_2\,]\,, \quad \Delta_1^q = 7.10^{-4}\,[\,2{,}25 g_o + 1180 g_2\,]\,,$$

$$\Delta_2^1 = 7.10^{-4}\,[-3{,}2 g_o + 2760 g_2]\,, \quad \Delta_2^q = 7.10^{-4}\,[\,-3{,}2 g_o + 2640 g_2\,]\,,$$

$$\Delta_3^1 = 7.10^{-4}\,[2{,}6 g_o + 1100 g_2\,]\,, \quad \Delta_3^q = 7.10^{-4}\,[\,2{,}16 g_o + 1160 g_2\,]\,;$$

$$\Big/\, 7.10^{-4} = -12\sqrt{3}\, a_o q_o \frac{G_\rho}{a} \frac{4}{\pi\left(1 + \frac{\beta_1^2}{4}\right)} \frac{k^3}{15} \,\Big/\,.$$

We see from /II,11/ and also from the Table II and from Figure 6, that the quadratic approximation - perhaps only fortitiously - makes the agreement with the experiment better and favours the big $g_o : g_2$ ratio, if we respect the experimental indication sign $\bar{\Delta}$ = sign Δ .

Appendix III. The $\Delta I = 1$ Way of C Violation.

In clarifying the isospin properties of the C violating millistrong

Hamiltonian undoubtedly the $\eta \to \pi^+ \pi^o \pi^-$ asymmetry is the most convenient tool, because the C violation is competing only with an I forbidden C conserving transition. The C violation is, however, restricted to the $\Delta I = 0$ and $\Delta I = 2$ channels. To learn something about the other isospin possibilities $\Delta I = 1$ and 3 is a much more difficult task. In principle any charge asymmetry in the $\omega \to \pi^+ \pi^o \pi^-$ decay would prove a C violation with $I = 1$ or 3, but the C violation is competing here with a C conserving strong interaction / $\Delta I = 0$ /, so the predicted value of the asymmetry is rather small. The centrifugal situation effects are more favourable, than in the $\eta \to \pi^+ \pi^o \pi^-$ decay, but due to the high Q value this help is not too effective. The larger ω width makes the elimination of the background more difficult. So one cannot expect any information from ω before we collect a statistics about 100 or 1000 times bigger than available now.

The $\Delta I = 3$ way of C violation has been analyzed already in details [6]. That possibility does not look very interesting because it predicts $|\varepsilon| << |\varepsilon'|$, and this is contradicted by the $K_L^o \to \pi^{\pm} \ell^{\mp} \nu$ asymmetry experiments. Here we are going to discuss the $\Delta I = 1$ case a bit.

CP violation with CPT symmetry means a violation of the time reversal invariance. The upper limit on the violation of the detailed balance is 4.10^{-3} in the nuclear reactions $^{24}Mg(d,p)^{25}Mg$ and $^{24}Mg(\alpha,p)^{27}Al$ [22]. This is not enough to prove or exclude the millistrong C violation in the reaction channel $\Delta I = 0$, because the C violation is competing here with the isosymmetric strong interaction. A more favourable situation may be expected in such nuclear reactions, which are pure $\Delta I = 1$ transitions. /The C violation is competing here with a coupling $z^2/137$./ The main difficulty in observing the reversibility of $\Delta I = 1$ transitions is the following: the isospin in the ground state of stable $I_3=0$ nuclei is always $I=0$, so it looks difficult to find a $0 \rightleftarrows 1$ transition, which can be observed in both directions with sufficient accuracy. / The nuclear reaction experiments are, on the other hand, sensitive only to long range interactions./

May be, the best way to put an upper limit on the C violating vertex with $\Delta I = 1$ is just the $\eta \to \pi^o e^+ e^-$ decay. A possible way of this transition is through the ω pole /Figure 4, with ω instead of the ρ meson/. By assuming a C violation of the type

$$H_1' = g_1 \, \omega_\mu \left(\pi^o \, \partial_\mu \, \eta - \eta \, \partial_\mu \, \pi^o \right) ,$$

we arrive at the following form of the $\eta \to \pi^o e^+ e^-$ width:

$$\Gamma(\eta \to \pi^o e^+ e^-) = 0{,}47 \text{ kev} \left[(g_o + 2g_2) f_\rho + g_1 f_\omega \right]^2 . \qquad /50/$$

Here f_ρ is the ρ-γ coupling constant, which is taken usually $f_\rho = e G_\rho^{-1} = 0{,}05$. f_ω is the ω-γ coupling, which is about 0,1 [20]. The upper limit /4/ gives now the restriction

$$\left| g_o + 2g_2 + \frac{f_\omega}{f_\rho} g_1 \right| < 0{,}4 . \qquad /51/$$

We have now two possibilities: If $g_o = 0$, g_2 and g_1 may be expected in the "millistrong" range $10^{-2} - 10^{-3}$, consequently /51/ is fulfilled. If g_o is the dominating coupling constant of C violation and g_1, g_2 are only "radiative corrections", /51/ is essentially a limitation on g_o, and the g_2, g_1 terms are negligible.

• - • - • - •

Table and Figure captions.

Table I : The various types of CP violation.

Table II: Experimental and theoretical values of the charge asymmetry parameters in the $\eta \to \pi^+ \pi^o \pi^-$ decay.

Figure 1: The Dalitz plot.

Figure 2: The Δ and the $\bar{\Delta}$ asymmetry parameters.

Figure 3: ρ meson dominance in the C-violating $\eta \to \pi^+ \pi^o \pi^-$ amplitude.

Figure 4: ρ meson dominance in the $\eta \to \pi^o e^+ e^-$ decay.

Figure 5: The domain of the allowed g_o and g_2 values /linear approximation/.

Figure 6: The domain of the allowed g_o and g_2 values /quadratic approximation/.

TABLE I

ΔS	0				1			2
ΔI	0	1	2	3	1/2	3/2	5/2	any
CP violating vertex works Only among hadrons	$\varepsilon \gg \varepsilon'$ n DIPOLE MOMENT $\eta \to \pi^+\pi^0\pi^-$ NUCLEAR REACTION [2,3,4]	$\varepsilon \sim \varepsilon'$ n DIPOLE MOMENT $\omega \to \pi^+\pi^0\pi^-$ NUCLEAR REACTION	$\varepsilon \sim \varepsilon'$ n DIPOLE MOMENT $\eta \to \pi^+\pi^0\pi^-$ [5,6]	$\varepsilon \ll \varepsilon'$ $\omega \to \pi^+\pi^0\pi^-$	$\varepsilon \gg \varepsilon'$ [7,8]	$\varepsilon \sim \varepsilon'$	$\varepsilon \ll \varepsilon'$ [5]	$\varepsilon \gg \varepsilon'$ [9]
	MILLISTRONG C VIOLATION				MILLIWEAK CP VIOLATION			SUPERWEAK
CP violating vertex among hadrons plus one photon	$\varepsilon \sim \varepsilon'$ n DIPOLE MOMENT $\eta \to \pi^+\pi^0\pi^-$ [10,11]	$\varepsilon \sim \varepsilon'$ n DIPOLE MOMENT $\eta \to \pi^+\pi^0\pi^-$ $\eta \to \pi^0 e^+ e^-$ $\eta \to \pi^+\pi^-\gamma$	$\varepsilon \sim \varepsilon'$ n DIPOLE MOMENT $\eta \to \pi^+\pi^0\pi^-$	$\varepsilon \sim \varepsilon'$ $\eta \to \pi^+\pi^0\pi^-$	$\varepsilon \sim \varepsilon'$ $K \to \pi\pi\gamma$ [12]	$\varepsilon \sim \varepsilon'$ $K \to \pi\pi\gamma$	$\varepsilon \sim \varepsilon'$	$\varepsilon \gg \varepsilon'$
	ELECTROMAGNETIC C VIOLATION				WEAK ELECTROMAGNETIC C VIOLATION			SUPERWEAK

TABLE II.

Asymmetry parameter	Experiment [17]	THEORY: Linear approximation			THEORY: Quadratic approximation		
		$g_0 = 22$, $g_2 = 0$ ($\lvert g_0 + 2g_2 \rvert = 22 > 0{,}4$)	$g_0 = 0$, $g_2 = 0{,}0124$ ($\lvert g_0 + 2g_2 \rvert = 0{,}0248 < 0{,}4$)	$g_0 = 25 g_2 = 0{,}3075$, $g_2 = 0{,}0123$ ($\lvert g_0 + 2g_2 \rvert = 0{,}33 < 0{,}4$)	$g_0 = 35{,}5$, $g_2 = 0$ ($\lvert g_0 + 2g_2 \rvert = 35{,}5 > 0{,}4$)	$g_0 = 0$, $g_2 = 0{,}0126$ ($\lvert g_0 + 2g_2 \rvert = 0{,}0252 < 0{,}4$)	$g_0 = 25 g_2 = 0{,}3125$, $g_2 = 0{,}0125$ ($\lvert g_0 + 2g_2 \rvert = 0{,}34 < 0{,}4$)
$10^3 \Delta$	$15{,}2 \pm 5$	15,2	15,2	15,2	15,2	15,2	15,2
$10^3 \bar{\Delta}$	$4{,}4 \pm 5$	45,6	0,13	0,82	64,5	−0,41	0,16
$10^3 \Delta_1$	$7{,}06 \pm 16$	60,8	15,3	16	56	10,4	10,8
$10^3 \Delta_2$	$21{,}6 \pm 9$	−48	24	22,3	−79	23,4	22,4
$10^3 \Delta_3$	$13{,}9 \pm 6{,}4$	39,5	9,6	10,1	52,5	10,2	10,6

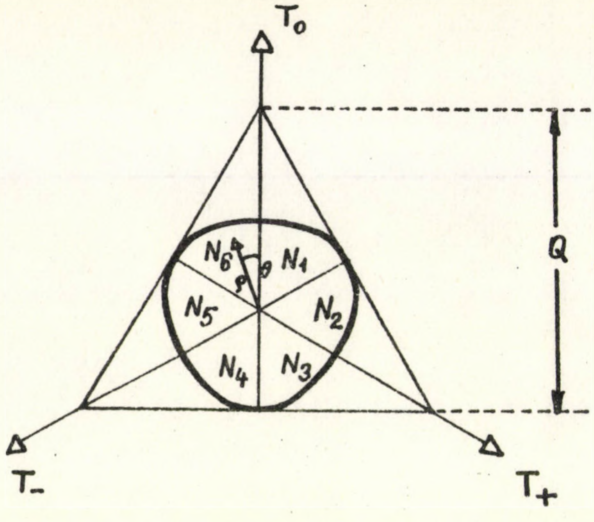


Figure 1

Figure 2

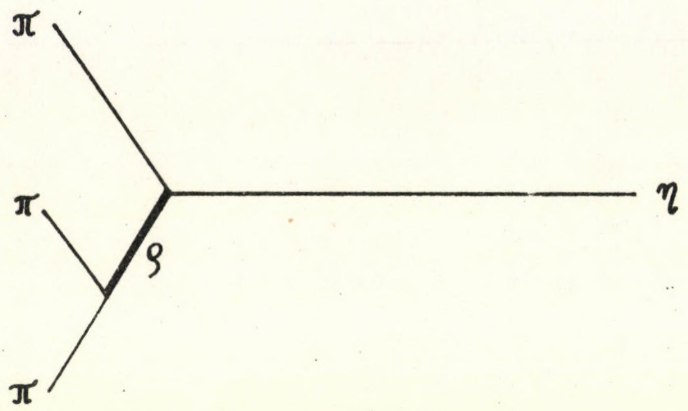


Figure 3

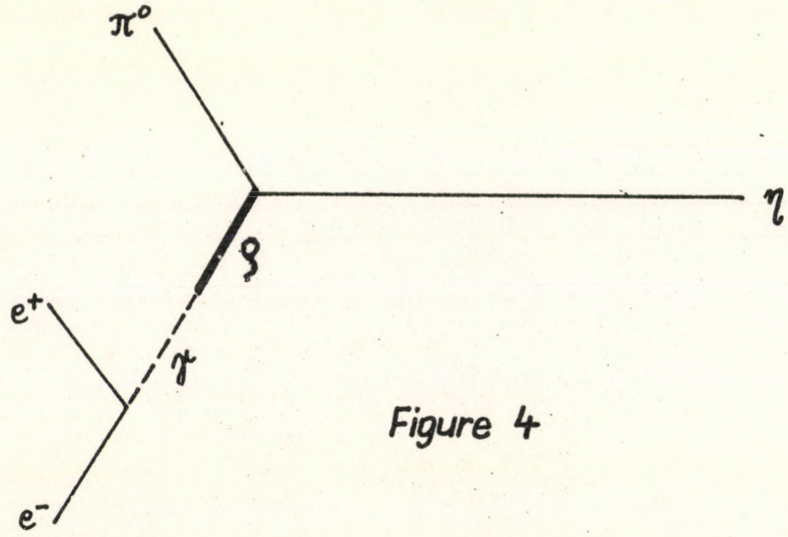


Figure 4

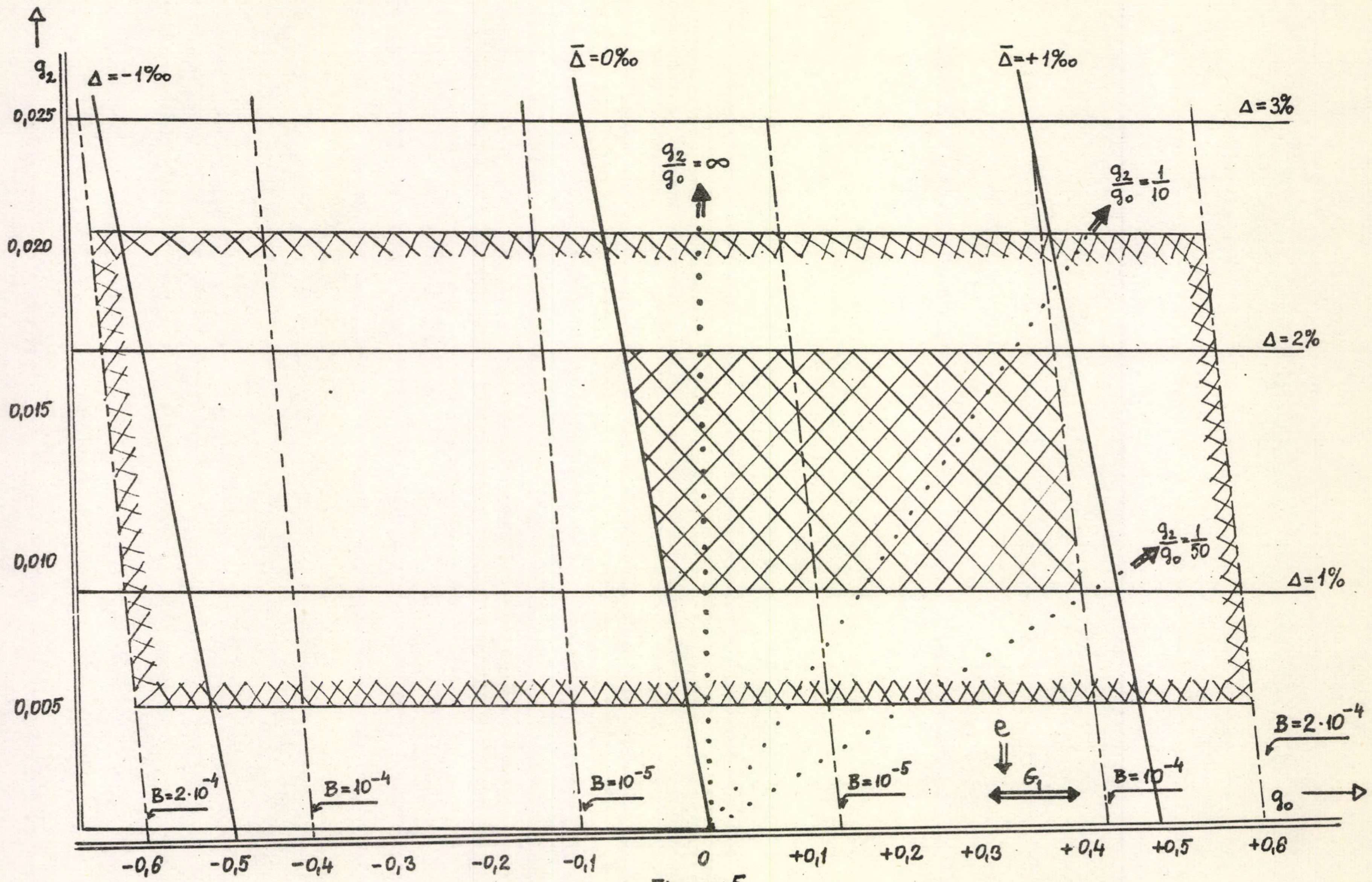
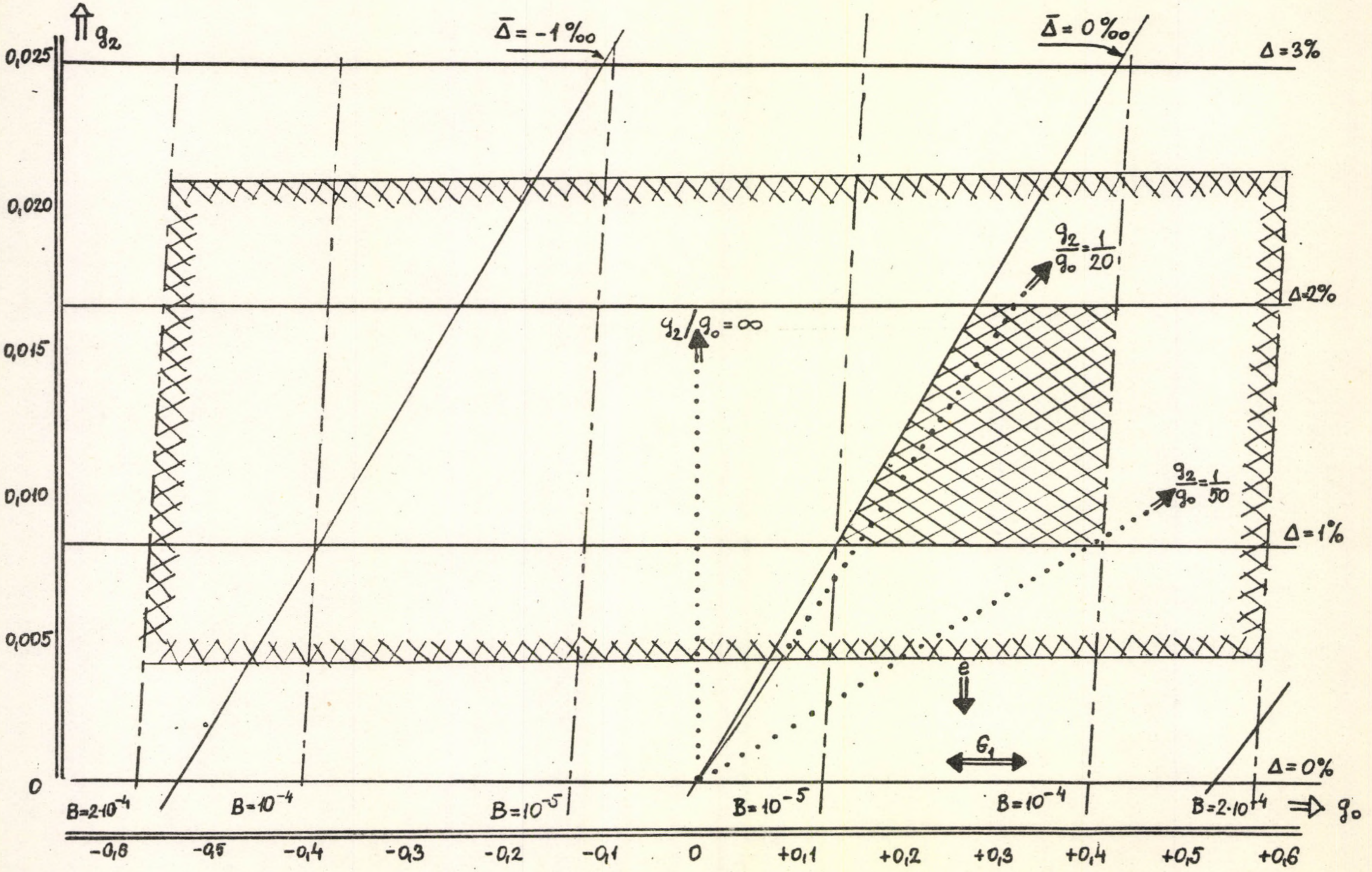

Δ
q2
Δ = -1‰
Δ̄ = 0‰
Δ̄ = +1‰
Δ = 3%
Δ = 2%
Δ = 1%
0,025
0,020
0,015
0,010
0,005
q2/q0 = ∞
q2/q0 = 1/10
q2/q0 = 1/50
B = 2·10^-4
B = 10^-4
B = 10^-5
e
G1
q0
-0,6
-0,5
-0,4
-0,3
-0,2
-0,1
0
+0,1
+0,2
+0,3
+0,4
+0,5
+0,6

Figure 5



g2
0,025
0,020
0,015
0,010
0,005
0
Δ̄ = -1 ‰
Δ̄ = 0 ‰
Δ = 3%
Δ = 2%
Δ = 1%
Δ = 0%
g2/g0 = 1/20
g2/g0 = 1/50
g2/g0 = ∞
e
G1
B=2·10^-4
B=10^-4
B=10^-5
B=10^-5
B=10^-4
B=2·10^-4
g0
-0,6
-0,5
-0,4
-0,3
-0,2
-0,1
0
+0,1
+0,2
+0,3
+0,4
+0,5
+0,6

Figure 6

References

1. Proceedings of the International Conference on High Energy Physics, Vienna /1968/

2. J.Prentki, M.Veltman, Physics Letters 15, 88, 1965

3. N.Cabibbo, Phys.Rev.Letters 14, 965, 1965

4. Y.Fujii, G.Marx, Physics Letters 17, 75, 1965

5. M.Veltman, Physics Letters 24B, 587, 1967

6. G.Marx, P.Surányi, Nuovo Cimento 54, 856, 1968

7. S.Glashow, Phys.Rev.Letters, 14, 35, 1964

8. T.N. Truong, Phys.Rev.Letters 11, 358, 1964

9. L.Wolfenstein, Nuovo Cimento 42A, 17, 1966

10. J.Bernstein, G.Feinberg, T.D.Lee, Phys.Rev. 139B, 1650, 1965
T.D.Lee, Phys.Rev. 140B, 959, 1965; 140B, 967, 1965

11. G.Feinberg, Phys.Rev. 140B, 1402, 1965
P.Babu, M.Suzuki, Phys.Rev. 162, 1359, 1967

12. B.A.Abruzov, A.T.Filippov, Physics Letters 20, 537, 1966

13. Review of Particle Properties, UCRL-8030 preprint /August 1968/

14. M.Gormley, E.Hyman, W.Lee, T.Nash, J.Peoples, C.Schultz S.Stein, Phys.Rev.Letters 21, 402, 1968

15. H.Yuta, S.Okubo, Phys.Rev.Letters 21, 781, 1968

16. M.Nauenberg, Physics Letters 17, 329, 1965

17. A.M.Cnops, G.Finocchiaro, P.Mittner, J.D.Dufey, B.Gobbi, M.A.Pouchon, A.Müller, Physics Letters 27B, 113, 1968

18. B.Barrett, M.Jacob, M.Nauenberg, T.N.Truong, Phys.Rev. 141, 1342, 1965

19. S.Weinberg, review talk in [1]

20. G.Marx, Phys.Rev. 140, 1068, 1965

21. M.J.Bazin, A.T.Goshaw, A.R.Zacher, C.R.Sun, Phys.Rev.Letters 20, 895, 1968

22. D.Bodanski, W.J.Braithwaite, D.C.Shreve, D.W.Storm, W.G.Weitkamp, Phys.Rev.Letters 17, 589, 1966.

Printed in the Central Research Institute of Physics, Budapest
Kiadja a MTA KFKI Könyvtár- és Kiadói Osztály. O.v.: Farkas Istvánné
Szakmai lektor: Surányi P. Nyelvi lektor: Hraskó P.
Példányszám 275 Munkaszám: 4169 Budapest, 1969. január 9.
Készült a KFKI házi sokszorositójában. F.v.: Gyenes Imre