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EXTENSION OF THE SL(2,C) SYMMETRY

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Abstract

The SL (2,C) trajectory generating algebra /TGA/ is extended to a SL(2,C)$\otimes$ SL (6,C) algebra in a natural way. Axial-, vector- and pseudoscalar meson trajectory families are generated by the same multiplet; the pion is automatically a conspiring particle with Toller quantum number j_o=1. Generalized SU (6) relations for Lorentz-pole residues are obtained.

I. Introduction and $SL(2,C)_L \otimes SL(2,C)_S$ Symmetry

Scattering processes, in which the mass of particles in the final state is equal to those in the initial state have an extra symmetry at zero momentum transfer. [1-4]. The group of this symmetry is defined as follows. If the scattering amplitude $T_{p_1p_2}$, $p_3 p_4$ is written after a formal crossing as

$$< - p_2, p_4 \ |T| \ p_1, -p_3> = <E,p' \ |T| \ E, p> = <p'|T|(E)| \ p> \quad , \qquad /1/$$

where $E = p_1-p_3 = p_4-p_2$, $p=\frac{1}{2}/p_1+p_3/$, $p'=\frac{1}{2}/-p_2-p_4/$, then we can define a SL(2,C) group as the simultaneous Lorentz-transformation of p and p', leaving E unchanged. The scattering amplitude is invariant under the transformations belonging to the aforementioned group, if $E = 0$. Otherwise it is invariant under a subgroup of SL(2,C) only, namely under the little group belonging to E . If we denote the elements of the corresponding SL(2,C) algebra by $M_{\mu\nu}$, $M_{\mu\nu} = -M_{\nu\mu}$, μ,ν= 0, 1, 2, 3, we may write

$$[\ T(0) \ , \ M_{\mu\nu}] = 0 \ . \qquad /2/$$

States can be classified according to irreducible representations of SL (2,C) , labelled by the quantum numbers σ and j_o [5]. $M_{\mu\nu} M_{\mu\nu} = 2 \ [\sigma(\sigma+2) + j_o^2 \ ]$, $\varepsilon_{\mu\nu\rho\kappa} \ M_{\mu\nu} \ M_{\rho\nu} = 8ij_o \ (\sigma + 1)$. Expanding the scattering amplitude, we find poles in the σ plane after an analytic continuation [1-4]. These, so-called Lorentz-poles generate an infinite number of Regge-poles. The contribution of such a pole to the scattering amplitude can be written as [5]

$$\langle p' ; s_2,\lambda_2 ; s_4,\lambda_4 |T(0)| p; s_1,\lambda_1 ; s_3,\lambda_3 \rangle$$

$$= \langle \bar{p}' ; s_2,\lambda_2 ; s_4,\lambda_4 |\{\sigma,j_0\}, s',\lambda' ; s' \rangle D^{\sigma j_0}_{s\lambda s'\lambda'}(g'^{-1}g) \qquad /3/$$

$$\cdot\ \xi_\sigma \lim (\bar{\sigma}-\sigma)\, T^{\bar{\sigma},j_0}_{ss'} \langle|\{\sigma,j_0\}, s,\lambda ; s \,|\, \bar{p} ; s_1,\lambda_1; s_3,\lambda_3 \rangle + \dots ,$$

where $T^{\sigma,j_0}_{ss'}$ is defined as

$$T^{\sigma,j_0}_{ss'} = \langle \{\sigma,j_0\} , j,.m; s' |\, T(0) |\{\sigma,j_0\}, j,m; s \rangle .$$

The ket $|\, \bar{p} ; s_1, \lambda_1; s_3\lambda_3 \rangle$ is a two- particle state with zero total four momentum and relative momentum $\bar{p} = (\bar{p},0,0,0)$; particles i having spin and spin projection to the z axis, denoted by s_i and λ_i respectively. $|\{\sigma,j_0\} , j,m ; s \rangle$ is the same type of two-particle state in SL(2,C) angular momentum picture, with total spin s and angular quantum numbers corresponding to the relative momentum $\{\sigma,j_0\},j,m$. The function $D^{\sigma j_0}_{s\lambda,s'\lambda'}(g'^{-1}g)$ is the matrix element of SL(2,C) transformation $g'^{-1}g$

$$D^{\sigma j_0}_{s\lambda s'\lambda'}(g'^{-1}g) = \langle\{\sigma,j_0\}, s',\lambda' |\, U(g'^{-1}g) |\{ \sigma,j_0\} , s,\lambda \rangle ,$$

where g and g' are boosts, transforming from $\bar{p}$ to p and from $\bar{p}'$ to p' , respectively. ξ_σ is the signature factor. The s and s' indices of the matrix element of T(0) indicate the s and s' dependence of Lorentz-pole vertices. Lorentz-poles will carry a dependence only on σ and j_0, the Casimir-operators of the maximal <u>conserved</u> group SL (2,C) . The role of s and s' can be understood very easily, if we couple together the spins s_1,s_3 and s_2,s_4 in the zero relative momentum frame

$$|\, p; s,\lambda,s_1,s_2\rangle = \sum_{\lambda i} (s\lambda|s_1\lambda_1 s_2 - \lambda_2)(-1)^{s_2-\lambda_2} |p;s_1,\lambda_1,s_2\lambda_2 \rangle$$

and expand in "angular momentum"

$$|p; s,\lambda,s_1,s_2\rangle \to |p^2, \{\sigma_0,0\} ,\ell,\mu; s,\lambda,s_1,s_2 \rangle$$

/the four- dimensional angular momentum has to be put into self-adjoint representations of SL(2,C) / . Putting the spin state into a Weinberg-

-type representation, [6] $\{s, \pm s\}$, we couple together the two SL(2,C) states to give

$$|p; \{\sigma,j_o\}, j,m; s,s_1,s_2 > = \sum_{\ell,\mu,\lambda} <\{\sigma,j_o\},j,m|\{\sigma_o,0\}\ \ell,\mu;\ \{s,s\}\ s\lambda >$$

$$|p;\{\sigma_o,0\},\ \ell,\mu;\ s,\lambda,s_1,s_2 > ,$$

where we denoted the Clebsch-Gordan coefficient of the Lorentz-group by

$$< \{\sigma,j_o\}j,m\ |\{\sigma_o,0\}\ ,\ell,\mu\ ;\ \{s,s\}\ s,\lambda\ > .$$

We did not indicate the dependence of the ket on the left hand side of the last equation on σ_o , because $\sigma - j_o = \sigma_o$ is always satisfied. The introduction of labels $\{\sigma_o,0\}$ $\{s,s\}$ instead of $\{\sigma,j_o\}$ corresponds to the splitting of the generators of SL (2,C) into two parts

$$M_{\mu\nu} = L_{\mu\nu} + S_{\mu\nu} \qquad /4/$$

Since the effect of the two SL(2,C) groups can be defined independently for the states in question,

$$[\ L_{\mu\nu}\ ,\ S_{\rho\kappa}\ ] = 0 \qquad /5/$$

is satisfied in the subspace in consideration.

A natural way of extending SL(2,C) symmetry is to assume that the operators $S_{\mu\nu}$ commute with T(O) as well, thus Regge poles are calssified according to the representations of $SL(2,C)_L \otimes SL(2,C)_S$.

Assuming the existence of one σ_o pole, expansion /3/will take the form

$$< p';\ s_2,\lambda_2;\ s_4,\lambda_4\ \ T(0)|\ p;\ s_1,\lambda_1;\ s_3,\lambda_3 >$$

$$= < \bar{p}';\ s_2,\lambda_2;\ s_4,\lambda_4\ |\{\sigma_o,0\},0,0;\ \{s,s\}\ s\lambda' > D^{\sigma_o 00}_{0000}\ (g'^{-1}g) \qquad /6/$$

$$D^{ss}_{s\lambda s\lambda'}\ (g'^{-1}g)\xi_\sigma\ \ \lim\ (\bar{\sigma}_o - \sigma_o)\ T^{\bar{\sigma}_o,s}$$

$$< \{\sigma_o,0\},\ 0,0;\ \{\ s,s\},\ s,\lambda\ |\ \bar{p}\ ;\ s_1,\lambda_1;\ s_3,\lambda_3 > + \dots$$

All the poles are now labelled by σ and s . It is clear that one σ_o pole with given s will correspond to a series of σ poles with given j_o values and vica versa. More precisely a pole at $\{\sigma_o,0\},\{s,s\}$ gives usual Lorentz-poles at

$$\{\sigma, j_o\} = \{\sigma_o + s, s\}\ ,\ \{\sigma_o + s-1,\ s-1\}, \ldots \{\sigma_o - s, -s\}\ .$$

Itt follows that with the exception of the trivial $s = 0$ case the leading trajectory $(\sigma = \sigma_o + s)$ has $j_o = s \neq 0$. On the other hand, we know that leading boson trajectories are coupled to two pseudoscalar boson systems, so they have $j_o = 0$. This observation requires a slight generalization of our symmetry scheme by allowing representations different from $\{s, \pm s\}$ for $SL(2,C)_S$. Then if we choose a representation of the type $\{S, 0\}$ the leading Lorentz trajectory will have quantum numbers $\{\sigma_o + S, 0\}$. However, a representation of $SL(2,C)_S$, which is not of the Weinberg-type [6] cannot realize on physical two-particle states. This can be prooved very easily by taking into account parity conservation. A state $|\{s_1+s_2, s_1-s_2\}, s, \lambda>$ $s_1 \neq 0$, $s_2 \neq 0$ has parity $\eta(-1)^s$ by construction, where η is some internal parity independent of s and λ. On the other hand, states $|s, \lambda>$ constructed from a given pair of particles at rest have parity η independently of s and λ. Degenerate parity doublets allow the construction of representations of the above type, e.g. the system of zero mass fermions /quarks or baryons/ can be classified according to such representations of $SL(2,C)_S$. Because of universality, we may assume that all Regge-poles are classified by the above group, while Regge-vertices which depend on the nature of external particles are not invariant under $SL(2,C)_S$ transformations. Of course the $SL(2,C)_M$ classification of vertices remains valid. When we go over to scattered particles of finite mass /from scattered particles of zero mass/ the parity degeneracy will be resolved and the <u>non-compact</u> generators of $SL(2,C)_S$ cannot be defined any more /they do not commute with parity/. Nevertheless, generators belonging to the maximal compact subgroup $SU(2)_S$ remain well-defined. So we can accept for particles of finite mass as approximate symmetry $SL(2,C)_L \otimes SL(2,C)_S$ for the Regge-trajectories and $SL(2,C)_M$ or $SU(2)_L \otimes SU(2)_S$ for the Regge-vertices.

As an illustration we may consider the pseudovector representation of $SL(2,C)_S$

$$P\ |\{\sigma_o, 0\}, 0, 0;\ \{1, 0\}\ ,\ s, \lambda > = -(-1)^s\ |\ \{\sigma_o, 0\}\ 0, 0;\ \{1, 0\}\ ,\ 0, 0 >\ .$$

The three states corresponding to $s = 1$ can be constructed from a $N\bar{N}$ two-particle state, unlike the state $s = 0$. The three $s = 1$ states are linear combinations of the states appearing in the f_o, f_1 and f_{22} helicity amplitudes [7], in $N\bar{N}$ scattering. A pole in the σ_o plane corresponds to the following poles in the σ plane

$$\{\sigma_o,0\} \otimes \{1,0\} = \{\sigma_o + 1,0\} \oplus \{\sigma_o,1\} \oplus \{\sigma_o, -1\} \oplus \{\sigma_o -1, 0\} \quad .$$

All the poles are coupled to the $s = 1$ states, which are physically realizable. If pions are coupled at zero momentum transfer, they are in the $\{\sigma_o, \pm 1\}$ representation. Then the $\{\sigma_o+1,0\}$ representation corresponds to A_1 mesons. In exact $SL(2,C)_L \otimes SL(2,C)_S$ symmetry limit their distance in the σ plane is exactly unity. The possible symmetry breaking mechanism will be discussed in section 3. If $SU(2)_S$ remains a good symmetry in the vertex functions, the ratio of the coupling of different Lorentz-poles is given simply by the Clebsch - Gordan coefficients of the Lorentz-group. It is worth while to remark that in our scheme conspiracies of the II. and III. class [7] appear necessarily <u>together</u>.

Since mesons are coupled to two pseudoscalar mesons at zero momentum transfer, the ρ -trajectory family has $j_o=0$. Probably it is the leading family of a $SL(2,C)_S$ multiplet. If we try to find some trace of $SU(6)$ symmetry /as we shall see later/ we have to put the ρ -family into a vector representation of $SL(2,C)_S$, for which

$$P \mid \{\sigma_o,0\}, 0,0 \; ; \{1,0\}. s,\lambda > = (-1)^s \mid \{\sigma_o,0\} , 0,0 \; ; \{1,0\} , s,\lambda > \cdot$$

Only the $s=0$ state is coupled to the $N\bar{N}$ system /this is the f_{11} helicity state/. The $s=0$ system is coupled to the $\{\sigma_o+1,0\}$ and $\{\sigma_o-1,0\}$ families only. In the framework of the $SL(2,C)_L \otimes SL(2,C)_S$ symmetry ρ -meson and pions are always in different multiplets /their G-parity is different, but G commutes with $SL(2,C)_S$ generators/.

2. <u>The Inclusion of Internal Symmetries</u>

If we consider the scattering of zero mass quarks or baryons, we can define easily $SL(6,C)_S$ transformations /one may think about an $U(6,6)_S$ extension of this group/, which commute with $SL(2,C)_L$ by construction. In this case we obtain a minimal trajectory generating algebra $SL(2,C)_L \otimes SL(6,C)_S$. [8]. This algebra is minimal in the following sense: if $T(0)$ commutes with $SL(2,C)_L \otimes G_S$, where G_S is a subalgebra of $SL(6,C)_S$, G_S contains at least one operator linearly independent of the generators of $SL(2,C)_S$ and $SU(3)$, the internal symmetry group, then $SL(2,C)_L \otimes SL(6,C)_S$ will commute with $T(0)$ as well. This statement is a simple consequence of the equations

$$[T(0), M_{\mu\nu}] = 0,$$

$$[T(0), T^A] = 0,$$

and the Jacobi-identity for double commutators of the type $[T(0),[M_{\mu\nu},G_s]]$ and $[T(0),[T^A, G_s]]$, where T^A are the generators of $SU(3)$.

For particles of finite mass the parity degeneracy is resolved, so the maximal symmetry for the vertices will be the maximal compact subgroup of $SL(2,C)_L \otimes SL(6,C)_S$: $SU(2)_L \otimes SU(6)_S$ /which commutes with the parity operator!/.

The trajectories, however, will be classified by the TGR $SL(2,C)_L \otimes SL(6,C)_S$

As evident from the above discussion, non-unitary representations realize for both commuting subalgebras. These are infinite-dimensional "self-adjoint" /$j_0 = 0$/ for $SL(2,C)_L$ and finite-dimensional for $SL(6,C)_S$.

First we shall consider mesons. They are probably in a $(6,\bar{6})$ /quark-antiquark/ representation of $SL(6,C)_S$. The $(6,\bar{6})$ representation goes over to $(\bar{6},6)$ under space reflection, thus, parity eigenstates can be constructed from the linear combination of these representations $(6,\bar{6})^{\pm}$

Trajectory families corresponding to positive and negative parity combinations will conspire with each other just like scalar and pseudoscalar trajectories in a $j_0 = 1$ representation in usual $SL(2,C)_M$. This fact is a simple consequence of the equality

$$\left(\langle 6,\bar{6}| + \langle\bar{6},6|\right) T \left(|6,\bar{6}\rangle + |\bar{6},6\rangle\right) = \left(\langle 6,\bar{6}| - \langle\bar{6},6|\right) T \left(|6,\bar{6}\rangle - |\bar{6},6\rangle\right) =$$
$$= T^{(6,\bar{6})} + T^{(\bar{6},6)}.$$

For determining the appearing $SL(2,C)_M$ families, we reduce first the $SL(6,C)_S$ representations to $SL(2,C)_S \otimes SU(3)$, then add the $SL(2,C)_S$ to the $SL(2,C)_L$ representations to give

$$\{\sigma_0, 0\} \otimes (6,\bar{6})^{\pm} = \{\sigma_0 + 1, 0\}^{\pm}_8 \oplus \{\sigma_0 + 1, 0\}^{\pm}_1 \oplus \{\sigma_0, 1\}^{\pm}_8$$

$$\oplus \{\sigma_0, -1\}^{\pm}_8 \oplus \{\sigma_0, 1\}^{\pm}_1 \oplus \{\sigma_0, -1\}^{\pm}_1 \oplus \{\sigma_0 - 1, 1\}^{\pm}_8 \oplus \{\sigma_0 - 1, -1\}^{\pm}_1,$$

where we denoted the representations of $SL(2,C)_M \otimes SU(3)$ by $\{\sigma, j_0\}^{\tau}_n$ where n is the dimension of the $SU(3)$ representation and the τ sign refers to the internal parity. The representations with different τ are of different C-parity /or G-parity/ as well. If the vector meson nonet is identified as $\{\sigma_0+1,0\}^{+}_8$ and $\{\sigma_0+1,0\}^{+}_1$ /so the σ_0 signature is even/, then $\{\sigma_0+1,0\}^{-}_0$ and $\{\sigma_0+1,0\}^{-}_8$ give an axial-vector meson family nonet, $\{\sigma_o; \pm 1\}_8^-$ and $\{\sigma_o, \pm 1\}_1^-$ conspiring scalar and pseudoscalar meson nonets, $\{\sigma_o, \pm 1\}_8^+$ and $\{\sigma_o, \pm 1\}_1^+$ conspiring scalar and pseudoscalar nonets of opposite G-parity, and some even lower lying vector and pseudovector nonet families.

Since the Lorentz quantum numbers of the ρ -meson and pion trajectory families are $\sigma_\rho \approx o,54$ and $\sigma_\pi \approx -o,o2$, respectively, their difference is less than unity, thus the $SL(6,C)_S$ symmetry must be broken even for the trajectories.

The vertices are invariant only under $SU(6)_S$. If we decompose the $(6,\bar{6})$ representation into $SU(6)_S$ representations of definite parity we obtain

$$(6,\bar{6})^+ \oplus (6,\bar{6})^- = \underline{35}^+ \oplus \underline{35}^- \oplus \underline{1}^+ \oplus \underline{1}^- \quad .$$

A $\underline{35}$ representation of definite parity is composed from both representations of definite parity. Only the states of positive parity are coupled to $B\bar{B}$ systems.

By coupling the positive and negative energy baryons at rest, we obtain the $SU(6)$ representations $\underline{56} \otimes \underline{56} = \underline{1} \oplus \underline{35} \oplus 4o5 \oplus 2695$. Representations $\underline{1}$ and $\underline{35}$ give a projection to the $(6,\bar{6})$ representation. The elements of the above $SU(6)$ representations will be the different helicity states. In contrast with static $SU(6)$, pseudoscalar meson pairs are coupled to the ρ -trajectory. The two-meson system contains the $\underline{35}$ representation of $SU(6)_S$ and the ρ -trajectory is coupled only to the s = O state $(j_o = o!)$.

As far as the baryons are concerned, they can be put into /56,1/ and /1,56/ representations of $SL(6,C)_S$. Then we get the following $SL(2,C)_M$ families

$$\{\sigma_o, 0\} \otimes [(56,1) \oplus (1,56)] = \{\sigma_o + \tfrac{3}{2}, + \tfrac{3}{2}\}_{1o} \oplus \{\sigma_o + \tfrac{3}{2}, - \tfrac{3}{2}\}_{1o}$$

$$\oplus \{\sigma_o + \tfrac{1}{2}, \tfrac{1}{2}\}_{1o} \oplus \{\sigma_o + \tfrac{1}{2}, - \tfrac{1}{2}\}_{1o} \oplus \{\sigma_o - \tfrac{1}{2}, + \tfrac{1}{2}\}_{1o} \oplus \{\sigma_o - \tfrac{1}{2}, - \tfrac{1}{2}\}_{1o}$$

$$\oplus \{\sigma_o - \tfrac{3}{2}, \tfrac{3}{2}\}_{1o} \oplus \{\sigma_o - \tfrac{3}{2}, - \tfrac{3}{2}\}_{1o} \oplus \{\sigma_o + \tfrac{1}{2}, \tfrac{1}{2}\}_8 \oplus \{\sigma_o + \tfrac{1}{2}, - \tfrac{1}{2}\}_8$$

$$\{\sigma_o - \tfrac{1}{2}, \tfrac{1}{2}\}_8 \oplus \{\sigma_o - \tfrac{1}{2}, - \tfrac{1}{2}\}_8 .$$

Our notations are similar to those used for the description of boson trajectory families. The representations $\{\sigma_o + \frac{3}{2}, \pm \frac{3}{2}\}_{1o}$ and $\{\sigma_o + \frac{1}{2}, \pm \frac{1}{2}\}_8$ can be identified as the known decuplet and octet trajectory families [9,10,11].

An interesting consequence of this classification scheme is that the Δ family has quantum number $|j_o| = \frac{3}{2}$, which forbids its coupling to the ΠN system in the symmetry limit. However, even $SL(2,C)_M$ is broken for the Δ residues due to unequal masses. Another consequence of this classification is that the Δ trajectory functions $\alpha_\Delta(w)$ cannot have a linear term in their Taylor-expansion [9,10], a fact, which was observed in various experimental fits [11,12].

If the $SL(6,C)_S$ symmetry were valid at non-zero momentum transfer, then at the point $\sigma_o=0$ -as easy to check from the Clebsch - Gordan coefficients - all representations but $\{\sigma_o + \frac{3}{2}, \pm\frac{3}{2}\}_{10}$ and $\{\sigma_o + \frac{1}{2}, \pm\frac{1}{2}\}_8$ would decouple, that is to say, we would get back the multiplets (and their parity partner) belonging to the usual $\underline{56}$ representation of $SU(6)$.

The coupling of Lorentz-poles to different two-particle states can be given most easily by writing down the contribution of a $SL(2,C)_L \otimes SL(6,C)_S$ pole to the scattering amplitude of particles α and β going over to particles γ and δ, as

$$
\begin{aligned}
&< p' \,; s', \lambda' \,; A \,|T|\, p \;\; s, \lambda \,; A > = \\
&= \sum_{B,B',\sigma,j_o} < \bar{p}' \,; s', \lambda', A \,|\, \{\sigma_o, 0\}, 0,0 \,; B', s', \lambda', A > f(B') \qquad /7/ \\
&< B', s', \lambda', A \,|\, C, \{s_1+s_2, s_1-s_2\} , s', \lambda', A > \\
&<\{\sigma_o,0\},0,0;\, C, \{s_1+s_2,s_1-s_2\} , s',\lambda', A|\, \{\sigma,j_o\}, s',\lambda';\, A > D^{\sigma j_o}_{s\lambda s'\lambda'}(g'^{-1}g) \\
&< \{\sigma,j_o\}, s,\lambda,A \,|\, \{\sigma_o,0\}, 0,0;\, C, \{s_1+s_2, s_1-s_2\} , s,\lambda, A > \\
&< C, \{s_1+s_2, s_1-s_2\} , s,\lambda,A \,|\, B,s,\lambda, A> f(B)<\{\sigma_o,0\}, 0,0;\, B,s,\lambda,A|\, \bar{p};\, s,\lambda;\, A> .
\end{aligned}
$$

In eq. 7 we used the following notations: A stands for the $SU(3)$ quantum numbers (Casimir-operators and sub-quantum numbers Y, I, I_3). B,B' and C denote the $SU(6)_S$ and $SL(6,C)_S$ representations in questión, respectively.

The first and last brackets under the sum in eq. 7 are the products of transition matrix elements $<\{\sigma_o,0\},0,0|\,\bar{p}> = \frac{\sigma_o+1}{2\Pi}$ and Clebsch - Gordan coefficients of $SU(6)_S$

$$< B,\ s,\ \lambda,\ A \mid \alpha\bar{\gamma};\ s,\ \lambda,\ A > ,$$

$f(B)$ is the reduced vertex function.

The second brackets towards the centre of the expression on the r.h.s. of eq. 7 are the transition matrix elements from $SU(6)_S$ representations B and B' to $SL(6,C)_S$ representation C . The brackets next to the function $D^{\sigma j_o}_{s\lambda s'\lambda'}$ are the products of transition matrix elements from $SL(6,C)_S$ to $SL(2,C)_S \otimes SU(3)$representations and Glebsch - Gordan coefficients of the Lorentz- Group.

$$< \{\sigma_o,0\}\ ,0,0\ ;\ \{s_1+s_2,\ s_1-s_2\},\ s,\lambda\ |\{\sigma,j_o\},\ s,\lambda > .$$

Using the known expressions for the above matrix elements we obtain

$$< p';\ s',\ \lambda';\ A\ |T|\ p;\ s,\lambda\ ;\ A >$$

$$= \frac{1}{(2\Pi)^2} \sum_{B,B',\sigma,j_o} (\sigma_o+1)(2s_1+1)(2s_2+1)\left[(\sigma+1)^2 - j_o^2\right] f(B')\ f(B)$$

$$< \bar{\beta}\delta\ ;\ s',\lambda'\ ;\ A \mid B',\ s',\lambda',\ A > < B,\ s,\ \lambda,\ A \mid \alpha\bar{\gamma}\ ;\ s,\lambda\ ;\ A > \qquad /8/$$

$$\begin{Bmatrix} \frac{\sigma_o}{2} & \frac{\sigma+j_o}{2} & s_1 \\ s & s_2 & \frac{\sigma-j_o}{2} \end{Bmatrix} \begin{Bmatrix} \frac{\sigma_o}{2} & \frac{\sigma+j_o}{2} & s_1 \\ s' & s_2 & \frac{\sigma-j_o}{2} \end{Bmatrix} D^{\sigma j_o}_{s\lambda s'\lambda'}\ (g'^{-1}g) ,$$

where we expressed the Clebsch - Gordan coefficients of the Lorentz-Group by 6j coefficients [13].

3. Symmetry breaking

As we have seen, the $SL(6,C)_S$ symmetry of Regge trajectory families is broken. First we shall consider the symmetry breaking mechanism for the $SL(2,C)_L \otimes SL(2,C)_S$ group. Since the symmetry breaking operator has to be invariant under $SL(2,C)_M$, the most general form of a symmetry breaking operator is

$$T' = \sum_{\sigma j_o j_m} C_{\sigma,j_o}\ T^{\{\sigma,j_o\}}_{S;j_m}\ T^{\{\sigma,j_o\}}_{L;j_m} , \qquad /9/$$

where $T^{\{\sigma,j_o\}}_{S;j_m}$ is a scalar operator under $SL(2,C)_L$ and a tensor-operator of representation $\{\sigma,\ j_o\}$ under $SL(2,C)_S$,while $T^{\{\sigma,j_o\}}_{L;j_m}$ is a scalar operator under $SL(2,C)_S$ and a tensor operator of representation $\{\sigma , j_o\}$ under $SL(2,C)_L$. Since the realized $SL(2,C)_S$ representations are of finite dimension, only few terms of sum 9. will give a contribution to a matrix element of the type

$$< \{\sigma,j_o\} , j,m \mid T' \mid \{\sigma,j_o\} , j,m > =$$

$$= \sum_{\ell,\mu,s,\lambda,\ell',\mu',s',\lambda'} < \{\sigma,j_o\} , j,m \mid \{\sigma_o,0\} ,\ell',\mu' ; \{s_1+s_2 , s_1-s_2\} , s',\lambda' > \qquad /10/$$

$$< \{\sigma_o,0\} , \ell',\mu' ; \{s_1+s_2, s_1-s_2\} ,s',\lambda' \mid T' \mid \{\sigma_o,0\} ,\ell,\mu ; \{s_1+s_2,s_1-s_2\},s,\lambda >$$

$$< \{\sigma_o,0\} , \ell, \mu ; \{s_1+s_2, s_1-s_2\} , s,\lambda \mid \{\sigma,j_o\} , j,m > .$$

The first order perturbation to the Lorentz-poles is given by the expression [9,10]

$$\sigma' = \sigma + \alpha < \{\sigma,j_o\} , j,m \mid T' \mid \{\sigma,j_o\} , j,m > . \qquad /11/$$

Using equations 9. and 10. we can obtain for σ' in a straightforward way

$$\sigma' = \sigma + \sum_{N,M} \bar{C}_{N,M} \, \tau_\sigma \begin{Bmatrix} \sigma_o/2 & \frac{N+M}{2} & \sigma_o/2 \\ s_1 & \frac{\sigma+j_o}{2} & s_1 \end{Bmatrix} \begin{Bmatrix} \sigma_o/2 & \frac{N-M}{2} & \sigma_o/2 \\ s_2 & \frac{\sigma-j_o}{2} & s_2 \end{Bmatrix} , \qquad /12/$$

where the constants $\bar{C}_{N,M}$ give the weight of the representation $\{N,M\}$ in the perturbation operator, τ_σ is the signature of the pole in question. E.g. a $\sigma_{\mu\nu} \; p_\mu p'_\nu$ type coupling of $SL(2,C)_L$ and $SL(2,C)_S$ corresponds to the $\{1,\pm 1\}$ representations of the perturbation. If one assumes the presence of only one irreducible representation in the perturbation operator, one can get useful relations among the perturbed σ values of a $SL(2,C)_S$ multiplet.

In the framework of the $SL(6,C)_S$ symmetry we can construct at first operators which break $SL(6,C)_S$ but commute with $SL(2,C)_S$. The degeneracy between the $(6,\bar{6})^+$ and $(6,\bar{6})^-$ representations can be resolved by a (21,21) type two-quark - two-antiquark perturbation, which contains the scalar representation of $SL(2,C) \otimes SU(3)$ and gives a transition between $(6,\bar{6})$ and $(\bar{6},6)$.

If we want to break $SL(2,C)_S$ as well, we can put the symmetry breaking operator into simple representations of $SL(6,C)_S$, e.g. (35,1), from which we may project out $SU(3)$ singulet representations and couple with an identical $SL(2,C)_L$ representation to give a $SL(2,C)_M$ scalar.

4. Discussion

The application of the above described symmetry scheme to actual processes will be discussed in a forthcoming paper. We hope to extend our considerations to non-vanishing momentum-transfer [9,10] as well. For non-vanishing momentum transfer, in the C.M.S. of the t channel we hope to classify trajectories according to the representations of $SU(2)_L \otimes SU(6)_S$, the maximal compact subgroup of our TGR, just like in the case of TGR $SL(2,C)_M$, when at non-zero momentum transfer the remaining symmetry was the maximal compact subgroup of the TGR, $SU(2)_M$.

We mention that Salam et al. [14] tired to extend $SL(2,C)_M$ symmetry in an entirely different manner by embedding $SL(2,C)_M$ in a higher group, $SL(6,C)_M$ and continuing some of the Casimir operators of the latter to complex values.

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