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MASS FORMULA IN BROKEN SL(2,C) AND ITS APPLICATION TO THE NUCLEON AND DELTA REGGE TRAJECTORIES

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BUDAPEST

MASS FORMULA IN BROKEN SL/2,C/ AND ITS APPLICATION TO THE NUCLEON AND DELTA REGGE TRAJECTORIES

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ABSTRACT

Consequences of a Regge-trajectory generating algebra SL/2,C/ are explored at nonvanishing energy. Nucleon and delta trajectories satisfy a second order "mass-formula" remarkably well.

In a series of papers[1-8], we investigated the concept and consequences of a trajectory generating algebra /TGA/, for hadron systems. The main result of the investigation is that - as a consequence of standard analyticity and invariance properties of the scattering amplitude and of crossed - channel unitarity, Regge trajectories occur in families; the corresponding "wave functions" of pseudostates[6/] at zero momentum transfer, t /i.e. vanishing mass of the exchanged Regge pole/ /Regge "vertices"/ transform according to an irreducible representation /IR/ of a trajectory generating algebra, larger than the Lie algebra of the little groups for space - or timelike momentum transfers. The role of the TGA is to make certain that at the contraction point, t=0 of the aforementioned little groups /SL/2,R/ and SU/2/, respectively, no unwanted singularities occur in any scattering amplitude. Generally speaking, the TGA does not generate extra symmetries of the scattering amplitudes but rather it expresses some properties of the bound state spectrum /"Regge-poles"/ in a compact form. It was shown in ref[4/] that the minimal algebra satisfying these requirements is SL/2,C/.

In this letter we explore the consequences of the TGA just mentioned for the algebraic classification of Regge trajectories off the point t=0.

It is intuitively clear and can be demonstrated by model calculations that off the point t=0 the family of pseudostates cannot transform anymore according to an IR of the TGA. Nevertheless, since the unsymmetric terms have definite - and simple - transformation properties under the TGA, one can make some definite predictions about the behaviour of individual members of a family of Regge trajectories generated by the TGA.

To fix the ideas, let us consider a Bethe-Salpeter /BS/ model of Regge trajectories. We write the homogeneous, partial wave BS equation in operator form:

$$H(w,j)|\varphi\rangle \equiv [F(w) - K(w,j)]|\varphi\rangle = 0, \qquad /1/$$

where F is the inverse of the disconnected part of the two-particle Green's function, K stands for the BS kernel. The total mass and spin of the bound state are $W = t$ and j, respectively. At $W = 0$ /because of the TGA/, the dependence of H on j enters only through the SL/2,C/ invariant, . We have namely for a bound state solution the relation: $j = \sigma - \varkappa$, where σ determines the position of the "Lorentz pole" and is the "order" of the bound state in the SL/2,C/ family in question. Expending now eq. /1/ around the point $W = 0$, we write:

$$(H_0 + (H_W + H_\sigma \alpha'_\varkappa)W + \ldots)(|\varphi\rangle + W|\varphi'\rangle + \ldots) = 0 \qquad /2/$$

corresponding to an expansion of the Regge trajectories:

$$\alpha_\varkappa(W) = \sigma - \varkappa + \alpha'_\varkappa W + \tfrac{1}{2}\alpha''_\varkappa W^2 + \ldots$$

On collecting the powers of W and multiplying /2/ with the bra-vector $\langle\varphi|$ we arrive at the following expressions of the derivatives $\alpha_\varkappa^{(n)}$:

$$\alpha'_\varkappa = \langle H_W \rangle ,$$

$$\alpha''_\varkappa = \langle H_{WW}\rangle - 2\langle H_W \frac{1-P_\varphi}{H_0} H_W\rangle - 2\alpha'_\varkappa \qquad /3/$$

$$\times\left[\langle H_j \frac{1-P_\varphi}{H_0} H_W\rangle + \langle H_W \frac{1-P_\varphi}{H_0} H_j\rangle\right] + (\alpha'_\varkappa)^2\left[\langle H_{jj}\rangle - \right.$$

$$\left. - 2\langle H_j \frac{1-P_\varphi}{H_0} H_j\rangle\right], \quad \ldots$$

In these formulae, $H_0 \equiv H$ ($W=0$, $\sigma = \sigma_i$/ σ_i being the position of a Lorentz-pole), the subscripts mean partial derivatives and P_φ is the projection operator onto the bound state at $W=0$. Expectation values, like $\langle H_W\rangle$, mean $\langle\varphi|H_W|\varphi\rangle$ taken at $W=0$ and $j = \sigma_i - \varkappa$. The bound state $|\varphi\rangle$ at $W=0$ is characterised by the eigenvalues of the Casimir operators of SL/2,C/ at $W=0$; conventionally, we chose the numbers $\{\sigma, j_0\}$ to label the bound state. The numbers σ and j_0 are connected with the Casimir operators of SL/2,C/ in its real form as follows:

$$\tfrac{1}{4}(\underset{\sim}{S} - i\underset{\sim}{T})^2 = j_1(j_1+1)\,\mathbb{1} ,$$

$$\tfrac{1}{4}\ S + iT^{\ 2} = j_2(j_2+1)\,\mathbb{1} ,$$

$$j_1 = \tfrac{1}{2}(\sigma + j_0), \qquad j_2 = \tfrac{1}{2}(\sigma - j_0),$$

where S and T stand for the "compact" and "non-compact" elements of SL/2,C/, respectively.

We now observe that H_W is an irreducible tensor operator $\sim\{1,0\}$ under SL/2,C/, similarly $H_{WW}\sim\{1,0\}\otimes\{1,0\}$, etc, while $H_0\sim\{0,0\}$, $H_\sigma\sim\{0,0\}$, i.e. they are invariants. Using this fact and the Wigner-Eckart theorem for SL/2,C/, we can extract the entire dependence on $\varkappa$ of the matrix elements into Clebsch-Gordan coefficients of the TGA. Inserting the values of the latter[8], we get the following phenomenological expressions for Regge trajectories that belong to one SL/2,C/ "family" /i.e. build up an IR of the TGA/ at W=0:

$$\alpha_\varkappa(W)=\sigma-\varkappa+A(\sigma-\varkappa+\tfrac{1}{2})W+$$
$$+\left[B_1+B_2(\sigma-\varkappa)(\sigma-\varkappa+1)+A^2(\sigma-\varkappa+1/2)\right]W^2+O(W^3) \qquad /4/$$

/Fermi trajectories with $|j_0|=1/2$/

$$\alpha_\varkappa(W)=\sigma-\varkappa+\left[b_1+b_2(\sigma-\varkappa)(\sigma-\varkappa+1)\right]W^2$$
$$+O(W^4) \qquad /5/$$

/Bose trajectories, with j_0 = integer/.

Thus in the Chew-Frautschi approximation /the power series of $\alpha(W)$/ broken off at the quadratic term/, a - generally infinite - family of fermion Regge trajectories is described by four independent parameters, whereas three parameters are sufficient for the description of a boson family.
/The parameters A, B_1, B_2, etc. are essentially invariant combinations of the reduced matrix elements of operators appearing in /3//.

The details of the theory of a broken TGA have been described elsewhere[7,8]; here we report the results of the analysis of the best - known Regge trajectories - the I=1/2 and I=3/2 nonstrange baryon trajectories - based on this theory.

a/ I=1/2. We assumed that the known N_α trajectory is the parent of an SL(2,C) family. Further, the positions and spin-parity assiquments of the $P_{11}(1466)$ and $S_{11}(1550)$ resonances suggest that the may lie on the $\varkappa$= 2 member of the same family. On making this hypothesis, we determined the parameters σ, A, B_1 and B_2 from the states just mentioned and from the data of $P_{11}(938)$ and $F_{15}(1688)$ (presumably the parent of $P_{11}(1466)$.) The resulting values of the parameters are:

$$\sigma = -0{,}37$$
$$A = 0{,}054\ \mathrm{GeV}^{-1}$$

$$B_1 = 1.04\ \mathrm{GeV}^{-2}, \qquad B_2 = 0.08\ \mathrm{GeV}^{-2}$$

and the trajectories according to eq. (4) are plotted in Fig. 1. We observe that the resulting value of σ is in good agreement with the zero intercept of the nucleon trajectory as obtained e.g. from the recent fit of Noirot et al[9] to backward π N scattering. Further, the $\varkappa = 1$ trajectory /which we have calculated/ appears to coincide precisely with the N_γ trajectory known phenomenologically. The resonant states plotted in Fig. 1. are taken from Rosenfeld's tables[10] and of Lovelace[11]. Plotted are also the states (?) which were classified as "resonance interpretation in doubt" in ref[11]. Spin-parity assignments of some higher resonances were taken from Barger[12]. The agreement between the observed resonances and their mass values resulting from eq. (4) is in general quite good[13].

b/ I=3/2. A similar procedure was applied to the observed Δ-resonances. Accepting - the exchange - degenerate - fit of ref[9] for Δ_α, the $\varkappa$ =1 trajectory was forced through D_{35}(1954); the resulting values of the parameters being:

$$\sigma = 0.15, \quad A \approx 0, \quad B_1=0.89 \text{ GeV}^{-2}, \quad B_2=0.07 \text{ GeV}^{-2}.$$

The corresponding curves are plotted in Fig. 2. The agreement between the predicted and observed mass values of the I=3/2 resonances in indeed encouraging.

To conclude, it seems that hadronic Regge trajectories can be classified succesfully according to representations of a broken SL(2,C) TGA. It is important to point out that a mass formula of the type /4/,/5/ is a necessary consequence of such a classification and it should be checked against the experimental data whenever one attempts to put some Regge trajectories into families of the TGA.

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Footnotes and references

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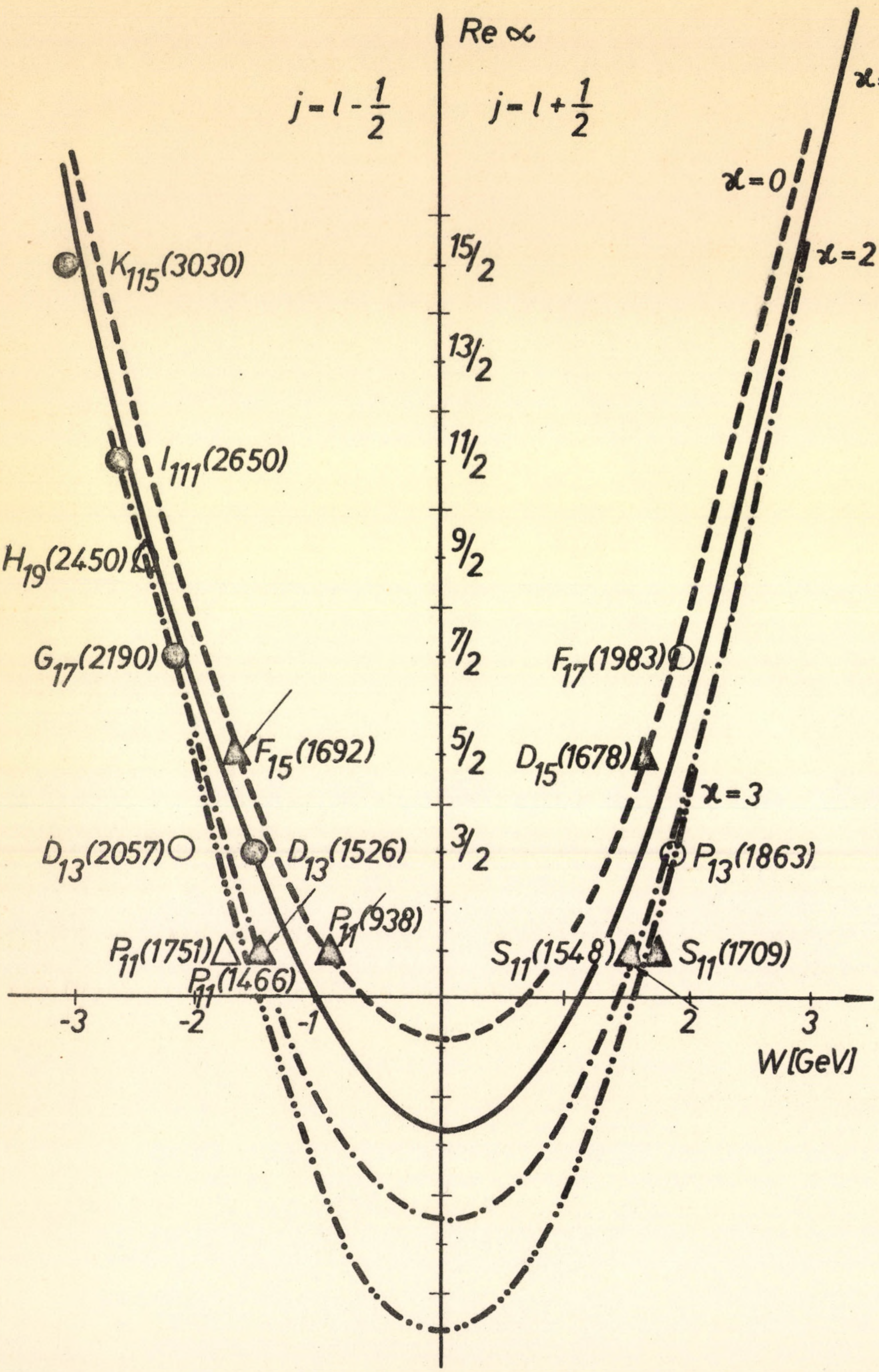
12./ V.Barger, Proc. Topical Conference on High Energy Collisions of Hadrons, CERN /1968/, to be published

13./ The fits used in this letter are somewhat uncertain due to ambiguities in determining the position of a resonance in the presence of other open channels /see the discussion of this point given in[10/] /and experimental errors. Here - as reliable error - estimates are not available - we simply accepted the data at their face - value. We tentatively ignored some troubles with SU/3/ assignments. If P_{11}/1466/ is a member of a $\overline{10}$ /as suggested by Lovelace, CERN rep. /1965/ TH628/, then it cannot be together with S_{11}/1548/ into one family which is almost certainly an octet. However, if P_{11}/1466/ and Z_o are in $\overline{10}$, then it is somewhat difficult to get rid of an unobserved stable Ξ^*. Also, the mass values and spin-parity assignments seem to suggest that P_{11}/1466/ and S_{11}/1548/ may be MacDowell partners, in which case also P_{11} would be an octet.

Figure captions

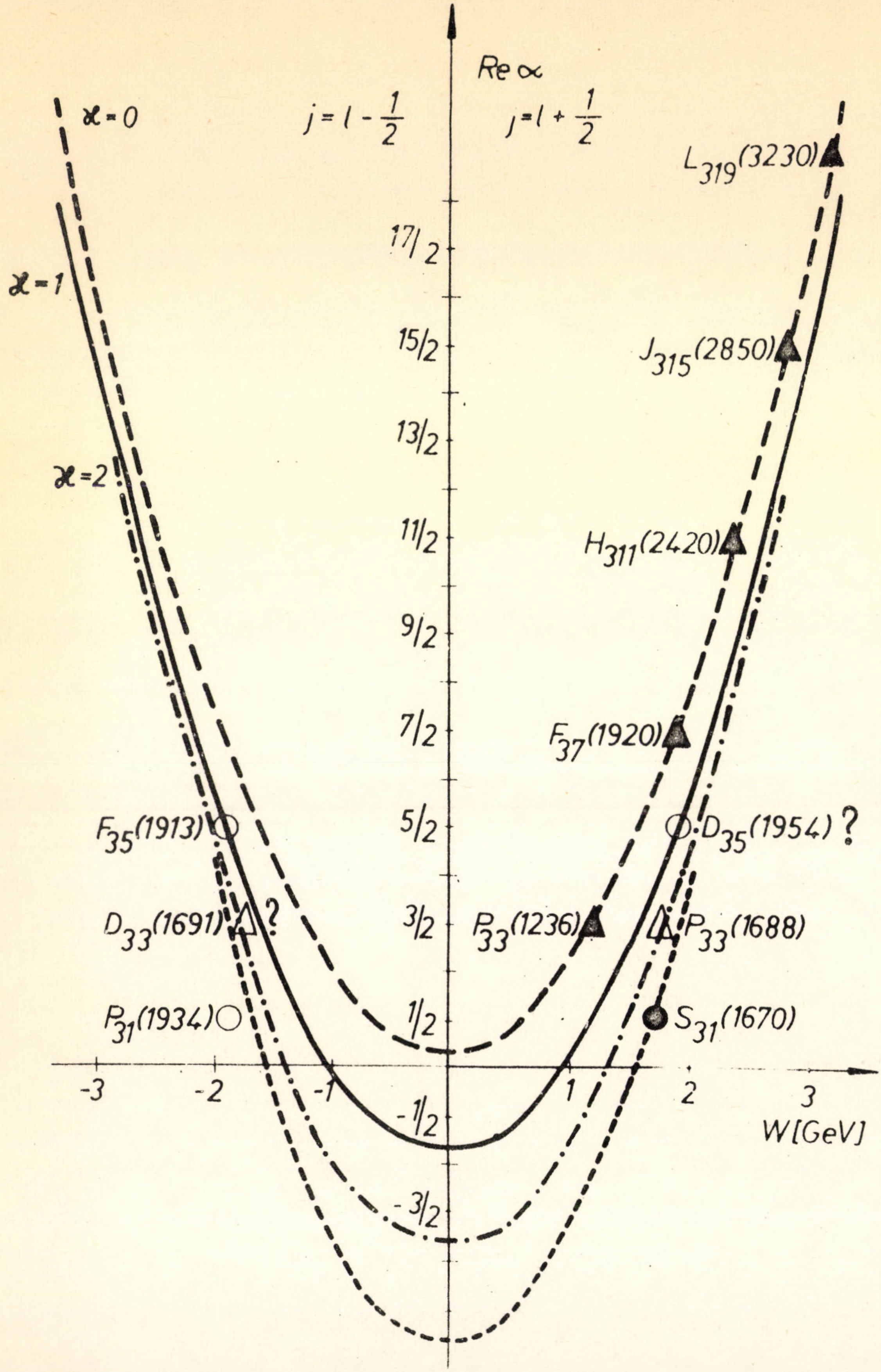
Fig. 1. Regge trajectories with the observed I=1/2 nucleon resonances. Resonances of positive signature are marked by triangles, these of negative signature by circles. Well-established resonances are drawn with full triangles and circles. Points classified as "resonance interpretation in doubt" in[11] are distinguished by a questionmark.

Fig. 2. Regge trajectories with the observed I=3/2 resonances. The notation is the same as in Fig. 1.



Re α
j = l - 1/2
j = l + 1/2
ϰ=1
ϰ=0
ϰ=2
ϰ=3
15/2
13/2
11/2
9/2
7/2
5/2
3/2
K115(3030)
I111(2650)
H19(2450)
G17(2190)
F15(1692)
D13(2057)
D13(1526)
P11(938)
P11(1751)
P11(1466)
F17(1983)
D15(1678)
P13(1863)
S11(1548)
S11(1709)
-3
-2
-1
2
3
W[GeV]

Fig. 1.



Re α
ϰ=0
ϰ=1
ϰ=2
j = l - 1/2
j = l + 1/2
17/2
15/2
13/2
11/2
9/2
7/2
5/2
3/2
1/2
-1/2
-3/2
-3
-2
-1
1
2
3
W[GeV]
L319(3230)
J315(2850)
H311(2420)
F37(1920)
F35(1913)
D35(1954) ?
D33(1691) ?
P33(1236)
P33(1688)
P31(1934)
S31(1670)

Fig. 2.

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