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WHO IS AFRAID OF W?

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BUDAPEST

WHO IS AFRAID OF W?*

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Abstract

The $SL(2,C)$ classification of Regge trajectories is extended to nonzero energies. Regge trajectories can be classified according to a "broken" $SL(2,C)$ algebra, each "Regge-particle" being a superposition of a few irreducible representations of $SL(2,C)$.

A "mass formula" is derived to describe Regge trajectories which belong to one $SL(2,C)$ family. The Δ-family turns out to be exchange-degenerate to a good approximation. The known nucleon trajectories can be fitted into one $SL(2,C)$ family well described by the mass formula. The mass formula predicts several new resonances, some of them without parity partners. Evidence is put forward for the existence of a new family of nucleon Regge trajectories.

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* Classification of Regge trajectories at nonzero energies $(W \neq 0)$ according to a broken $SL(2,C)$ algebra.

1. INTRODUCTION

The $SL(2,C)$ classification scheme of Regge intercepts at zero mass [1,2,3] begins to produce some spectacular results, such as the prediction of a forward peak in pion photoproduction, a uniform asymptotic behaviour of two-body inelastic processes, the resolution of the Gribov-Volkov conspiracy problem, etc.

It has been realized [4] that the role of $SL(2,C)$ is - generally speaking - not of a symmetry algebra /SA/ but of a spectrum generating algebra /SGA/, since e.g. physical two-particle scattering states do not furnish a basis for its representations. This recognition paved the way for a model - independent proof [5] of the $SL(2,C)$ classification scheme within the framework of S-matrix theory. An attempt to use this algebra for the classification of entire Regge trajectories [5] revealed that such a scheme cannot be very useful unless one includes the breaking of the zero-mass SGA in a systematic way. Here we report the first results of an investigation into the role of a broken $SL(2,C)$ in classifying Regge trajectories at nonzero energies.

The symmetry breaking operator turns out to be quite complicated, and in general cannot be given in closed form; nevertheless, if the Regge trajectories are given as a power series in the mass, one can make quite definite predictions about the behaviour of the members of an " $SL(2,C)$ - family", i.e. of trajectories which at zero mass transform according to an irreducible representation /IR/ of the $SL(2,C)$ SGA.

The main results are the following:

a./ The I=1/2 nucleon trajectories $(N_\alpha, \dots N_\gamma)$ can be fitted into one $SL(2,C)$ family, characterised by an IR $\{-0.33, 1/2\}$. Restricting ourselves to terms not higher than $O(W^2)$ in the expansion of the trajectory, /the familiar Chew-Frautschi approximation/, we are able to describe all the trajectories belonging to the family remarkably well by a formula containing only four adjustable parameters.

We find indication for the existence of a second $SL(2,C)$ family, transforming according to the IR $\{0.11, 1/2\}$.

b./ We show that in the expansion of the Δ -trajectories - provided they are classified at $W=0$ according to an IR with $|j_0| = 3/2$, the terms linear in W are absent; therefore, in the Chew-Frautschi approximation these trajectories are exchange-degenerate.

In the following section we briefly describe a perturbation formalism devised to determine Regge trajectories. Section 3. is devoted to the phenomenological investigation of the nucleon family, while in Sec 4. the results are summarized and discussed. Our notation is the following. We write $|(j_1 j_2) j m\rangle$ for a vector transforming according to the IR $(j_1 j_2)$ of $SL(2,C)$ or, equivalently, $|\{\sigma j_0\} j m\rangle$ where $j_1 = 1/2(\sigma + j_0)$, $j_2 = 1/2(\sigma - j_0)$. For a finite dimensional IR $\sigma \geq j \geq j_0$ therefore we write $j = \sigma - \varkappa$ and use $\varkappa$ for the labelling of the members of an $SL(2,C)$ family of Regge trajectories. Here σ can be any complex number, while j_0 is an integer or half integer. We define the reduced matrix element of an irreducible tensor $T^{(j_1 j_2)}_{jm}$ as follows:

$$\langle (j_1 j_2) j m | T^{(j_1'' j_2'')}_{j'' m''} | (j_1' j_2') j' m' \rangle = \langle (j_1 j_2) \| T^{(j_1'' j_2'')} \| (j_1' j_2') \rangle \times$$

$$\times \langle (j_1 j_2) j \| (j_1' j_2') j' ; (j_1'' j_2'') j'' \rangle (j' m' ; j'' m'' | j m),$$

where the last factor is a Clebsch-Gordan coefficient /CGC/ of $SU(2)$ and the second factor can be expressed by a 9-j symbol:

$$\langle (j_1 j_2) j \| (j_1' j_2') j' ; (j_1'' j_2'') j'' \rangle =$$

$$= [(2j'+1)(2j''+1)(2j_1+1)(2j_2+1)]^{1/2} \begin{Bmatrix} j_1' & j_1'' & j_1 \\ j_2' & j_2'' & j_2 \\ j' & j'' & j \end{Bmatrix}.$$

2. REPRESENTATION MIXING THEORY OF REGGE TRAJECTORIES

a./ Perturbation theory

In a field theoretical formalism, Regge trajectories and residues can be determined as the solutions of a homogeneous Bethe-Salpeter equation. In what follows, we are not interested in specific forms of the Bethe-Salpeter kernel, numerical values of the trajectories, but want to extract some general information only about the form of the Regge trajectories and Regge residues near zero mass.

We write the partial wave Bethe-Salpeter equation for a bound state in operator form as follows:

$$GK \| E_\mu , j m , \nu \rangle\!\rangle = \| E_\mu , j m , \nu \rangle\!\rangle \qquad /2.1/$$

where G is the disconnected part of the two-particle Green operator, K is the Bethe-Salpeter kernel.

The operators G , K , and hence the "state vector" are functions of E_μ , the total energy-momentum vector of the bound state and of j its spin. One can go over to the rest frame of the bound state, where $E_\mu = (\underline{0}, W)$, W being the mass of the bound state. Evidently, G , K are independent of the projection m of the spin; ν stands for the rest of the quantum numbers necessary to determine the state vector.

We assume that the operator GK is well-behaved so that e.g. in momentum representation eq. /2.1/ gives an integral equation with a Hilbert-Schmidt kernel, further that the operator GK is analytic in the parameters W and j in a suitable domain. /Of these assumptions, the first one can be somewhat relaxed,[1] the second one can be verified in perturbation theory./

A similarity transformation

$$|W, \mu\rangle = G^{-1/2} \| W, jm, \nu \rangle\!\rangle \qquad /2.2/$$

reduces the problem to a self-adjoint one. It can be shown that in the final results all the square roots drop out, so the analyticity properties of the matrix elements are not spoiled; moreover, the spectrum of the self-adjoint problem is the same as that of the original one.

Define the operator:

$$H(W, j) = G^{1/2} K G^{1/2} - 1$$

then eq. /2.1/ after the transformation /2.2/ can be written as follows:

$$H(W, j) |W, \mu\rangle = 0 \qquad /2.3/$$

We want to determine the Regge-trajectories: $j = \alpha(W)$ as eigenvalues of eq. /2.3/. The unusual feature of /2.3/ is that the dependence on the eigenvalue is nonlinear. Nevertheless, the problem can be solved in the form of a power series.

Let us assume that we know the solution at zero mass, there - as shown in refs[1,4] - the dependence of H on the spin j enters only through the $SL(2,C)$ invariant, σ . Therefore $H(0, j) \equiv H(0, \sigma)$ Let the eingenvectors be $|0, \mu\rangle$; the intercepts of the Regge trajectories $\alpha_i^x = \sigma_i - \varkappa$, where σ_i is the position of the i^{th} "Lorentz-pole".

Write:

$$\lambda_i^{x} \equiv \lambda(\mu) = \alpha_i^{x}(W) - \alpha_i^{x}$$
$$= \lambda_1(\mu) W + \tfrac{1}{2}\lambda_2(\mu) W^2 + \ldots$$

Expanding into a power series, we rewrite eq. /2.3/ as follows:

$$\begin{aligned}&(H_0 + W H_W + \tfrac{1}{2} W^2 H_{WW} + \cdots)(|0,\mu\rangle + W|1,\mu\rangle + \tfrac{1}{2}W^2|2,W\rangle + \ldots)\\ &+ (H_\sigma + W H_{\sigma W} + \tfrac{1}{2} W^2 H_{\sigma WW} + \ldots)(\lambda_1 W + \tfrac{1}{2}\lambda_2 W^2 + \ldots)\\ &\times(|0,\mu\rangle + W|1,\mu\rangle + \tfrac{1}{2}W^2|2,\mu\rangle + \ldots) + \ldots = 0,\end{aligned} \qquad /2.4/$$

where

$$H_0 = H(0,0),$$
$$H_W = \left[\frac{\partial H}{\partial W}\right]_{W=0,\ \sigma=\sigma_i},$$
$$H_\sigma = \left[\frac{\partial H}{\partial \sigma}\right]_{W=0,\ \sigma=\sigma_i},$$

etc.

On multiplying eq. /2.4/ by the vector $\langle 0,\mu|$ and equating powers of W, we arrive at the following set of equations:

$$\langle 0,\mu'|H_W|0,\mu\rangle + \lambda_1(\mu)\,\delta(\mu',\mu)\,h_1(\mu) = 0,$$

$$\begin{aligned}&h_1(\mu')(\lambda_1(\mu') - \lambda_1(\mu))\langle 0,\mu'|1,\mu\rangle + \tfrac{1}{2}\langle 0,\mu'|H_{WW}|0,\mu\rangle\\ &+ \lambda_2(\mu) h_1(\mu)\delta(\mu',\mu) + \lambda_1(\mu)\langle 0,\mu'|H_{\sigma W}|0,\mu\rangle + \tfrac{1}{2}(\lambda_1(\mu))^2 h_2(\mu)\delta(\mu',\mu) = 0,\end{aligned} \qquad /2.5/$$

....

In writing down the eqs. /2.5/ we took into account that the operators H_σ, $H_{\sigma\sigma}$, ..., being derivates of an $SL(2,C)$ invariant operator with respect to an invariant /i.e., σ /, are themselves invariant operators; hence /assuming that there is no degeneracy in σ / their matrix in the unperturbed basis is diagonal:

$$\langle 0,\mu'|H_\sigma|0,\mu\rangle \equiv h_1(\mu)\delta(\mu',\mu), \quad \langle 0,\mu'|H_{\sigma\sigma}|0,\mu\rangle \equiv h_2(\mu)\delta(\mu',\mu), \quad \text{etc.}$$

further that - by definition -,

$$H_0|0,\mu\rangle = 0.$$

From the system /2.5/ we can successively determine the derivatives of the Regge trajectories, λ_i , and the corrections to the "state vectors" of the Regge poles:

$$\lambda_1(\mu) = \frac{-1}{h_1(\mu)} \langle 0,\mu | H_w | 0,\mu \rangle,$$

$$\lambda_2(\mu) = \frac{-1}{h_1(\mu)} \Big\{ \frac{1}{2} \langle 0,\mu | H_{ww} | 0,\mu \rangle$$

$$+ \lambda_1(\mu) \langle 0,\mu | H_{\sigma w} | 0,\mu \rangle + \frac{1}{2} (\lambda_1(\mu))^2 h_2(\mu) \Big\},$$

$$\cdots$$

$$\langle 0,\mu' | 1,\mu \rangle = \frac{\frac{1}{2}\langle 0,\mu' | H_{ww} | 0,\mu \rangle - \lambda_1(\mu) \langle 0,\mu' | H_{\sigma w} | 0,\mu \rangle}{h_1(\mu)(\lambda_1(\mu') - \lambda_1(\mu))} \qquad (\mu' \neq \mu),$$

$$\langle 0,\mu | 1,\mu \rangle = 0, \qquad /2.6/$$

$$- - - -$$

/Analogous formulae can be written down if the first order correction to the trajectory, $\lambda_1(\mu)$ vanishes./

As a result we obtain the Regge trajectory and vertex in the form of a power series:

$$\alpha_i^{\varkappa}(W) = \sigma_i - \varkappa + \lambda_1 W + \lambda_2 W^2/2 + \ldots ,$$

$$\langle A | W,\mu \rangle = \langle A | 0,\mu \rangle + W \sum_{\mu' \neq \mu} \langle A | 0,\mu' \rangle \langle 0,\mu' | 1,\mu \rangle + \ldots , \qquad /2.7/$$

where $\langle A|$ stands for a two-particle state. We see that at nonzero energy the Regge trajectory is a definite superposition of Lorentz trajectories. In a moment it will become evident that at sufficiently small energies, when the power series can be broken off, as a consequence of the selection rules we have to superpose only a relatively small number of $SL(2,C)$ representations.

b./ Group structure of the perturbed Regge trajectory

The quantity W is the $j = m = 0$ component of a tensor transforming according to the IR $(\frac{1}{2},\frac{1}{2})$ of $SL(2,C)$. Consequently / H_0 being an invariant operator/, the derivates of H have the following transformation properties:

$$H_W \sim \left(\tfrac{1}{2},\tfrac{1}{2}\right),$$
$$H_{\sigma W} \sim \left(\tfrac{1}{2},\tfrac{1}{2}\right),$$
$$H_{WW} \sim \left(\tfrac{1}{2},\tfrac{1}{2}\right) \otimes \left(\tfrac{1}{2},\tfrac{1}{2}\right), \text{etc.}$$

Using the reduction formula

$$\left(\tfrac{1}{2},\tfrac{1}{2}\right) \otimes \left(\tfrac{1}{2},\tfrac{1}{2}\right) = (1,0)\oplus(0,1)\oplus(1,1)\oplus(0,0)$$

we see that only $(0,0)$ and $(1,1)$ contribute to the matrix elements /since $(1,0)$ and $(0,1)$ are antisymmetric tensors/. In virtue of the Wigner-Eckart theorem we rewrite the matrix elements as follows.

$$\langle (j_1' j_2') j' m' | H_W | (j_1 j_2) j m \rangle = \langle (j_1' j_2') \| H_W \| (j_1 j_2) \rangle \times \left[(2j+1)(2j_1'+1)(2j_2'+1)\right]^{1/2} \begin{Bmatrix} j_1 & \tfrac{1}{2} & j_1' \\ j_2 & \tfrac{1}{2} & j_2' \\ j & 0 & j \end{Bmatrix} \delta_{jj'} \delta_{mm'} \qquad /2.8/$$

and similarly for the matrix elements of the other operators. /In the vectors $|0,\mu\rangle$ we exhibited the $SL(2,C)$ transformation property and suppressed every other index/.

The selection rules for H_W and H_{WW} are summarized in Table I.

TABLE I.

Selection rules for the perturbation operators

H_W:

σ'	$\sigma-1$	σ	σ	$\sigma+1$
j_0'	j_0	j_0+1	j_0-1	j_0

H_{WW}:

σ'	$\sigma+2$	$\sigma+1$	σ	$\sigma+1$	σ	$\sigma-1$	σ	$\sigma-1$	$\sigma-2$
j_0'	j_0	j_0+1	j_0+2	j_0-1	j_0	j_0+1	j_0-2	j_0-1	j_0

The perturbation operators transform according to self-conjugate $(j=0)$ representations of $SL(2,C)$, hence $\sigma' \equiv \sigma \pmod 1$ and $j_0' \equiv j_0 \pmod 1$. /The superselection rule of the conservation of particles with half-integer spin, generalised to Regge trajectories./ Further, as in the diagonal elements $(\sigma' = \sigma)$ of the perturbing operator we must also have $|j_0'| = |j_0|$, it follows for the derivatives of the trajacteries:

$$\left[\frac{d^{2n+1}\alpha(W)}{dW^{2n+1}}\right]_{W=0} = 0 \qquad \text{for bosons} \quad /j_0 = \text{integer}/ ,$$

$$\left[\frac{d^{2n+1}\alpha(W)}{dW^{2n+1}}\right]_{W=0} = 0 \qquad \text{for fermions} \quad /j_0 = \text{half integer}/ .$$

$$(n < j_0 - 1/2)$$

In particular, as the family of Δ -trajectories transforms according to a representation with $|j_0| = 3/2$ at $W=0$, it follows that terms of $O(W)$ in the expansion of the trajectories are absent; in other words, if the Δ -family can be approximated by a parabola, then /to this approximation/, the Δ -trajectories are exchange-degenerate, in very good agreement with a phemomenological Chew-Frautschi plot.

3. PHENOMENOLOGICAL DESCRIPTION OF THE NUCLEON FAMILY: THE MASS FORMULA

Let us apply the formalism developed in the previous Section to the $I = 1/2$ nucleon trajectories. There are at least two nucleon trajectories (N_α, N_γ) known phenomenologically; their intercepts at $W=0$ are separated approximately /or exactly?/ by one unit of angular momentum and the trajectories are of opposite signature.

This situation suggests that the known N -trajectories are numbers of one $SL(2,C)$ family. Evidently this family should transform according to an I.R. of $SL(2,C)$ with $|j_0| = \frac{1}{2}$ at $W=0$. From eq. /2.6/ after taking out the j -dependence into the CGC of $SL(2,C)$ and some rearrangements, one can derive the following phenomenological formula for the family of trajactories:

$$\alpha_\varkappa(W) = \sigma - \varkappa + A(\sigma + \tfrac{1}{2} - \varkappa)W + [B_1 + B_2(\sigma-\varkappa)(\sigma-\varkappa+1)]W^2 + O(W^3), \qquad /3.1/$$

where the parameters A, B_1, B_2 are essentially reduced matrix elements. We determined the values of σ, A, $B_1 + \sigma(\sigma+1)B_2$ by comparing eq. /3.1/ with a recent fit to the N_α trajectory[8/].

A further condition was obtained by forcing the first daughter /supposedly the N_γ trajectory/ through the $D_{13}(1541)$ resonance. As a result we find the following values of the parameters:

$$\sigma = -0{,}33 ,$$
$$A = 0{,}65 \ GeV^{-1} ,$$
$$B_1 = 0{,}99 \ GeV^{-2} ,$$
$$B_2 = -0{,}33 \ GeV^{-2}$$

The resulting curves for $\varkappa$ = 0, 1, 2 are plotted in Fig. 1, together with the observed πN resonances as compiled in Lovelace[9/]; a few higher lying resonances, not contained in ref.[9/] were taken from Rosenfeld's tables[10/].

The agreement between the predicted and observed positions of the resonances in most of the cases is good, in particular for the well-established resonances.

It is interesting to observe that none of the observed resonances lies on the trajectory with $\varkappa=2$. In fact this trajectory is rather flat, it rises until W = -19GeV and should give the following four resonances:

$$P_{11}(2500) , \ F_{15}(1800) , \ H_{19}(7000) , \ J_{113}(9400).$$

At W = -19GeV the trajectory turns back.

Observation of these resonances would be interesting, since according to the theory, they do not have parity partners. The trajectories of higher order of the nucleon family $\varkappa$ = 3,4... never reach the physical region.

There are five I = 1/2 resonances which can not be fitted into this scheme with any reasonable choise of the parameters. These are:

$P_{11}(1466)$ /the Roper resonance/, $P_{11}(1751)$, $S_{11}(1548)$, $S_{11}(1700)$ and $D_{13}(2057)$. These resonances or some of them may start a new $SL(2,C)$ family. If one accepts the argument of Lovelace[11/] that $P_{11}(1456)$ is a member of a $\overline{10}$ of $SU(3)$, then this fact alone excludes it from the nucleon family.

We tentativly determined the parameters of this hypothetical new family in eq. (3.1) by assuming that $P_{11}(1466)$ and $S_{11}(1548)$ lie on a trajectory with $\varkappa=0$ and $P_{11}(1751)$ together with $S_{11}(1700)$ on its second daughter $(\varkappa=2)$. The result is seen on Fig. 2. The values of the parameters for this family are:

$$\sigma = 0{,}11\ ,$$
$$A = -0{,}03\ GeV^{-1}\ ,$$
$$B_1 = 0{,}12\ GeV^{-2}\ ,\ B_2 = 0{,}40\ GeV^{-2}.$$

It should be pointed out that σ is slightly positive: therfore this trajectory may have a considerable influence on backward πN scattering. Another important remark is that if P_{11} /1466/ belongs to a $\overline{10}$ of SU_3, then so does its parity partner S_{11} /1548/ and its daughters, $P_{11}(1751)$ and $S_{11}(1700)$. This could be decided by photoproducing these resonances, cf. ref.[11/]

No other observed resonance seems to fall onto the trajectories of this family /if it is indeed a family..../, but to draw a final conclusion would be premature at present.

4. CONCLUSIONS

To summarize, we established that at nonzero energies the Regge trajectories can be classified according to a broken $SL(2,C)$ algebra.

Near zero mass the "states" along a Regge trajectory can be composed of a few IR-s of $SL(2,C)$. The theory leads to a "mass formula" describing the behaviour of the members of an $SL(2,C)$ family in terms of a few parameters. In our opinion, the application of the theory to the nucleon trajectories is definitely succesful; it would be interesting to see if the predictions of the theory /e.g. the existence of high-mass resonances without parity partners, the existence of a second family of nucleon trajectories/ can be verified experimentally.

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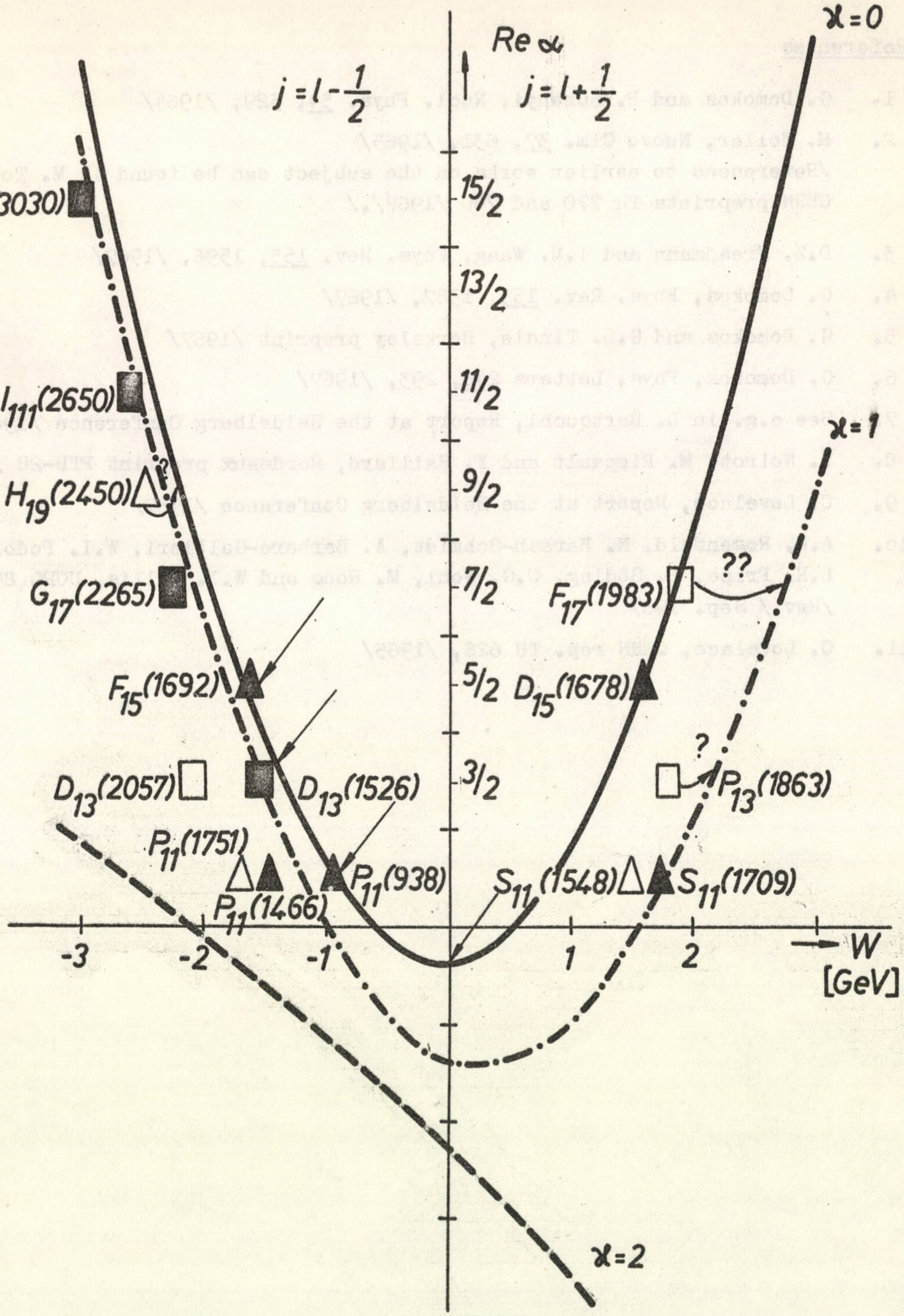
Re α
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j = l + 1/2
χ=0
χ=1
χ=2
15/2
13/2
11/2
9/2
7/2
5/2
3/2
K115(3030)
I111(2650)
H19(2450)
G17(2265)
F15(1692)
D13(2057)
D13(1526)
P11(1751)
P11(1466)
P11(938)
S11(1548)
S11(1709)
F17(1983)
D15(1678)
P13(1863)
??
?
-3
-2
-1
1
2
W
[GeV]

Fig. 1.

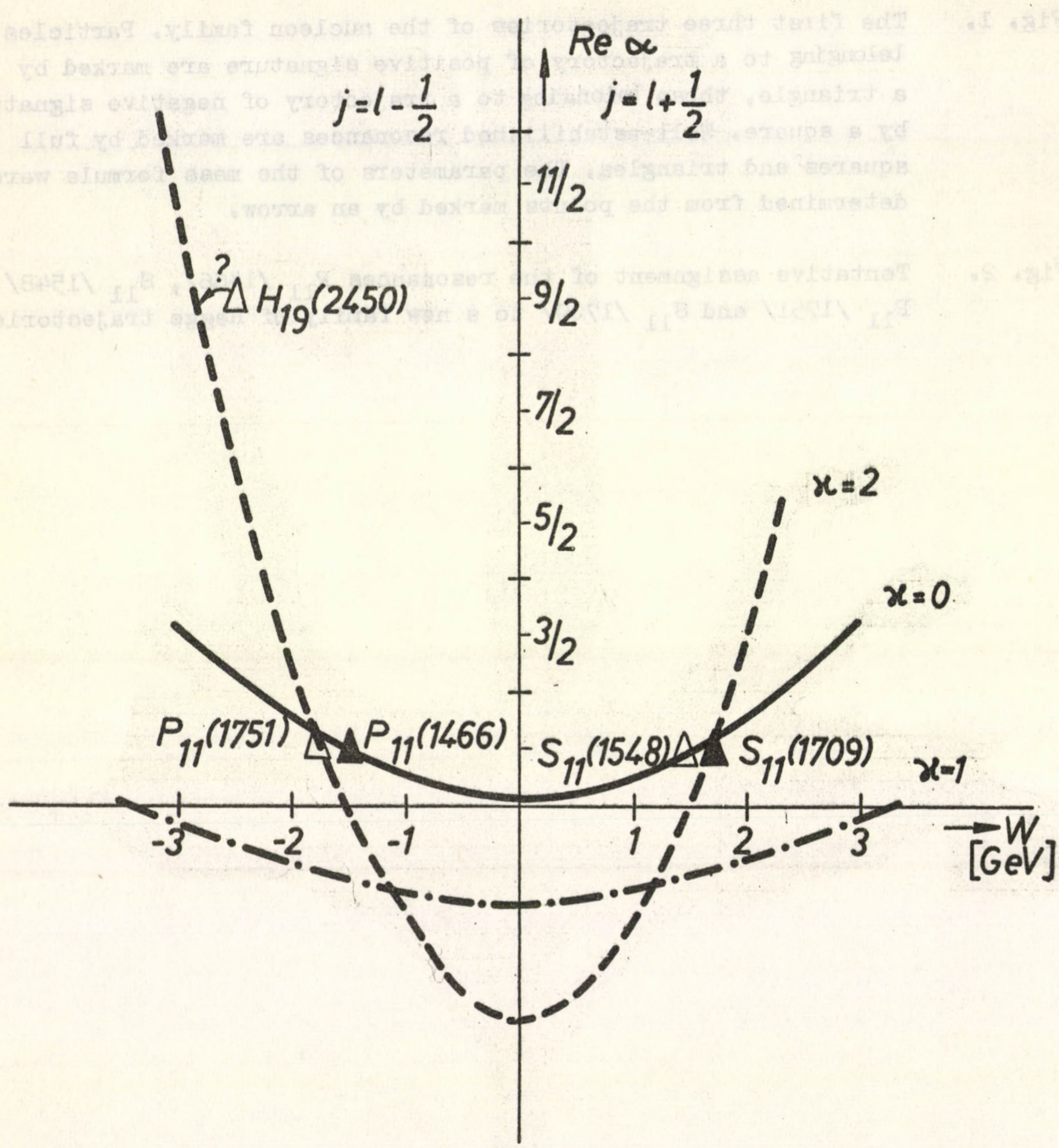
Re α
j = l - 1/2
j = l + 1/2
11/2
9/2
7/2
5/2
3/2
? H19(2450)
P11(1751)
P11(1466)
S11(1548)
S11(1709)
κ=2
κ=0
κ-1
-3
-2
-1
1
2
3
W
[GeV]

Fig. 2.

Figure Captions

Fig. 1. The first three trajectories of the nucleon family. Particles belonging to a trajactory of positive signature are marked by a triangle, those belonging to a trajectory of negative signature by a square. Well-estabilished resonances are marked by full squares and triangles. The parameters of the mass formula were determined from the points marked by an arrow.

Fig. 2. Tentative assignment of the resonances P_{11} /1466/, S_{11} /1548/ P_{11} /1751/ and S_{11} /1709/ to a new family of Regge trajectories.

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