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THEORY OF THE TEMPERATURE DEPENDENCE OF THE ROTON ENERGY

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SUMMARY

The temperature dependence of the roton energy is examined considering its collision with any other thermally excited roton and the interaction between one excitation and two rotons. The final state interaction strongly modifies the two-roton density of states and therefore the Hartree-Fock approximation is not valid for the roton-roton scattering. Using point interaction model for the roton-roton coupling and examining only the self-energy processes for the one particle Green function linear in the number of rotons it is shown that the amplitude of this temperature dependence changes sign as the roton-roton coupling strength is increased and an upper bound is obtained for the temperature dependence of the roton energy. As this upper bound is only 1/7 part of the experimental result, they can be interpreted only by assuming more than seven independent channels with different angular dependence.

РЕЗЮМЕ

Исследована температурная зависимость энергии ротона, рассматривая его столкновение с другими термически возбужденными ротонами и переходы двух ротонов в одно возбуждение. Взаимодействие в конечном состоянии сильно модифицирует плотность двухротонных состояний, и поэтому приближение Хартри-Фока неприменимо для ротон-ротонного рассеяния. При вычислении одночастичной функции Грина были приняты во внимание только собственные энергетические диаграммы, пропорциональные числу ротонов, и было использовано приближение тесного взаимодействия ротонов. Таким образом было показано, что амплитуда этой температурной зависимости меняет знак при увеличении константы ротон-ротонной связи и был найден верхний предел температурной зависимости энергии ротона. Верхний предел составляет 1/7-ую часть экспериментального значения, что может быть объяснено только предположением существования семи независимых каналов с различной угловой зависимостью.

KIVONAT

A rotonoknak egymással való ütközését és az egyrészecskés gerjesztések és a kétrotonos állapotok közötti kölcsönhatást figyelembe véve megvizsgáltuk a rotonenergia hőmérsékletfüggését. A két roton állapotsürüséget a fellépő végállapotkölcsönhatás erősen megváltoztatja, ezért a Hartree-Fock közelítés nem alkalmazható a roton-roton szórásnál. A rotonok között kontakt kölcsönhatást feltételezve és az egyrészecskés Green függvényben csak a rotonszámmal lineáris sajátenergiás folyamatokat tekintve megmutattuk, hogy a hőmérsékletfüggés amplitudója a csatolás erősségének növelésével előjelet vált és a rotonenergia hőmérsékletfüggésére egy felső korlátot kaptunk. Mivel ez a felső korlát a kisérleti értéknek csupán 1/7-ed része, fel kell tételezni, hogy több mint hét egymástól független különböző szögfüggésü csatorna létezik.

Introduction

In order to interpret the thermodynamical properties of superfluid helium Landau[1] has proposed the existence of two types of elementary excitations in HeII. There are long wavelength density fluctuations, the phonons, and elementary excitations with wavelength corresponding to the mean atomic distance in the liquid, the rotons. The best description of rotons has been given by Feynman[2] and later in an improved form by Feynman and Cohen[3]. Starting from first principles they were able to derive the elementary excitation spectrum shown in Fig.1. This theory, however, due to the applied variational method could not say anything about the interaction between the elementary excitations.

Examining the interaction between the elementary excitations of HeII Landau and Khalatnikov[4] have shown that at low temperatures ($T < 1^{o}K$) the interaction between phonons, and at higher temperatures (below the λ point) the interaction between rotons play fundamental role. These interactions were calculated in Hartree-Fock (H-F) approximation with the assumption that there is a direct interaction between rotons.

Recently, numerous experimental and theoretical results have shown that the roton-roton interaction cannot be described in H-F approximation. Thus, Raman scattering experiments in superfluid helium done by Greytak et.al.[5] have shown that the two-roton density of states (for total momentum K near zero) has a strong deformation. This experiment was explained by Ruvalds and Zawadowski[6,7] and independently by Iwamoto[8], with the supposition that there is a direct attractive interaction between the rotons. An arbitrarily weak attraction between two rotons gives rise to a two-roton bound state[6,7,8] which is split off below the two-roton continuum. The above theoretical predictions have been verified by more recent experiments of Greytak et.al.[9] which yield a binding energy $E_B=0.37^oK$ for the bound pair with K=0.

Neutron scattering experiments have shown the appearence of two branches in the elementary excitation spectrum[10,11], which may be interpreted as a consequence of an interaction between the one particle and two-roton states[6,7]. Both light scattering and neutron scattering experiments as well as viscosity measurements[12] show a simple dependence of the roton linewidth on the number of rotons. This fact can be explained reasonably by assuming that the dominant interaction at $T > 1^oK$ is the roton-roton scattering.

Recent neutron scattering experiments done by Dietrich et.al.[13] (see Fig.2.) show, that the roton energy as the function of temperature decreases proportionally to the number of rotons. The experiments have been done in a wide interval of pressure. As it can be seen from Fig.2. the temperature dependence of the roton energy can be written in the form:

$$\Delta(T)=\Delta_0 - 39\sqrt{T}\, e^{-\frac{\Delta(T)}{T}} \quad {}^{o}K \qquad (1)$$

where T is the temperature, and only Δ_0 depends on the pressure (in atm) in the following way: $\Delta_0=8.75-0.12p$ Assuming a direct interaction between rotons with coupling constant g_4, Ruvalds[14] has calculated the temperature dependence of the roton energy in H-F approximation. In order to fit the experimental results he had to suppose $g_4= -3.7\text{x}10^{-38}\,\text{ergcm}^3$. It is worth mentionong that this value of the roton-roton coupling is close to that value which is needed in H-F approximation to fit the experimental roton lifetime. Taking into account this temperature dependence of the roton energy Ruvalds has been able to calculate the temperature dependence of the superfluid part of HeII in agreement with experiment.

The appearence of the bound state of two rotons and the hybridization between the one and two particle states show that the roton-roton interaction should be studied in a better approximation than the H-F one. Taking into account the final state interaction, a consequent investigation of the roton lifetime has been done first by Fomin[15]. Similar result have been obtained by Yau and Stephen[16] and Nagai et.al.[17] The interaction of rotons with the particles of the condensate have also been taken into account by Solana et.al.[18] The above authors find that due to the roton-roton interaction, the two-roton density of states is strongly deformed, and the lifetime of a free roton goes to a constant value as the strength of the attractive interaction increases (see Fig.3.). This upper bound value remains four times less than the experimental result.

The intention of this paper is to investigate the influence of the final state interaction on the temperature dependence of the roton energy to decide, whether the result of H-F calculation[14] is satisfactory, or the higher order processes play an equally important role as in the case of the roton lifetime theory.

Chapter I

The formalism applied in what follows is the same as in Ref.6,7,18. The basic point is that the rotons are taken as well defined one particle excitations below the λ point, where the inverse roton lifetime remains much smaller than the roton energy. In this way employing the finite temperature Green function technique we take the roton propagator in the form:

$$G_1(k;i\omega n)=\frac{1}{i\omega_n - E_k - \Sigma(k;i\omega_n)} \qquad (2)$$

where $\omega_n=2\pi nT$ (n is an integer and T is the temperature). The roton dispersion curve is choosen in the form:

$$E_k=\Delta_o+\frac{(k-k_o)^2}{2\mu_o} \quad \text{,where} \quad \begin{array}{l} \Delta_o=8.67\ {}^oK \\ k_o=1.92\ \text{Å}^{-1} \\ \mu_o=0.16m_{He^4} \end{array} \qquad (3)$$

As these parameter values come from the experiment, they refer to interacting rotons already and therefore in the self-energy the temperature independent real energy shift corrections should be neglected.

As it has been pointed out in Ref.18 it is a good approximation to neglect the real part of Σ in the further calculation[19]. The imaginary part of the self-energy for energies near to Δ_o and for momenta near to k_o can be considered as energy and momentum independent. The experimental value is for the single roton width ($\frac{\Gamma}{2} = \mathrm{Im}\Sigma$) is:

$$\frac{\Gamma}{2} = 42\sqrt{T}\, e^{-\frac{\Delta(T)}{T}} \ {}^{o}K \qquad (4)$$

The temperature dependence of the roton energy will be determined by the following equation:

$$\Delta(T) = \Delta_o + \mathrm{Re}\Sigma(k_o;\Delta(T)) \qquad (5)$$

Because we are interested only in that part of the self-energy which is proportional to the number of rotons, only the processes which are first order in the number of rotons will be examined. It is not a bad approximation of the self-energy because below the λ point the number of rotons is proportional to $\sqrt{T}\, e^{-\frac{\Delta(T)}{T}}$ which is a small parameter. In terms of Feynman graphs we will consider the diagrams containing only one "backward-going" roton line which corresponds to a thermally excited roton. The higher order diagrams containing more than one "backward-going" roton line will be neglected.

Let us suppose, that the interaction between the rotons may be described by a contact coupling g_4, which is independent of the energies and momenta of the scattered rotons. In this case there are only two diagrams with one "backward-going" roton line (see Fig.4). The diagram in Fig.4/a corresponds to the H-F approximation, while the diagram in Fig.4/b represents a process where in the intermediate state the two rotons interact. This interaction given by means of the two roton Green function is shown in turn in Fig.4/c. The analytical form of the roton self-energy given by Fig.4. is:

$$\Sigma(K;i\omega_n)=-T\sum_{n'}\int\frac{d\vec{k}}{(2\pi)^3}\,G_1(\vec{k};i\omega_{n'})\left\{2g_4+G_2(\vec{K}+\vec{k};i\omega_n+i\omega_{n'})\right\} \tag{6}$$

The two-roton Green function G_2 includes the roton-roton scattering to infinite order.

As a consequence of the used separable interaction the infinite geometrical series for the two-roton Green function can be summed up and we obtain[7,18]:

$$G_2(K;i\omega_n)=\frac{G_2^{(0)}(K;i\omega_n)}{1-\frac{g_4}{2}G_2^{(0)}(K;i\omega_n)} \tag{7}$$

Here the unperturbed two-roton Green function is introduced in the following way:

$$G_2^{(0)}(K;i\omega_n)=-T\sum_{n'}\int\frac{d\vec{k}}{(2\pi)^3}\,G_1(\vec{k};i\omega_{n'})G_1(\vec{K}-\vec{k};i\omega_n-i\omega_{n'}) \tag{8}$$

After performing the sum over frequency $\omega_{n'}$, and the integration over $\vec{k}$ we obtain for $G_2^{(0)}$:

$$G_2^{(0)}(K;i\omega_n)=-4\varrho_2^{(0)}(K)\ln\frac{i\omega_n-2\Delta_1+i\Gamma}{i\omega_n-2\Delta_0+i\Gamma} \tag{9}$$

where $\varrho_2^{(0)}(K)=\frac{\mu_0 k_0^2}{4\pi K}$ is the density of states of noninteracting rotons for momenta of interest, i.e. $0.1k_o < K < 2k_o$, and Δ_1 is the energy of a "maxon" (see Fig.1).

In Eq.6. the summation over $\omega_{n'}$ can be easily done by transforming the sum to a contour integral. We may restrict ourselves to the cut of the one particle Green function. As at small temperatures the occupation number for energies near to the two-roton energy is much smaller than that for energies near to Δ_o, the contribution coming from the pole of the two particle Green function - corresponding to two thermally excited rotons - can be neglected. From this it follows that the roton self-energy can be written as:

$$\sum(K;\Omega)=\int\frac{d\vec{k}}{(2\pi)^3}\int d\omega\,\frac{2g_4 n_B(\omega)\,\varrho_1(k;\omega)}{1-\frac{g_4}{2}G_2^{(0)}(\vec{K}+\vec{k};\Omega+\omega)} \qquad (10)$$

where we introduced the one particle spectral function:

$$\varrho_1(k;\omega)=-\frac{1}{\pi}\,\mathrm{Im}\,G_1(k;i\omega_n\to\omega+i\delta)$$

and the Bose function:

$$n_B(\omega)=\left[e^{\frac{\omega}{T}}-1\right]^{-1}$$

In the temperature region $1 < T < 1.8^{o}K$ it is true from Eq.4. that $\Gamma \ll T$, therefore the following approximation can be used: $n_B(\omega)\,\varrho_1(k;\omega)\approx n_B(E_k)$. With this form of ϱ_1 we obtain:

$$\sum(K;\Omega)=\int\frac{d\vec{k}}{(2\pi)^3}\,n_B(E_k)\,\frac{2g_4}{1-\frac{g_4}{2}G_2^{(0)}(\vec{K}+\vec{k};\Omega+E_k)} \qquad (11)$$

First let us consider the properties of the roton self-energy given by Eq.11.

Neglecting the intermediate state interaction, i.e. leaving off the denominator of the integrand in Eq.11. we get back the H-F approximation. After making the integration over k, we get for the real part of the roton self-energy in this approximation the following expression:

$$\mathrm{Re}\sum{}^{H-F} = 2g_4 N_r(T) \qquad (12)$$

where

$$N_r(T) = \frac{2\mu_0^{1/2}k_0^2}{(2\pi)^{3/2}}\sqrt{T}\, e^{-\frac{\Delta(T)}{T}} \qquad (13)$$

is the number of rotons. Ruvalds[14] could fit this expression to the experimental results on the temperature dependence of the roton energy with $g_4 = -3,7 \times 10^{-38}\,\mathrm{ergcm}^3$. The H-F approximation however, as it can be seen from Eq.11, can be applied in the weak coupling limit only.

Let us consider the strong coupling limit. In this case in the denominator of Eq.11. we can neglect the 1 , so the real part of the self-energy becomes:

$$\mathrm{Re}\sum(K;\Omega) = \int \frac{d\vec{k}}{(2\pi)^3}\,\frac{2n_B(E_k)}{\varsigma_2^{(0)}(\vec{K}+\vec{k})}\,\frac{f_1(\Omega+E_k)}{f_1^2(\Omega+E_k)+f_2^2(\Omega+E_k)}\ , \qquad (14)$$

where

$$f_1(\Omega) = \ln\frac{(2\Delta_1-\Omega)^2+\Gamma^2}{(\Omega-2\Delta_o)^2+\Gamma^2}\ ,$$

$$f_2(\Omega) = \pi + 2\,\mathrm{arctg}\frac{(2\Delta_1-\Omega)(\Omega-2\Delta_o)+\Gamma^2}{2(\Delta_1-\Delta_o)\Gamma}$$

For $g_4 \rightarrow \infty$, independently of the sign of the coupling $\mathrm{Re}\Sigma$ tends to a finite positive value, which leads to the increase of the roton energy with the temperature in contrast to the experimental fact.

For the real part of the roton self-energy - at larger energies than the roton energy - we get an upper limit on $\mathrm{Re}\Sigma$, in analogy with the upper limit for the imaginary part of the self-energy (see Fig.2.) which Fomin[15] and Yau-Stephen[16] have pointed out. From Eq.11, $\mathrm{Re}\Sigma$ is:

$$\mathrm{Re}\Sigma(K;\Omega) = \int \frac{d\vec{k}}{(2\pi)^3}\, n_B(E_k) 2g_4 \frac{1-\frac{g_4}{2}\mathrm{Re}G_2{}^o}{\left[1-\frac{g_4}{2}\mathrm{Re}G_2{}^o\right]^2 + \left[\frac{g_4}{2}\mathrm{Im}G_2{}^o\right]^2} \qquad (15)$$

Using the inequality:

$$\left|\frac{x}{x^2+a^2}\right| < \frac{1}{2|a|}$$

we obtain:

$$\left|\mathrm{Re}\Sigma(K;\Omega)\right| < \int \frac{d\vec{k}}{(2\pi)^3}\, 2n_B(E_k)\frac{1}{\left|\mathrm{Im}G_2{}^o(\vec{K}+\vec{k};\Omega+E_k)\right|}$$

After making the integration over k, we get the following upper limit for the real part of the roton self-energy:

$$\left|\mathrm{Re}\Sigma(K;\Omega\sim\Delta_o)\right| < \frac{8}{3\mu_o k_o} N_r(T) \qquad (16)$$

This limit is 6 times less than the experimental value!

Returning to the expression of the $\mathrm{Re}\Sigma$ given by Eq.11, the integration over angles can be carried out and at the roton momentum we have:

$$\sum(k_o;\Omega)=2g_4\int\frac{k^2dk}{2\pi^2}\, n_B(E_k)\left\{1-2g_4'(f_1+if_2)+\right. \tag{17}$$

$$\left.+2g_4'^2(f_1+if_2)^2\ln\frac{1-g_4'(f_1+if_2)}{-g_4'(f_1+if_2)}\right\}_{\Omega+E_k}$$

where the dimensionless coupling $g_4'=g_4\,\varrho_2^{(0)}(2k_o)$ has been introduced. It is worth mentioning that the value of the dimensionless coupling at a typical value of g_4 remains small enough (if $g_4=1\times10^{-38}$ ergcm^3 than $g_4'=0.075$) which justifies the applied perturbational method.

The remaining integration over k can be made numerically. The study of the imaginary part of the self-energy was given in Ref.18. At energies larger than the roton energy the Im$\sum$ can be seen in Fig.3. as a function of the coupling parameter. As the value of the coupling increases the Im$\sum$ at small couplings like than in the H-F approximation - proportional to the square of the coupling - and at larger values it tends to a constant value - to the Yau-Stephen limit - which is only 1/4 of the experimental result.

The energy dependence of Re$\sum$ is shown in Fig.5. for a few values of the coupling. In the knowledge of Re$\sum$ the roton energy is given as a function of the roton-roton coupling. This is represented in Fig.6. For small coupling the H-F approximation is valid. Its range of validity is $|g_4| < 0.5\times10^{-38}$ ergcm^3. At a given temperature the roton energy is minimal for $g_4 = -0.9\times10^{-38}$ ergcm^3, but this decrease is too small, being 7 times less than the experimental value. For larger couplings the temperature dependent correction to Δ_o s still smaller and above a certain value of g_4 it even ecomes positive.

Consequently, supposing a contact $\delta(\vec{r}-\vec{r}\,')$ type interaction between rotons the experimental decrease of the roton energy cannot be interpreted. It can give only 1/7 part of the experimental result. The reason of this is the exsistence of a strong final state interaction in the roton collision. The attractive roton-roton interaction shifts the energy of the two-roton towards lower energies and results in a two roton bound state. Increasing the strength of the coupling, the two-roton continuum is depopulated and the weight of the two roton bound state in the two roton density of states increases. As the contribution of the two-roton continuum and that of the two roton bound state to the real part of the roton self-energy has opposite sign, due to the above mentioned depopulation of the continuum ReΣ has a peculiar dependence on g_4. This can be seen in Fig.5. and Fig.6.

The decrease of the number of states in the two-roton continuum reflected in the behaviour of ImΣ as a function of the roton-roton coupling as well as, it can see on Fig.3. For energies higher than the roton energy, ro-

A. The influence of the interaction between the one particle and two-roton states on the roton self-energy.

Besides the ordinary roton-roton scattering the collision of a roton with a particle in the condensate can also give contribution to the roton self-energy. The influence of this process - i.e. that of the hybridisation between the one particle and two-roton states - on the elementary excitation spectrum of the HeII was investigated by Ref.6,7. This interaction can be characterized by a coupling g_3, which obviously will depend on the number of particles in the condensated state. The processes contributing to the roton self-energy are represented in Fig.7. It is worth noting that the influence of the hybridization have to be considered in the diagrams of Fig.4/b as well. In this diagram the one (two) roton Green function G_1 (G_2) should be repleaced by $\tilde{G}_1$ ($\tilde{G}_2$) which allows for two (one) particle states in the intermediate state. The total contribution to the roton self-energy is:

$$\sum(K;i\omega_n) = -T\sum_{n'}\int\frac{d\vec{k}}{(2\pi)^3}\,\tilde{G}_1(k;i\omega_{n'})\Big\{2g_4 + g_4^2\tilde{G}_2 + (2g_3)^2\tilde{G}_1 + 2g_3^2 g_4(\tilde{G}_1G_2 + G_1\tilde{G}_2)\Big\}_{\vec{K}+\vec{k};\,i\omega_n+i\omega_{n'}} \qquad (18)$$

where the Green function are renormalised due to the hybridization in the following way:

$$G_1(k;i\omega_n)=\frac{G_1(k;i\omega_n)}{1-g_3^2G_1G_2}\ ,\ G_2(k;i\omega_n)=\frac{G_2(k;i\omega_n)}{1-g_3^2G_1G_2} \qquad (19)$$

In Eq.18. similary as earlier only the cut of the "backward-going" single roton propagator should be considered. After making the sum over energy, and applying the approximation $\varrho_1\ \delta(\omega-E_k)$ it gives the following expression:

$$\sum(K;\Omega)=\int\frac{d\vec{k}}{(2\pi)^3}\ n_B(E_k)\ \frac{2g_{eff}}{1-\frac{g_{eff}}{2}\ G_2^{(0)}(\vec{K}+\vec{k};\Omega+E_k)} \qquad (20)$$

i.e. the structure of the self-energy is the same as if only the roton-roton collision is included (see Eq.11). The effect of the coupling g_3 can be incorporated into an effective coupling g_{eff}:

$$g_{eff}=g_4\left(1+\frac{2g_3^2}{g_4}G_1(\vec{K}+\vec{k};\Omega+E_k)\right) \qquad (21)$$

This effective coupling is complex due to the imaginary part of the one particle Green function. The imaginary part, however, can be neglected.

The upper limit for the real part of the self-energy given in Eq.16, is independent of the coupling g_4, and it is valid even if hybridization is taken into account. Neglecting the contribution of the diagram in Fig.7/b, there are two independent scattering channels represented by the couplings g_3 and g_4 respectively. This would give approximately twice higher value for the self-energy. The diagram on Fig.7/b represents the interference term between the two interaction, and as a consequence of this the self-energy shift is reduced as if there were one interaction only.

B. Roton-roton collision taking into account the states with higher angular momenta

Up till now, it has been assumed that the roton-roton vertex can be repleaced by an $\delta(\vec{r}-\vec{r}')$ type interaction, which corresponds to the scattering with angular momentum $l=0$. As this attempt to explain the temperature dependence of the roton energy failed, a better description of the angular dependence of the roton-roton vertex should be taken into account, as proposed by Fomin[15]. Due to the special form of the phonon-roton dispersion the motion of the center of mass of the scattered rotons can not be separated from the relative motion. Expanding the roton-roton vertex with respect to the sperical harmonics, the scattering states having different angular momenta will be mixed. Therefore, it is more useful to expand the roton-roton vertex with respect to an angle:

$$\gamma(\vec{k}_1;\vec{k}_2;\vec{k}_3;\vec{k}_4)=\sum_m g_4^{(m)}\, e^{im\varphi} \qquad (22)$$

where the angle φ characterises the rotation of the plane determined by the momenta of the rotons before and after the scattering. As the rotons are Bose particles, the expression of the roton-roton vertex given by Eq.22. has even m only.

The coupling constant $g_4^{(m)}$ belonging to the different channels m are supposed to be independent of the momenta for the following reasons: if only those collisions are considered where all the particles are rotons, they will have approximately the same momenta in absolute value, and therefore the dependence of the roton-roton vertex on the angle between the incoming, as well as between the outgoing rotons will be negligible.

The dependence of the coupling constants $g_4^{(m)}$ on the total momentum of the scattered rotons can not change the character of the behaviour of the roton self-energy as it was pointed out in the previous chapter.

Using the above approach, it is easily to see that the roton self-energy can be expressed as the sum of the self-energy contributions corresponding to different channels in the quantum number m:

$$\sum(K;\Omega)=\sum_{m}\int\frac{d\vec{k}}{(2\pi)^3}\,n_B(E_k)\,\frac{2g_4^{(m)}}{1-\frac{g_4^{(m)}}{2}G_2^{(0)}(\vec{K}+\vec{k};\Omega+E_k)} \qquad (23)$$

For each terms of the sum in Eq.23 the limit given in Eq.16 is valid. In order to get the experimental value of the shift of the roton energy, the coupling constants $g_4^{(m)}$ should be different from zero several value of m. The smallest number of channels, which is necessary to get the observed lifetime is four, while for the real part of the self-energy it is seven. For instance, assuming that for the first seven channels $g_4^{(m)}=-1x10^{-38}$ergcm^3 and

Chapter II

In order to explain the temperature dependence of the roton energy there remain the following two possibilities:

A. Maintaining the assumption that the roton-roton interaction is $\delta(\vec{r}-\vec{r}\,')$ type, we should considered the roton-roton collisions with one elementary excitation in the final state. This process leads to an interaction between the one particle and two-roton states. Its contribution can be important, as the appearence of the two branch structure in the neutron scattering experiments can be explained with this coupling[6,7].

B. Another possibility to explain the observed shift of the roton energy was proposed by Fomin[15]. The roton-roton vertex can be expanded with respect to the m-th Fourier coefficient of the rotation of the plane determined by the momenta of the scattered rotons. In this way the roton self-energy will be given as the sum of the single self-energy contributions corresponding to the different quantum number m.

In what follows the above two possibilities will be considered.

$g_4^{(m)}=0$ otherwise, both the lifetime and the energy shift of the rotons is describable reasonably.

ughly we have:

$$\mathrm{Im}\sum(k_0;\ \Omega>\Delta_0)\propto g_4^2\,\rho_2(k_0;2\Omega)$$

It is worth nothing, that the above mentioned behaviour of the self-energy will be not change if the roton-roton coupling is allowed to depend on the total momentum of the two scattered rotons. The expression for the self-energy given in Eq.11 and the upper limit for Re $\sum$ given in Eq.16 remain unchanged.

Conclusions

Assuming a $\delta(\vec{r}-\vec{r}')$ type interaction, that part of the roton self-energy has been examined which is proportional to the number of rotons, as the experiments show this behaviour for the lifetime and the energy shift of the rotons. The processes that contribute are those in which one roton is excited thermally. It has been show that the H-F approximation is applicable for the roton-roton interaction in the weak coupling limit only, and the final state interaction has an essential role. The formation of the bound state of two rotons - which comes into being at arbitarily weak attractive interaction - changes fundamentaly the two-roton density of states. With increasing strength of the roton-roton interaction the states from the two-roton continuum are transfered into the bound state and accordingly the density of states in the two-roton continuum decreases. Since the imaginary part of the self-energy on the energy shell is proportional to the product of the two-roton density of states in the two-roton continuum and of g_4 , increasing the interaction Im $\sum$ goes to a constant value which remains only 1/4 part of the experimental value. The real part of the roton self-energy to which the states in the two-roton continuum and in the bound state contribute with opposite sign, with the increase of the strength of the interaction changes sign and tends to a finite positive value.

The maximum value of the real part of the roton self-energy is only 1/7 part of its experimental value.

Both real and imaginary parts of the roton self-energy have a similar behaviour in the case that the roton-roton interaction is separable; thus the successive roton-roton collisions happen independently of one another, i.e. both the Green function of two rotons and the roton self-energy can be expressed as the sum of a geometrical series.

It is known, that the scattering of rotons on the condensate leads to a hybridization between the one particle and the two-roton states. This hybridization gives rise to the two branch structure observed in neutron scattering experiments. The temperature dependent contribution of the roton-condensate scattering to the roton self-energy is a nonlinear function of the coupling constant for the following reason: the self-energy contribution coming from the hybridization process is proportional to the one particle density of states near the two roton energy. However, the hybridization process expells the states from this place to the lower and the higher energies, producing the two branch structure. In this way the roton self-energy contribution resulting from the hybridisation process has a similar structure, than if only the direct roton-roton interaction is taken into account. Considering these two possible roton-roton interaction processes together, the total contribution due to the interference will be the same as if there were only one interaction process.

As with a $\delta(\vec{r}-\vec{r}')$ type interaction it is not possible to describe the temperature shift of the roton energy, following Fomin, a strong angular dependence has been supposed for the roton-roton interaction. The self-energy being given as the sum of the partial self-energy contributions labelled by the quantum number m, the experimental value can be fitted by supposing that there are at least seven sufficiently strong channels. For example, assuming for the first seven channels that $g_4^{(m)}=-1\mathrm{x}10^{-38}\mathrm{ergcm}^3$ and that for the other channels $g_4^{(m)}$ is much smaller, we shall get for both the imaginary and real part of the self-energy a value which is in agreement with the experiments.

This sharp angular dependence of the roton-roton interaction means at the same time that the strength of the roton-roton interaction will be decreasing as the total momentum of the scattered rotons desreases. This may be the reason why in light scattering experiments (where the bound state at K=0 is examined) the coupling constant appears to be very small.

Acknowledgements

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19. Footnote (best to calculate $\sum$ self-consistently but, as in Ref.18, self-consistency is readily achieved)

Figure Captions

Fig.1. Comparison of the Landau-Feynman excitation spectrum (dotted curve) with spectrum obtained from neutron data (solid lines).

Fig.2. The roton energy as the function of $\sqrt{T}\, e^{-\frac{\Delta(T)}{T}}$ for different pressures estimated from the results of neutron scattering experiments done by Dietrich et. al.[13]

Fig.3. The calculated imaginary part of the roton self-energy (at $\Omega = \Delta + T$) divided by $\sqrt{T}\, e^{-\frac{\Delta(T)}{T}}$ versus the roton-roton coupling is represented by the solid line. The dotted line represents the Yau-Stephen limit.

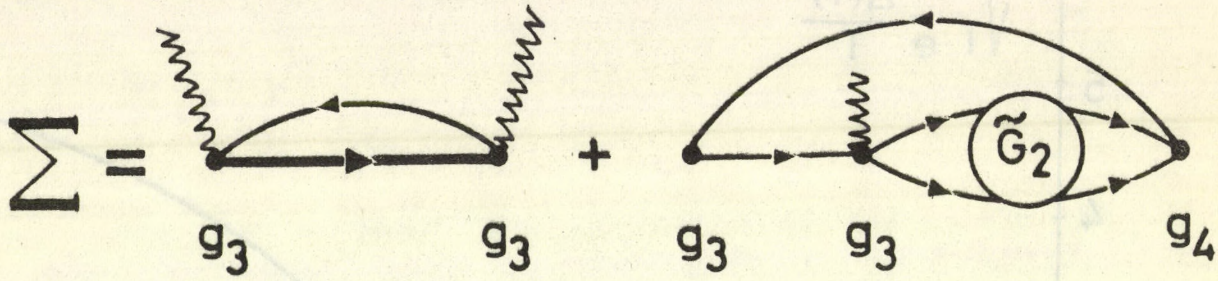
Fig.4. The self-energy diagrams for the one particle Green's function: a) Hartree-Fock approximation, b) self-energy diagram where G_2 includes two roton bound states, c) Diagrammatic representation of the Bethe-Salpeter equation for two-roton propagator G_2.

Fig.5. The real part of the roton self-energy divided by the result of the Hartree-Fock approximation versus energy for different coupling constants.

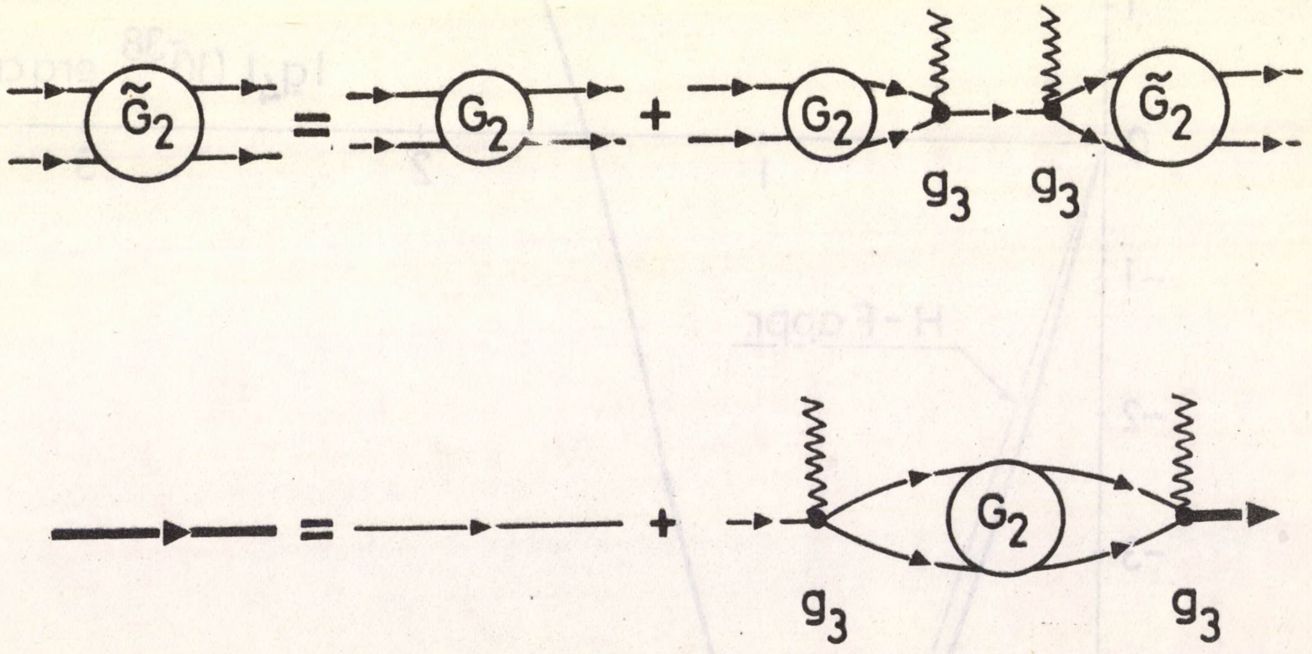
Fig.6. The calculated roton energy divided by $\sqrt{T}\, e^{-\frac{\Delta(T)}{T}}$ as the function of the value of roton-roton coupling (solid line). The dotted line represents the result of the Hartree-Fock approximation.

Fig.7. a) and b) Diagrammatic representation of the self-energy contributions coming from the interaction of rotons with particles in the condensed state.

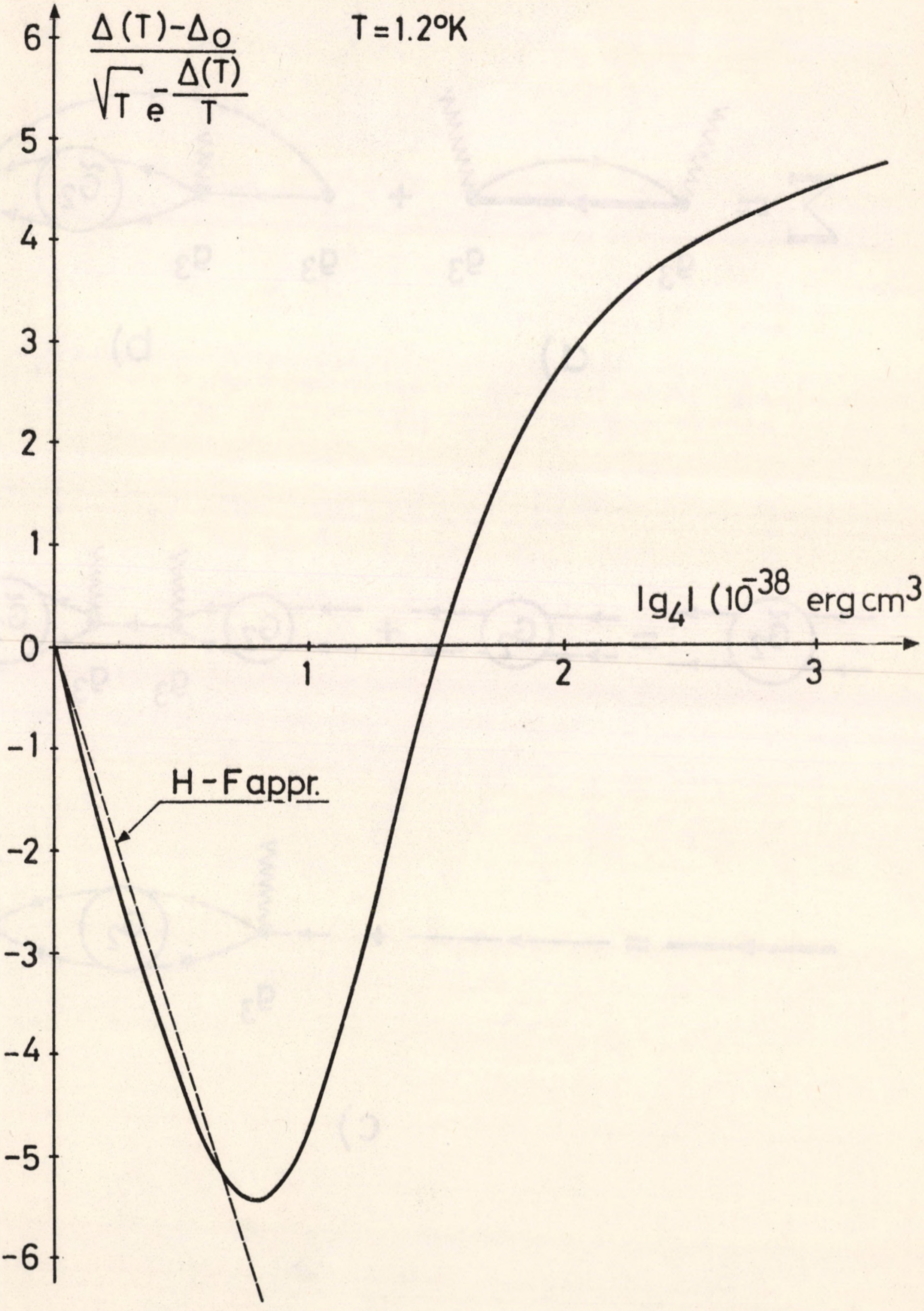
c) The diagrammatic representation of the influence of the hybridization process of the two- and one-particle Green's functions.



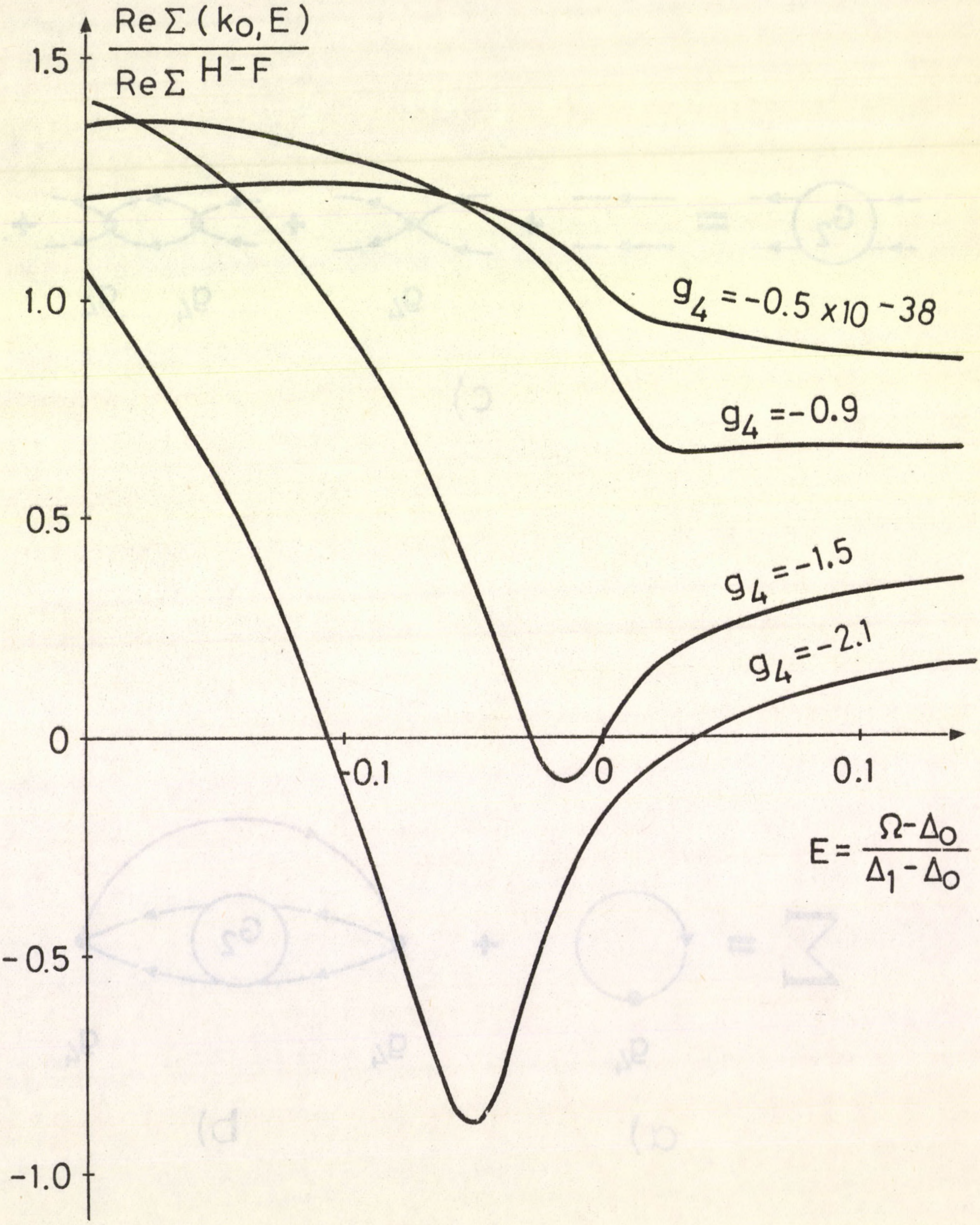
a) b)

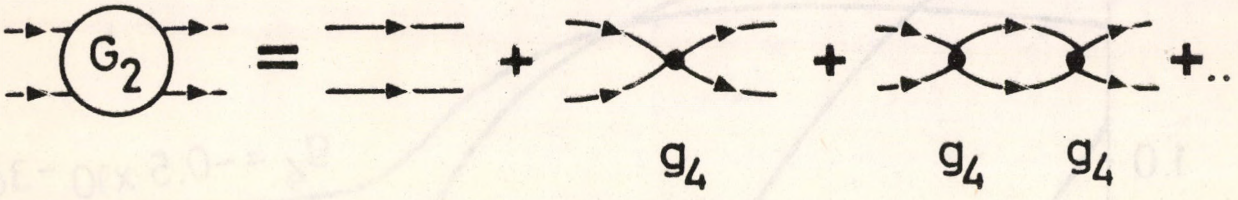


c)

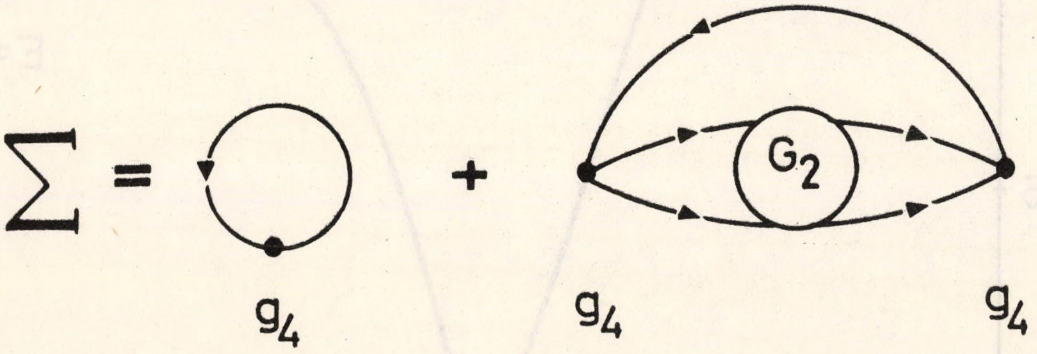


$\frac{\Delta(T)-\Delta_0}{\sqrt{T}\, e^{-\frac{\Delta(T)}{T}}}$
T=1.2°K
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
$|g_4|$ (10^{-38} erg cm^3)
1
2
3
H-F appr.

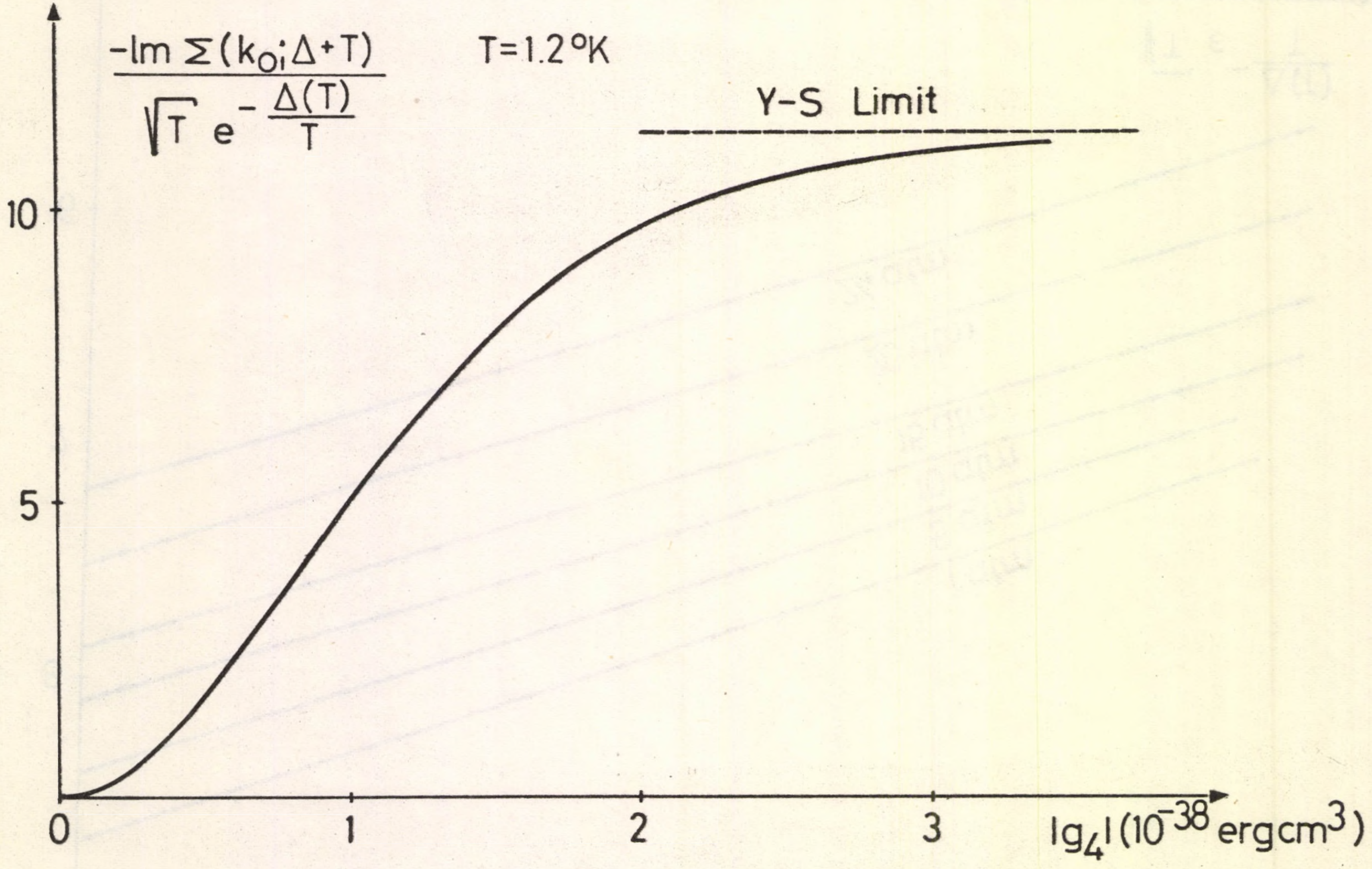

$\frac{\mathrm{Re}\,\Sigma(k_0, E)}{\mathrm{Re}\,\Sigma^{H-F}}$
1.5
1.0
0.5
0
-0.5
-1.0
-0.1
0
0.1
$g_4 = -0.5 \times 10^{-38}$
$g_4 = -0.9$
$g_4 = -1.5$
$g_4 = -2.1$
$E = \frac{\Omega - \Delta_0}{\Delta_1 - \Delta_0}$

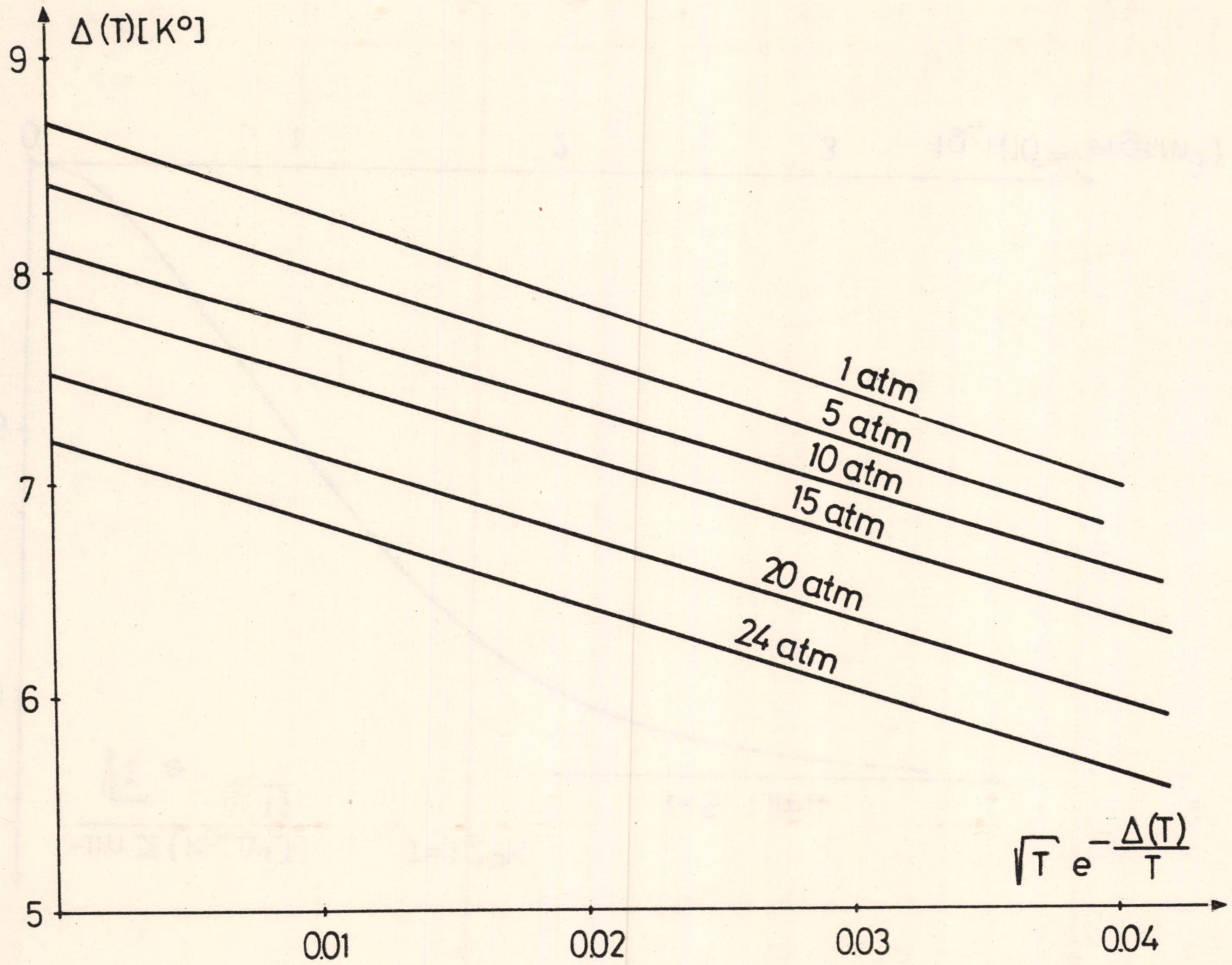


c)

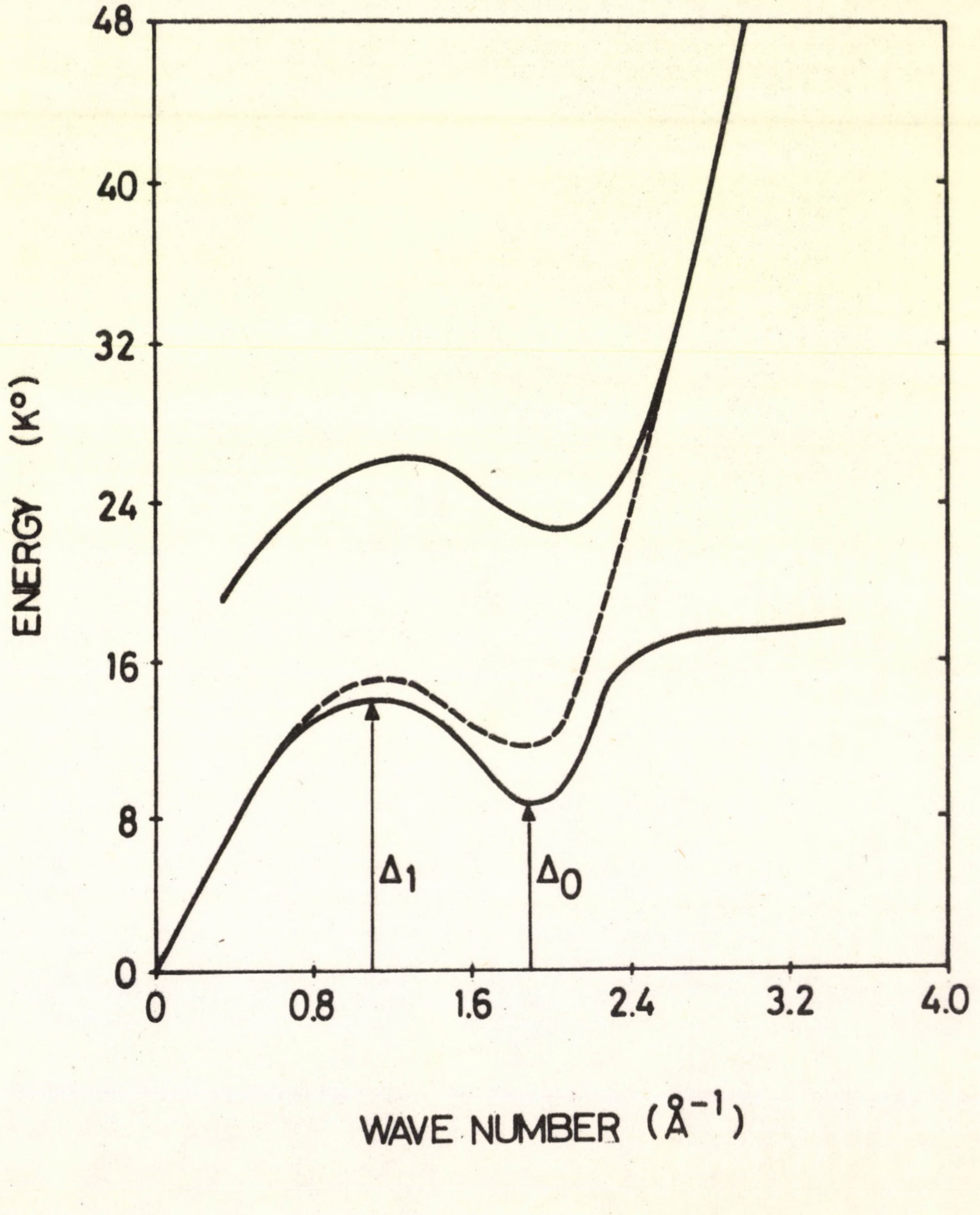


a) b)


$\frac{-\mathrm{Im}\,\Sigma(k_{0i};\Delta+T)}{\sqrt{T}\,e^{-\frac{\Delta(T)}{T}}}$
T=1.2 °K
Y-S Limit
10
5
0
1
2
3
$\lg_4 l\,(10^{-38}\,\mathrm{erg\,cm^3})$



Δ (T)[K°]
9
8
7
6
5
0.01
0.02
0.03
0.04
$\sqrt{T}\, e^{-\frac{\Delta(T)}{T}}$
1 atm
5 atm
10 atm
15 atm
20 atm
24 atm


48
40
32
24
16
8
0
ENERGY (K°)
0
0.8
1.6
2.4
3.2
4.0
WAVE NUMBER (Å^{-1})
Δ_1
Δ_0

Kiadja a Központi Fizikai Kutató Intézet
Felelős kiadó: Tompa Kálmán, a KFKI Szilárdtestfizikai Tudományos Tanácsának elnöke
Szakmai lektor: Zawadowski Alfréd
Nyelvi lektor: Sólyom Jenő
Példányszám: 335   Törzsszám: 72-7187
Készült a KFKI sokszorositó üzemében
Budapest, 1972. augusztus hó