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A GENERAL APPROACH FOR DETERMINISTIC ADAPTIVE REGULATORS BASED ON EXPLICIT IDENTIFICATION

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1.INTRODUCTION

The automatic adjustment of regulators for industrial processes has received particular attention for many years. Apart from the complexity of the regulator, there are indeed practical situations requiring automatic tuning or adaptivity in order to ensure better control performance:

- the parameters of the system to be controlled vary from time to time
- the system itself is actually nonlinear and the parameters of the linearized process model depend on the operating points
- an automatic adjustment of the regulator parameters is required, because of the complexity or slow dynamics of the process.

The present state of the research works in the field of the adaptive control reflect a synthesis of the classical principles and the results of the seventies, when the idea of self-tuning control got so attractive and popular.

The subject of this work is the adaptive servo-regulatory problem. Aiming the practical applicability pole/zero assignment regulators will be considered rather than optimal strategies. The noise rejection will be designed for deterministic, stepwise disturbances, however to show the generality of the presented design base, the stochastic noise rejection problem will be shortly discussed,as well.

Since the early paper of Peterka [38] and Åström and Wittenmark [5] the self-tuning regulators mean a very attractive class of adaptive controllers.The basic self-tuning control algorithms were developed for the rejection of stochastic disturbances acting on the process to be controlled. There was some early try to extend the self-tuning regulators to include constant set point, too [25]. Aiming the practical applicability of the self-tuning regulators the basic self-tuning idea was extended by Clarke and Gawthrop [14,15,16].Their generalized self-tuning controller minimizes a loss function incorporating terms relating to the servo and regulatory performance, thus it can be interpreted as a model-reference strategy, as well [22].

A different suboptimal approach was given by Wellstead et al. [47,49]. This method detunes the optimal, minimum variance controller by prescribing poles and zeros for the closed loop system. The servo problem was also solved [48], but in a rather complex way.

A general and systematic approach was given by Åström et al. [7,8] for the deterministic servo problem. In addition to the general pole/zero placement design structures a classification for the adptive schemes was also suggested [7], namely the explicit and implicit schemes were introduced. Unlike the implicit schemes the explicit schemes require the identification of the plant, thus the explicit and implicit schemes are often referred as indirect or direct schemes, respectively[36].

It is well known [6,7,33,40,42,43,44] that the cancellation controllers which are to cancel poorly damped zeros or zeros of inverse unstable processes result in rippling intersampling behaviour and exhibit the infinite sensitivity problem, respectively. To avoid these difficulties a number of design strategies have been developed so far, which can be essentially classified as cancellation methods and methods minimizing a general loss function. The cancellation controllers allow the process zeros outside a given design region to appear in the closed loop system, while the general loss functions incorporate terms relating to the control effort, but in this latter case the loss function adjustment is far not a trivial design step [2,3].

The relationships between the mentioned design methods and some links with the frequency domain considerations were discovered in a very interesting way by Allidina and Hughes [1,2,3,27]. A new pole placement design method suitable for the adaptive control of nonminimum phase systems was presented by Berger[11], where the pole placement was also achieved by the choice of an appropriate loss function, but the suggested method requires a matrix inversion in every step.

Regarding to the cancellation controllers the adaptive control can be based on both implicit and explicit schemes. As far as the implicit or direct methods are concerned, the fundamental difficulty is caused by the fact, that the parametrization leads to an estimation problem bilinear in parameters. The solution given by Åström needs further analysis, while another approach presented by Hetthéssy et al.[24] suffers from the numerical difficulty of finding the common roots of two updated regulator polynomials. A direct adaptive control strategy for continuous nonminimum phase systems has been developed recently by Elliott [18]. All these direct methods estimate an extended parameter vector. A quasi-direct method by Lozano and Landau [36] updates the process and regulator parameters in each step and the estimated process parameters are used to filter the input/output observations.

The explicite schemes are usually based on a Diophantine equation to be solved. In addition to this, in general case the estimated process numerator has to be factorized to separate the process zeros to be and not to be cancelled [7,8]. If no process zeros are to be cancelled the factorization is not required. For this latter case Goodwin [23] gave a detailed analysis including a local convergence proof. Similar results were presented by Boland and Giblin [12] using appropriate saturation for the control action.

However, an early attempt is known to compute the regulator parameters in a very simple way, without solving any identity[30]. A number of works [19,26,29,35,51] have recalled this explicit method recently, but only stable processes have been considered so far.

In this work the deterministic servo-regulatory problem is considered and solved, treating well known results and giving some innovations. Two basic control structures are established in different forms to control both stable and unstable, as well as minimum and nonminimum phase processes. The first structure requires moderate computations, and the known results are extended for integrating and unstable systems, while the second structure creates a predictive form based on a polynomial equation to be solved. In both structures prespecified zeros and poles can be placed into the closed loop system, regarding to follow a reference model and the noise rejection, as well. The influence of the prespecified zeros and poles on the control action is shown. For the adaptive solution the explicite scheme is proposed, however the implicit adaptive solutions are also overviewed and a new derivation of the direct adaptive algorithm by Åström is also presented.

As an important motive from practical point of view, saturations in the control input are also taken into consideration. The theoretical results are illustrated by hybrid simulations.

2. DETERMINISTIC SERVO REGULATOR DESIGN FOR PROCESSES WITH KNOWN PARAMETERS

Consider the single input single output, discrete time linear systems given by

$$y(t) + a_1 y(t-1) + \ldots + a_n y(t-n) = b_o u(t-d) + b_1 u(t-d-1) + \ldots + b_m u(t-d-m)$$

where $t=0,1,2,\ldots$ denotes the discrete time instants, u and y stand for the system input and output, respectively. The discrete time delay d is assumed to be $d>0$.
Using the backward shift operator z^{-1} and introducing the polynomials

$$A(z^{-1}) = 1 + a_1 z^{-1} + \ldots + a_n z^{-n}$$

and

$$B(z^{-1}) = b_o + b_1 z^{-1} + \ldots + b_m z^{-m}$$

the discrete transfer function of the process is obtained by

$$y(t) = \frac{B(z^{-1})}{A(z^{-1})} z^{-d} u(t) , \qquad (2-1)$$

where $A(z^{-1})$ and $B(z^{-1})$ are relatively prime polynomials (they have no common roots).

The aim of the control is to ensure a closed loop system to follow the output of a reference model $N(z^{-1})/M(z^{-1})$ as close as possible. A general control structure involving a precompensator W_1, a serial regulator W_2 and a feedback compensator W_3 is shown in Figure 2-1. For the sake of simplicity the argument z^{-1} will be dropped in the sequel. As the reference model driven by a reference signal $y_r(t)$ is placed into the feedforward path, the optimal overall transfer function between the reference model and the process output is z^{-d}, which gives

$$\frac{W_1 W_2 \frac{B}{A} z^{-d}}{1+W_2 W_3 \frac{B}{A} z^{-d}} = z^{-d}$$

or equivalently

$$W_2 B\ (W_1 - W_3 z^{-d}) = A \quad . \qquad (2-2)$$

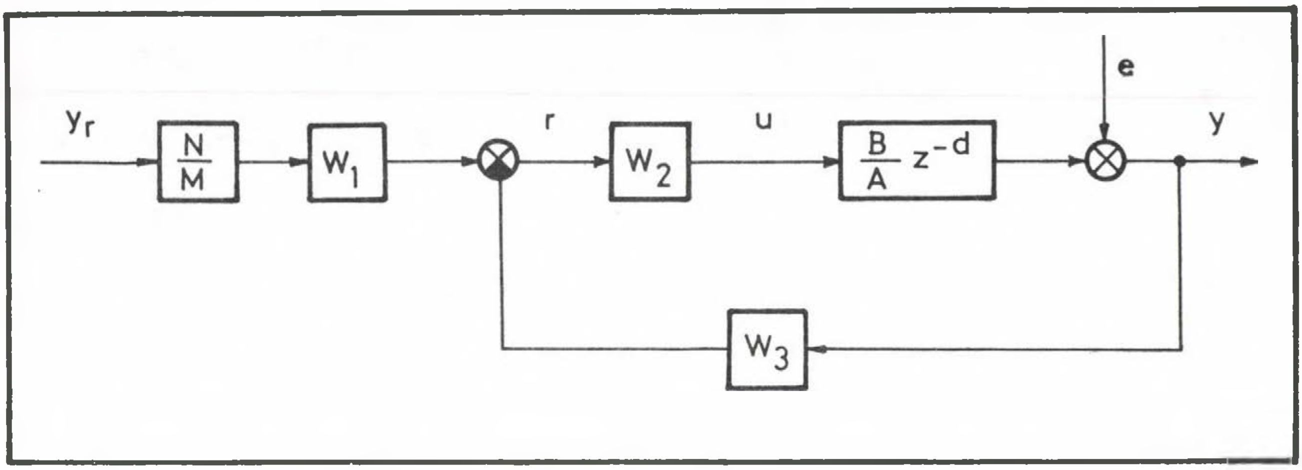


Figure 2-1.

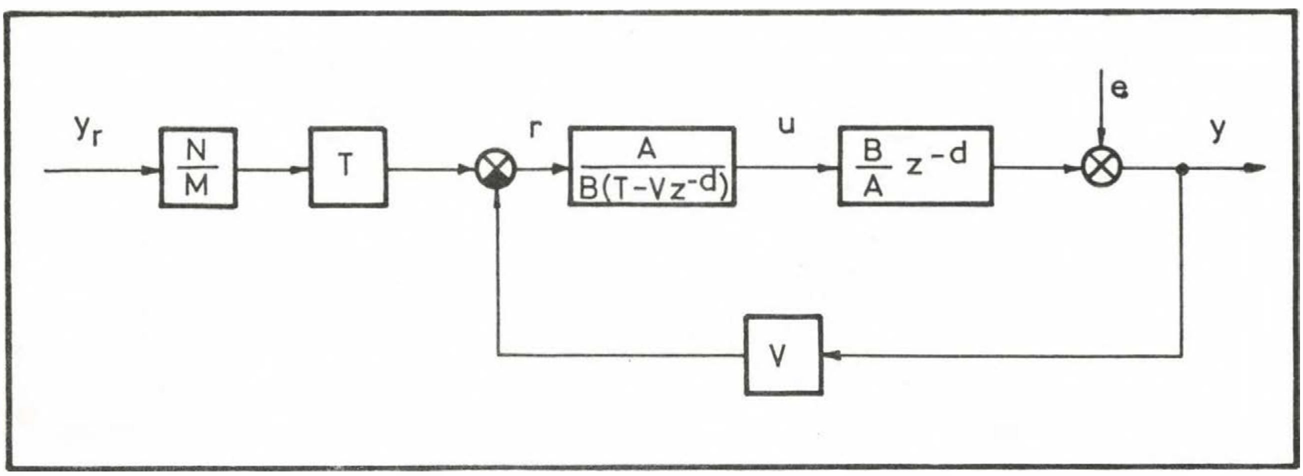


Figure 2-2.

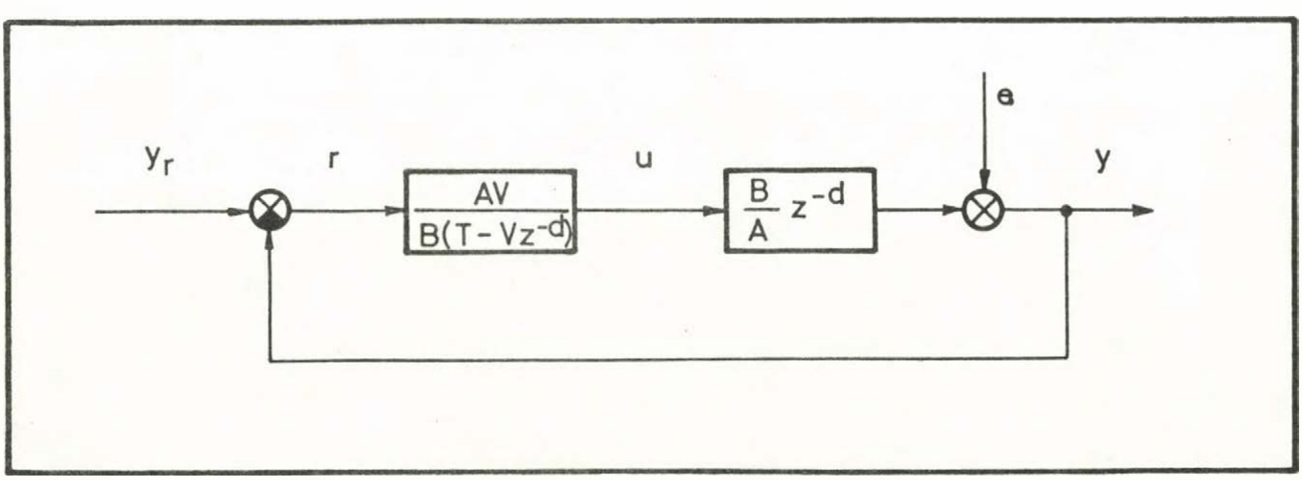


Figure 2-3.

Eq. (2-2) shows that an infinite set of $\{W_1, W_2, W_3\}$ exists to meet the optimality, however, they ensure different noise rejection, as W_1 does not appear in the closed loop. Assuming deterministic, step-wise disturbances acting on the process output, the complete noise rejection takes d steps in the optimal case. Designing also the dynamics of the noise rejection we have

$$\frac{1}{1+ W_2 W_3 \frac{B}{A} z^{-d}} = 1 - \frac{V}{T} z^{-d} \qquad (2-3)$$

where the polynominals

$$V = v_o + v_1 z^{-1} + \ldots + v_{n_V} z^{-n_V}$$

and

$$T = t_o + t_1 z^{-1} + \ldots + t_{n_T} z^{-n_T} \qquad (2-4)$$

contain prespecified zeros and poles in the noise rejection by Eq.(2-3). Now Eq.(2-2) and Eq.(2-3) lead to

$$\frac{W_1}{W_3} = \frac{T}{V} \quad , \tag{2-5}$$

thus choosing

$$W_1 = T \tag{2-6}$$

and

$$W_3 = V \quad , \tag{2-7}$$

a serial regulator

$$W_2 = \frac{A}{B\ (T - Vz^{-d})} \tag{2-8}$$

is obtained (Figure 2-2).

Eq.(2-3) shows that to avoid the steady-state errors of the noise rejection

$$V(1) = T(1) \tag{2-9}$$

must hold, as

$$1 - \frac{Vz^{-d}}{T}\Bigg|_{z=1} = 0. \qquad (2\text{-}10)$$

It is remarkable that the condition by Eq. (2-9) causes the serial regulator W_2 to be of integrating type, namely the denominator $B(T-Vz^{-d})$ has a root of $z_1 = 1$, as $T(1) - V(1).1 = 0$.

The closed loop properties can be summarized as follows:

	Poles	Zeros	Overall transfer function
$y_r \to y$	M	N	$\frac{N}{M}$
$e \to y$	T	V	$1 - \frac{Vz^{-d}}{T}$.

It is seen that all the closed loop zeros and poles can be prespecified by using appropriate design polynomials M,N,T and V.
Results for the special choice of N=V and M=T,

as well as for N=V=M=T=1 are shown in Figure 2-3 and 2-4 , respectively.

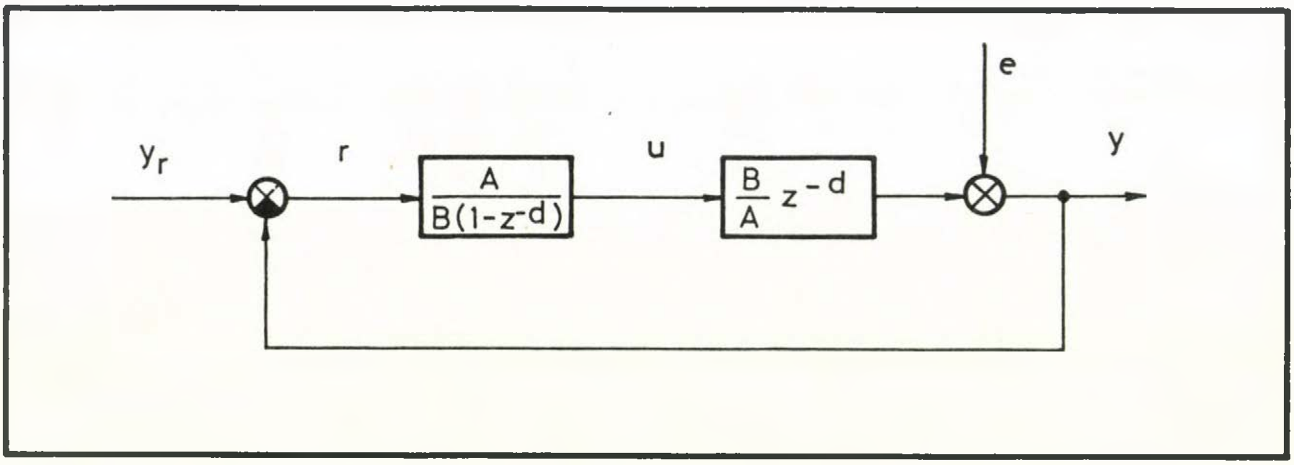
Based on Figure 2-2 the characteristic equation is

$$1 + \frac{A}{B(T-Vz^{-d})} \cdot \frac{B}{A} z^{-d} = 0$$

or equivalently

$$ABT = 0 \qquad (2\text{-}11)$$

which shows that because of the infinite sensitivity problem [7,14,33,34,43] the previously proposed algorithm is applicable only for stable, minimum phase systems. To make the citation easier the strategy shown in Figure 2-2 will be referred as optimal β structure, or simply as β_0 structure in the sequel. To summarize its applicability it should be underlined that it is <u>inadmissably sensitive</u> when controlling unstable or nonminimum phase systems. On the other hand, in the knowledge of the process parameters, the <u>regulator parameters can be directly computed</u> by



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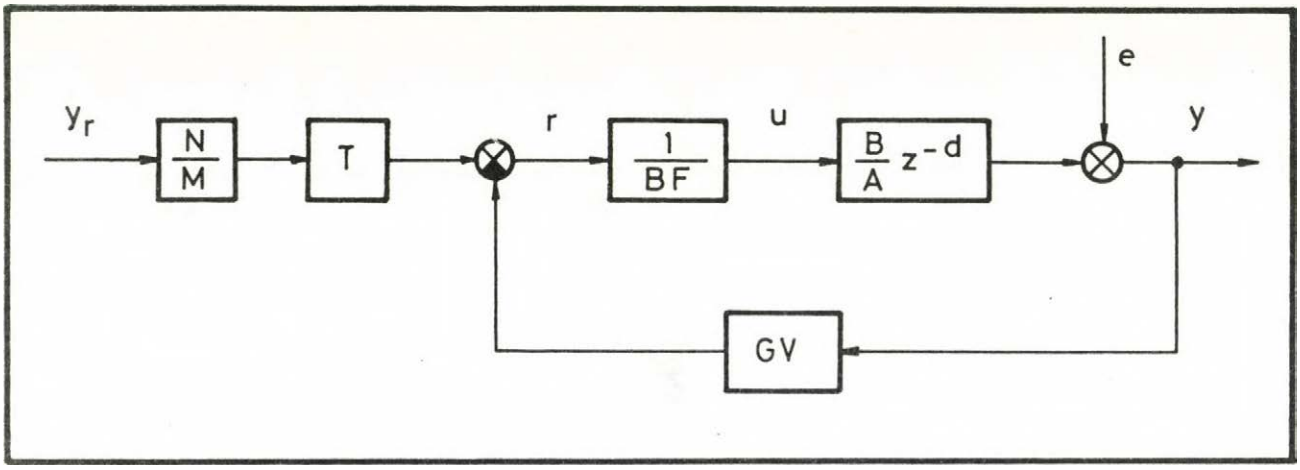

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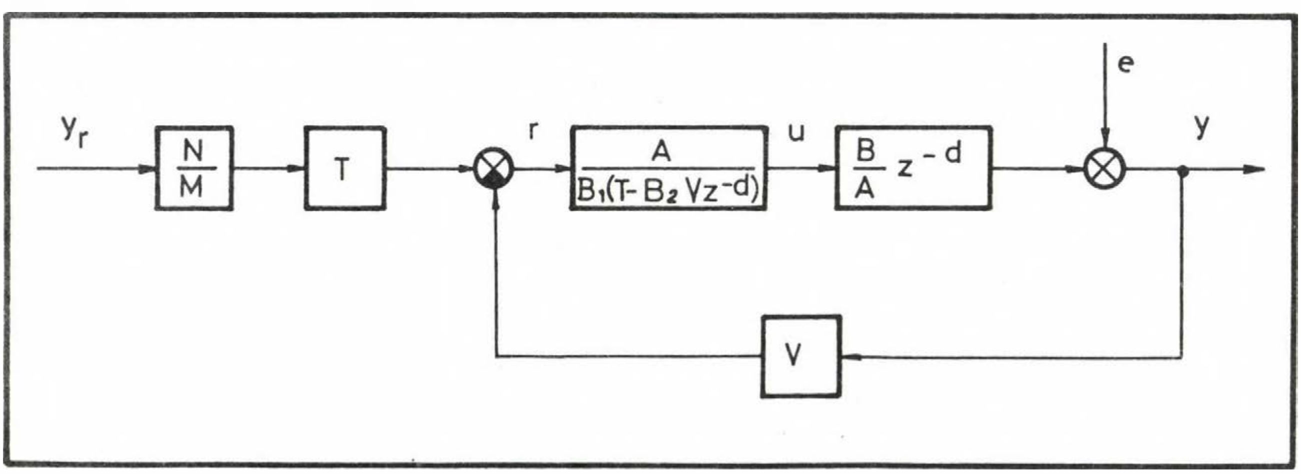

Figure 2-6.

$$W_1 = T$$

$$W_2 = \frac{A}{B(T-Vz^{-d})} \qquad (2\text{-}12)$$

$$W_3 = V$$

to ensure the prespecified closed loop zeros and poles.

In the sequel it will be shown that the poles of the process do not appear necessarily in the serial regulator. Define polynominals F and G by

$$T = AF + VGz^{-d} \quad , \qquad (2\text{-}13)$$

then applying Eq. (2-13) - similarly to the multivariable case [32] - for the process output

$$Ty(t+d) = (AF+VGz^{-d})\ y(t+d) =$$
$$= AFy(t+d)+VGy(t)=BFu(t)+VGy(t), \qquad (2\text{-}14)$$

is obtained as an errorless prediction form for $Ty(t+d)$. The polynominals introduced by Eq.(2-13) are unique using F and G with degrees

$$F = f_o + f_1 z^{-1} + \ldots + f_{n_v+d-1} z^{-(n_v+d-1)}$$
$$G = g_o + g_1 z^{-1} + \ldots + g_{n-1} z^{-(n-1)} \quad . \qquad (2\text{-}15)$$

As the optimality has been defined by

$$y(t+d) = \frac{N}{M} y_r(t) \quad , \qquad (2\text{-}16)$$

Eqs. (2-14) and (2-16) give

$$u(t) = \frac{\frac{N}{M} T\, y_r(t) - VGy(t)}{BF} \qquad (2\text{-}17)$$

for the control input (Figure 2-5). The condition defined by Eq. (2-16) can be considered as a trivial design step, as well.

The overall transfer function relating to the output noise is determined by

$$\frac{1}{1 + \frac{VG}{BF} \cdot \frac{B}{A} z^{-d}} = \frac{AF}{T} = 1 - \frac{VG}{T} z^{-d} \quad , \qquad (2\text{-}18)$$

which shows that in addition to the design polynomials the noise rejection depends on G, too. For example in case of $V=T=1$ and $G(1)=1$ the complete noise rejection takes (n_G+d) steps.

The presented algorithm will be referred optimal α, or α_o strategy in the sequel. Summing up its main properties it is seen from its characteristic equation

$$BT = 0 \tag{2-19}$$

that it can be applied for minimum phase systems and the regulator parameters are to be computed by solving a polynomial equation (Bezout identity).

It has been shown so for that the α_o and β_o strategies are equally unable to control nonminimum phase systems. To clear up the importance of this problem it has to be emphasized that unlike continuous systems, nonminimum phase discrete systems do not mean a narrow class at all, as sampled minimum phase continuous systems exhibit nonminimum phase properties rather frequently [7,8,33,34,41,49].

To control nonminimum phase discrete or sampled continuous systems <u>suboptimality</u> in the design procedure will be introduced in the sense of allowing the unstable or poorly damped zeros to appear as zeros of the closed loop system [40].

Factorize the numerator of the process by

$$B = B_1 \cdot B_2 \tag{2-20}$$

with $B_2(1)=1$, where B_2 contains the zeros not to be cancelled [7,8] :

$$B_1 = B_1(z^{-1}) = b_{10}+b_{11}z^{-1}+\ldots+ b_{1,m_1} z^{-m_1}$$

$$B_2 = B_2(z^{-1}) = b_{20}+b_{21}z^{-1}+\ldots+ b_{2,m_2} z^{-m_2} \tag{2-21}$$

$$m_1 + m_2 = m$$

$$b_{20} + b_{21} +\ldots+ b_{2,m_2} = 1 \ .$$

To ensure an overall transfer function of B_2z^{-d} between the reference model and the process output we have

$$\frac{W_1W_2 \frac{A}{B} \quad z^{-d}}{1 + W_2W_3 \frac{B}{A} \quad z^{-d}} = B_2 \ z^{-d} \ ,$$

or equivalently

$$W_2 B_1 (W_1 - B_2 W_3 z^{-d}) = A \quad . \tag{2-22}$$

Similarly, B_2 is allowed to appear in the noise rejection in the following way:

$$\frac{1}{1 + W_2 W_3 \frac{B}{A} z^{-d}} = 1 - \frac{VB_2}{T} z^{-d} \quad , \tag{2-23}$$

wherefrom

$$\frac{W_1}{W_3} = \frac{T}{V} \tag{2-24}$$

is obtained again. By choosing

$$W_1 = T \tag{2-25}$$

and

$$W_3 = V \tag{2-26}$$

the serial regulator given by Eq. (2-22) is

$$W_2 = \frac{A}{B_1 (T - B_2 V z^{-d})} \tag{2-27}$$

The presented suboptimal β strategy (β_s) is shown in Figure 2-6. Results with N=V and M=T, as well as with N=V=M=T=1 are shown in Figure 2-7 and 2-8 , respectively.
The characteristic equation for the general case is

$$AB_1T = 0 \quad , \qquad (2\text{-}28)$$

while the closed loop properties can be summarized as follows:

$$\begin{array}{llll} & \text{Poles} & \text{Zeros} & \text{Overall transfer function} \\ y_r \to y & M & N.B_2 & \dfrac{NB_2}{M} z^{-d} \\ e \to y & T & V.B_2 & 1 - \dfrac{VB_2}{T} z^{-d} \quad . \end{array} \qquad (2\text{-}29)$$

prespecified derived

By Eq. (2-28) it can be stated that the β_s strategy can be applied for any stable process, and the regulator parameters can directly be computed having the process numerator appropriately factorized.

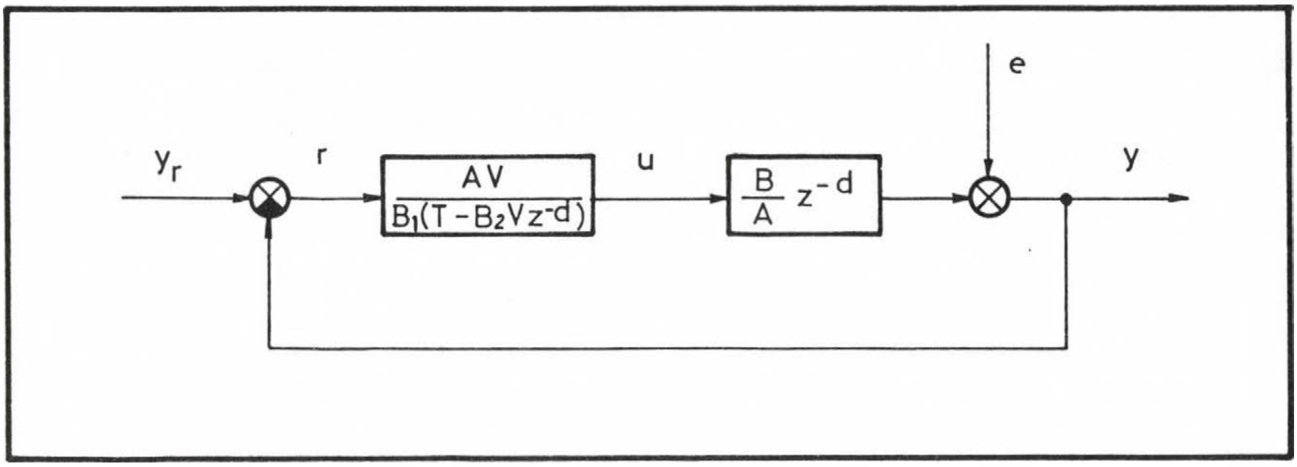

Figure 2-7.

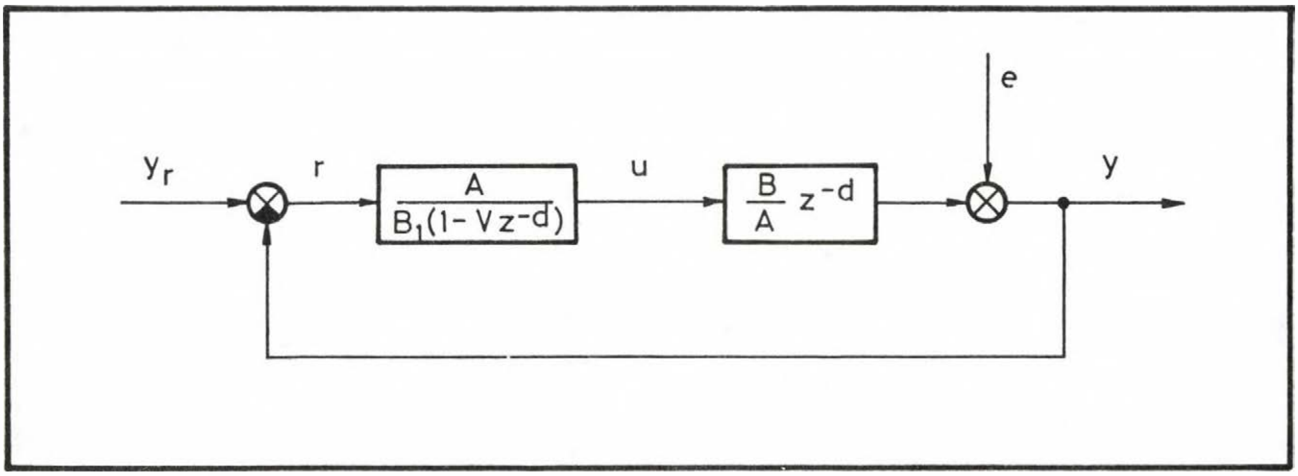

Figure 2-8.

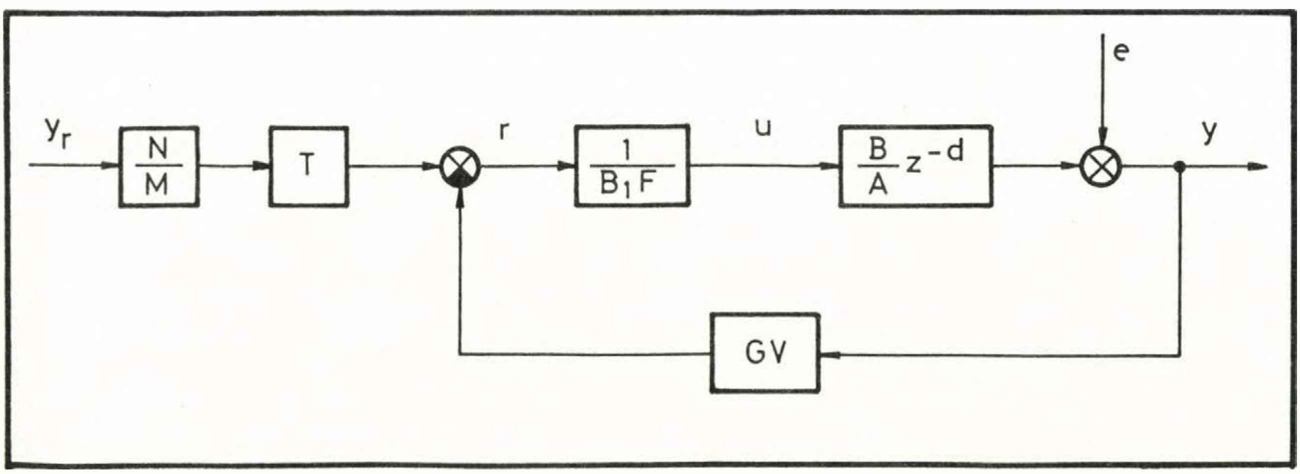

Figure 2-9.

The control input now can be expressed by

$$u(t) = \frac{[\frac{N}{M} T y_r(t) - Vy(t)] A}{B_1 (T - VB_2 z^{-d})} = \frac{A}{B_1} [\frac{N}{M} y_r(t) - \frac{V}{T} e(t)] \tag{2-30}$$

which clearly shows the direct influence of the design polynomials on the control action.

It is remarkable that if no process zeros are to be cancelled the factorization $B=B_1B_2$ is not required, as in this case

$$B_2 = \frac{B}{B(1)} \tag{2-31}$$

and

$$B_1 = B(1) \quad . \tag{2-32}$$

This special, practical minded choice is discussed by several authors [19,29,35,51].
Introducing further speciality by $N=V$ and $T=1$ W_2W_3 becomes

$$W_2W_3 = \frac{AV}{B(1) - VBz^{-d}} \quad . \tag{2-33}$$

This result was introduced by Kalman [30] with $V=1$, treated by Favier and Guillermin [19] , used by Kurz et al. [35] with $V=1$ as DB1 and with $V=v_o+v_1z^{-1}$ as DB2 dead-beat regulator, as well as applied by Zimmermann [51] with a general, parameter optimizable plynomial V.

To illustrate the practical possibilities offered by the design polynomials and to show that the general approach includes well-known algorithms [29, 35] as special cases, consider a design with

$$M = T = 1$$

$$N = V = v_o + v_1 z^{-1}$$

$$v_o + v_1 = 1$$

$$y_r(t) = Y_o \quad , \quad \text{if} \quad t \geqq 0 \quad \text{and}$$

$$y_r(t) = 0 \quad \text{otherwise,}$$

$$B_2 = \frac{B}{B(1)}$$

$$B_1 = B(1) .$$

Using Eq. (2-30) with $e(t)=0$ we have

$$u(t) = \frac{A}{B(1)} (v_o + v_1 z^{-1}) \; y_r(t) \quad , \qquad (2\text{-}34)$$

which gives

$$u(0) = \frac{v_o Y_o}{B(1)} \tag{2-35}$$

$$u(1) = \frac{a_1 v_o Y_o}{B(1)} + \frac{v_o Y_o}{B(1)} + \frac{v_1 Y_o}{B(1)} =$$

$$= a_1 u(0) + \frac{Y_o}{B(1)} \quad . \tag{2-36}$$

It is seen that the first value of the control input can be influenced by v_o, however choosing too small a $u(0)$, a large $u(1)$ could be required. A reasonable choice for v_o is obtained if

$$u(1) \leqq u(0) \tag{2-37}$$

is ensured. Combining Eqs. (2-36) and (2-37)

$$\frac{1}{1-a_1} \leqq v_o < 1 \tag{2-38}$$

gives a practical region for v_o.

In the sequel a predictive suboptimal strategy (α_s) will be derived. Define F and G by

$$T = AF + B_2GVz^{-d} \qquad (2\text{-}39)$$

with

$$F = f_0+f_1z^{-1} +\ldots+ f_{n_v+m_2+d-1}z^{-(n_v+m_2+d-1)} \qquad (2\text{-}40)$$

and

$$G = g_o+g_1z^{-1} +\ldots+ g_{n-1}\ z^{-(n-1)}\ , \qquad (2\text{-}41)$$

Using Eq. (2-39) we have

$$\begin{aligned} Ty(t+d) &= (AF + B_2GVz^{-d})y(t+d) = \\ &= AFy(t+d)+B_2GVy(t) = BFu(t)+B_2GVy(t) = \\ &= B_2\ [B_1Fu(t) + GVy(t)]\ . \end{aligned} \qquad (2\text{-}42)$$

Combining Eq. (2-42) and

$$y\ (t+d) = \frac{N}{M}\ B_2y_r(t) \qquad (2\text{-}43)$$

as a condition of the suboptimality the control input is given by

$$u(t) = \frac{\frac{N}{M} \cdot T\, y_r(t) - GVy(t)}{B_1 F} , \qquad (2\text{-}44)$$

and the closed loop system is shown in Figure 2-9. The noise rejection of the α_s structure is

$$\frac{1}{1 + \frac{BGVz^{-d}}{AB_1F}} = 1 - \frac{B_2GVz^{-d}}{T} , \qquad (2\text{-}45)$$

and thus

$$T(1) = G(1)V(1) , \qquad (2\text{-}46)$$

will ensure the elimination of steady-state errors, as $B_2(1)=1$. Taking Eq. (2-46) into consideration Eq. (2-39) gives

$$A(1)F(1) = 0 \qquad (2\text{-}47)$$

again. For processes with no integrator(s) $F=F'(1-z^{-1})$ should be used to satisfy the above requirement. The closed loop properties ensured by the α_s strategy can be summarized as follows:

$$
\begin{array}{lccl}
 & \text{Poles} & \text{Zeros} & \text{Overall transfer function} \\
y_r \to y & M & N \cdot B_2 & \dfrac{NB_2 z^{-d}}{M} \\
e \to y & T & V \cdot B_2 G & 1 - \dfrac{B_2 G V z^{-d}}{T} \\
 & \underbrace{\qquad\qquad}_{\text{prespecified}} & \underbrace{\qquad}_{\text{derived}} &
\end{array} \qquad (2\text{-}48)
$$

The characteristic equation is

$$B_1 T = 0 \qquad (2\text{-}49)$$

wherefrom it is seen that the α_s structure can be used to control any kind of processes.

It is remarkable that assuming stochastic output disturbances driven by white noise through a noise model of C/A [4], the closed loop transfer function is

$$\frac{C}{A}\left(1 - \frac{B_2 GV}{T} z^{-d}\right) = \frac{C(T-B_2GVz^{-d})}{AT} = \frac{CAF}{AT} = \frac{CF}{T} , \qquad (2\text{-}50)$$

wherefrom a choice by

$$T = T'C \qquad (2\text{-}51)$$

gives the detuned suboptimal control introduced by Wellstead et. al [47, 49] . See also in Clarke [17].

Note that the choice of $B_2 = 1$ reproduces the optimal solution, i.e. all the process zeros are to be cancelled, while specifying $B_2 = B/B(1)$ corresponds to a design principle of no process zero cancellation. This latter case could be very advantageous from practical point of view, namely the factorization $B=B_1B_2$ does not need to solve the equation of $z^mB=0$.

To improve the β_s structure for the control of unstable processes define the serial regulator by

$$W_2 = \frac{A_1}{B_1F} , \tag{2-52}$$

where

$$A = A_1A_2(1-z^{-1})^k \tag{2-53}$$

and A_2 contains all the unstable poles and it is normalized by $A_2(1)=1$. Using Eq. (2-22)

$$W_2B_1(W_1-B_2W_3z^{-d}) = A_1A_2(1-z^{-1})^k \tag{2-54}$$

is obtained, and choosing $W_1=T$ and $W_3=GV$ a polynomial equation

$$T = A_2F\ (1-z^{-1})^k + B_2GVz^{-d} \qquad (2-55)$$

determines the regulator polynomials F and G.
From the condition of steady-state errors for

$k=0$

$$F = F'(1-z^{-1})$$

has to be supposed again, while for $k > 0$ the condition by

$$A(1)F(1) = 0$$

is automatically fulfilled.

This solution exhibits the properties of both the α and β structures, thus it will be referred as a γ_s structure (Figure 2-10).

As a special case consider stable systems ($A_2=1$) with one integrator ($k=1$). From Eq. (2-55) $G=g_o$ is obtained and

$$T = F(1-z^{-1}) + g_oB_2Vz^{-d} \quad . \qquad (2-56)$$

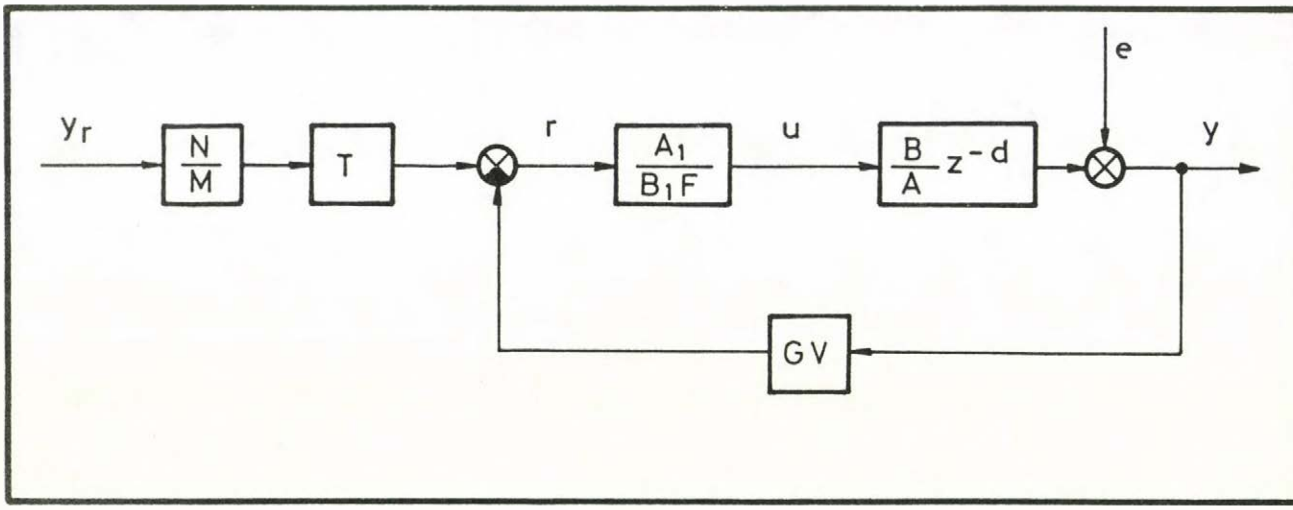

Figure 2-10.

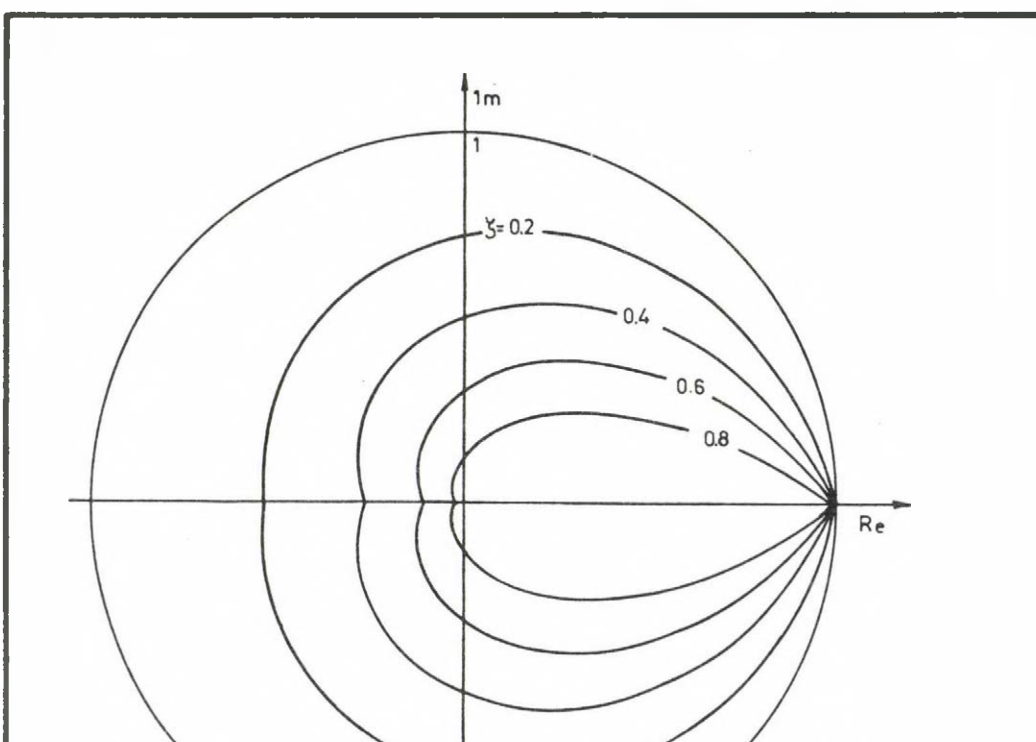

Figure 2-11.

Substituting $z=1$ into the above equation we have

$$T(1) = g_o V(1) . \tag{2-57}$$

This implies that $z_1=1$ is a root of $T-g_oB_2Vz^{-d}$ and the polynomial F can be computed explicitely by

$$F = \frac{T - g_oB_2Vz^{-d}}{1-z^{-1}} = f_0+f_1z^{-1}+...+f_{n_v+m_2+d-1}z^{-(n_v+m_2+d-1)} . \tag{2-58}$$

Summing up the applicability of the γ_s structure, the characteristic equation by

$$A_1B_1T = 0 \tag{2-59}$$

shows that it can be applied for all kind of processes. Note that the order of the polynomial equation (2-55) to be solved is lower by the number of the stable poles than in case of the α_s strategy.
In addition to this it gives an explicit expression for the regulator polynomials in case of stable processes with one integrator.

All the presented structures are reviewed in Table

Table 2-I.

	Type of design	W_1	W_2	W_3	u(t)	$y_r \rightarrow y$	$e \rightarrow y$	Char. eq.	Applic. cond.	Cond. of zero steady-state error	Polynomial equation
optimal	α_o	T	$\frac{1}{BF}$	GV	$\frac{\frac{N}{M} Ty_r(t) - GVy(t)}{BF}$	$\frac{N}{M} z^{-d}$	$1 - \frac{GVz^{-d}}{T}$	BT=0	min. phase process	G(1)V(1)=T(1)	$T = AF + GV z^{-d}$ $k=0:\ F = (1-z^{-1})F'$, $n'_F = n_V + d - 1$, $n_G = n$ $k>0:\ n_F = n_V + d - 1$, $n_G = n-1$
optimal	β_o	T	$\frac{A}{B(T - Vz^{-d})}$	V	$\frac{\left[\frac{N}{M} Ty_r(t) - Vy(t)\right] A}{B(T - Vz^{-d})}$	$\frac{N}{M} z^{-d}$	$1 - \frac{Vz^{-d}}{T}$	ABT=0	min. phase stable proc. with no int.	V(1)-T(1)	∅
suboptimal	α_s	T	$\frac{1}{B_1 F}$	GV	$\frac{\frac{N}{M} Ty_r(t) - GVy(t)}{B_1 F}$	$\frac{NB_2}{M} z^{-d}$	$1 - \frac{B_2 GVz^{-d}}{T}$	$B_1 T = 0$	any linear process	G(1)V(1)=T(1)	$T = AF + B_2 GVz^{-d}$ $k=0:\ F = (1-z^{-1})F'$, $n' = n_V + m_2 + d - 1$, $n_G = n$ $k>0\ \ n_F = n_V + m_2 + d - 1$, $n_G = n-1$
suboptimal	β_s	T	$\frac{A}{B_1 (T - B_2 Vz^{-d})}$	V	$\frac{\left[\frac{N}{M} Ty_r(t) - Vy(t)\right] A}{B_1 (T - B_2 Vz^{-d})}$	$\frac{NB_2}{M} z^{-d}$	$1 - \frac{B_2 Vz^{-d}}{T}$	$AB_1 T = 0$	stable proc. with no int.	V(1)=T(1)	∅
suboptimal	γ_s	T	$\frac{A_1}{B_1 F}$	GV	$\frac{\left[\frac{N}{M} Ty_r(t) - GVy(t)\right] A_1}{B_1 F}$	$\frac{NB_2}{M} z^{-d}$	$1 - \frac{B_2 GVz^{-d}}{T}$	$A_1 B_1 T = 0$	any linear process	G(1)V(1)=T(1)	$T = A_2 F (1-z^{-1})^k + B_2 GVz^{-d}$ $k=0:\ F = (1-z^{-1})F'$, $n'_F = n_V + m_2 + d - 1$, $n_G = n_2$ $k>0:\ n_F = n_V + m_2 + d - 1$, $n_G = n_2 + k - 1$
suboptimal	γ'_s	T	$\frac{A_1}{B_1 F}$	$g_o V$	$\frac{\left[\frac{N}{M} Ty_r(t) - g_o Vy(t)\right] A_1}{B_1 F}$	$\frac{NB_2}{M} z^{-d}$	$1 - \frac{g_o B_2 Vz^{-d}}{T}$	$A_1 B_1 T = 0$	stable proc. with one int.	$g_o V(1) = T(1)$	$F = \frac{T - g_o VB_2 z^{-d}}{1 - z^{-1}} = f_o + f_1 z^{-1} + \ldots f_{n_V + m_2 + d - 1} z^{-(n_V + m_2 + d - 1)}$

Process:

$$\frac{B}{A} z^{-d} = \frac{B_1 B_2 z^{-d}}{A_1 A_2 / 1 - z^{-1} /^k}$$

2-I. Note that the polynomial equations lead to a set of linear equations, however the resolution could be very sensitive [9, 17].

2.1. Design considerations for the cancellation of the process zeros

In this chapter the design regions for the process zeros to be cancelled will be considered. As the process zeros to be cancelled mean poles for the regulator, the dynamic behaviour of the control input depends on the cancelled zeros. It is well-known that any regulator pole outsied the unit cirle causes an exponentially growing control input. However, regulator poles within the unit circle can also cause ripples in the continuous process output [34, 40, 43, 44], if the oscillation of the control input is not damped appropriately. To avoid the disadvantageous intersampling behaviour of the continuous process output the discrete process zeros to be cancelled have to lie inside a design region.

For continuous time systems the angle between the negative real axis and the line from the origin to the dominant poles determines the damping.

Similar criteria exist for sampled data systems. To show the critical curves of constant damping consider a continuous system of a dominant pole pair

$$\frac{1}{1 + 2\zeta Ts + T^2 s^2} , \qquad (2\text{-}60)$$

which has a step response equivalent discrete form by

$$\frac{b_o + b_1 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}} z^{-1} , \qquad (2\text{-}61)$$

where

$$a_1 = -2e^{-\zeta x} \cos x\sqrt{1-\zeta^2} \qquad (2\text{-}62)$$

$$a_2 = e^{-2\zeta x} \qquad (2\text{-}63)$$

and x stands for the relative sampling [33] :

$$x = \frac{h}{T} .$$

Now the poles of the sampled system are given by the roots of

$$z^2 A = z^2 + a_1 z + a_2 = 0. \tag{2-64}$$

The roots of Eq. (2-64) are

$$z_{1,2} = e^{x(-\zeta + j\sqrt{1-\zeta^2})}, \tag{2-65}$$

and the critical curves for some given ζ are shown in Figure 2-11.

When selecting the zeros of the process to be controlled a damping for the control input has to be chosen and then

$$z^m B = b_o z^m + b_1 z^{m-1} + \ldots + b_m = 0 \tag{2-66}$$

has to be solved. If a given root of Eq. (2-66) is inside the design region corresponding to the given damping it is qualified as a process zero to be cancelled, otherwise it belongs to the zeros not to be cancelled.

If all the process zeros not to be cancelled have negative real part then the step response of the closed loop system is monotonous in the sampling instants. For the proof consider

$$z^{m_2} \cdot B_2 = b_{2o} z^{m_2} + b_{21} z^{m_2 - 1} + \ldots + b_{2,m_2} =$$

$$= b_{2o} (z - z_1)(z - z_2) \ \ldots \ (z - z_{m_2}) \ . \qquad (2\text{-}67)$$

The coefficients of B_2 coming from the negative real roots are obviously positive. The complex conjugate roots give positive coefficients, as well, because having

$$z_i = -a + jb$$
$$z_j = -a - jb$$

$(z-z_i)(z-z_j)$ gives

$$(z-z_i)(z-z_j) = z^2 + 2az + a^2 + b^2 \ . \qquad (2\text{-}68)$$

On the other hand the discrete step response of the closed loop system is

$$y(t) = B_2 \cdot 1(t-d) \ , \qquad (2\text{-}69)$$

if no design polynomials are taken into considera-

tion. As Eq. (2-69) gives a step response of finite settling time and the coefficients of the polynomial B_2 mean the increments of the step response, it is proved that the step response is monotonous.
The intersampling behaviour of the continuous process output will depend on the damping of the control input and on the frequeincy response of the system corresponding to the sampling time.

The design regions for B_1 and B_2 require a detailed analysis, as the roots of B_1 determine the damping of the control input, while the roots of B_2 have an effect for the step response of the closed loop system. However, an essential difference is seen between the unstable and poorly damped roots,because the effect of the poorly damped roots can be modified by choosing appropriate design polynomials [7] . If positive unstable roots are included in B_2, the step response of the closed loop system exhibits typical nonminimum phase properties. In this latter case both the undershoot and the overshoot of the step response can be effected by appropriately inserted zeros and/or poles [46] .

2.2. Constrained control input

As a consequence of the particular nature of the controlled plant and the applied actuator, actual physical limits exist in any practical control system. This means that the relationship

$$U_{MIN} \leq u(t) \leq U_{MAX} \qquad (2\text{-}70)$$

must hold for $u(t)$ in every step. If the serial regulator W_2 produces a control input $u(t)$ out of the region given by Eq. (2-70) then the actual control input $u'(t) \neq u(t)$ should be $u'(t)=U_{MIN}$ $(u(t) < U_{MIN})$ or $u'(t)=U_{MAX}$ $(u(t) > U_{MAX})$.
If the control input determined by W_2 is constrained before sending it as a command to the acutator, W_2 no longer controls a linear plant, and a very slow output transient, that is a very poor control performance can be observed [28] . In order to control the plant in a linear way by W_2 , the saturation has to be transformed to proceed W_2 (see Figure 2-12 and 2-13). This idea [28] has been called "self-limiting", as the tranformed saturation levels usually vary from time to time.

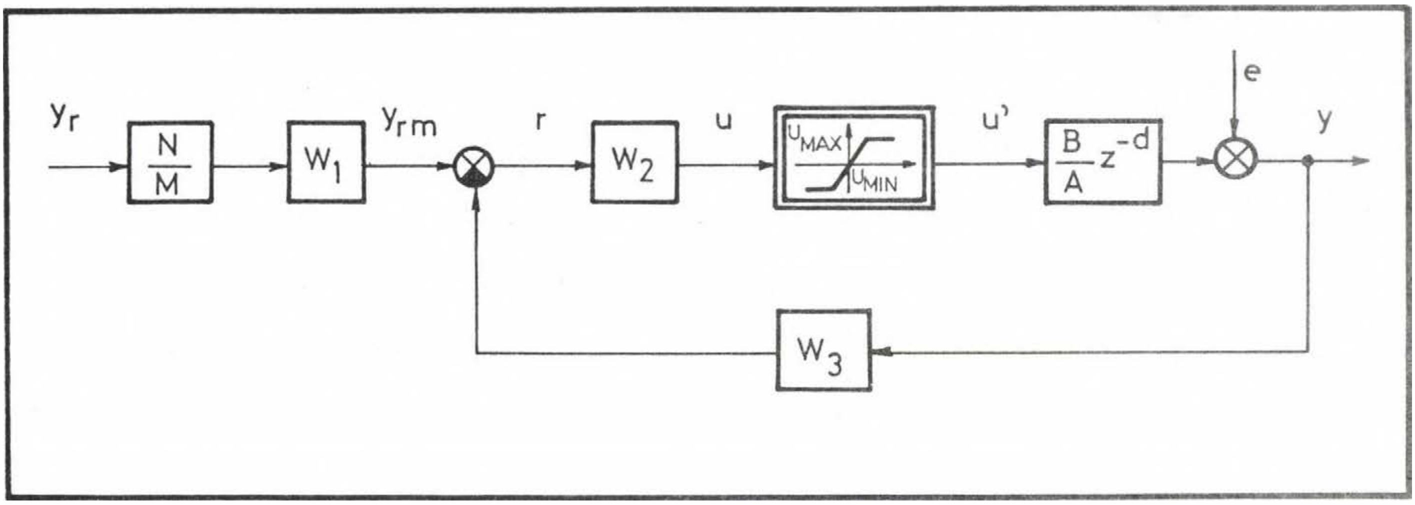

Figure 2-12.

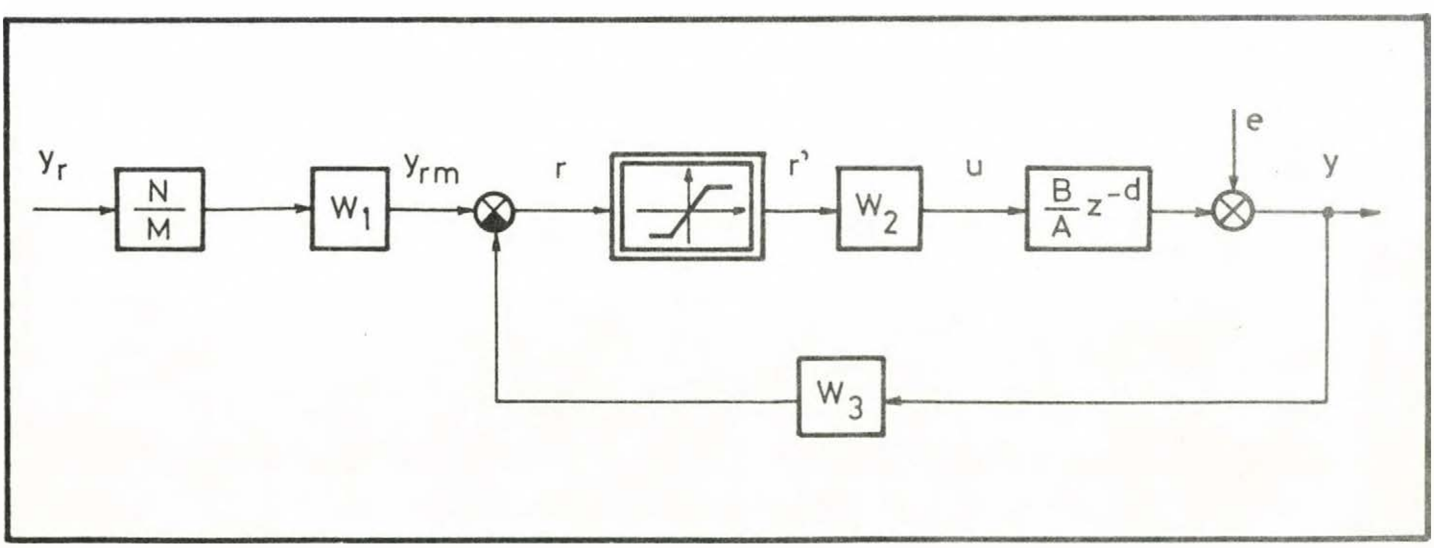

Figure 2-13.

It has turned out that using the self limiting principle - limiting $r(t)$ - the control performance decreases much less than limiting $u(t)$. Moreover, the mentioned transformation of the saturation can be performed in the real-time control program in a very simple way, as it will be shown in the sequel.

Introduce

$$W_2 = \frac{W_2^N}{W_2^D} \tag{2-71}$$

normalised by

$$W_2^N = w_{20}^N + w_{21}^N z^{-1} + \ldots$$

$$W_2^D = 1 + w_{21}^D z^{-1} + \ldots \quad .$$

Thus

$$u(t) = W_2\, r(t) = \frac{W_2^N}{W_2^D}\, r(t) = \underline{p}^T \underline{x}(t), \tag{2-72}$$

where $\underline{p}$ is a vector of the regulator parameters

$$\underline{p}^T = [w_{2o}^N \; w_{21}^N \; \dots \; w_{21}^D \; w_{22}^D \; \dots] \quad , \tag{2-73}$$

while $\underline{x}$ (t) is a memory vector

$$\underline{x}^T (t) = [r(t) \; r(t-1) \; \dots \; -u(t-1) \; -u(t-2) \dots] . \tag{2-74}$$

If u(t) $\{U_{MIN} , U_{MAX}\}$ an r'(t) is to be recalculated as a value producing a control input just on the given limit (U_{MIN} cr U_{MAX}):

$$r'(t) = \frac{U_M - \underline{\tilde{p}}^T \underline{\tilde{x}}(t)}{w_{20}^N} \quad , \tag{2-75}$$

where

$$\underline{p}^T = [w_{2o}^N \; \underline{\tilde{p}}^T] \tag{2-76}$$

and

$$\underline{x}^T(t) = [r(t) \; \underline{\tilde{x}}^T(t)] \quad . \tag{2-77}$$

In the memory vector this recalculated value and the actual limit (U_{MIN} or U_{MAX}) have to be shifted when preparing the next step. Thus at (t+1) the control input is determined by

$$u(t+1) = \underline{p}^T \underline{x}\,(t+1) \quad , \tag{2-78}$$

where

$$\underline{x}^T(t+1) = [r(t+1) \quad r'(t) \ \dots \ -U_M \ -u(t-1) \dots] \ . \tag{2-79}$$

The limit checking of u(t+1) has to be done in the same way. It is seen that the limited value of r(t) is a function of not only U_{MIN} and U_{MAX}, but also that of the situation determined by t.

3. THE ADAPTIVE DETERMINISTIC SERVO PROBLEM

In the previous chapter some suboptimal control algorithms for the deterministic servo problem were presented in a unified form.
Suboptimality was just introduced in order to take into consideration the uncertainty in the knowledge of the process parameters given by an identification procedure. At the same time it was shown that the design polynominals included by the control algorithms give a wide range for the design of the closed loop performance.

The adaptive schemes are classified [7] as explicit and implicit methods, depending on the previous identification of the process. In this chapter both implicit and explicit schemes will be considered for the deterministic servo problem.

3.1 Implicit (direct) methods

Based on the general design principles given previously several implicit adaptive schemes will be summarized in a uniform way, and a new derivation of a direct algorithm [9] will be presented, as well. All the methods to be considered aim to give an estimation for the controller parameters of the α structure defined in the former chapter. The estimation procedures will not be considered in details here, suggestions for using the recursive least squares method [9, 16] or its modified version [36] as well as extended least squares [17] are well known.

As an introduction recall the polynomial equation of the α design structure

$$T = AF+B_2GVz^{-d} \quad . \tag{3-1}$$

Combining the above equation with the process equation

$$Ay(t+d) = Bu(t)$$

we have a predictive form for $Ty(t+d)$

$$Ty(t+d) = B_2 \; [B_1Fu(t)+GVy(t)] \quad .$$

It is seen that this predictive form is bilinear in the parameters. Define $y^F(t)$ as a filtered output by the design polinomial V

$$y^F(t) = Vy(t) \quad ,$$

thus

$$Ty(t+d) = B_2 \; [B_1 Fu(t) + Gy^F(t)] \qquad (3-2)$$

serves as a base for the further considerations.

Direct algorithm by Åström [9]

Define $h(t)$ by

$$\begin{aligned} h(t) &= B_1 F\, u(t) + G\, y^F(t) \\ &= Q\, u(t) + G\, y^F(t) \quad , \qquad (3-3) \end{aligned}$$

thus the residual corresponding to the predictive form can be written as

$$\varepsilon(t) = T\,y(t) - B_2\,[B_1 Fu(t-d) + Gy^F(t-d)]$$

$$= T\,y(t) - B_2 h(t-d) \quad . \qquad (3\text{-}4)$$

Introducing the parameter vector

$$\underline{p}^T = [q_o \;\cdots\; q_{m_2+n_v+d-1}\; g_o \;\cdots\; g_{n_G}\; b_{2o} \cdots\; b_{2,m_2}]$$

and the memory vector

$$\underline{x}^T(t) = [B_2 u(t) \ldots B_2 u(t-m_2-n_v-d+1)\, B_2 y^F(t) \ldots B_2 y^F(t-n_G)$$

$$h(t) \;\ldots\; h(t-m_2)]$$

the following recursive estimation is suggested [9] for the parameters:

$$\hat{\underline{p}}_t = \hat{\underline{p}}_{t-1} + \underline{\underline{R}}_t\, \underline{x}(t)\, \varepsilon(t) \quad , \qquad (3\text{-}5)$$

where

$$\underline{\underline{R}}_t = \frac{1}{\lambda}\left\{ \underline{\underline{R}}_{t-1} - \frac{\underline{\underline{R}}_{t-1}\, \underline{x}(t-d)\, \underline{x}^T(t-d)\, \underline{\underline{R}}_{t-1}}{\lambda + \underline{x}^T(t-d)\, \underline{\underline{R}}_{t-1} \underline{x}(t-d)} \right\}$$

includes a forgetting factor $0<\lambda\leqq 1$.

<u>Quasi-direct algorithm by Åström [9]</u>

Estimate the process parameters using the residuals

$$\varepsilon_1(t) = y(t) - \hat{B}u(t-d) - \hat{\hat{A}}y(t-1) \quad , \qquad (3-6)$$

where

$$\hat{A} = 1 + \hat{\hat{A}}\, z^{-1} \quad .$$

Factorizing $\hat{B}$ by $\hat{B}=\hat{B}_1\hat{B}_2$ the parameters of the regulator polynomials Q and G can be estimated using the residuals

$$\varepsilon_2(t)=Ty(t)- [\hat{Q}\hat{B}_2u(t-d)+\hat{G}\hat{B}_2y^F(t-d)]$$

$$=Ty(t)- [\hat{Q}u^F(t-d)+\hat{G}y^{FF}(t-d)]$$

$$=Ty(t)-\underline{p}^T\underline{x}(t-d) \qquad (3-7)$$

where

$$\underline{p}^T= [q_o\ldots q_{m_2+n_v+d-1}\ g_o\ldots g_{n_G}]$$

and

$$\underline{x}^T(t) = [u^F(t) \ldots u^F(t-m_2-n_V-d+1) \; y^{FF}(t) \ldots y^{FF}(t-n_G)] \quad .$$

Quasi-direct algorithm by Lozano and Landau [36]

Choosing

$$B_2 = \frac{B}{B(1)}$$

and

$$B_1 = B(1)$$

no process zeros are to be cancelled. As $d>0$, it follows from the polynomial equation

$$T = AF + B_2 GVz^{-d}$$

that

$$f_o = t_o$$

where

$$T = t_o + \tilde{T}z^{-1}$$

and

$$F = f_o + \tilde{F}z^{-1} \quad .$$

Now the residual has the following form

$$\varepsilon(t) = Ty(t) - \frac{B}{B(1)} [B(1)Fu(t-d) + Gy^F(t-d)]$$

$$= Ty(t) - f_o Bu(t-d) - B\tilde{F}u(t-d-1) - \frac{BG}{B(1)} y^F(t-d)$$

$$= (T - t_o A)y(t) - \tilde{F}Ay(t-1) - \frac{BG}{B(1)} y^F(t-d)$$

$$= (T - t_o A)y(t) - \underline{p}^T \underline{x}(t-1) \quad , \qquad (3\text{-}8)$$

where

$$\underline{p}^T = [f_1 \ldots f_{m_2+n_v+d-1} \quad g_o \ldots g_{n_G}]$$

and

$$\underline{x}^T(t-1) = [Ay(t-1) \ldots Ay(t-m_2-n_v-d+1) \frac{B}{B(1)} y^F(t-d) \ldots$$

$$\ldots \frac{B}{B(1)} y^F(t-m-d)] \quad .$$

Direct algorithm using stochastic approximation

Determine the elements of the parameter vector

$$\underline{p}^T = [q_o \ldots q_{m_2+n_v+d-1} \; g_o \ldots g_{n_G} \; b_{2o} \ldots b_{2,m_2}]$$

by minimizing the loss function

$$\underline{L}(\underline{p}) = \frac{1}{2} \underline{\varepsilon}_t^T \underline{\varepsilon}_t \rightarrow \min_{\underline{p}} \quad , \tag{3-9}$$

where

$$\underline{\varepsilon}(t) = T\, y(t) - B_2 \,[Q\, u(t-d) + G y^F(t-d)] \tag{3-10}$$

and

$$\underline{\varepsilon}_t^T = [\, \varepsilon(1) \quad \varepsilon(2) \; \ldots\ldots \; \varepsilon(t)] \quad .$$

Using the canonical form of stochastic approximation [45]

$$\underline{p}_t = \underline{p}_{t-1} - \underline{\underline{H}}^{-1} (L, \underline{p}_{t-1}) \, \underline{\nabla}_p \, L(\underline{p}_{t-1}) \tag{3-11}$$

where

$$\underline{\nabla}_{\underline{p}} \quad L\ (\underline{p}) = \frac{dL(\underline{p})}{d\ \underline{p}} = \frac{d\ \underline{\varepsilon}_t^T}{d\ \underline{p}^T}\ \underline{\varepsilon}_t = \underline{\underline{J}}^T\ (\underline{\varepsilon}_t,\ \underline{p})\ \underline{\varepsilon}_t$$

(3-12)

and

$$\underline{\underline{H}}\ (L,\underline{p}) = \frac{d}{d\underline{p}}\ \frac{dL(\underline{p})}{d\ \underline{p}^T}$$

$$= \underline{\underline{J}}^T(\underline{\varepsilon}_t\ ,\ \underline{p}\ )\ \underline{\underline{J}}(\ \underline{\varepsilon}_t,\ \underline{p}) + \frac{d\ \underline{\underline{J}}^T(\underline{\varepsilon}_t,\ \underline{p})}{d\ \underline{p}}\ \underline{\varepsilon}_t$$

$$= \underline{\underline{H}}^{(1)} + \underline{\underline{H}}^{(2)} \qquad (3\text{-}13)$$

To determine $dL/d\underline{p}$ and $d^2L/d\underline{p}\ d\underline{p}^T$ the following derivatives should be taken into consideration:

$$\frac{\partial\ \varepsilon(t)}{\partial q_i} = -\ B_2\ z^{-(d+i)}\ u(t) = -\ u^F\ (t-d-i)\ ;$$

$$\frac{\partial\ \varepsilon(t)}{\partial g_i} = -B_2\ z^{-(d+i)}\ \ y^F(t) = -\ y^{FF}\ (t-d-i)\ ;$$

$$\frac{\partial\ \varepsilon(t)}{\partial b_{2i}} = -z^{-(d+i)}\ [Q\ u(t) + G\ y^F\ (t)] = -\ h(t-d-i)\ ;$$

$$\frac{\partial^2 \varepsilon(t)}{\partial q_i \, \partial q_j} = 0; \quad \frac{\partial^2 \varepsilon(t)}{\partial q_i \, \partial g_j} = 0; \quad \frac{\partial^2 \varepsilon(t)}{\partial g_i \, \partial g_j} = 0$$

$$\frac{\partial^2 \varepsilon(t)}{\partial q_i \, \partial b_{2j}} = -z^{-(d+i+j)} \; u(t) = -u(t-d-i-j) \; ;$$

$$\frac{\partial^2 \varepsilon(t)}{\partial g_i \, \partial b_{2j}} = -z^{-(d+i+j)} \; y^F(t) = -y^F(t-d-i-j);$$

$$\frac{\partial^2 \varepsilon(t)}{\partial b_{2i} \, \partial b_{2j}} = 0$$

Using the above equations

$$\frac{dL(\underline{p})}{d\underline{p}} = \begin{bmatrix} \dfrac{\partial L}{\partial q_i} \\ \dfrac{\partial L}{\partial g_i} \\ \dfrac{\partial L}{\partial b_{2i}} \end{bmatrix} = \begin{bmatrix} -\sum_{k=1}^{t} \varepsilon(k) \; u^F(k-d-i) \\ -\sum_{k=1}^{t} \varepsilon(k) \; y^{FF}(k-d-i) \\ -\sum_{k=1}^{t} \varepsilon(k) \; h \; (k-d-i) \end{bmatrix}$$

$$\underline{\underline{H}}^{(1)} = \begin{bmatrix} \sum_{k=1}^{t} u^F(k-d-i)u^F(k-d-j) & \sum_{k=1}^{t} u^F(k-d-i)y^{FF}(k-d-j) & \sum_{k=1}^{t} u^F(k-d-i)h(k-d-j) \\ \sum_{k=1}^{t} y^{FF}(k-d-i)u^F(k-d-j) & \sum_{k=1}^{t} y^{FF}(k-d-i)y^{FF}(k-d-j) & \sum_{k=1}^{t} y^{FF}(k-d-i)h(k-d-j) \\ \sum_{k=1}^{t} h(k-d-i)u^F(k-d-j) & \sum_{k=1}^{t} h(k-d-i)y^{FF}(k-d-j) & \sum_{k=1}^{t} h(k-d-i)h(k-d-j) \end{bmatrix}$$

$$H^{(2)} = \begin{bmatrix} \underline{0} & \underline{0} & -\sum_{k=1}^{t} u(k-d-i-j)\ \varepsilon(k) \\ \underline{0} & \underline{0} & -\sum_{k=1}^{t} y^F(k-d-i-j)\ \varepsilon(k) \\ -\sum_{k=1}^{t} u(k-d-i-j)\ \varepsilon(k) & -\sum_{k=1}^{t} y^F(k-d-i-j)\ \varepsilon(k) & \underline{0} \end{bmatrix}$$

Approximating $\underline{\underline{H}} = \underline{\underline{H}}^{(1)} + \underline{\underline{H}}^{(2)}$ by $\underline{\underline{H}} \cong \underline{\underline{H}}^{(1)}$ $\underline{\underline{H}}_t^{-1}$ can be expressed in a recursive way:

$$\underline{\underline{H}}_t^{-1} = [\underline{\underline{H}}_{t-1} + \underline{x}(t-d)\ \underline{x}^T(t-d)]^{-1}$$

$$= \underline{\underline{H}}_{t-1}^{-1} - \frac{\underline{\underline{H}}_{t-1}^{-1}\ \underline{x}(t-d)\ \underline{x}^T(t-d)\ \underline{\underline{H}}_{t-1}^{-1}}{1 + \underline{x}^T(t-d)\ \underline{\underline{H}}_{t-1}^{-1}\ \underline{x}(t-d)} \qquad (3\text{-}14)$$

where

$$\underline{x}^T(t) = [\ldots.\ u^F(t-i)\ \ldots.\ y^{FF}(t-i)\ \ldots.\ h(t-i)\ \ldots.] \quad .$$

Using the recursive form of $\underline{\underline{H}}_t^{-1}$ and

$$\frac{dL\ (\underline{p}_t)}{d\underline{p}_t} = \frac{dL\ (\underline{p}_{t-1})}{d\ \underline{p}_{t-1}} - \varepsilon(t)\ \ \underline{x}(t-d) \qquad (3\text{-}15)$$

after some computation we obtain

$$\hat{\underline{p}}_t = \hat{\underline{p}}_{t-1} + \underline{\underline{H}}_t^{-1}\ \underline{x}(t-d)\ \varepsilon(t) \qquad (3\text{-}16)$$

which is actually the direct algorithm by Åström [9] .

A more accurate form for $\underline{\underline{H}}_t^{-1}$ can be obtained if $\underline{\underline{H}}^{(2)}$ is not neglected. To show how to update $\underline{\underline{H}}_t^{-1}$ recursively in this case first a recursive updating of $\underline{\underline{H}}_t$ is given by

$$\underline{\underline{H}}_t = \underline{\underline{H}}_{t-1} + \begin{bmatrix} u^F(t-d-i) \\ y^{FF}(t-d-i) \\ h(t-d-i) \end{bmatrix} [u^F(t-d-j) \quad y^{FF}(t-d-j) \quad h(t-d-j)] \; +$$

$$\begin{bmatrix} u(t-d-i-j) \\ y^F(t-d-i-j) \\ 0 \end{bmatrix} [0 \quad 0 \quad -\varepsilon(t)] + \begin{bmatrix} 0 \\ 0 \\ -\varepsilon(t) \end{bmatrix} [u(t-d-i-j) \quad y^F(t-d-i-j) \quad 0]$$

Having the above form of $\underline{\underline{H}}_t$ the tripple use of the matrix inversion lemma

$$(\underline{\underline{A}} + \underline{\underline{B}}\ \underline{\underline{C}}\ \underline{\underline{D}})^{-1} = \underline{\underline{A}}^{-1} - \underline{\underline{A}}^{-1}\ \underline{\underline{B}}\ (\underline{\underline{C}}^{-1} + \underline{\underline{D}}\ \underline{\underline{A}}^{-1}\ \underline{\underline{B}})^{-1}\ \underline{\underline{D}}\ \underline{\underline{A}}^{-1}$$

with $\underline{\underline{C}} = 1$ leads to a recursive form of $\underline{\underline{H}}_t$.

3.2 Explicit (indirect) methods

The explicit adaptive schemes are based on the model of the process to be controlled. The parameters of the process model are usually updated by an appropriate on-line identification procedure. As the excitation for the identification is generated by the regulator, the identification has to be performed in the closed loop. Thus first the conditions for the closed loop identifiability will be considered.

Assume that the process is described by

$$y(t) = \frac{B}{A}\ u(t-d) \tag{3-17}$$

where

$$A = 1 + a_1 z^{-1} + \ldots + a_n z^{-n}$$

$$B = b_o + b_1 z^{-1} + \ldots + b_m z^{-m}$$

$$d > 0$$

and the regulator is given by

$$u(t) = \frac{P}{Q}\,[y_r(t) - y(t)] \tag{3-18}$$

where

$$P = p_o + p_1 z^{-1} + \ldots + p_{n_P} z^{-n_P}$$

$$Q = q_o + q_1 z^{-1} + \quad + q_{n_Q} z^{-n_Q} \quad .$$

The memory vector for the explicit identification is

$$\underline{x}_I^T(t)=[u(t-d)\ u(t-d-1)\ \ldots\ u(t-d-m)\ -y(t-1)\ -y(t-2)$$

$$\ldots\ -y(t-n)] \tag{3-19}$$

and thus the residual can be expressed by

$$\varepsilon(t) = y(t) - \underline{x}_I^T(t)\ \underline{p}_p$$

where $\underline{p}_p$ stands for the process parameters

$$\underline{p}_p^T = [b_o\ \ b_1\ \ldots\ b_m\ a_1\ a_2\ \ldots\ a_n] \quad .$$

As long as the changes in the set point y_r (reference signal) produce sufficient perturbation there are no difficulties when estimating the process parameters. For lack of external perturbation ($y_r=0$) Eq. (3-18) gives

$$u(t) = \frac{-P}{Q}\, y(t) = \frac{-1}{q_o}\, \underline{x}_c^T(t)\, \underline{p}_c \tag{3-20}$$

where

$$\underline{x}_c^T(t) = [y(t)\ y(t-1) \ldots y(t-n_P)\ \ u(t-1) \ldots u(t-n_Q)] \tag{3-21}$$

and

$$\underline{p}_c^T = [p_o\ \ p_1\ \cdots\ p_{n_P}\ \ \ q_1\ \cdots\ q_{n_Q}] \ .$$

Now using Eq. (3-19) and Eq. (3-20)

$$\underline{x}_I^T(t) = [\ \frac{-1}{q_o}\, \underline{x}_c^T\ (t-d)\ \underline{p}_c\ u(t-d-1) \ldots u(t-d-m)\ -y(t-1)$$

$$\ldots\ldots\ -y(t-n)] \ .$$

It is seen that the first element of $\underline{x}_I(t)$ depends on observation not appearing in $\underline{x}_I(t)$, if

$$m < n_Q \tag{3-22}$$

or

$$n < n_p + d \quad . \tag{3-23}$$

This means that if the condition by Eqs.(3-22) or (3-23) are fulfilled then the process can be identified in closed loop [29]. Table 2-I. shows that the closed loop identifiability conditions are met in general, however for processes with integrator(s) and $d=1$ the α and γ stratigies can be used only with $n_v > 0$.

In this work the identification step is not considered in details. The recursive version of the least squares method usually gives acceptable results, but particular care should be taken when updating the weighting matrix used for the parameter estimation [16].

Based on the estimated process parameters the regulator parameters can be computed following the design strategies summarized in Table 2-I. As far as the computational aspects are concerned, the β structures are more advantageous, because no polynomial equation has to be solved when using them.

If the regulator paramteres are determined by solving a polynomial equation (α or γ strategies), the computational efforts can be reduced significantly, if in every sampling and control step only one iteration is done instead of the exact resolution [17].

Consider a general polynomial equation by

$$T = AF + B_2GVz^{-d} = AF + SGz^{-d} \, . \qquad (3\text{-}24)$$

Equating the powers of z^{-1} a set of linear equations is obtained for the regulator parameters

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdot\cdot \\ a_1 & 1 & & & & & & \\ a_2 & a_1 & & & s_o & 0 & & \\ \cdot & \cdot & & & s_1 & s_o & & \\ \cdot & \cdot & & & & & & \\ \cdot & \cdot & & & & & & \\ \cdot & \cdot & & & & & & \\ a_n & a_{n-1} & & & & & & \\ 0 & a_n & & & s_{n_s} & s_{ns-1} & & \\ \cdot & \cdot & & & & & & \\ \cdot & \cdot & & & 0 & s_{n_s} & & \\ \cdot & \cdot & & & & & & \\ 0 & 0 & \cdot\cdot & a_n & 0 & 0 & \cdot\cdot & s_{n_s} \end{bmatrix} \begin{bmatrix} f_o \\ f_1 \\ \vdots \\ \\ g_o \\ g_1 \\ \vdots \\ \\ \end{bmatrix} = \begin{bmatrix} t_o \\ t_1 \\ \\ \vdots \\ \\ \\ 0 \end{bmatrix}$$

where

$$n_s = m_2 + n_v \quad . \tag{3-25}$$

Introducing the vector of the regulator parameters

$$\underline{p}^T = [f_o \; f_1 \; \dots \; g_o \; g_1 \; \dots \;]$$

the previously given equations have the following from

$$\underline{\underline{K}} \; \underline{p} = \underline{t} \tag{3-26}$$

or equivalently

$$\underline{p} = \underline{t} - (\underline{\underline{K}} - \underline{\underline{I}}) \; \underline{p} \quad . \tag{3-27}$$

Based on this latter form a simple recursive form by

$$\hat{\underline{p}}_i = \underline{t} - (\underline{\underline{K}} - \underline{\underline{I}}) \; \hat{\underline{p}}_{i-1} \tag{3-28}$$

can be set up. Using Eq. (3-28) in real-time simulations promising results were obtained, however the convergence proof is missing yet. Note that many other iterative solutions could be taken into consideration, but both the convergence and sesitivity analysis seem to be rather difficult.

4. SIMULATION RESULTS

Simulation has a particular importance in the development of adaptive controllers. However it should be emphasized that the pure discrete simulations for continuous processes have no special importance, because the intersampling behaviour of the closed loop system remains hidden. Using hybrid simulations, where the process was represented by an analogue model and an I8080 based microcomputer with real-time facilities performed the discrete adaptive control task via a zero order holding unit [24], a number of simulation runs were investigated.

In this work only a few real-time experiments will be presented to show the basic advantageous properties of the proposed control algorithms.

Consider a third order continuous process given by

$$\frac{1}{(1+2s)(1+5s)(1+10s)} \quad . \tag{4-1}$$

The step response equivalent discrete transfer function of the above process exhibits critical zeros at the usual sampling rates [33]:

h	z_1	z_2
2 sec	-0.18	-2.52
3 sec	-0.14	-2.12
4 sec	-0.11	-1.79

Qualifying all the process zeros as zeros not to be cancelled the following experiments were done

Figure No.	h	N	M
4-1	2	1	1
4-2	3	1	1
4-3	4	1	1
4-4	2	$0.5+0.5z^{-1}$	1
4-5	4	$0.33+0.33z^{-1}+0.33z^{-2}$	1

The simulation experiments were performed by the explicit β_s structure defined previously.

The process identification was started from zero elements, and the recursive least squares method was used. The Figures clearly show the influence of the sampling time and that of the design polynomial N. At the same time the fast convergence is also seen.

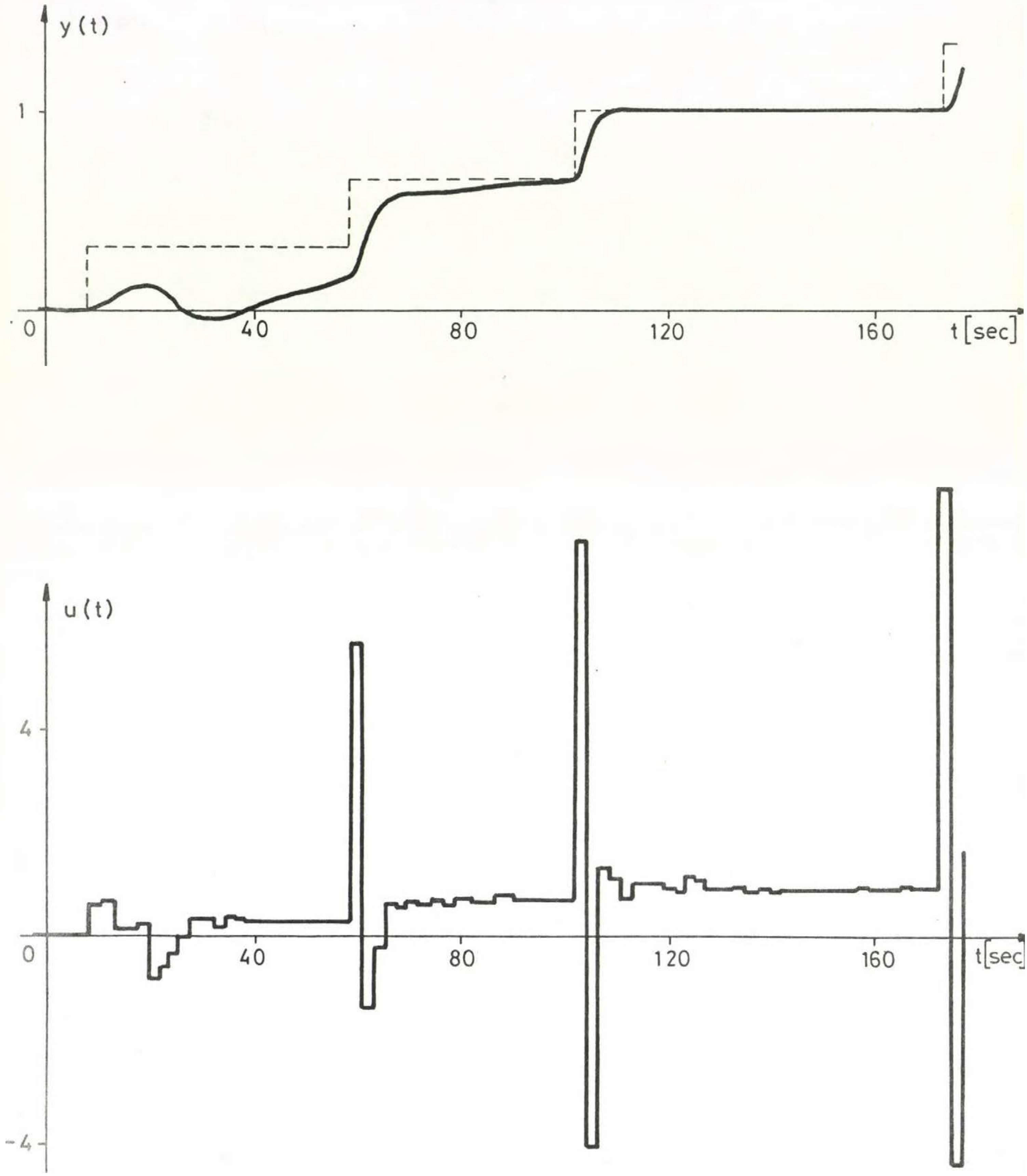

Figure 4-1.

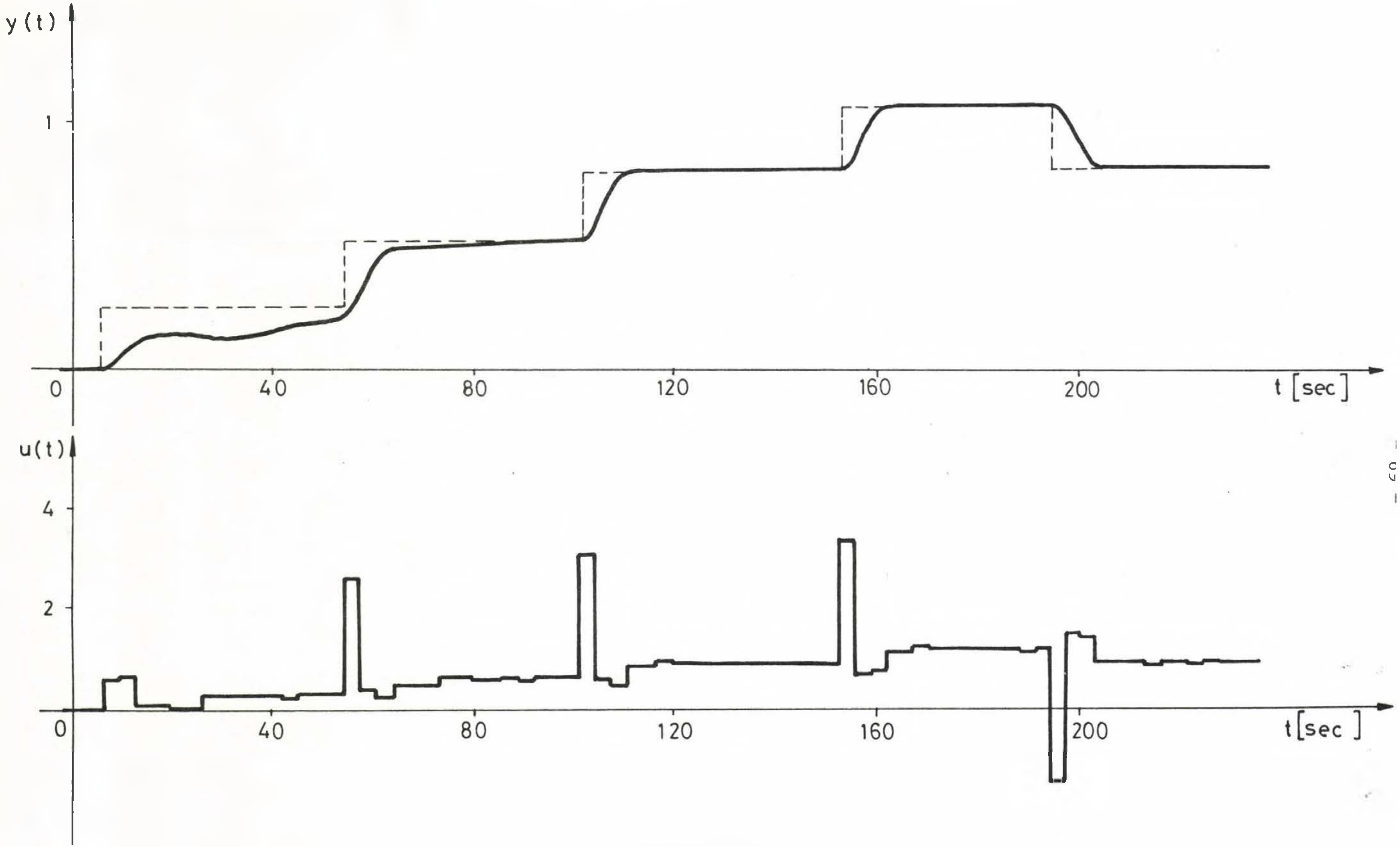

Figure 4-2.

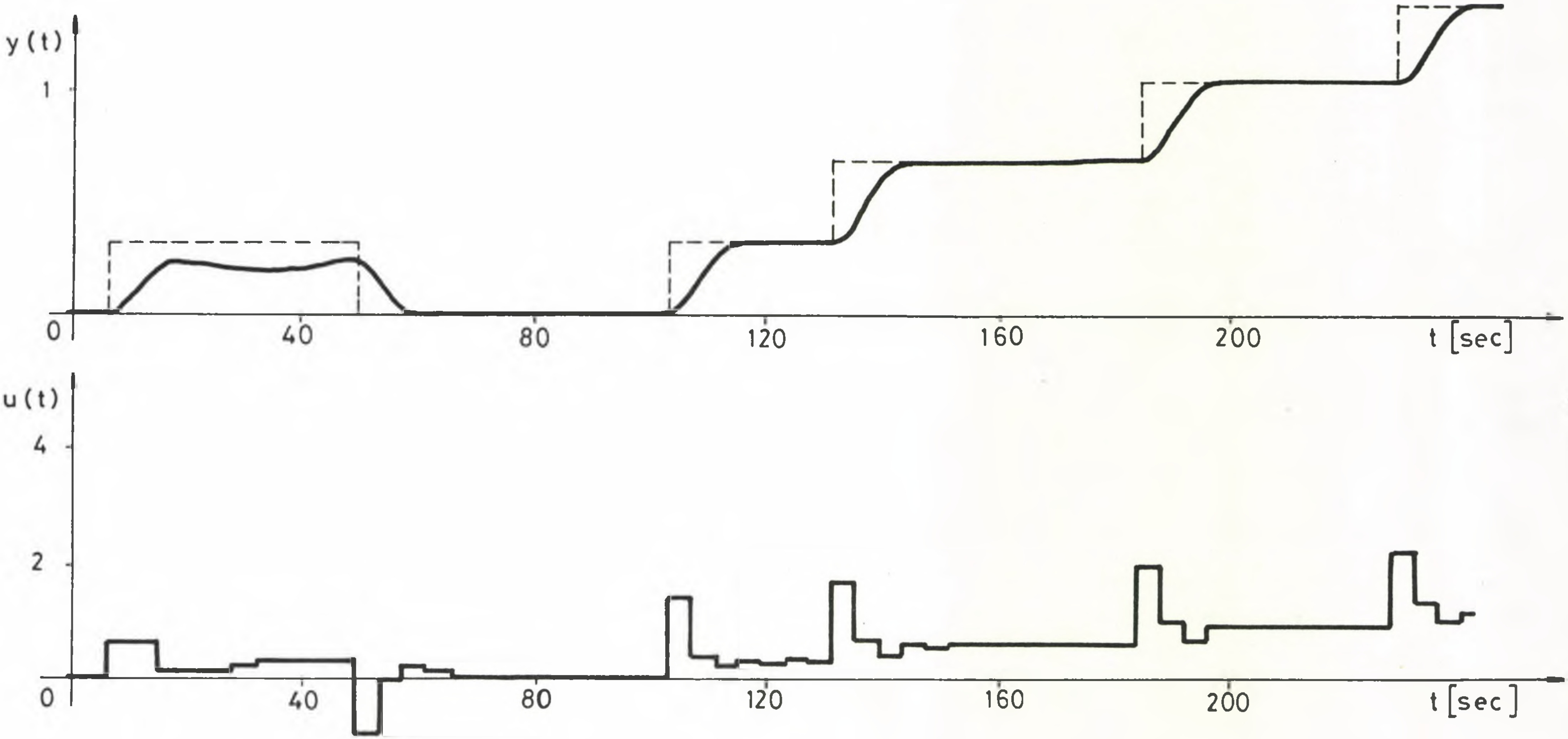

Figure 4-3.

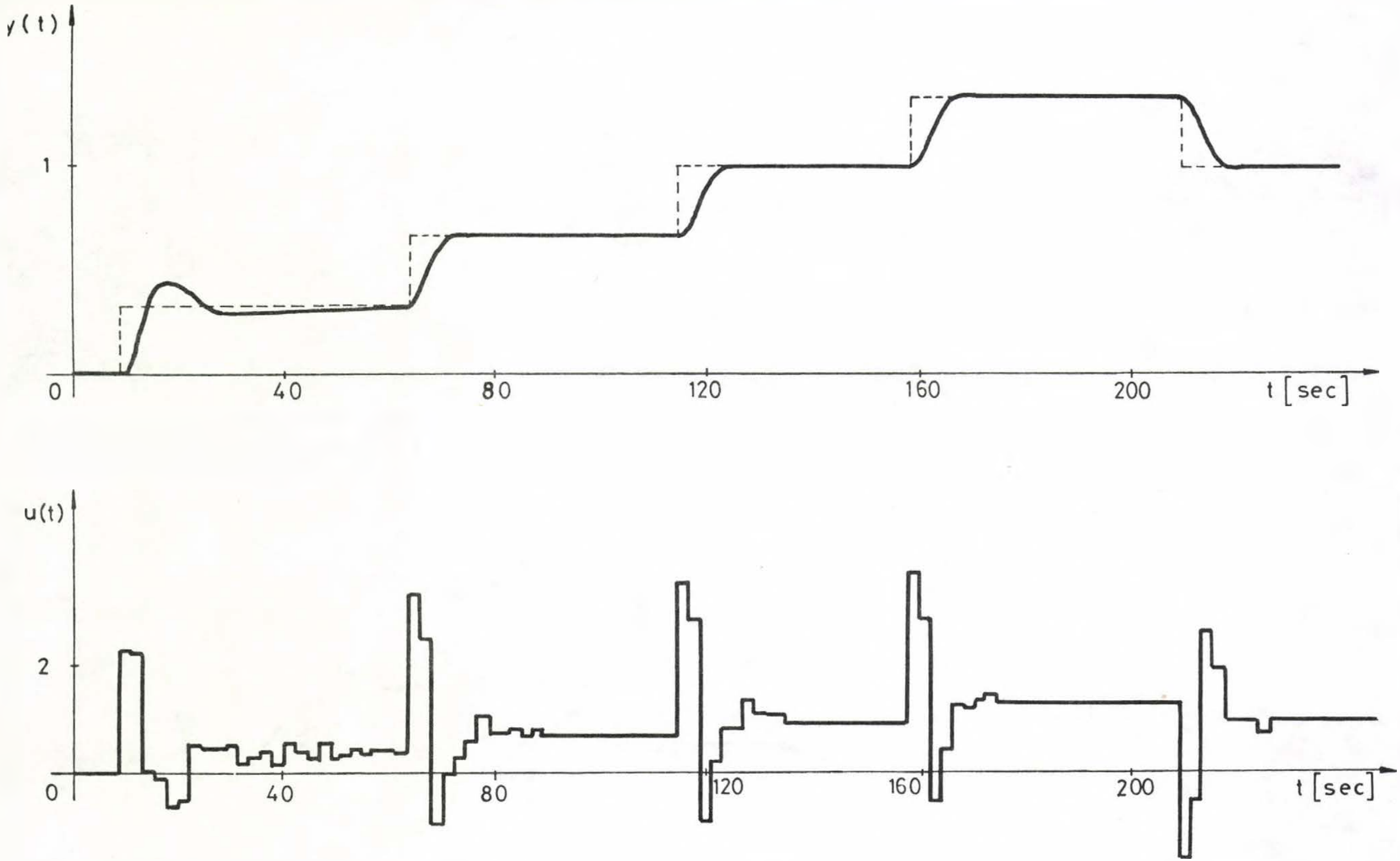

Figure 4-4.

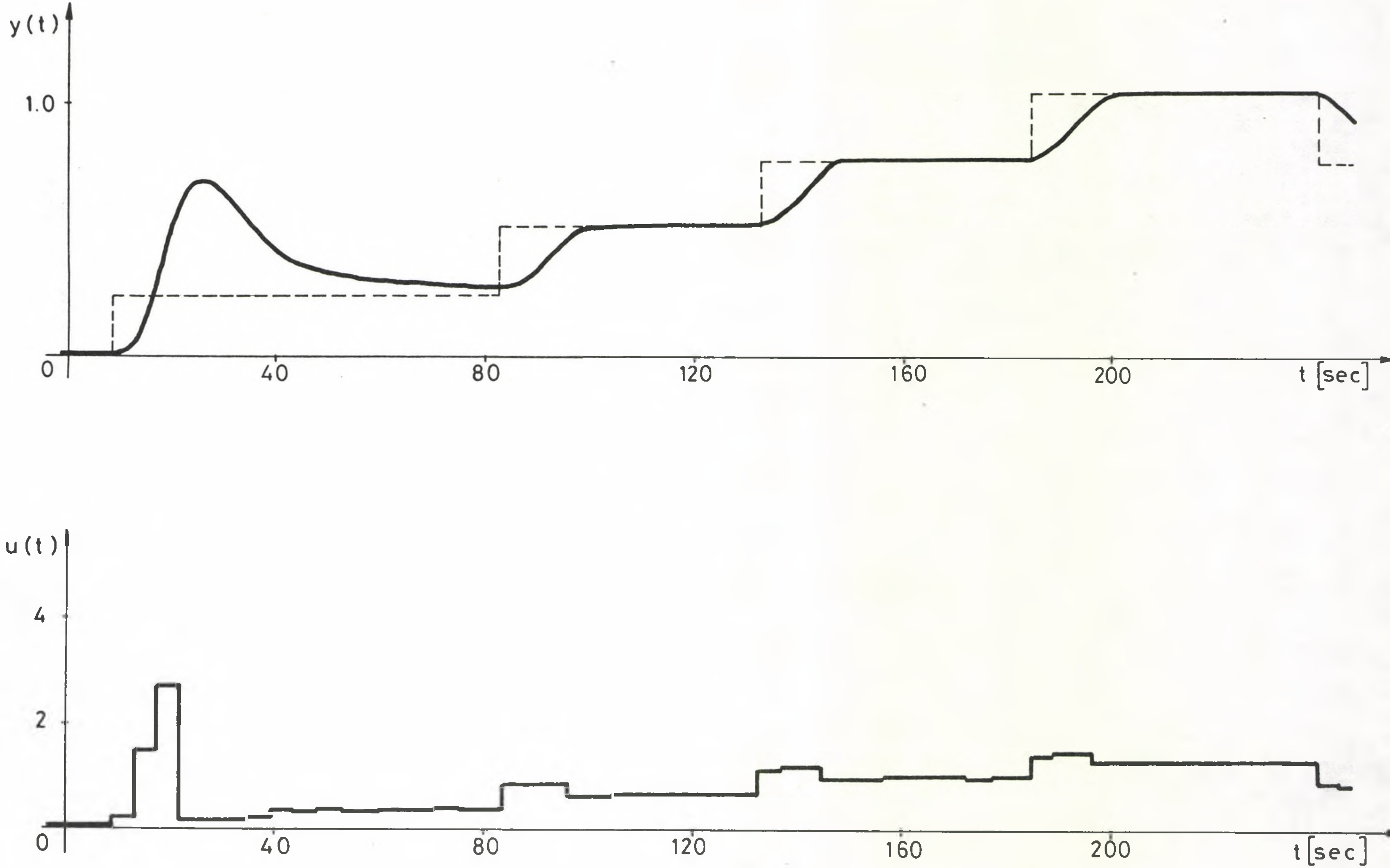

Figure 4-5.

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