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VELOCITY AND CURRENT DETERMINATION FOR STEADILY TRAVELLING DOMAINS IN SEMICONDUCTORS

I. RECOMBINATION INSTABILITY

by

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1. Introduction

It is accepted now, that in systems having negative differential conductivity there is a possibility for electrical domain formation [1] - [2]. In case of recombination instability, the negative differential conductivity is due to the field dependence of the electron capture cross section of multielectron centers. At proper conditions, the domains can steadily travel along the sample, say, from the cathode to the anode, and cause in the external circuit a characteristic oscillation. If we neglect the times of domain formation and disappearence, two parameters seem to be important for characterizing the system. The first parameter is the domain velocity $/u_o/$ which determines the oscillation frequency if the geometry is fixed, the second one is the current $/j_s/$, which characterizes the amplitude of oscillations.

Recently, several papers were published on the theory of domain formation and domain dynamics for recombination instability [3] - [11]. It seems that the conditions for the onset of instability one may calculate from the linear theory correctly, but this is not the case for domain velocity and current, though, since the first observations of current oscillations [12] - [17], new and more precise experimental data on recombination instability are available [18] - [24]. The discrepancy between the experiment and the theory is related to the fact, that the steadily travelling domains cannot be considered as a small perturbation and, on the other hand, in the dynamics of slowly moving domains,

the domain shape, which was taken into account only partly in the existing non-linear theories, plays a non-negligible role.

In the present paper a method is proposed to determine the domain velocity /u_o/ and the current /j_z/ in the presence of steadily travelling domains. The essential point of the method is the determination of charge densities in the conduction band and in the localized states / ϱ_i ; i = 1,2/ as a function of electric field E and its derivatives, or more precisely the functions $\varrho_i(E, E', E'')$ /the $'$ means the differentiation with respect to $z = x - u_o t$/. Then a system of nonlinear <u>algebraic</u> equation for u_o and j_z will be given, introducing the parameters $S_i \equiv \frac{\varepsilon}{4\pi} E_o^{(i)}$ /i = 1,2,..,4./. They characterize the domain shape in the <u>neutrality point</u> where $E'/z_o/ = 0$. We consider the case, when the concentration of empty and filled centers is much larger than the electron concentration in the band. For the determination of possible domain velocity values u_o, an algebraic equation of fourth degree will be derived. The actual domain shape /and hence the domain shape parameters S_i/ may be calculated from a nonlinear differential equation of third order. Instead of numerical or analitical examination of this equation we shall consider /with some arbitrariness/ the following hypothetical domain shapes: flat domains; almost flat domains; symmetrical domains and domains of arbitrary shape. The calculation led to the following results. Several domain modes exist, which, in general, are sensitive to the domain shape, that is they depend on the shape parameters S_i. From the expression of

domain velocity given here, the related expressions of previous papers may be deduced.

On the basis of the available experimental data, it is possible to decide which mode is realized in a given experiment. Starting from the slowest mode and assuming a proper /non-symmetrical/ domain shape, the velocity in correct order of magnitude is found. The current j_s is only slightly sensitive to the domain shape.

2. The basic equations for the problem

Let denote the charge densities in the high and low mobility states ϱ_1 and ϱ_2, respectively. In the case of recombination instability the mobility μ_2 and the diffusion constant D_2 are zero, therefore, using the usual notations* $\mu_1 \equiv \mu$ and $D_1 \equiv D$, the equation of continuity, the Poisson equation and the equation of recombination kinetics may be written in the form [6], [9]:

$$\frac{\partial \varrho}{\partial t} + \frac{\partial j}{\partial x} = 0 \qquad (1)$$

$$j = \varrho_1 \mu E + D \frac{\partial \varrho_1}{\partial x} \qquad (2)$$

$$\frac{\partial E}{\partial x} = \frac{4\pi}{\varepsilon} (\varrho_0 - \varrho_1 - \varrho_2) \equiv \frac{4\pi}{\varepsilon} \varrho \qquad (3)$$

$$\frac{\partial \varrho_2}{\partial t} = C_0 \left[\varrho_1 (N_t - \frac{\varrho_2}{e}) f(E) - n_1 \varrho_2\right] \equiv \Psi(\varrho_1, \varrho_2, E) \qquad (4)$$

where, is the compensating positive charge density; - the dielectric constant; ϱ - the space charge density; C_0 - the capture parameter at zero field; $C/E/ = C_0 f/E/$ - the same in the presence of electric field E; n_1 - the usual

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* For the simplicity's sake, μ and D are assumed to be independent from the field. There is no difficulty to extend the calculation for the field dependent μ and D.

parameter of the S-R-H theory: $n_1 \equiv n_o P_{to}/n_{to}$, where p_{to} and n_{to} are the concentrations of empty and filled centers, resp., and $n_{to} + p_{to} = N_t$, / $\varrho_1 = en$; $\varrho_2 = en_t$/. The background illumination may be taken into consideration by introducing an effective S-R-H factor, defined by $n_1^* = n_1 + \frac{SI}{c_o}$ where S is the photon capture cross section of the recombination centers, I - the photon flux density of incident light [7]. To complete the equation /1/ - /4/ the proper boundary conditions are to be given. Assuming the electrical neutrality of the whole sample, and using the Kirchoff's law, we have [6], [9]:

$$\int_0^L \varrho \, dx = 0 \tag{5a}$$

$$\int_0^L E \, dx + JR = \mathcal{E} \tag{5b}$$

Here L is the length of the sample, J - the cufrent in the circuit of resistivity R, and $\mathcal{E}$ is the e.m.f. of the battery. We note, that if the sample has a net charge [20], the condition /5a/ should be changed.

To describe the steadily travelling domains we shall use the so called Riemann-type solution [7], [9], when the electric field E and the components of space charge density depend only on the variable $z = x - u_o t$, that is, the domain represents a space charge wave propagating with constant velocity u_o. In this case the differentiation rules are $\frac{\partial}{\partial t} = -u_o \frac{d}{dz}$; $\frac{\partial}{\partial x} = \frac{d}{dz}$ and denoting by $'$ the differentiation with respect to the z, one obtains the following system of equations:

$$j' = u_o \varrho' \tag{6}$$

$$j = u_o \varrho + j_s \tag{7}$$

$$E' = \frac{4\pi}{\varepsilon}(\varrho_o - \varrho_1 - \varrho_2) \equiv \frac{4\pi}{\varepsilon}\varrho \tag{8}$$

$$-u\varrho_2' = \Psi(\varrho_1, \varrho_2, E) \tag{9}$$

Integrating the equ./6/ it was taken into account, that for the wave-like solution the total current /j_s/ is constant [9]:

$$j_s \equiv j + \frac{\varepsilon}{4\pi}\frac{\partial E}{\partial t} = \text{const.}$$

To simplify the further calculations we shall linearize the function Ψ in the equ. /9/ with respect to the concentration, but not to the electric field. To do this, we assume that the concentration of empty and filled centers is much larger than the electron concentration in the band. This condition is fulfilled almost in every experiments on Au and Cu centers /except for perhaps the case of too large background illumination/. We have then, with the notation $\Psi \equiv \Psi_\ell$, for the kinetics of recombination:

$$-u_o \varrho_2' = \Psi_\ell \equiv \frac{1}{\tau_r}\varrho_1 - \frac{1}{\tau_g}\varrho_2 \qquad (10)$$

where τ_r and τ_g are the characteristic times of recombination and generation, respectively:

$$\tau_r = \frac{1}{C_o p_{to} f(E)} \; ; \quad \tau_g = \frac{1}{C_o [n_1 + n_o f(E)]} \approx \frac{1}{C_o n_1}. \qquad (10)'$$

The last approximation means that the field dependence of τ_g is neglected [4].

3. Determination of the functions /E,E',E''/ and the differential equation for field E/z/.

The charge densities ϱ_i/i = 1,2/ and the electric field strength E are connected by the differential equation system /6/, /7/, /8/ and /10/. As basic function, it is suitable to choose the field E, expressing the other quantities in term of the field and its derivatives. The only reason of doing so is that the electric field is generally known from experiments. In order to determine

the function $\varrho_i(E, E', E'')$ one obtains, using equ./8/ repeatedly, the following system of equations:

$$\begin{aligned} \varrho_1 + \varrho_2 &= T \\ \varrho_1 \mu E + D\varrho_1' &= \frac{u_0 \varepsilon E'}{4\pi} + j_s \\ -u_0 \varrho_2' &= \frac{\varrho_1}{\tau_+} - \frac{\varrho_2}{\tau_-} \\ \varrho_1' + \varrho_2' &= T' \end{aligned} \qquad (11)$$

where $T \equiv \varrho_0 - \frac{\varepsilon E'}{4\pi}$ is the total negative charge density. After simple calculation we obtain:

$$\varrho_1 = \frac{R + TM}{L + M} \; ; \qquad \varrho_2 = \frac{-R + TL}{L + M} \qquad (12)$$

where:

$$L = \mu E u_0 + \frac{D}{\tau_+} \; ; \quad M = \frac{D}{\tau_-}$$

$$R = u_0 \left(j_s + \frac{u_0 \varepsilon E'}{4\pi} + \frac{D\varepsilon E''}{4\pi} \right) \qquad (12')$$

The equs./12/ in fact give the expressions $G_i \equiv \varrho_i$ /E,E,E/ with the unknown parameters /integration constants/ u_0 and j_s. It is quite easy now to write the differential equation for E/z/, using equs. /12/ and /7/. We have then:

$$F(E, E', E'', E''') \equiv G_1 \mu E + D \sum_{i=0}^{2} \frac{\partial G_1}{\partial E^{(i)}} E^{(i+1)} - j_s - \frac{u_0 \varepsilon E'}{4\pi} = 0. \qquad (13)$$

Instead of analitical or numerical investigation of this differential equation, our purpose is to determine the domain velocity u_0 and current j_s assuming the electric field and its derivatives to be known, say, in the point of electrical neutrality defined evidently by $E'/z_0/ = 0$.

4. Determination of u_0 and j_s for domains of arbitrary shape

The calculation is based on the equs./6/ and /7/. In fact, if $\varrho_i(z)$ and $E(z)$ were known the velocity and current could be determined from these equations. Let us introduce

the domain shape parameters $S_i \equiv \frac{\varepsilon}{4\pi} E_o^{(i)}$, which characterize the domain shape in the neutrality point $z = z_o$. Then we have

$$j_s = \varrho_{1o} v_o + D \varrho'_{1o} \tag{14}$$

$$S_2 u_o = \varrho'_{1o} v_o + D \varrho''_{1o} \tag{15}$$

The index "o" means from now on that the function should be taken in the neutrality point. It is easy now to determine the mobile charge density and its derivatives in the point $z = z_o$, using equation /12/. Calculating u_o and j_s the following notations were introduced:

$$\frac{1}{\tau} = \frac{1}{\tau_+} + \frac{1}{\tau_g}\ ; \quad \gamma = -\frac{E_o}{\tau_{+o}} \left(\frac{d\tau_+}{dE}\right)_o\ ; \quad \tau_{Mo} = \frac{\varepsilon}{4\pi \varrho_o \mu}\ ; \quad v_o = \mu E_o\ ;$$

then we have

$$M_o + L_o = u_o v_o + \frac{D}{\tau_o}\ ; \quad L''_o = \left(\mu u_o + \frac{\gamma D}{E_o \tau_{+o}}\right) E''_o\ ; \quad M' = 0,$$

where the last equation is the consequence of the assumption /10/. For the current one obtains from equ. /14/:

$$j_s = \frac{\tau_o}{\tau_g} v_o \varrho_o - \frac{\tau_o}{\tau_g} D S_2 + \tau_o u_o \left(v_o S_2 + D S_3\right) + \tau_o u_o^2 S_2 \tag{16}$$

A simple but longer calculation gives the algebraic equation for domain velocity:

$$b_3 u_o^3 + b_2 u_o^2 + b_1 u_o + b_o = 0 \tag{17}$$

where

$$b_3 = v_o S_3$$

$$b_2 = \frac{D}{\tau_o} S_3 + v_o P - \frac{4\pi\mu S_2}{\varepsilon} \left(j_s + D S_2\right)$$

$$b_1 = \frac{D}{\tau_o} P - v_o K - S_2 \left[\frac{D}{\tau_g \tau_{Mo}} + \frac{4\pi\mu \gamma D}{\varepsilon v_o \tau_{+o}} \left(j_s + D S_2\right)\right]$$

$$b_o = -\frac{D}{\tau_o} K - \frac{S_2 \gamma D^2}{v_o \tau_{+o} \tau_g \tau_{Mo}}$$

and here

$$P = -\frac{S_2}{\tau_o} + v_o S_3 + D S_4$$

$$K = \frac{v_o}{\tau_g} S_2 + \frac{D}{\tau_g} S_3$$

Evidently, equs. /16/ and /17/ are coupled algebraic equations for the velocity and the current in terms of domain shape parameters S_i. As one can easily see, if j_s is inserted in b_1 and b_2 the system leads to an algebraic equation of fourth degree for u_o alone. Doing this calculation and, at the same time, introducing the dimensionless velocity $x \equiv u_o/w_o$, where by definition

$$w_o = \frac{D}{v_o \tau_o} \tag{18}$$

one obtains

$$a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_o = 0 \tag{19}$$

and the coefficients are

$$a_4 = -\alpha_o \alpha_1$$

$$a_3 = -\alpha_o(1+\alpha_3) - \alpha_o \alpha_1 \alpha_+ \gamma + \alpha_1 \alpha_3$$

$$a_2 = -\alpha_1 K_1 - \alpha_o \alpha_+ - \alpha_o \alpha_+ \gamma (1+\alpha_3) + \alpha_1 \alpha_3$$

$$a_1 = -\alpha_1 K_1 - \alpha_g K_2 - \alpha_o \alpha_+^2 \gamma$$

$$a_o = -\alpha_g K_2$$

where

$$\alpha_o = \frac{E_o'' D^2}{E_o v_o^2}; \quad \alpha_1 = \frac{D}{v_o^2 \tau_o}; \quad \alpha_2 = v_o \tau_o \frac{S_3}{S_2}; \quad \alpha_3 = \frac{D S_3}{v_o S_2};$$

$$\alpha_4 = D \tau_o \frac{S_4}{S_2}; \quad \alpha_+ = \frac{\tau_o}{\tau_{+o}}; \quad \alpha_g = \frac{\tau_o}{\tau_g}; \quad \alpha_M = \frac{\tau_o}{\tau_{Mo}}$$

$$K_1 = 1 - \alpha_2 - \alpha_4 + \alpha_g \alpha_M; \quad K_2 = 1 + \alpha_3 + \alpha_1 \alpha_+ \alpha_M \gamma.$$

Knowing the above parameters one may determine the possible values of domain velocity. The model, adopted in the present paper, gives four domain modes. It was tacitly assumed, that $S_2 \neq 0$. If $S_2 = 0$ one has to solve the equ. /17/ directly. The equ. /19/ may be used for the exact numerical determination of domain velocity. It is more convenient, however, to give approximative formulae for domain velocities, as they allow to draw some useful conclusions on the behaviour of slow domains.

5. Approximative expressions for j_s and u_o.

It is easy to see that at usual values of parameters* in equ. /19/, the roots are well separated, i.e. $|x_4| \gg |x_3| \gg |x_2| \gg |x_1|$. In this case the following approximation holds for x_i:

$$x_i \approx - \frac{a_{i-1}}{a_i} \qquad (20)$$

Using equ./20/ we may obtain the approximative roots immediately. In what follows, however, we shall use a plausible physical approximation, namely, the diffusion component of the current $D\varrho'_{10}$ will be neglected compared to the drift term $\varrho_{10} v_o$ in equ./14/. Then, instead of /16/ we have:

$$j_s = \frac{\tau_o}{\tau_g} v_o \varrho_o + v_o u_o v_o S_2 \qquad (21)$$

In connection with above approximation the following remark is necessary. The neglect of $D\varrho'_{10}$ is not equivalent with the limit $D \to 0$, because the derivative ϱ'_{10} contains the

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* To estimate the order of magnitude of the parameters in question, one may use the following values: $E_o = 10^3\ Vcm^{-1}$; $\mu = 5 \cdot 10^{-12}\ cm^2V^{-1}sec^{-1}$; $D = 10\ cm^2sec^{-1}$; $\varrho_o/e = 10^{15}\ cm^{-3}$; $\tau_{No} = 2 \cdot 10^{-12}$ sec; $d = 10^{-2}$ cm; $S_3/S_2 = -2/d$; $S_4/S_2 = 3/d^2$; /see equ./28/ for r=1/, $\tau_{ro} = 10^{-5}$ sec; $\tau_g = 1$ sec; $\gamma = 2$. The parameters of recombination centers refer to $Au^=$ at about 30°K. [18] [19] [24] The value of γ is correct both for $Au^=$ and $Cu^=$ centers [22]. Then we have: $\alpha_o = 4 \cdot 10^{-8}$; $\alpha_1 = 4 \cdot 10^{-8}$; $\alpha_2 = -10^4$; $\alpha_3 = -4 \cdot 10^{-4}$; $\alpha_4 = 3$; $\alpha_g = 10^{-5}$; $\alpha_M = 5 \cdot 10^6$; $\alpha_r = 1$; $K_1 = 10^4$; $K_2 = 1{,}4$.

diffusion constant D and the current j_g implicitly. Indeed, if we compare the expressions /16/ and /21/, it is seen that in equ. /21/ the term with u_o^2 does not appear, though it does not contain the diffusion constant. Actually, the neglect of term in question involves two approximations: the neglect of diffusion and the assumption $\left|\frac{U_o}{v_o}\right| \ll 1$. As the largest root of equ./19/ is $x_4 = -\frac{1}{\alpha_1}$, i.e. $u_o^{(IV)} = -v_o$, the assumption $\left|\frac{U_o}{v_o}\right| \ll 1$ means that the domain velocity should be much less than the drift velocity of electrons at the peak field.

Then, instead of equ. /19/ we obtain:

$$a_3' x^3 + a_2' x^2 + a_1' x + a_o' = 0 \qquad (22)$$

where:

$$a_3' = \alpha_1 \alpha_3 - \alpha_o$$
$$a_2' = \alpha_1 \alpha_3 - \alpha_o - \alpha_1 K_1 - \alpha_o \alpha_+ \gamma$$
$$a_1' = -\alpha_1 K_1 - \alpha_g K_2 - \alpha_o \alpha_+ \gamma$$
$$a_o' = -\alpha_g K_2$$

$x = -1$ is a root of equ./22/, hence a possible domain velocity is:

$$U_o^{(III)} = -\frac{D}{v_o \tau_o} \qquad (23)$$

The velocity given by /23/ is an exact root of equ./22/, but not that of the original equation. This fact is very important, because if $u_o = u_o^{(II)}$, then $M_o + L_o = 0$ and $\mathcal{S}_i$ and its derivatives would have singularities in the neutrality point. These singularities, however, disappear, if we make the approximation carefully, taking into account that $\left|\frac{U_o}{v_o}\right| \ll 1$.

The two other roots we may obtain from the equation:

$$(\alpha_1 \alpha_3 - \alpha_o) x^2 - (\alpha_1 K_1 + \alpha_o \alpha_+ \gamma) x - \alpha_g K_2 = 0 \qquad (24)$$

Thus the possible domain velocities are obtained, assuming that the diffusion term in the current is negligible. The calculation is valid for arbitrary domain shapes if $S_2 \neq 0$. For the slowest mode equ./24/ gives /$|\alpha_1 K_1| \gg |\alpha_0 \alpha_+ \gamma|$; $K_1 \neq 0$/:

$$-u_0^{(1)} = \frac{\tau_0}{\tau_g} v_0 \frac{\frac{DS_3}{v_0 S_2} + \frac{\gamma D \tau_0}{v_0^2 \tau_{M0} \tau_{T0}}}{1 - v_0 \tau_0 \frac{S_3}{S_2} - D\tau_0 \frac{S_4}{S_2} + \frac{\tau_0^2}{\tau_g \tau_{M0}}} \qquad (25)$$

Before discussing the consequences of expression /25/, the following remark is necessary. The domain velocity generally depends on the domain shape, therefore, one has to apply /25/ carefully. Thus, if we are interested in the peak field dependence of the domain velocity it is to remember that also the domain shape parameters S_i may vary with E_0. In what follows, therefore, it is more convenient to define /with some arbitrariness/ different domain shapes for the determination of domain velocity, using e.g. equs./24/ or /25/. As all quantities refer to the neutrality point, it is sufficient to know the function E/z/ only around this point. The hypothetical field and space charge distributions are illustrated on Fig.1.

For <u>flat domains</u>, all domain shape parameters are zero, by definition, therefore the velocity remains undetermined. In this very special, if not oversimplified case, instead of electrical neutrality point an other one has to be chosen. The case of <u>almost flat domain</u> may be treated again on the basis of equ./17/, as $S_2 = 0$. We have the result immediately: there exists only a single domain mode with velocity $u_0^{(1)}$.

Suppose that the domain around its neutrality point may be described by symmetrical function. Two special cases will be considered: the Gaussian domain $E = E_o \exp\left[-\left(\frac{z}{d}\right)^2\right]$ and a quadratic domain $E = E_o\left[1-\left(\frac{z}{d}\right)^2\right]$. For the Gaussian domain the equ./25/ gives:

$$-u_{oG}^{(1)} = \frac{\tau_o}{\tau_g} v_o \frac{1 + \frac{\gamma D \tau_o}{v_o^2 \tau_{Mo} \tau_{ro}}}{1 + \frac{6 D \tau_o}{d^2} + \frac{\tau_o^2}{\tau_g \tau_{Mo}}} \qquad (26)$$

Let $\frac{\gamma D}{v_o^2 \tau_{Mo}} \gg 1$ and $\frac{6 D \tau_o}{d^2} \gg 1 + \frac{\tau_o^2}{\tau_g \tau_{Mo}}$ then /26/ gives /using $\tau_o \approx \tau_{ro}$ /:

$$d \simeq \sqrt{3}\left(\frac{|u_{oG}^{(1)}| 2 \tau_{Mo} \tau_g v_o}{\gamma}\right)^{1/2} \qquad (26')$$

This expression offers a relationship between the domain width d and domain velocity u_o. A similar expression was given first in [7], based on the so called quasi linear approximation, except for a $\sqrt{3}$ factor and for the fact that in /26/ all quantities refer to the neutrality point, while in equ./16/ of paper [7] to the average field in the sample. If the domainshape is quadratic and thus $S_3 = S_4 = 0$; $S_2 \neq 0$ we have:

$$-u_{oq}^{(1)} = \frac{\tau_o}{\tau_g} v_o \frac{1 + \frac{\gamma D}{v_o^2 \tau_{Mo}}}{1 + \frac{\tau_o^2}{\tau_g \tau_{Mo}}} \qquad (27)$$

and the inconvenient S_2 falls out. It can be easily seen, that the equ./27/, with the except of the correction factor is identical with the formula /19/ of paper[4]*. In case of domains of arbitrary shape it is necessary to allow for all domain shape parameters to be different from zero. Its important consequence is that, if S_3 differs but slightly from zero, it may alter the domain velocity essentially,

* To make it more clear we remind that the definition of τ_{Mo} and τ_{kd} of paper [4] differ by a factor of $\frac{\tau_o}{\tau_g}$, i.e. $\tau_{Mo} \approx \frac{\tau_o}{\tau_g} \tau_{kd}$

since in the denominator of /25/ S_3/S_2 is multiplied by the large $v_o \tau_o$ factor. If all parameters were known, the domain velocity could be determined. For a rough estimate of domain velocity it is sufficient to assume $\left|\frac{S_3}{S_2}\right| \sim \frac{1}{d}$; $\left|\frac{S_4}{S_2}\right| \sim \frac{1}{d^2}$. It is more convenient, however, to use a concrete function. Suppose, for simplicity's sake that in the neutrality point the field distribution behaves like a function of type $y = y_o \left(\frac{x}{d}\right)^r e^{-\frac{x}{d}}$ around its maximum. Then we have

$$\frac{S_3}{S_2} = -\frac{2}{rd} \; ; \quad \frac{S_4}{S_2} = -\frac{3}{rd^2}\left(1 - \frac{2}{r}\right) \tag{28}$$

For r = 0,5; $\frac{S_3}{S_2} = -\frac{4}{d}$; $\frac{S_4}{S_2} = \frac{18}{d^2}$. In this special case the expression /25/ gives:

$$-u_{oa}^{(I)} \approx \frac{d}{4\tau_g}\left(1 + \frac{\gamma D}{v_o^2 \tau_{mo}}\right) \tag{29}$$

as in the denominator, the term $v_o \; _o S_3/S_2$ dominates. Other empirical domain shapes may be examined in a similar way. Let us illustrate this by the function $y = \frac{1-\exp(-\alpha x)}{1+\exp[\beta(x-d)]}$. On Fig.2. the function $y(x')$ / $x' = \frac{x}{d}$ / is plotted for different values of parameters $\alpha' = \alpha d$; $\beta' = \beta d$: In Table I. the four domain velocities are given, calculated from equs./24/, /23/ and using the shape parameters of the above functions /d = 10^{-2}cm, $u_o^{(IV)} = -v_o$/.

It should be noted, that neither the values nor the signs of the ratios S_3/S_2 and S_4/S_2 could be determined from the present theory itself. /Probably, S_3/S_2 and S_4/S_2 should be negative in order to make all terms of the denominator positive/. It is known from experiments that generally the domain shape and the propagation direction are closely related /see Fig.1a./, but on the influence of a proper /inhomogeneous/ background illumination the domain may be stopped or it may get started in any direction along the sample [16], [22].

6. Quasi-equilibrium between band and traps

Let us consider the case, when the distribution of electrons over the zero and high-mobility states takes place in a short time compared to other characteristic times in the problem. The previously used method may be applied in a similar way. The actual calculation is quite simple. In fact, for the case considered, one may write:

$$\frac{\varrho_2}{\varrho_1} = \frac{\tau_g}{\tau_+} \tag{30}$$

i.e. the ratio of the localized and the moving parts of space charge density depends on the field E alone. Using the equs./8/ and /30/ for ϱ_1 and ϱ_2 we obtain:

$$\varrho_1 = \frac{\tau}{\tau_g} T \quad ; \qquad \varrho_2 = \frac{\tau}{\tau_+} T \tag{31}$$

The equs./31/ show that and depend both on E and E. For the current j_s and the velocity u_o one obtains:

$$j_s = \frac{\tau_o}{\tau_g} \varrho_o v_o - \frac{\tau_o}{\tau_g} D S_2 \tag{32}$$

$$-u_o = \frac{\tau_o}{\tau_g} v_o + \frac{\gamma D \tau_o}{v_o \tau_{mo} \tau_g} + \frac{S_3}{S_2} \cdot \frac{\tau_o D}{\tau_g} \tag{33}$$

In the second term of equ./33/ the approximation $\tau_o \approx \tau_{+o}$ was used. According to the difference between the definitions of τ_{mo} here and τ_m in paper [5]/see footnote at equ./27/ /, the expression /33/ may be considered as a generalization of related result of paper [5] in two respects: in cited paper the domain was assumed to be symmetrical, therefore the third term of equ. /33/ was omitted, secondly, here a correction factor γ arose with a value of about 2. For the usual parameter values, the third term is small compared to the first one, thus in the approximation used, the domain velocity turned out to be insensitive to the domain shape.

From these calculations follows that the assumption of quasi-equilibrium results in omission of generally non-negligible terms in the expression for domain velocity /e.g. $v_o \tau_o S_3/S_2$/.

7. Conclusions and comparison with experiments

Before surveying the results of the calculations, it is worthwhile to repeat the most important assumptions and simplifications. It was assumed that a./ there exists, in the originally homogeneous sample, a steadily propagating domain which can be described by a wave-like solution /Riemann-type solution/; b./ the negative differential conductivity is due to the field induced transfer of electrons from high-mobility states to zero-mobility-states. The effects of traps and deeper levels of multielectron centers were neglected. For the sake of simplicity a./ the mobility and the diffusion coefficient were assumed to be field independent and b./ the equation of the recombination kinetics was linearized with respect to the concentrations, but not to the field.

The main results of the paper may be summarized as follows. /i/ The mobile and the localized components /ϱ_1, ϱ_2/ of the space charge wave are given in terms of electric field E/z/ and its derivatives, or more precisely, the functions $G_i = \varrho_i/E, E', E''$ /, together with the nonlinear differential equation $F/E, E', E'', E'''/ = 0$ using u_o and j_s as parameters. /ii/ Introducing the domain shape parameters S_i /up to S_4/, an algebraic equation system is derived to determine the domain velocity and the current. /iii/ There exist different domain modes with noncommensurable velocities. [9] In general, the domain velocity depends on the actual domain shape, i.e. on the domain shape parameters S_i. /iv/ Based on the experimental

data it is possible to decide which mode will be realized. Assuming a proper domain shape the theory gives the domain velocity in correct order of magnitude /10^{-3}cmsec^{-1}/.

In connection with /i/ it is important to remark that the existence of the $\varrho_i(E, E', E'')$ functions is a direct consequence of the wave-like solution* and it holds for more general case, when the equ./ / is not linearized with respect to the concentrations and, further, for field dependent μ and D as well. This result shows, that for general case, one cannot expect the components ϱ_i to be the function of electric field alone in every point of the domain /7/.

From the point of view of comparison of the present theory with experimental results, the first problem is to decide, which mode is realized in a given experiment. The velocity of the fourth, omitted mode is equal to the electron drift velocity at the peak-field. One can hardly expect such a high velocity for a space charge wave having a zero-mobility component /ϱ_2/. The second mode may be excluded on the basis of the temperature dependence of domain velocity. In fact, according to [24] the domain velocity and the current have similar temperature dependence, namely both are determined by $\frac{\tau_{to}}{\tau_g}$. It is obvious, that $u_o^{(II)}$ does not show a temperature dependence like this. On the other hand the order of magnitude of $u_o^{(II)}$ is different from that of the measured velocity for

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* The wave-like solution is not the unique possibility for domain motion. One can imagine that the domain "migration" is accompanied by a small change of the domain shape itself. "caterpillar-like" movement

$Au^{=}$ centers in Ge at about 30°K [18], [19], [24]. The same is true for u_o. In fact, $|u_o^{(II)}| = 0{,}2$ cmsec^{-1}; $|u_o^{(III)}| = 2{,}6.10^2$cmsec^{-1}. Similarly, the Gaussian and quadratic domains lead to too high velocities, since $|u_{oG}^{(I)}| \approx |u_{oq}^{(I)}| = 1{,}4$cmsec^{-1}. The correct velocity value may be obtained from the slowest domain mode, assuming $S_3 \neq 0$. From equ./29/ $|u_{oq}^{(I)}| = 3{,}5.10^{-3}$cmsec^{-1} follows, while for more complex domain shapes /Fig.2/ see data in Table 1. Finally, the following question is to be answered: what is the physical reason for the domain velocity being so sensitive to the domain shape, at least for two domain modes. By a simple consideration this is connected probably with the fact that the low-mobility states are actually zero-mobility states /u =0/. Let us assume momentarily that $\mu_2 \neq 0$. It is clear physically that a condition for stabèè domain propagation is $\mu_2 E_o \approx \mu_1 E_\infty$ where E_o is the peak-field, E_∞ the low field at the front of the domain. In this case, one may expect, that the domain shape has no important role in the determination of domain velocity, as after all <u>both</u> components of space charge wave are mobile. For recombination instability, however, $E_o \mu_2 \equiv 0$ for arbitrary but finite values of E_o, therefore, requiring an <u>invariable domain shape</u>, the domain "migration" may be determined by the domain shape itself. It is, in our opinion, the reason for the high sensitivity of domain velocity on its shape, especially for the parameter S_3.

To answer this question more precisely one has to analize, at which conditions the quantities $\alpha_2, \alpha_3 \alpha_4$ /containing the S_i parameters/ are small, say, compared to unity, or $\frac{\tau_o^2}{\tau_q \tau_{mo}}$. In this respect τ_o has an important role, as decreasing the life-time τ_o , also α_2 and α_4 will decrease /see the case of quasi-equilibrium/, and the domain velocity will insensitive

to the domain shape parameter.

The proposed theory, in its present form, does not yield the shape parameters S_i the disadvantage of which is obvious.

From the above treatment of the non-linear problem, however, one may conclude, that in the dynamics of steadily travelling recombination domains the domain shape plays basic role.

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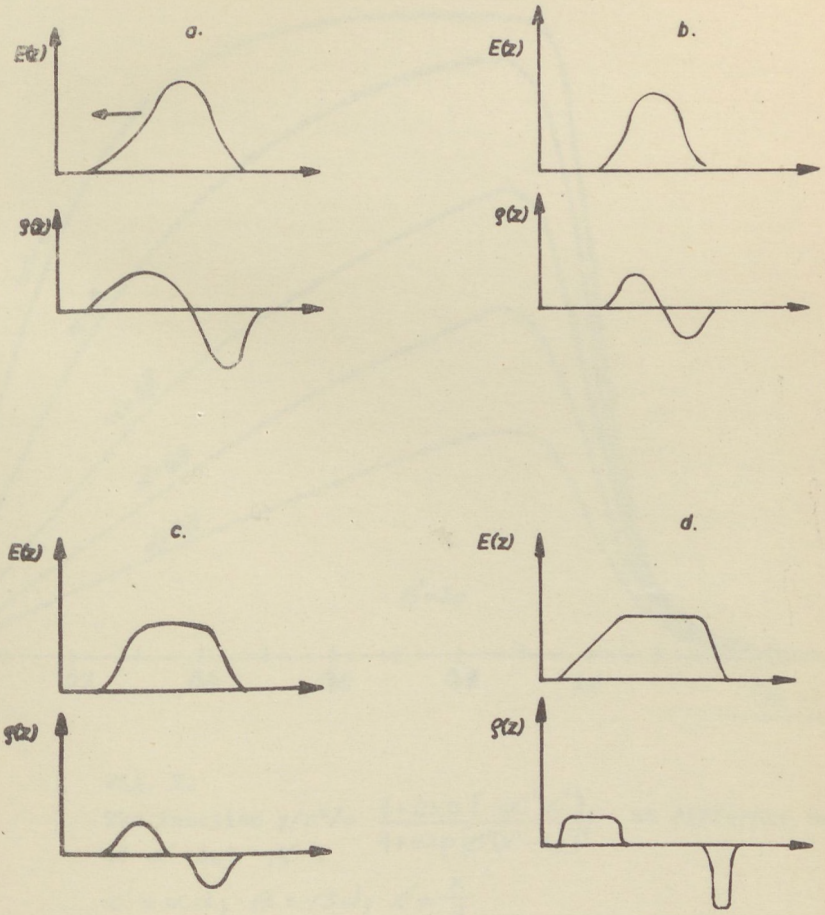

Fig. 1.
Different possible domain shapes E/z/ and the distribution of space charge ρ /z/ ~ E'/z/.

a. arbitrary domain shape /$S_1 \neq 0$/;

b. symmetrical domain /$S_{2i+1}=0$/;

c. almost flat domain /$S_2=S_3=0$, $S_4 \neq 0$/

d. flat domain /$S_1=0$/

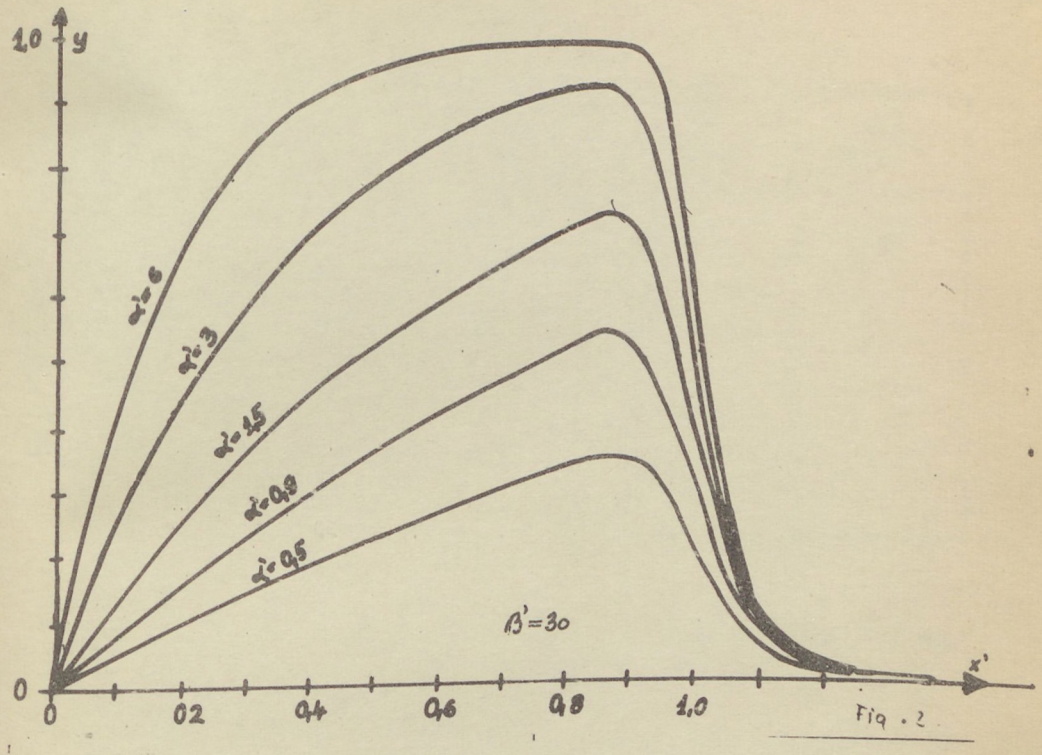

Fig. 2.
The function $y/x' = \dfrac{1-\exp(-\alpha' x')}{1+\exp[\beta'(x'-1)]}$ **at different values of** α' **and** β' **.**

$\alpha' = \alpha d$; $\beta' = \beta d$; $x' = \dfrac{x}{d}$

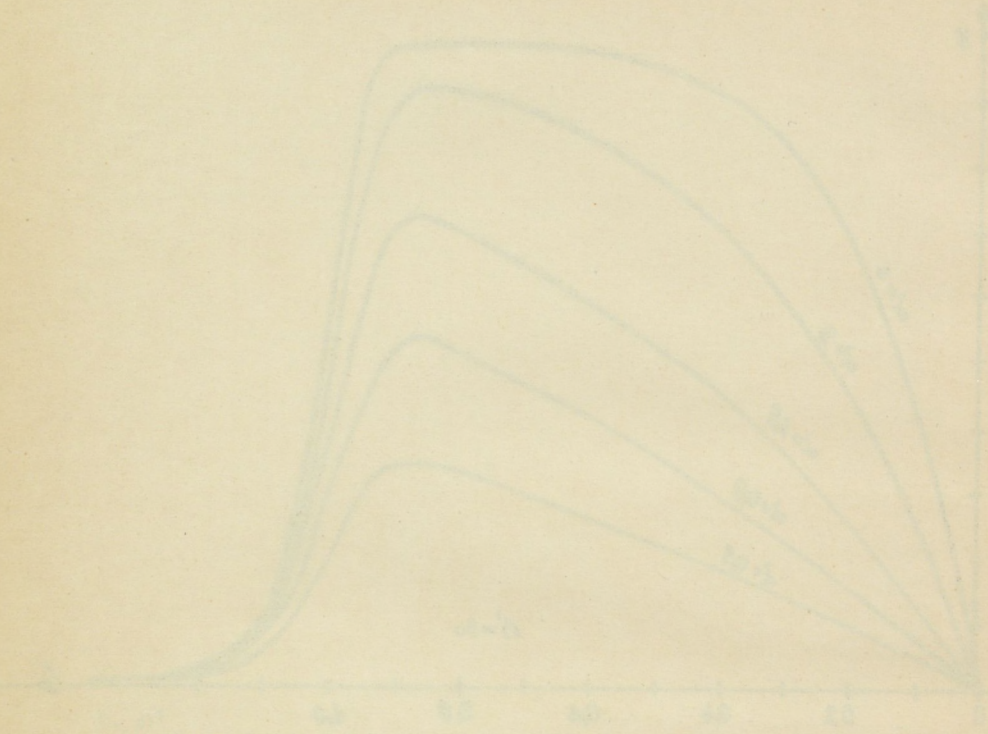

Table I.

α'	β'	$\lvert u_o^{(I)}\rvert$ $.10^{-3}$ cmsec^{-1}	$\lvert u_o^{(II)}\rvert$ cmsec^{-1}	$\lvert u_o^{(III)}\rvert$ $.10^{2}$ cmsec^{-1}	$\lvert u_o^{(IV)}\rvert$ cmsec^{-1}
0,5	30	0,522	0,2	9,817	5.10^6
1,5	30	0,522	"	15,19	"
3,0	30	0,533	"	30,47	"
6,0	30	0,728	"	100,2	"
6,0	10	4,021	"	12,597	"
0,9	30	0,521	"	11,635	"
0,9	20	0,854	"	6,924	"
0,9	10	2,190	"	6,851	"

Approximative domain velocities, calculated on basis of equs. /23/ and /24/, using the function $y(x')=\frac{1-\exp(-\alpha' x')}{1+\exp[\beta'(x'-1)]}$ for the determination of domain shape parameters S_i /d=10^{-2}cm/.
For some values of α' /β'=30/ y/x'/ is plotted on Fig. 2. / $\lvert u_o^{(IV)}\rvert \approx v_o$/

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